Valence Parton Distribution of Pion

GHP Workshop 2019

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BNL

In Collaboration with T. Izubuchi, L. Jin, C. Kallidonis, S. Mukherjee, P. Petreczky, C. Shugert and S. Syritsyn

First principle understanding of Parton structure?

Asymptotic freedom at short distances and Confinement at long distances

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Lattice

(Mass of proton, finite T crossover etc.)

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Perturbation theory

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PDF+

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PDF+ $\longleftarrow$ Lattice

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(Mass of proton, finite T crossover etc.)

LaMET formalism and assumptions

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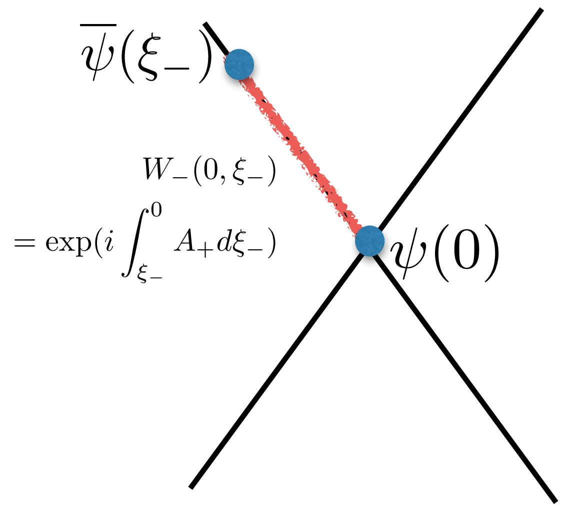
PDF :

(Soper '77)



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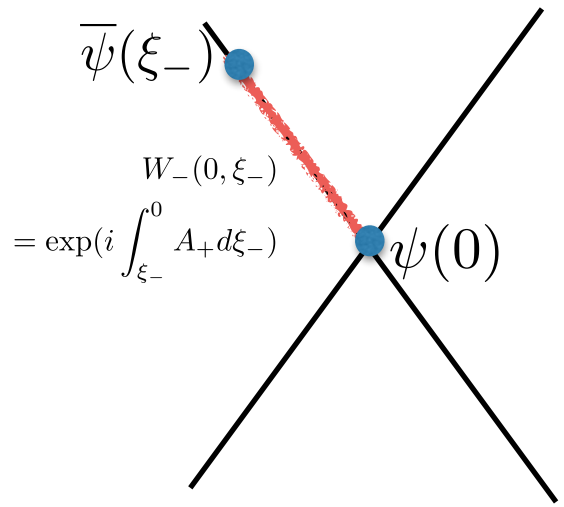
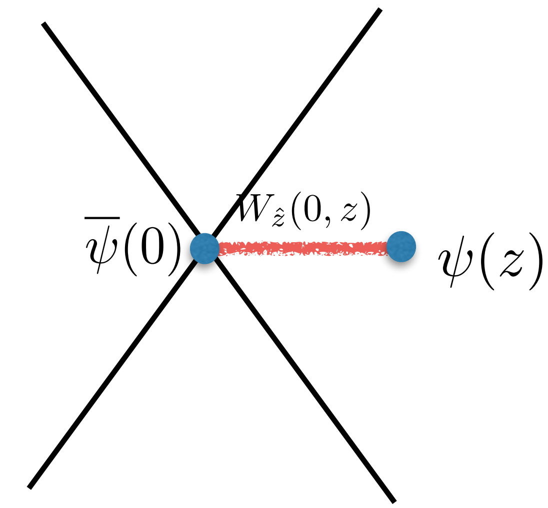


quasi-PDF :

(Ji '15)



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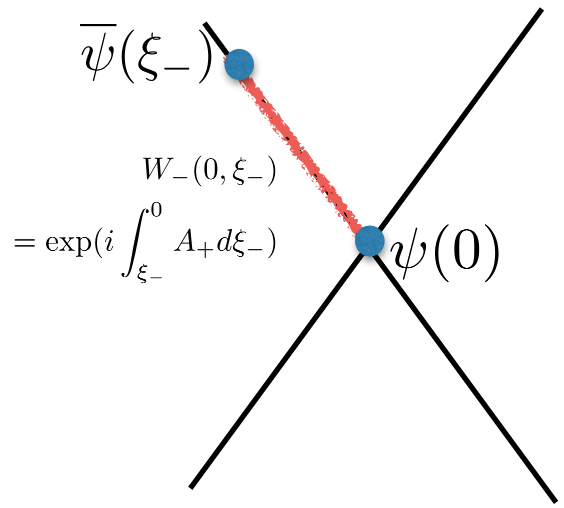

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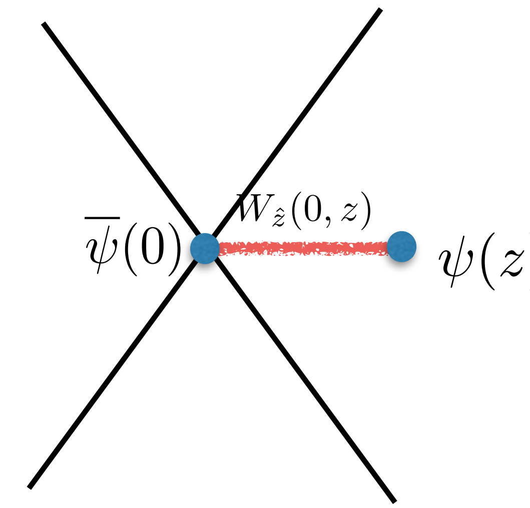
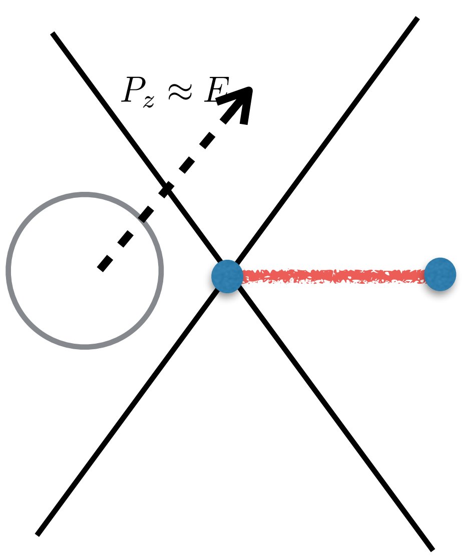


Rest frame of operator



(RI-MOM

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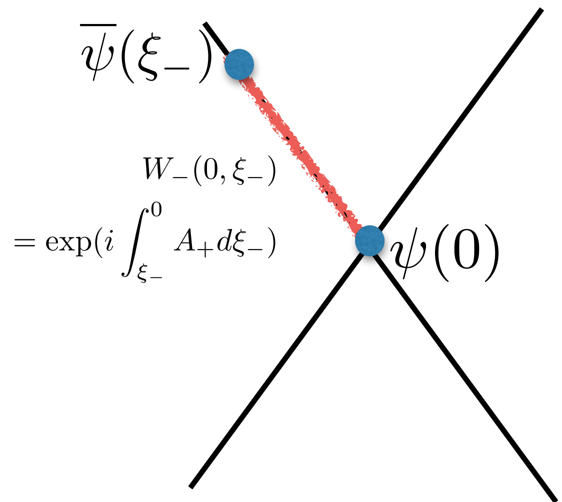

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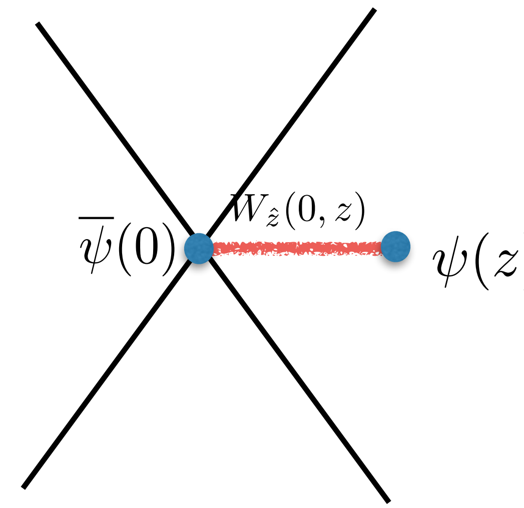

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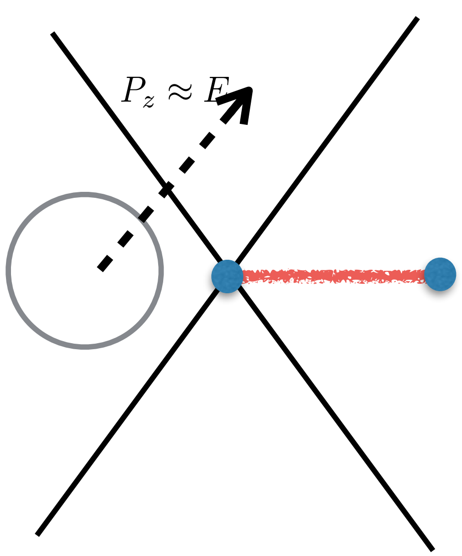

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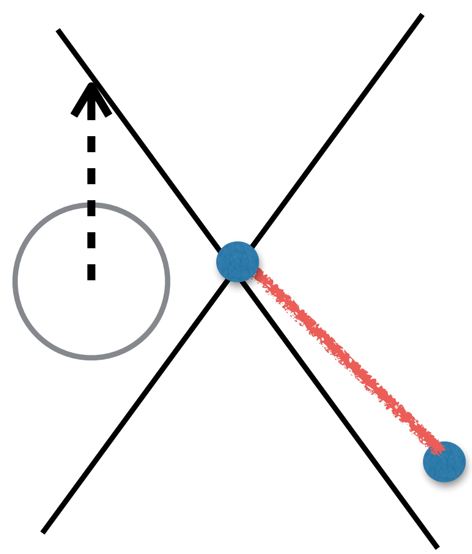

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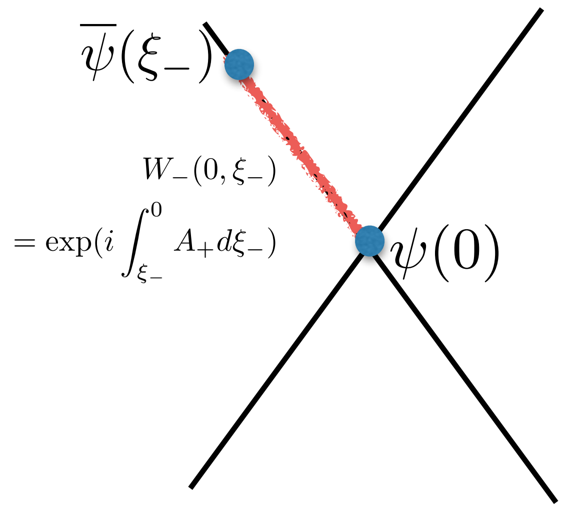

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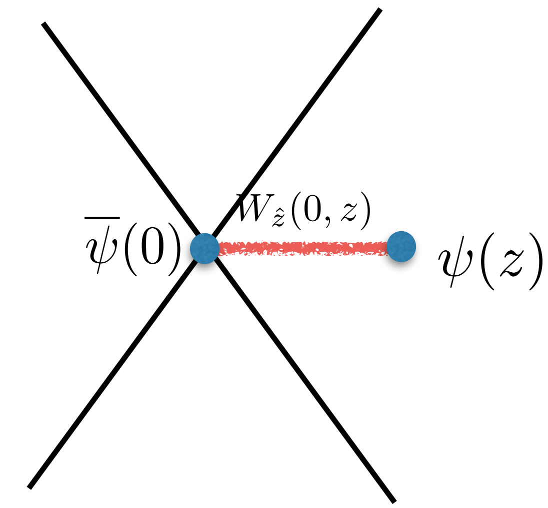

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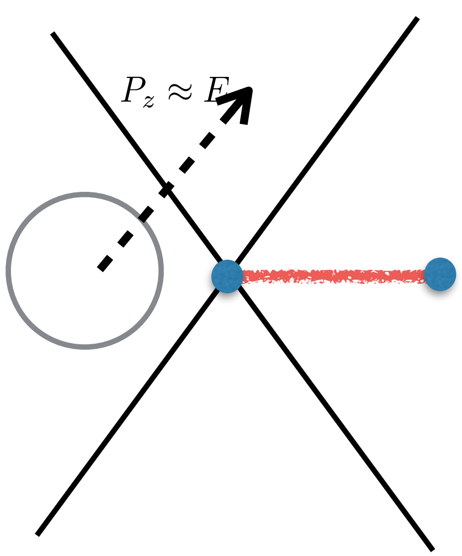

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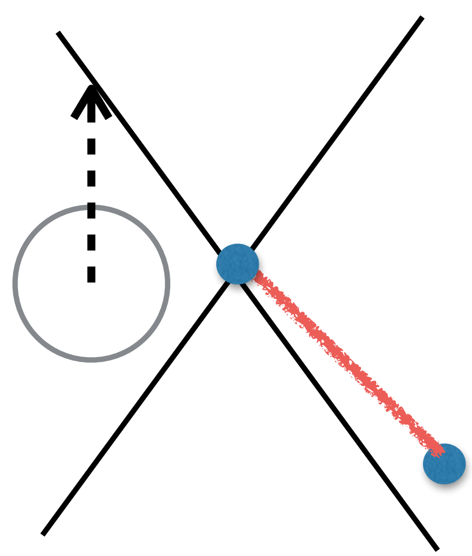
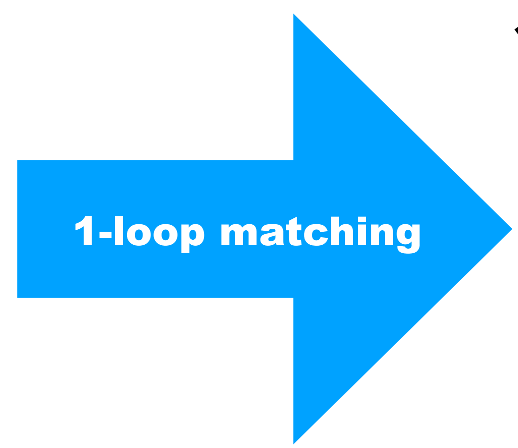
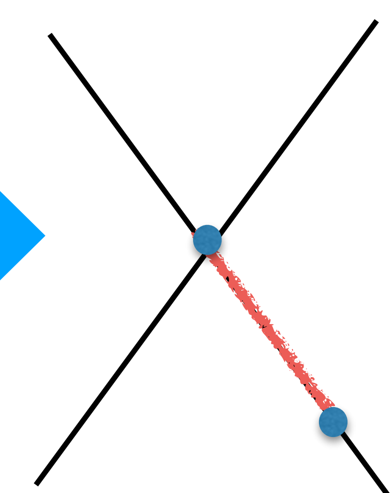




(Stewart and Zhao



(RI-MOM

MS-bar

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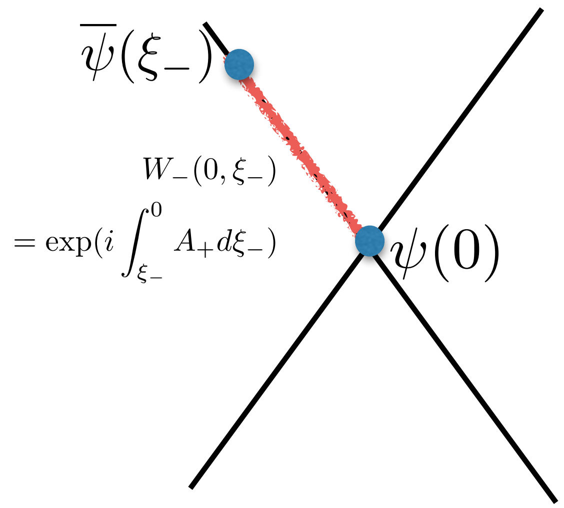

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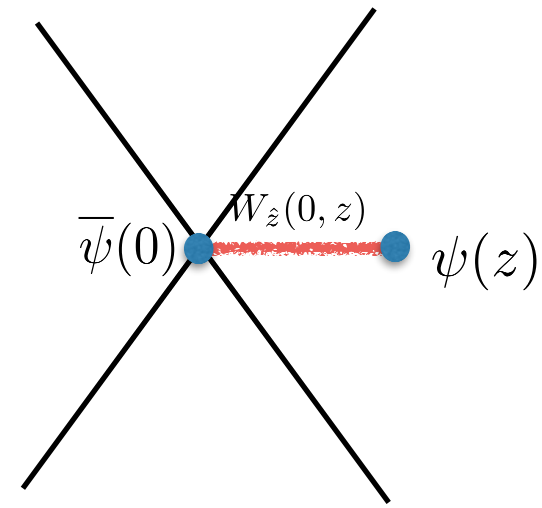

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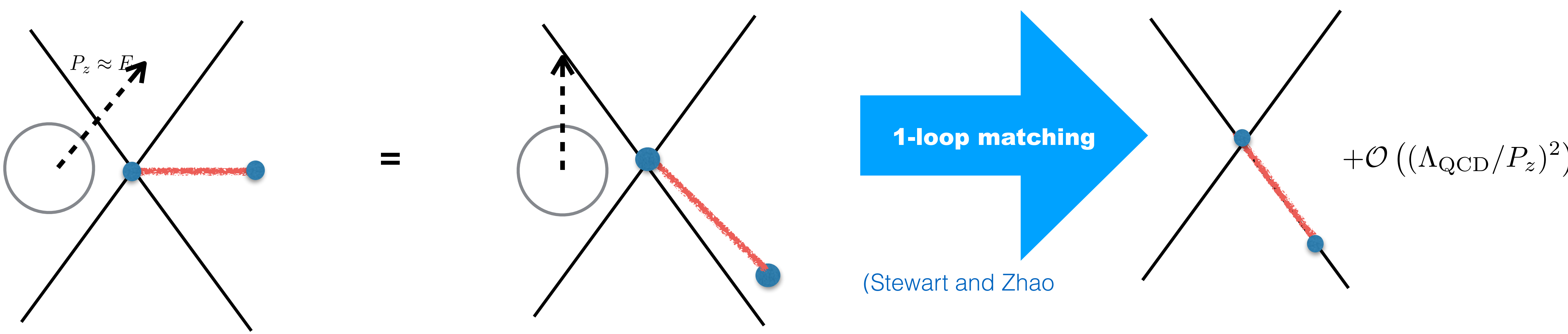


(RI-MOM

MS-bar

Valence PDF of $\pi^+(u\bar{d})$

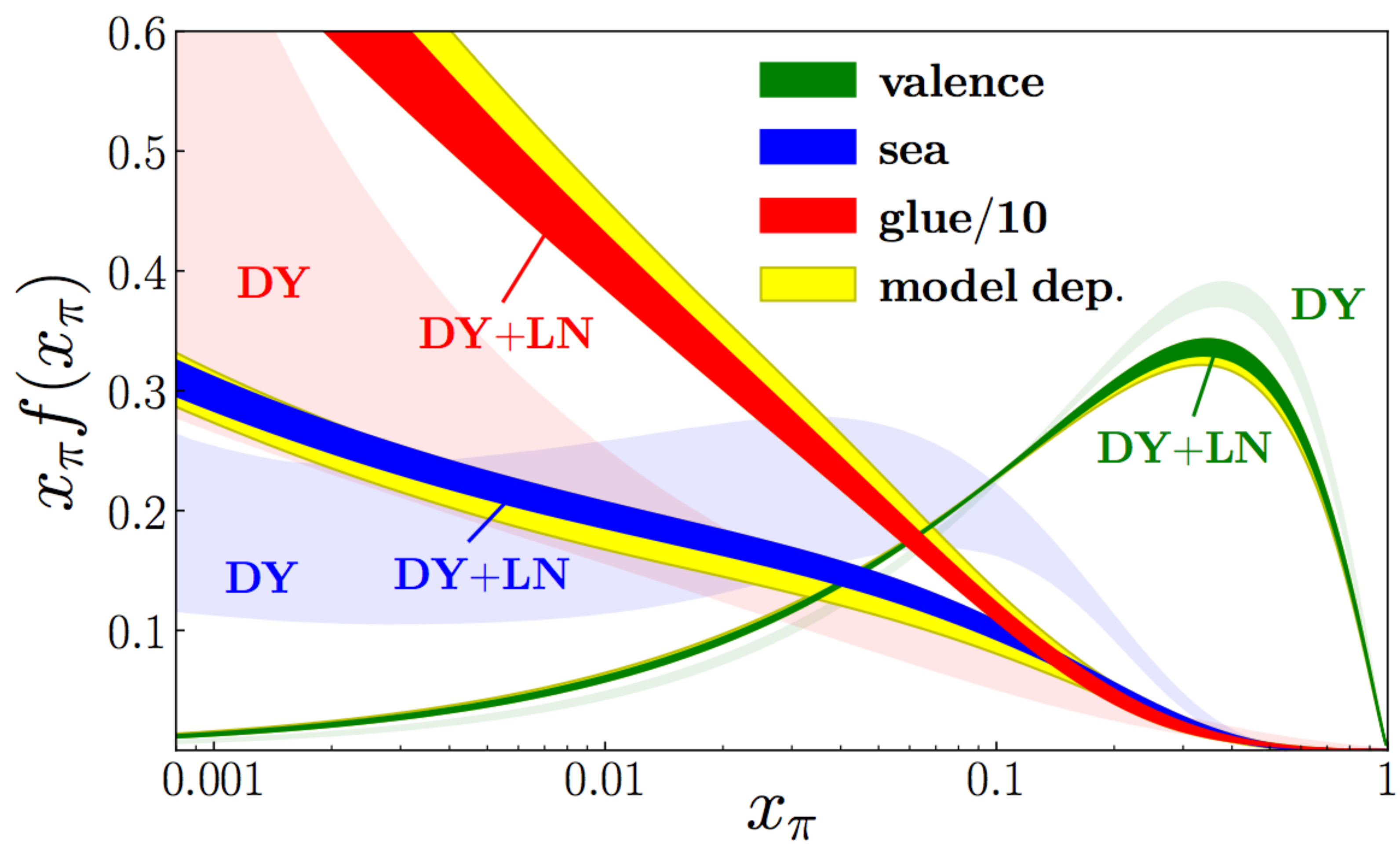
We measure the valence PDF of charged pion:

$$f_v^\pi(x) = f_u(x) - f_{\bar{u}}(x) = f_u(x) - f_d(x), \quad 0 < x < 1$$

Flavor non-singlet → No mixing with glue

no disconnected fermion diagrams

P. C. Barry et al, 2018

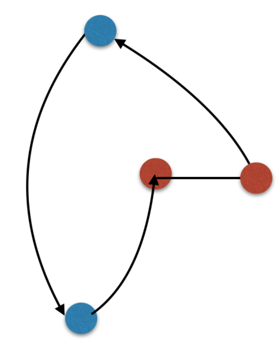


Lattice Set-up

- a=0.06 fm, 48^3 x 64 lattice (HotQCD ensemble) with HISQ sea quarks and Wilson-Clover valence quarks.

