

Single and Double Meson Photo-Production

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GHP-APS

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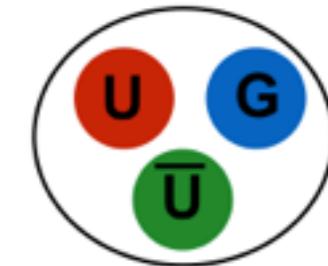
The Factorization Hypothesis



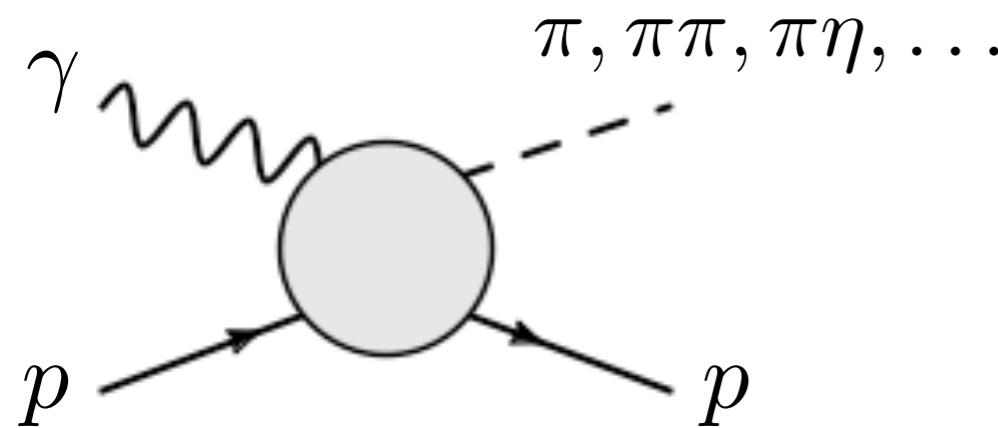
Photoproduction of mesons at $E_\gamma = 6 - 12$ GeV

Study photoproduction of mesons

Search for exotic resonances



Special interest in mesons:



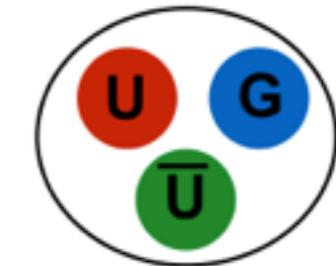
The Factorization Hypothesis



Photoproduction of mesons at $E_\gamma = 6 - 12 \text{ GeV}$

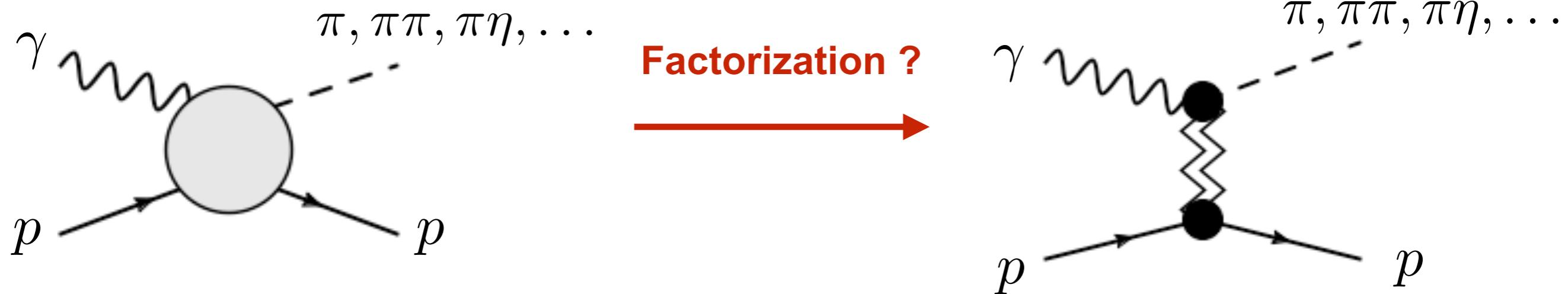
Study photoproduction of mesons

Search for exotic resonances



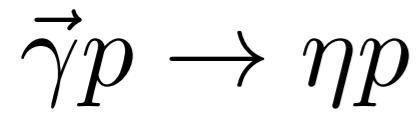
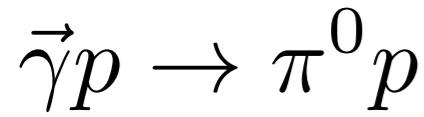
Special interest in mesons:

Does the target decouple at JLab energies ?

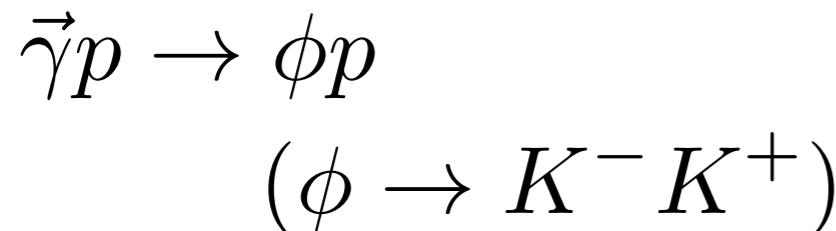
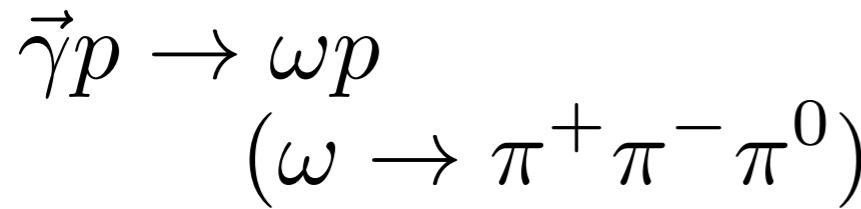
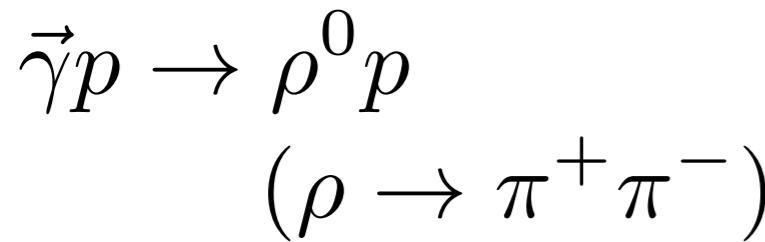


Outline

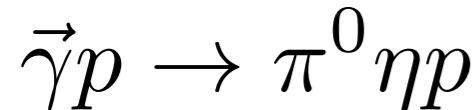
Single Meson Photoproduction:



Vector Meson Photoproduction:

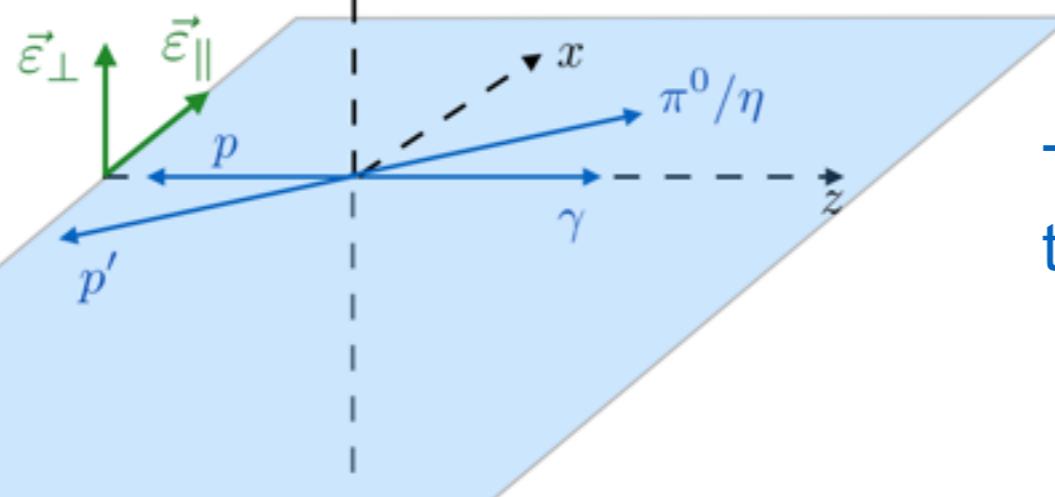


Double Mesons Photoproduction:

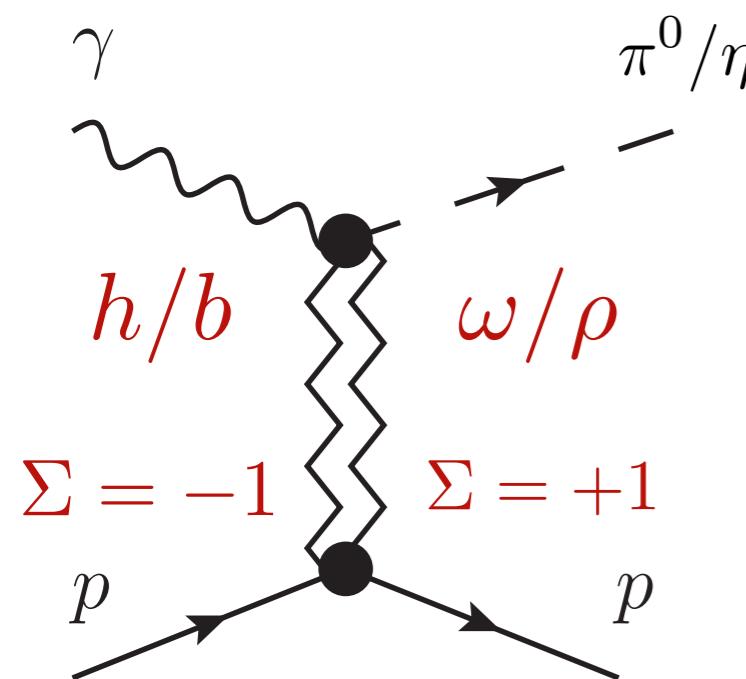


Single Meson Photoproduction

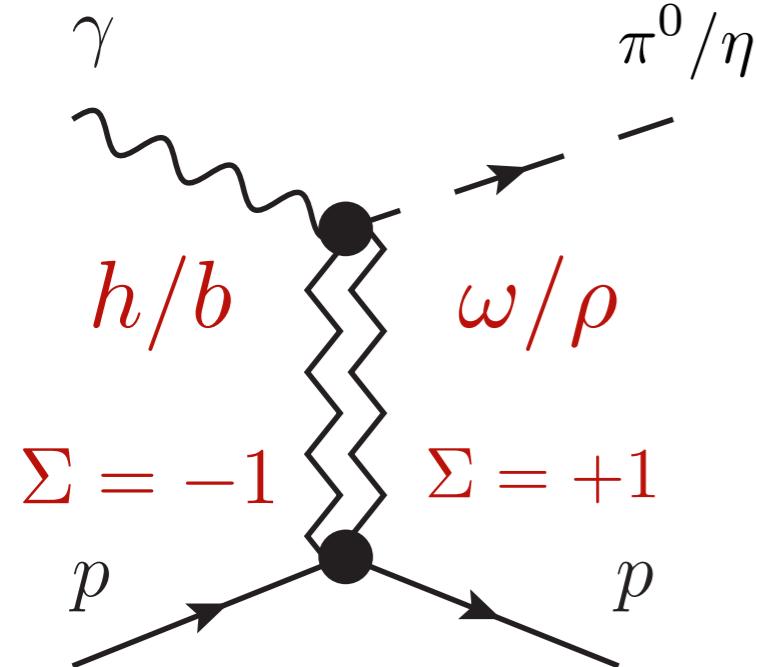
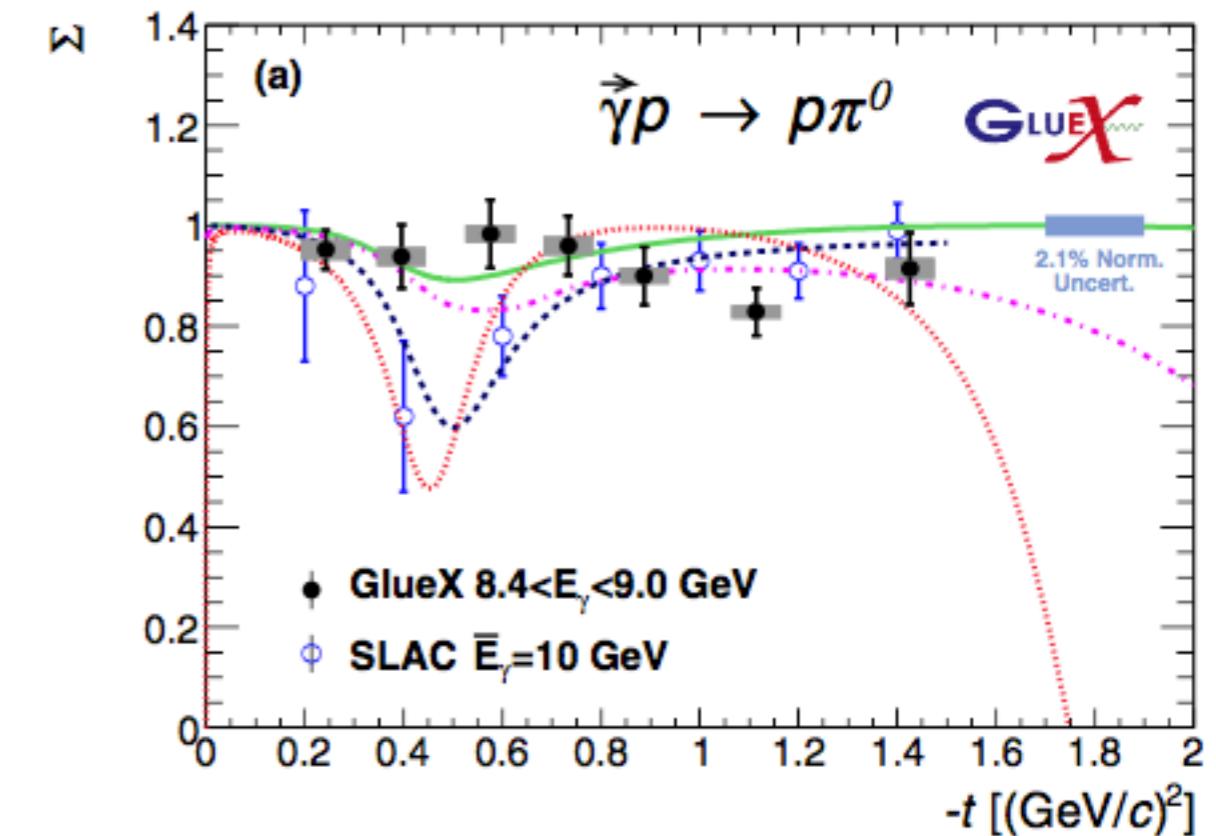
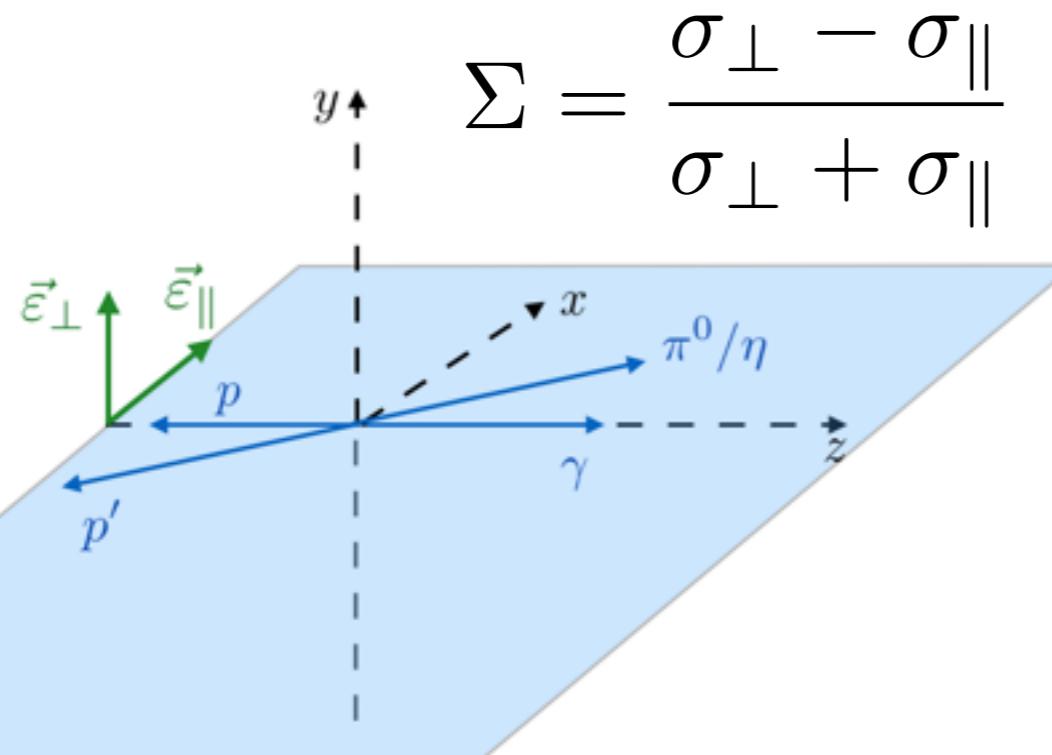
$$\Sigma = \frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}}$$



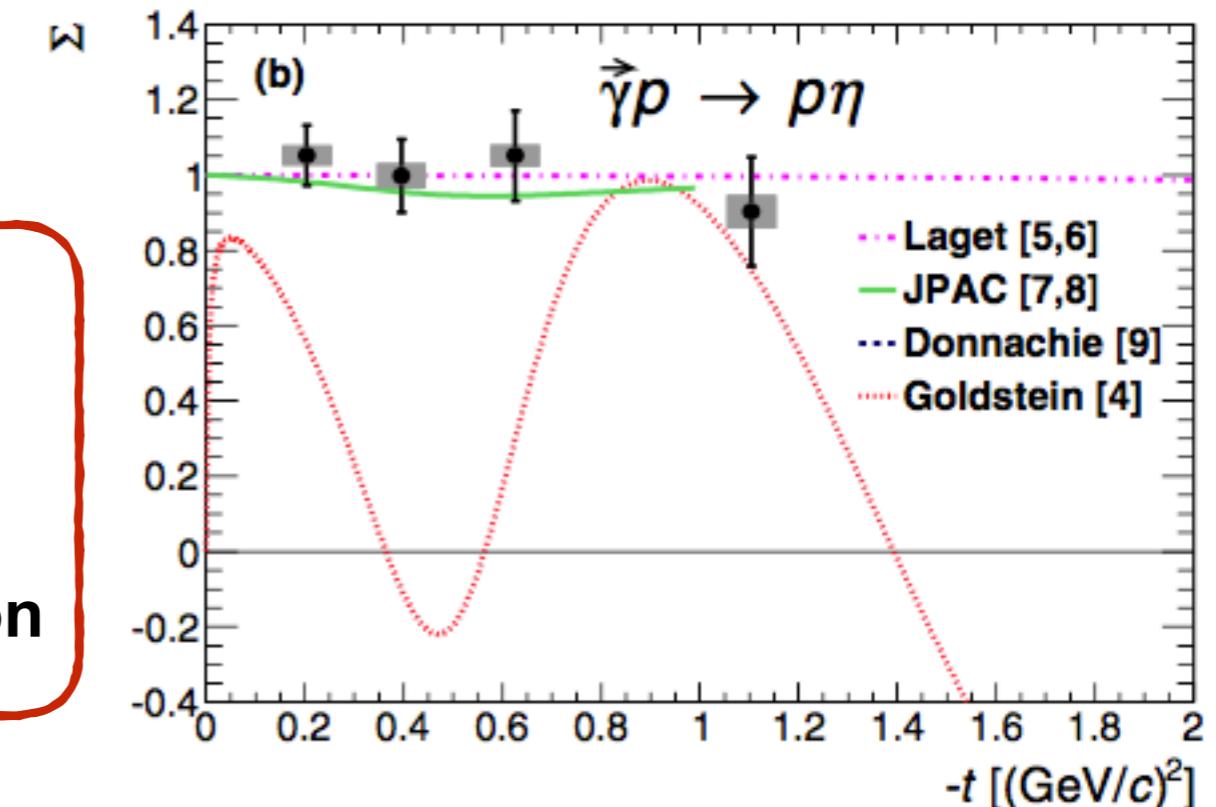
The beam asymmetry Σ is related to the reflection through the reaction plane



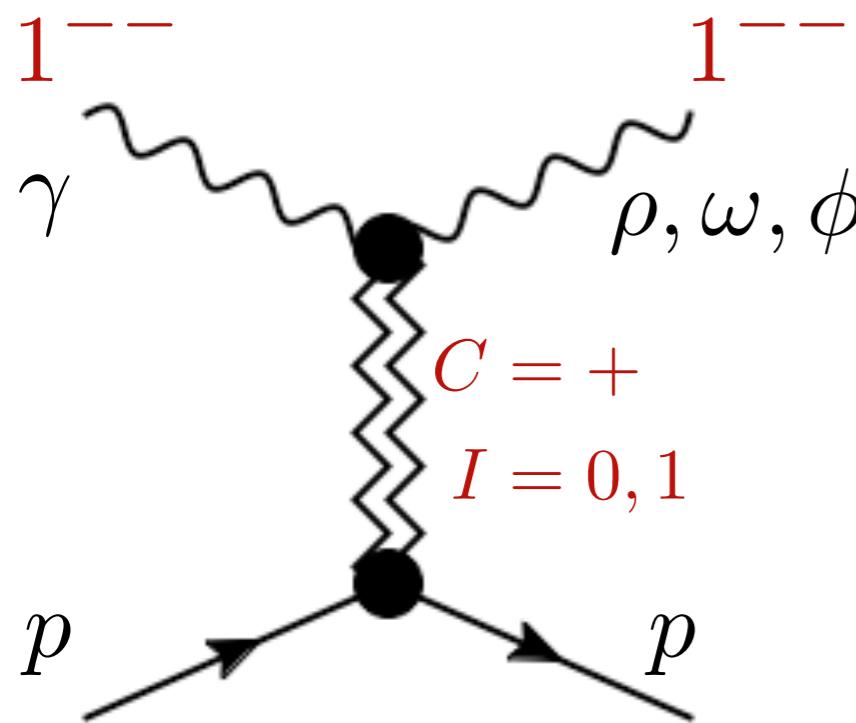
Single Meson Photoproduction



Dominance of vector meson exchange in both π^0/η photoproduction



Vector Meson Photoproduction



Probe different exchanges by combined analysis of ρ, ω, ϕ

Pomeron dominates at high energies

Use the angular distribution of the vector to extract spin density matrix elements

$$\frac{8\pi}{3} \frac{d\sigma}{d\Omega} = 1 - \rho_{00}^0 + (3\rho_{00}^0 - 1) \cos^2 \theta - 2\sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos \phi - 2\rho_{1-1}^0 \sin^2 \theta \cos 2\phi$$

