3-body quantization condition in a minimal unitary relativistic approach

— Maxim Mai —

Eur.Phys.J. A53 (2017)
Eur.Phys.J. A53 (2017)
Phys.Rev. D97 (2018)
Phys.Rev.Lett. 122 (2019)

3-BODY DYNAMICS AND QCD PUZZLES

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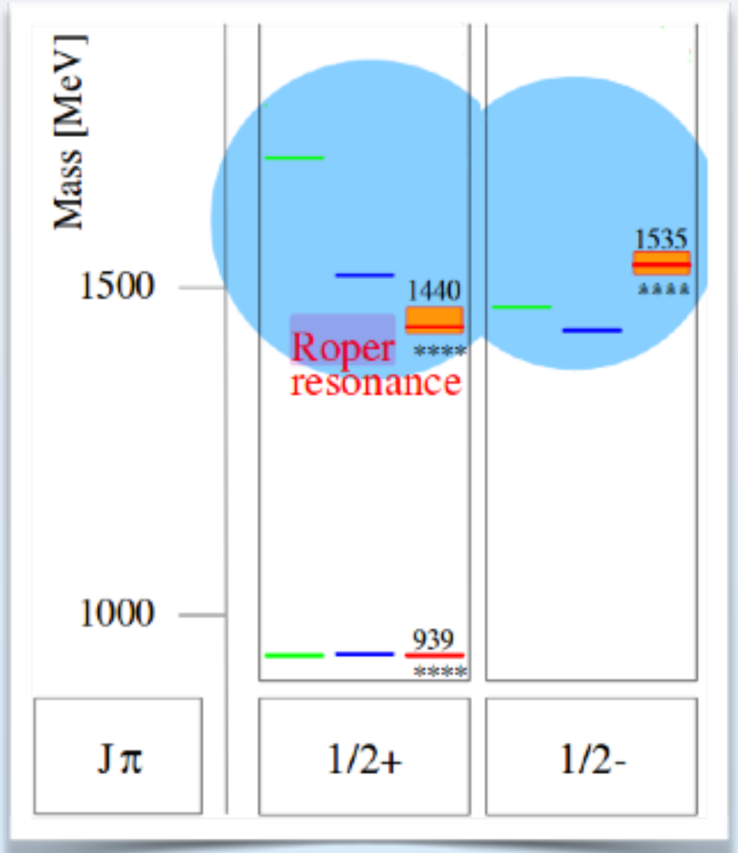

Loring et al. EPJA10 (2001)

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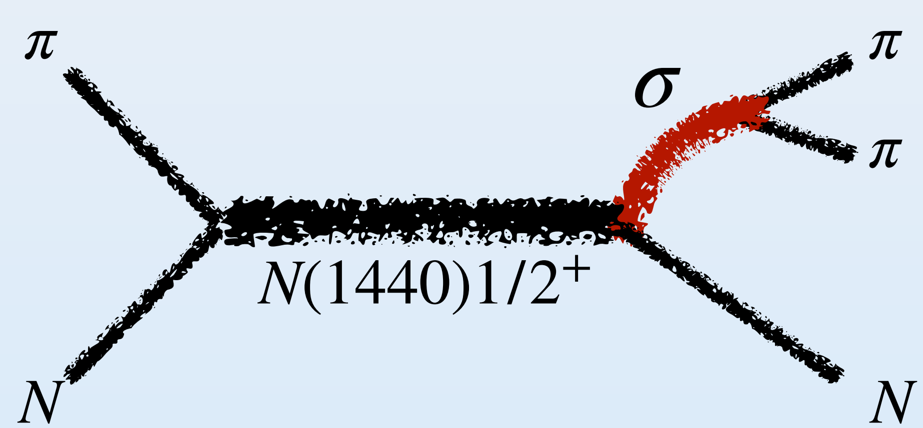

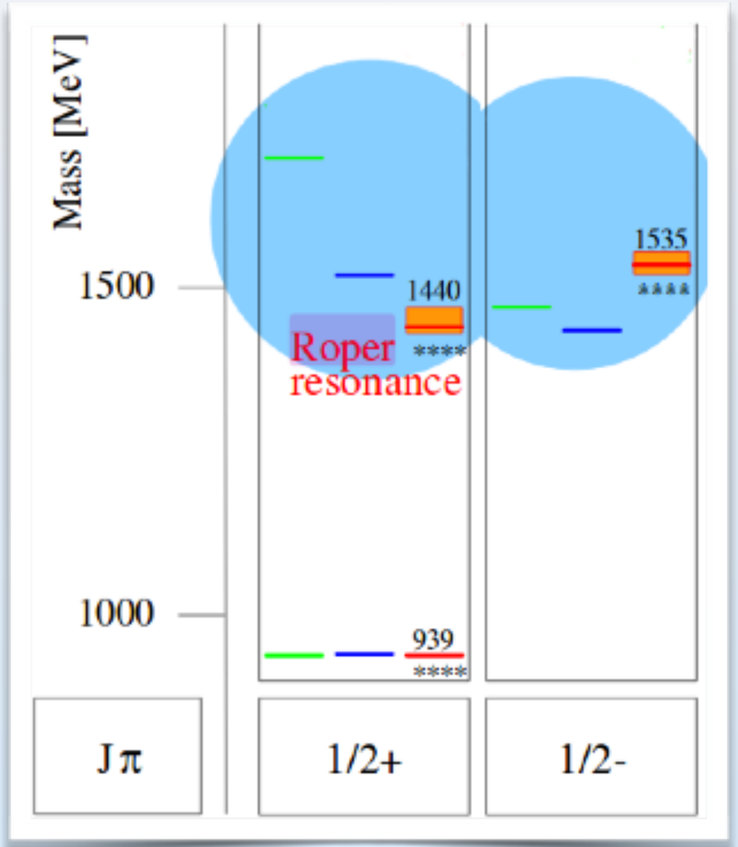

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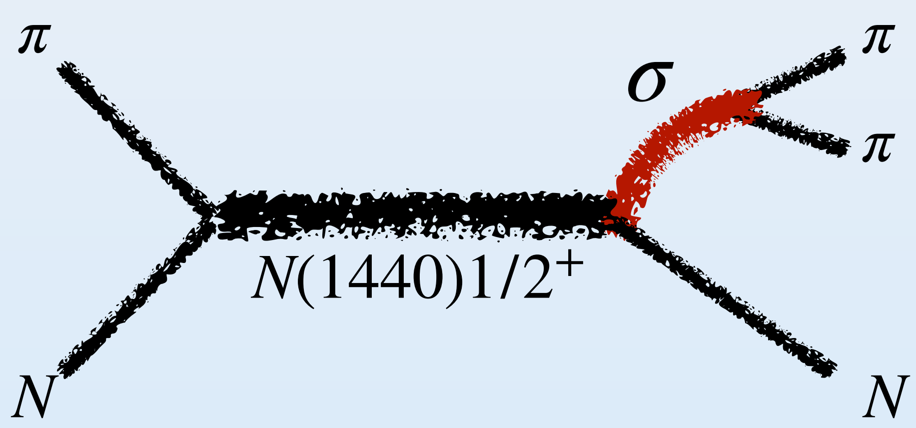

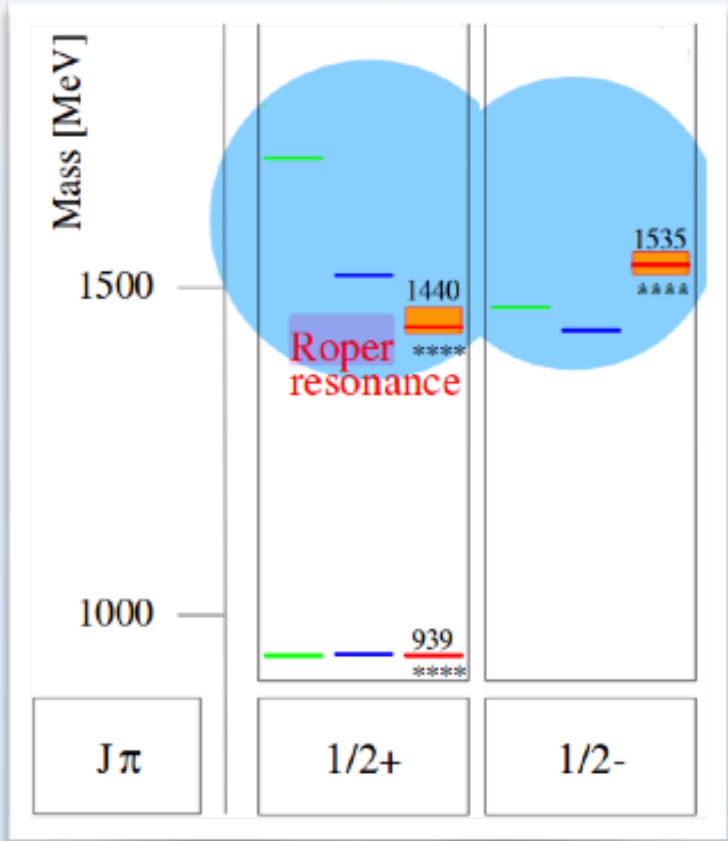

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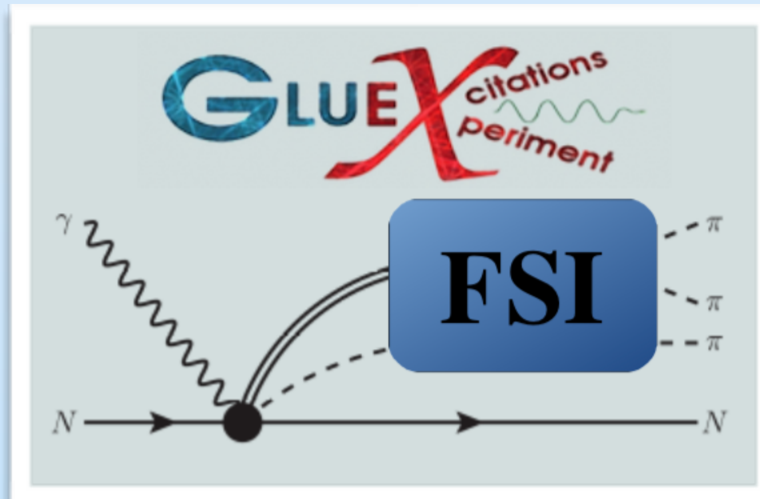

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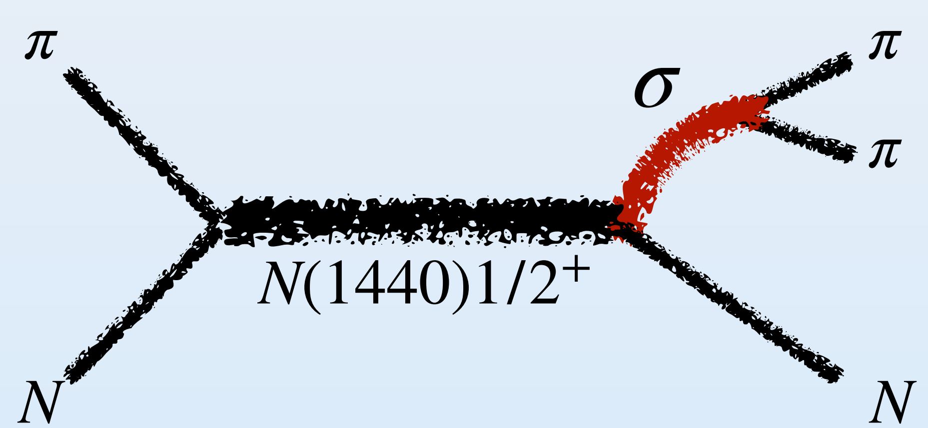

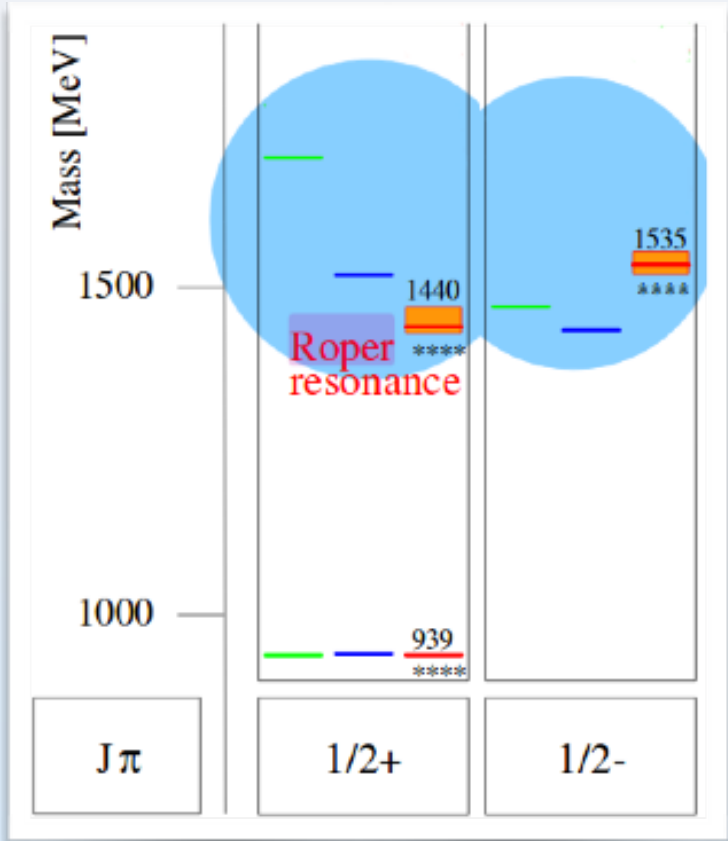

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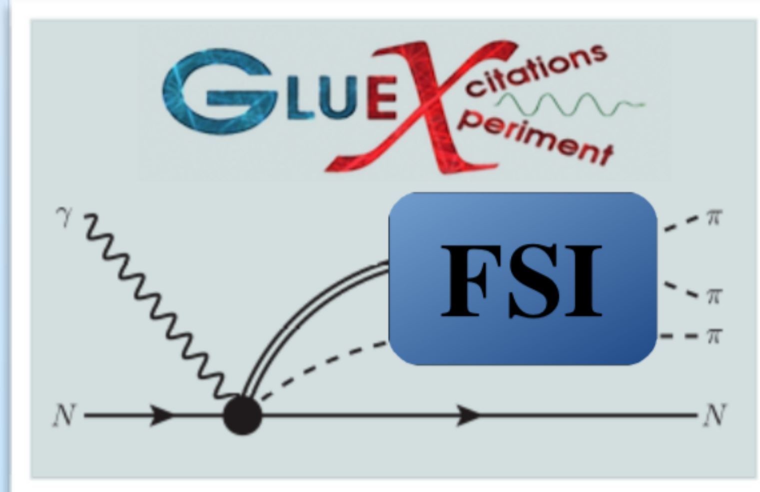

- ***X(3872)***
  - *decays dominantly into* $\boldsymbol{D\bar{D}\pi}$

- **...**

☞ Talk by J. Messchendorp

AB-INITIO APPROACH TO QCD: LATTICE QCD

- *first Lattice results are available*

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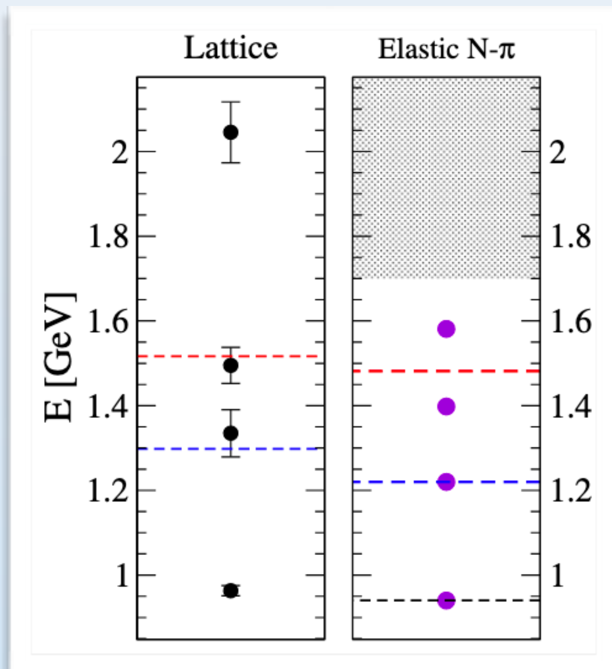

Lang et al. PRD 95(2017)

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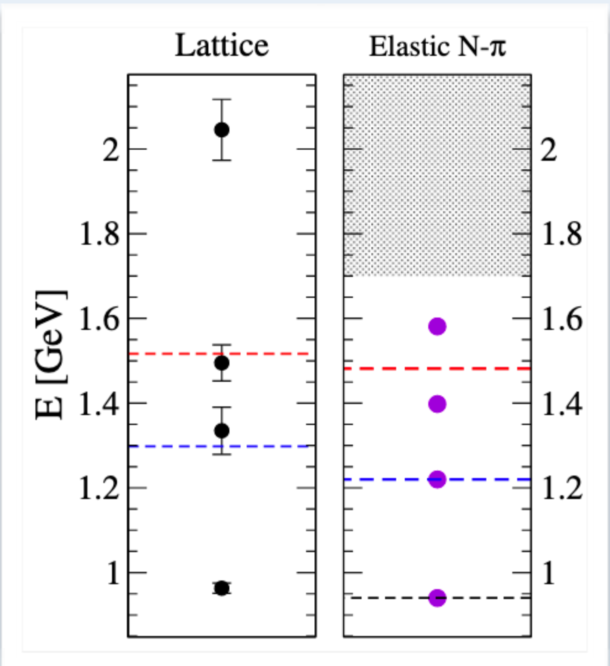

Lang et al. PRD 95(2017)