- Valence quark mass tuned to $m_\pi = 300\ \mathrm{MeV}$

- Three-point function we compute: real part of



- 1-HYP smeared Wilson line in quasi-PDF operator

- Boost smeared pion source and sink operator.

- Statistics ~168 gauge configurations. All-Mode Averaging (AMA) using 1-exact and 32 sloppy quark propagators.

Extraction of bare qPDF matrix element

$$h^B(z, P_z) = \lim_{t \to \infty} \frac{\left\langle \pi(t; P_z) O_\Gamma(z; \tau) \pi^\dagger(0; P_z) \right\rangle}{\left\langle \pi(t; P_z) \pi^\dagger(0; P_z) \right\rangle}$$

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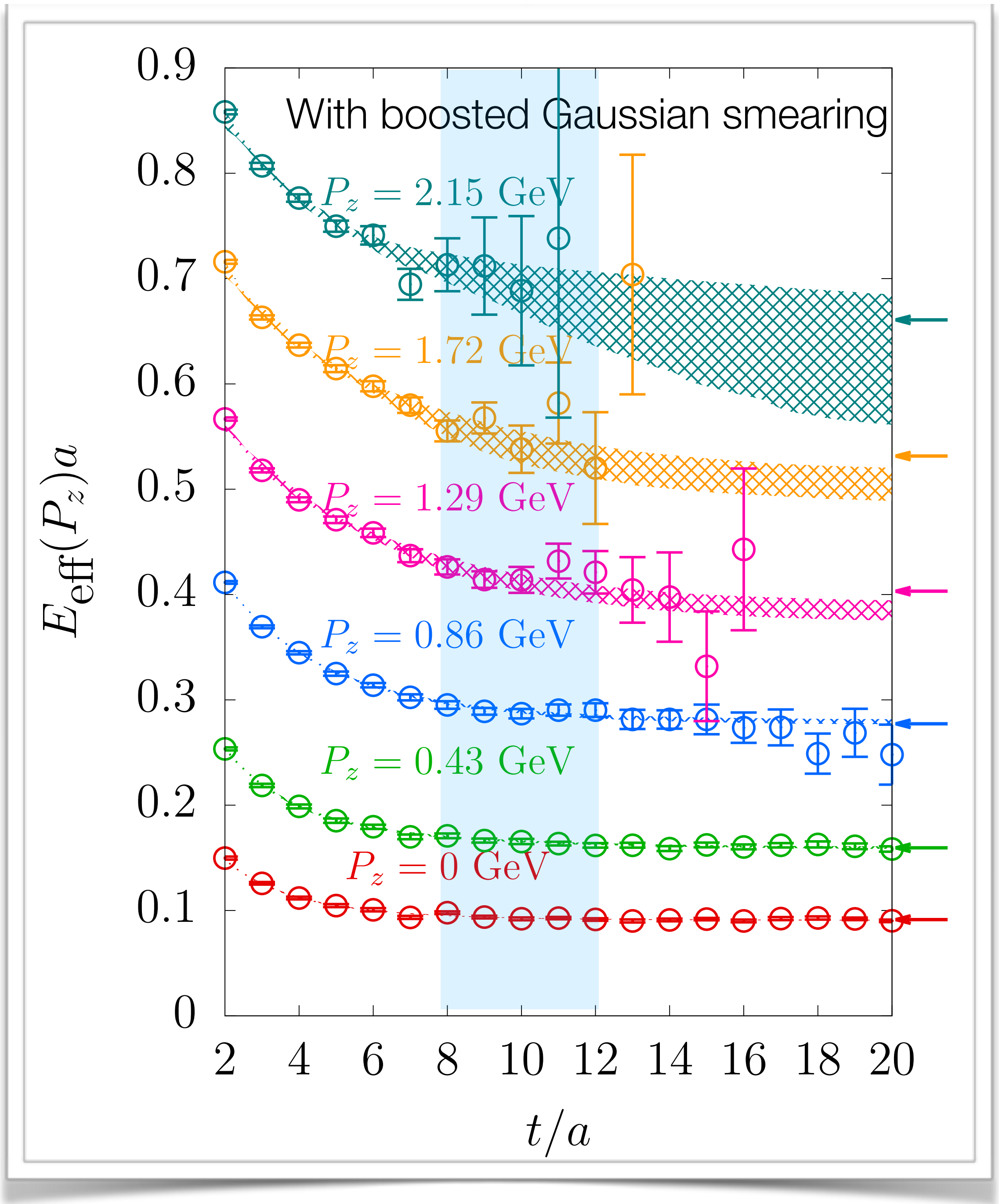

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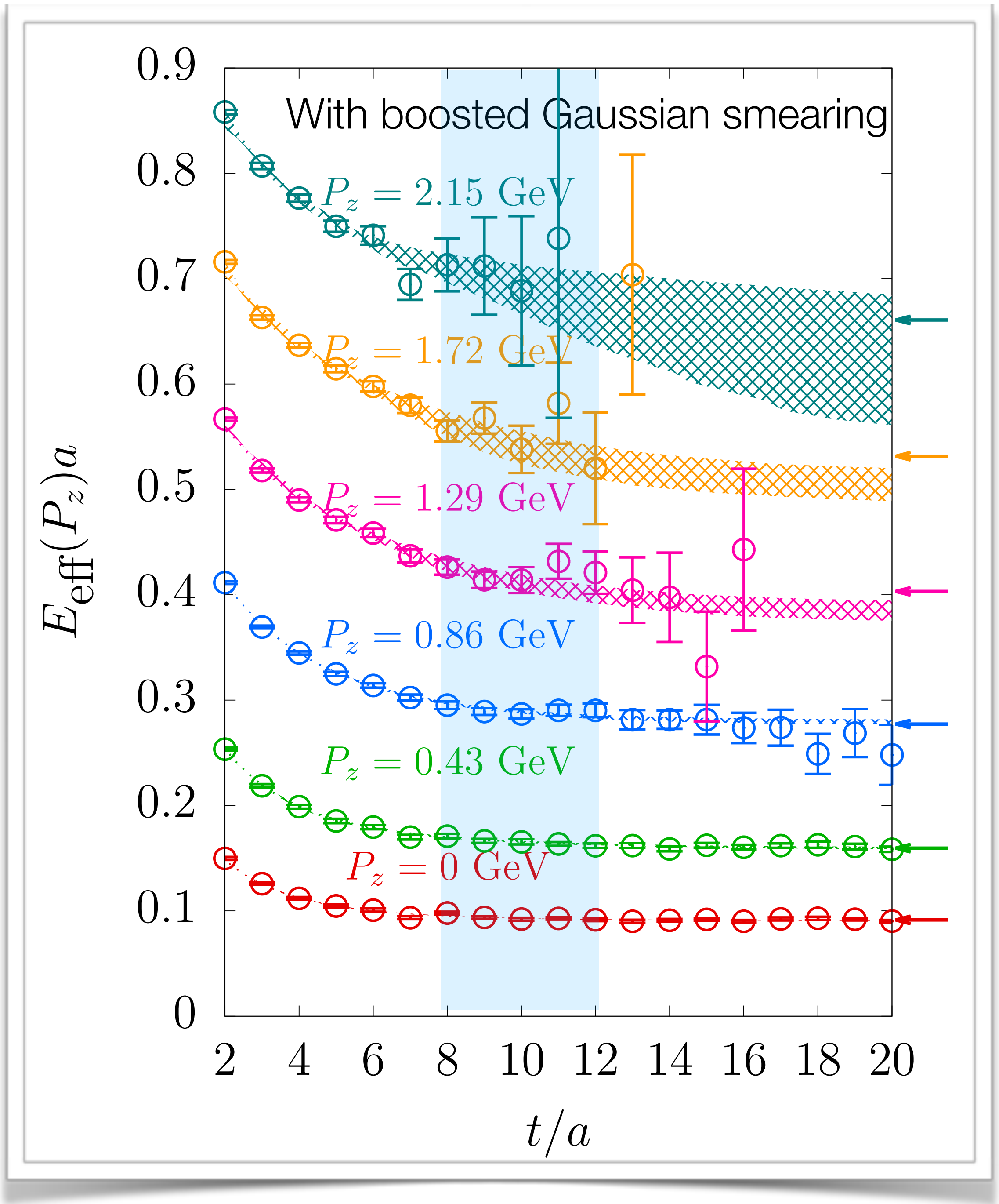

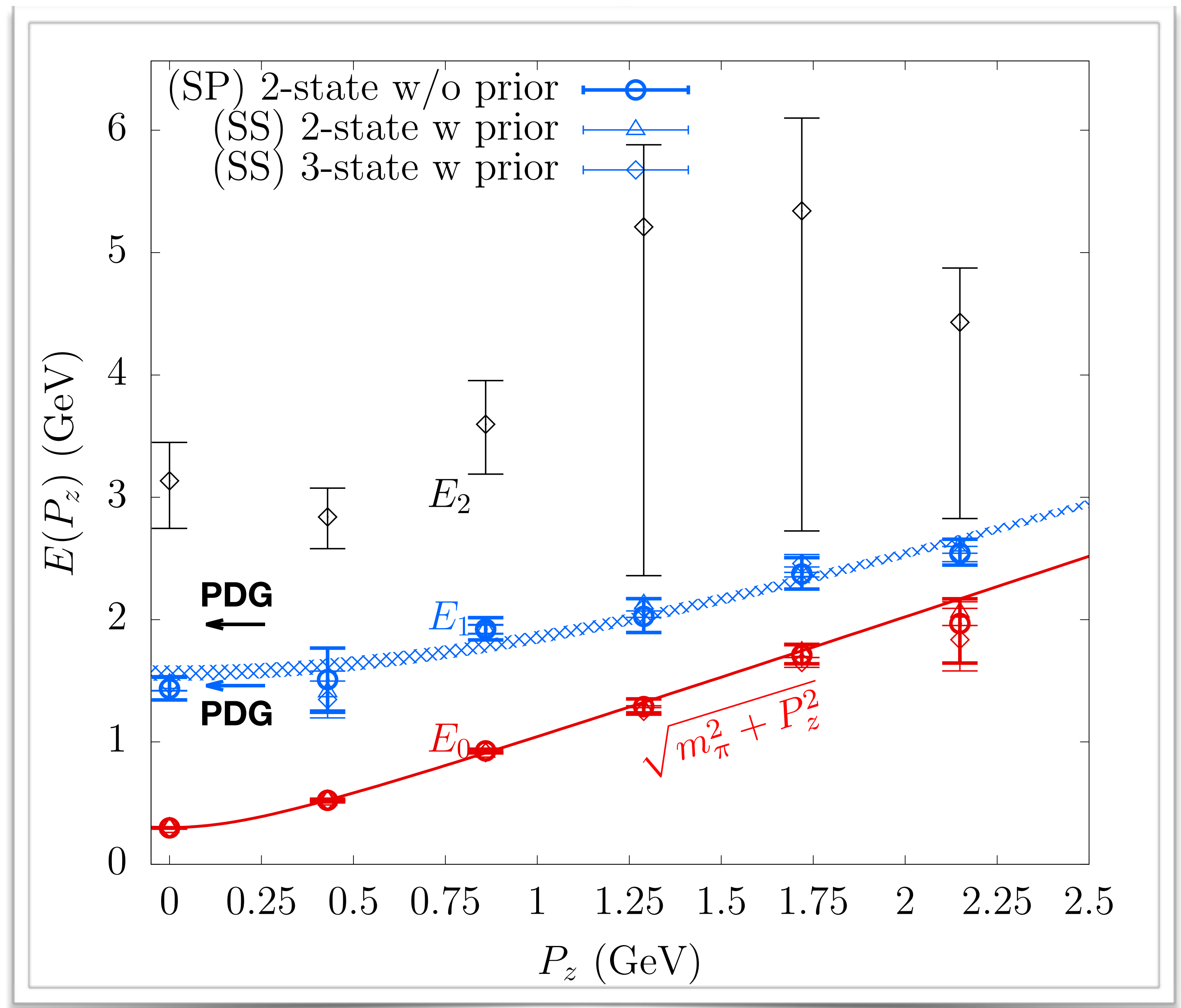

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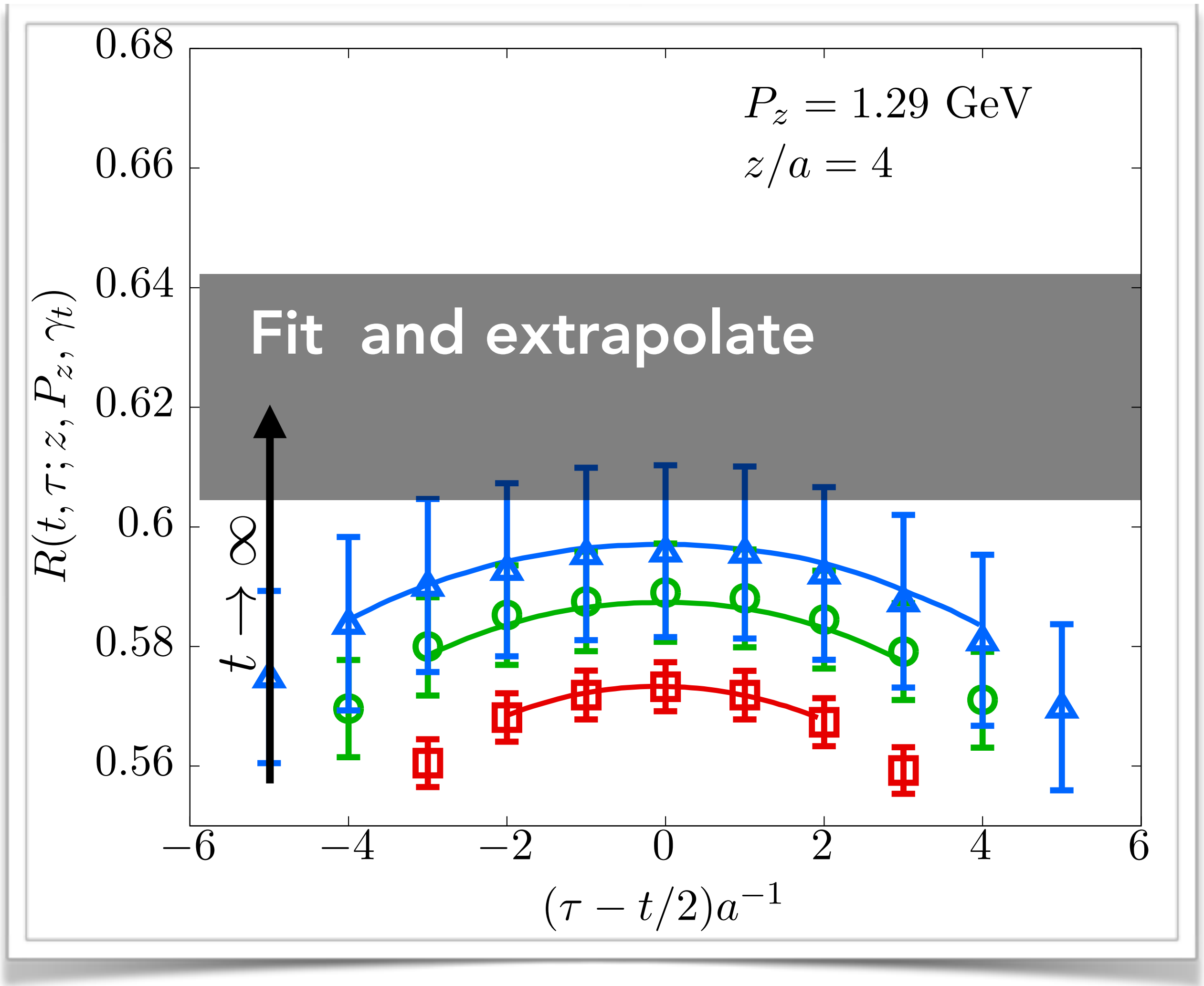