9 SDME accessible with linearly polarized beam

$$\rho_{00}^0$$

$$\operatorname{Re} \rho_{10}^0$$

$$\rho_{1-1}^0$$

$$\rho_{11}^1$$

$$\operatorname{Re} \rho_{10}^1$$

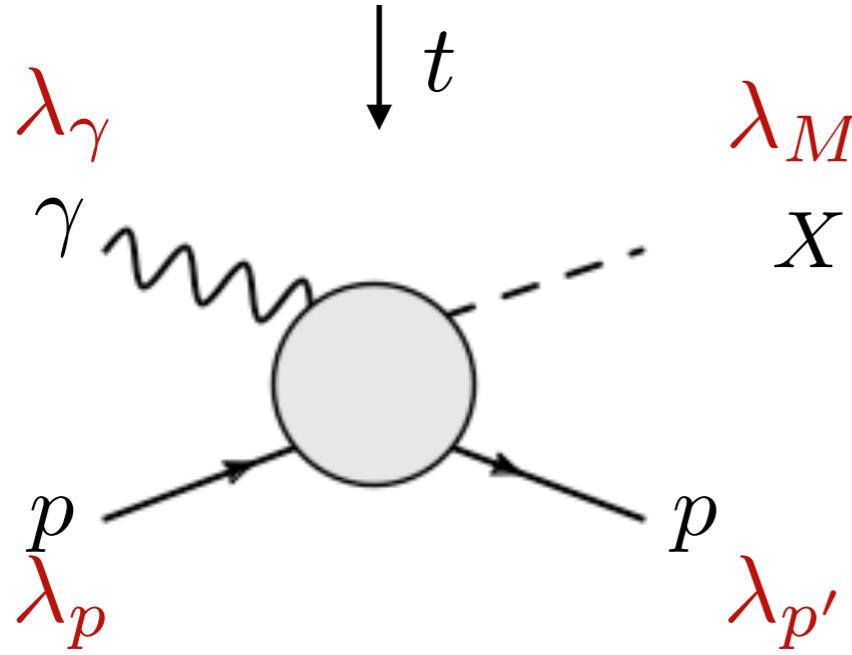
$$\rho_{11}^0$$

$$\rho_{1-1}^1$$

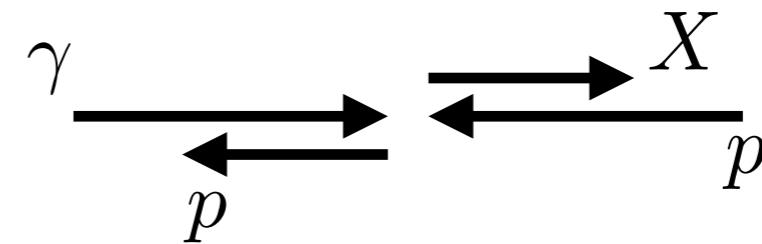
$$\operatorname{Im} \rho_{10}^2$$

$$\operatorname{Im} \rho_{1-1}^2$$

Factorization



Angular mom. conservation in forward direction:

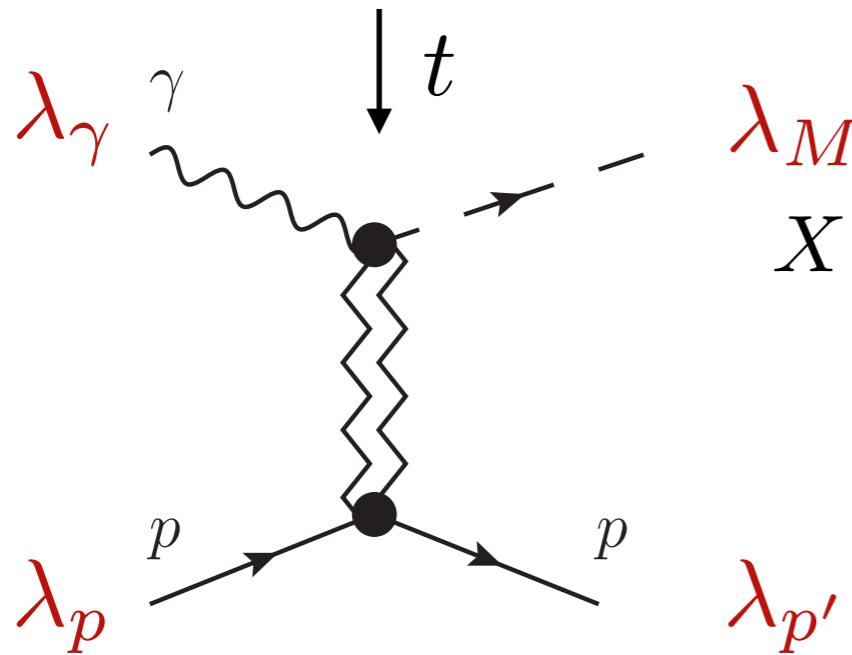


$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \underbrace{(\sin \theta/2)}_{\sqrt{-t}}^{|\lambda_\gamma - \lambda_M| - |\lambda_p - \lambda_{p'}|}$$

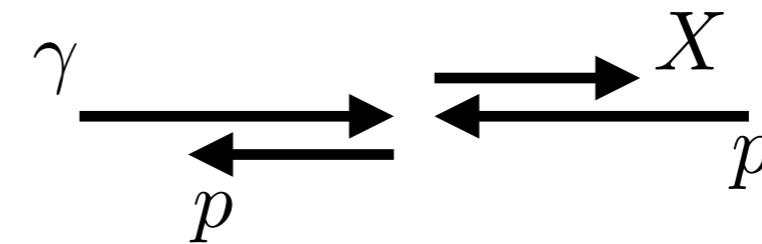
Leading order in the energy :

$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \gamma(t) (\sqrt{-t})^{|\lambda_\gamma - \lambda_M| - |\lambda_p - \lambda_{p'}|}$$

Factorization



Angular mom. conservation in forward direction:



$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \underbrace{(\sin \theta/2)}_{\sqrt{-t}}^{|\lambda_\gamma - \lambda_M|}$$

Leading order in the energy :

$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \gamma(t) (\sqrt{-t})^{|\lambda_\gamma - \lambda_M| - |\lambda_p - \lambda_{p'}|}$$

Factorization implies angular mom. conservation at each vertex:

$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M} \propto \gamma(t) (\sqrt{-t})^{|\lambda_\gamma - \lambda_M|} \times (\sqrt{-t})^{|\lambda_p - \lambda_{p'}|}$$

top vertex

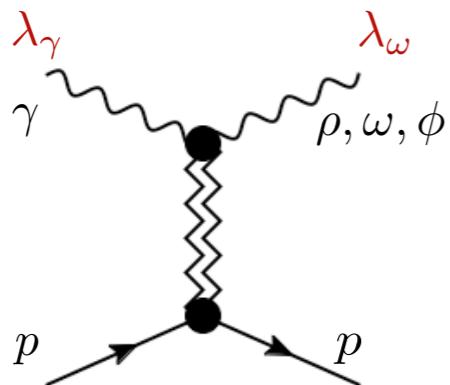
bottom vertex

Spin Density Matrix Elements

7

Use the angular distribution of the vector
to extract spin density matrix elements

$$\frac{8\pi}{3} \frac{d\sigma}{d\Omega} = 1 - \rho_{00}^0 + (3\rho_{00}^0 - 1) \cos^2 \theta - 2\sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos \phi - 2\rho_{1-1}^0 \sin^2 \theta \cos 2\phi$$



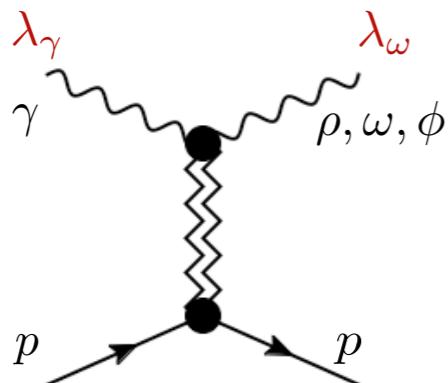
Structure at the top vertex:

$$T_{\lambda_\gamma \lambda_\omega} = \beta_0 \left(\delta_{\lambda_\gamma}^{\lambda_\omega} + \beta_1 \frac{\sqrt{-t}}{m_\omega} \delta_0^{\lambda_\omega} + \beta_2 \frac{-t}{m_\omega^2} \delta_{-\lambda_\gamma}^{\lambda_\omega} \right)$$

Spin Density Matrix Elements

Use the angular distribution of the vector
to extract spin density matrix elements

$$\frac{8\pi}{3} \frac{d\sigma}{d\Omega} = 1 - \rho_{00}^0 + (3\rho_{00}^0 - 1) \cos^2 \theta - 2\sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos \phi - 2\rho_{1-1}^0 \sin^2 \theta \cos 2\phi$$



Structure at the top vertex:

$$T_{\lambda_\gamma \lambda_\omega} = \beta_0 \left(\delta_{\lambda_\gamma}^{\lambda_\omega} + \beta_1 \frac{\sqrt{-t}}{m_\omega} \delta_0^{\lambda_\omega} + \beta_2 \frac{-t}{m_\omega^2} \delta_{-\lambda_\gamma}^{\lambda_\omega} \right)$$

$$\rho_{00}^0 = \frac{2}{N} \sum_{\lambda, \lambda'} \left| T_{\lambda, \lambda'}^{1,0} \right|^2$$

$$\operatorname{Re} \rho_{10}^0 = \frac{1}{N} \operatorname{Re} \sum_{\lambda, \lambda'} \left(T_{\lambda, \lambda'}^{1,1} - T_{\lambda, \lambda'}^{-1,-1} \right) T_{\lambda, \lambda'}^{*,1,0}$$

$$\rho_{1-1}^0 = \frac{2}{N} \operatorname{Re} \sum_{\lambda \lambda'} T_{\lambda, \lambda'}^{1,1} T_{\lambda, \lambda'}^{*,1,-1}$$

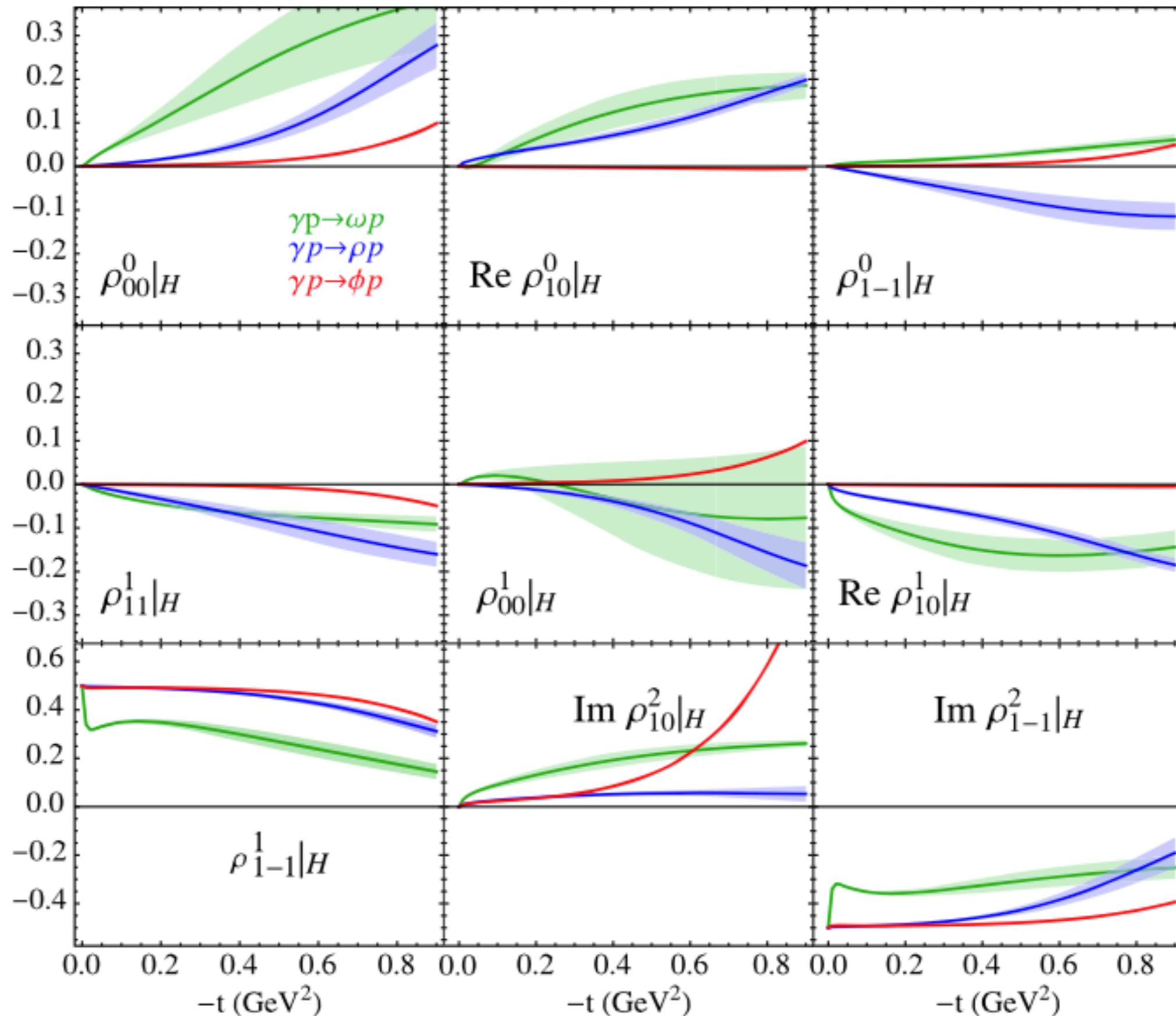
$$\rho_{00}^0 \propto \beta_1^2 \frac{-t}{m_\omega^2}$$

$$\operatorname{Re} \rho_{10}^0 \propto \frac{1}{2} \beta_1 \frac{\sqrt{-t}}{m_\omega}$$

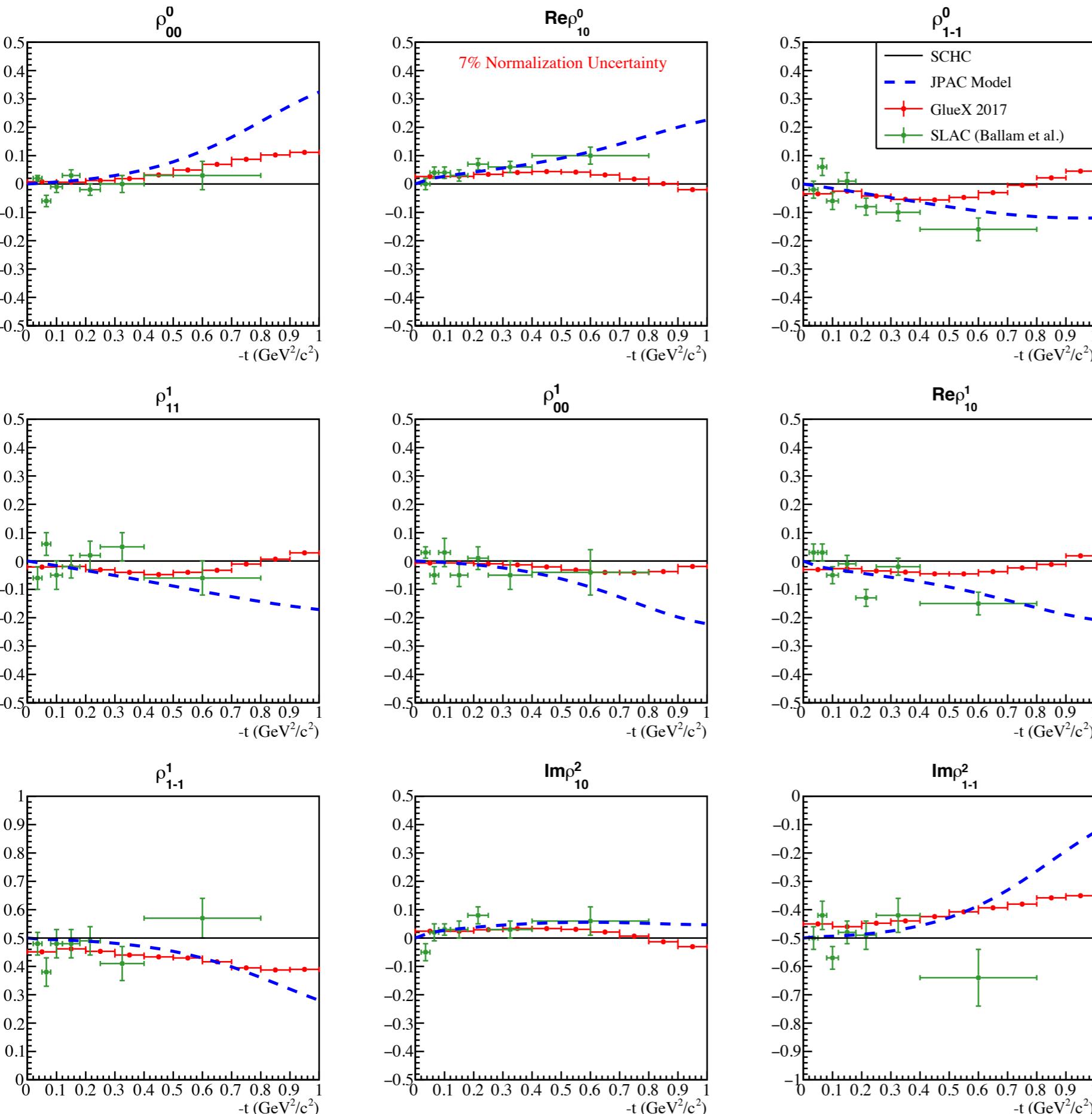
$$\rho_{1-1}^0 \propto \beta_2 \frac{-t}{m_\omega^2}$$

Predictions for Vector Meson SDME

VM et al (JPAC), PRD97 (2018)



$$\vec{\gamma}p \rightarrow \rho^0 p$$



GLUEX
Preliminary
Courtesy of A. Austregesilo

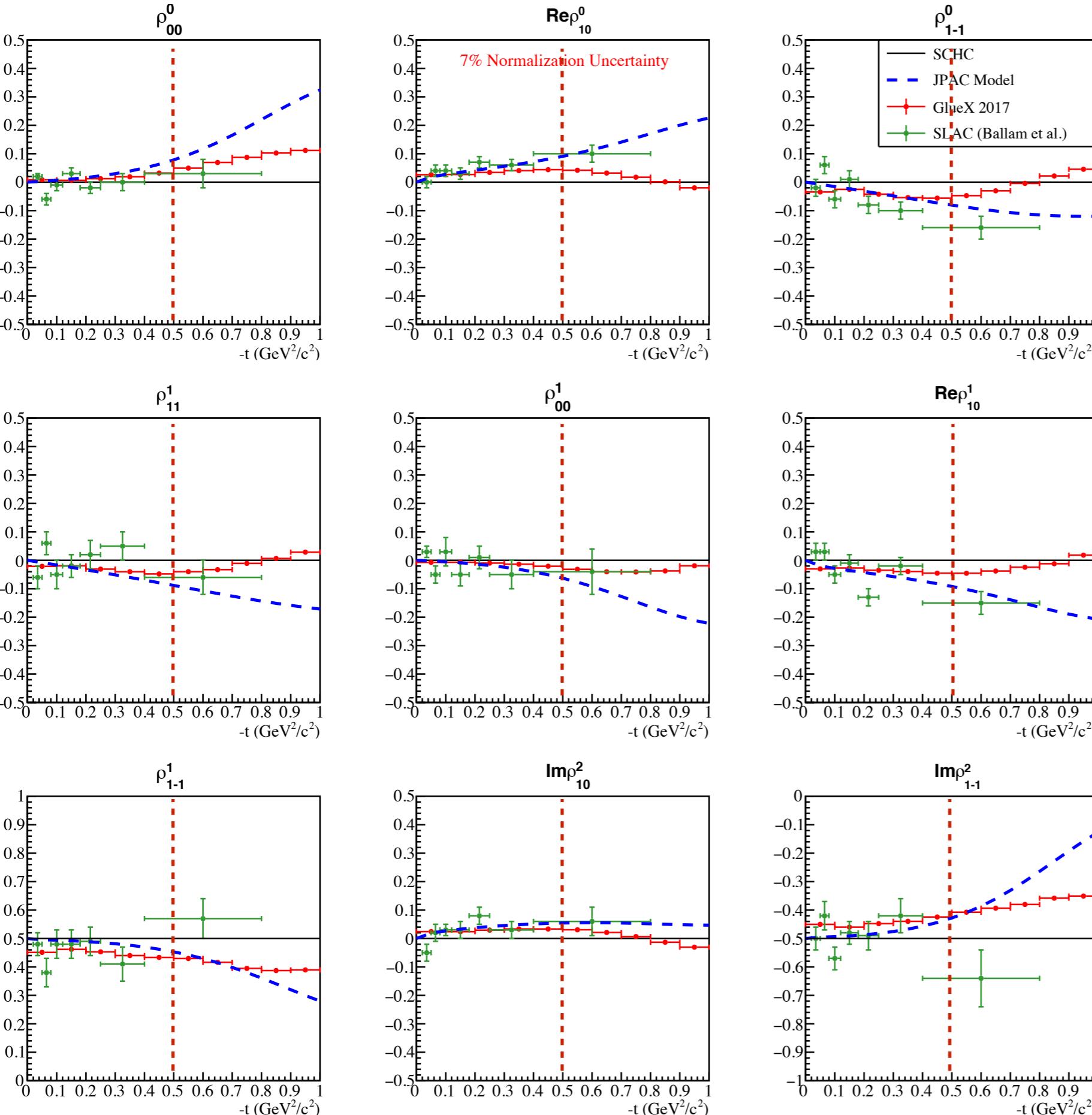
kinematics expanded
in power of $\frac{-t}{m_\rho^2}$

$$\rho_{1-1}^1 = \pm \frac{1}{2} + \mathcal{O}(t^2)$$

$$\text{Im } \rho_{1-1}^2 = \mp \frac{1}{2} + \mathcal{O}(t^2)$$

top sign for natural exchange
bottom sign for unnatural exch.