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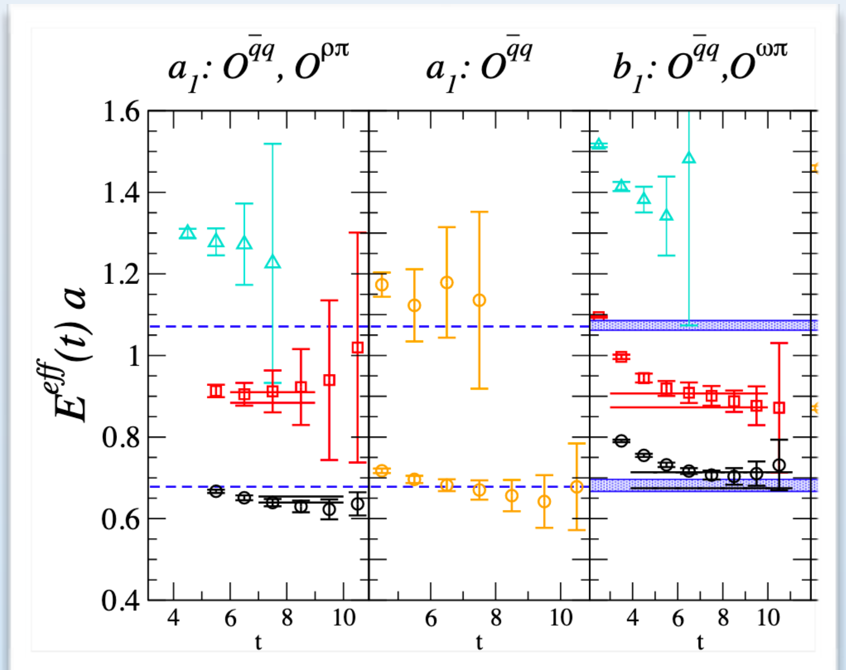

Lang et al. JHEP 1404

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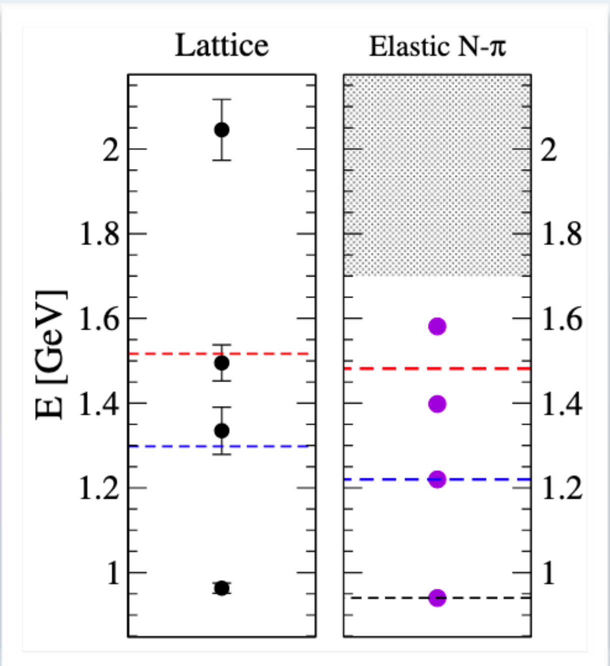

Lang et al. PRD 95(2017)

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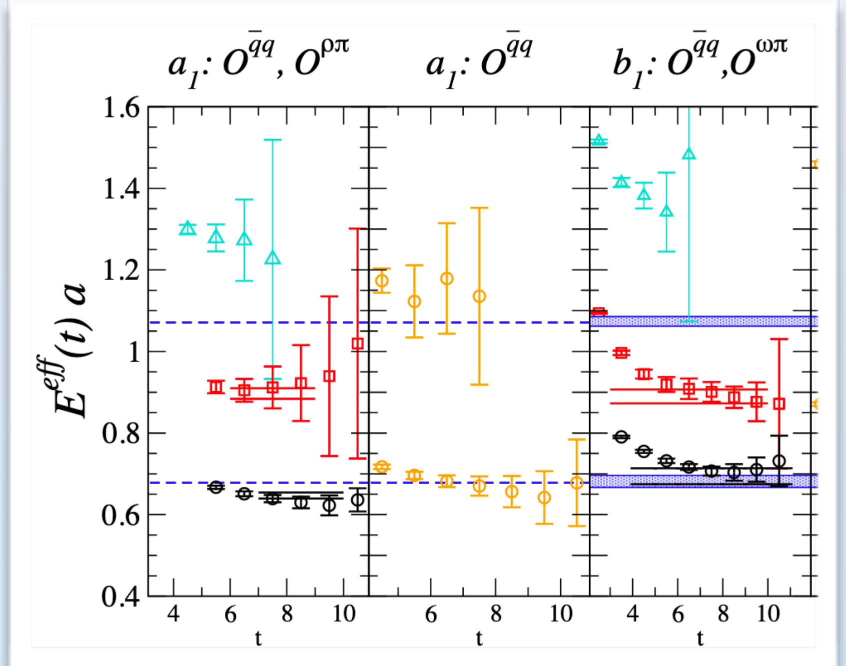

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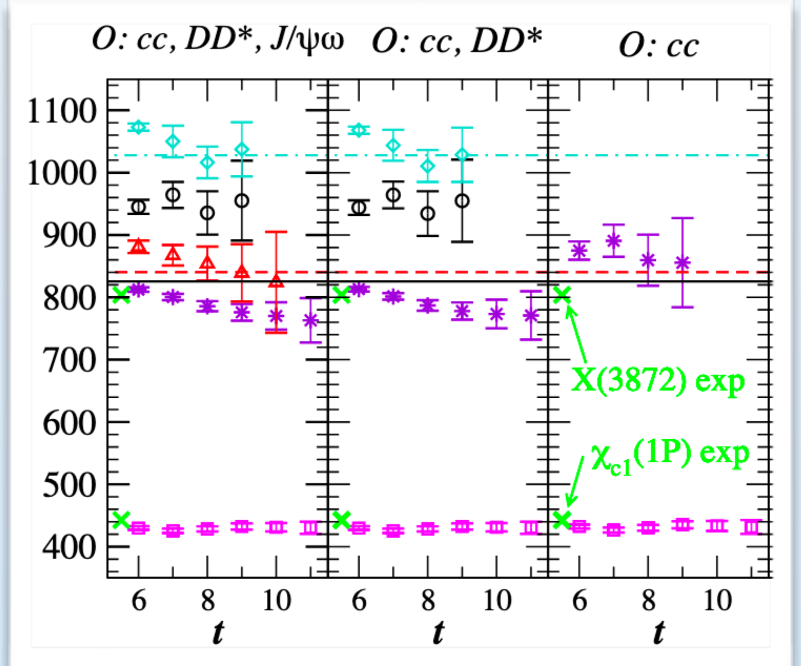

Prelovsek, Leskovec PRL111 (2013)

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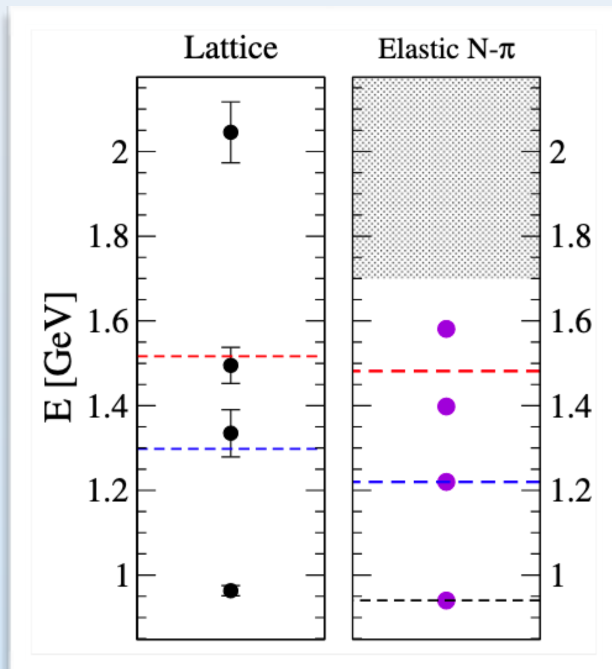

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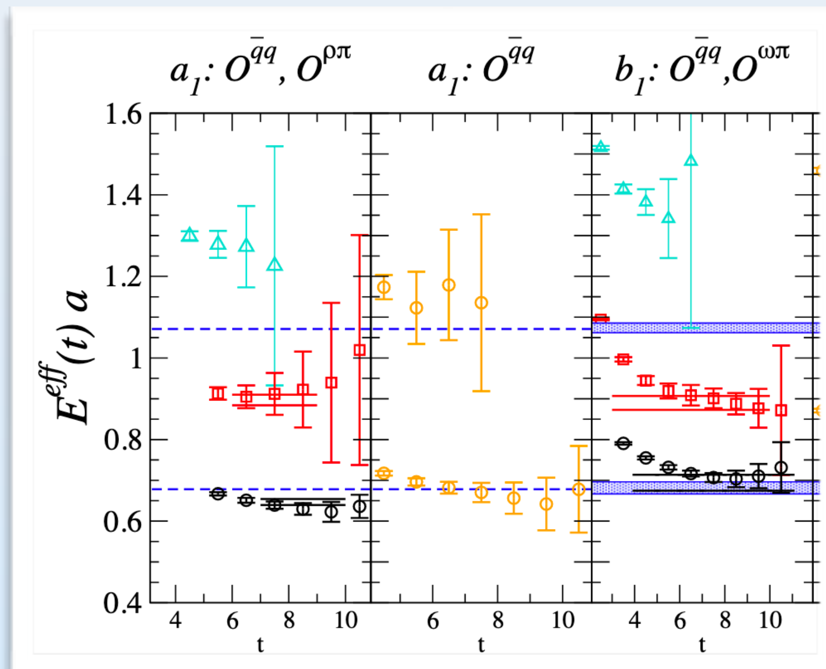

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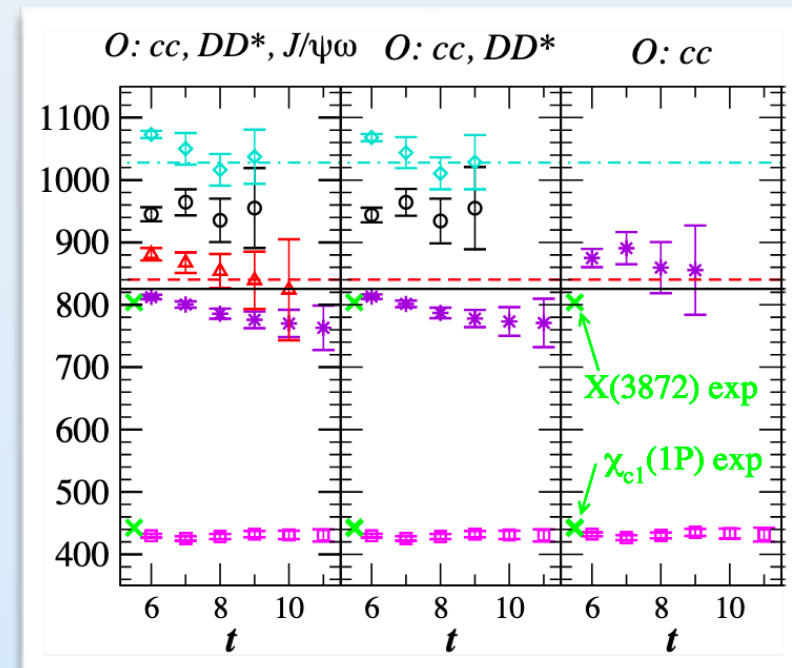

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◉ ***I=2 πρ*** Woss et al. JHEP 1807 [Hadspec]

◉ **...** more to come

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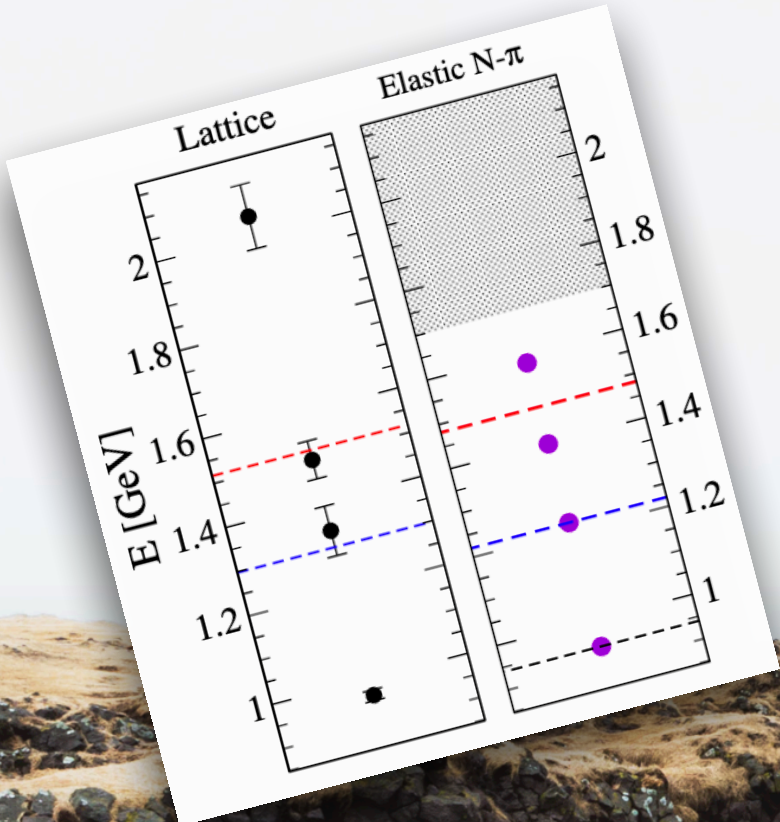

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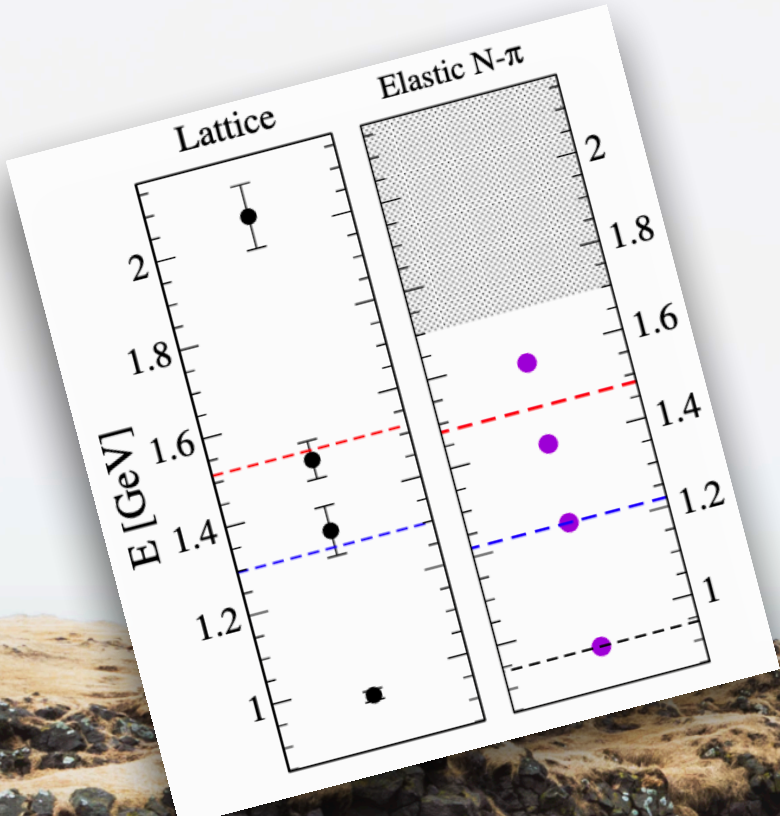

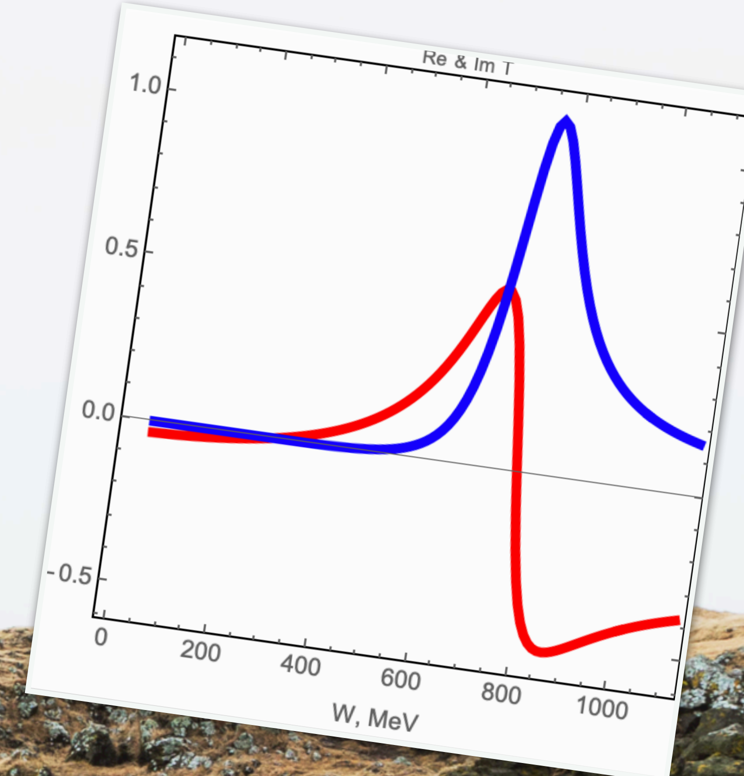

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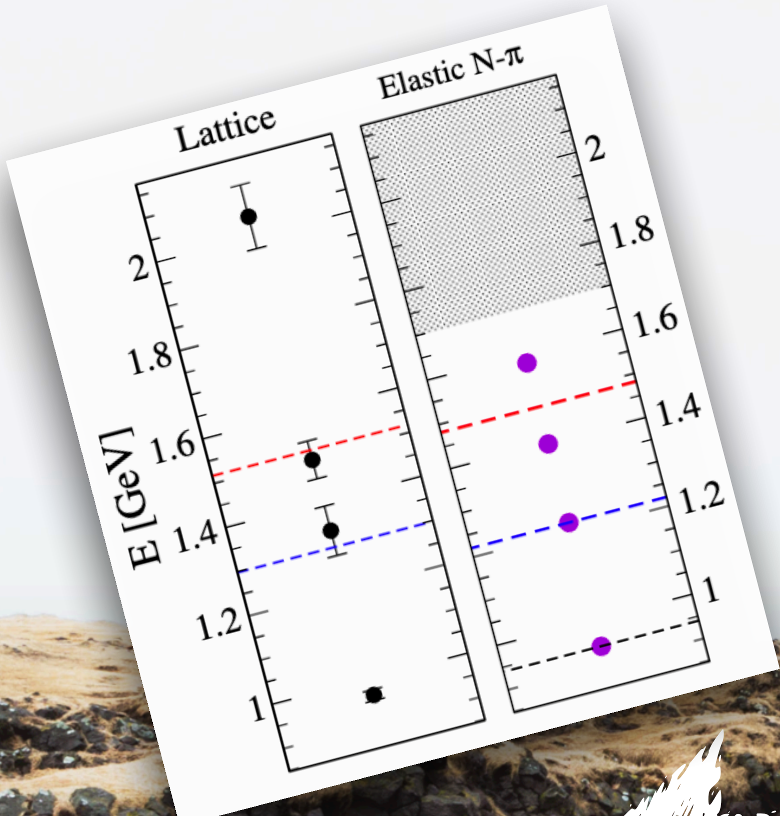