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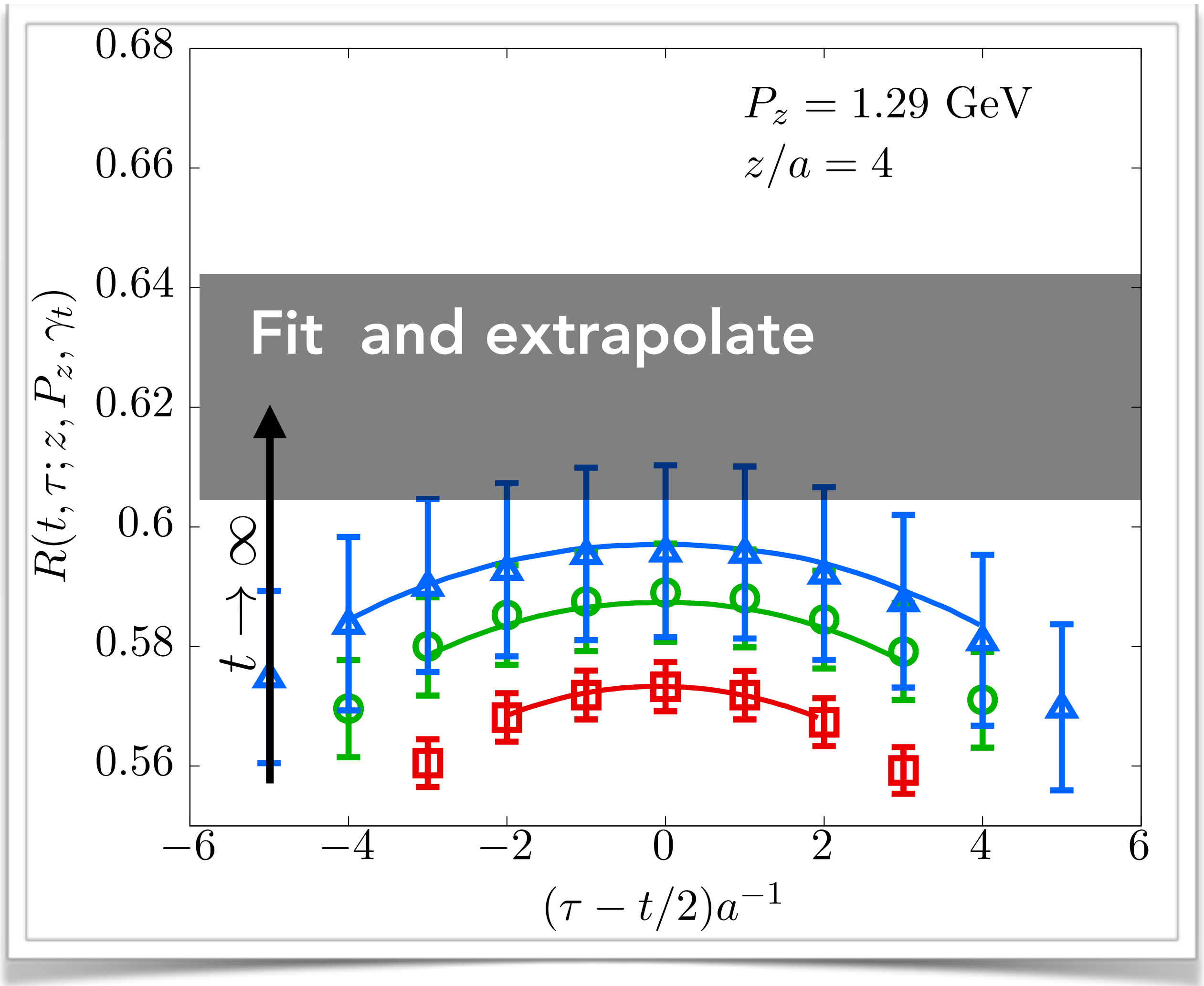

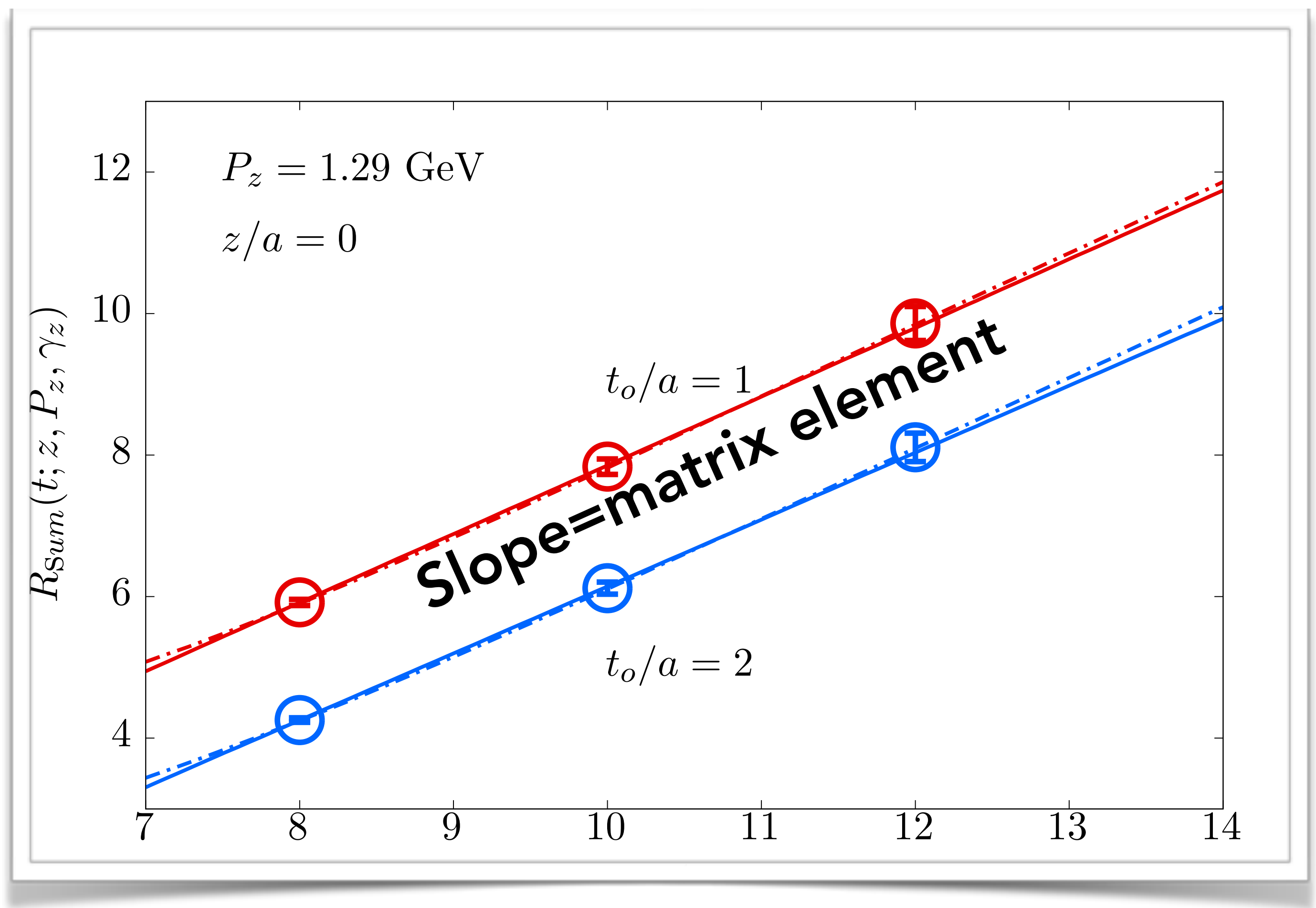

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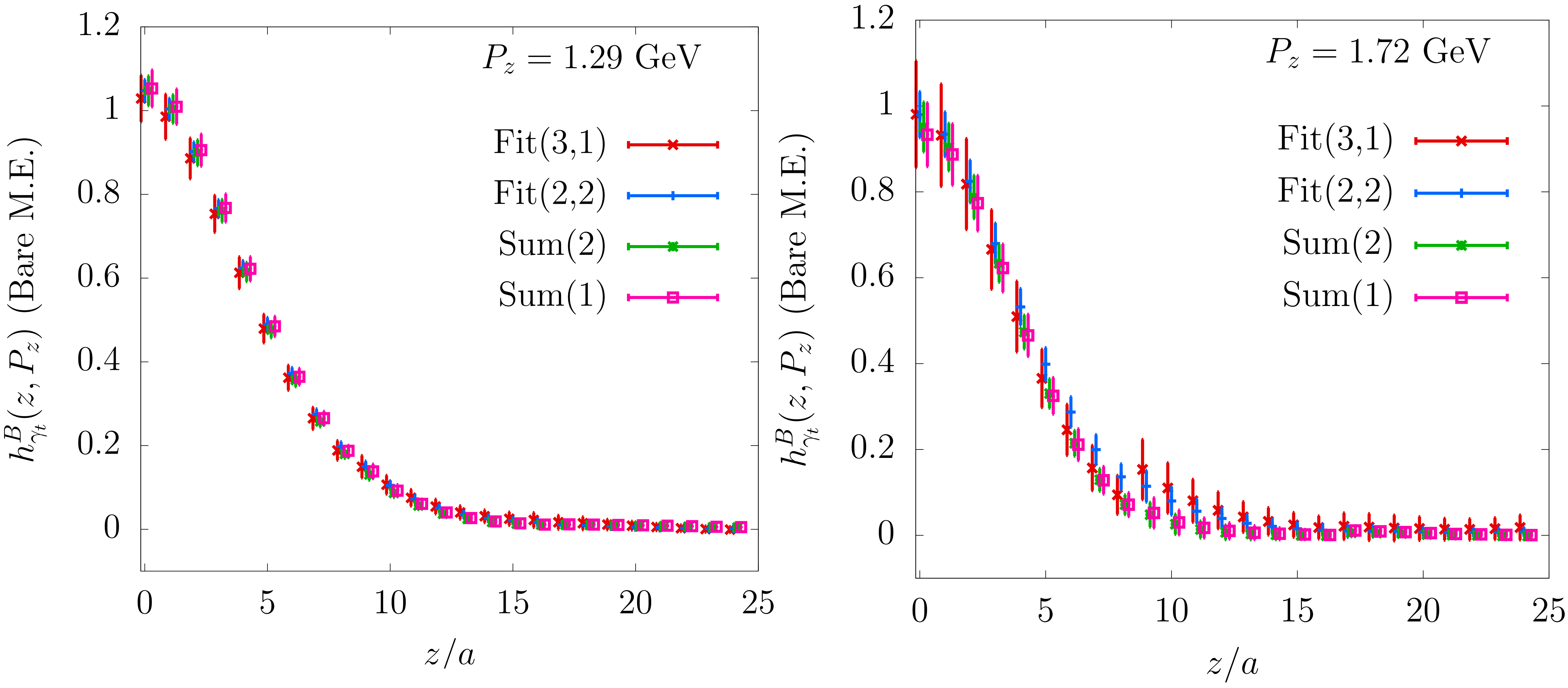
Matrix element after extrapolations



Renormalization of bare qPDF matrix element

(Ishikawa et al ’17)

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qPDF can be multiplicatively renormalized: (Ishikawa et al '17)

$$h^R_{\gamma_t}(z; P_z, P^R) = Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{\gamma_t}(z; P_z, a)$$

renormalized hadron qPDF

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renormalized hadron qPDF

bare hadron qPDF

Renormalization scheme independent conditions:

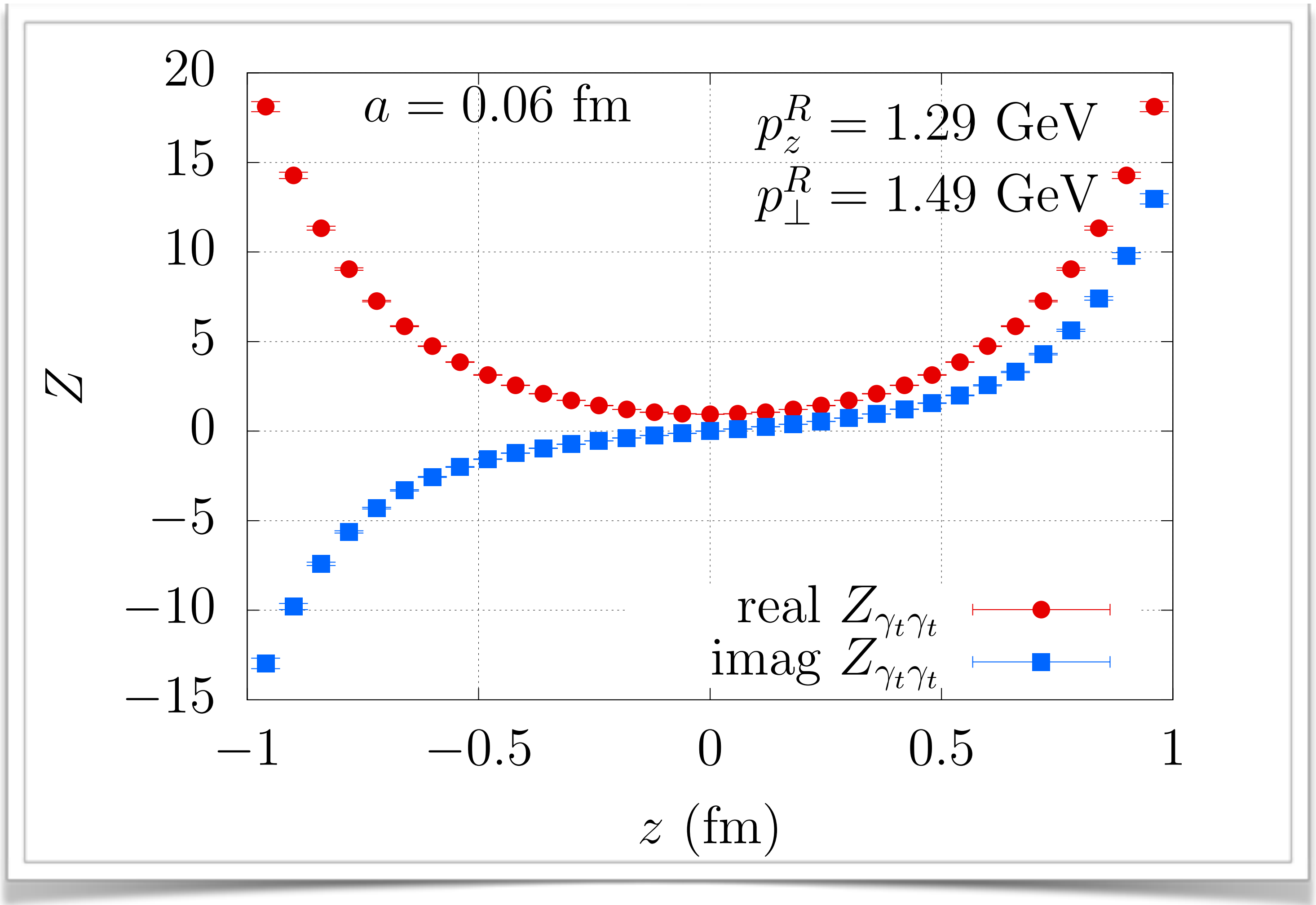
$$Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{quark}(z; P = P^R, a) = h_{free}(z; P^R)$$

qPDF with quark external states in full QCD

Tree level value

Implementable in lattice as well as pert. theory with off shell quark with $P^2 > 0$

RI-MOM renormalization factor



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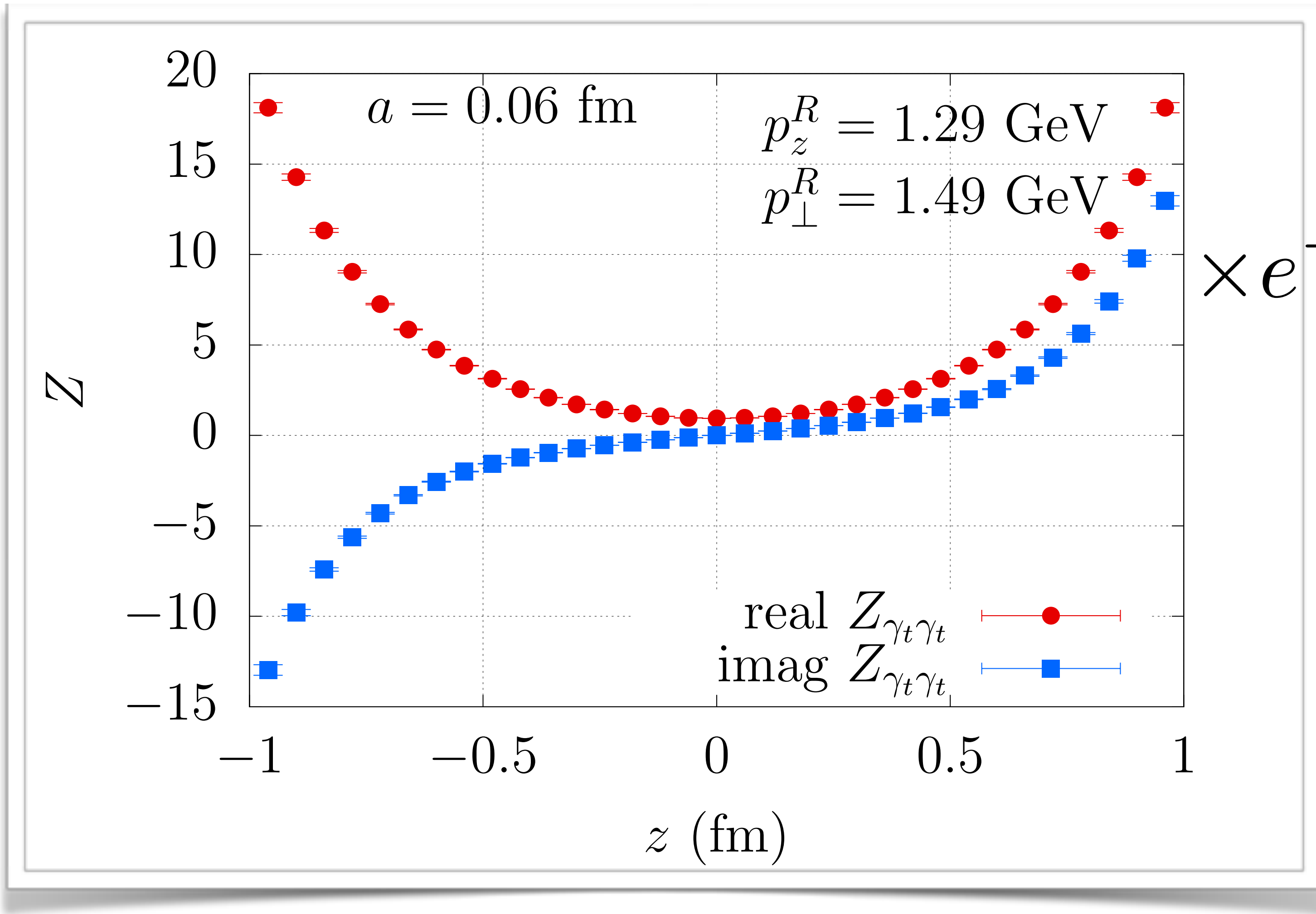
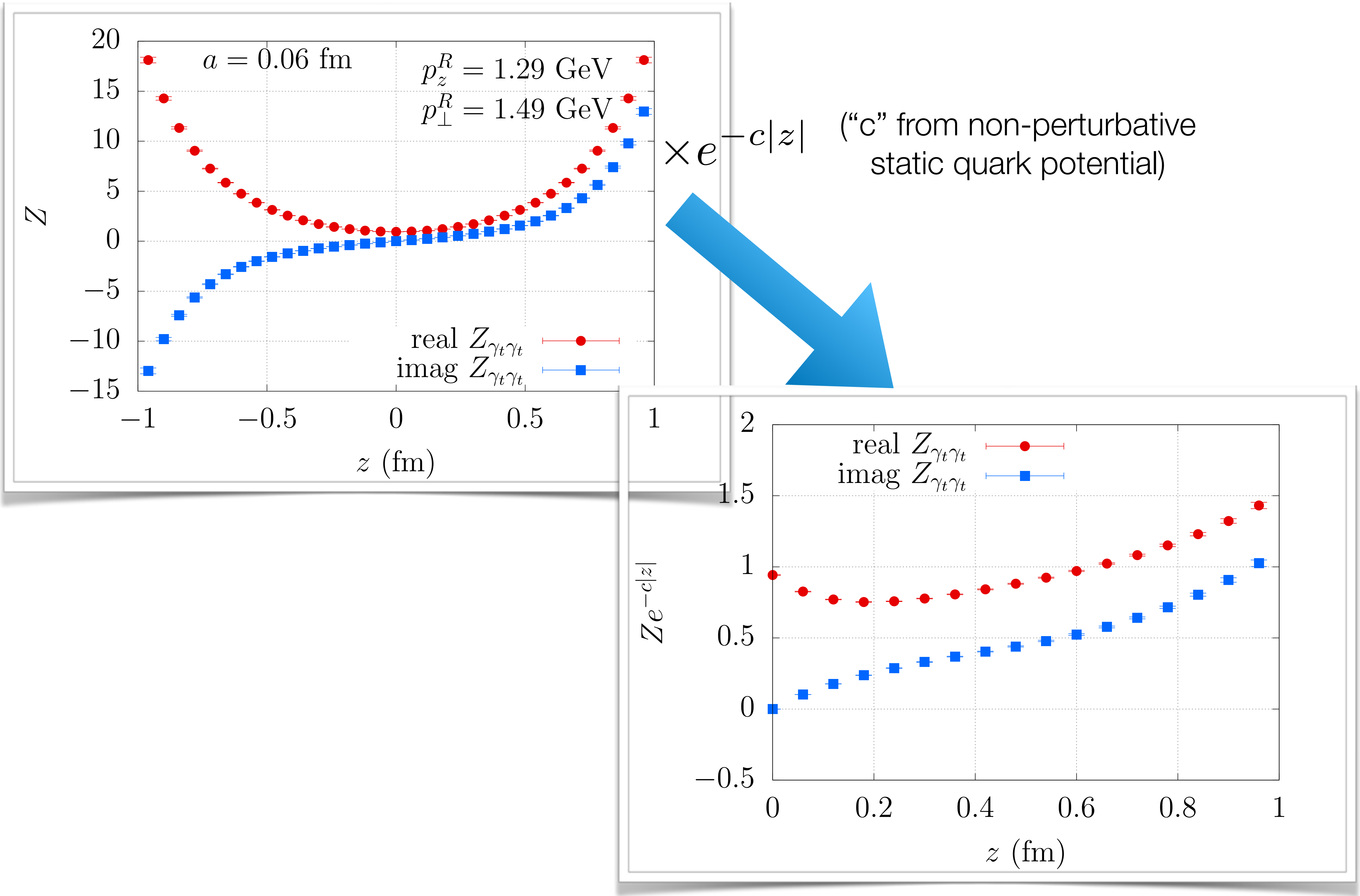


