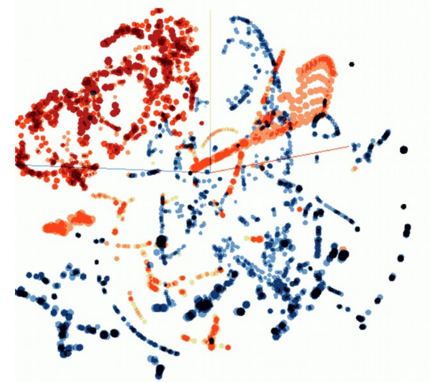
# mapping the coming PDF-Lattice synergy



T.J. Hobbs

…with Bo-Ting Wang, Pavel Nadolsky, and Fred Olness

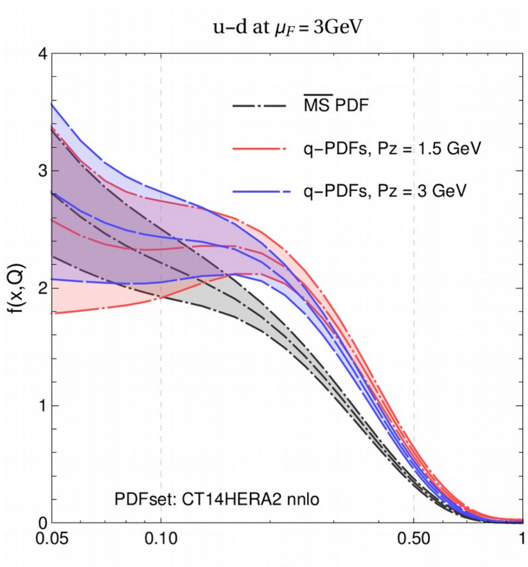
arXiv:1904.00022.

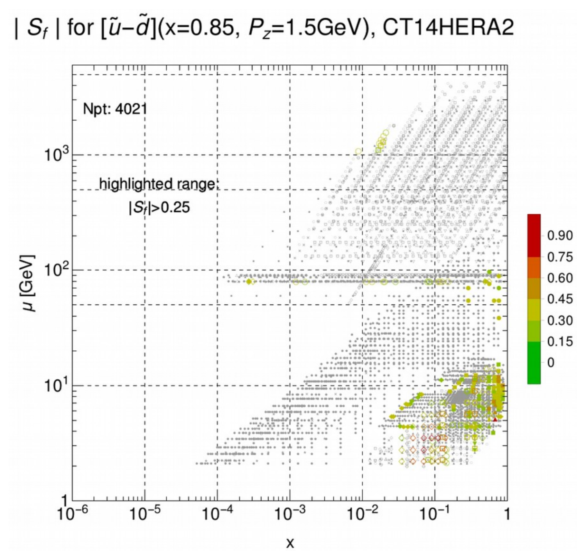
comprehensive repository of results: http://metapdf.hepforge.org/PDFSense/Lattice/

April 10th 2019

8th Meeting of the APS Topical Group on Hadronic Physics

Southern Methodist Univ.
and EIC Center@JLab

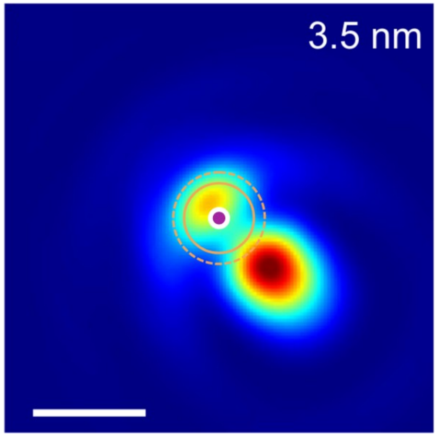



EIC Center at Jefferson Lab

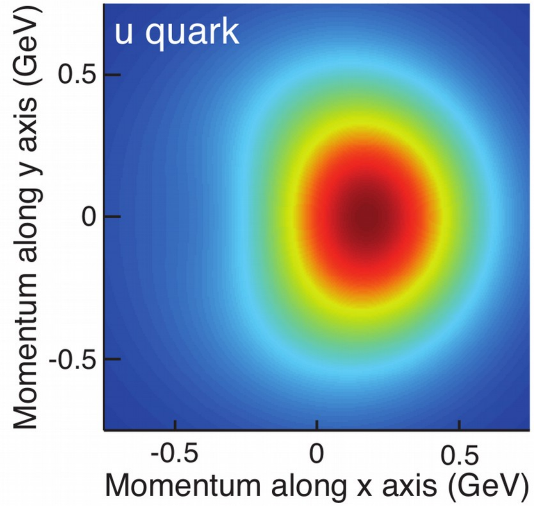
- hadronic physics is quickly becoming a **precision field**, with the prospect of detailed tomographic images of hadron structure a realizable goal

Jeong et al., PRB93, 165140



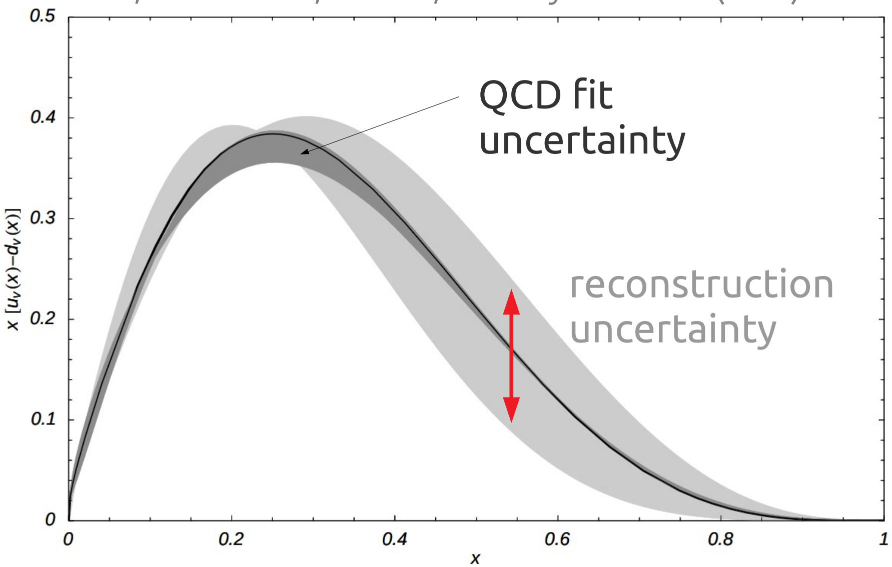

Sr STEM simulated image

Accardi et al., EPJA**52** (2016) no.9, 268.