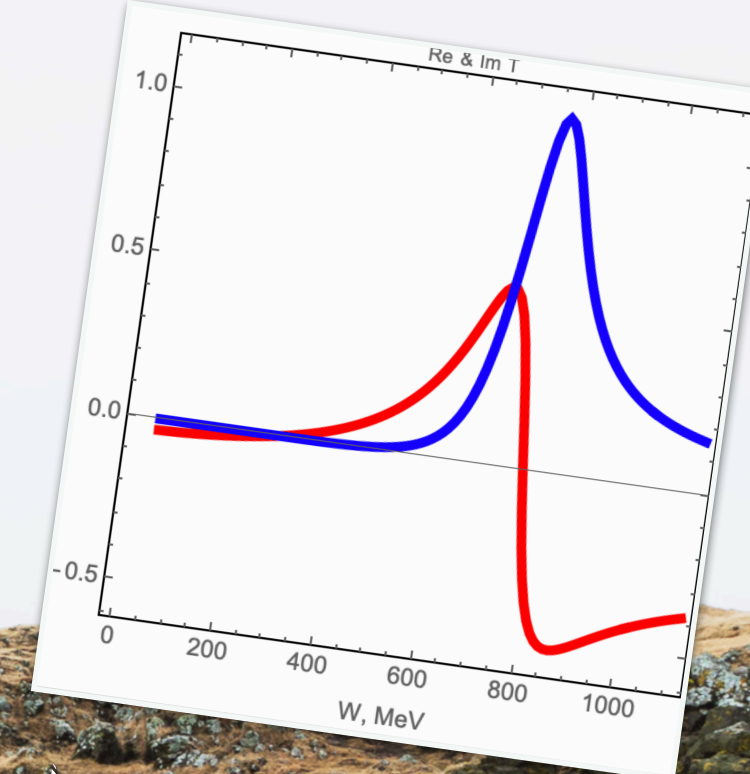

THIS TALK:

QUANTIZATION CONDITION FOR 3-BODY SYSTEMS

STATE OF THE ART

- **2-body case — Lüscher's method**
  - *one-to-one mapping*
  - *Various extensions: multi-channels, spin, ...*

Gottlieb, Rummukainen, Feng, Meißner, Li, Liu, Doring, Briceno, Rusetsky, Bernard...

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- **3-body case**
  - *presumably no one-to-one mapping:*

    complex kinematics (8 variables)

    sub-channel dynamics
  - *theoretical developments and pilot numerical investigation*

Sharpe, Hansen, Briceno, Hammer, Rusetsky, Polejaeva, Griesshammer, Davoudi, Guo...

MM, Doring(2017)

Pang, Hammer, Rusetsky, Wu(2017)

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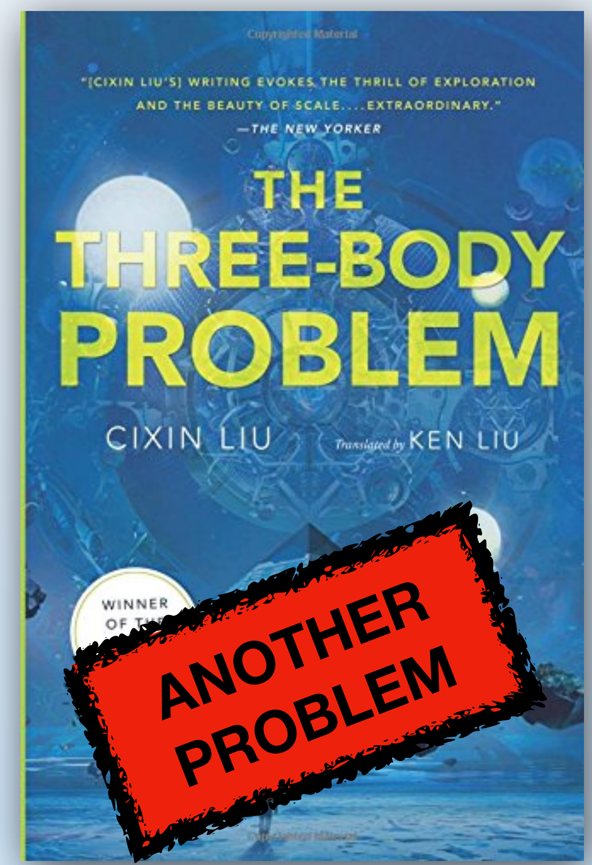
Blanton,Romero-López, Sharpe (2018)

Pang, Wu, Hammer, Meißner, Rusetsky (2019)



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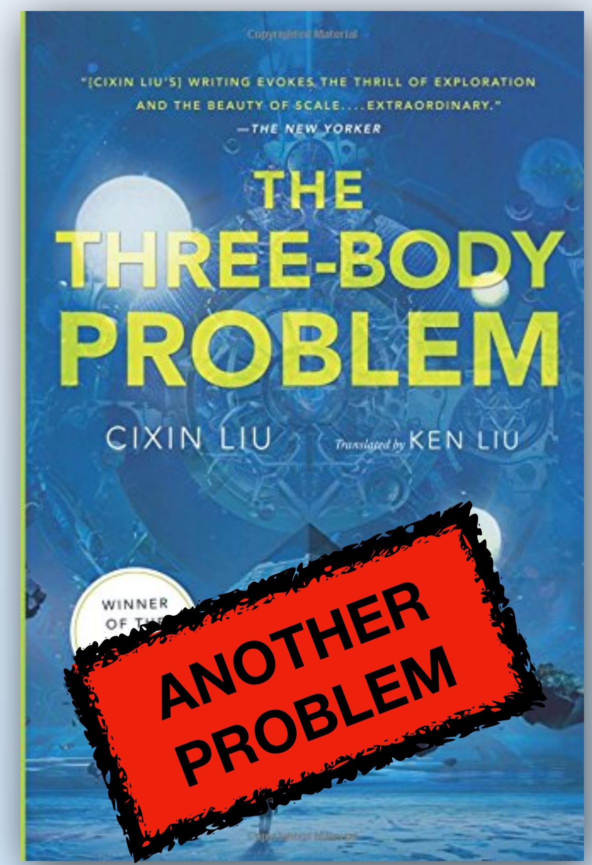

- *First data driven study of the finite volume spectrum*

  ($\pi^+\pi^+$) and ($\pi^+\pi^+\pi^+$) systems

  comparison with Lattice QCD results MM, Doering (2018) → Phys.Rev.Lett. 122 (2019)

>> THIS TALK <<

UNITARY ISOBAR INFINITE VOLUME AMPLITUDE

MM, Hu, Doring, Pilloni, Szczepaniak Eur.Phys.J. A53 (2017)

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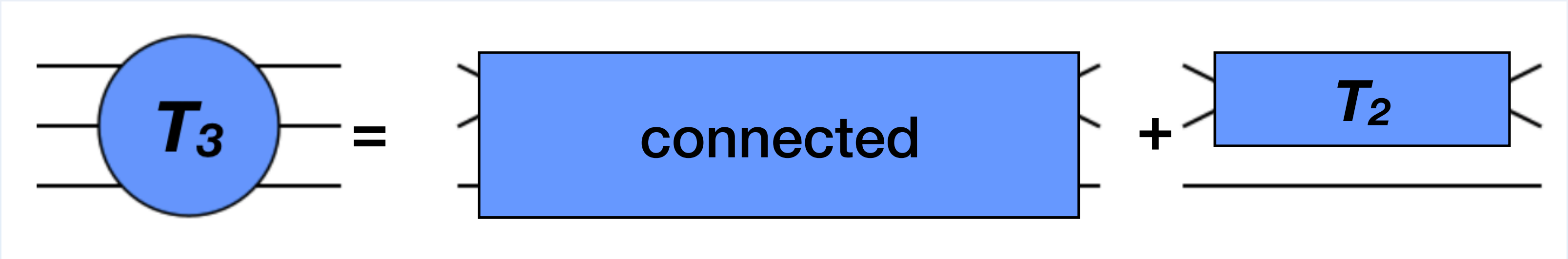

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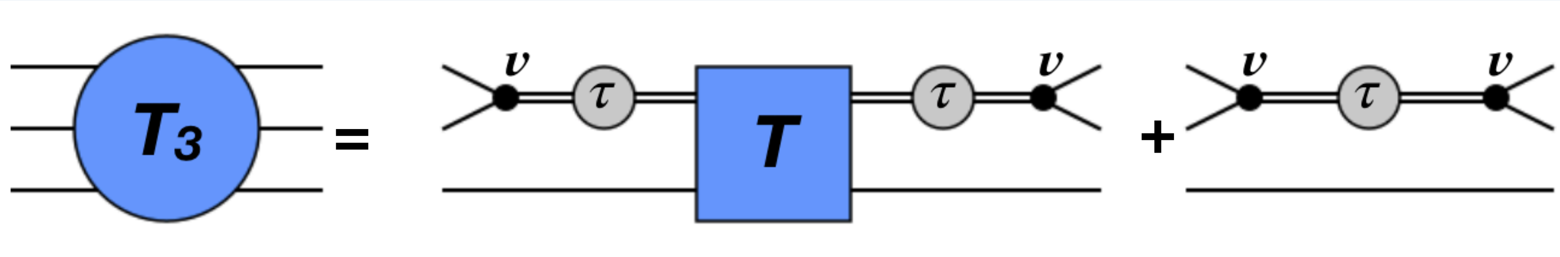

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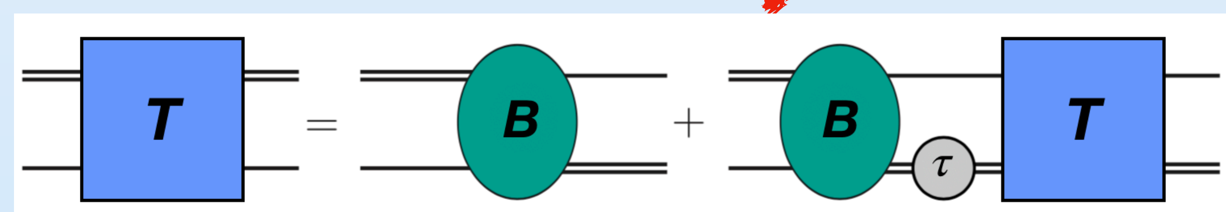

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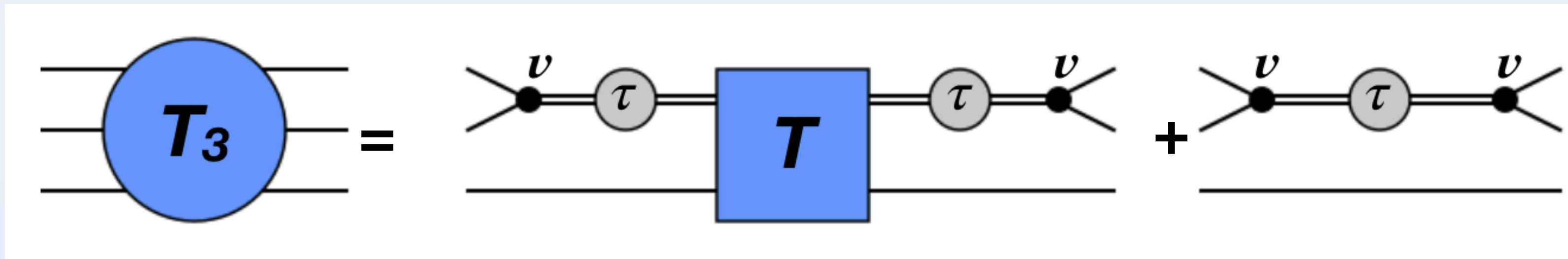

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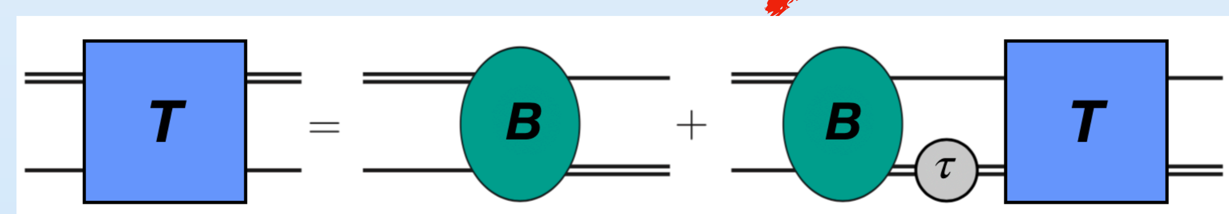

4) 3-body unitarity

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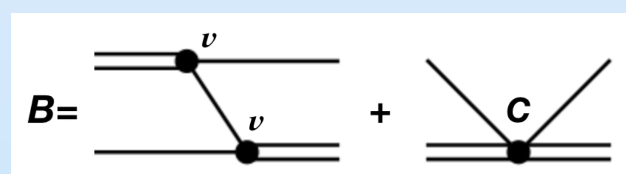

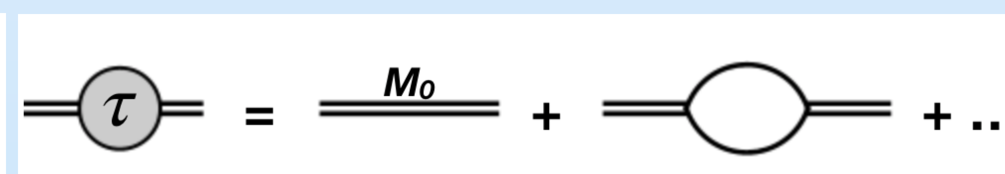

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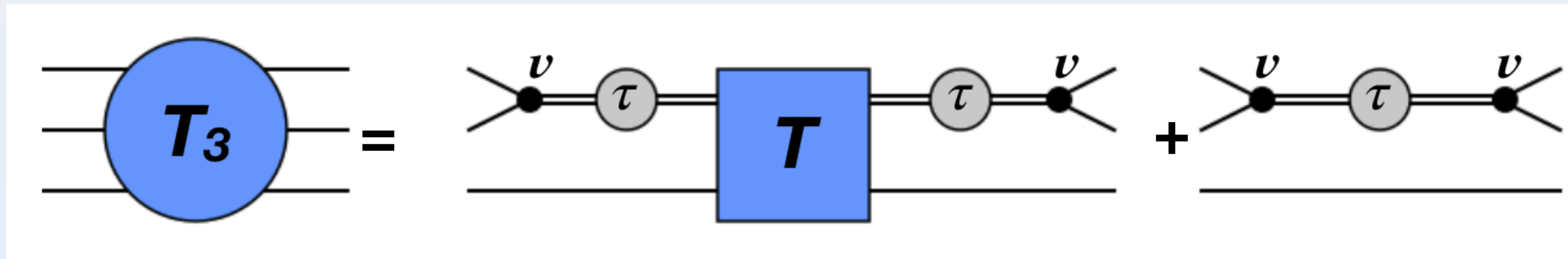

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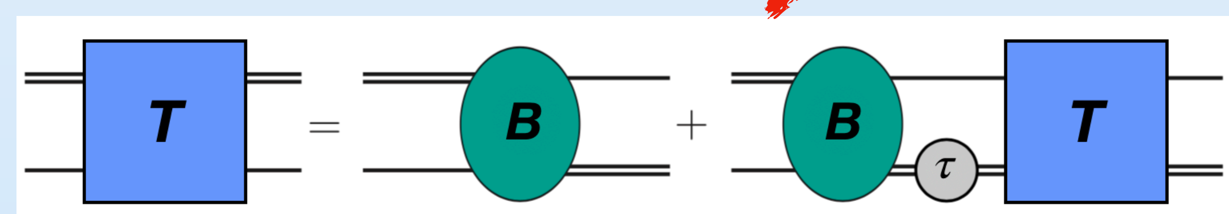

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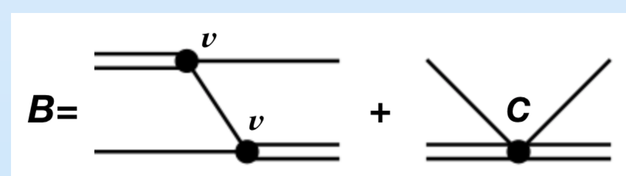