$$\vec{\gamma}p \rightarrow \rho^0 p$$



GLUEX
Preliminary
Courtesy of A. Austregesilo

kinematics expanded
in power of $\frac{-t}{m_\rho^2}$

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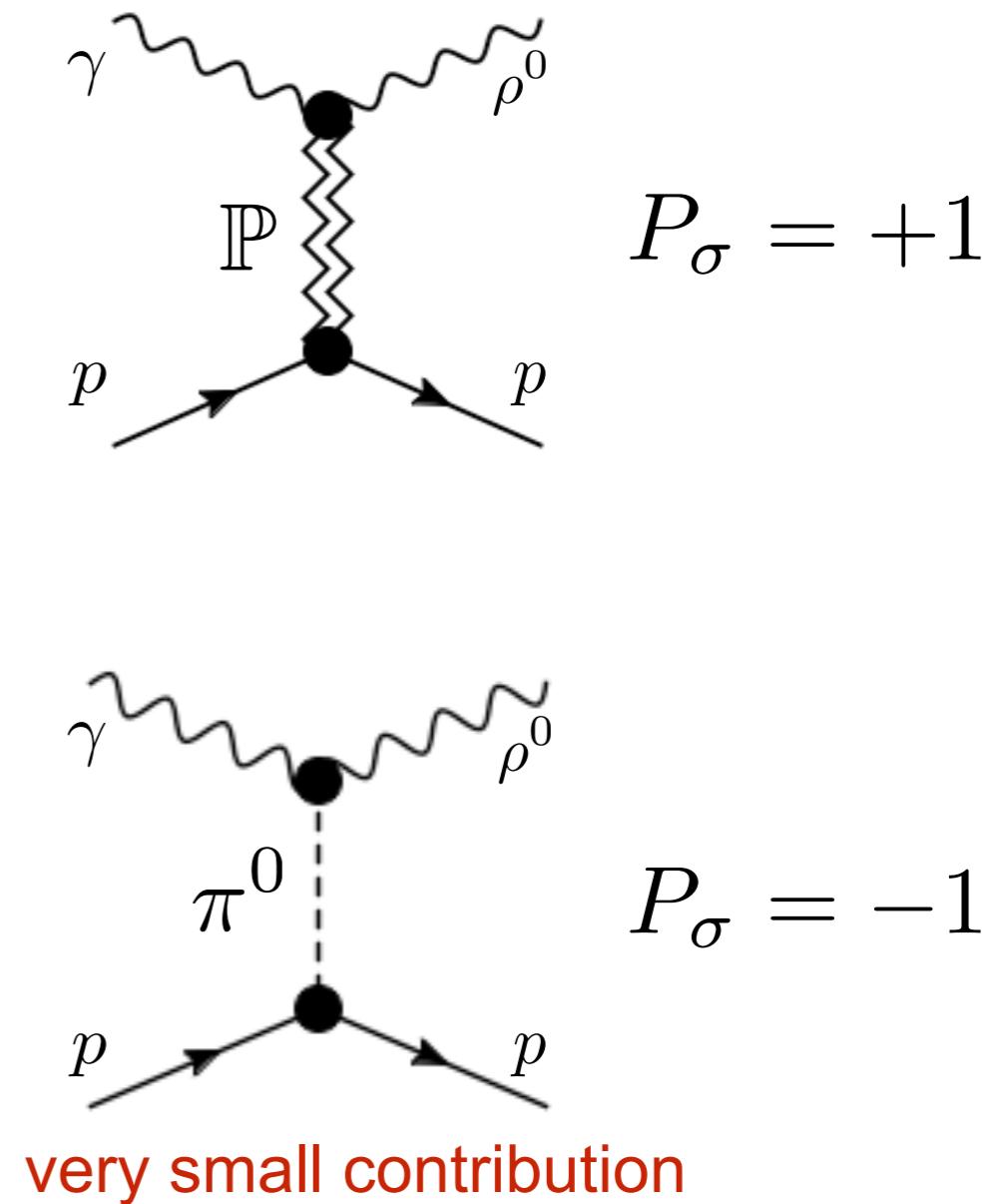
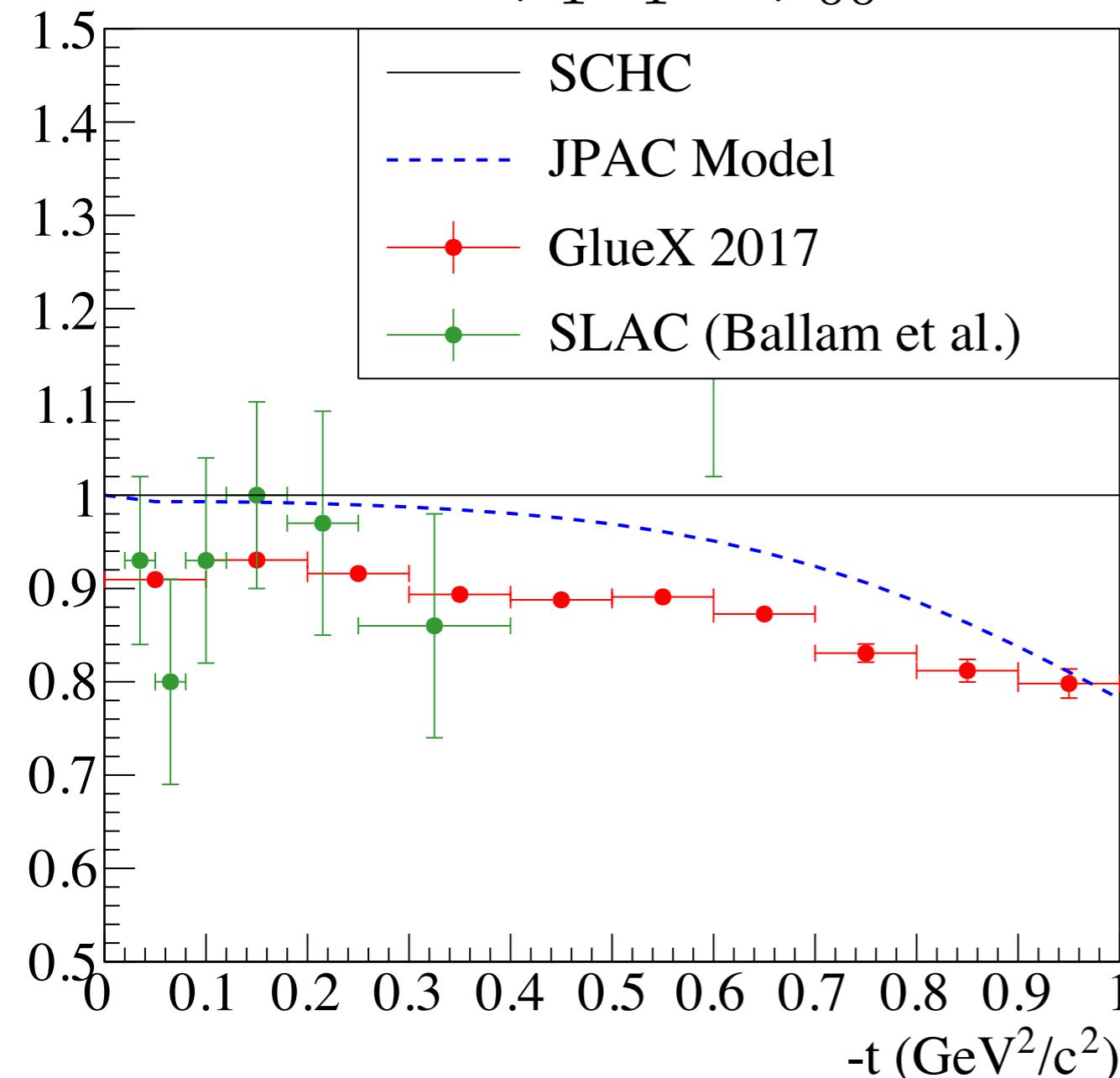
$$\text{Im } \rho_{1-1}^2 = \mp \frac{1}{2} + \mathcal{O}(t^2)$$

top sign for natural exchange
bottom sign for unnatural exch.

$$\vec{\gamma}p \rightarrow \rho^0 p$$

10

$$P_\sigma = 2\rho_{1-1}^1 - \rho_{00}^1$$



$P_\sigma \sim 0.9$ is the difference between natural vs unnatural exchanges

Outline

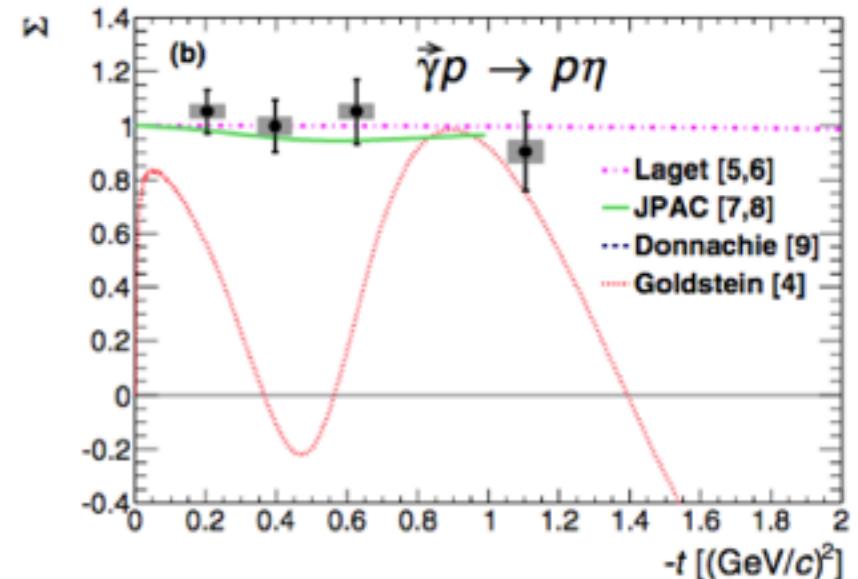
Conclusion

Single Meson Photoproduction:

$$\vec{\gamma}p \rightarrow \pi^0 p$$

$$\vec{\gamma}p \rightarrow \eta p$$

Dominance of natural exchanges in both π^0/η photoproduction



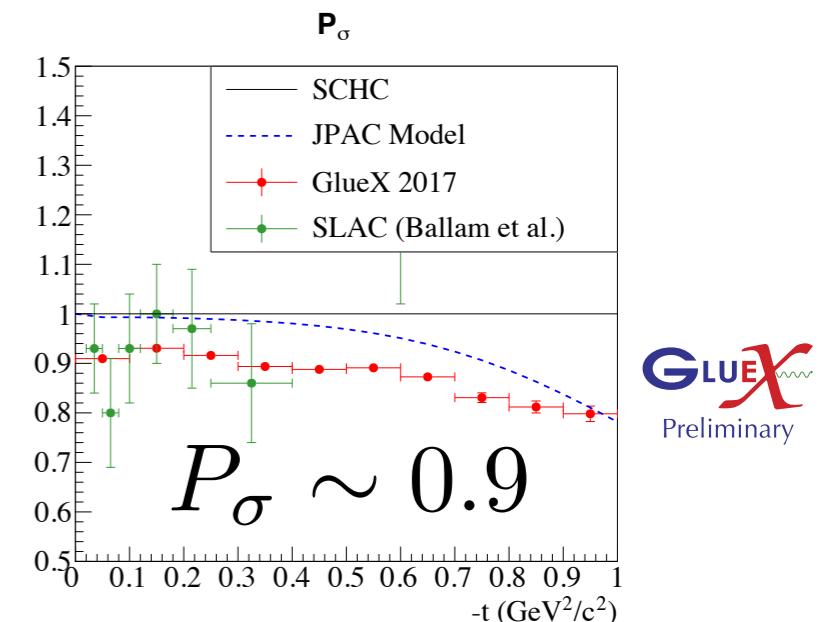
Vector Meson Photoproduction:

$$\vec{\gamma}p \rightarrow \rho^0 p$$

$$\vec{\gamma}p \rightarrow \omega p$$

$$\vec{\gamma}p \rightarrow \phi p$$

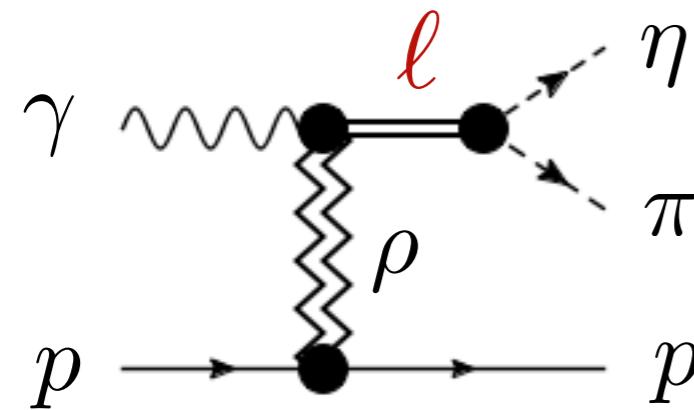
Consistent with factorization
Dominance of natural exchanges



Double Mesons Photoproduction:

$$\vec{\gamma}p \rightarrow \pi^0 \eta p$$

Model



$$R = \{ \underbrace{a_0(980)}, \underbrace{\pi_1(1600)}, \underbrace{a_2(1320)}, \underbrace{a_2(1700)} \}$$

$$S_0^{(+)} \quad P_{0,1}^{(+)} \quad D_{0,1,2}^{(+)}$$

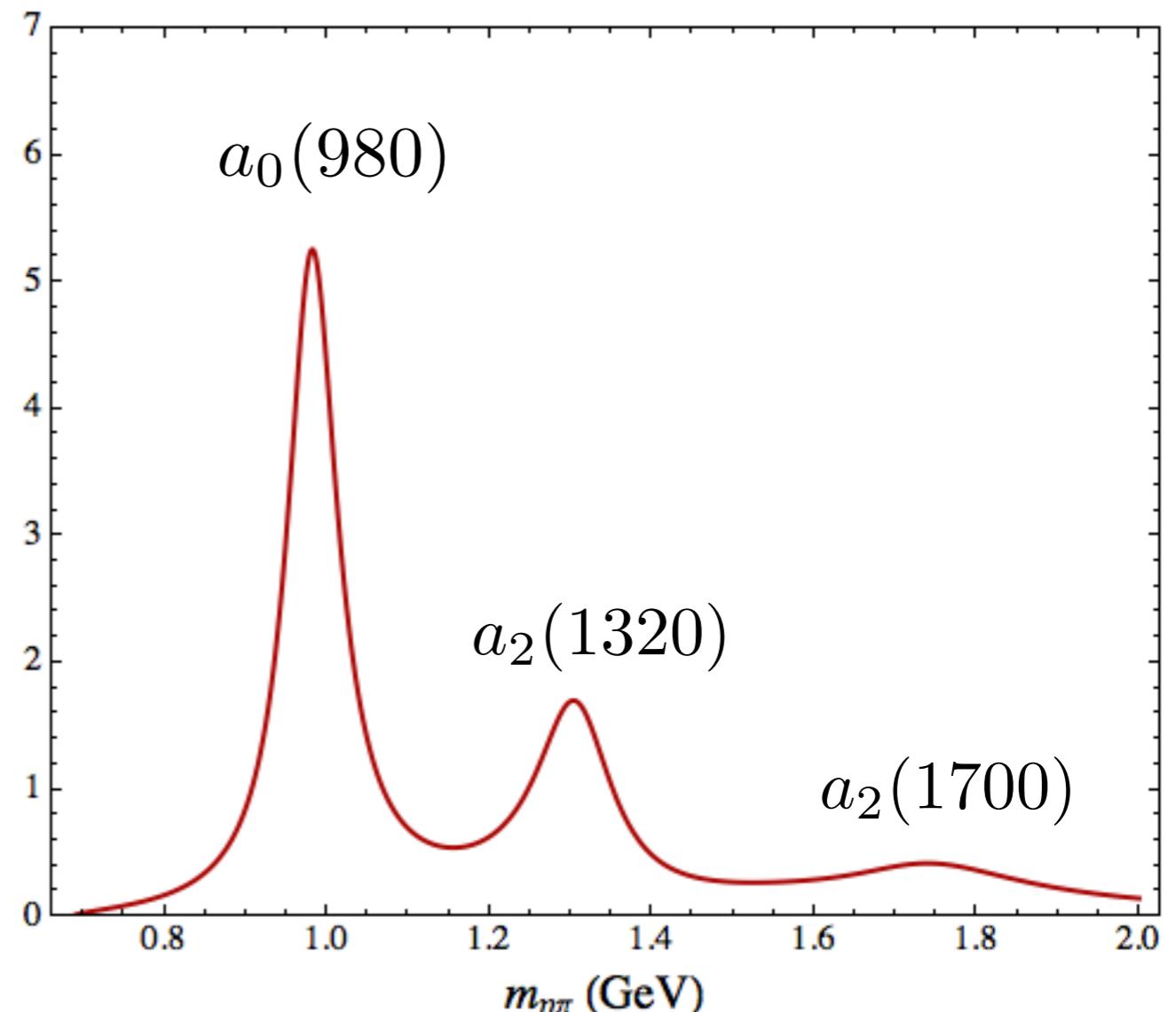
Quark content of the $a_0(980)$ meson: a red u quark, a blue g quark, and a green \bar{u} quark.

production: natural exchanges

line shape: Breit-Wigner form

parameters: arbitrary

Small exotic wave,
not apparent in the diff. cross. section

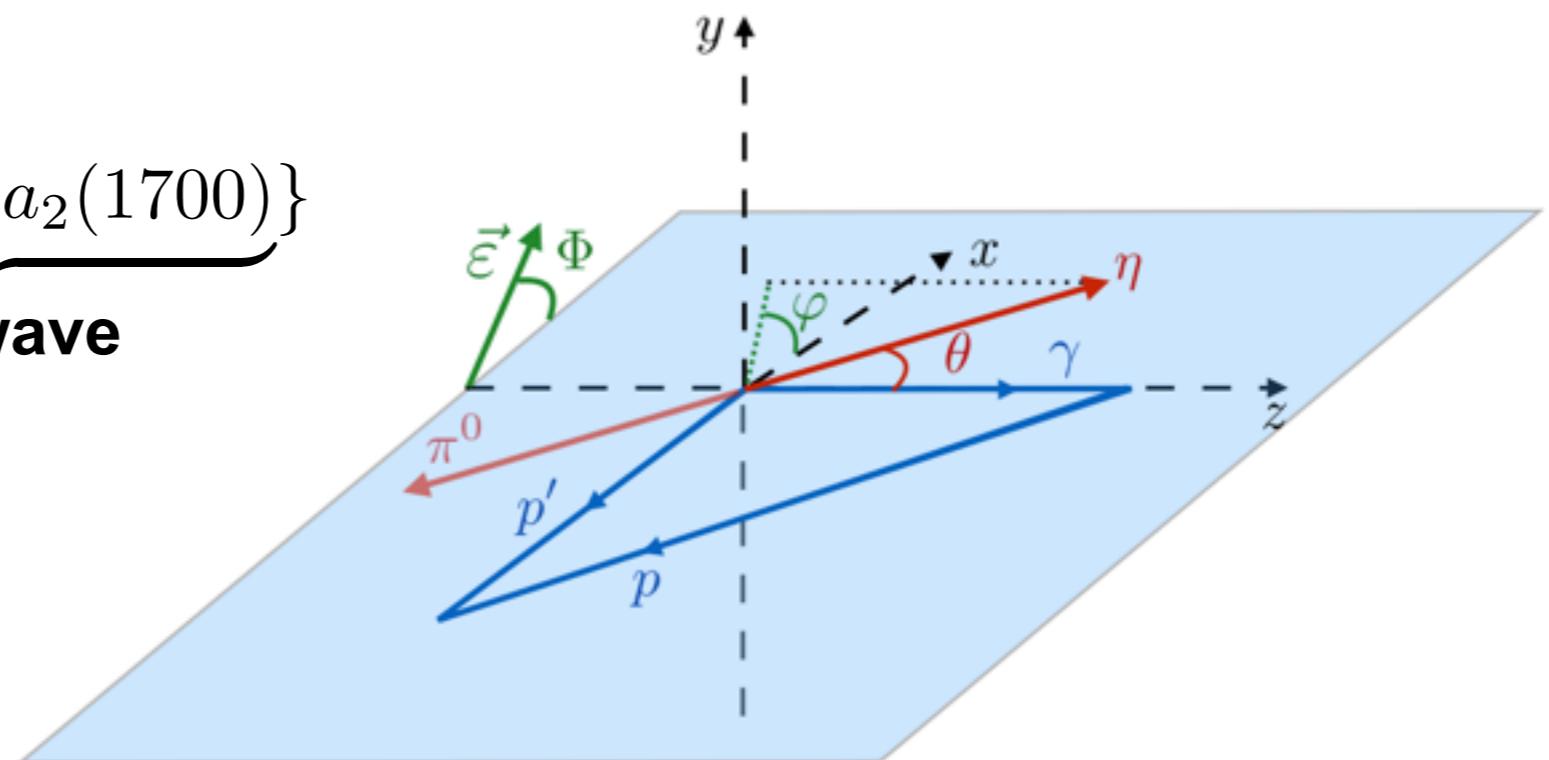
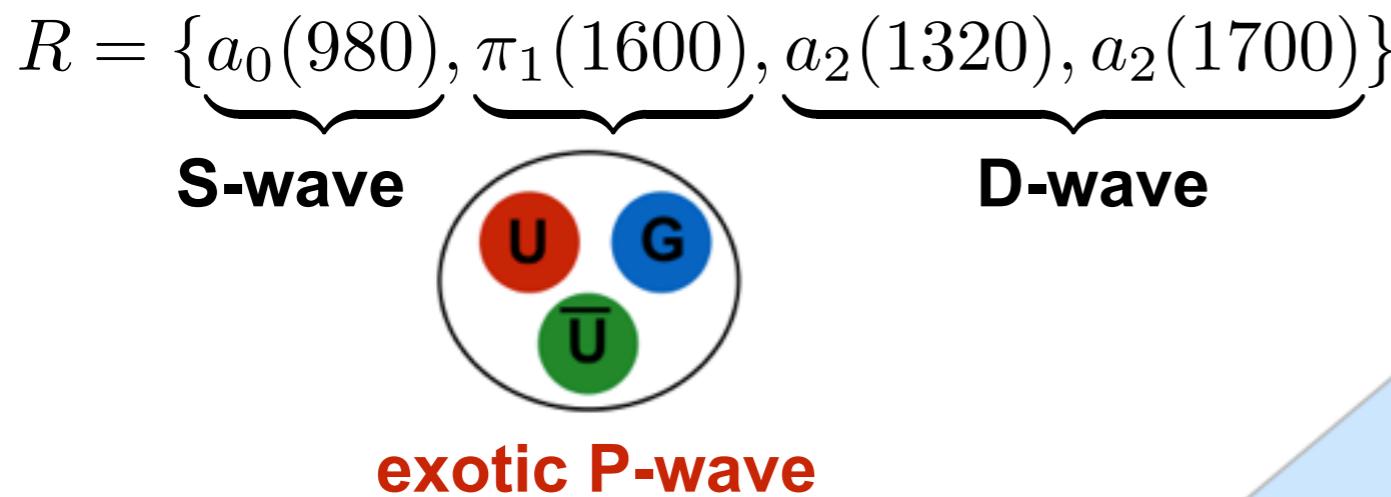


Observables: Moments of Angular distribution

13

$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega) \cos 2\Phi - P_\gamma I^2(\Omega) \sin 2\Phi$$