- Lattice QCD will be crucial to achieving this era of precision hadronic physics, and has made great strides in recent years – this will extend to collinear observables

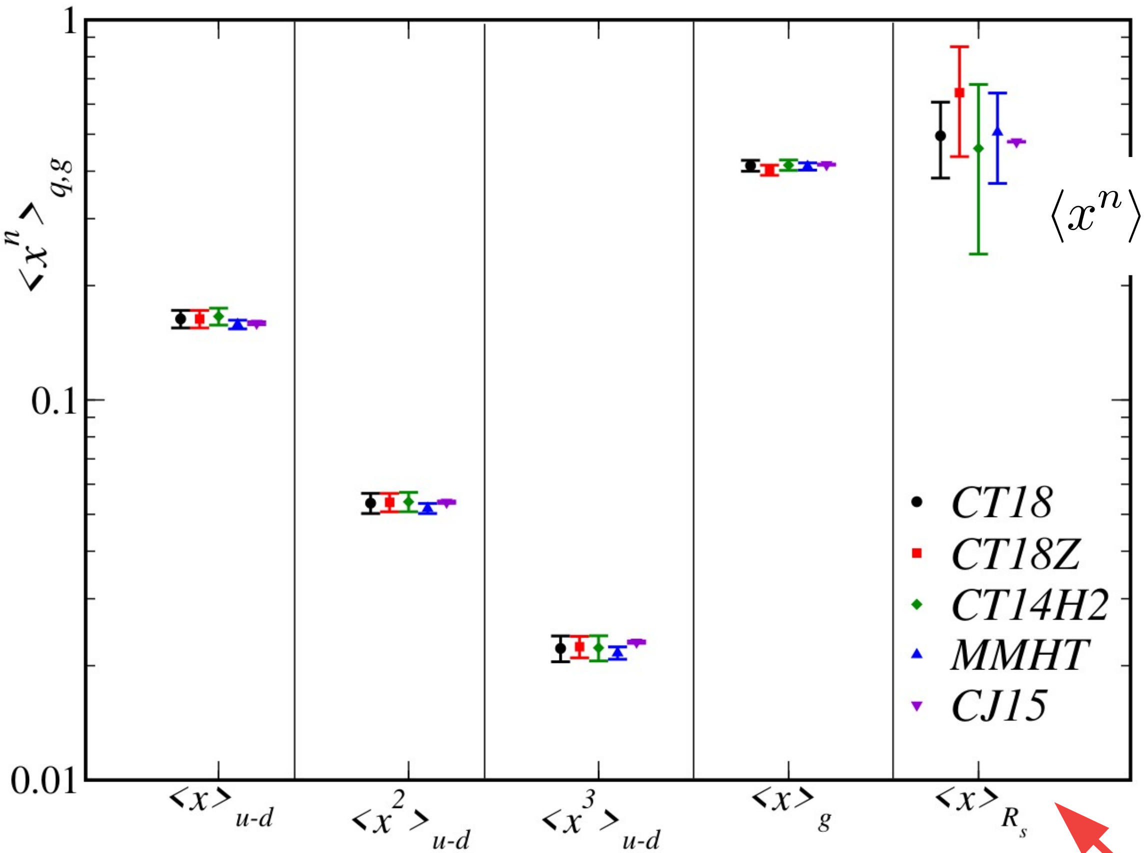
Detmold, Melnitchouk, Thomas, Mod.Phys.Lett. **A18** (2003) 2681.




- historically, there were efforts to reconstruct PDFs from their lattice-calculable moments (but resulting uncertainties were large!)
- various improvements have been made in the meantime
  - ➞ technical advances: more (and better) Mellin moment determinations
  - ➞ formal advances: quasi- and pseudo-PDFs

(...see talks by H.-W. Lin, C. Egerer.)

# PDF moments from CT18




$$\langle x^n \rangle_{q,g} = \int dx\, x^n f_{q,g}(x, \mu = 2\,\mathrm{GeV})$$

- progress in lattice QCD is compelling PDF phenomenologists to sharpen their benchmarks – especially for lower moments of light quarks

  → good agreement among phenom predictions of isovector, gluon moments!

→ constraints are significantly weaker for moments of the light quark sea distributions, *e.g.*, the strangeness suppression ratio, $\langle x \rangle_{R_s} \equiv \langle x \rangle_{s+\bar{s}} / \langle x \rangle_{\bar{u}+\bar{d}}$

**preliminary** CT results

| PDF moment | CT18 | CT18Z | CT14H2 |
|---|---|---|---|
| $\langle x \rangle_{u-d}$ | $0.163 \pm 0.009$ | $0.163 \pm 0.009$ | $0.166 \pm 0.008$ |
| $\langle x^2 \rangle_{u-d}$ | $0.054 \pm 0.003$ | $0.054 \pm 0.003$ | $0.054 \pm 0.003$ |
| $\langle x^3 \rangle_{u-d}$ | $0.022 \pm 0.002$ | $0.022 \pm 0.001$ | $0.022 \pm 0.002$ |
| $\langle x \rangle_g$ | $0.413 \pm 0.014$ | $0.402 \pm 0.012$ | $0.415 \pm 0.013$ |
| $\langle x \rangle_{R_s}$ | $0.496 \pm 0.112$ | $0.643 \pm 0.206$ | $0.459 \pm 0.217$ |

- lattice QCD calculations continue to improve and will be increasingly useful as inputs into QCD global analyses

PDF-Lattice whitepaper – Lin et al., PPNP100, 107 (2018); arXiv:1711.07916.

- the PDF-Lattice relationship will be *synergistic* :

→ PDF phenomenologists deliver improving benchmarks to challenge the Lattice

↕

→ Lattice calculations for PDF Mellin **moments** and **quasi-PDFs** can be theoretical priors for QCD global fits

PDFSense analysis – Hobbs, Wang, Nadolsky and Olness, arXiv:1904.00022.

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- moments from lattice can help unravel PDF flavor dependence, constrain phenom. PDFs:

**(i)**
$$\langle x^n \rangle_q = \int_0^1 dx\ x^n \left[q(x) + (-1)^{n+1}\, \overline{q}(x)\right] \ \rightarrow \langle x^{1,3,\dots}\rangle_{q^+},\ \langle x^{2,4,\dots}\rangle_{q^-}$$

$$\mu_F = \mu^{\mathrm{lat.}} = 2\,\mathrm{GeV}$$

- lattice can also now compute x-dependent quantities – the quasi-PDFs (qPDFs):

**(ii)**
$$\widetilde{q}(x, P_z, \widetilde{\mu}) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{ixP_z z} \langle P|\overline{\psi}(z)\gamma^z U(z,0)\psi(0)|P\rangle$$

# (i) PDF moments can be evaluated on the QCD lattice

- in lattice gauge theory, the accessible moments are C-odd/even combinations,

$$\langle x^n \rangle_q = \int_0^1 dx \; x^n \left[ q(x) + (-1)^{n+1} \, \overline{q}(x) \right]$$

$$\begin{cases} \langle x^n \rangle_{q^+} = \langle x^n \rangle_q \quad \text{for} \quad n = 2\ell - 1 \\ \langle x^n \rangle_{q^-} = \langle x^n \rangle_q \quad \text{for} \quad n = 2\ell \end{cases} \qquad \ell \in \mathbb{Z}^+$$

- the PDF moments are related to hadronic matrix elements of twist-2 operators:

PDF Mellin moments

$$\frac{1}{2} \sum_s \langle p, s | \mathcal{O}^q_{\{\mu_1, \cdots, \mu_{n+1}\}} | p, s \rangle = 2 v_q^{n+1} \left[ p_{\mu_1} \cdots p_{\mu_{n+1}} - \text{traces} \right]$$

$$\mathcal{O}^q_{\{\mu_1, \cdots, \mu_{n+1}\}} = \left( \frac{i}{2} \right)^n \bar{q}(x) \gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \ldots \overleftrightarrow{D}_{\mu_{n+1}} q(x)$$