$\times e^{-c|z|}$

("c" from non-perturbative static quark potential)

RI-MOM renormalization factor



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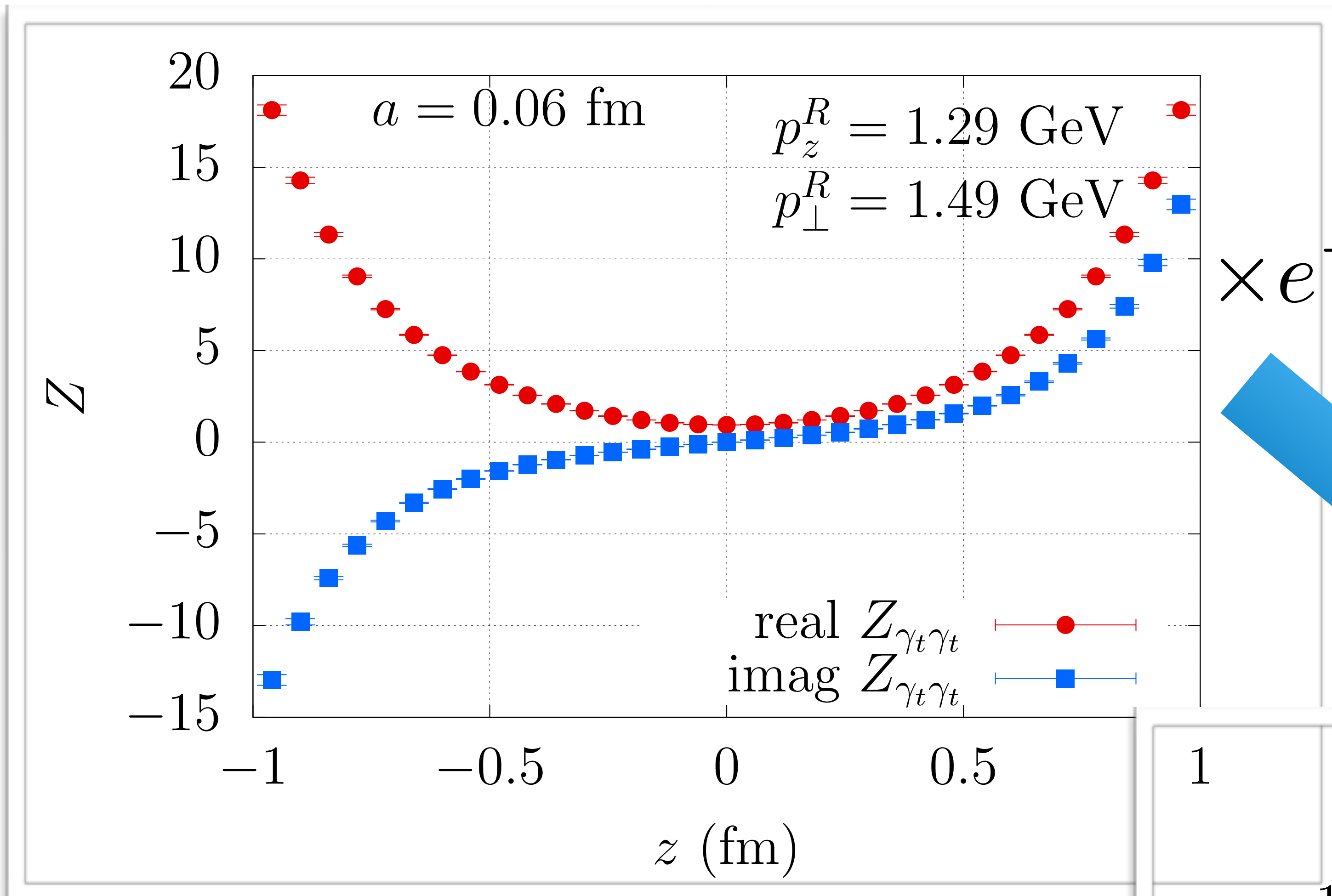

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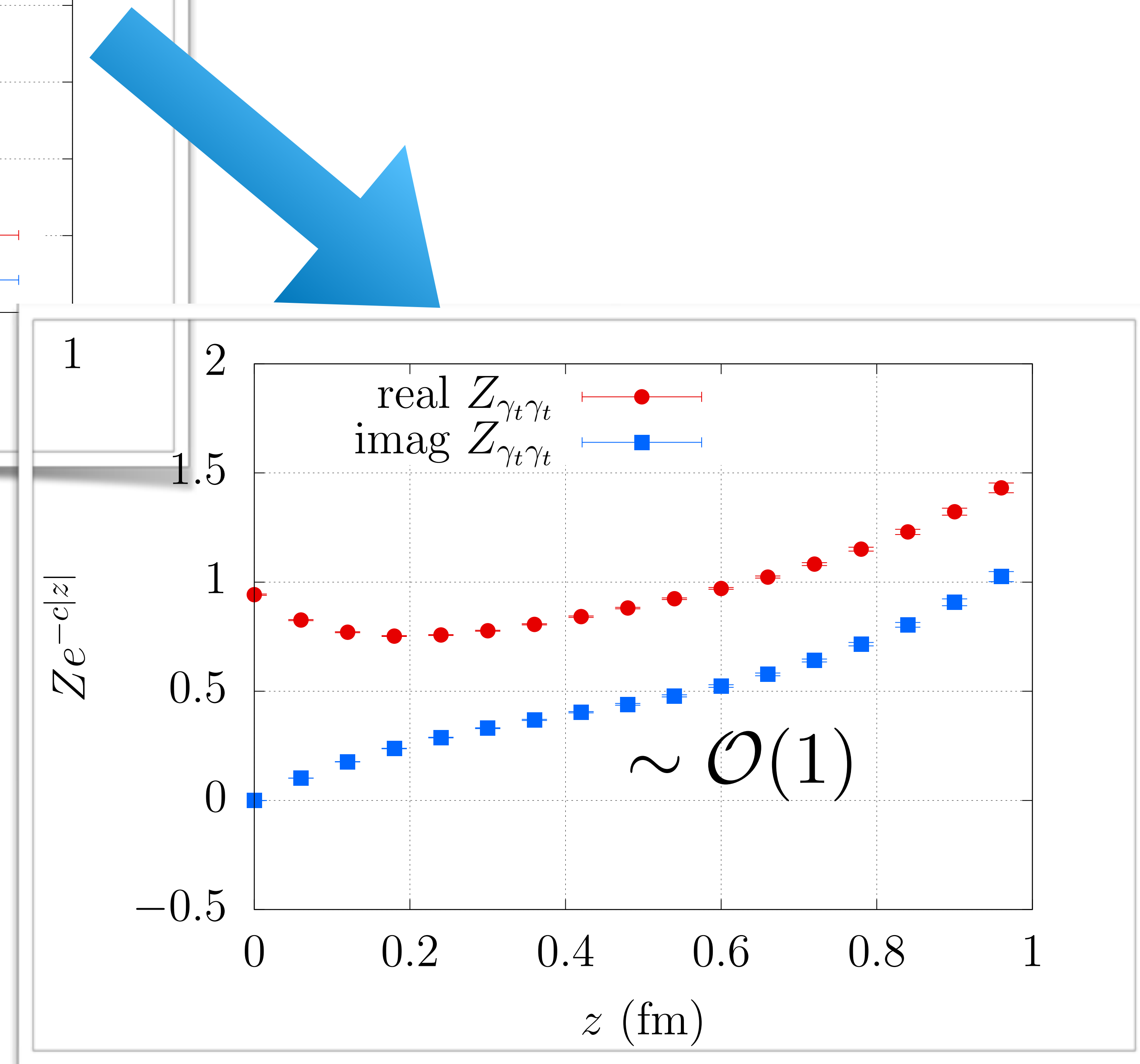

Quark external states in RI-MOM

Test validity of 1-loop results that enter matching

Quark external states in RI-MOM

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$$\zeta(z,p,p^R) \equiv \frac{Z(z,p) - Z(z,p^R)}{Z(z,p^R)} \stackrel{??}{=} \frac{q^{(1)}(z,p,p^R)}{q^{\mathrm{tree}}(z,p^R)}$$

One can think of ζ as the discrete analog of $\dfrac{\partial \log Z(p)}{\partial p}$

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$\boxed{\textbf{Compute on lattice}}$

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Compute on lattice

1-loop pert. theory

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Lattice and 1-loop running of ζ at shorter z

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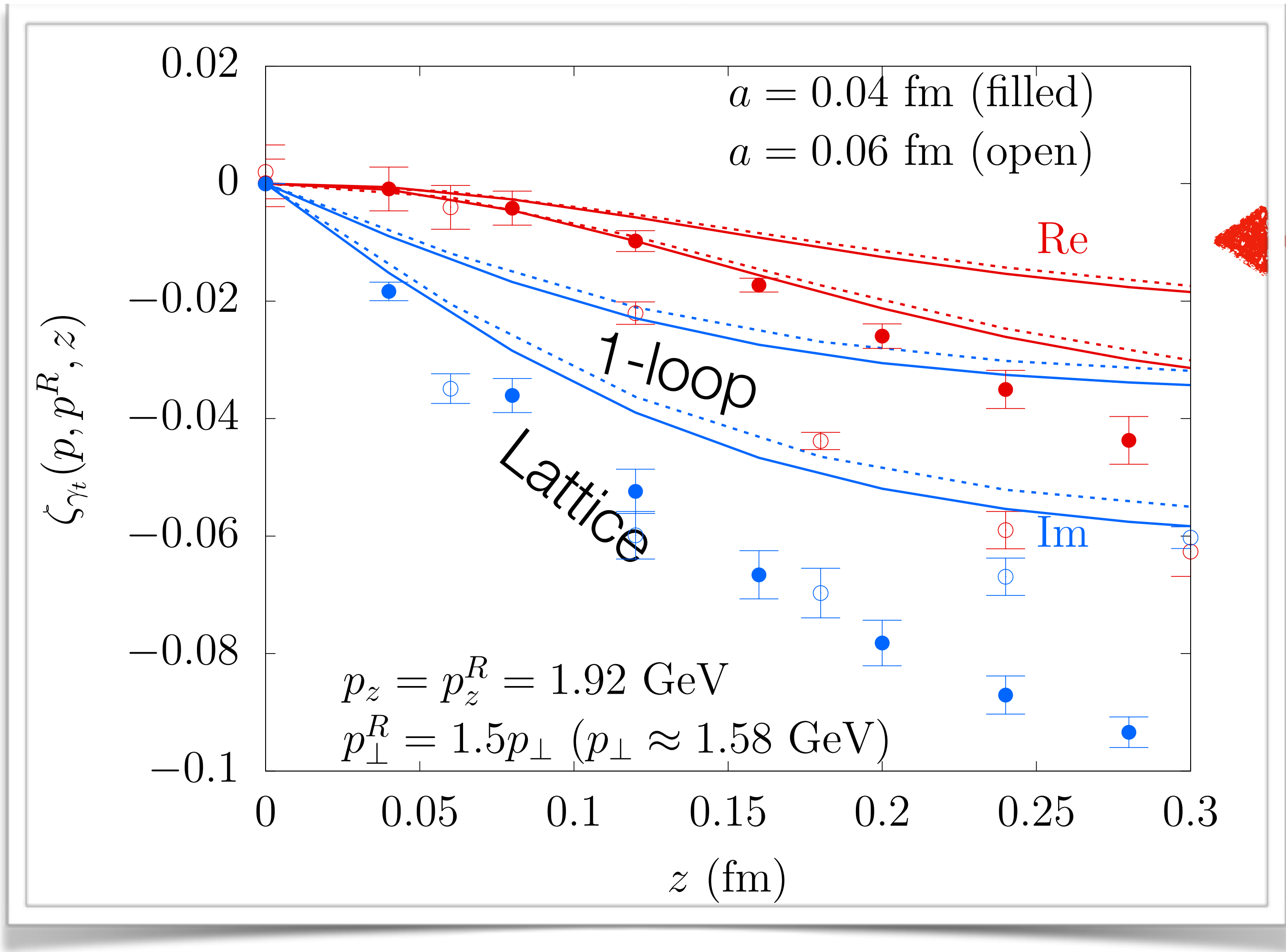
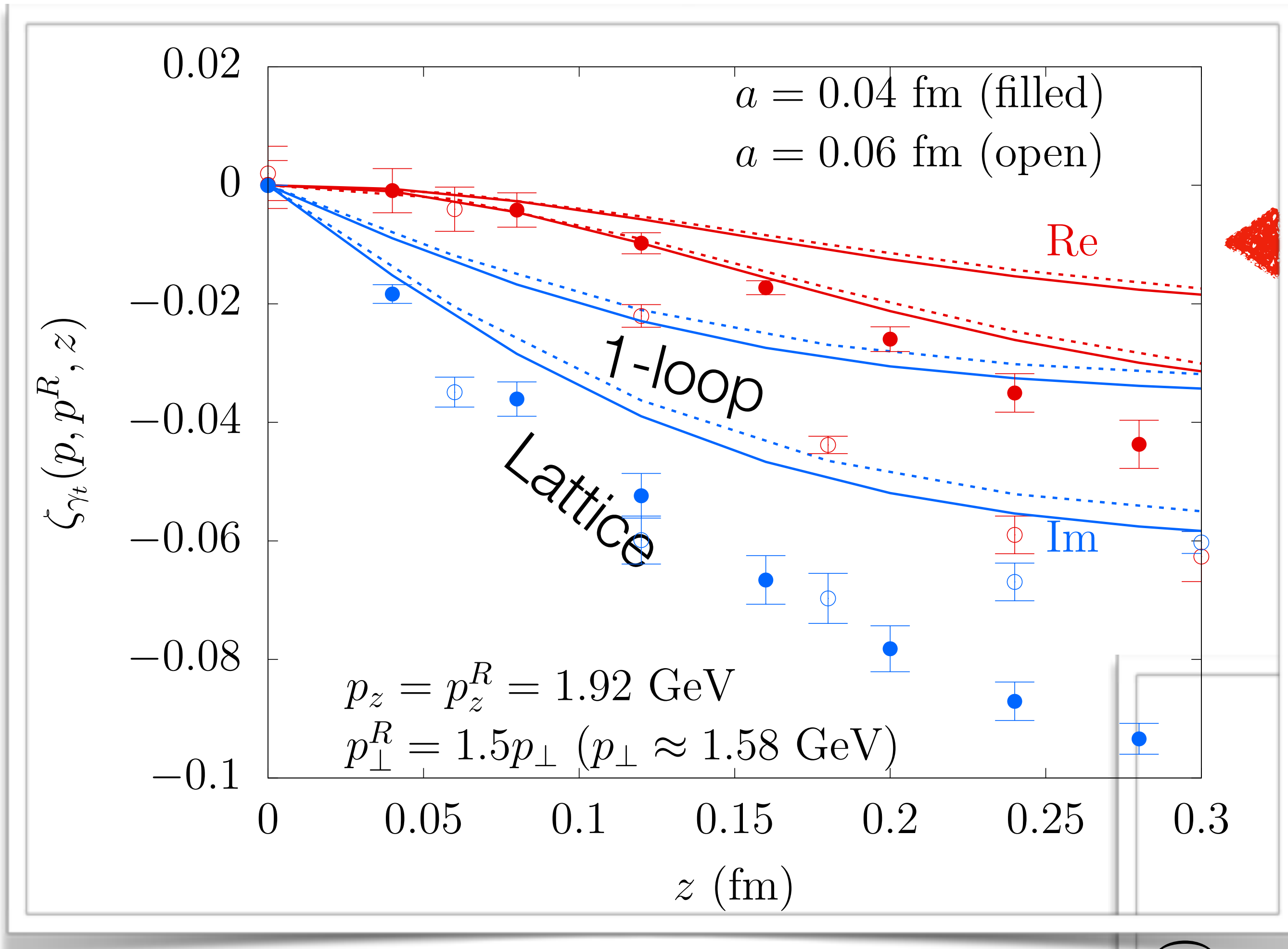


Comparison at different quark-antiquark separations z

Lattice and 1-loop running of ζ at shorter z



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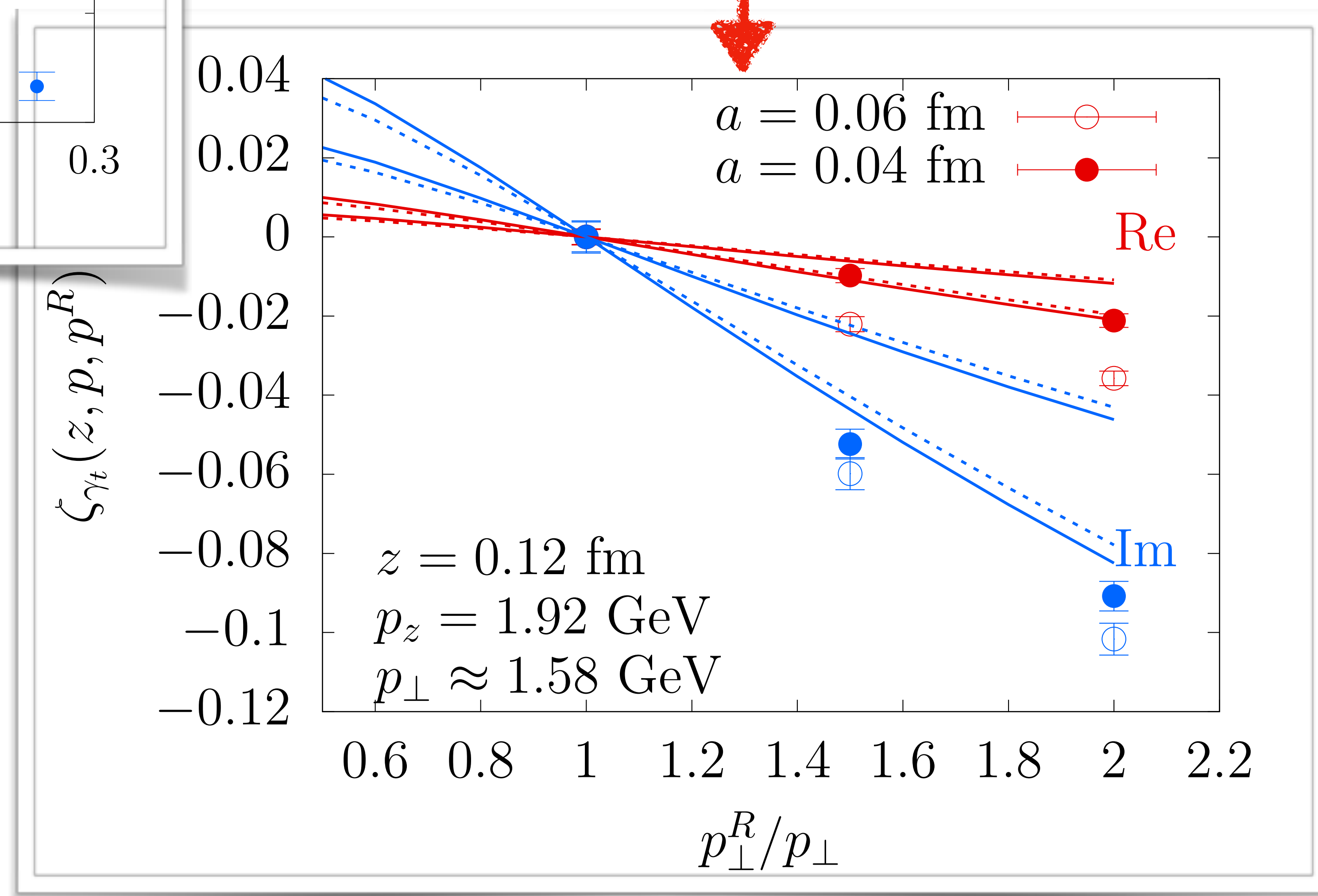
Running with quark momentum