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$



Observables: Moments of Angular distribution

13

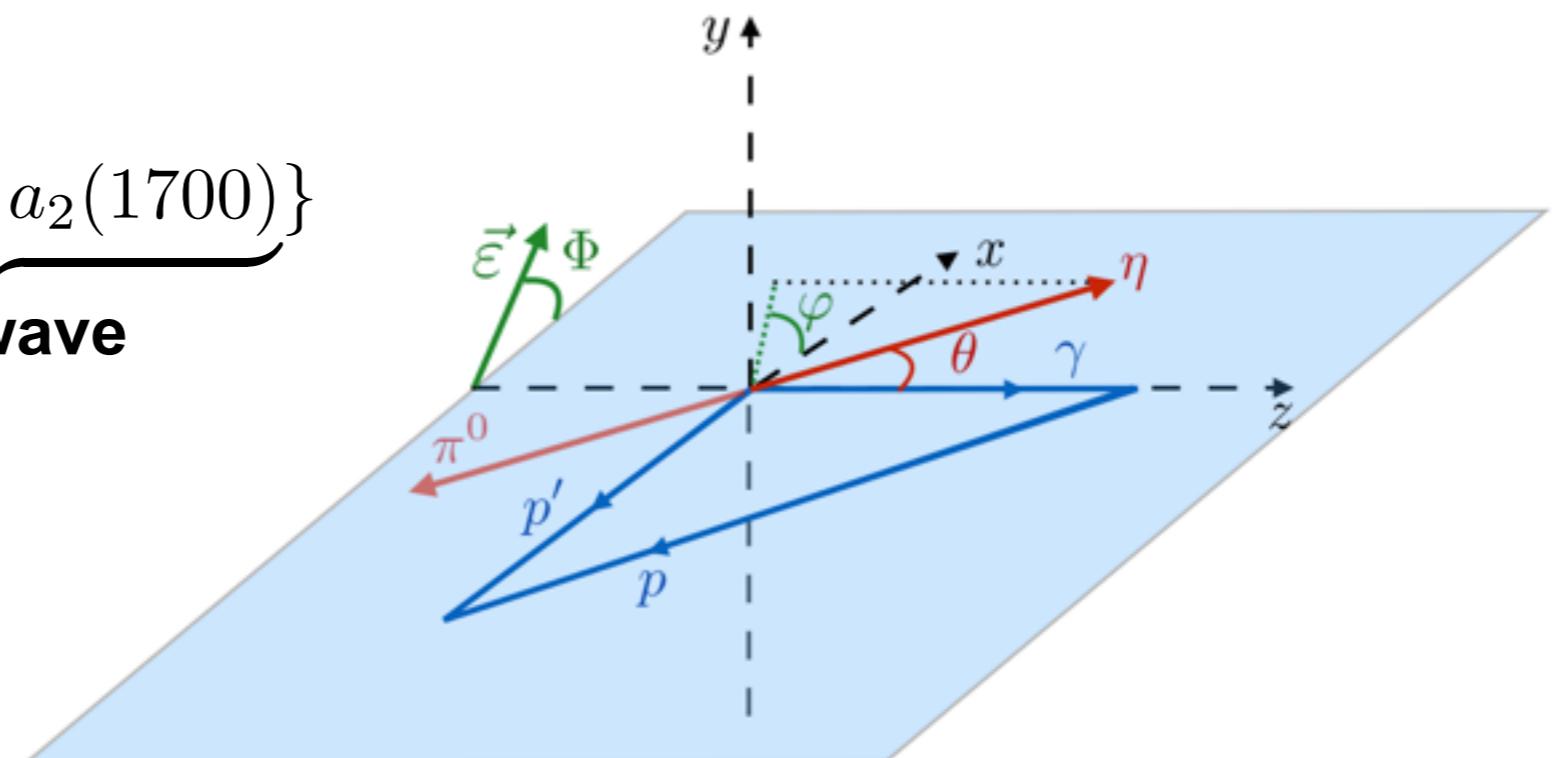
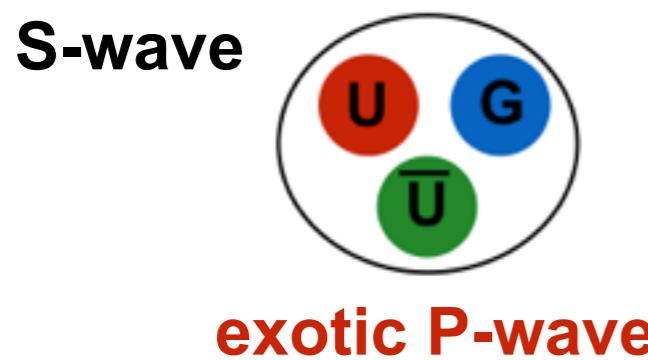
$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega) \cos 2\Phi - P_\gamma I^2(\Omega) \sin 2\Phi$$

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \boxed{\cos 2\Phi} d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

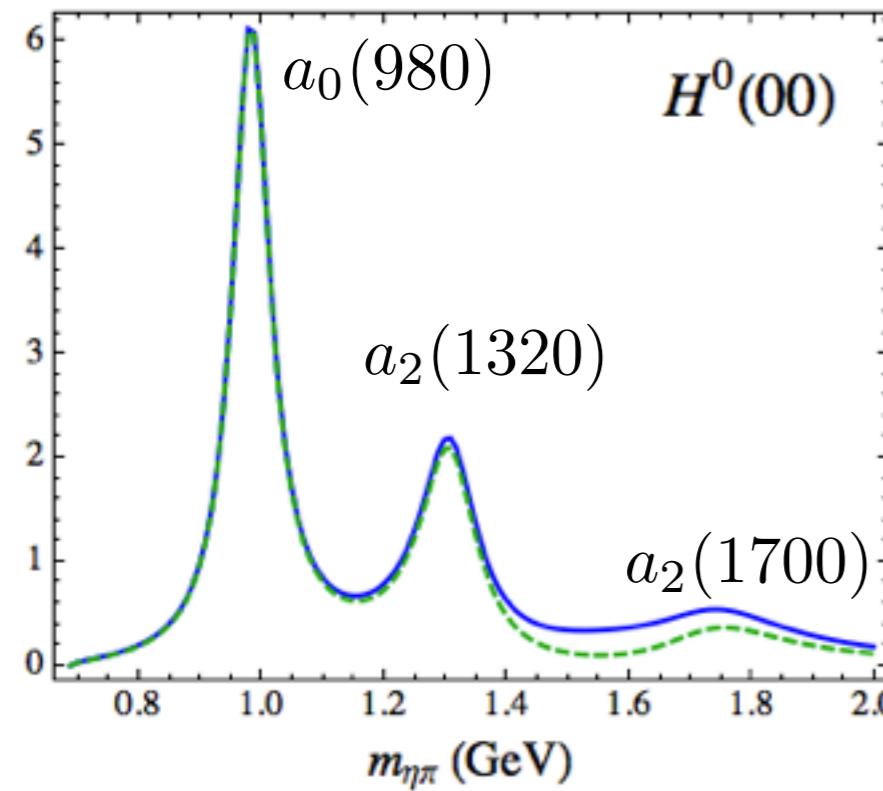
$$\text{Im } H^2(LM) = \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \boxed{\sin 2\Phi} d_{M0}^L(\theta) \sin M\phi \, d\Omega d\Phi$$

$$R = \underbrace{\{a_0(980), \pi_1(1600)\}}_{\textbf{S-wave}}, \underbrace{\{a_2(1320), a_2(1700)\}}_{\textbf{D-wave}}$$



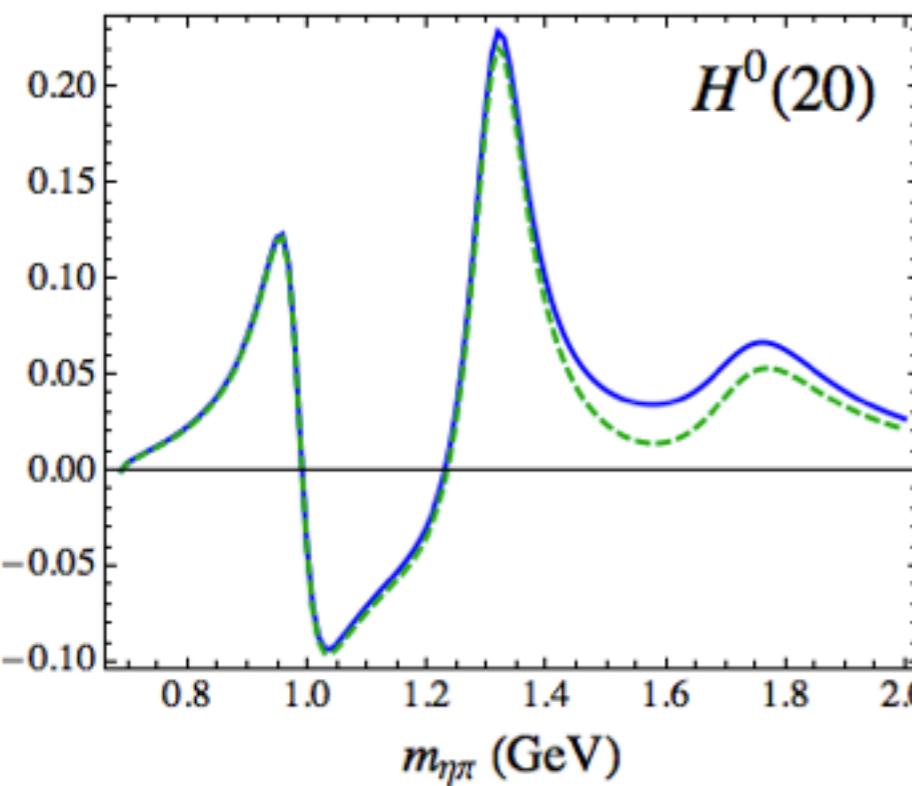
Moments

14



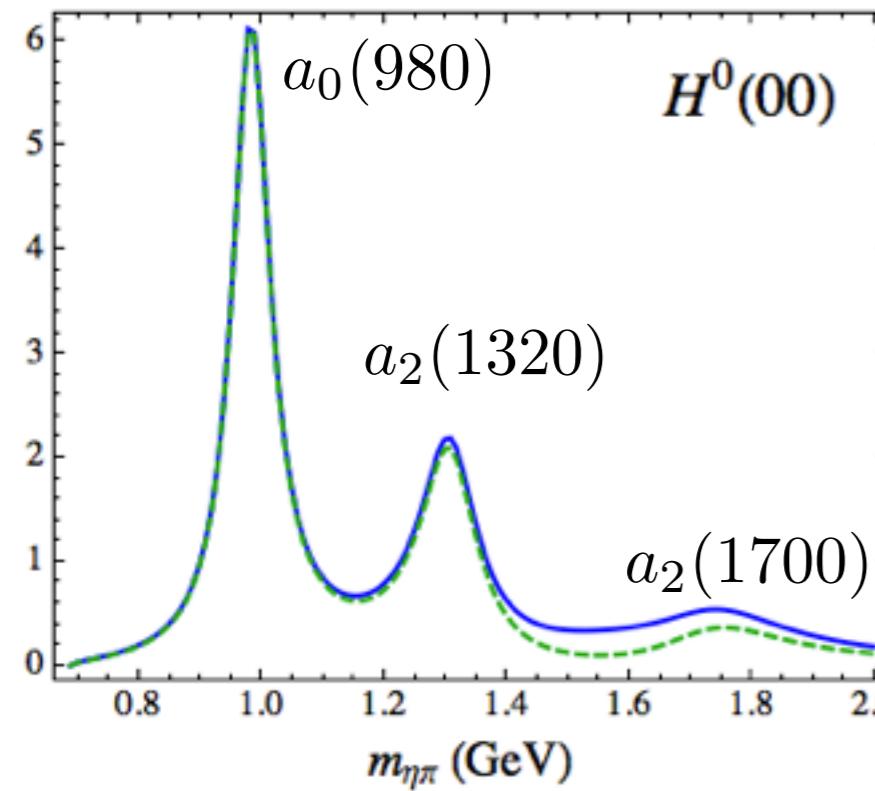
$$\begin{array}{lll} |S|^2 & |D|^2 & |P|^2 \\ (S+D)(S+D)^* \end{array}$$

solid lines: $S + P + D$ waves
dashed lines: $S + D$ waves

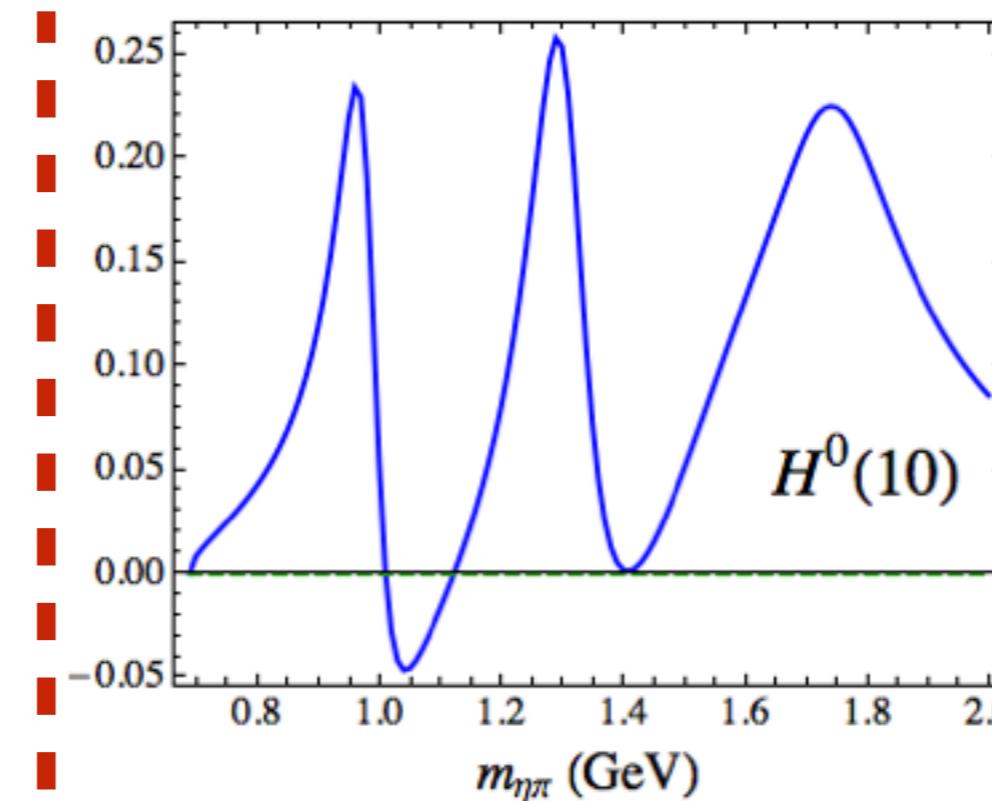


Moments

14

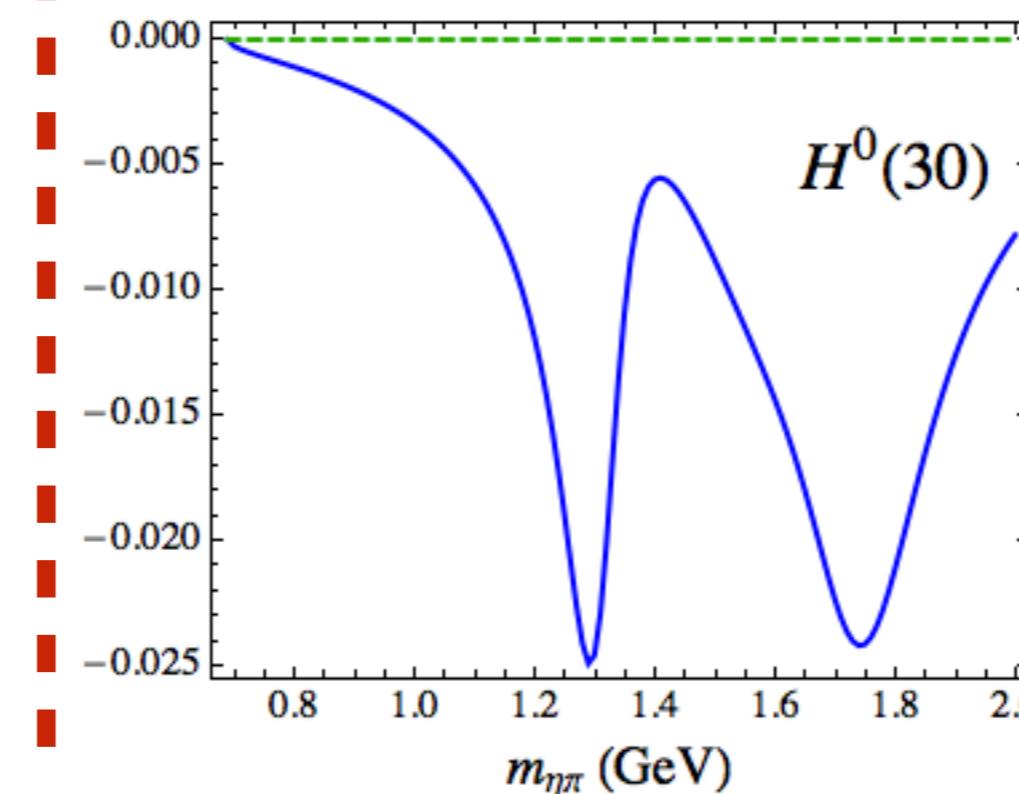
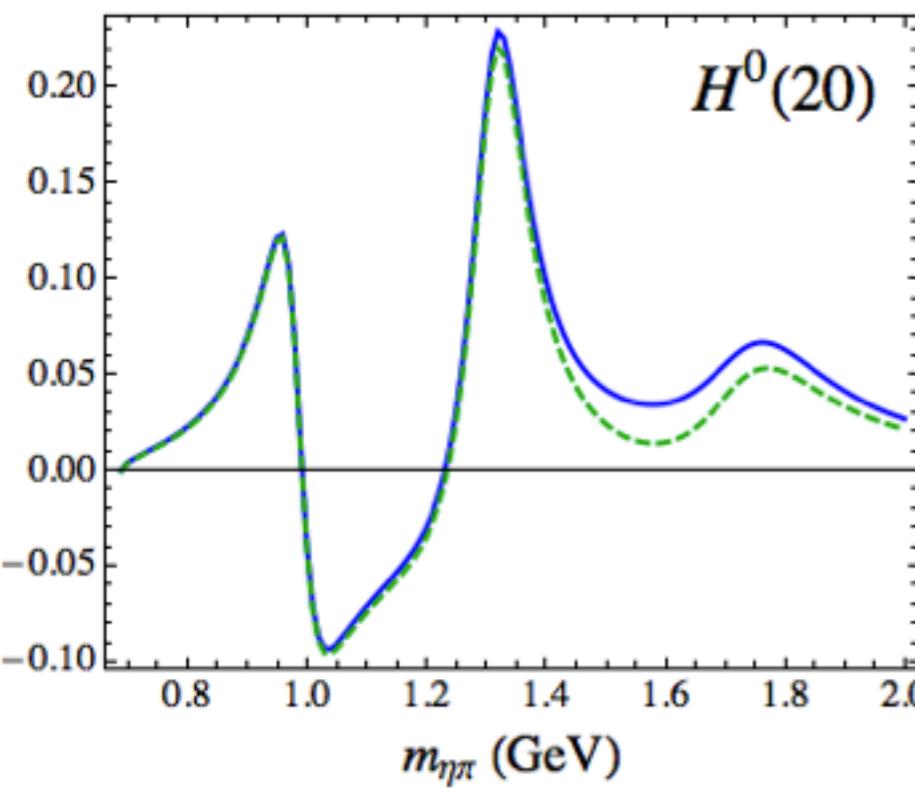


$$\begin{array}{ccc} |S|^2 & |D|^2 & |P|^2 \\ (S+D)(S+D)^* & & \end{array}$$



solid lines: $S + P + D$ waves
dashed lines: $S + D$ waves

**P- wave apparent as
an interference
in **odd moments** but
not in even moments**

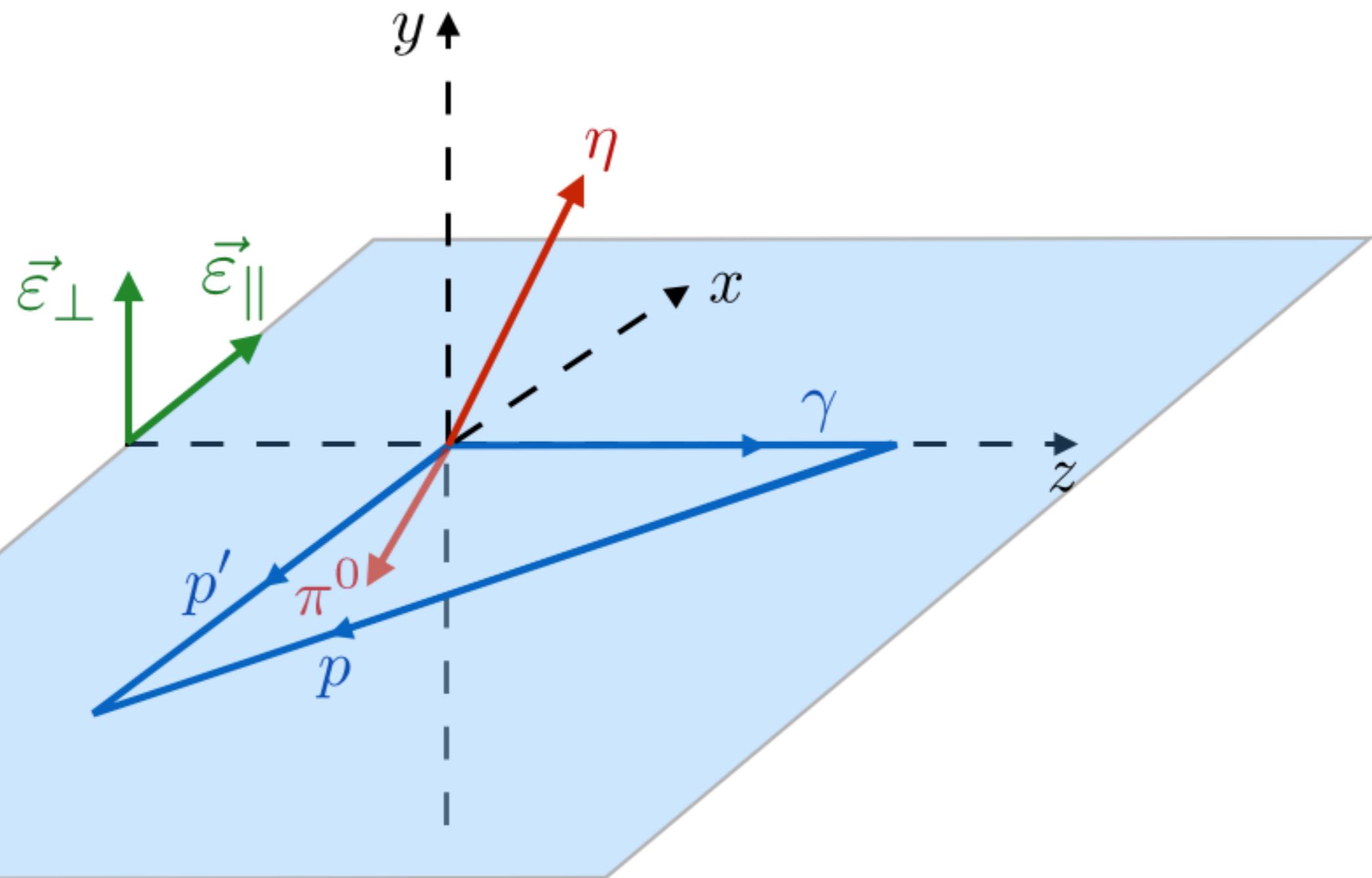


Beam Asymmetries

15

$$\Sigma_{\mathcal{D}} = \frac{1}{P_\gamma} \frac{\int_{\mathcal{D}} I^{\parallel}(\Omega) - I^{\perp}(\Omega) d\Omega}{\int_{\mathcal{D}} I^{\parallel}(\Omega) + I^{\perp}(\Omega) d\Omega}$$