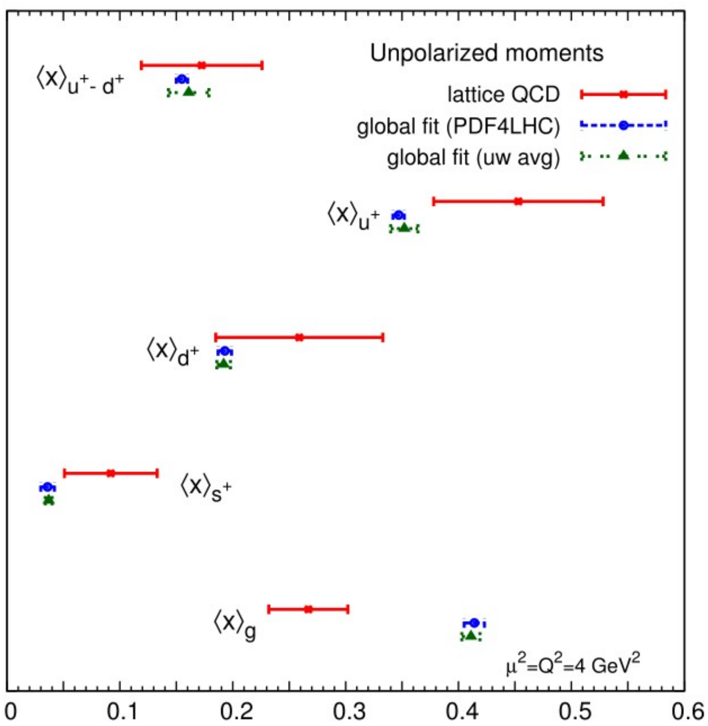
gauge-covariant derivatives

# the status of lattice QCD calculations

PDF-Lattice whitepaper – Lin et al., Prog. Part. Nucl. Phys. **100**, 107 (2018).

| Mom. | Collab. | Ref. | $N_f$ | Status | Disc [fm] | QM | FV | Ren | ES |
|---|---|---|---|---|---|---|---|---|---|
| $\langle x \rangle_{u^+-d^+}$ | ETMC 15 | [263] | 2+1+1 | P | 0.06, 0.08 | – | ■, ★ | ★, ★ | ■, ★ |
| | ETMC 15 | [263] | 2 | P | 0.06–0.09 | – | ○ | ★ | ■ |
| | RQCD 14 | [251] | 2 | P | 0.06–0.08 | – | ○ | ★ | ○ |

| Mom. | Collab. | Ref. | $N_f$ | Status | Disc | QM | FV | Ren | ES | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\langle x^2 \rangle_{u^--d^-}$ | LHPC and SESAM 02 | [279] | 2 | P | ■ | ■ | ■ | ○ | ■ | 0.145(69) |
| | QCDSF 05 | [93] | 0 | P | ■ | ■ | ■ | ★ | ■ | 0.083(17) |
| | LHPC and SESAM 02 | [279] | 0 | P | ■ | ■ | ■ | ○ | ■ | 0.090(68) |




- depending upon flavor and order, lattice extractions of Mellin moments have varying status (above, FLAG evaluations)
    - → e.g., the first isovector moment has been computed by numerous groups
    - → but the second, by relatively few
- systematic lattice effects are similarly widely varied

however, improvements are being made rapidly!

# toward a PDF-lattice working relationship

- what are the prospects for actually building a lattice-PDF synergy --- i.e., what must be done (aside from lattice improvements in the [unpolarized] moments)?

> *"Although the studies presented here are still in an initial exploratory phase, they provide strong motivation for global fitters to begin consider incorporating lattice-QCD constraints into their global analyses."*
>
> – Prog. Part. Nucl. Phys. **100**, 107 (2018).

→ PDF phenomenologists must understand which lattice output would be most beneficial – and where the greatest impact would be felt

**for this, a detailed accounting of how phenomenological knowledge of lattice-calculable moments is derived will be essential**

The problem with a seesaw is you're always off balance.

→ this understanding must be communicated to lattice practitioners, who must continue building a common framework for assessing lattice systematics/artifacts

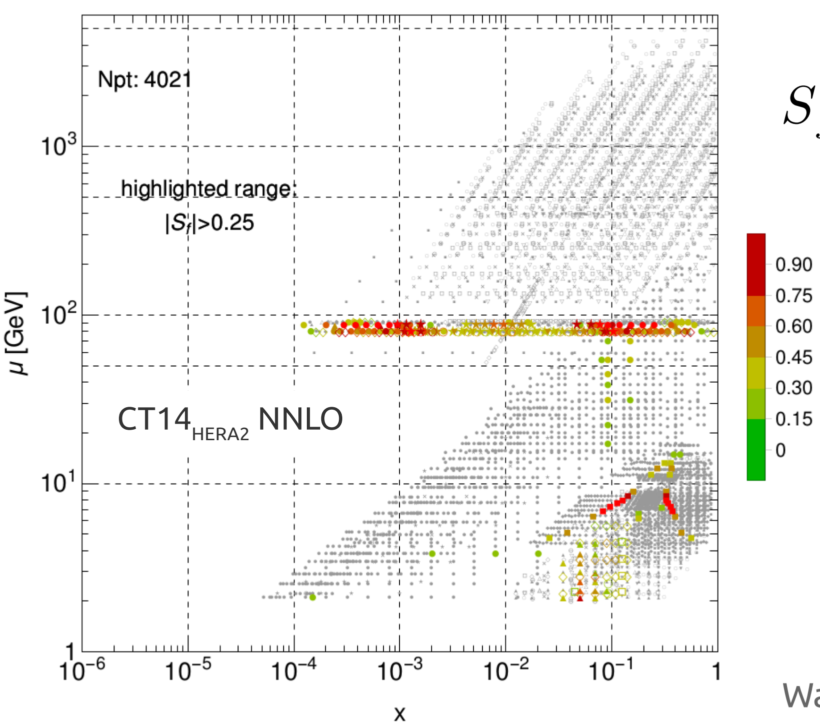
# assessing empirical constraints with PDFSense

- a QCD analysis produces an **ensemble of error PDFs** over which we may evaluate the fit quality (residual), and quantities derived from the PDF (e.g., the moments),

$$q^{j\in\{2N\}}(x,\mu^{\rm lat}) \;\longrightarrow\; \langle x^n\rangle^{j\in\{2N\}}_{q^\pm,\,\mu^{\rm lat}} = \int_0^1 dx\, x^n \left(q(x,\mu^{\rm lat}) \pm \overline{q}(x,\mu^{\rm lat})\right)_{j\in\{2N\}}$$

- define a generalized correlation – the sensitivity – as a statistical metric for the impact of the i[th] datum to a PDF or PDF-derived quantity:

| $S_f$ | for $\langle 1\rangle_{u^+-d^+}$




functional of the PDFs, $\langle x^n\rangle_{q^\pm,\mu^{\rm lat}}$

$$S_f = \frac{\Delta r_i}{\langle r_0\rangle_E}\,{\rm Corr}[\mathcal{F}\{q\}, r_i(x_i,\mu_i)]$$