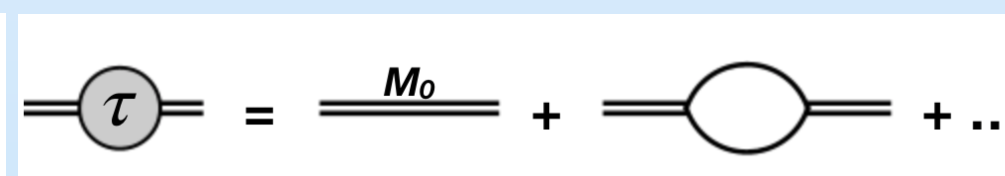

RESULT:

relativistic 3d integral-equation — ***v,C,M₀*** *to be fixed from data*

useful for phenomenological applications

Sadasivan, MM, Doering… in progress

- *recent study of analytic properties*

Jackura et al. [JPAC] Eur.Phys.J. C79 (2019)

3-BODY QUANTIZATION CONDITION

Power-law finite-volume effects

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on-shell configurations in T

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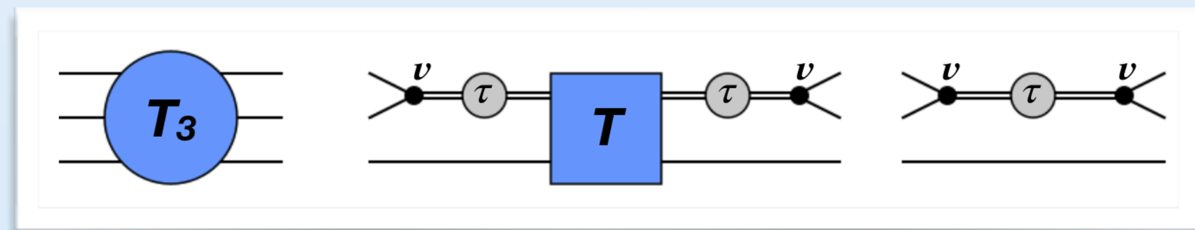
- ***Replace integrals by sums***
  - *Energy eigenvalues in a box:* $\{\, E^* \mid T(E^*) = \infty \,\}$



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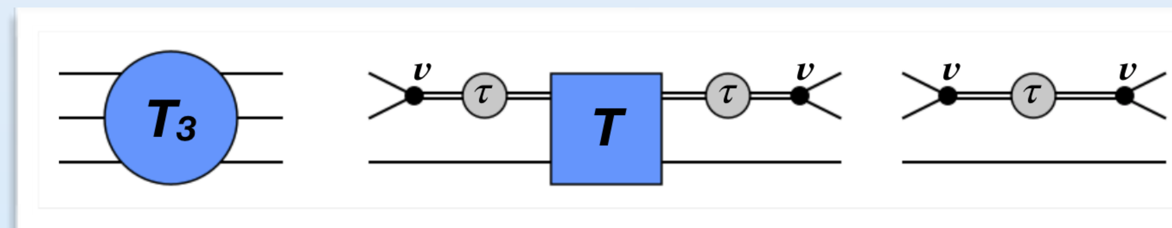

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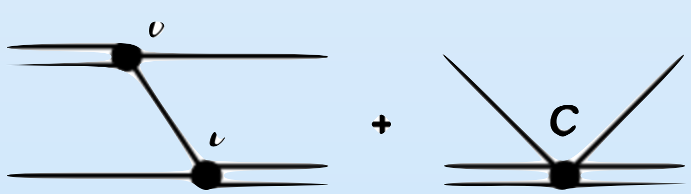

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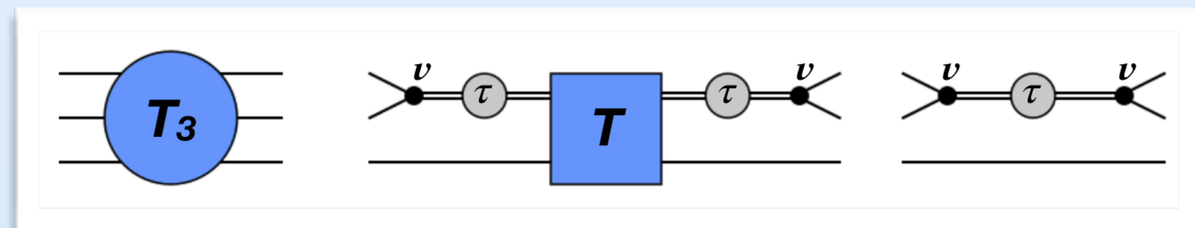

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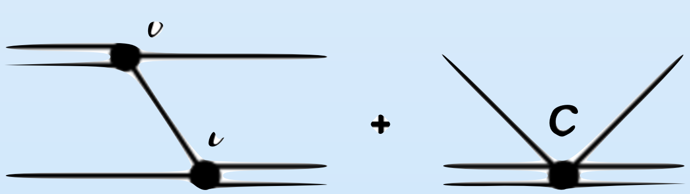
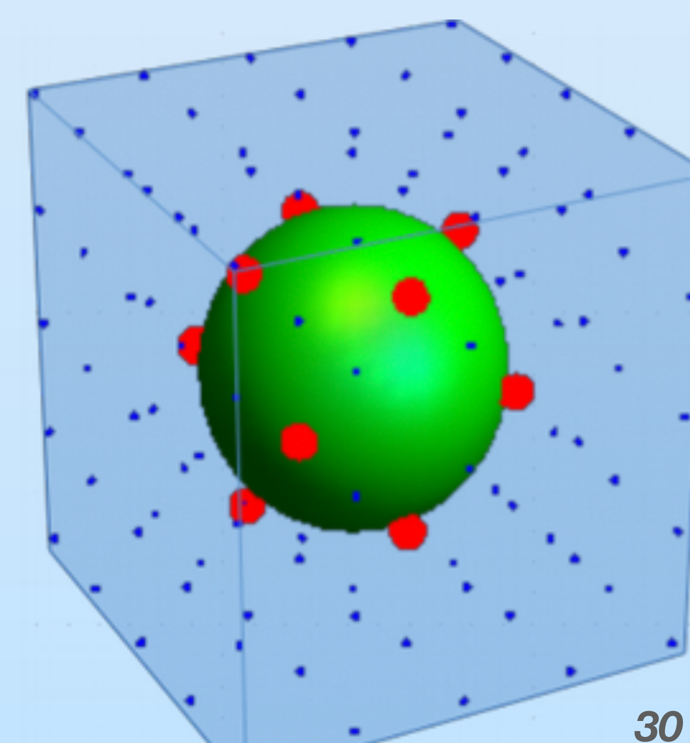


  - *projection to irreps is required*
  - *similar to* ***partial wave projection:****introduce shells*

Doring, Hammer, MM, PRD97(2018)



3-BODY QUANTIZATION CONDITION

- **Final result on shells, projected to irreps**

$$\boxed{\mathrm{Det}\left(B^{\Gamma ss'}_{uu'}(W^2)+\frac{2E_sL^3}{\vartheta(s)}\tau_s(W^2)\,\delta_{ss'}\delta_{uu'}\right)=0}$$

W *– total energy* ***s***$^{(')}$ *- shell index* ***u***$^{(')}$ *- basis index* **$\boldsymbol{\theta}$** *– multiplicity*

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EXPERIMENT

Infinite volume 3-body

Infinite volume 2-body

LATTICE QCD

Finite volume 3-body

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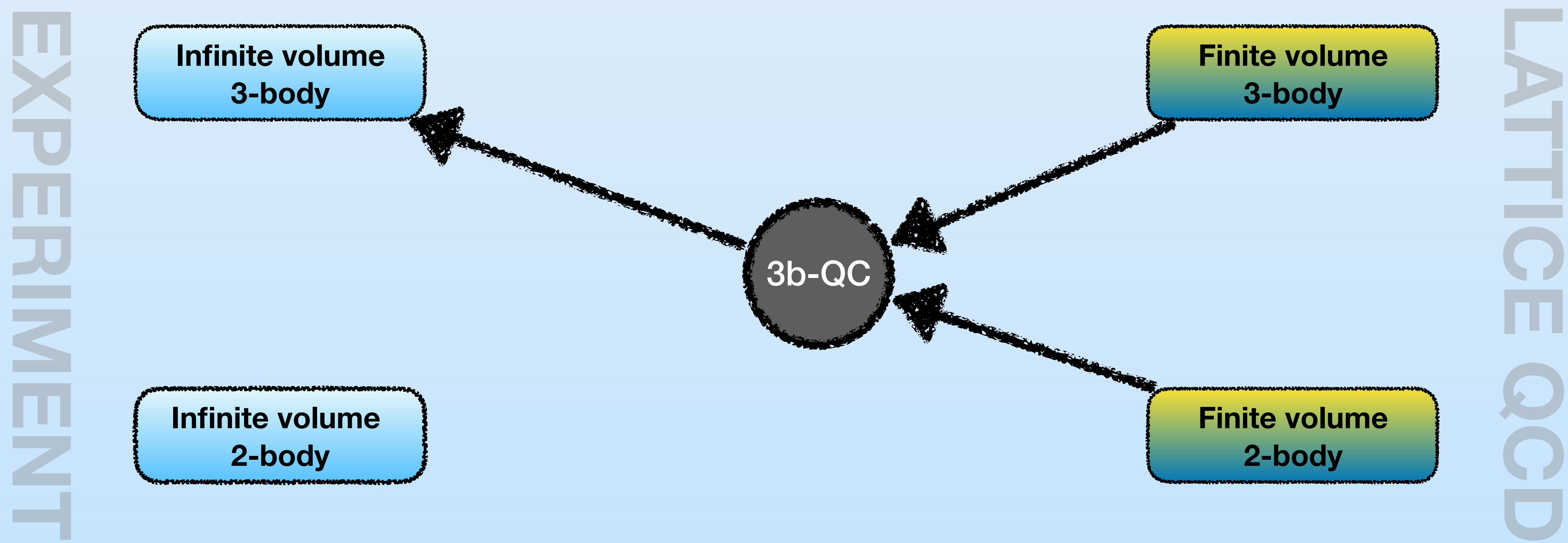

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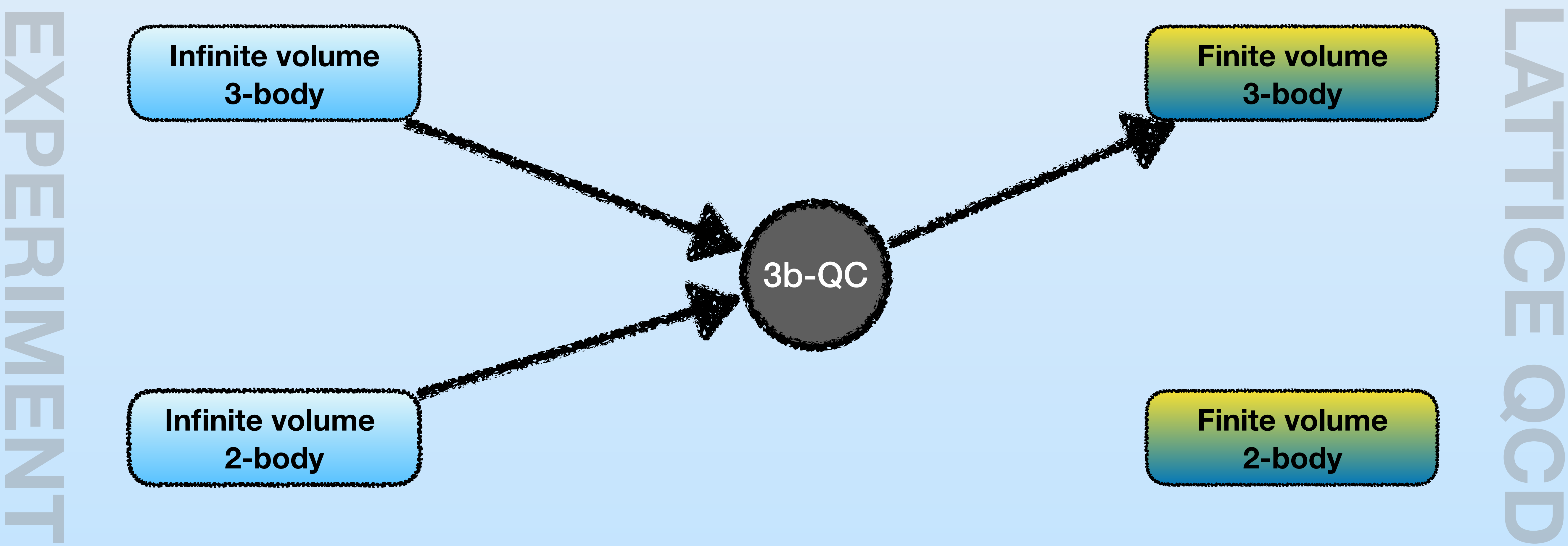

3-BODY QUANTIZATION CONDITION

- **Final result on shells, projected to irreps**

$$\boxed{\mathrm{Det}\left(B^{\Gamma ss'}_{uu'}(W^2)+\frac{2E_sL^3}{\vartheta(s)}\tau_s(W^2)\,\delta_{ss'}\delta_{uu'}\right)=0}$$

W *– total energy* ***s***$^{(\prime)}$ *- shell index* ***u***$^{(\prime)}$ *- basis index* ***θ*** *– multiplicity*

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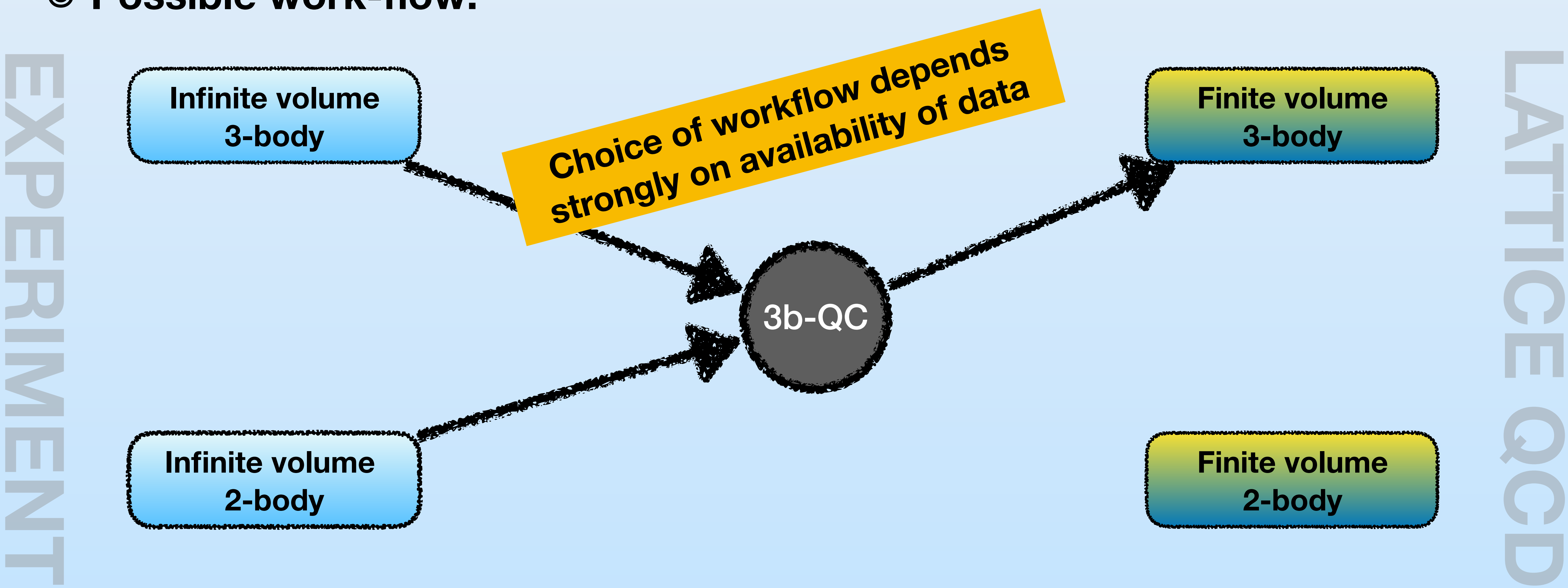

PHYSICAL APPLICATION

MM, Doring PRL 122 (2019)

- Interesting system to study: $\pi^+\pi^+\pi^+$

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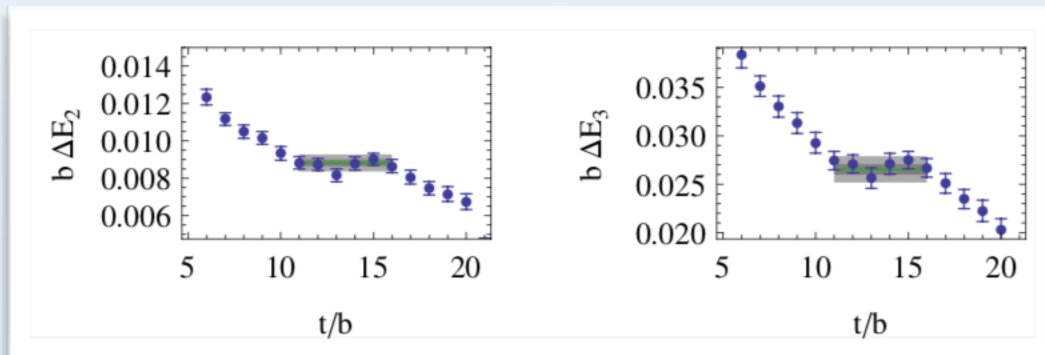