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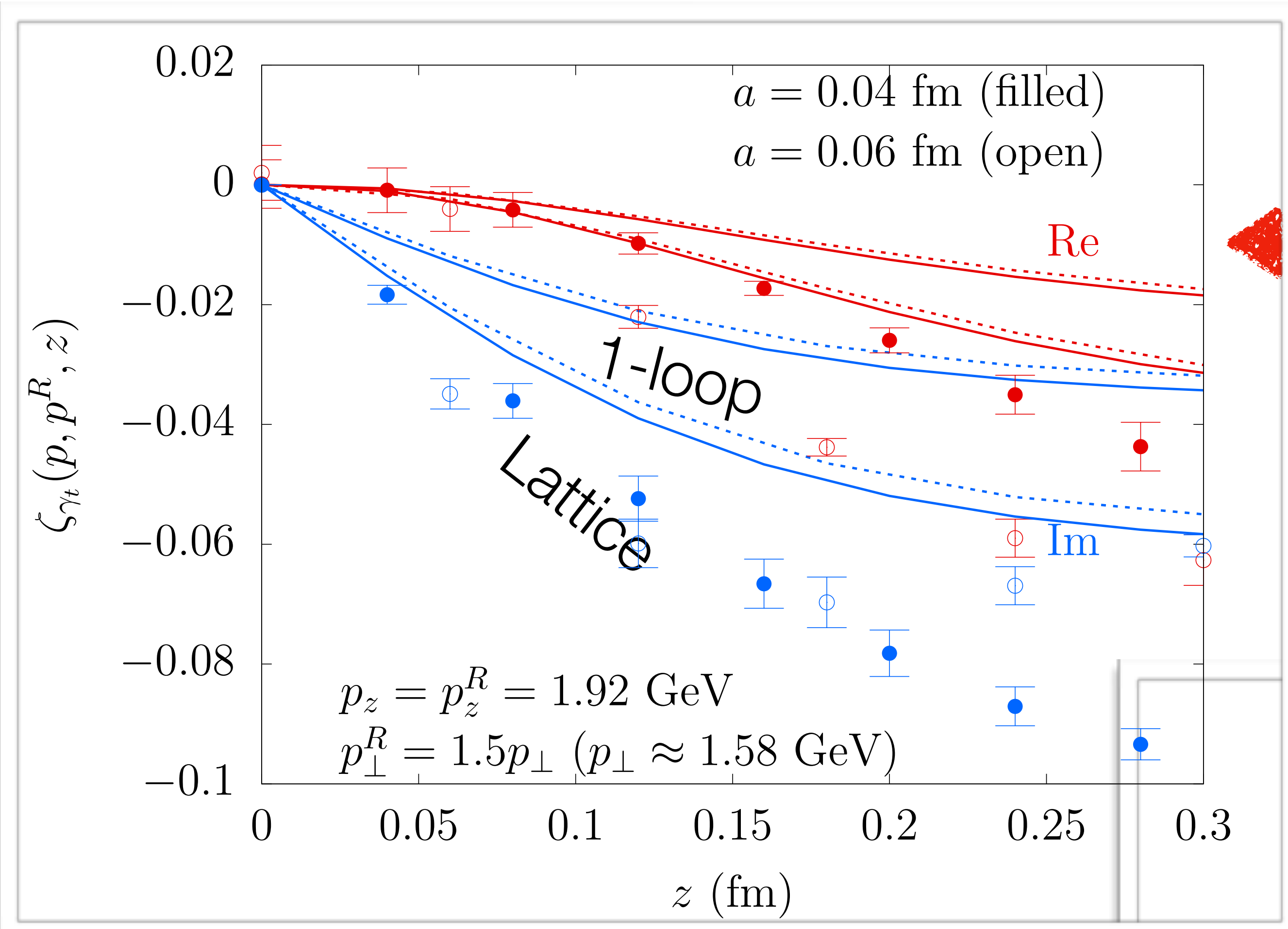

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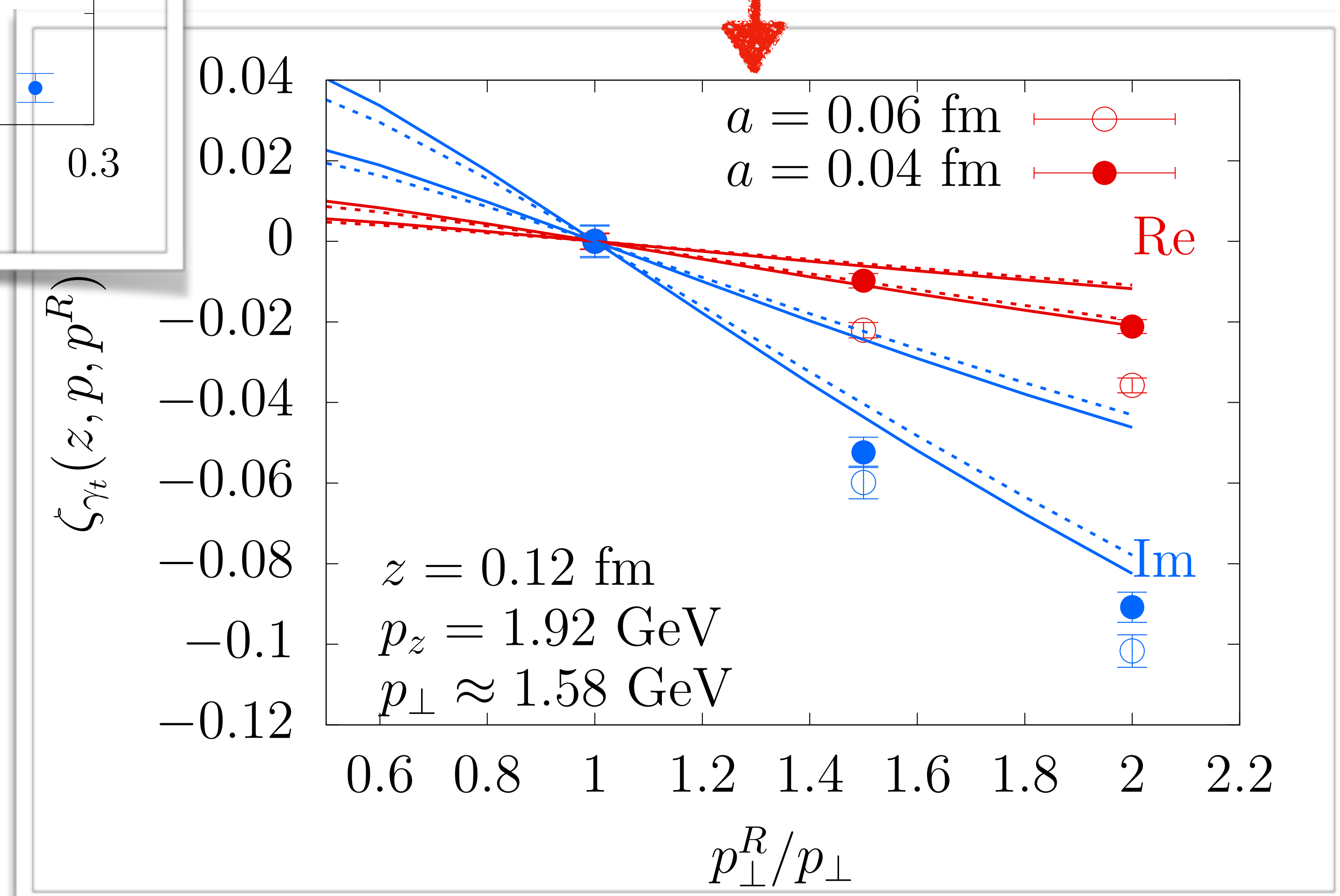
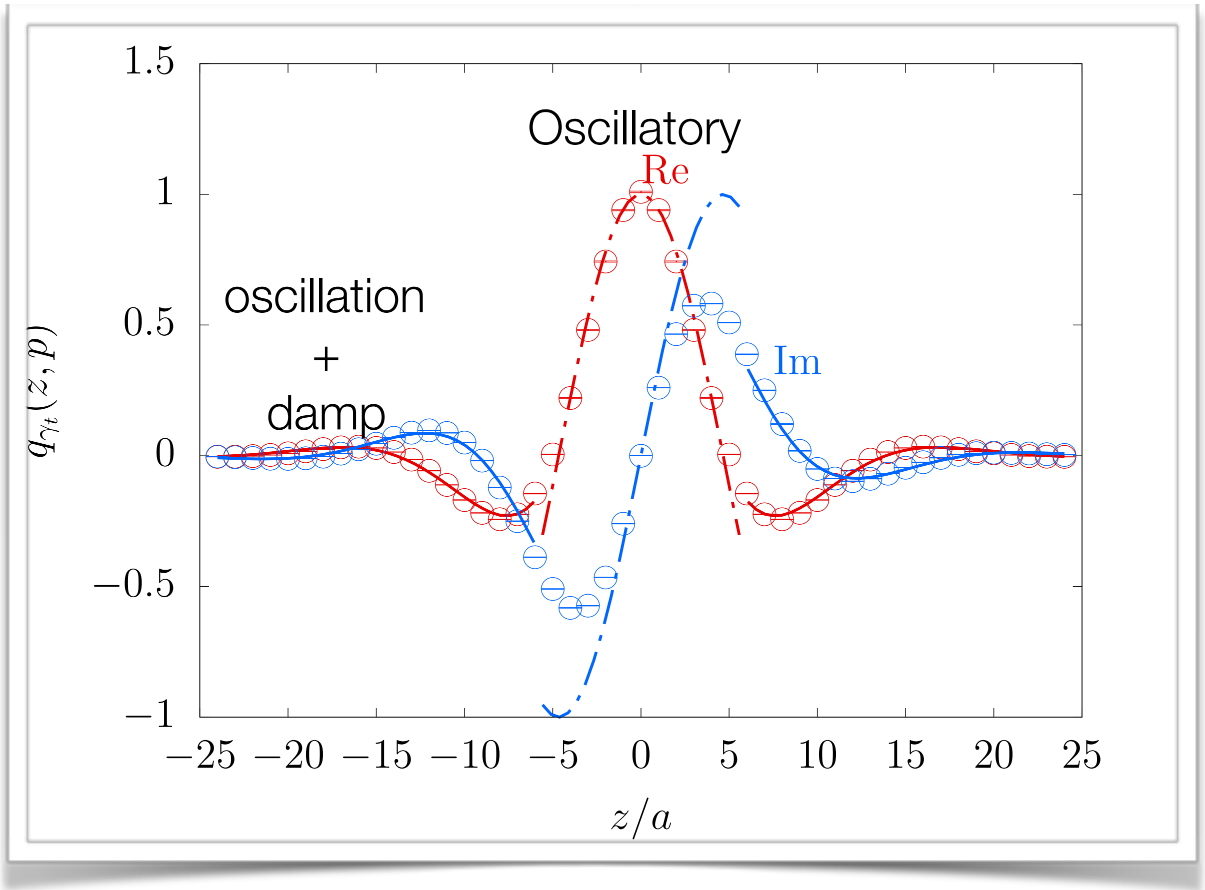


Two effects here

1. Better agreement as a is reduced
2. Disagreement as z is increased

Is it OK to use 1-loop qPDF results at all z?



$$m_{\rm scr} + i\omega \equiv -\log\left[\frac{q(z+1)}{q(z)}\right]$$

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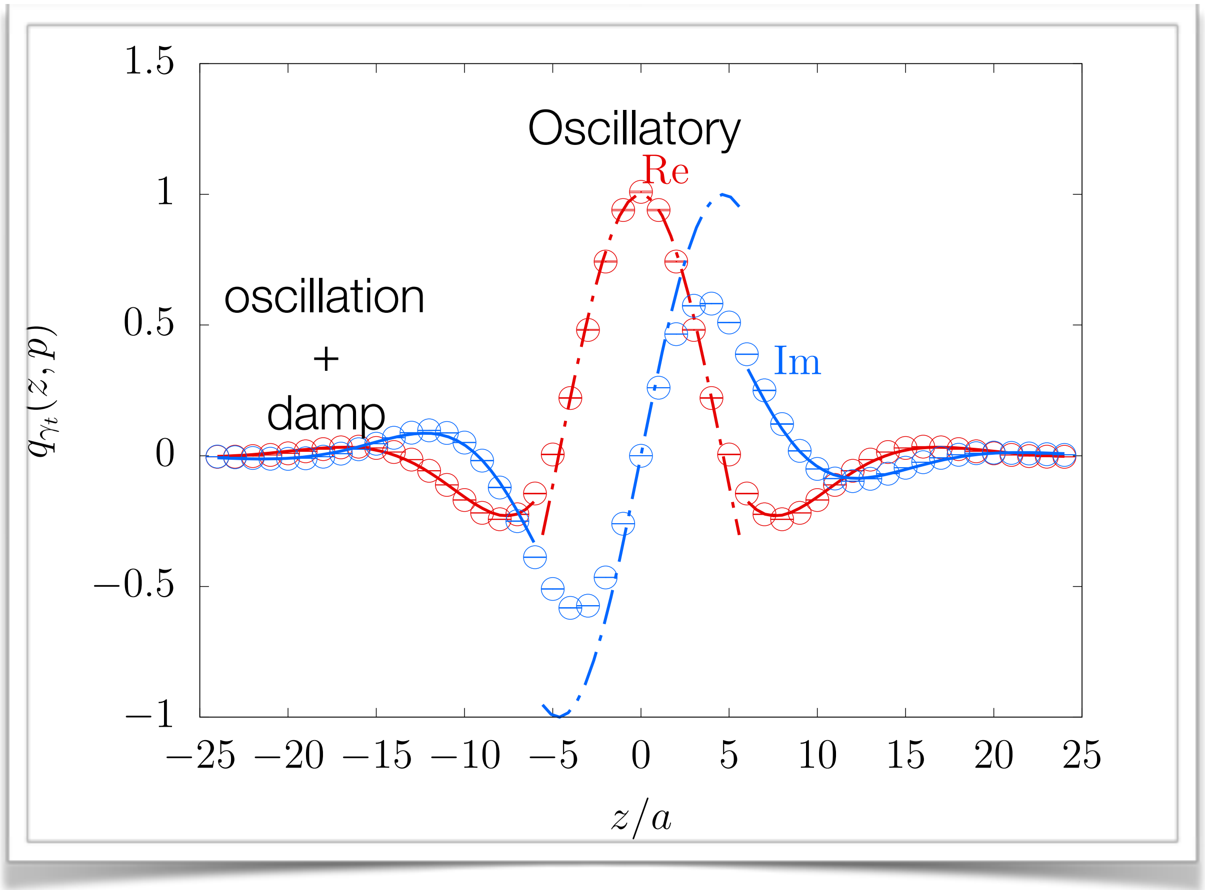

Damped oscillation ansatz: $q(z,p_z) \sim e^{i\omega z} e^{-m_{\rm scr} z} e^{-cz} \longrightarrow m_{\rm scr} + i\omega \equiv -\log\left[\frac{q(z+1)}{q(z)}\right]$

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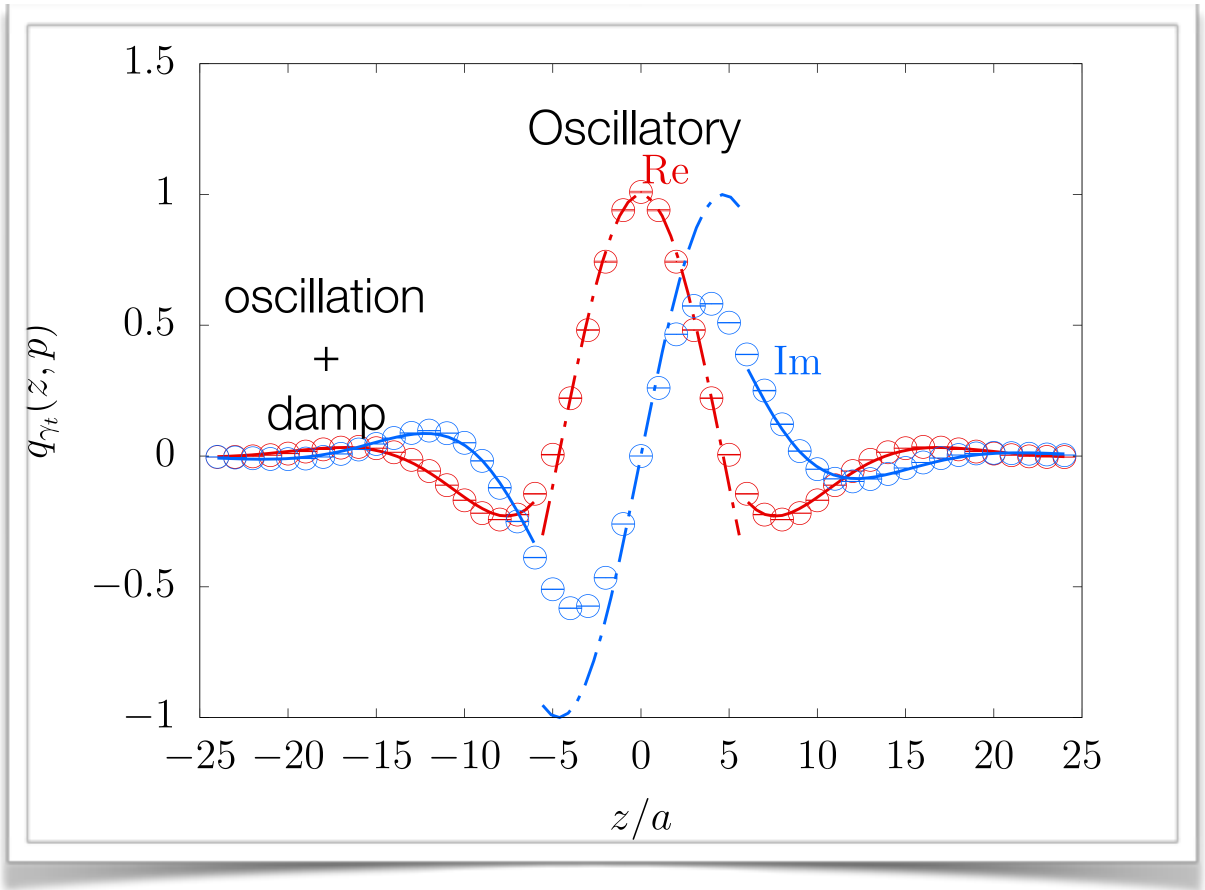