the residual, $r_i = \frac{1}{s_i}\left(T_i - D_i^{sh}\right)$

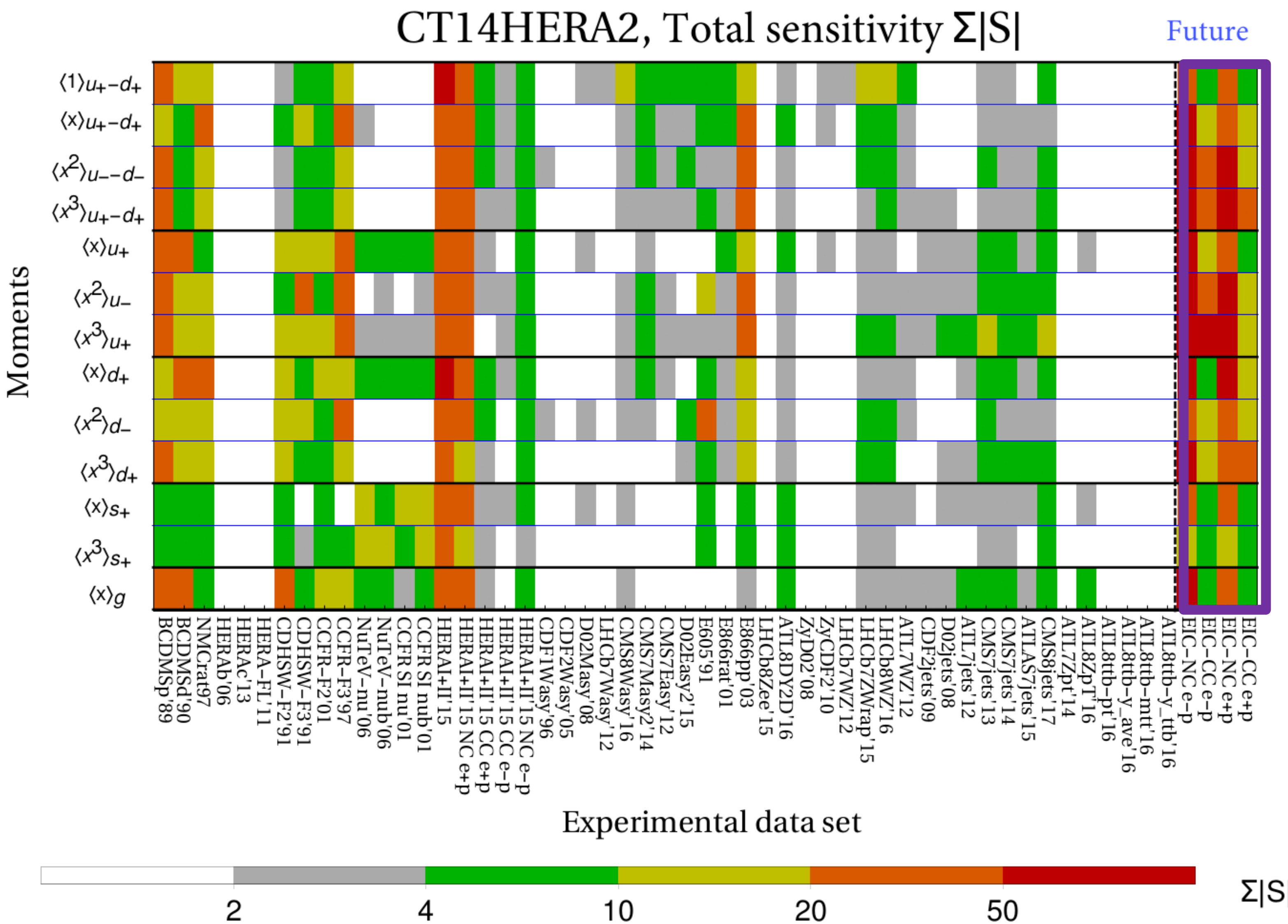
developed to quickly identify high-impact data in lieu of a full global analysis

→ allows kinematic **mapping** of PDF constraints

→ avoids various ambiguities involved in fitting

Wang, TJH, Doyle, Gao, Hou, Nadolsky, Olness, PRD**98**, 094030 (2018).

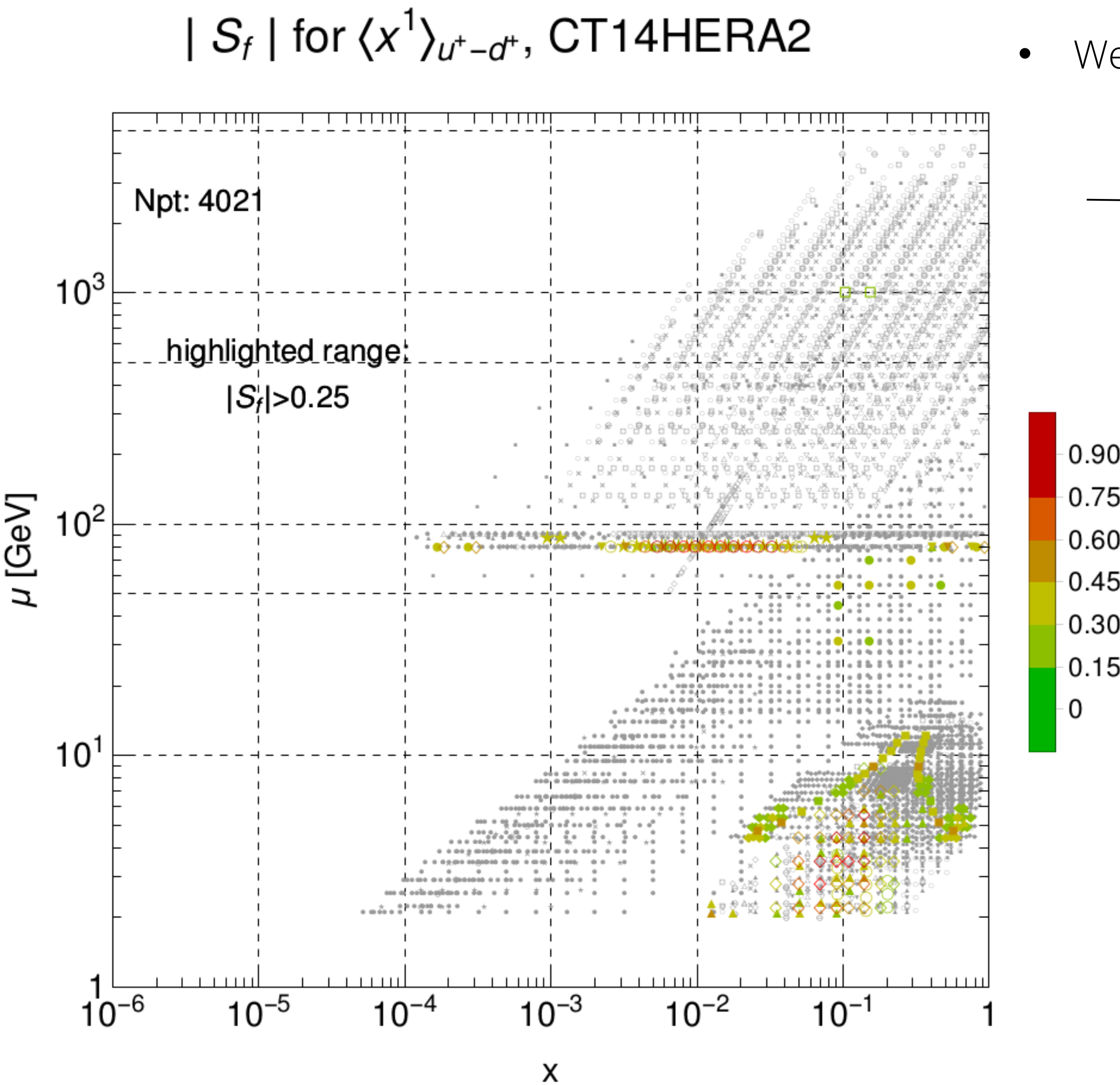
# sensitivity to Mellin moments




we show Mellin moments computable on the lattice at the scale $\mu_F = 2\,\mathrm{GeV}$