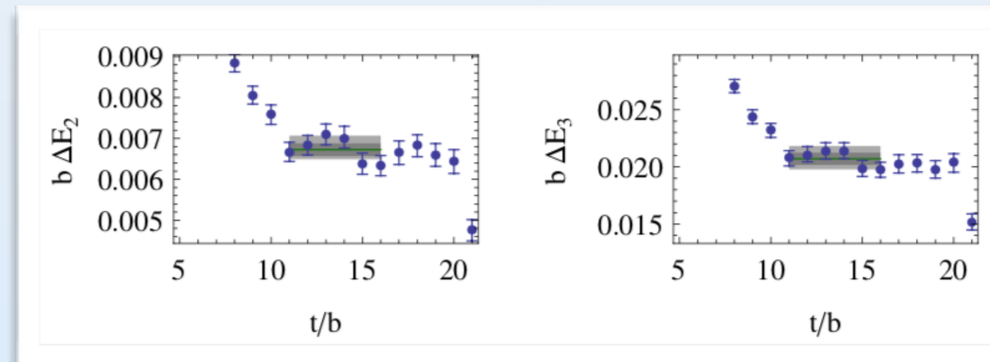

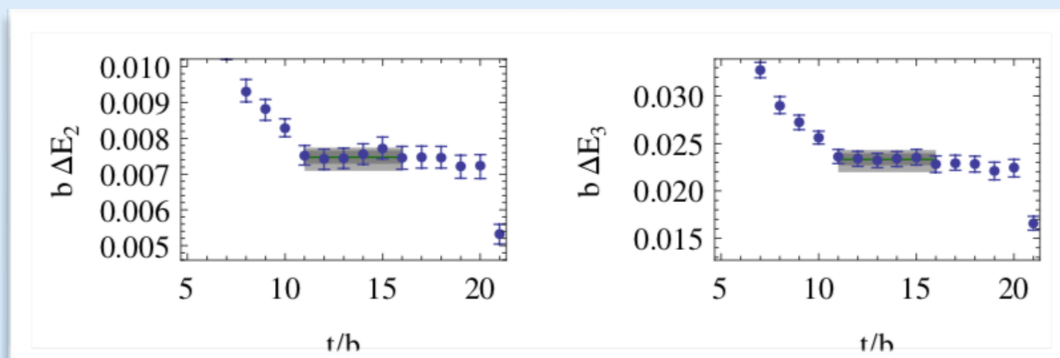

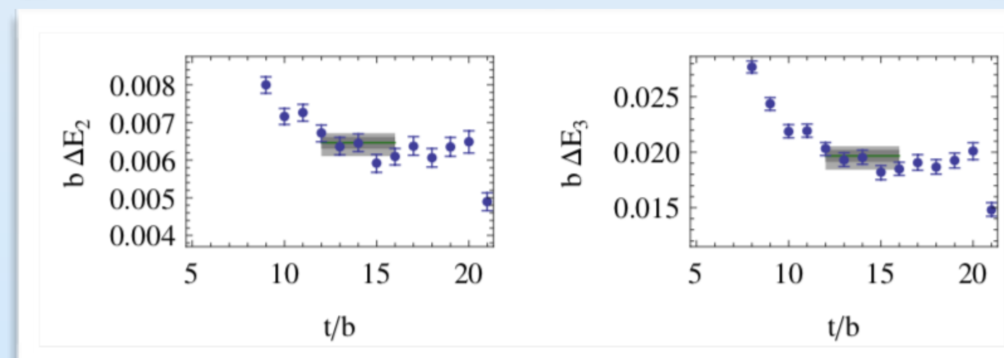

Beane et al. [NPLQCD] PRL100 (2008)   Detmold et al. [NPLQCD] PRD78 (2008)

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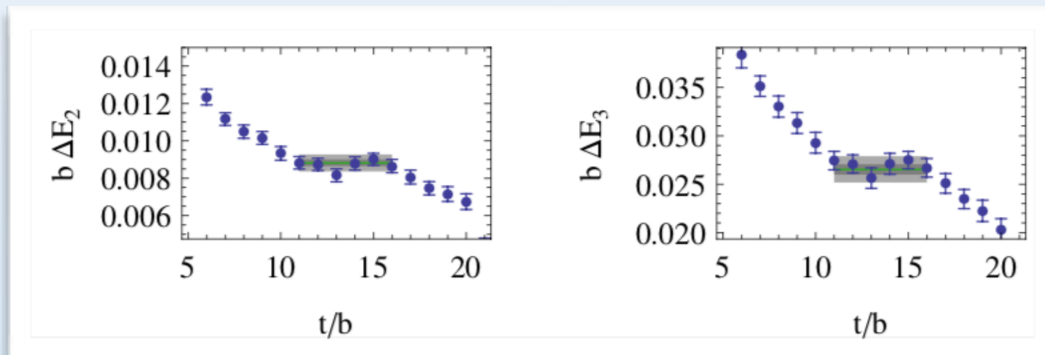

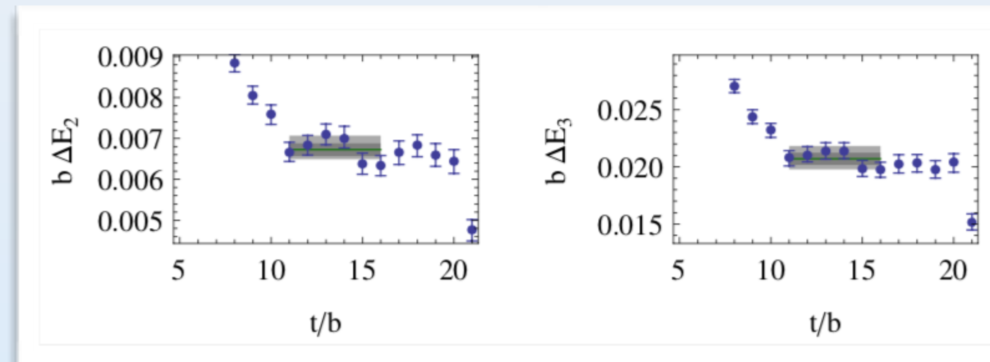

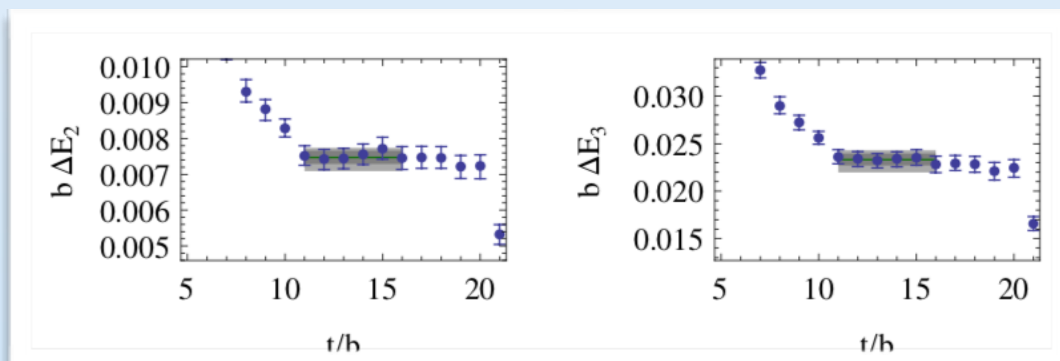

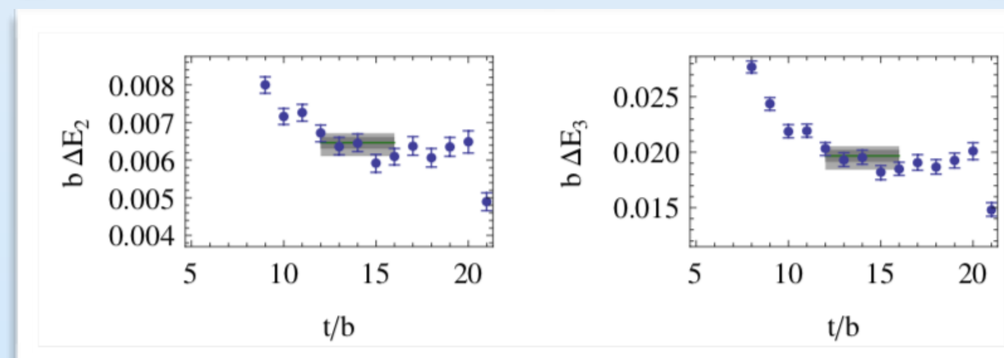

Beane et al. [NPLQCD] PRL100 (2008)   Detmold et al. [NPLQCD] PRD78 (2008)

  - *Repulsive channel* → **Does the “isobar” picture hold?**
  - ***m_π=291 / 352 / 491 / 591 MeV*** → **Chiral extrapolation in 3body system?**

PHYSICAL APPLICATION

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◉ **2-body sub-channel:**

- *One-channel problem —* $\pi^+\pi^+$ *system in S-wave*

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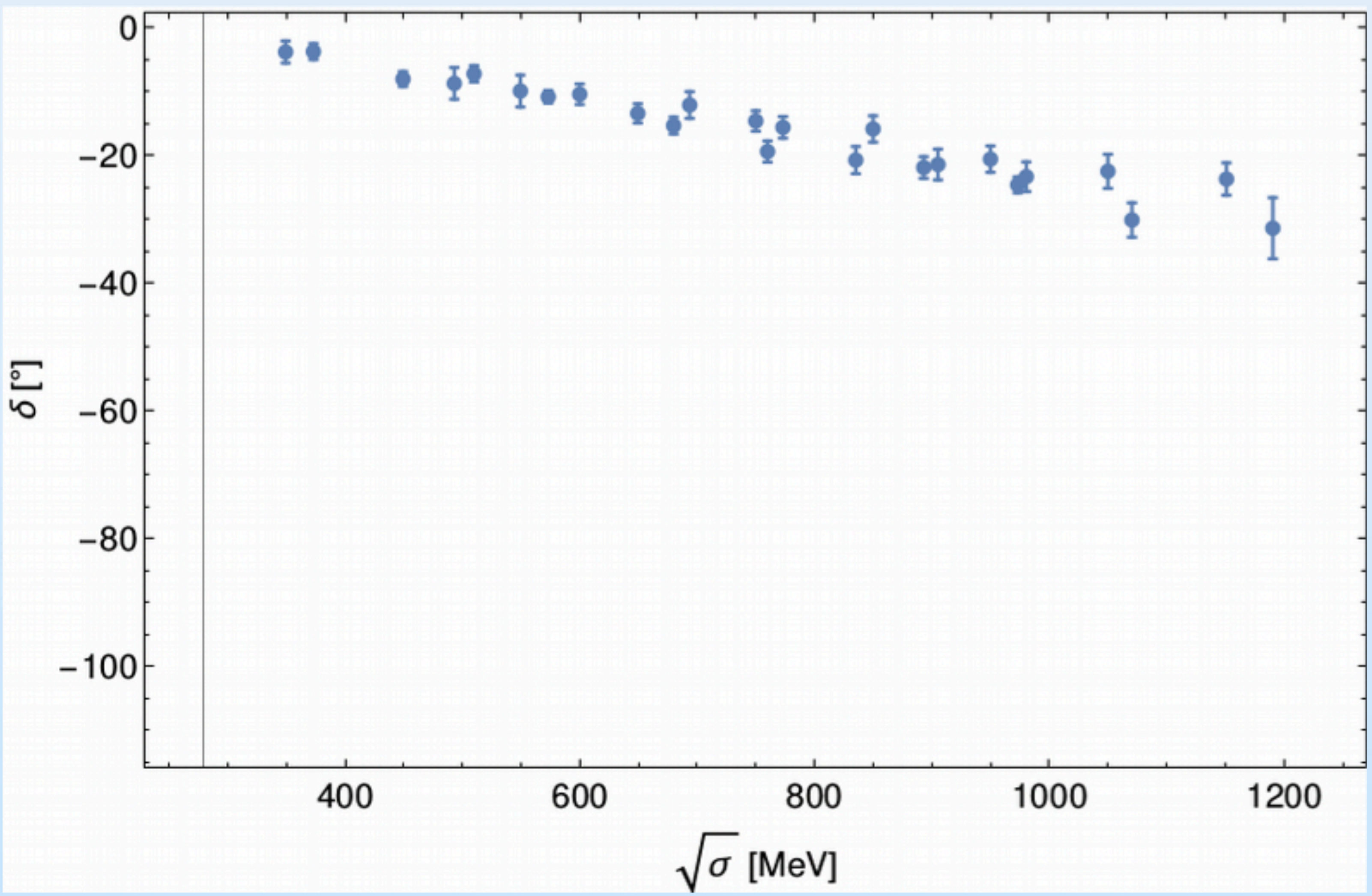

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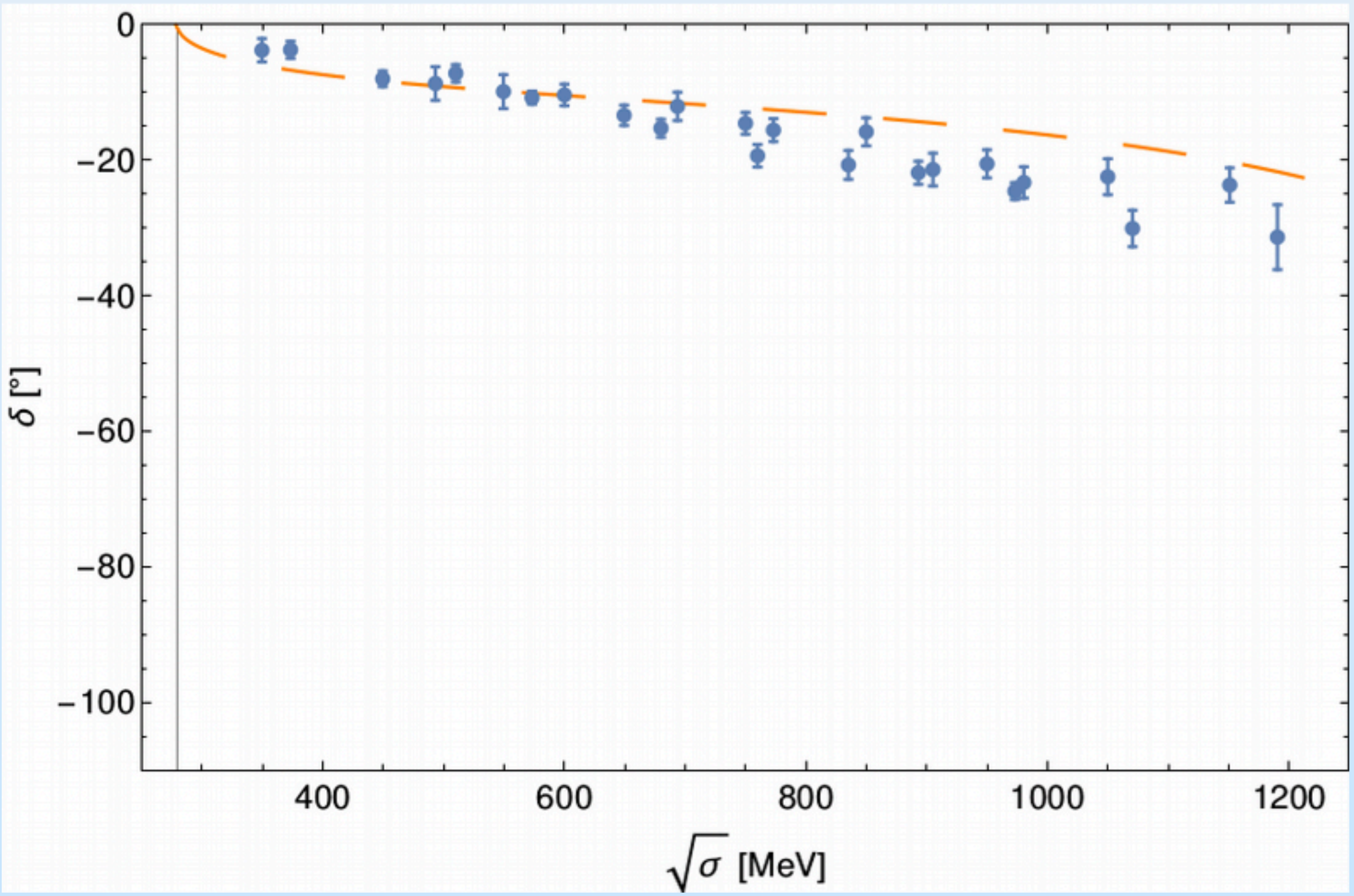

1) $$T_2 = \frac{-\lambda^2/(32\pi)}{\sigma - M_0^2 - \sum_\pm \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{\lambda^2}{4E_{\boldsymbol{k}}\sqrt{\sigma}(\sqrt{\sigma}\pm 2E_{\boldsymbol{k}})}}$$

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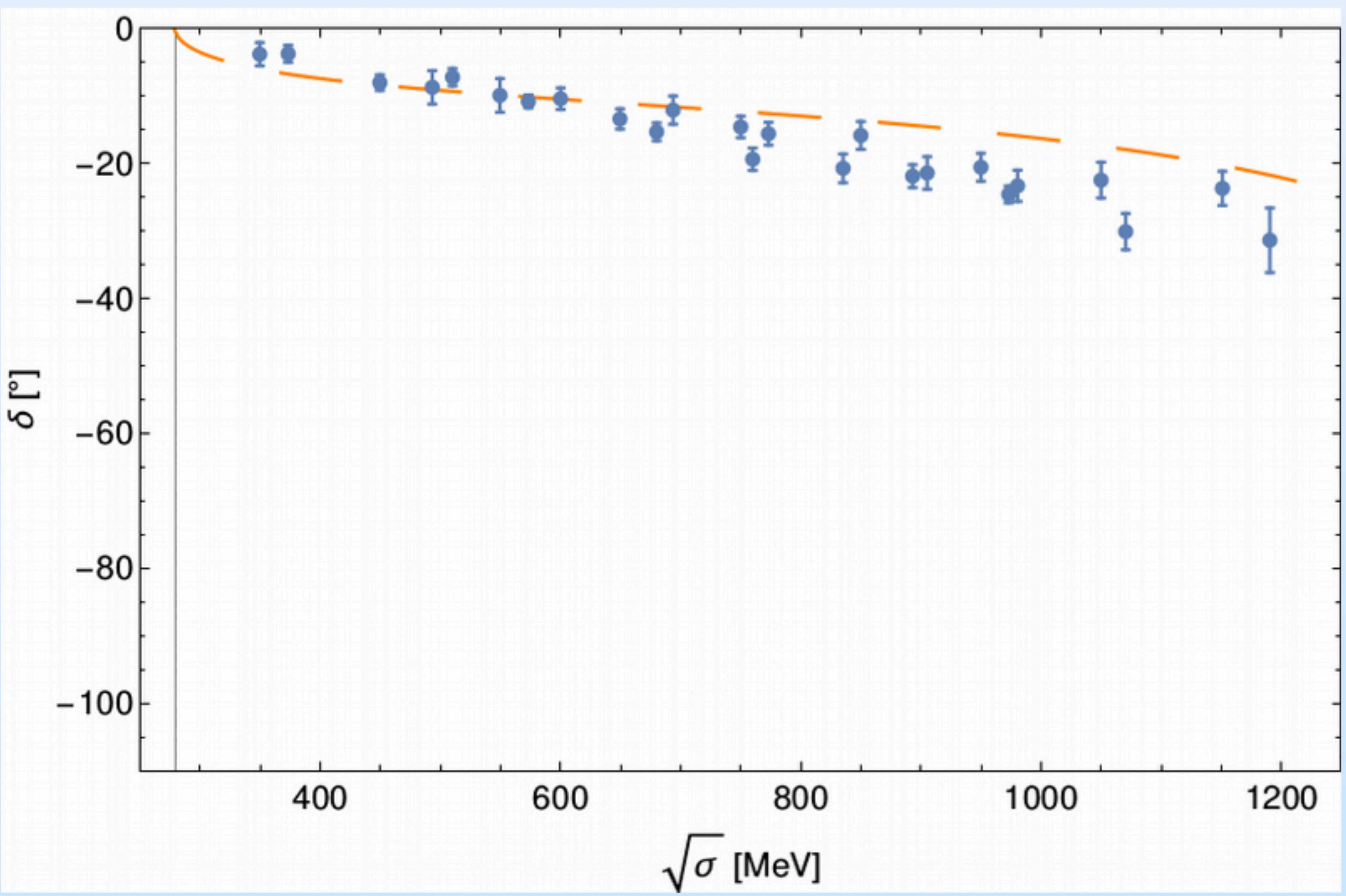