$\Sigma_{4\pi}$ = fully integrated

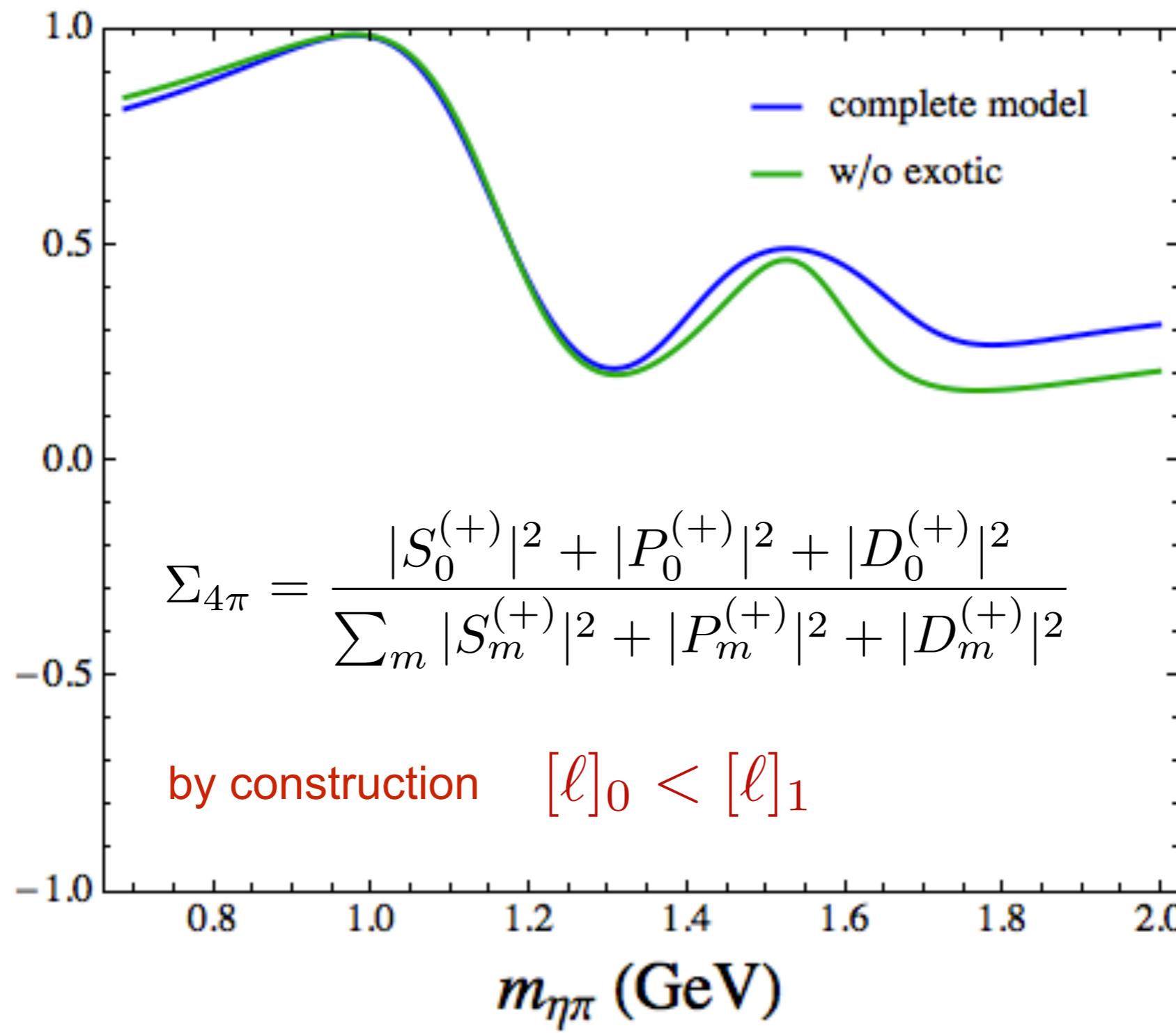


Beam Asymmetries

15

$$\Sigma_{\mathcal{D}} = \frac{1}{P_\gamma} \frac{\int_{\mathcal{D}} I^{\parallel}(\Omega) - I^{\perp}(\Omega) d\Omega}{\int_{\mathcal{D}} I^{\parallel}(\Omega) + I^{\perp}(\Omega) d\Omega}$$

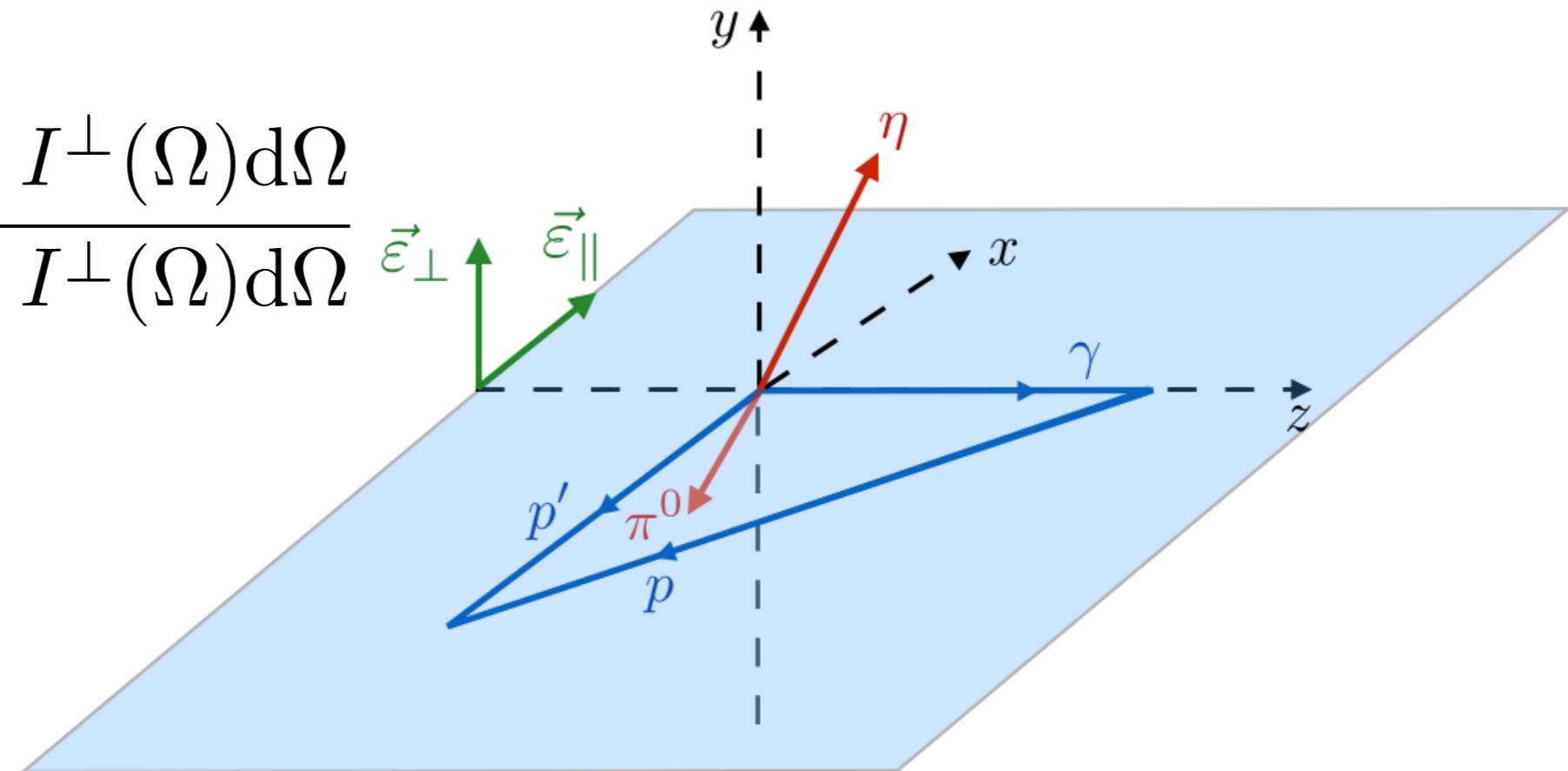
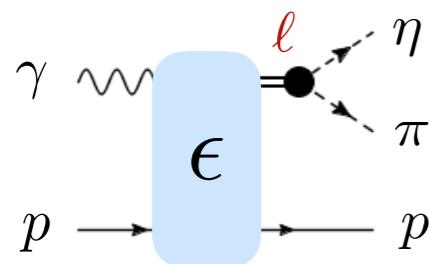
$\Sigma_{4\pi}$ = fully integrated



Beam Asymmetries

$$\Sigma_{\mathcal{D}} = \frac{1}{P_\gamma} \frac{\int_{\mathcal{D}} I^{\parallel}(\Omega) - I^{\perp}(\Omega) d\Omega}{\int_{\mathcal{D}} I^{\parallel}(\Omega) + I^{\perp}(\Omega) d\Omega}$$

**amplitude:
production x decay**



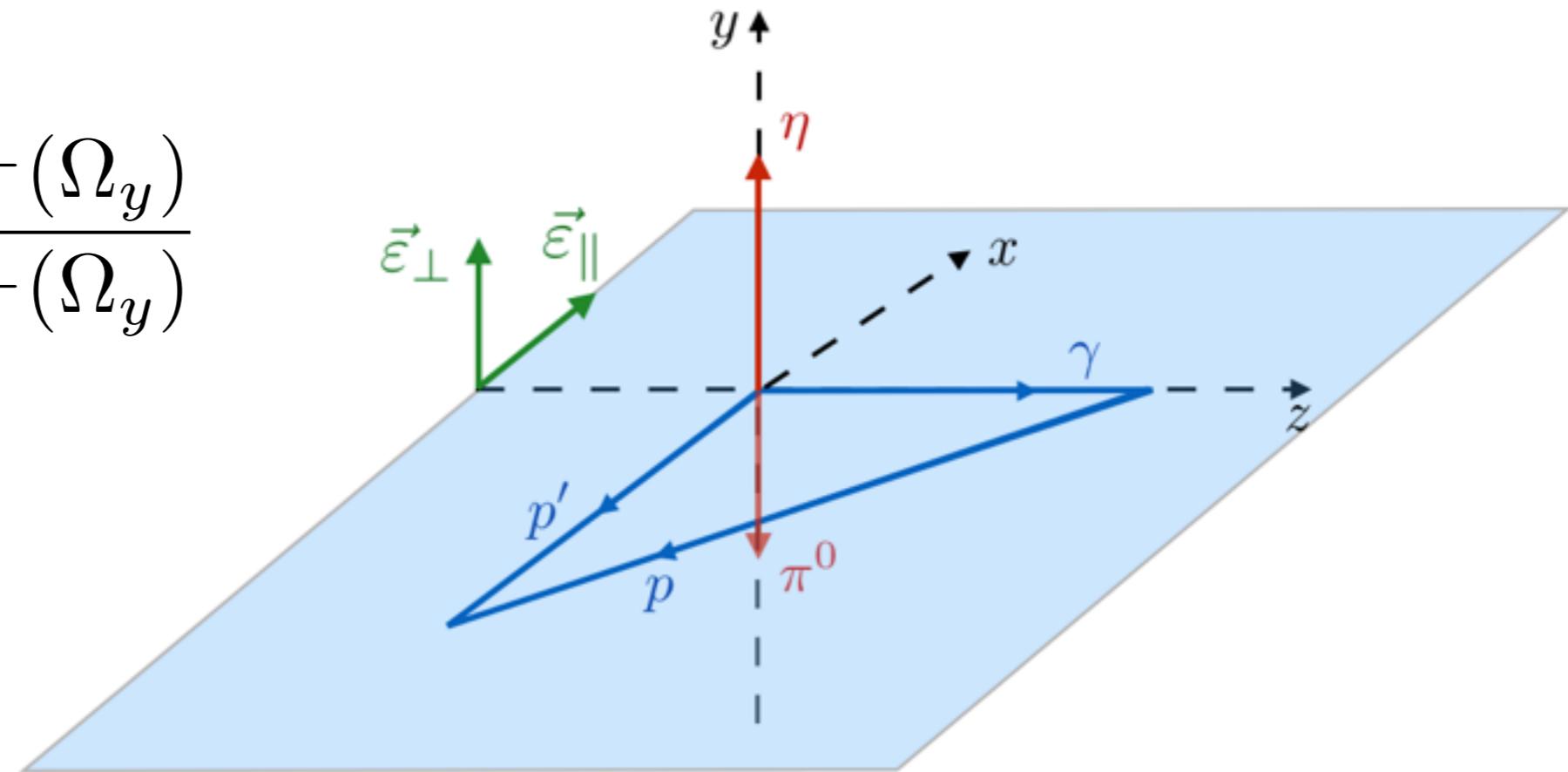
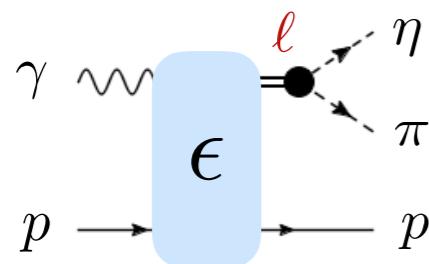
Beam asymmetry sensitive to reflection through the reaction plane

use reflection operator = parity followed by 180° rotation around Y-axis

Beam Asymmetries

$$\Sigma_y = \frac{1}{P_\gamma} \frac{I^{\parallel}(\Omega_y) - I^{\perp}(\Omega_y)}{I^{\parallel}(\Omega_y) + I^{\perp}(\Omega_y)}$$

**amplitude:
production x decay**



Beam asymmetry sensitive to reflection through the reaction plane

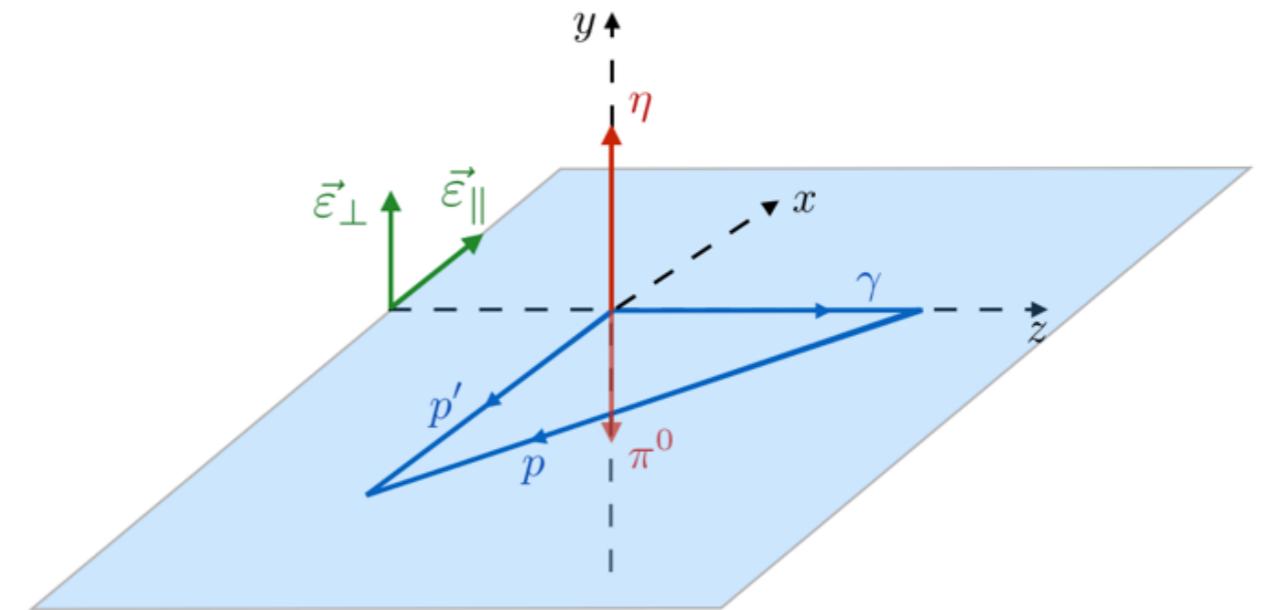
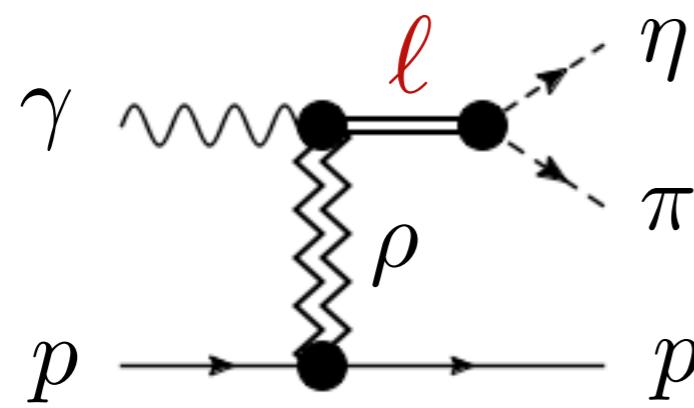
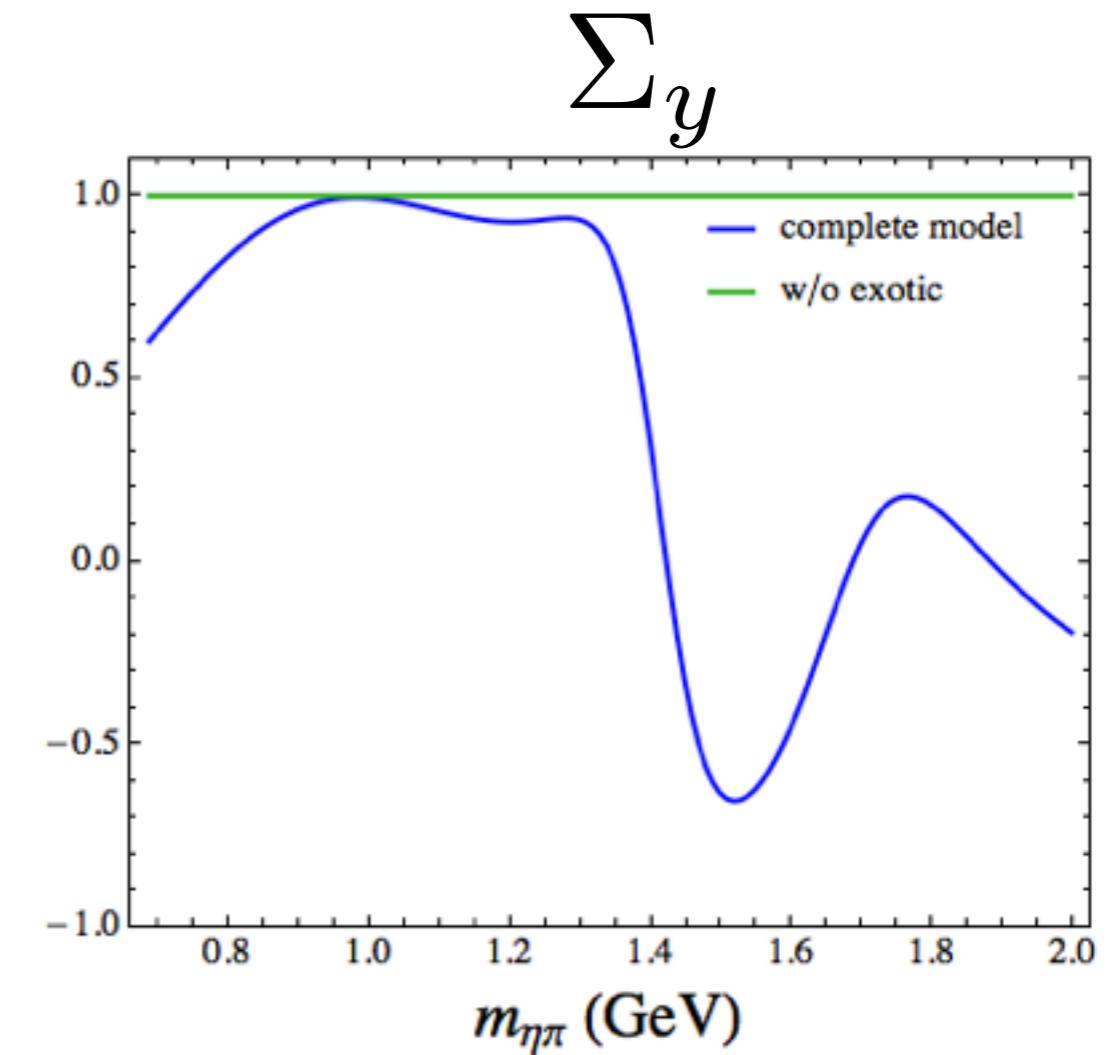
use reflection operator = parity followed by 180° rotation around Y-axis

$$[\ell]_m^{(\epsilon)} \longrightarrow \Sigma_y = \epsilon(-1)^\ell$$

Odd waves change sign!!!