HERA, BCDMS, NMC, E866 DY pair production are most sensitive to the moments

# sensitivity maps: isovector moments




$$\overline{x}_{|S_f|} = \sum_i x_i |S_f^i| \Big/ \sum_i |S_f^i|$$

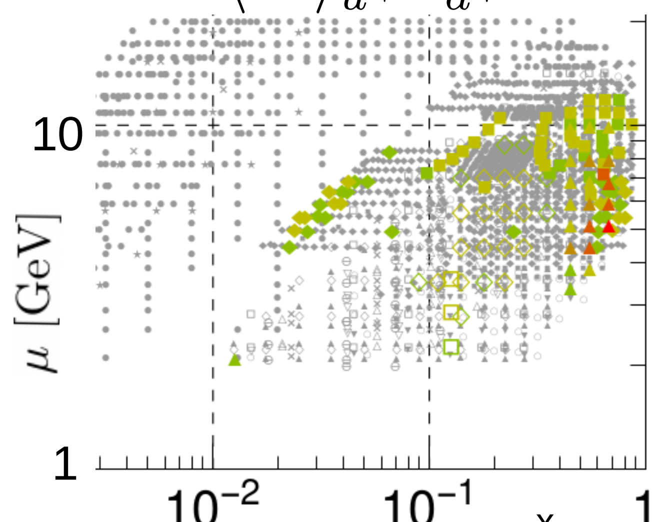
(grows with moment order, n)

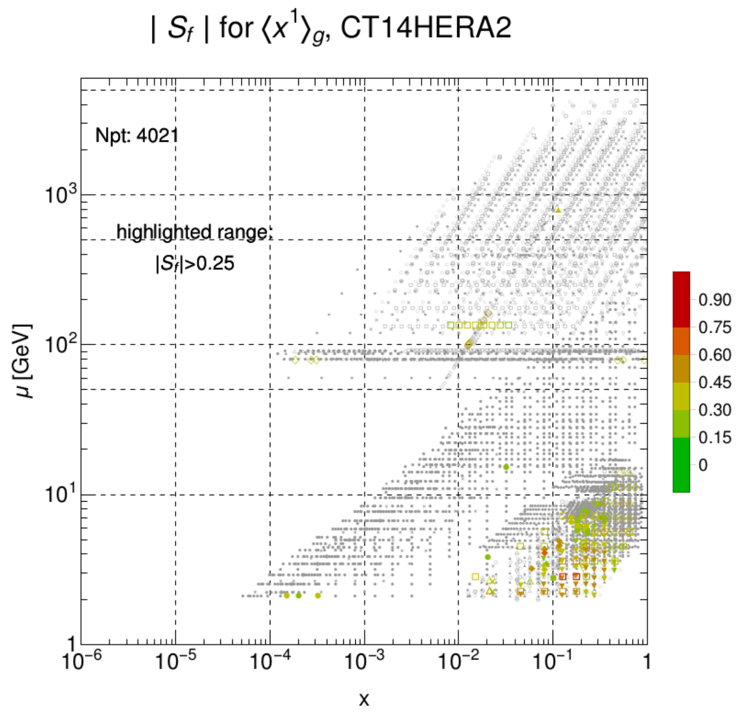
- We focus on **isovector** (u-d) PDF combinations

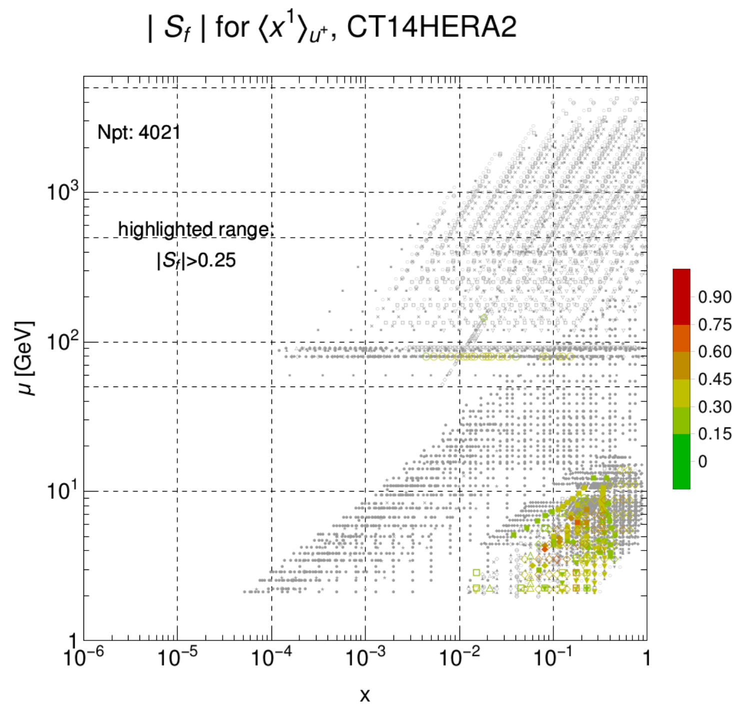
→ on the lattice, these are more readily computed since flavor non-singlet combinations do not receive disconnected insertions

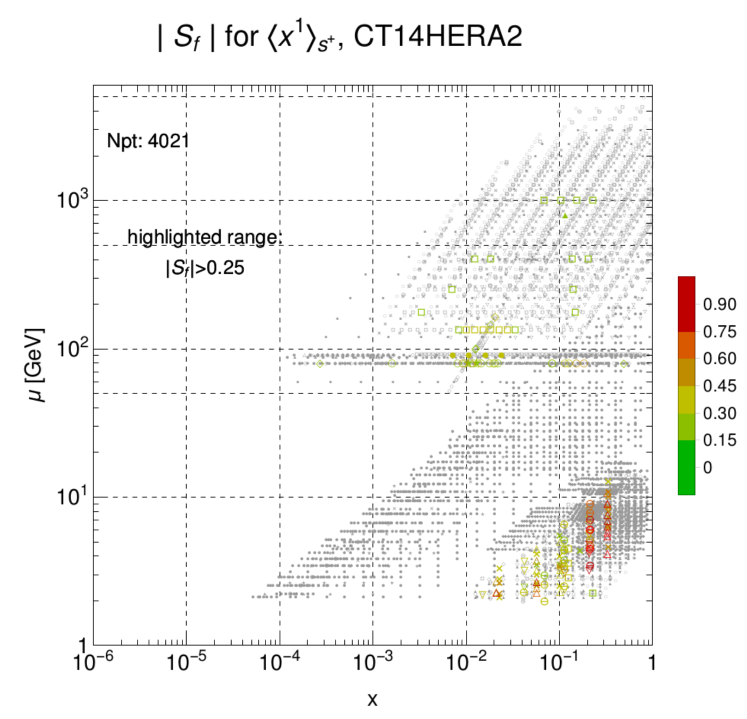
- Moments of higher order are constrained by higher $x_i$ fixed-target data:

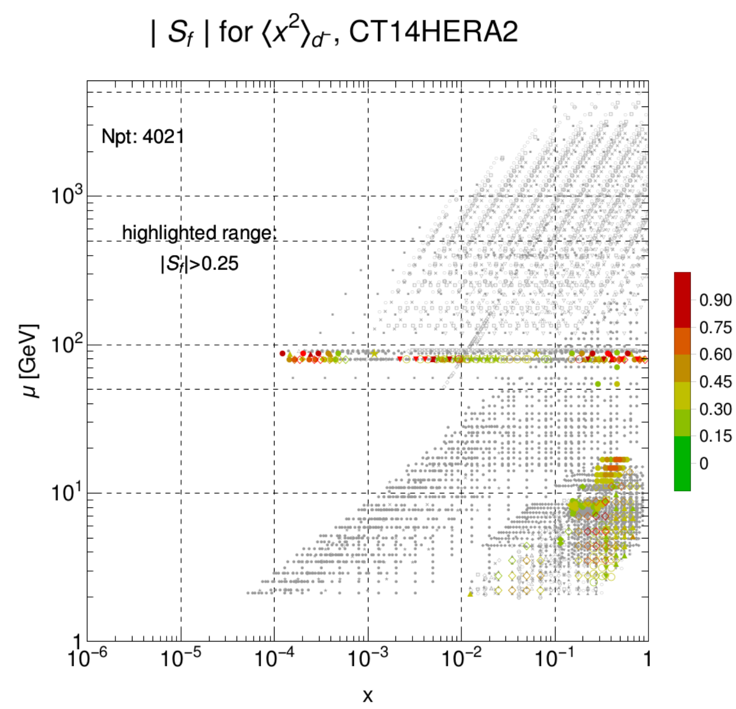
$\langle x^3 \rangle_{u^+-d^+}$






→ the pulls for most PDF moments are dominated by small clusters of experiments, with a roughly power-law falloff in their impact when ranked in descending order

→ data from fixed-target DIS and Drell-Yan [often on nuclear targets] are crucial!