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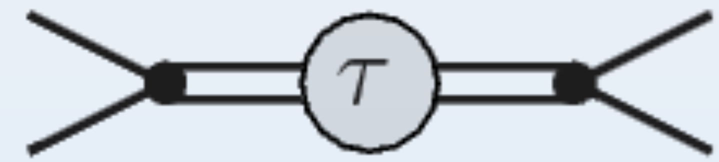

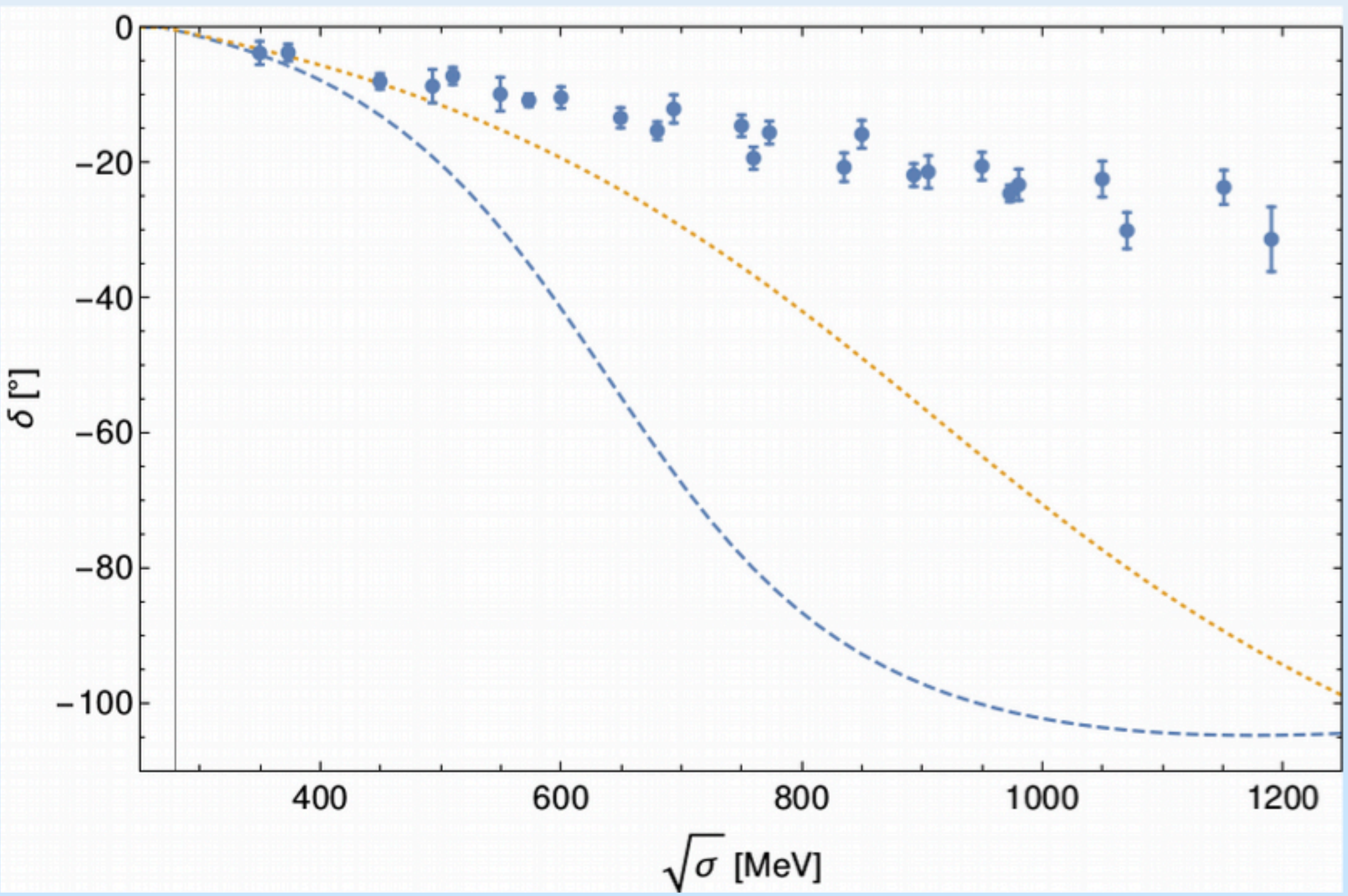

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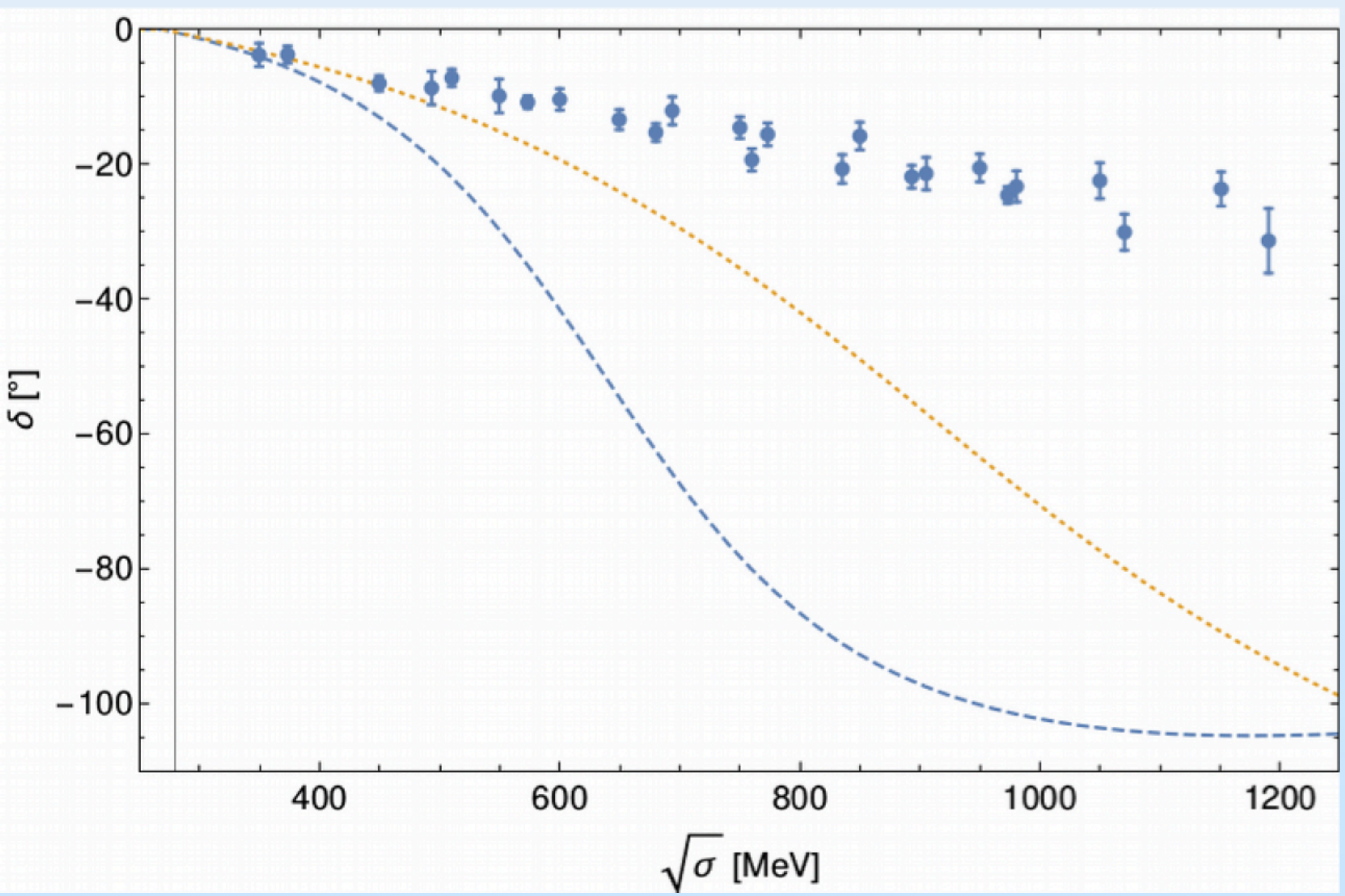

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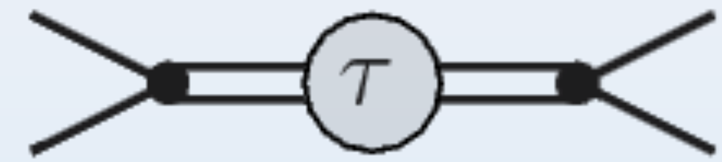

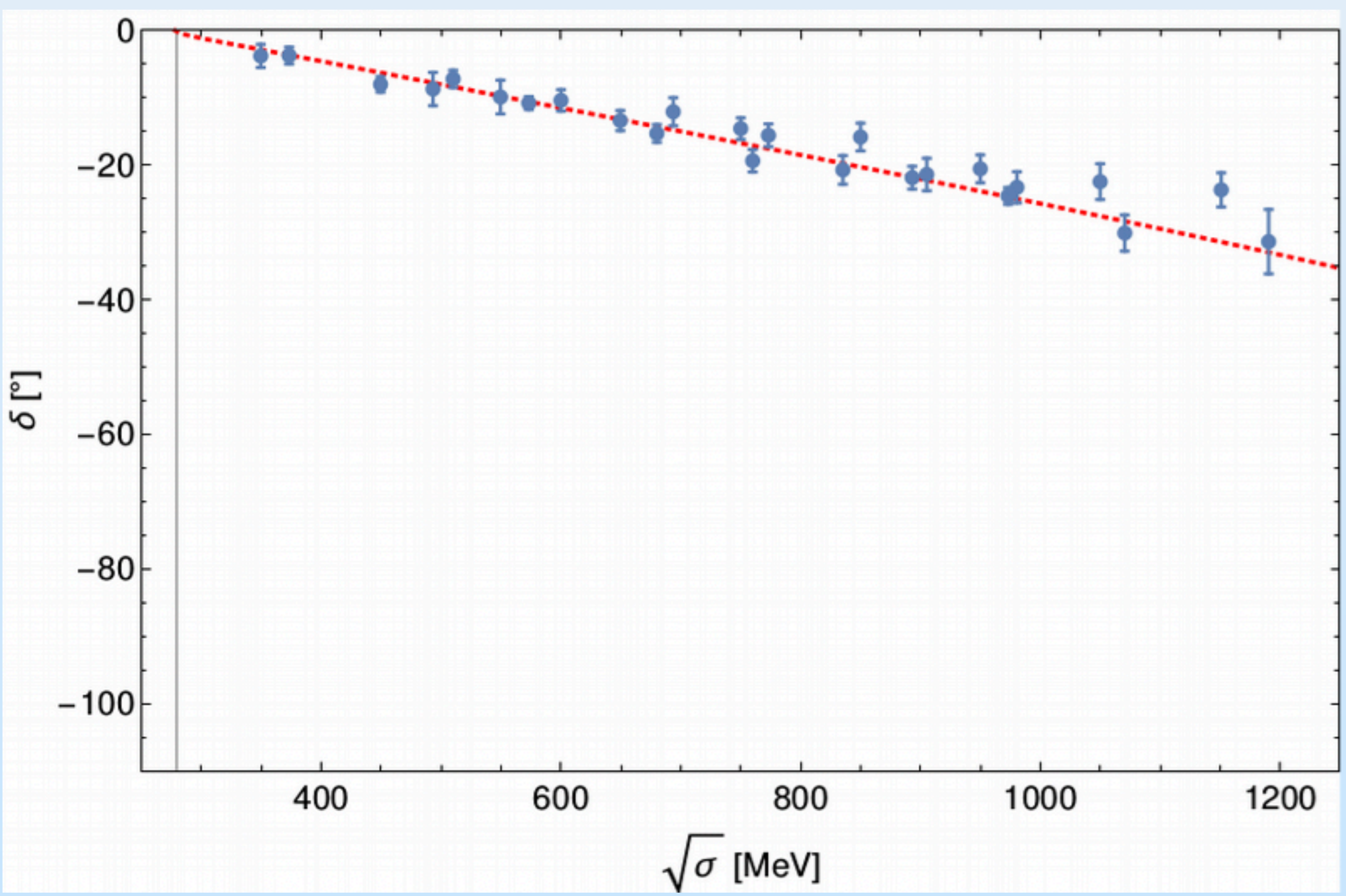

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Truong(1988)

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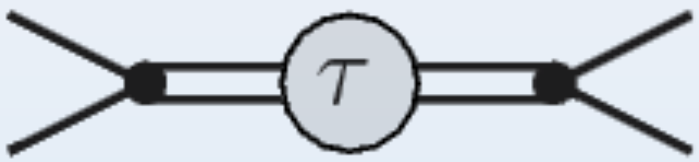

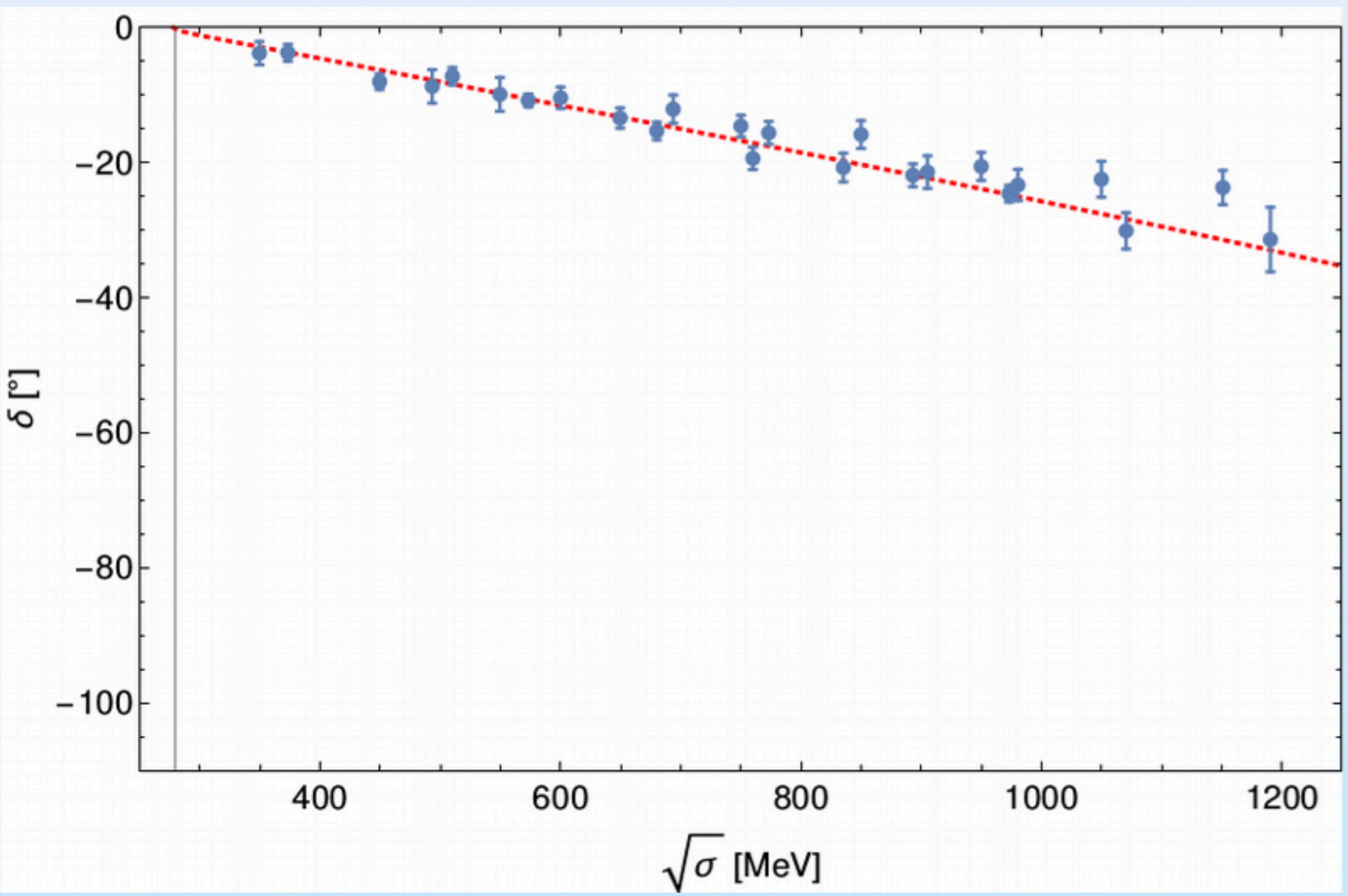

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Truong(1988)