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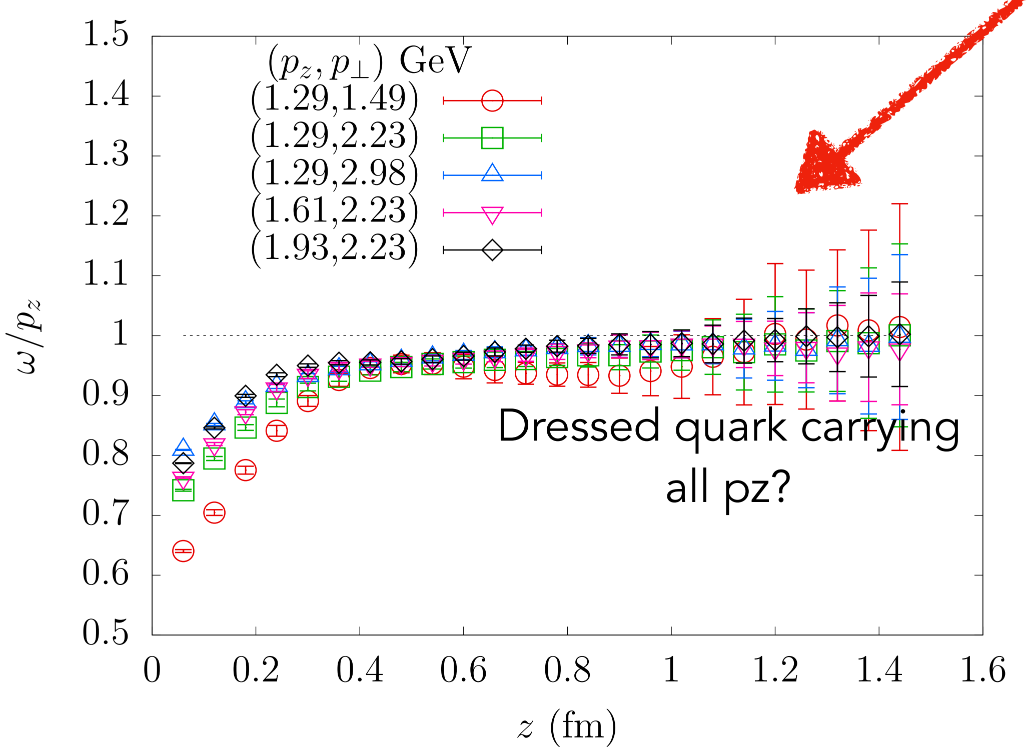

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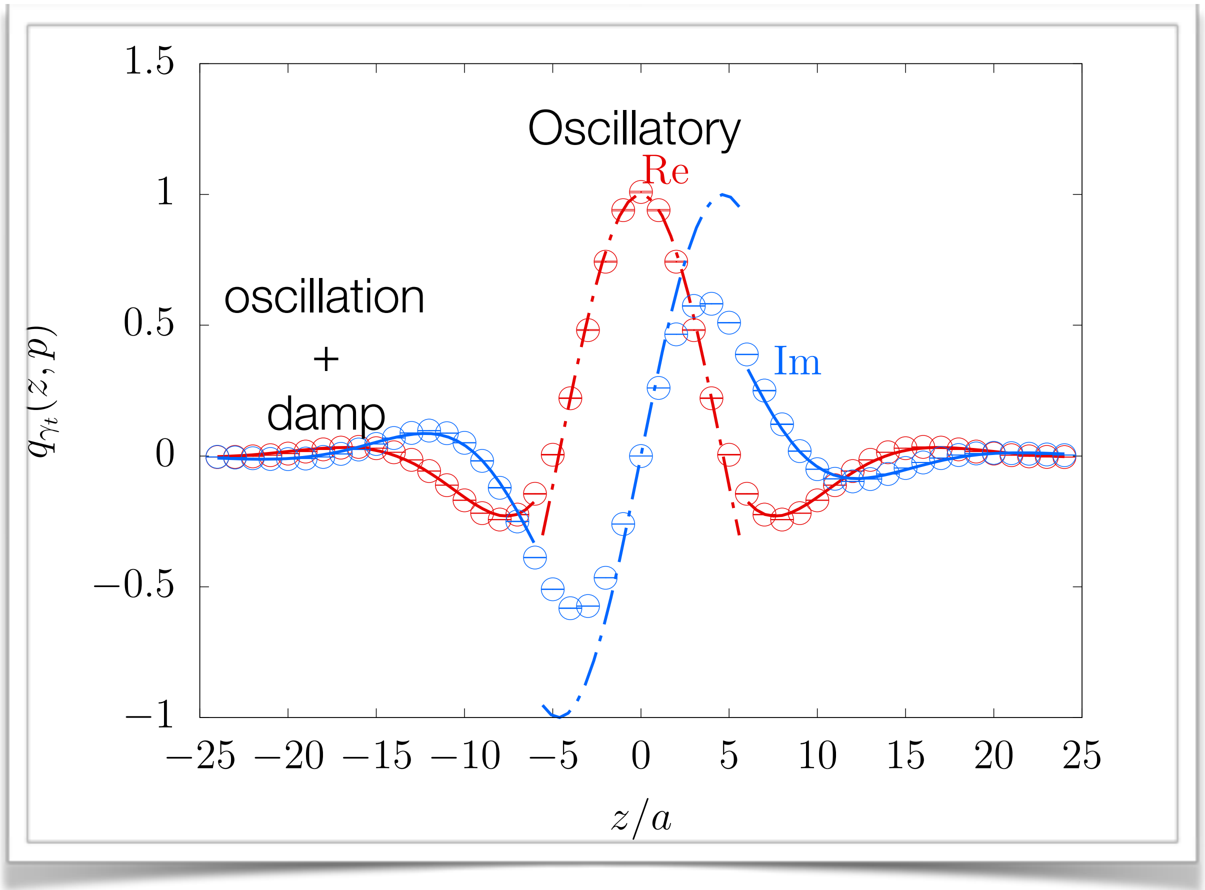
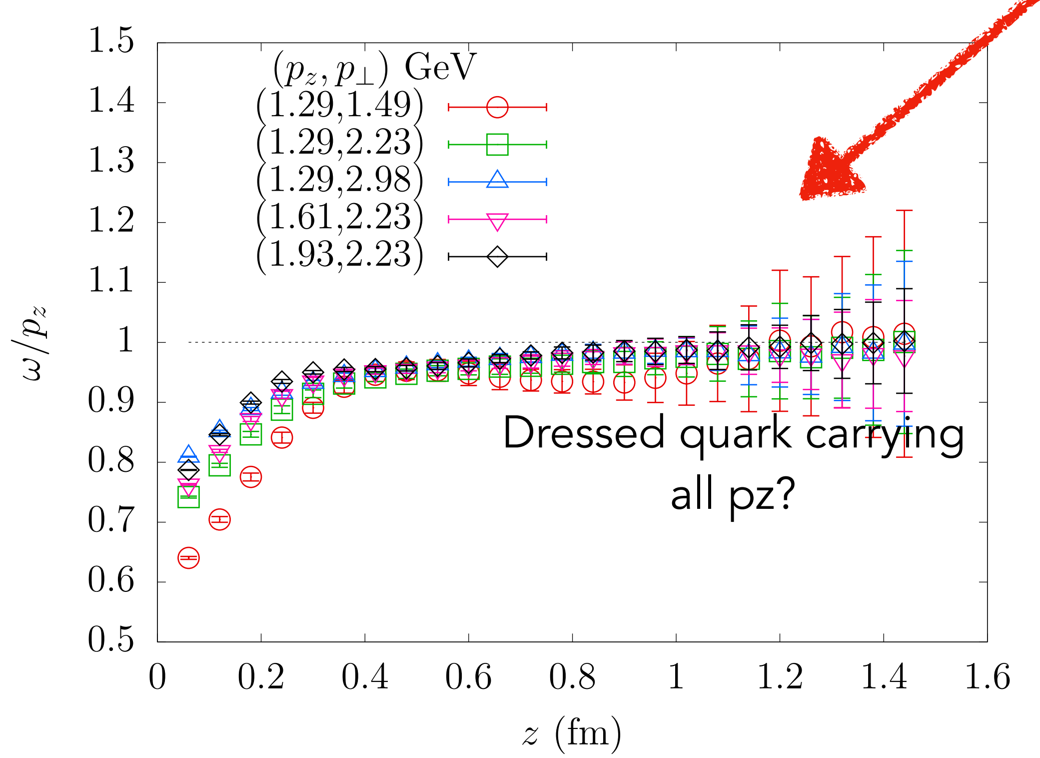
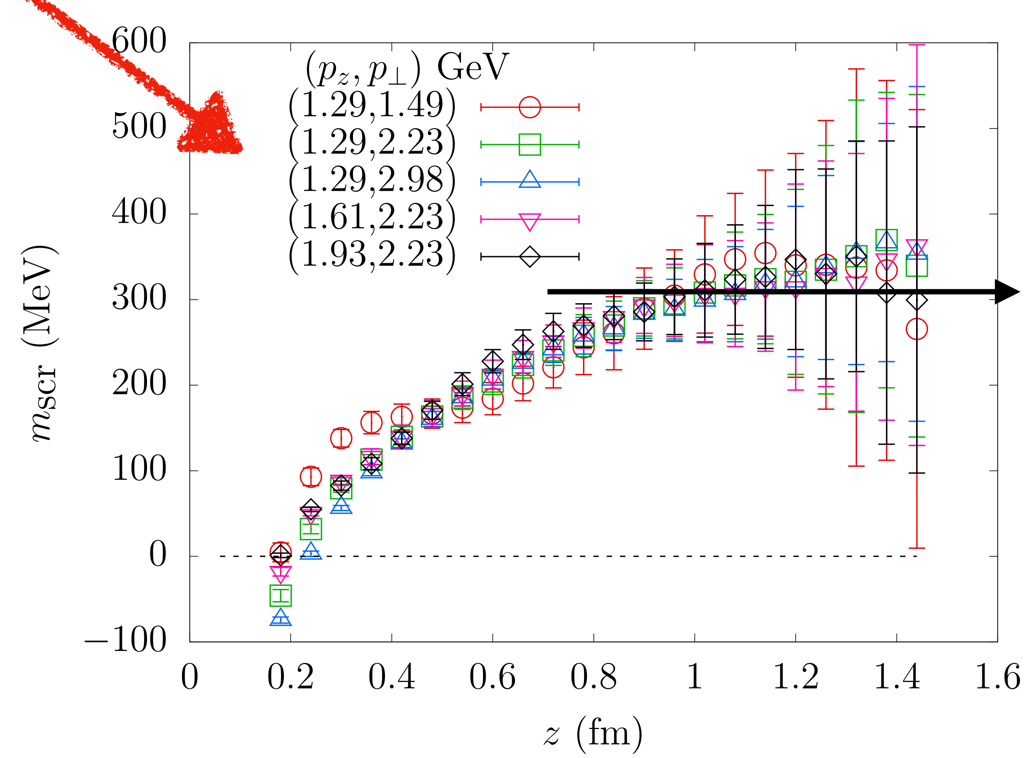


Damped oscillation ansatz: $q(z,p_z) \sim e^{i\omega z} e^{-m_{\rm scr} z} e^{-cz}$ → $m_{\rm scr} + i\omega \equiv -\log\left[\frac{q(z+1)}{q(z)}\right]$





Method 1:

Renormalized qPDF Matrix element
$h^R(z, P_z, P^R)$

$\xrightarrow{\text{Fourier}}$ Interpolate z and extrapolate to larger z

Quasi PDF
$\tilde{q}(x, P_z, P^R)$

$\xrightarrow{\text{Inverse 1-loop matching}}$

PDF
$f(x, \mu)$

Method 1:

Renormalized qPDF Matrix element $h^R(z, P_z, P^R)$ $\xrightarrow[\text{Interpolate z and extrapolate to larger z}]{\text{Fourier}}$ **Quasi PDF** $\tilde{q}(x, P_z, P^R)$ $\xrightarrow{\text{Inverse 1-loop matching}}$ **PDF** $f(x, \mu)$

Method 2:

Renormalized qPDF Matrix element $h^R(z, P_z, P^R)$ $\xleftarrow{\text{Fit over range of z}}$ **Model qPDF Matrix element** $h(z, P_z, P^R; a, b)$ $\xleftarrow{\text{Inv Fourier}}$ **Quasi PDF** $\tilde{q}(x, P_z, P^R; a, b)$ $\xleftarrow{\text{1-loop matching}}$ **Pheno inspired PDF**

$$f(x, \mu; a, b) = Ax^a(1-x)^b$$

Method 1:

Advantage: Model independent(?)
Disadvantages: No control over range of z

Renormalized qPDF Matrix element
$h^R(z, P_z, P^R)$

$\xrightarrow[\text{Interpolate z and extrapolate to larger z}]{\text{Fourier}}$

Quasi PDF
$\tilde{q}(x, P_z, P^R)$

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$\tilde{q}(x, P_z, P^R; a, b)$

$\xleftarrow{\text{1-loop matching}}$

Pheno inspired PDF
$$f(x, \mu; a, b) = Ax^a(1-x)^b$$

Method 1:

Advantage: Model independent(?)
Disadvantages: No control over range of z

Renormalized qPDF Matrix element
$h^R(z, P_z, P^R)$

$\xrightarrow{\text{Fourier}}$ Interpolate z and extrapolate to larger z

Quasi PDF
$\tilde{q}(x, P_z, P^R)$

$\xrightarrow{\text{Inverse 1-loop matching}}$

PDF
$f(x, \mu)$

Method 2:

Disadvantage: Model dependent, but same as what expts do
Advantage: Control over range of z