# (ii) quasi-PDFs allow access to PDFs' x dependence

- still, resolving the $P_z$ dependence of qPDFs remains an important theoretical issue

Ji, PRL**110**, 262002 (2013).

$\sim \langle P | \overline{\psi} \gamma^{z,t} \psi | P \rangle$ — matching kernel (pQCD) — higher-twist corrections

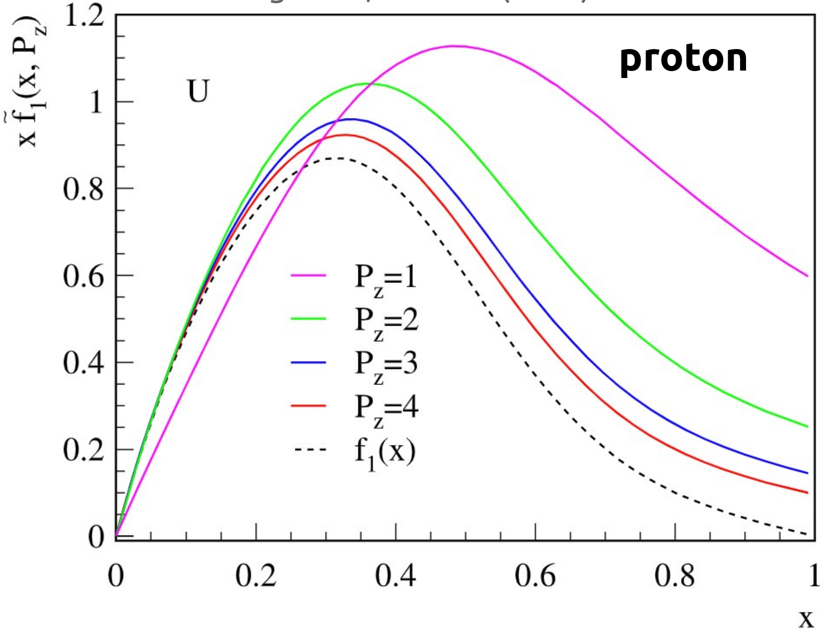
$$\widetilde{q}(x, P_z, \widetilde{\mu}) = \int dy\, Z\left(\frac{x}{y}, \frac{\Lambda}{P_z}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M^2}{P_z^2}\right)$$

$\sim \langle P | \overline{\psi} \gamma^{+} \psi | P \rangle$

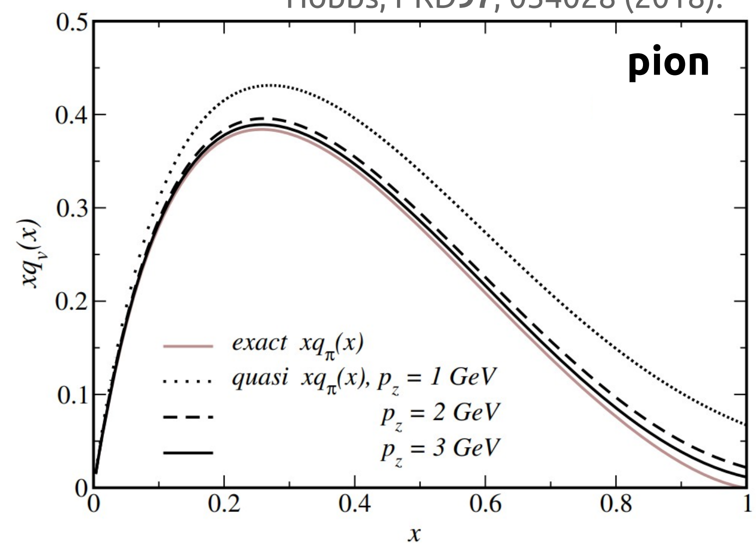
- ultimately, the x- and $P_z$ dependence of the qPDFs are informative of **hadronic wave functions**

→ this can be demonstrated with simplified models for the nucleon…

Gamberg et al., PLB**743** (2015) 112.

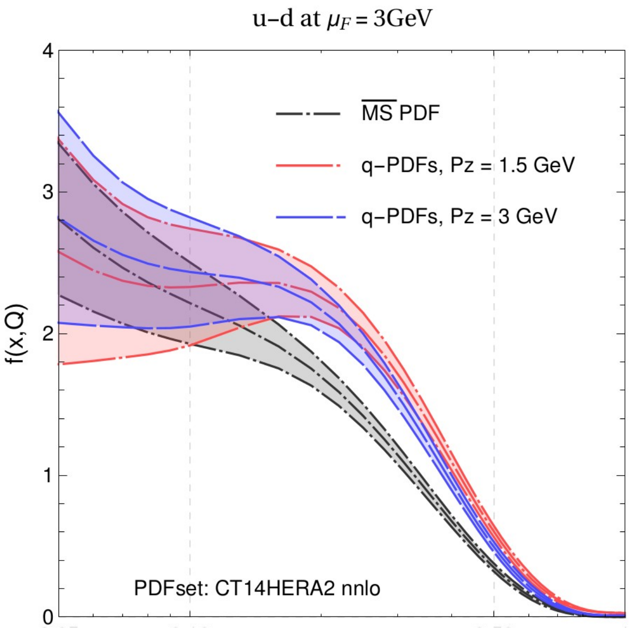


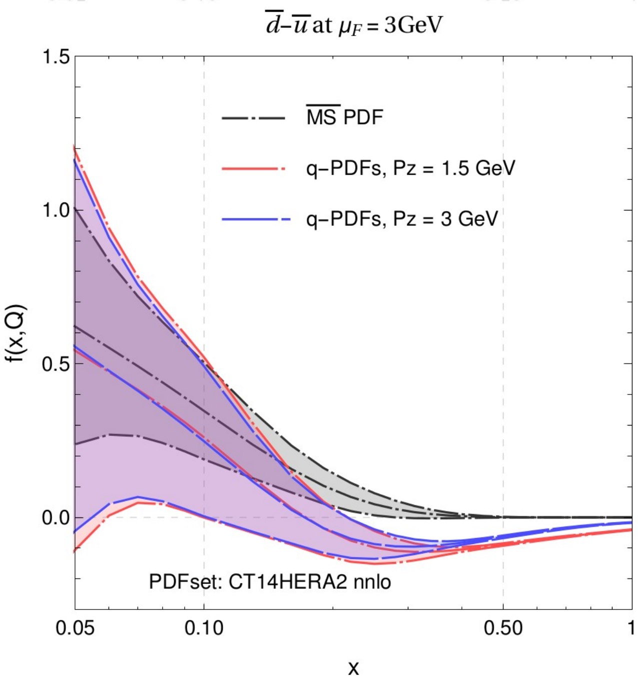

Hobbs, PRD**97**, 054028 (2018).




… and similar considerations hold for the pion.