- *correct **σ** & **m_π** behavior*
- *parameters known*

Gasser/Leutwyler(1984)

PHYSICAL APPLICATION

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- **2-body sub-channel:**
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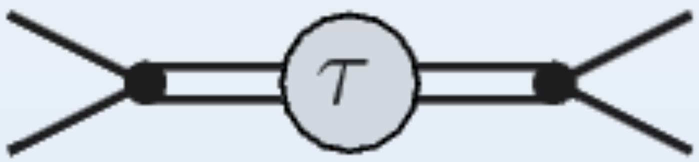

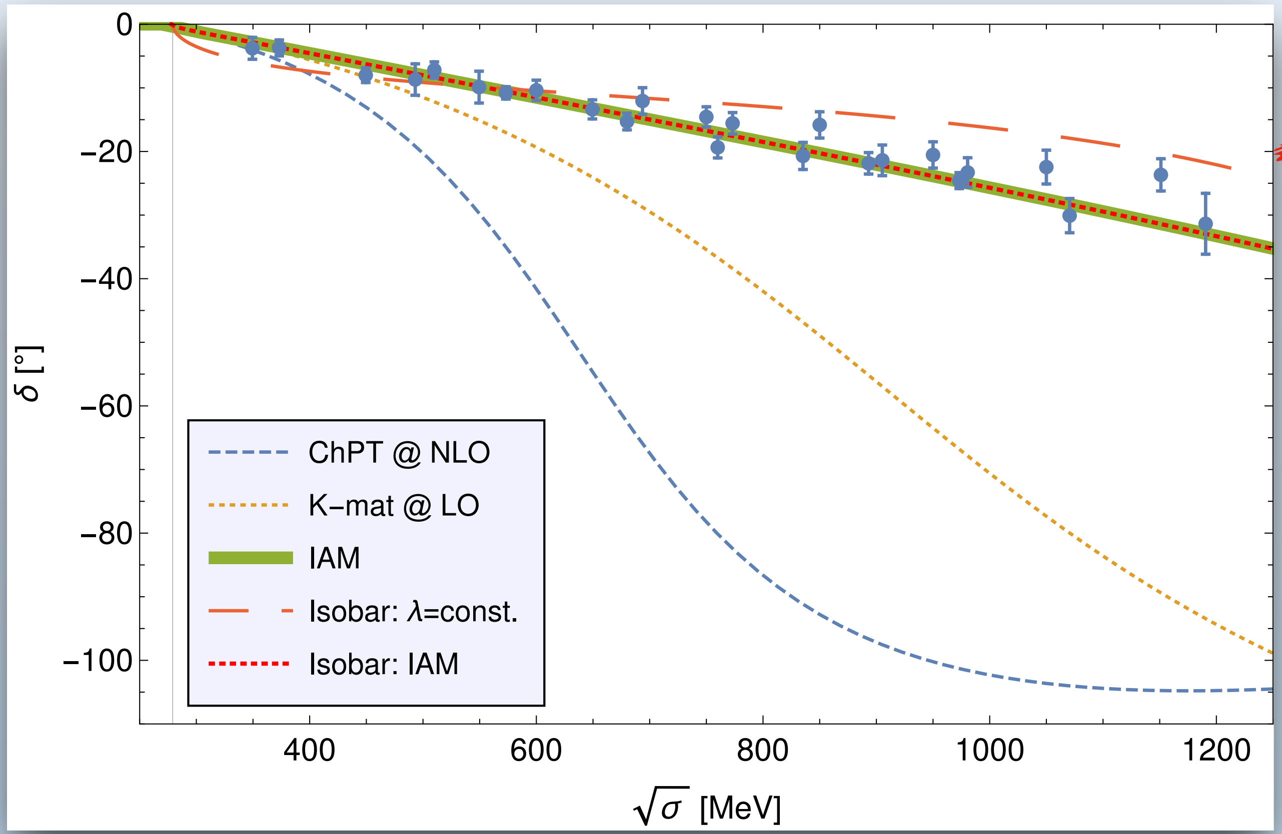

Discretize momenta = Luescher + $O(e^{-ML})$

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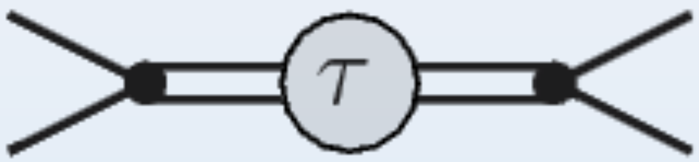

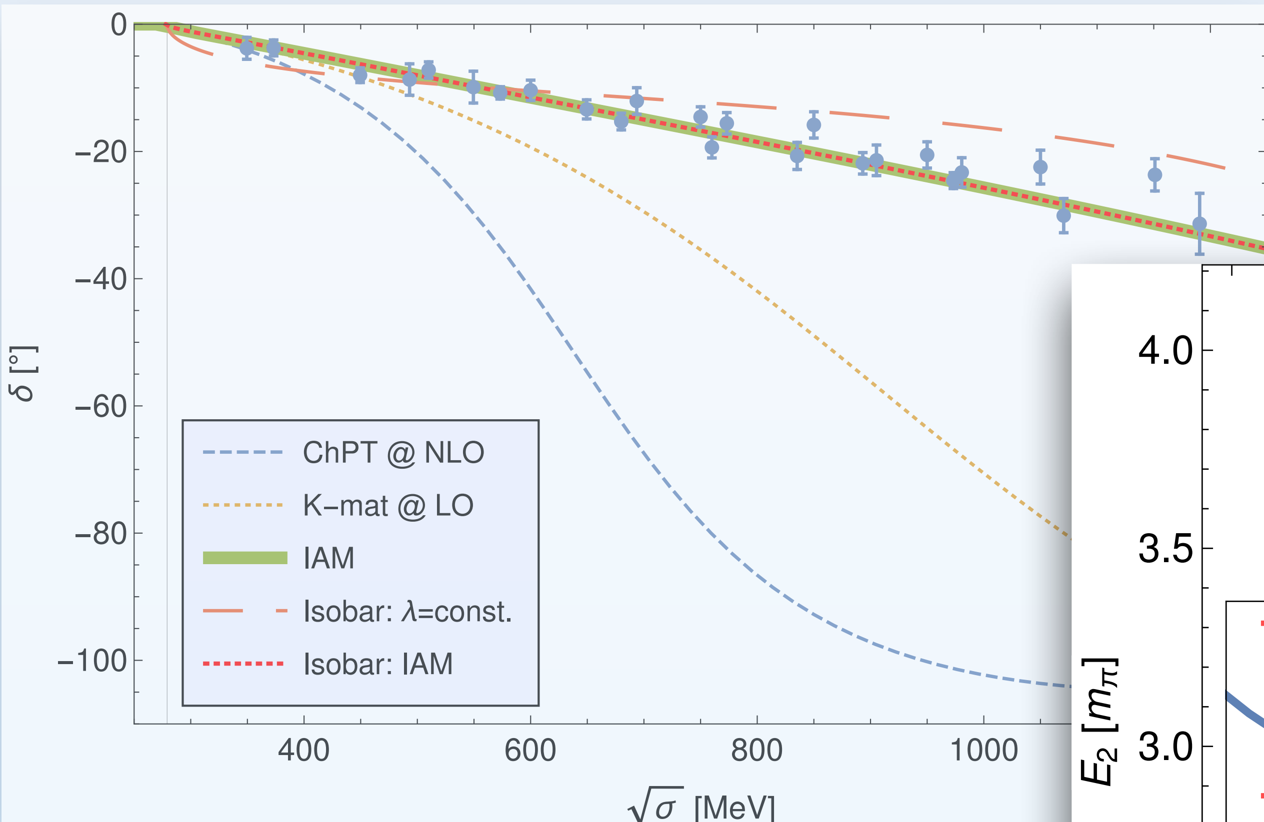

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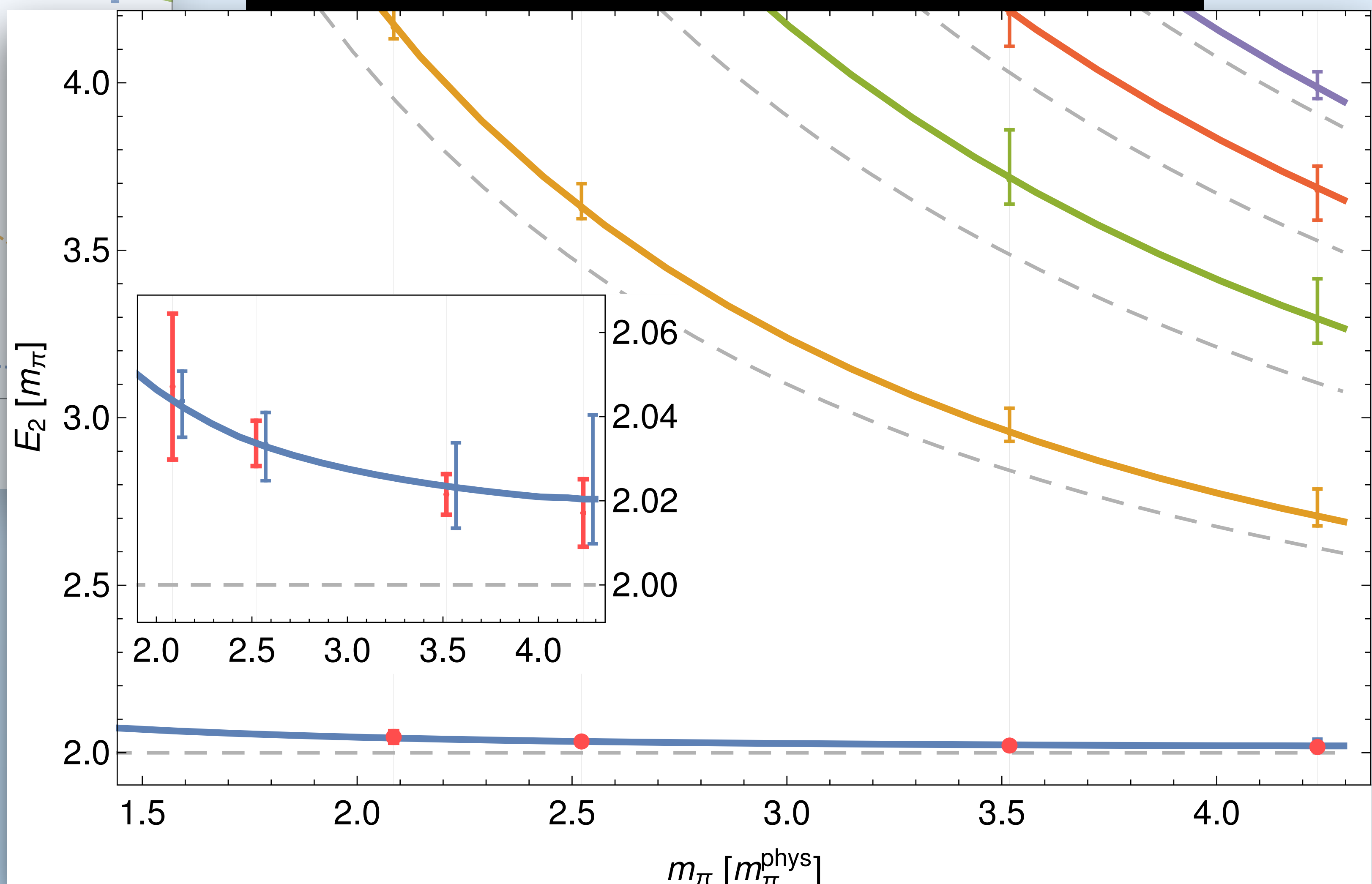

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  - *momenta dependent function*

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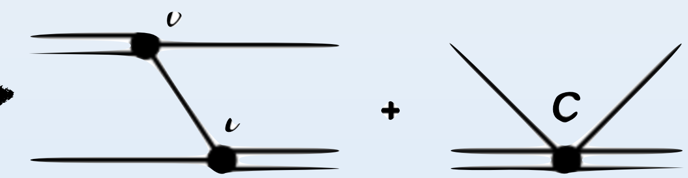

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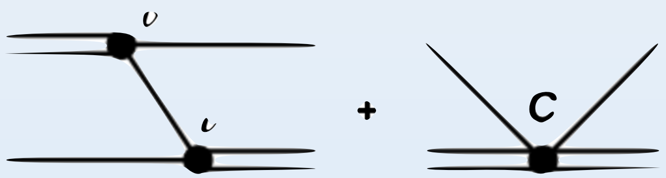
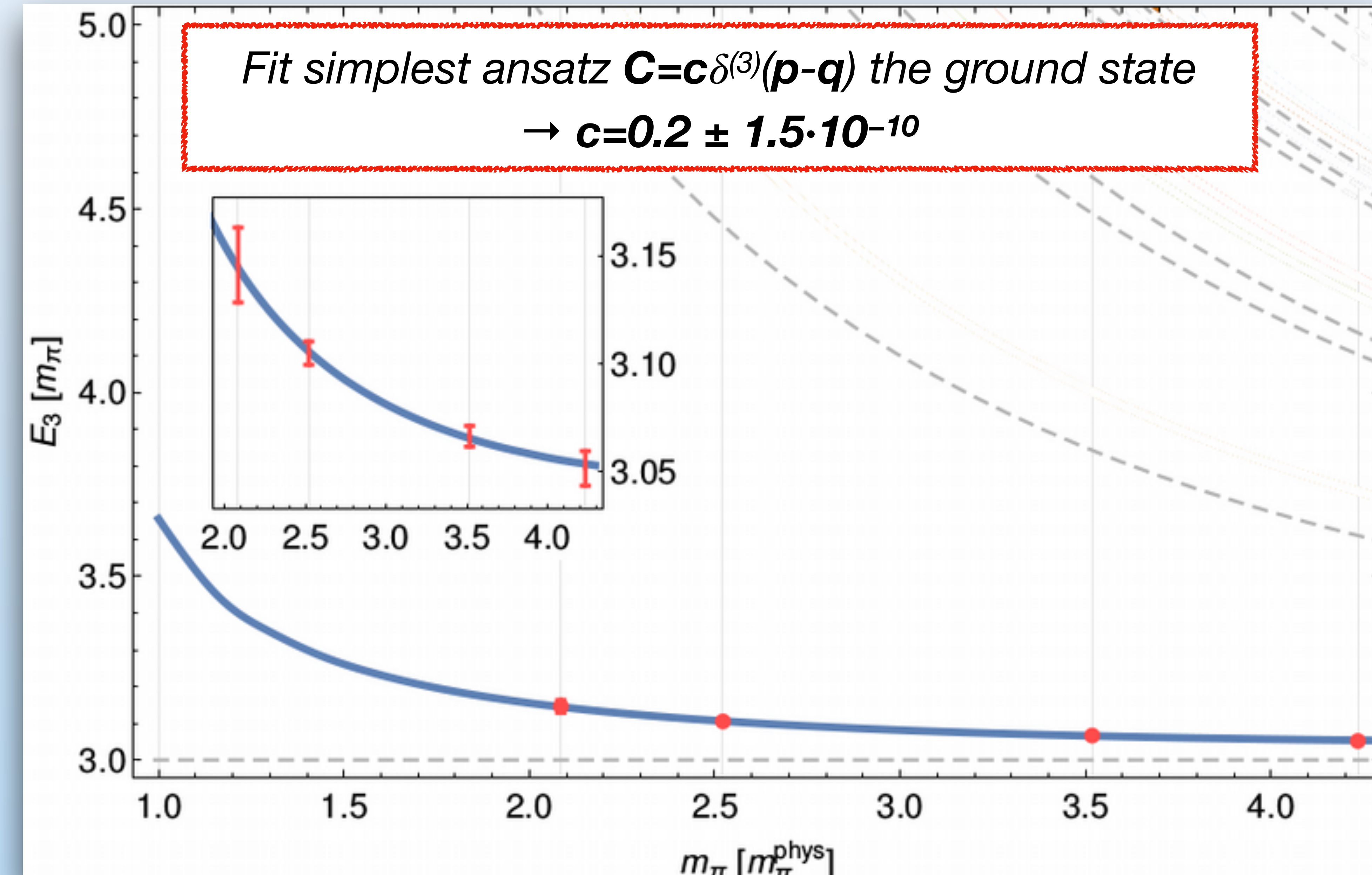