Renormalized qPDF Matrix element
$h^R(z, P_z, P^R)$

$\xleftarrow{\text{Fit over range of z}}$

Model qPDF Matrix element
$h(z, P_z, P^R; a, b)$

$\xleftarrow{\text{Inv Fourier}}$

Quasi PDF
$\tilde{q}(x, P_z, P^R; a, b)$

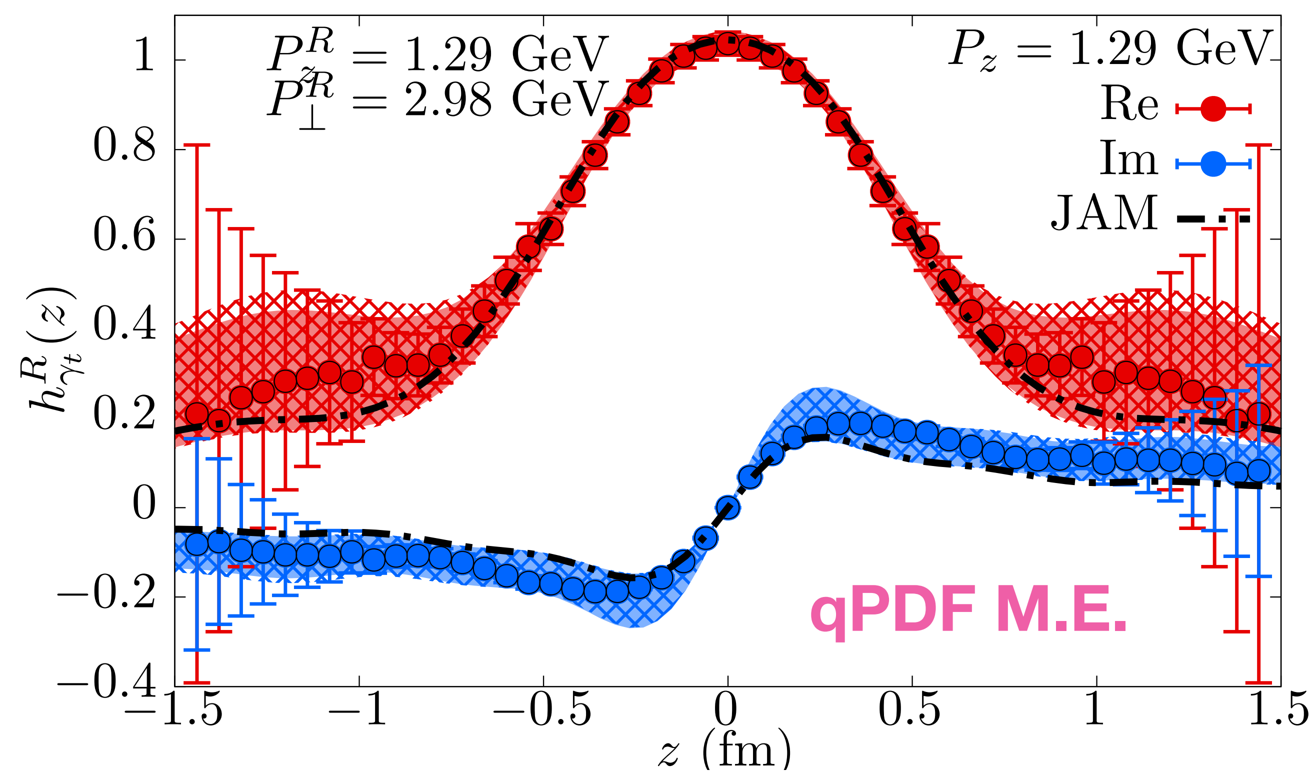
$\xleftarrow{\text{1-loop matching}}$

Pheno inspired PDF
$f(x, \mu; a, b) = Ax^a(1-x)^b$

$$h(z, P_z, P^R; a, b)$$

Fit over range of z

$$h^R(z, P_z, P^R)$$



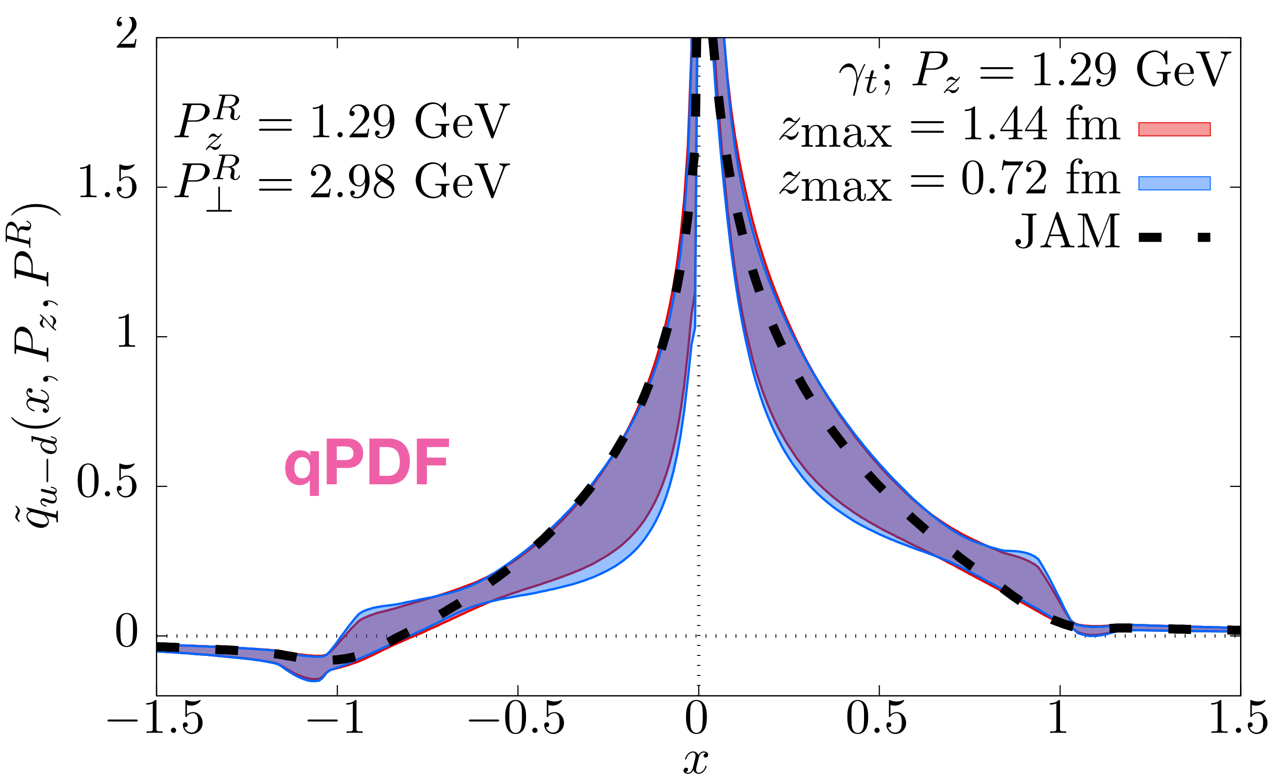
$$\tilde{q}(x, P_z, P^R; a, b)$$

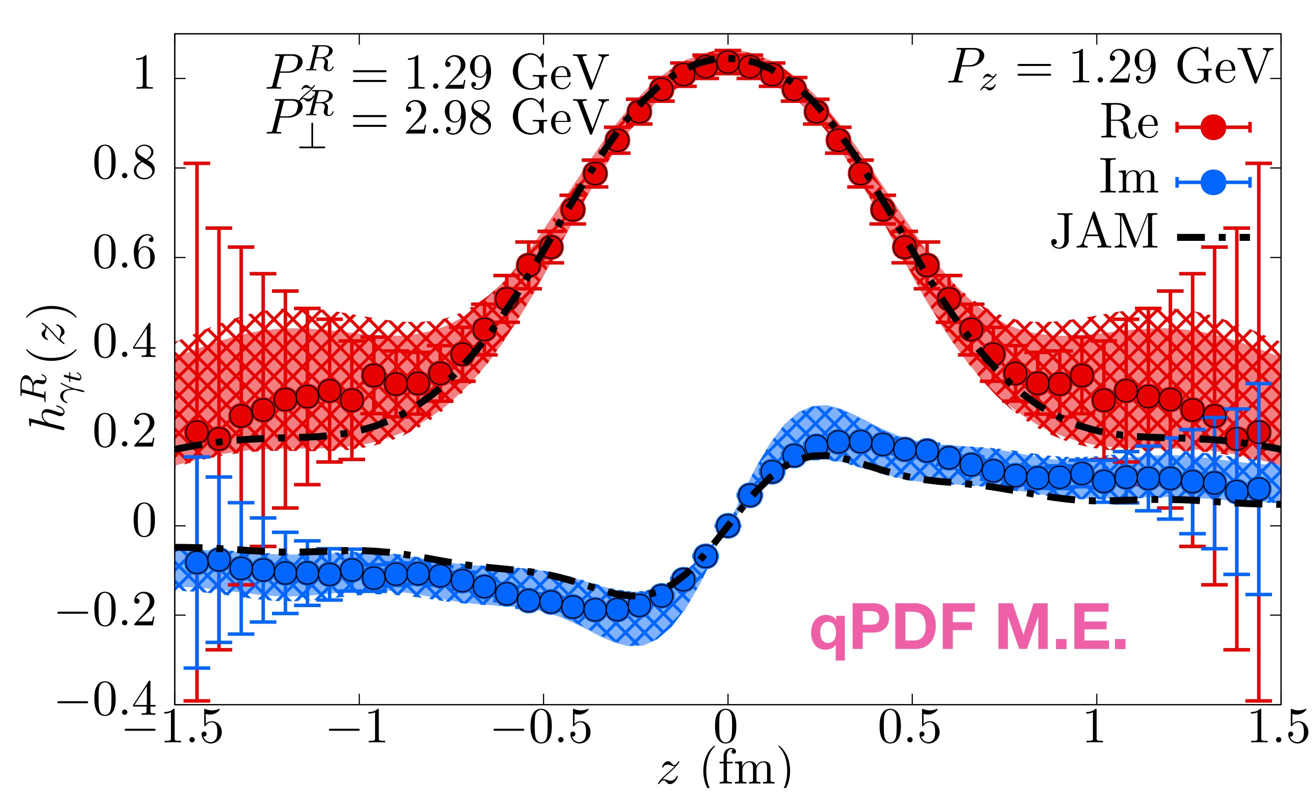
Inv Fourier

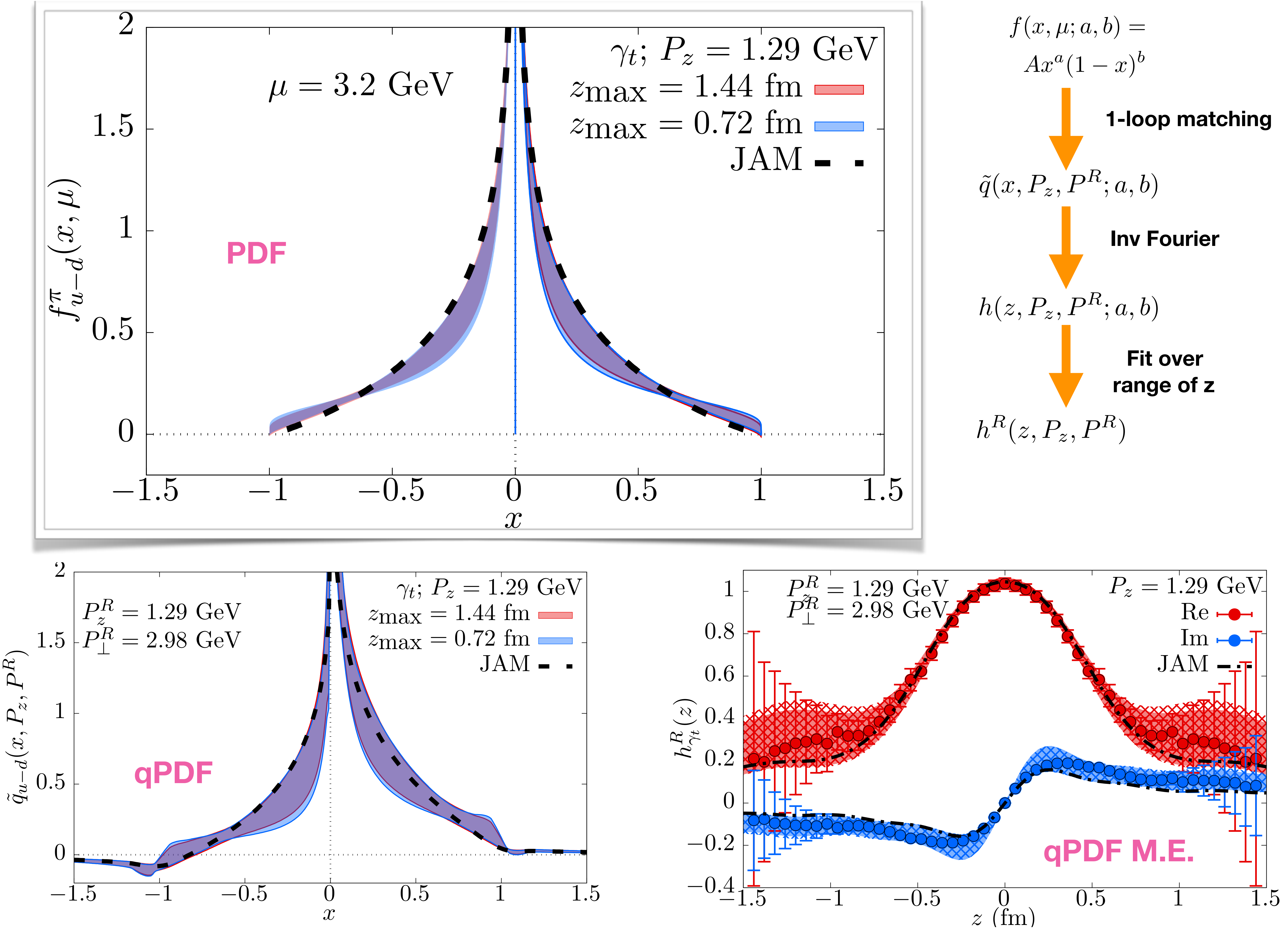
$$h(z, P_z, P^R; a, b)$$

Fit over range of z

$$h^R(z, P_z, P^R)$$

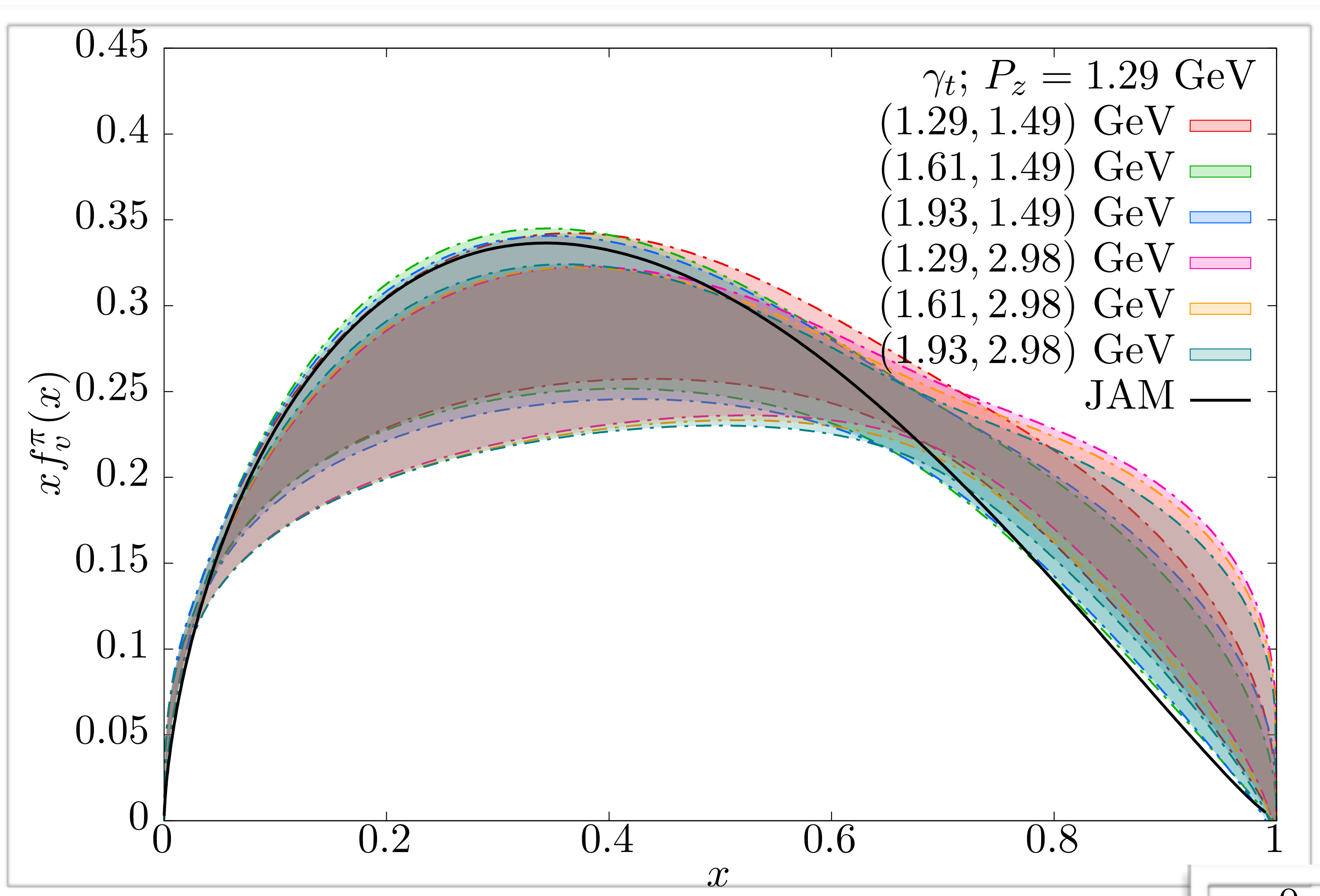




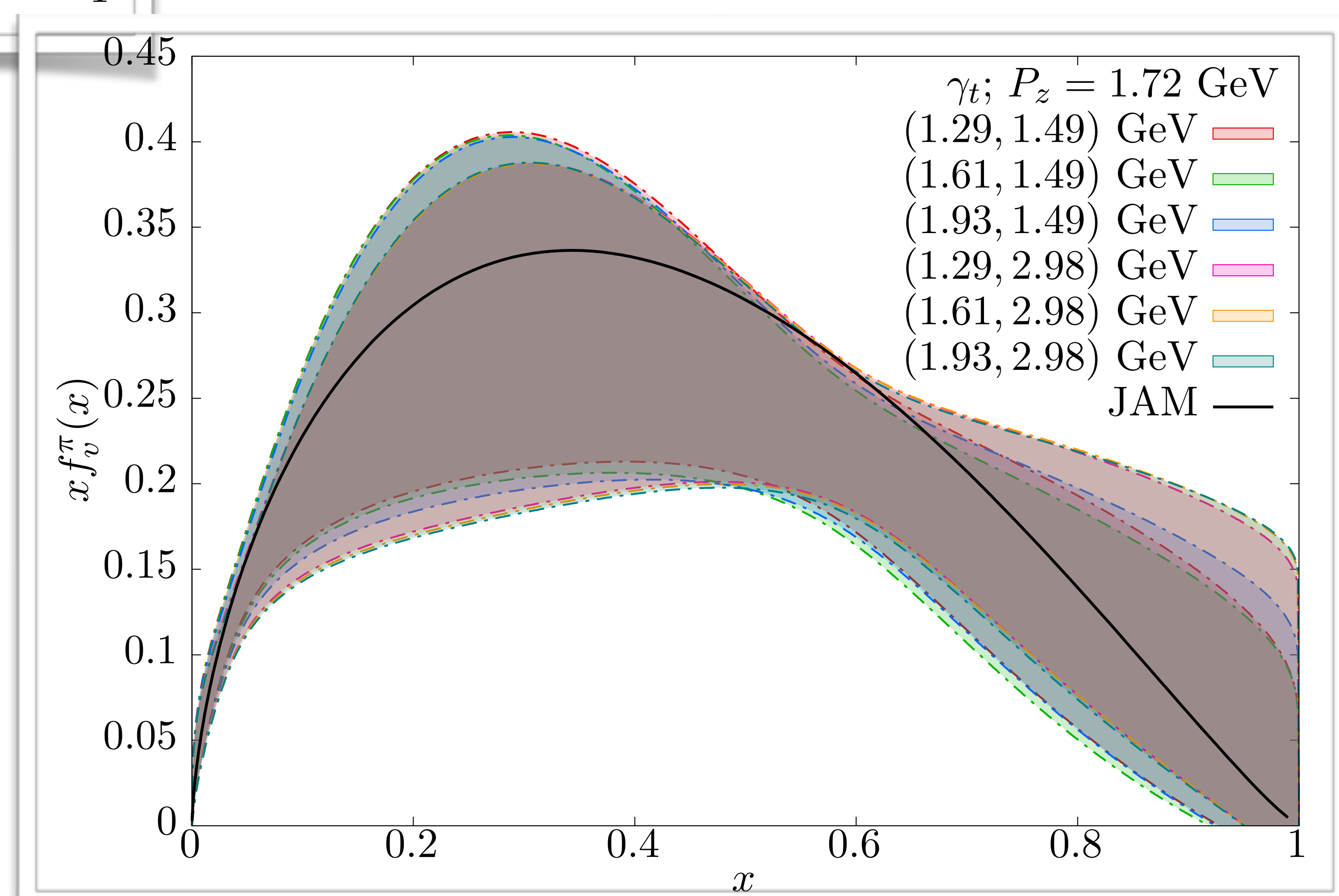


$\mu = 3.2$ GeV
γ_t; $P_z = 1.29$ GeV
$z_{\max} = 1.44$ fm
$z_{\max} = 0.72$ fm
JAM
PDF
$f^{\pi}_{u-d}(x,\mu)$
x
$f(x,\mu;a,b) =$
$Ax^a(1-x)^b$
1-loop matching
$\tilde{q}(x, P_z, P^R; a, b)$
Inv Fourier
$h(z, P_z, P^R; a, b)$
Fit over
range of z
$h^R(z, P_z, P^R)$
$P_z^R = 1.29$ GeV
$P_\perp^R = 2.98$ GeV
qPDF
$\tilde{q}_{u-d}(x, P_z, P^R)$
$P_z = 1.29$ GeV
Re
Im
qPDF M.E.
$h^R_{\gamma_t}(z)$
z (fm)