# total sensitivity to matched quasi-PDFs

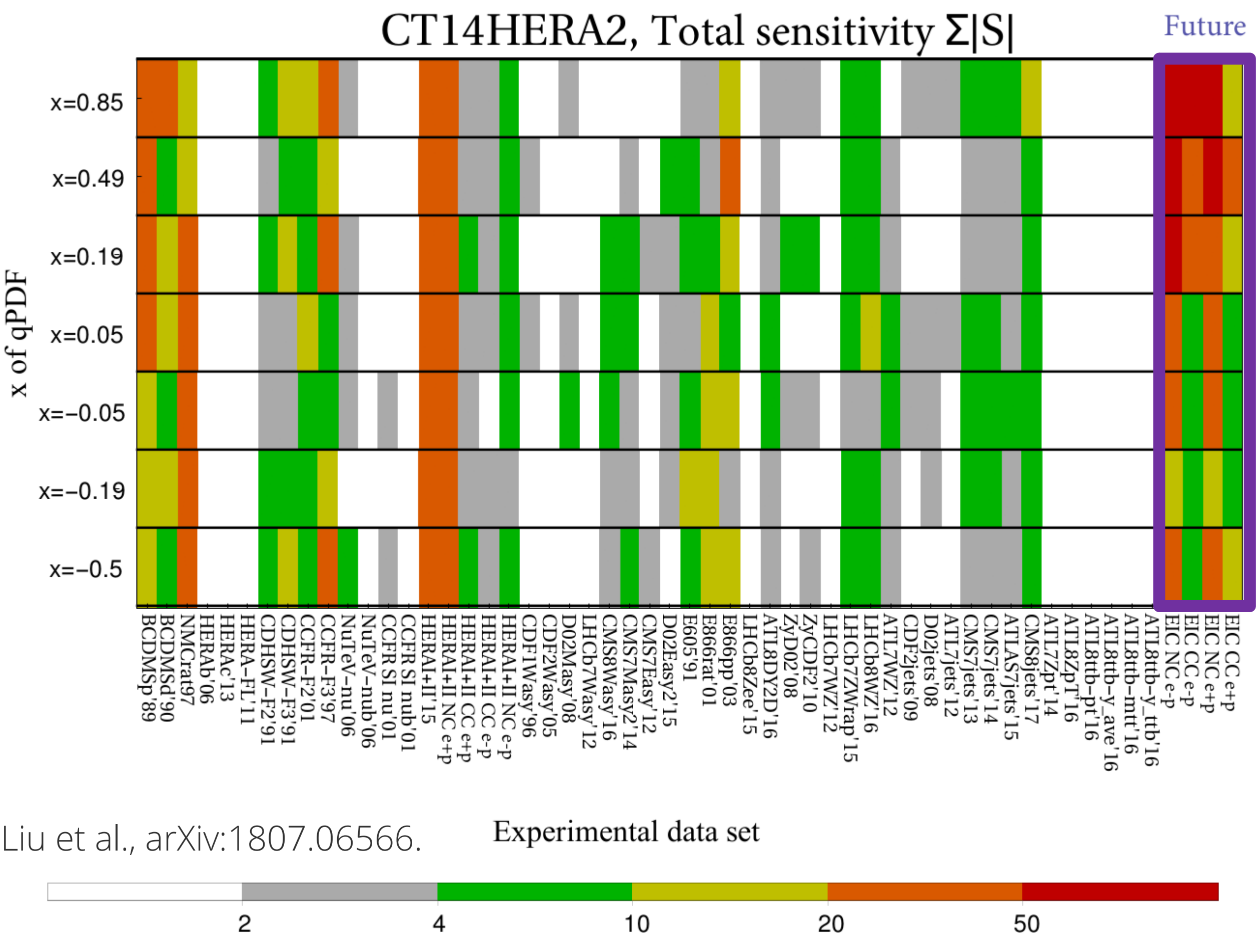



$$\bar{q}(x) = -q(-x) \;\rightarrow$$

$$q(x) \equiv \begin{cases} u(x) - d(x), & x > 0 \\ \bar{d}(|x|) - \bar{u}(|x|), & x < 0 \end{cases}$$

$$P_z = 1.5\,\text{GeV};\;\; \mu_F = 3\,\text{GeV}$$




Liu et al., arXiv:1807.06566.

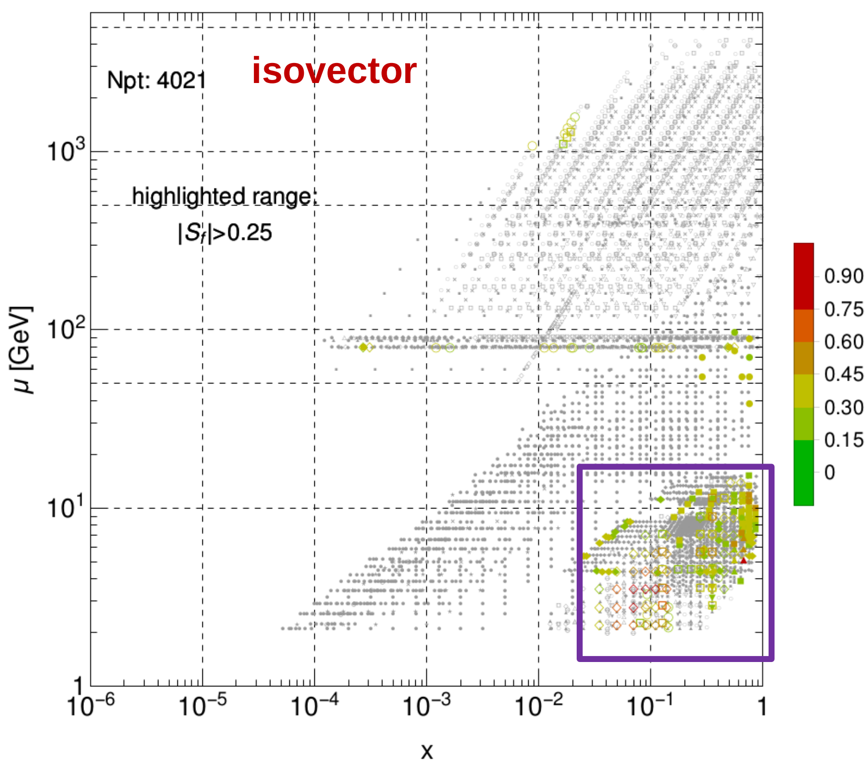
- Recovering PDFs from qPDFs requires the inversion of still-developing *matching relations*,

$$\widetilde{q}(x, P_z, \widetilde{\mu}) = \int dy\, Z\left(\frac{x}{y}, \frac{\Lambda}{P_z}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2}, \frac{M^2}{P_z^2}\right)$$