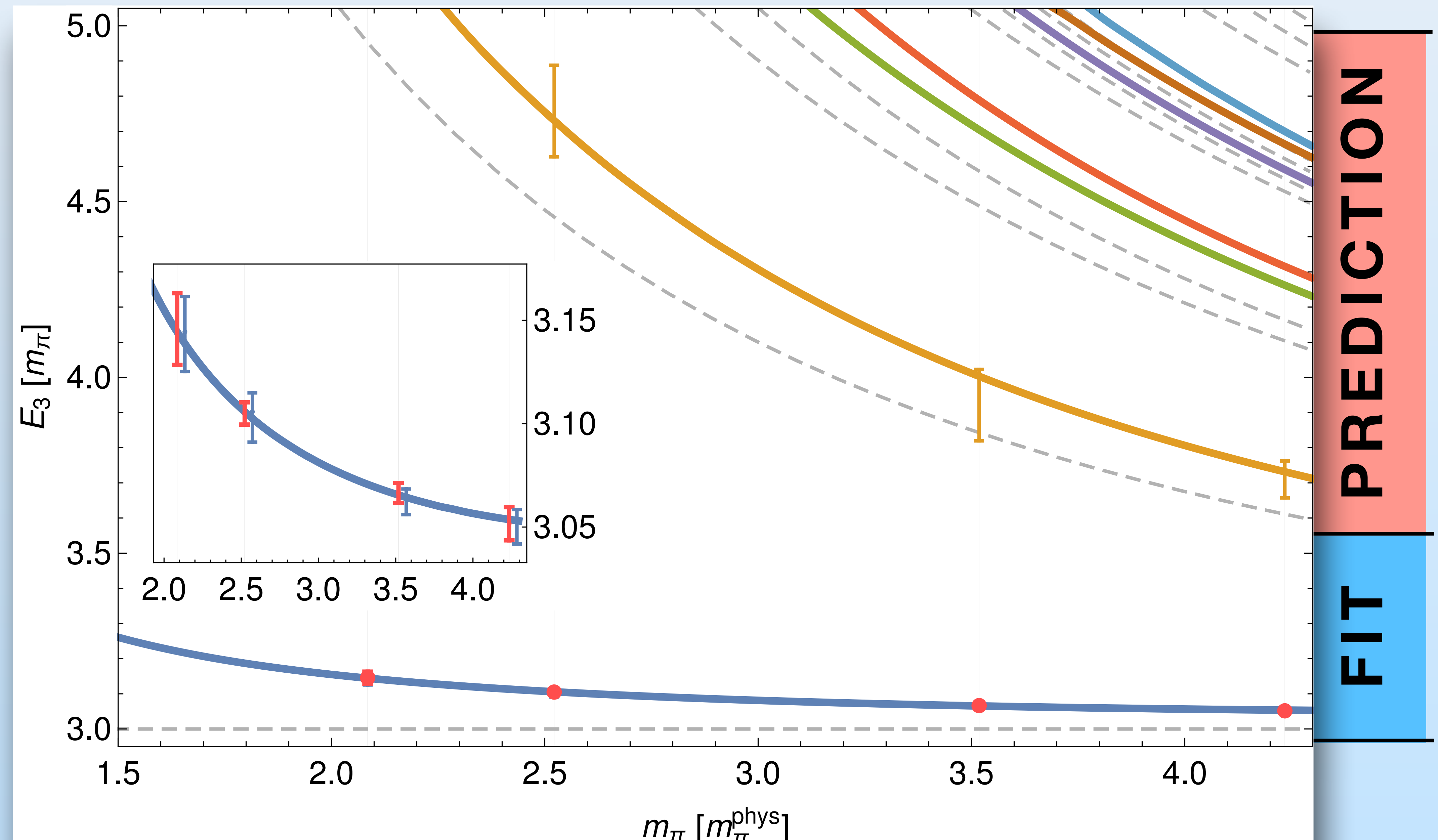


PHYSICAL APPLICATION

MM, Doring PRL 122 (2019)

◉ **3-body spectrum — predictions — excited level spectrum**

1. ***Novel pattern:*** *1-to-1 correspondence of interacting and non-interacting levels*
2. *Energy levels are shifted block-wise*
3. *Corresponding poles are simple*



SUMMARY

- Parametrization via 2-b. sub-channel amplitudes
- Relativistic integral equation **EPJA53 (2017)**

- Discretization & Projection to irreps of O_h leads to a relativistic 3body QC
EPJA53 (2017) PRD97 (2018)

- excited spectrum of $\boldsymbol{\pi^+\pi^+}$ & $\boldsymbol{\pi^+\pi^+\pi^+}$
- ground level compared with ***NPLQCD*** results
- predictions at physical pion mass

PRL 122 (2019)

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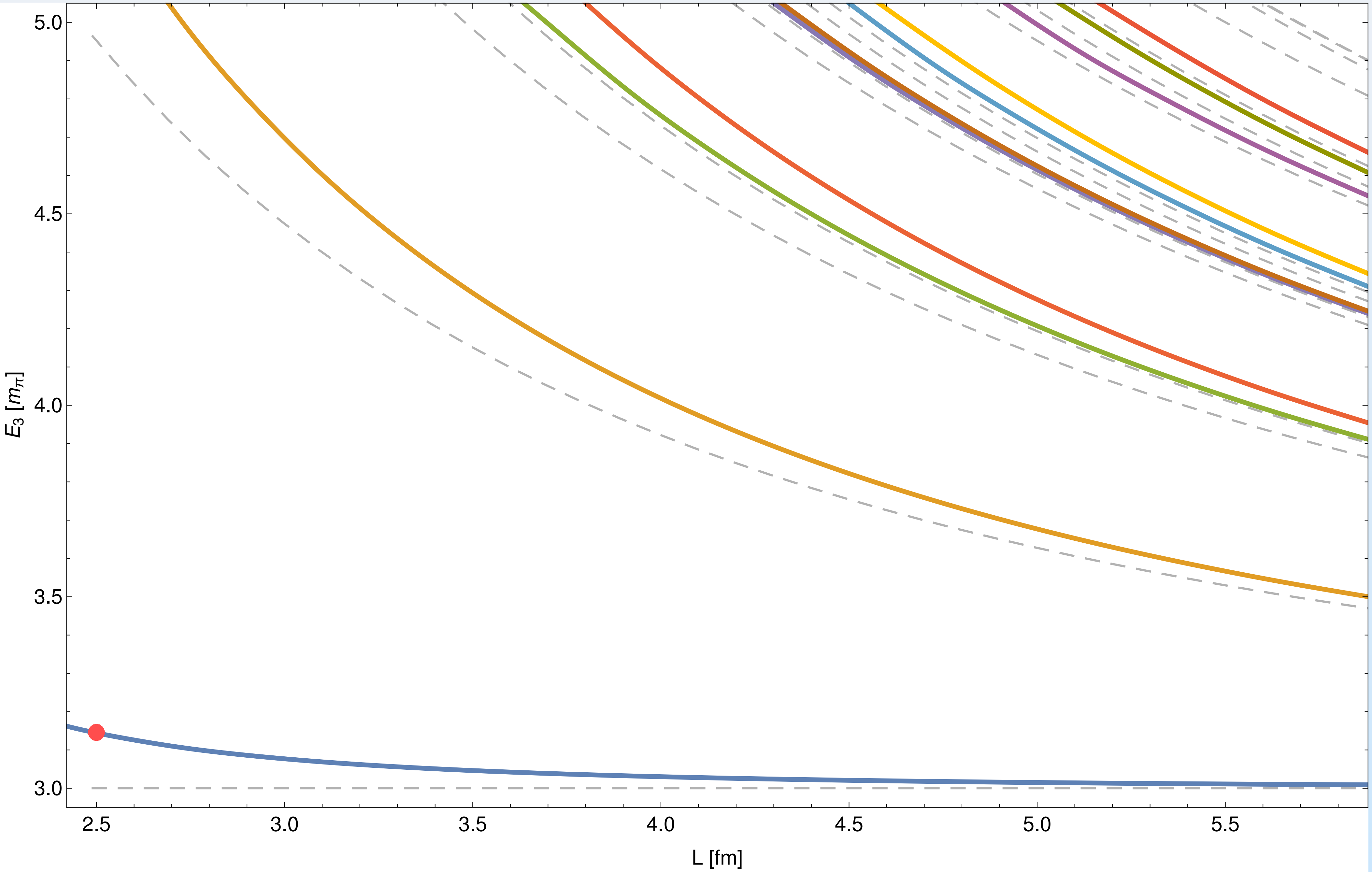
  PRL 122 (2019)

IN PROGRESS

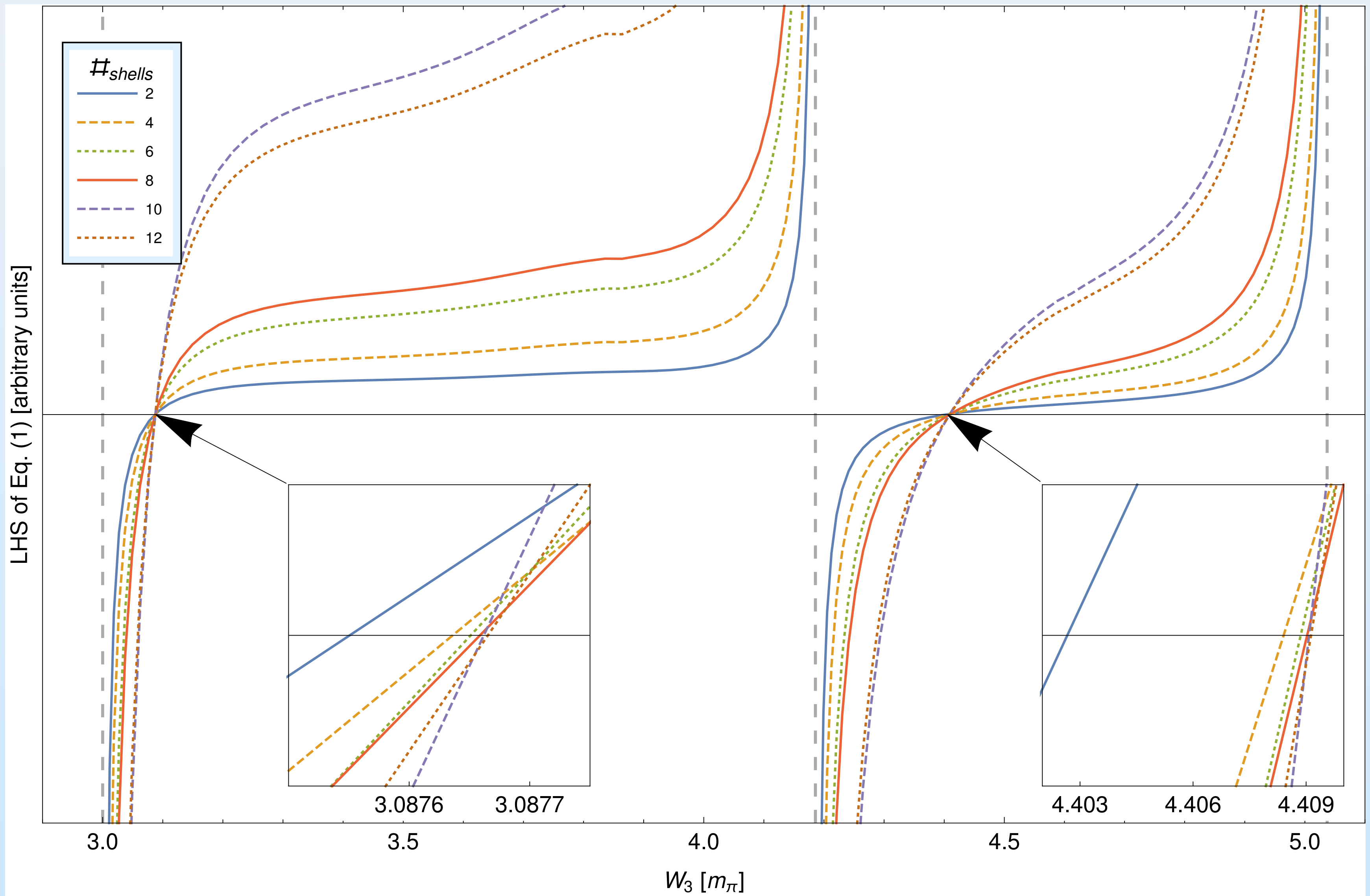
Extension to multi-channels — applications — a1(1260), N*(1440), ...

SPARES

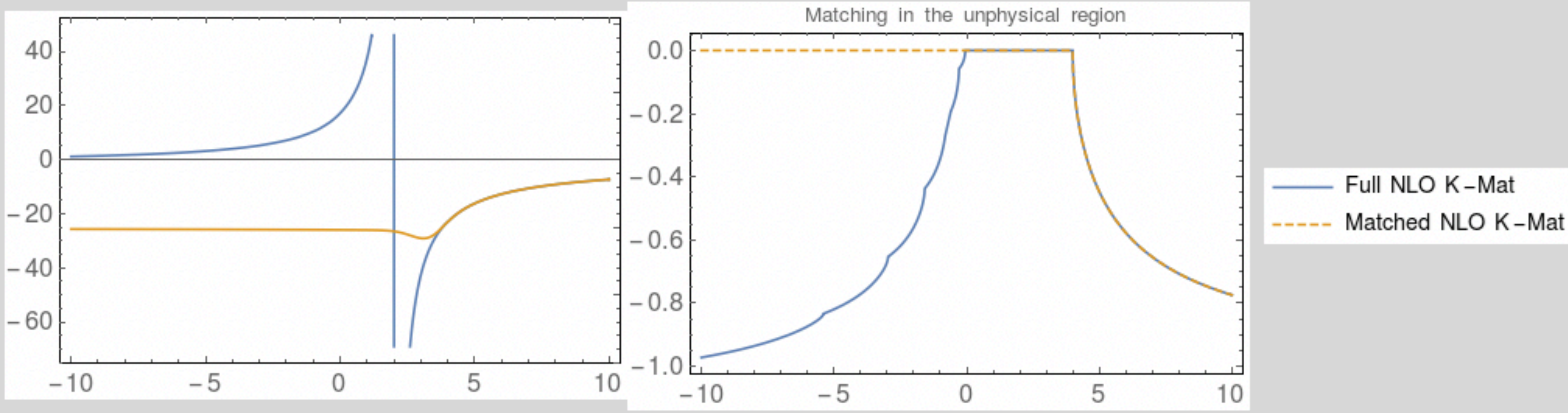
L-DEPENDENCE (M=291 MEV)

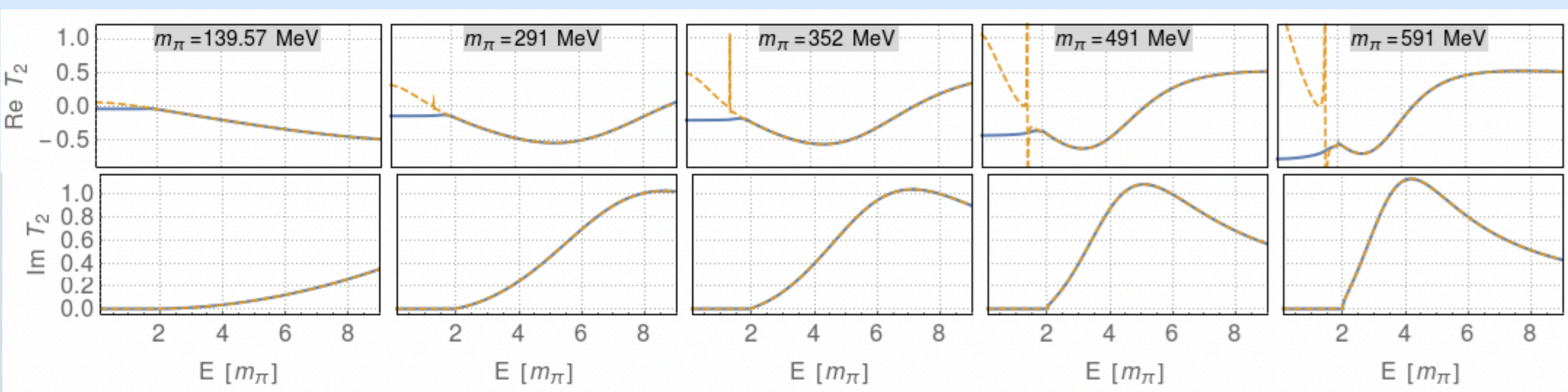


“SHELL-CUTOFF” DEPENDENCE



m_π [MeV]	139.57	291	352	491	591
E_2^1 $[m_\pi]$	$2.1228^{+0.0068}_{-0.0069}$	$2.0437^{+0.0071}_{-0.0086}$	$2.0334^{+0.0076}_{-0.0086}$	$2.0233^{+0.0105}_{-0.0098}$	$2.0204^{+0.0200}_{-0.0106}$
Refs. [24, 25]	–	$\mathbf{2.0471(27)(65)}$	$\mathbf{2.0336(22)(22)}$	$\mathbf{2.0215(16)(13)}$	$\mathbf{2.0171(16)(19)}$
E_2^2 $[m_\pi]$	–	–	$3.6245^{+0.0746}_{-0.0299}$	$2.9556^{+0.0728}_{-0.0263}$	$2.7045^{+0.0827}_{-0.0271}$
E_2^3 $[m_\pi]$	–	–	–	$3.7114^{+0.1482}_{-0.0737}$	$3.2911^{+0.1241}_{-0.0688}$
E_2^4 $[m_\pi]$	–	–	–	–	$3.6802^{+0.0707}_{-0.0902}$
E_2^5 $[m_\pi]$	–	–	–	–	$3.9829^{+0.0500}_{-0.0299}$
E_3^1 $[m_\pi]$	$3.6564^{+0.1014}_{-0.0847}$	$^*3.1444^{+0.0171}_{-0.0192}$	$^*3.1058^{+0.0091}_{-0.0147}$	$^*3.0655^{+0.0029}_{-0.0095}$	$^*3.0537^{+0.0048}_{-0.0119}$
Refs. [24, 25]	–	$\mathbf{3.1458(49)(125)}$	$\mathbf{3.1050(27)(27)}$	$\mathbf{3.0665(26)(22)}$	$\mathbf{3.0516(27)(53)}$
E_3^2 $[m_\pi]$	–	–	$4.7301^{+0.1577}_{-0.1027}$	$4.0031^{+0.0196}_{-0.1836}$	$3.7315^{+0.0309}_{-0.0742}$
E_3^3 $[m_\pi]$	–	–	–	$4.7043^{+0.0126}_{-0.5923}$	$4.2621^{+0.0001}_{-0.1739}$
E_3^4 $[m_\pi]$	–	–	–	$4.7890^{+0.0506}_{-0.1722}$	$4.3155^{+0.0837}_{-0.1341}$
E_3^5 $[m_\pi]$	–	–	–	–	$4.5913^{+0.0001}_{-0.1995}$
E_3^6 $[m_\pi]$	–	–	–	–	$4.6634^{+0.0001}_{-0.1070}$
E_3^7 $[m_\pi]$	–	–	–	–	$4.6995^{+0.0001}_{-0.0661}$


Matching in the unphysical region
Full NLO K−Mat
Matched NLO K−Mat


m_π =139.57 MeV
m_π =291 MeV
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Re T_2
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E [m_π]