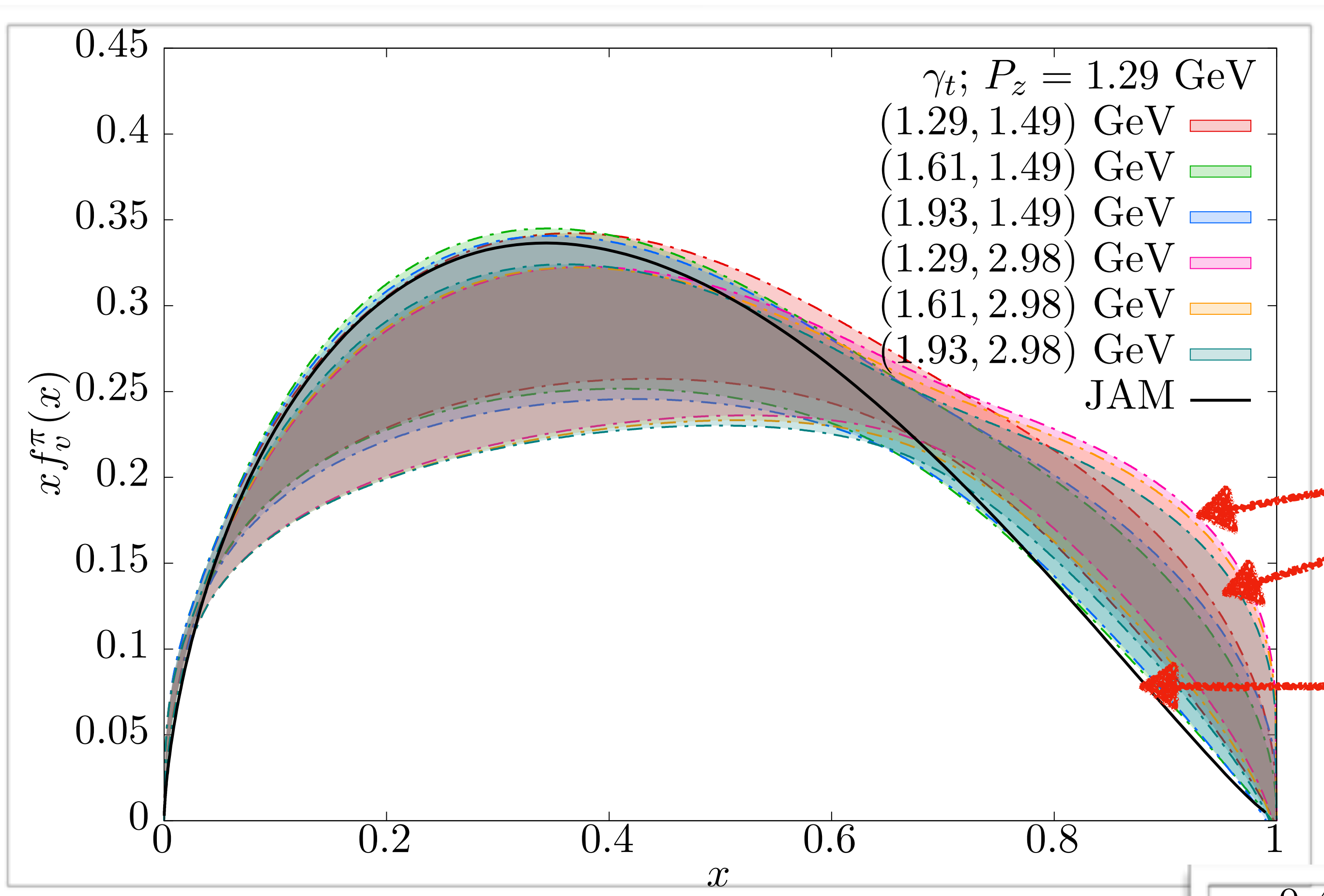
Valence PDF of pion from lattice



Using qPDF at two different momenta



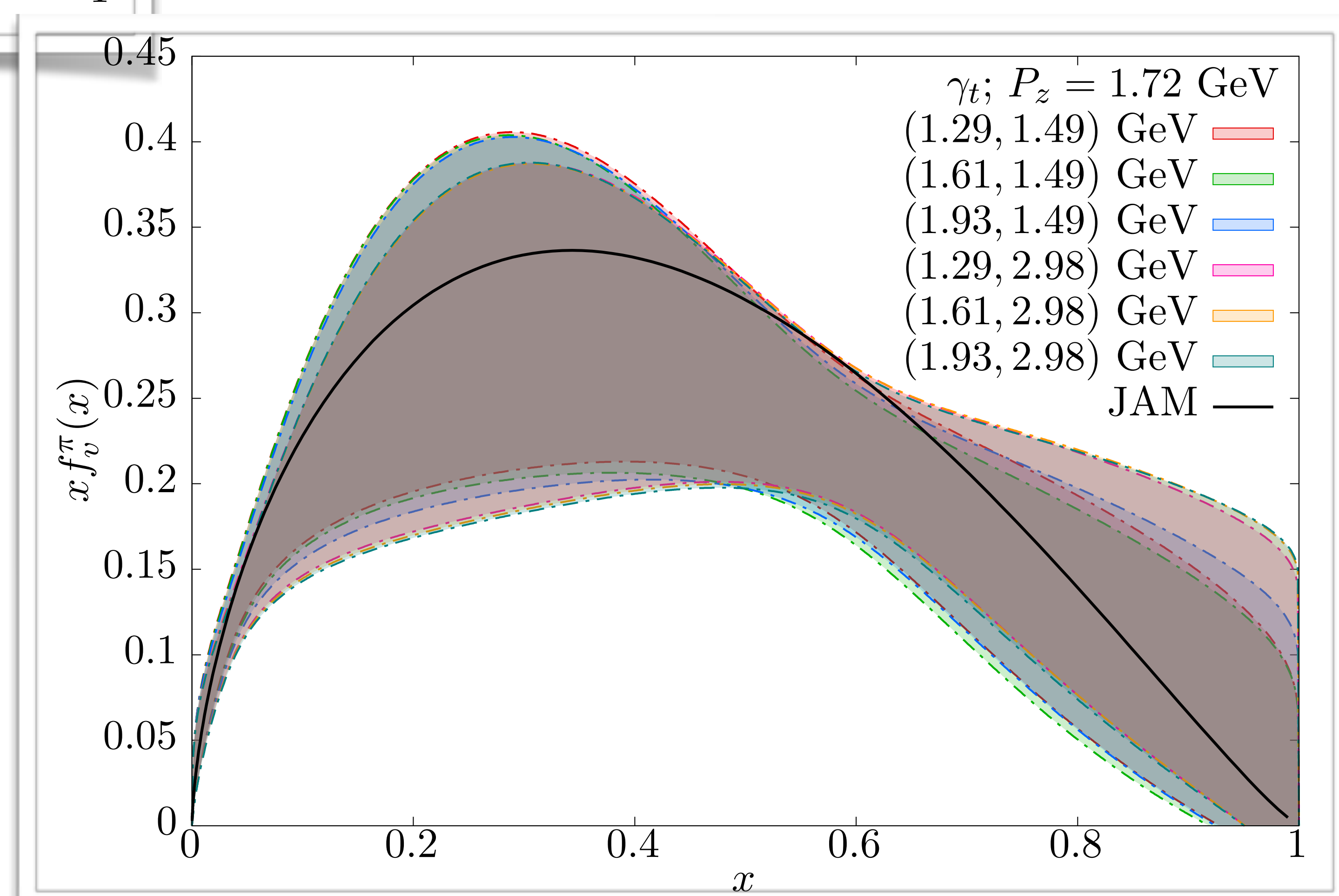
Valence PDF of pion from lattice



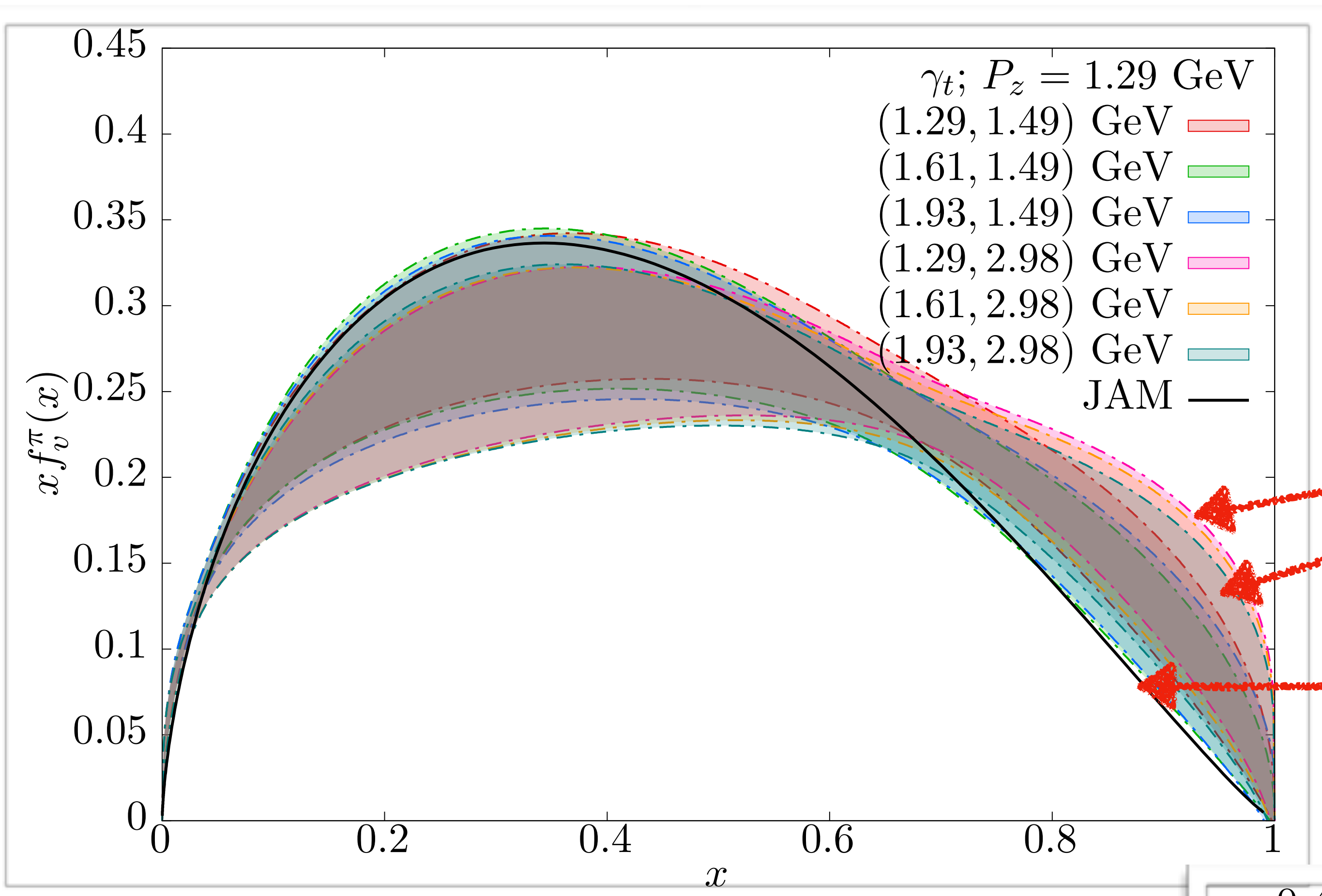
Using qPDF
at different
RI-MOM scales

JAM data
at 3.2 GeV

Using qPDF at
two different momenta

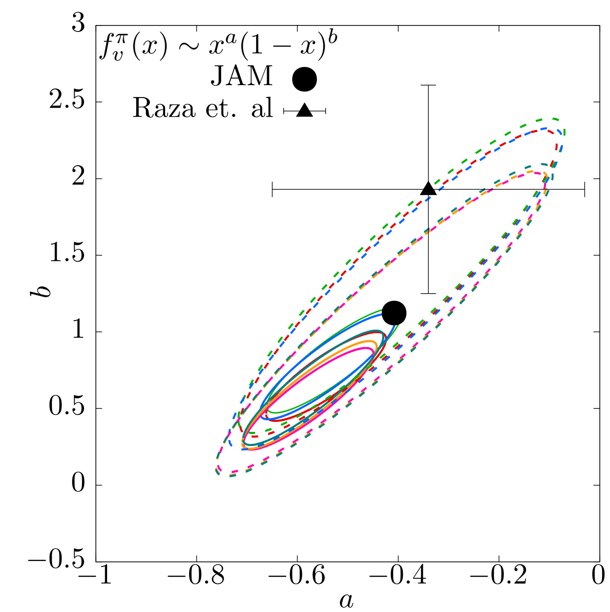


Valence PDF of pion from lattice

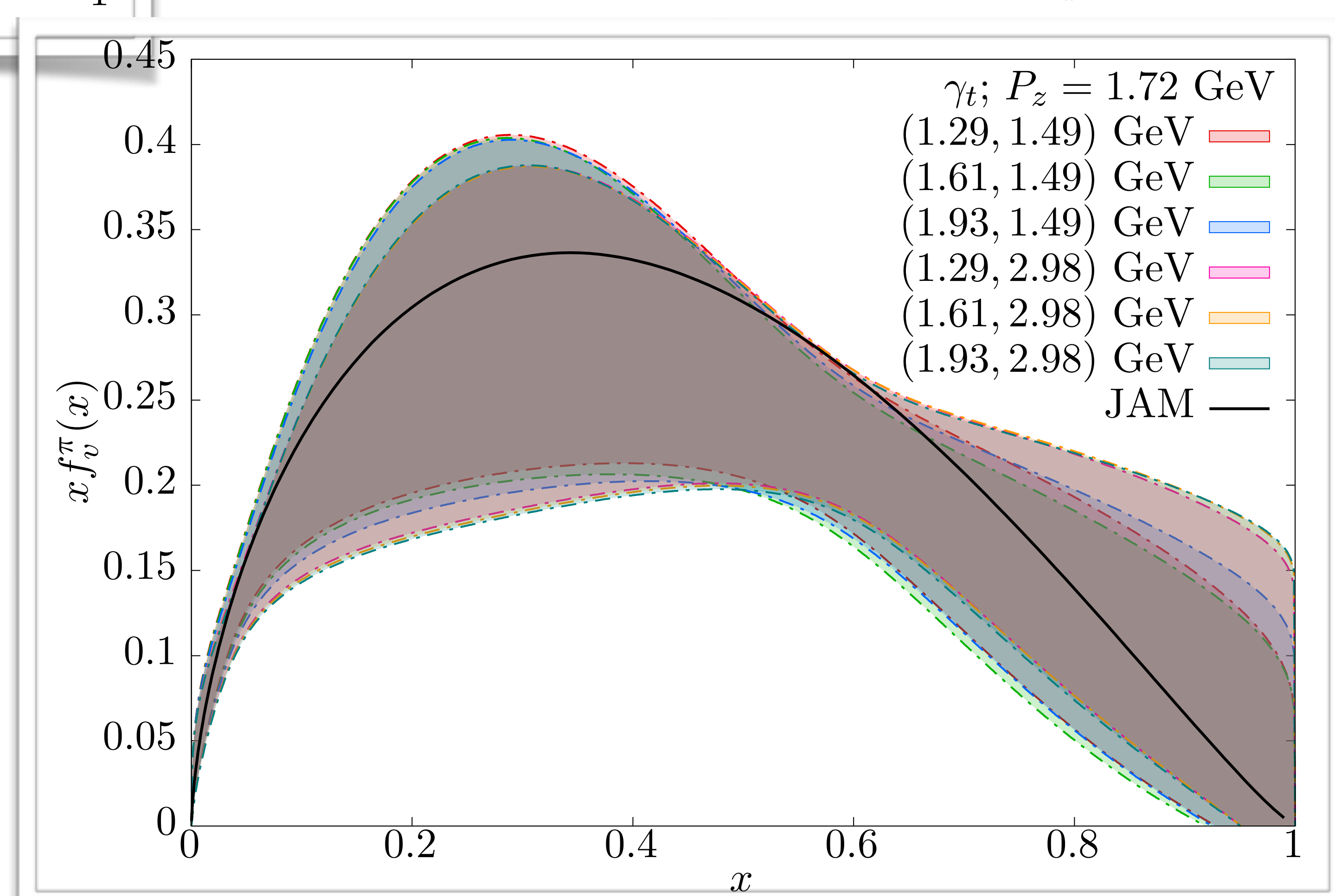


Using qPDF
at different
RI-MOM scales

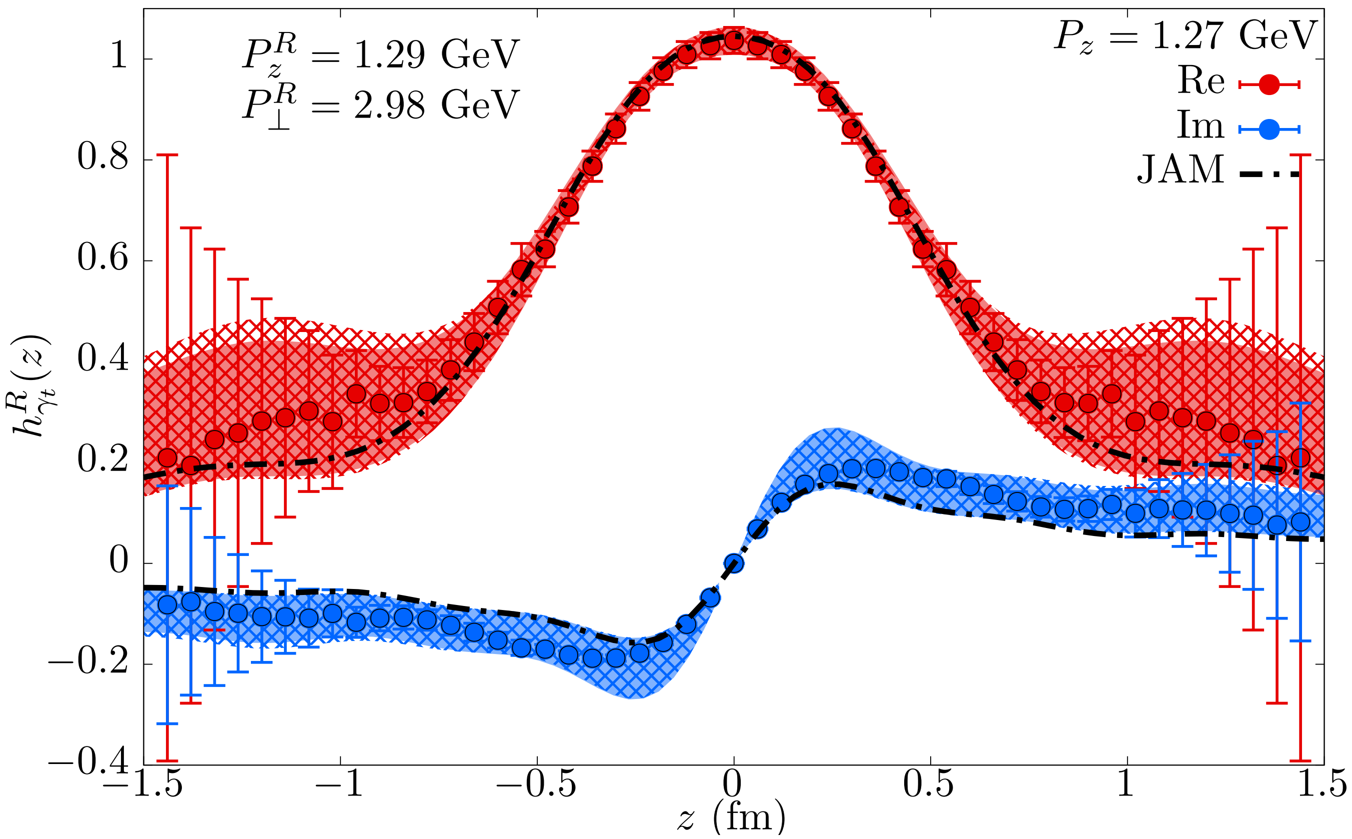
JAM data
at 3.2 GeV



Using qPDF at
two different momenta



A web of qPDF Matrix Elements to cross-check

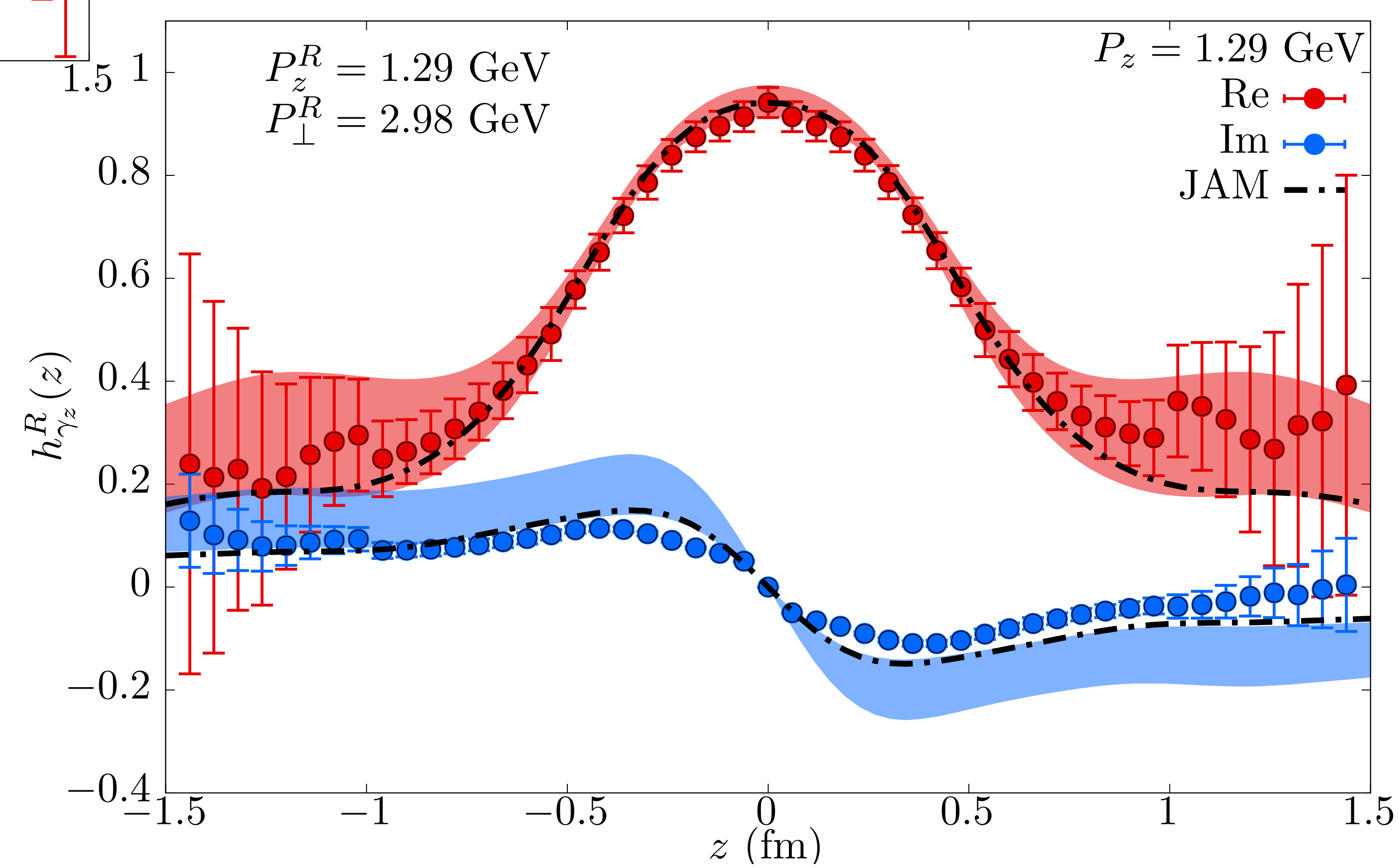


Input:

γ_t Quasi PDF matrix element

Output:

γ_z Quasi PDF matrix element



Conclusions

- Lot of progress in PDF extraction since the proposal of quasi distributions supplemented with LaMET matching.

- We studied the methodology and assumptions that go into the LaMET formalism applied to the quasi-PDF approach using valence PDF of pion as a case study using a=0.06 fm (and partly using a=0.04 fm) fine lattices.

- At nonzero pion momentum, a double problem: increase in noise and decrease in GS-excited state gap $\longrightarrow$ excited state extrapolations important and optimized boost smeared quark sources almost a requirement.

- Quasi-PDF with off-shell quark external states $\longrightarrow$ Consistency check for LaMET. Compared lattice and 1-loop running of quark quasi-PDF for z ~ 0.3 fm. Two competing effects — disagreement at larger z (*improved by higher order?*) and better agreement at finer lattice spacing (*lattice spacing finer than 0.04 fm required?*). For z > 1fm, clear indications of nonperturbative screening effects seen in quark quasi-PDF.

- Using 1-loop matching, described the steps to obtain the valence PDF of 300 MeV pion by boosting it to 1.29 and 1.72 GeV momenta on 0.06fm lattice using only quasi-PDF with z < 1fm. A good agreement with the JAM data was found. We also described how to use multiple "quasi-PDF lattice cross-sections" to check for internal consistency of the framework.

- On going: continuum limit is important => computation using a=0.04 fm lattice.