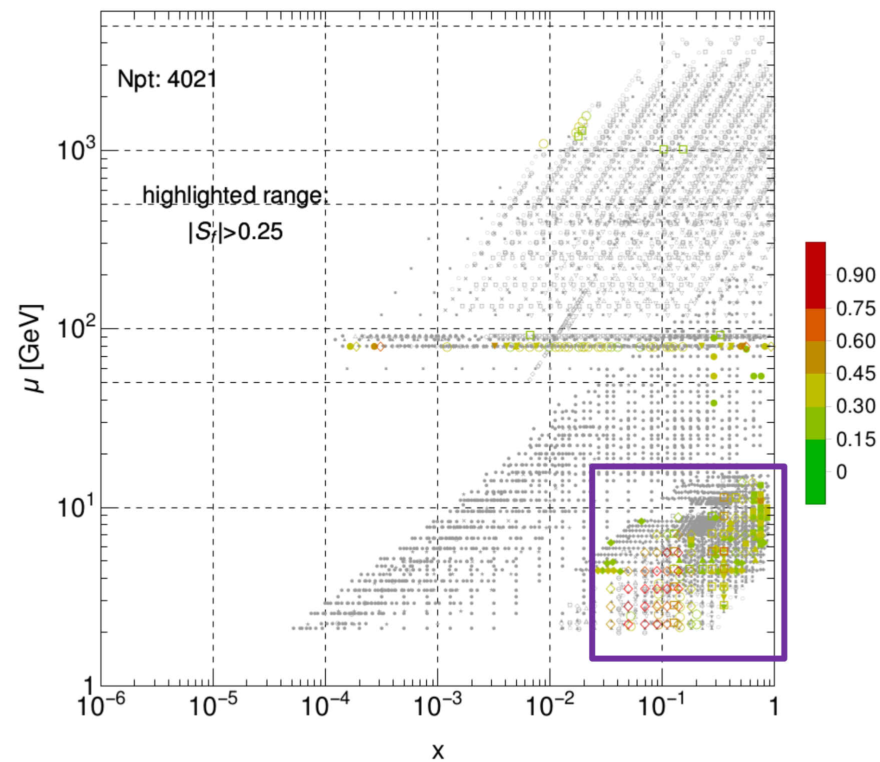
↓ qPDF ↓ ordinary PDF

- The matching formalism depends crucially on the nucleon boost, $P_z$; fixed-target DIS data at high $x_i$ are mildly sensitive to this $P_z$ dependence, and can aid theory developments in qPDFs:

| $S_f$ | for $[\tilde{u}-\tilde{d}]$(x=0.85, $P_z$=1.5GeV), CT14HERA2



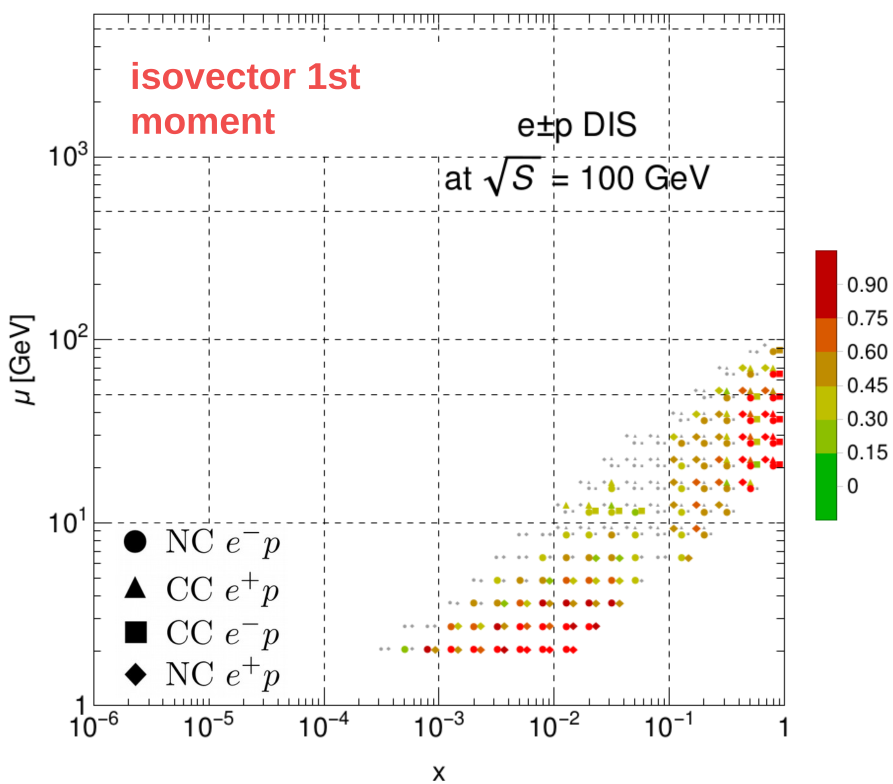

| $S_f$ | for $[\tilde{u}-\tilde{d}]$(x=0.85, $P_z$=3GeV), CT14HERA2



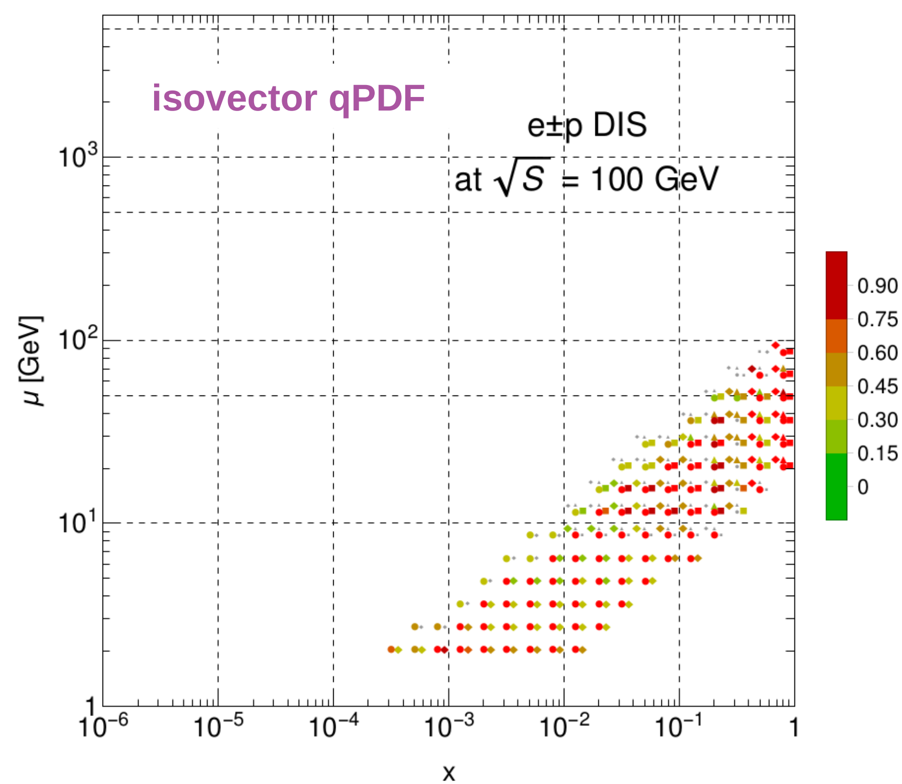
# An EIC would drive lattice phenomenology

- A high-luminosity lepton-hadron collider will impose very tight constraints on many lattice observables; below, the isovector first moment and qPDF
- Many of the experiments most sensitive to PDF Mellin moments and qPDFs involve nuclear targets → eA data from EIC would sharpen knowledge of nuclear corrections

$|S_f|$ for $\langle x^1 \rangle_{u^+-d^+}$, CT14HERA2




$|S_f|$ for $[\tilde{u}-\tilde{d}]$(x=0.85, $P_z$=1.5GeV), CT14HERA2




→ "EIC impact on tomography and HEP phenom.," Session **C09**: Sat., April 13th – 1:30pm

# **conclusions** – and next steps

- implementation of lattice results as theoretical priors into PDF analyses will benefit from inclusion of moments of various order, which more widely constrain x dependence

- better understanding of **nuclear-medium** effects will likely be necessary to improve phenom. knowledge of many flavor-separated moments

- modern data in typical NNLO global fits are already sufficiently precise to exhibit $P_z$ dependence in their sensitivity to **matched qPDFs**; improved data (esp. from EIC!) could help further unravel issues in qPDF theory

- many of the observations made here generalize to other nonperturbative functions – e.g., the GPDs, TMDs

→ extensions to similar analyses for these objects are possible!

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thanks!