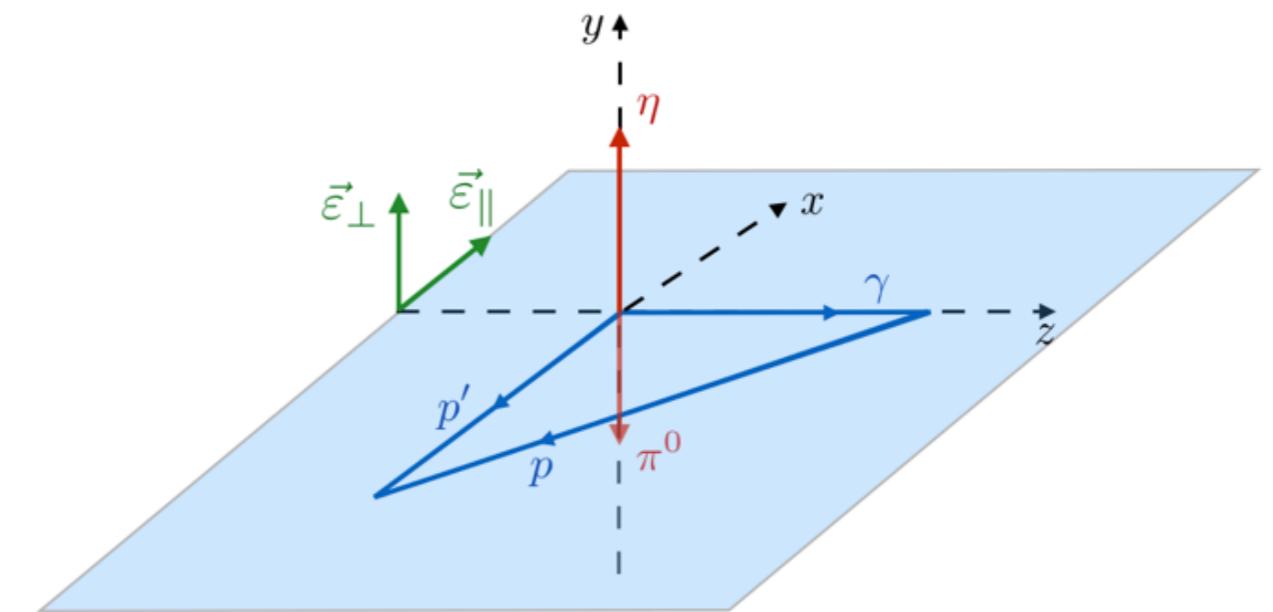
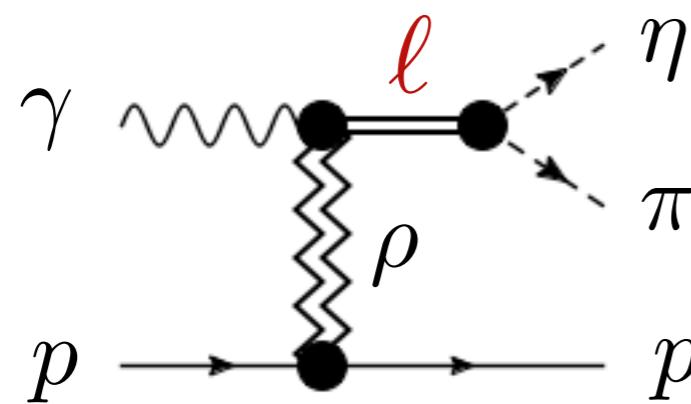
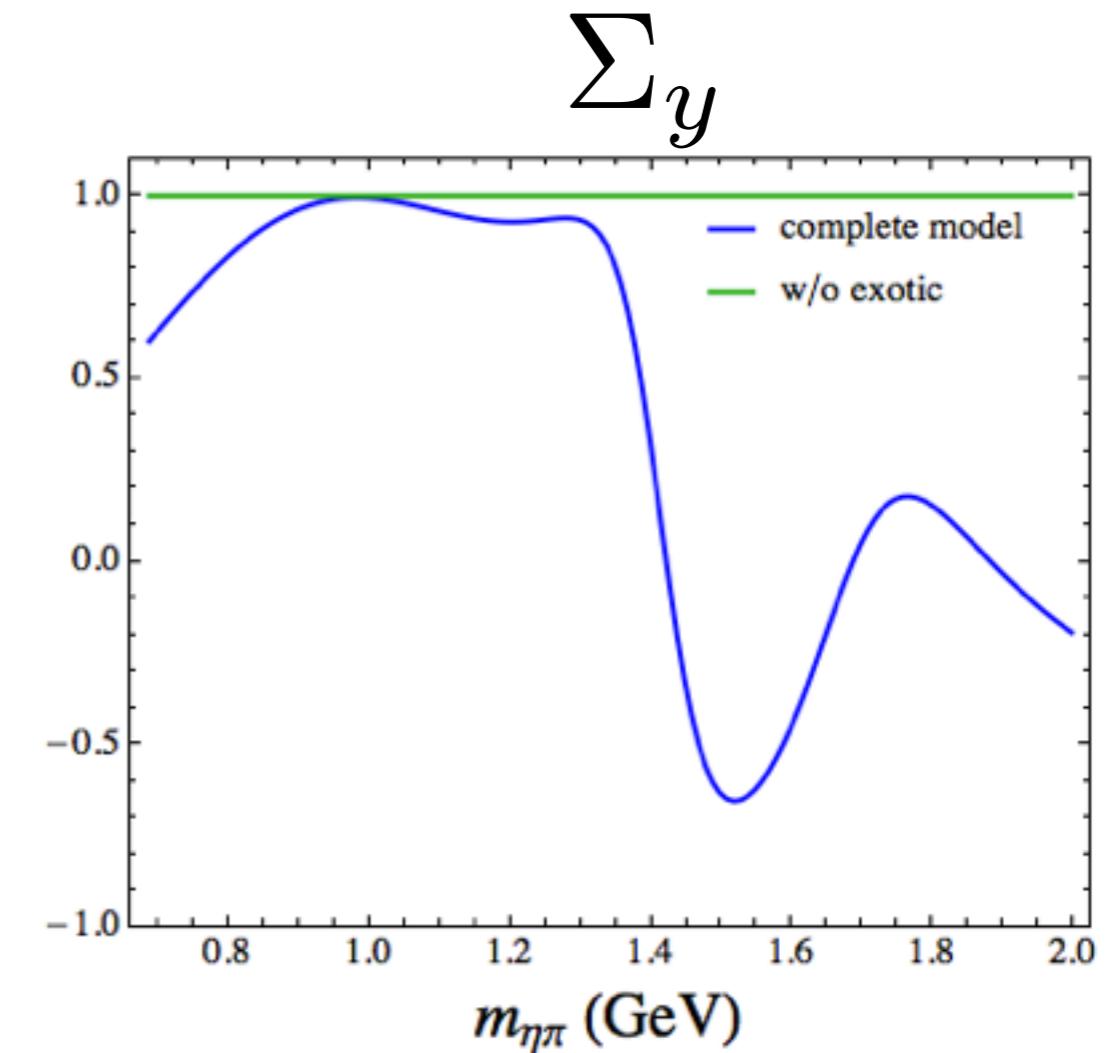
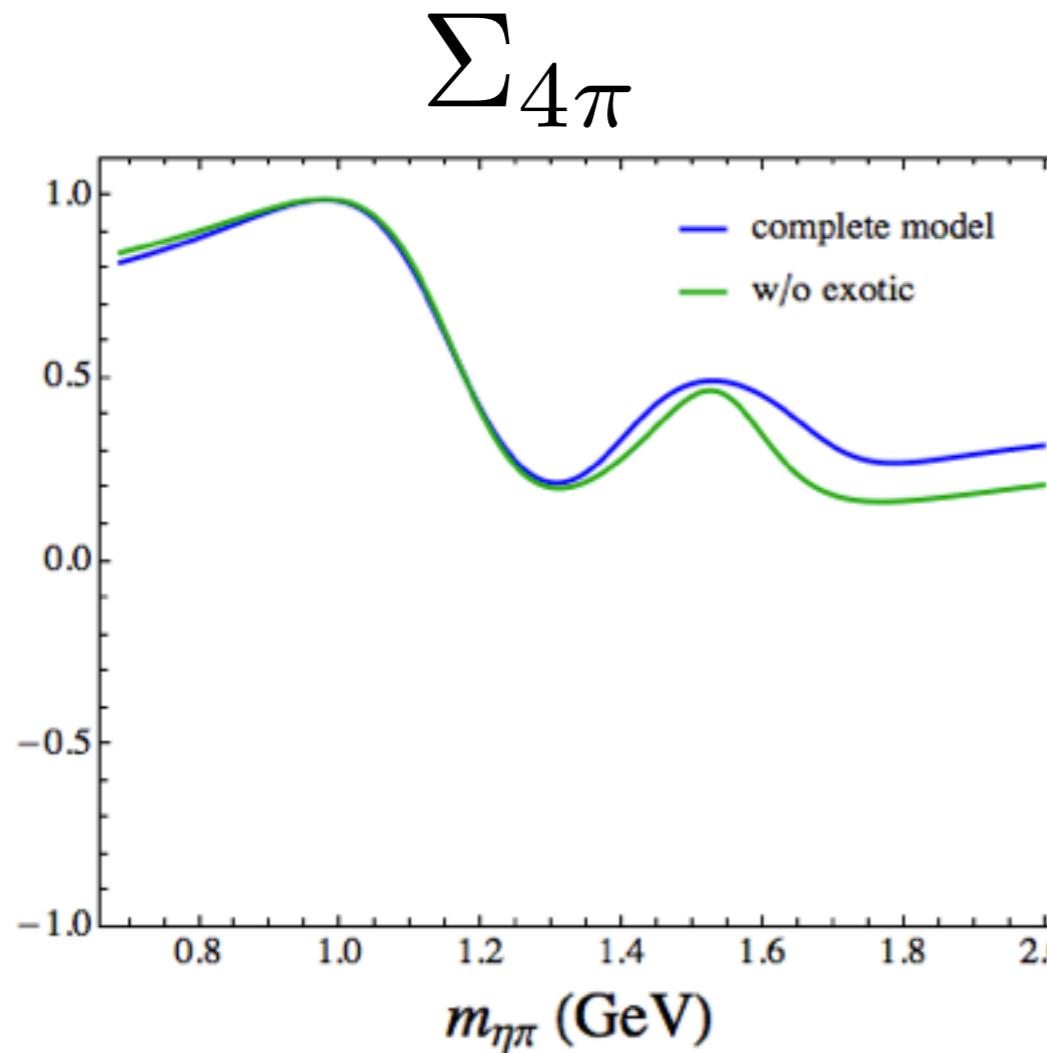
Beam Asymmetries

17



Beam Asymmetries

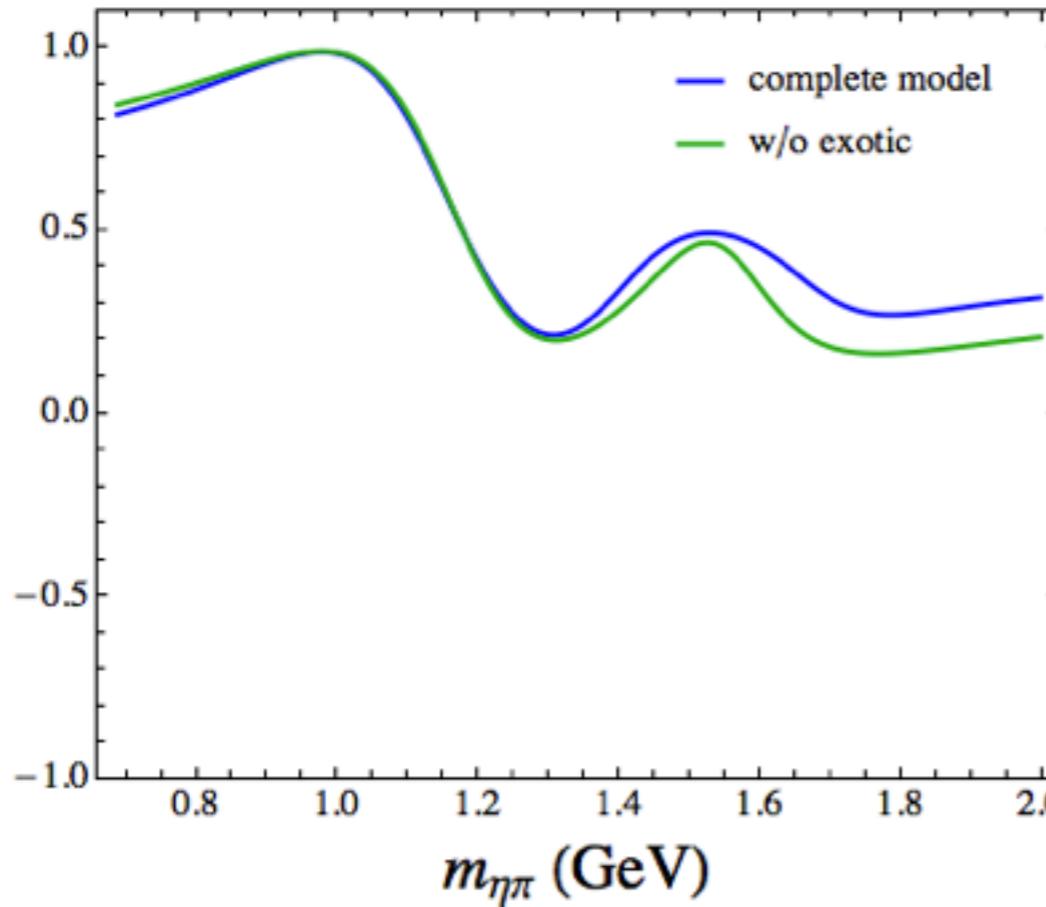
17



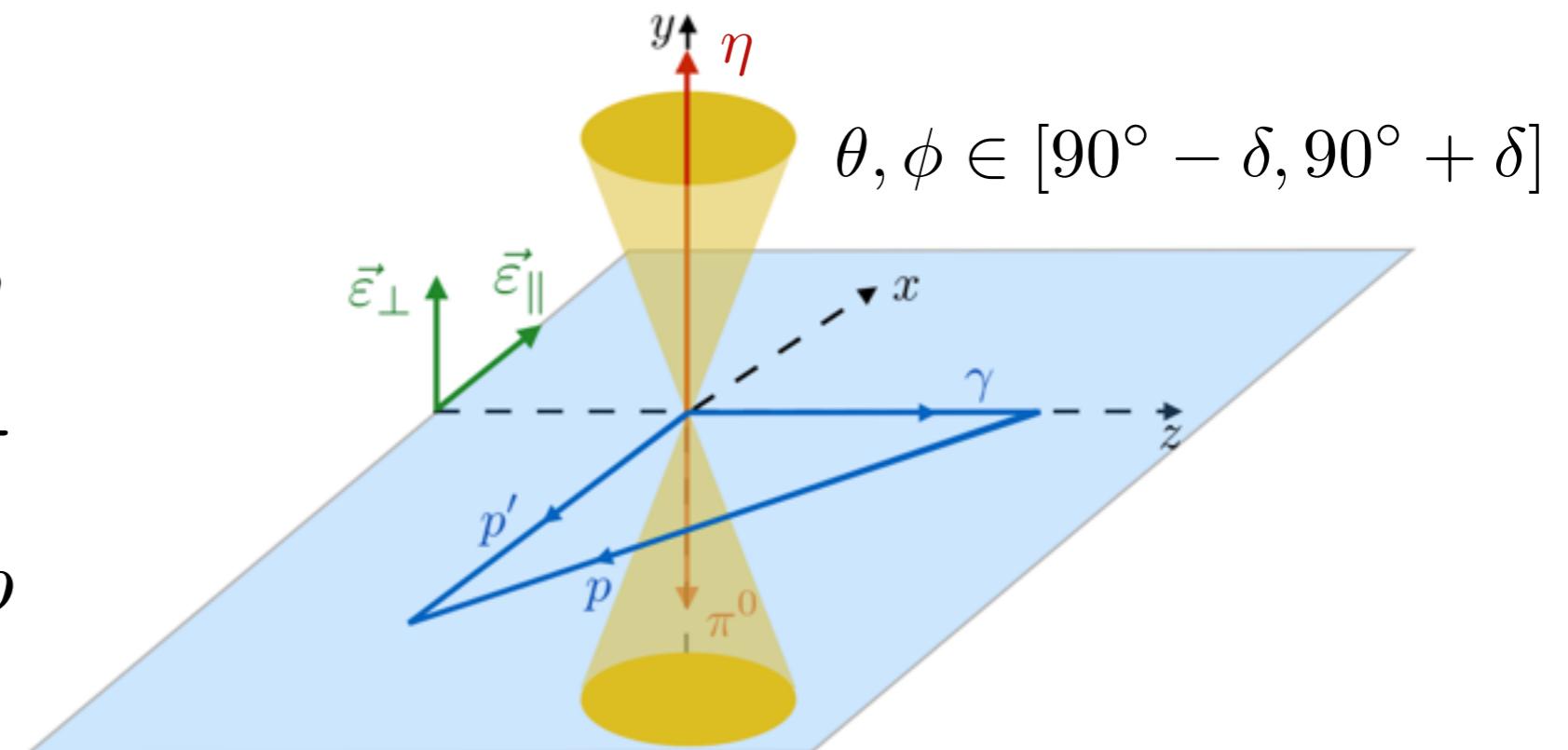
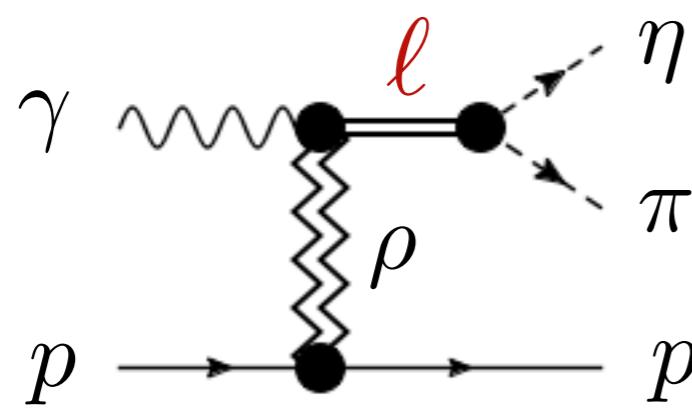
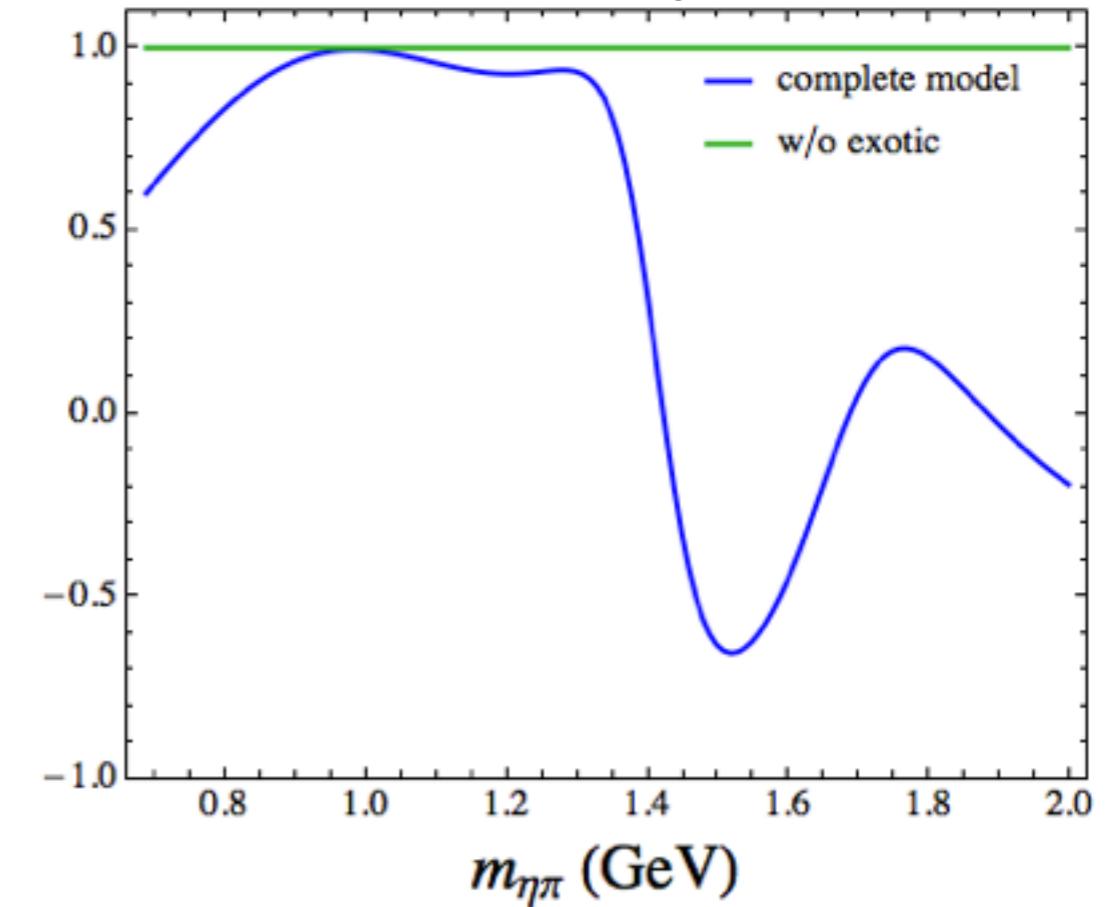
Beam Asymmetries

17

$$\Sigma_{4\pi}$$



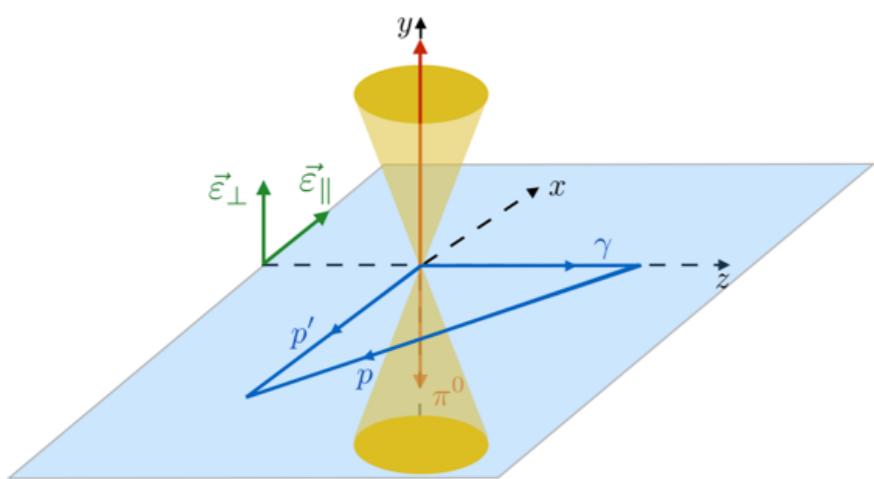
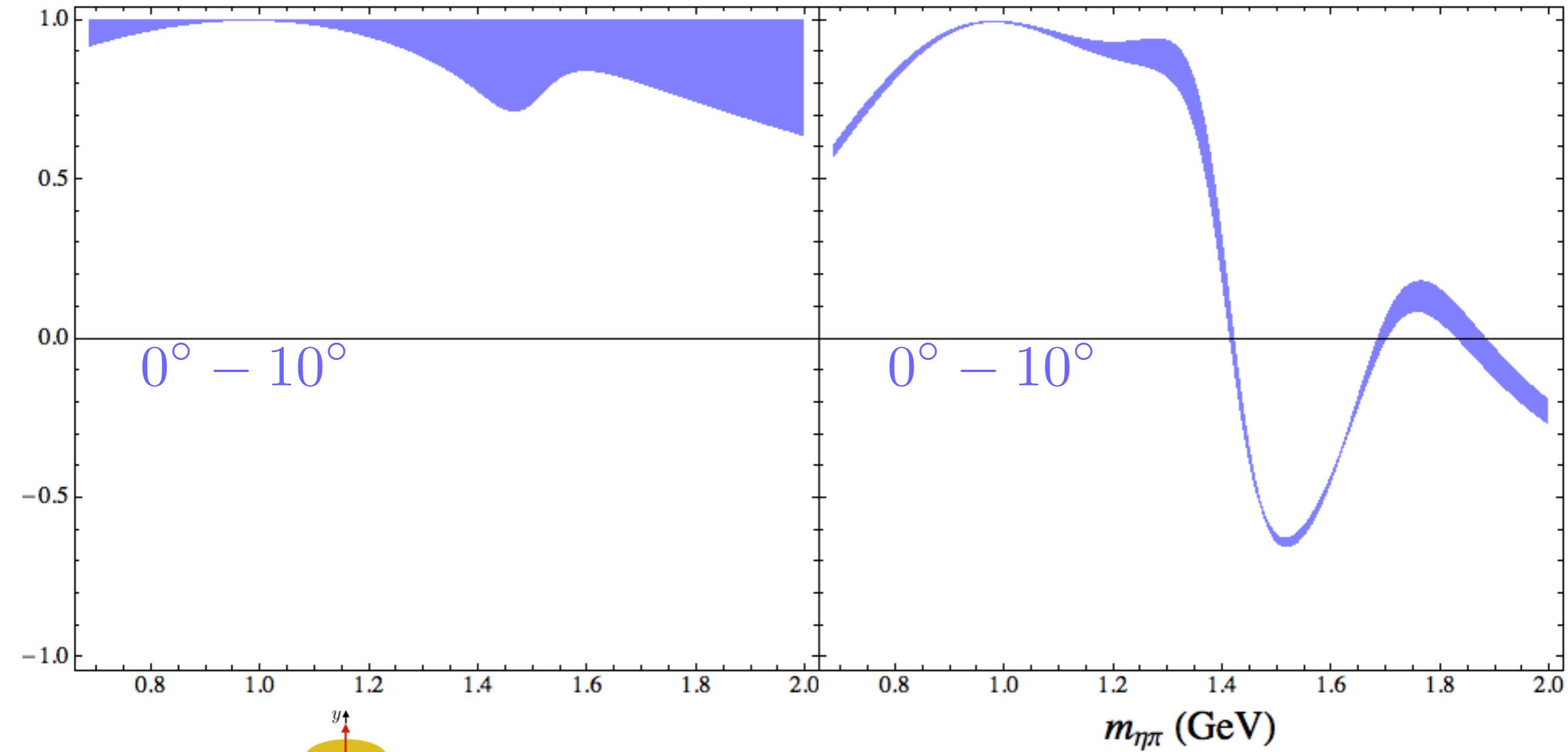
$$\Sigma_y$$



Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

only S and D waves

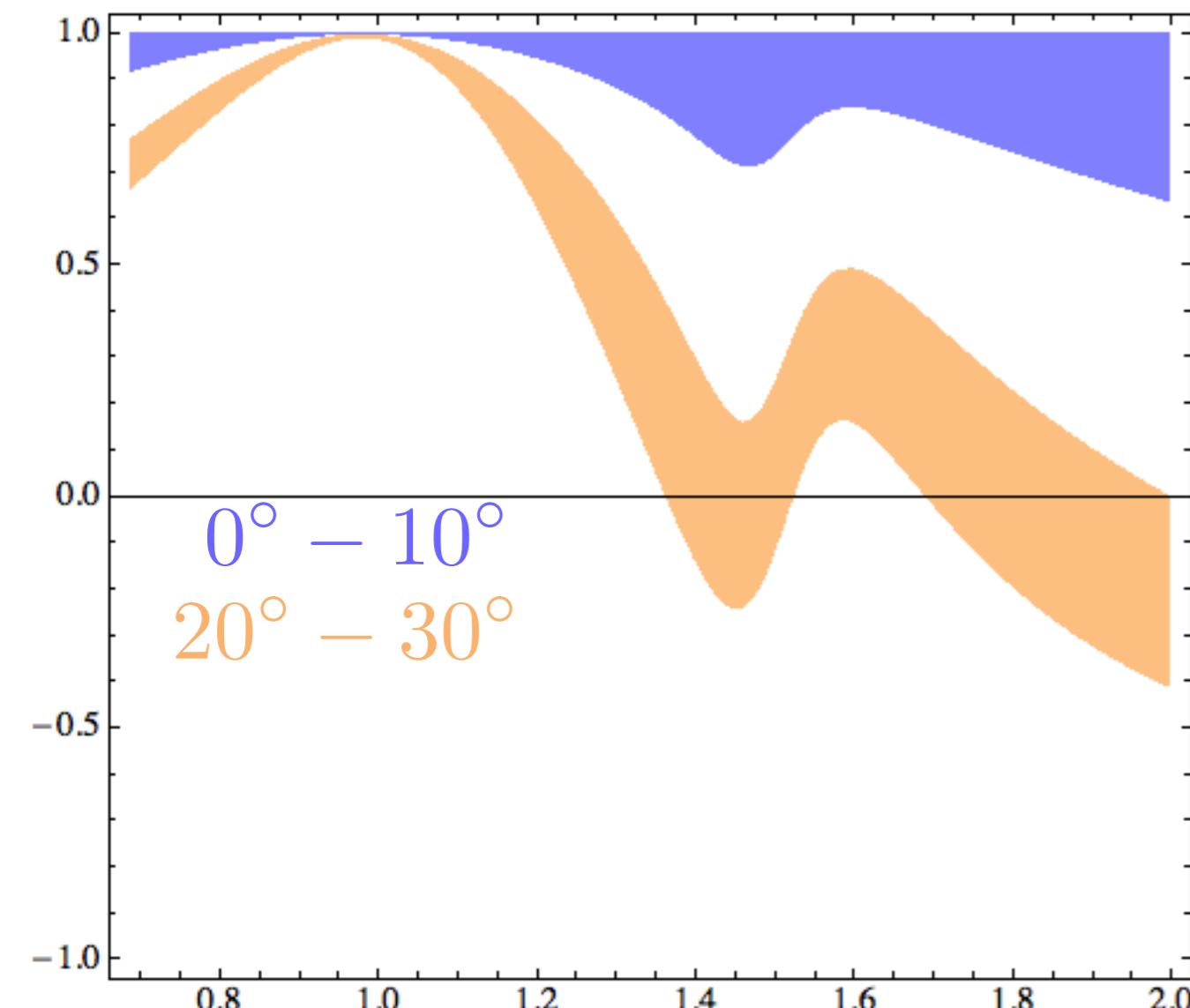
S, P and D waves



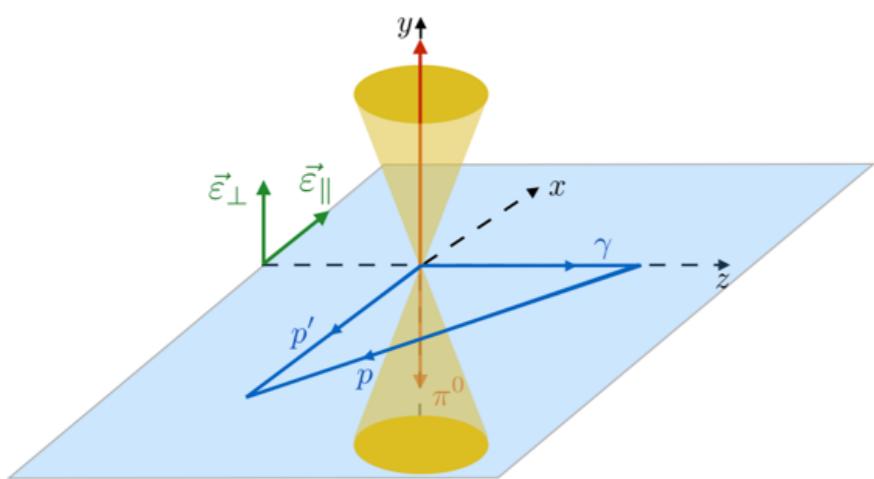
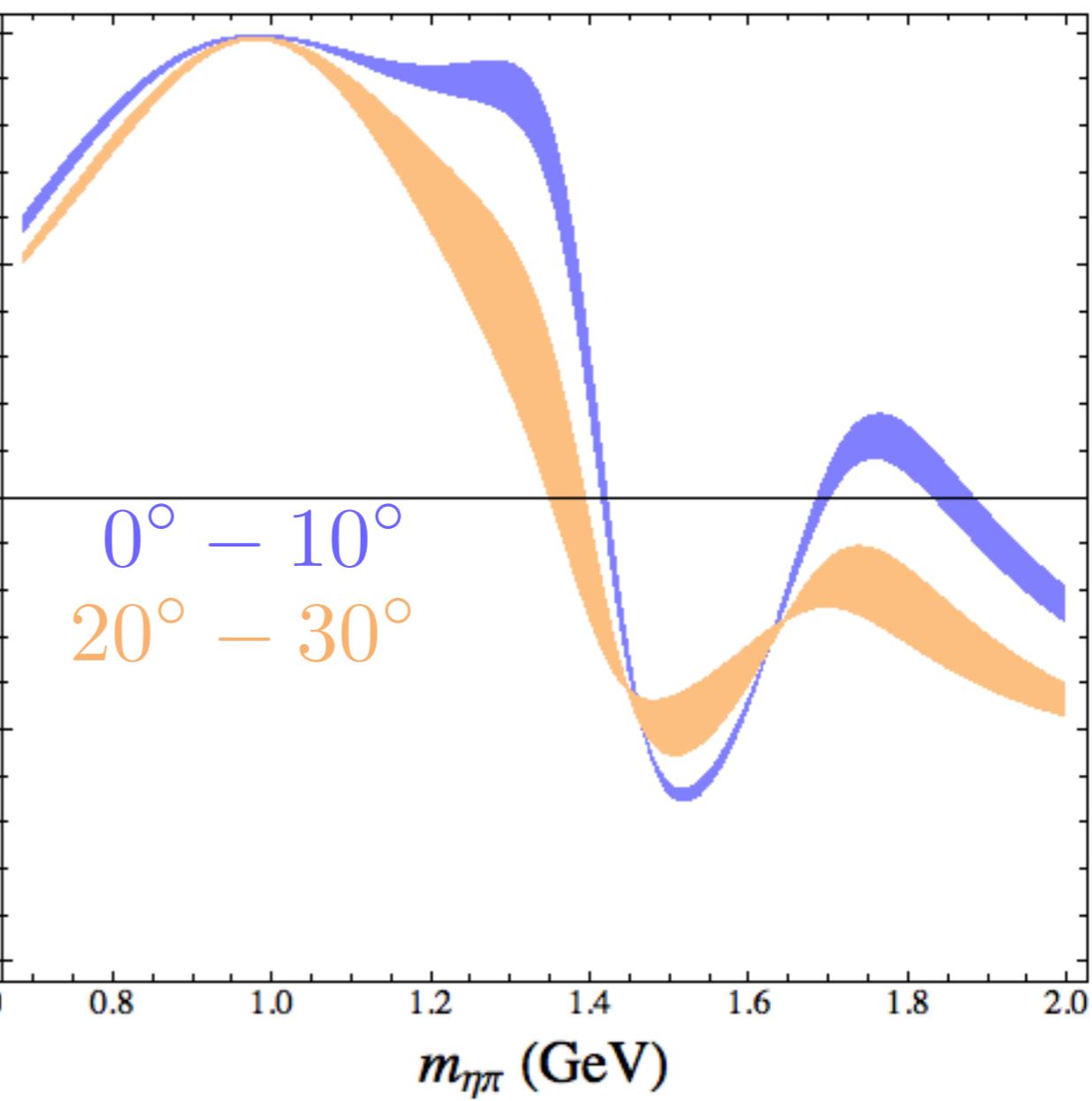
Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

18

only S and D waves

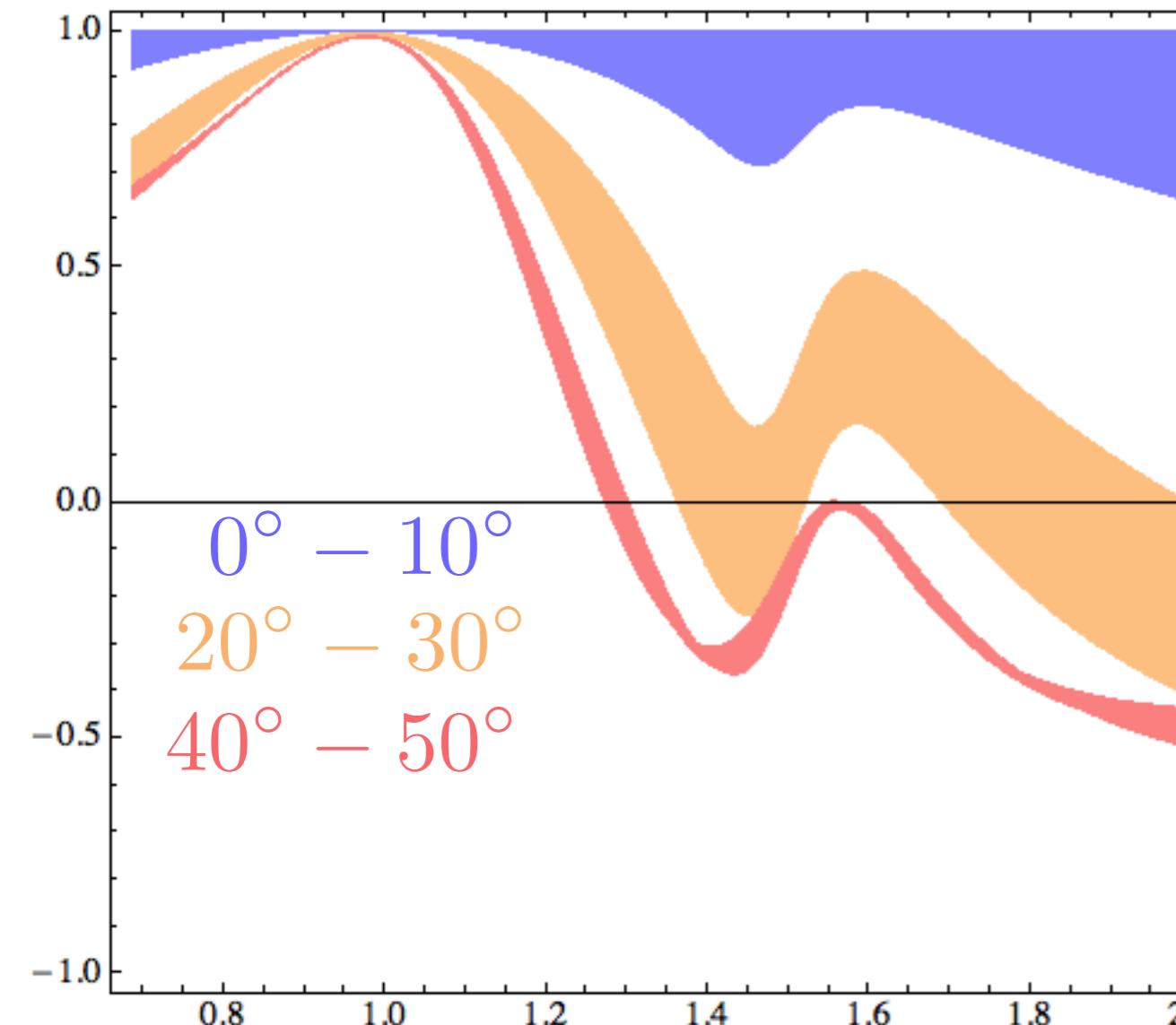


S, P and D waves

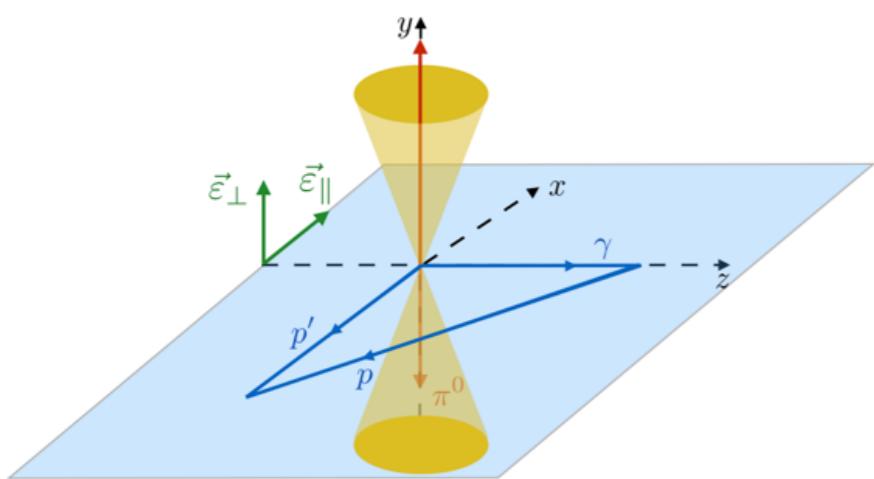
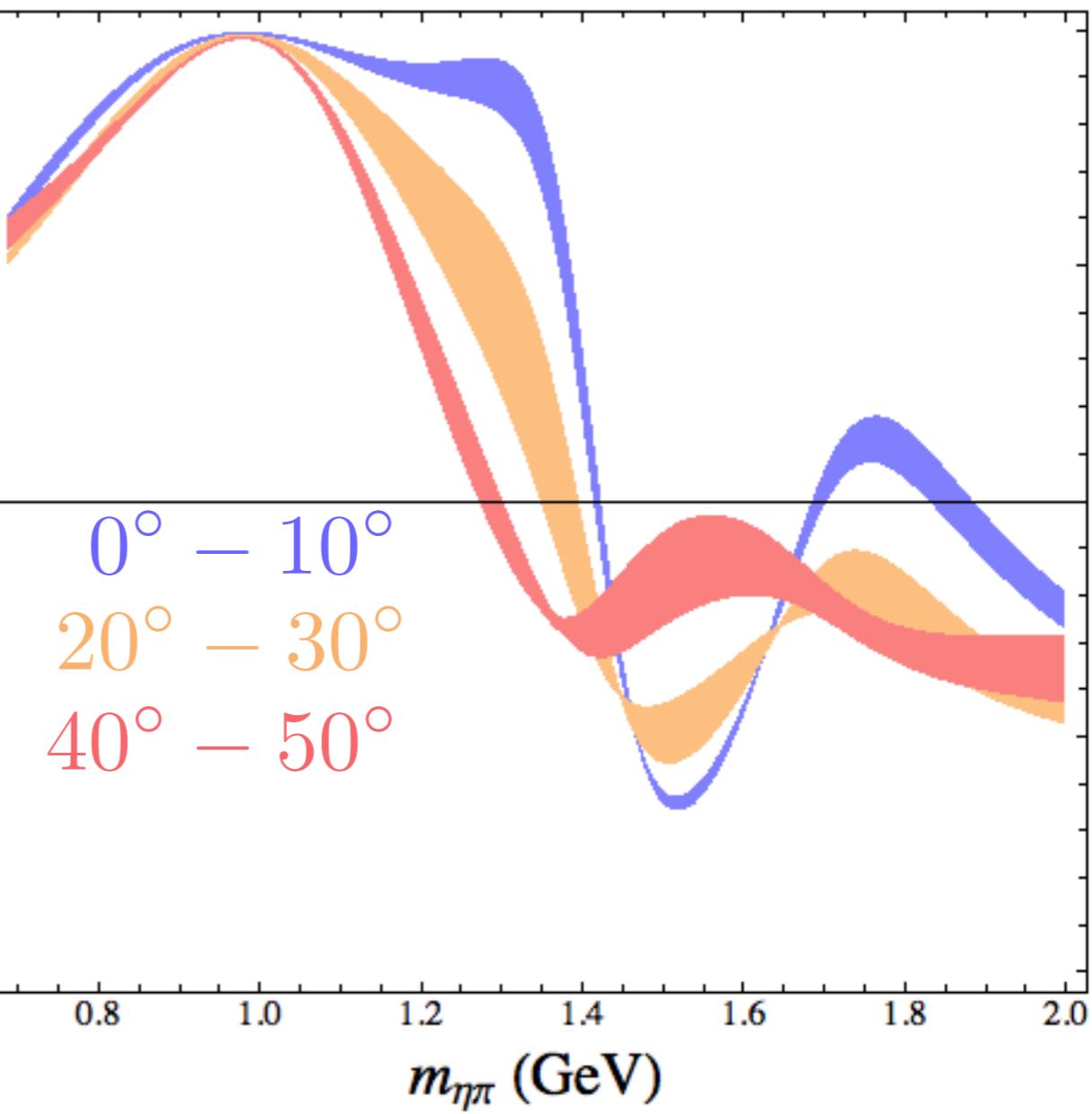


Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

only S and D waves



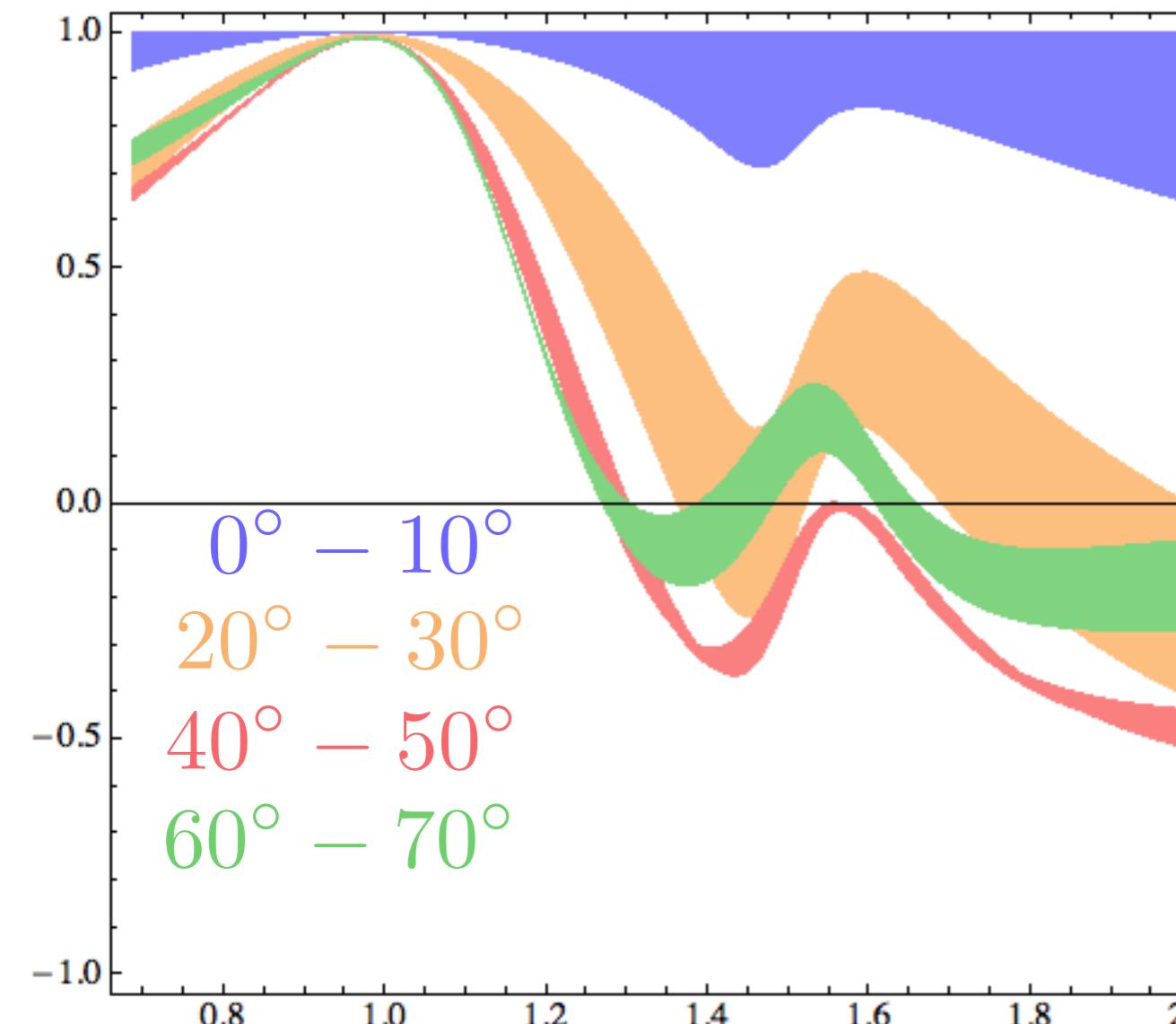
S, P and D waves



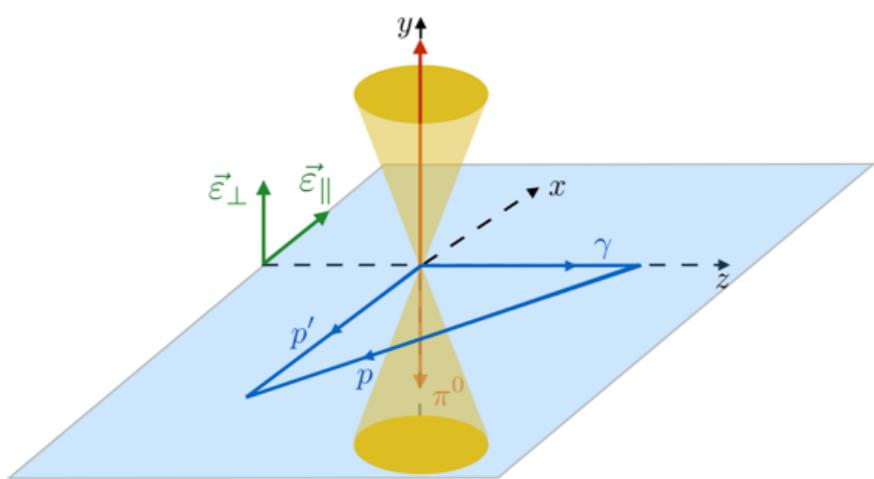
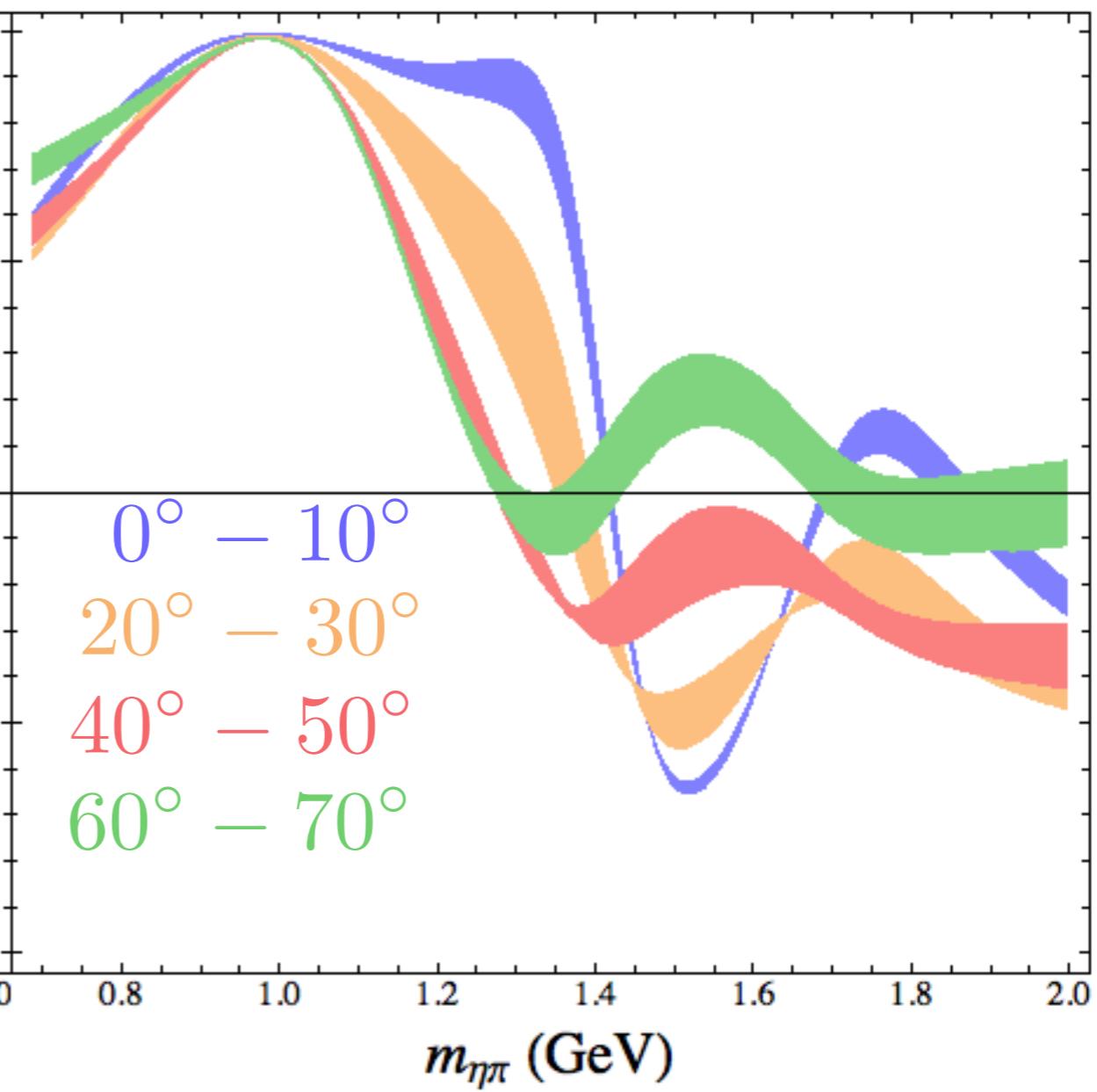
**with an opening angle greater than 30°
the observables is not sensitive to the P-wave
(with our model)**

Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

only S and D waves



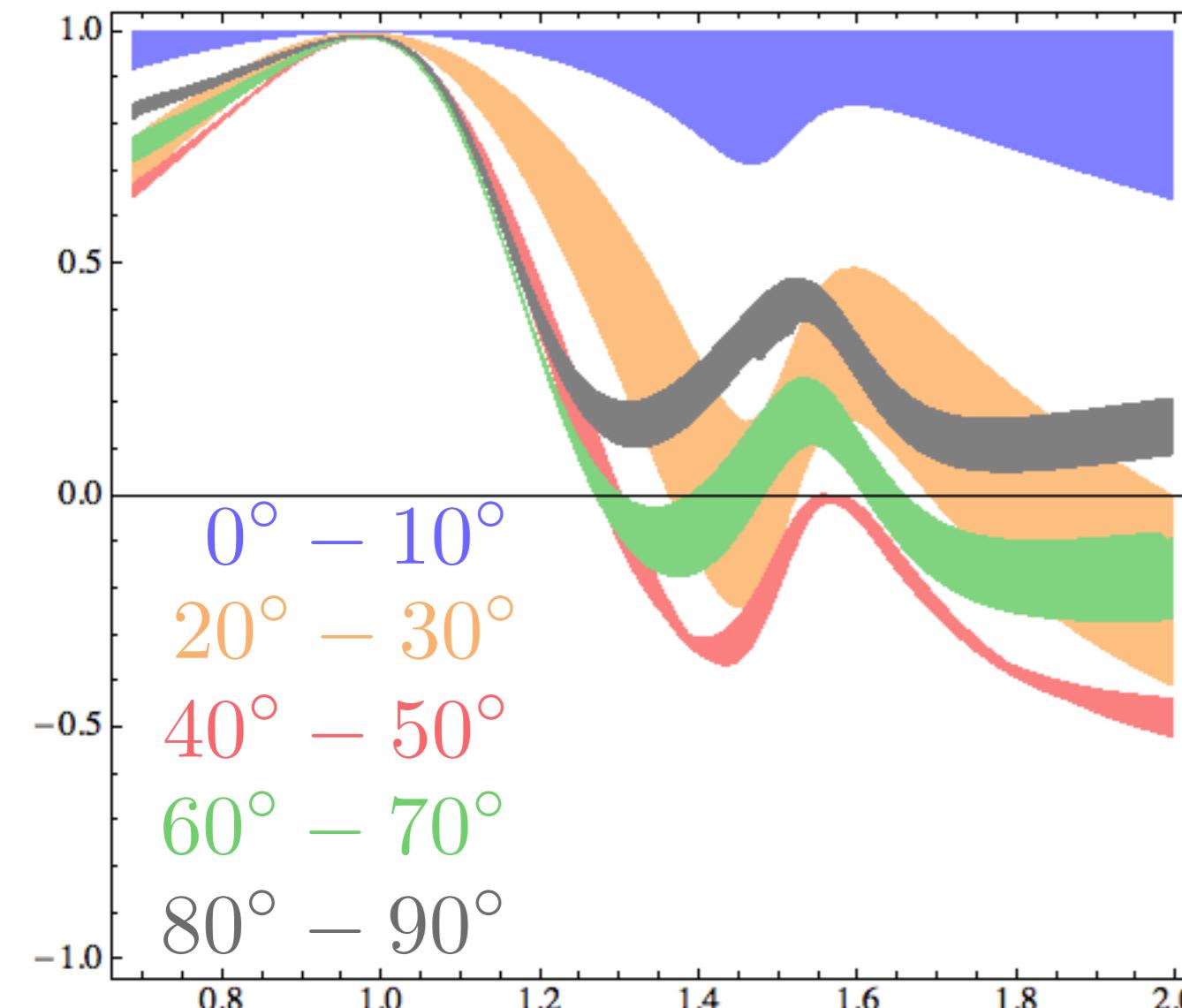
S, P and D waves



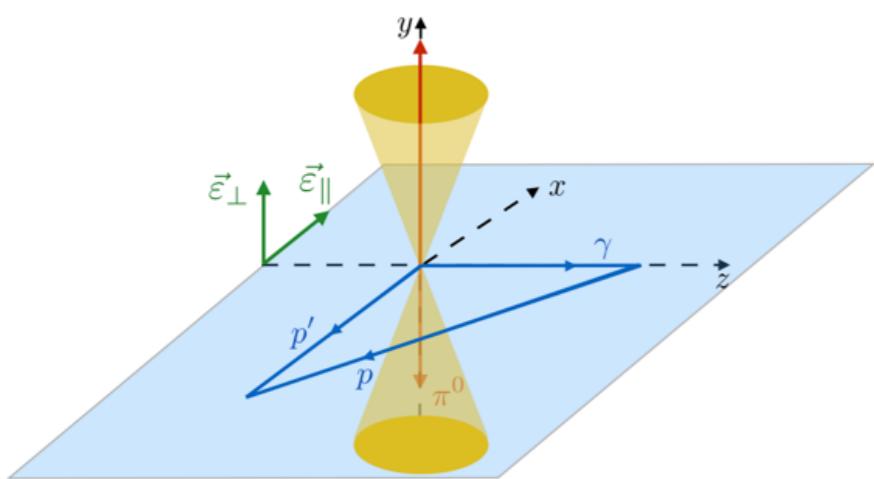
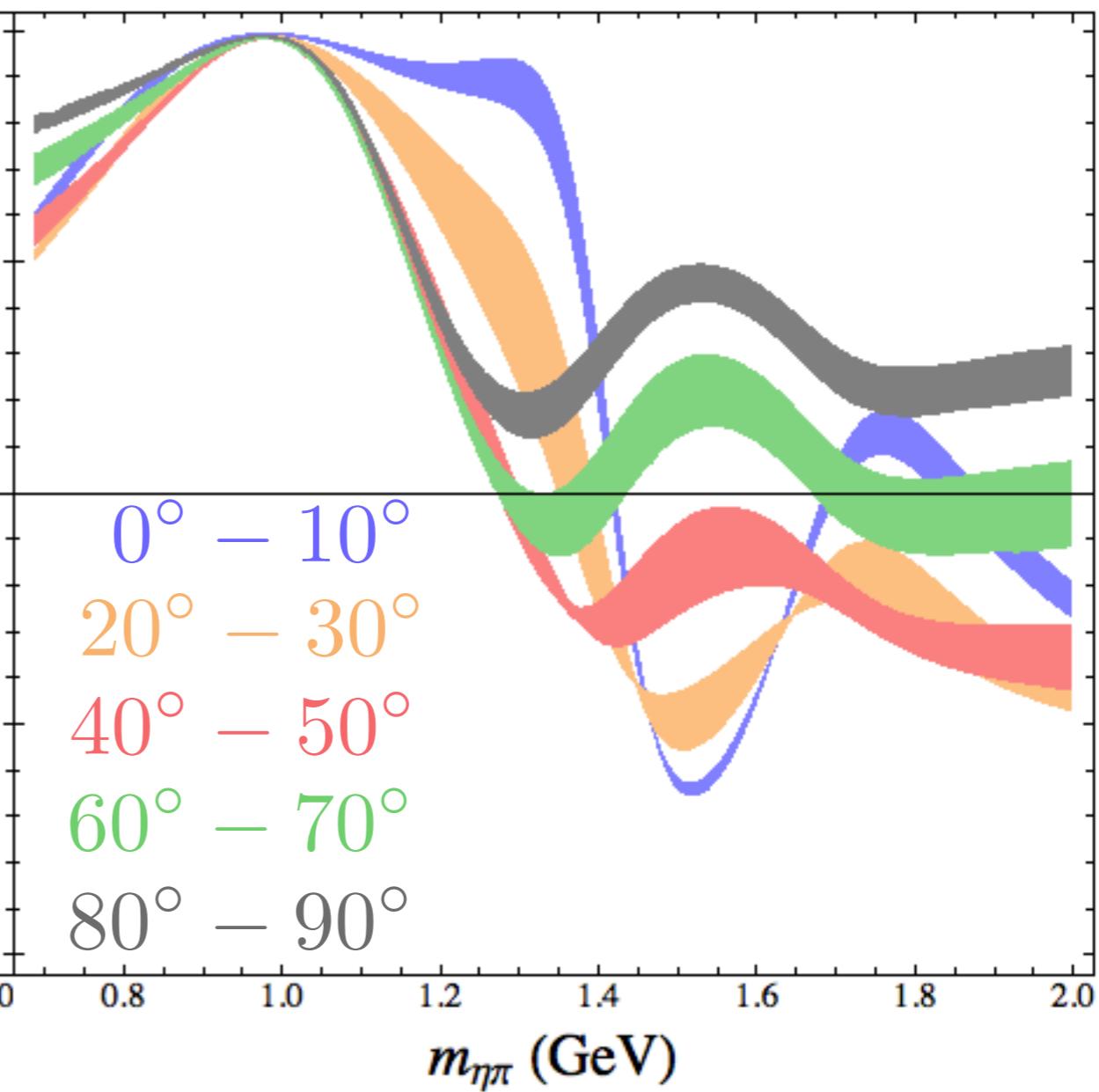
**with an opening angle greater than 30°
the observables is not sensitive to the P-wave
(with our model)**

Beam Asymmetries: $\Sigma_{y \pm \delta^\circ}$

only S and D waves



S, P and D waves



**with an opening angle greater than 30°
the observables is not sensitive to the P-wave
(with our model)**

Outline

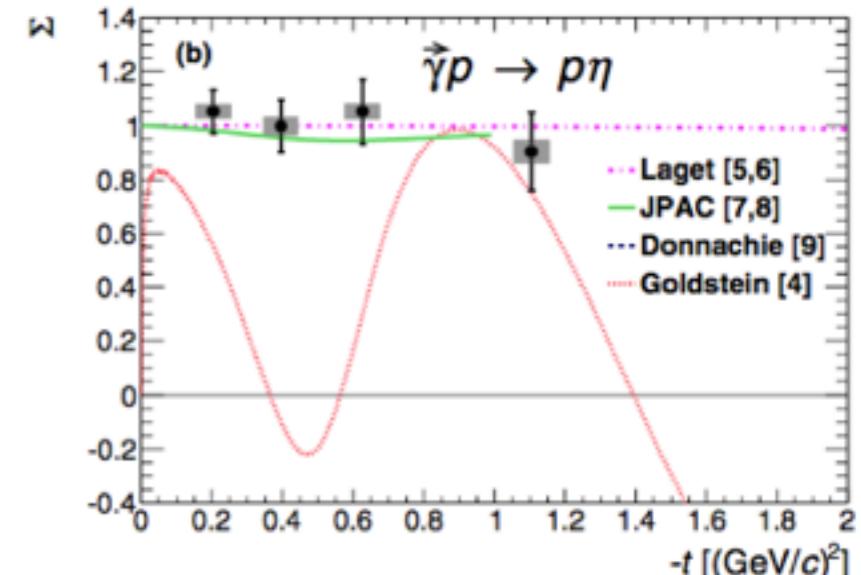
Conclusion

Single Meson Photoproduction:

$$\vec{\gamma}p \rightarrow \pi^0 p$$

$$\vec{\gamma}p \rightarrow \eta p$$

Dominance of natural exchanges in both π^0/η photoproduction



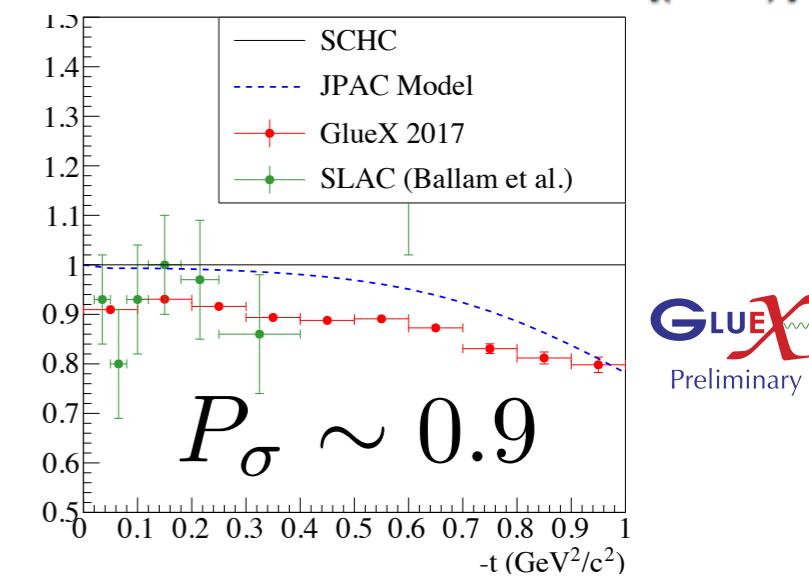
Vector Meson Photoproduction:

$$\vec{\gamma}p \rightarrow \rho^0 p$$

$$\vec{\gamma}p \rightarrow \omega p$$

$$\vec{\gamma}p \rightarrow \phi p$$

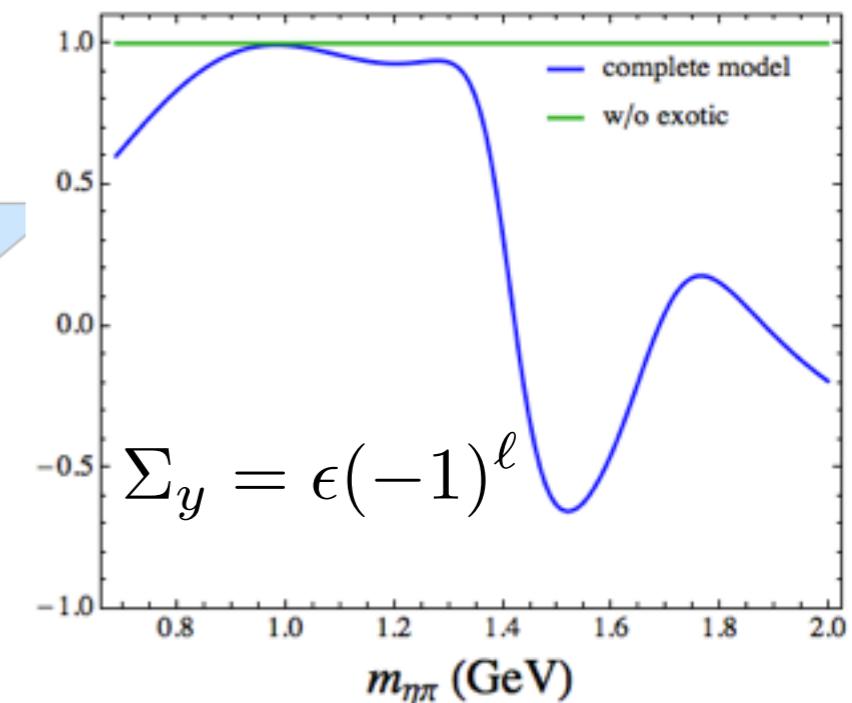
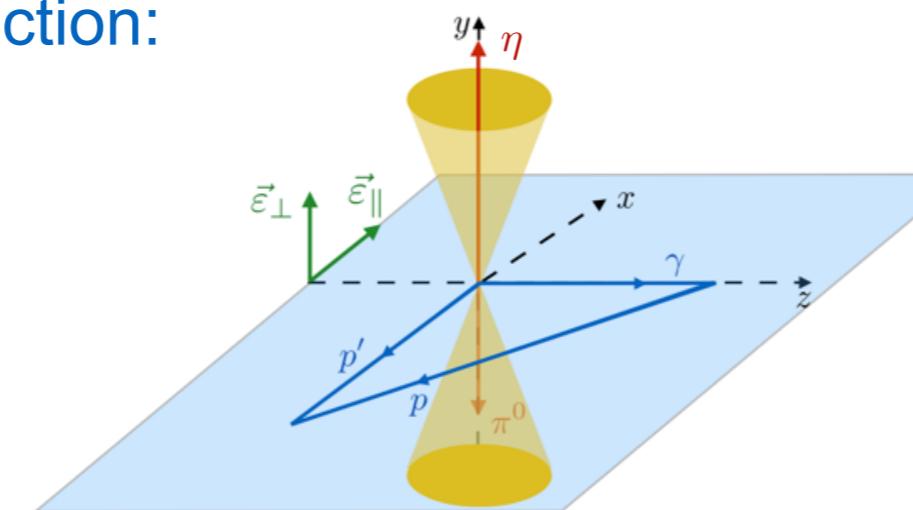
Consistent with factorization
Dominance of natural exchanges



Double Mesons Photoproduction:

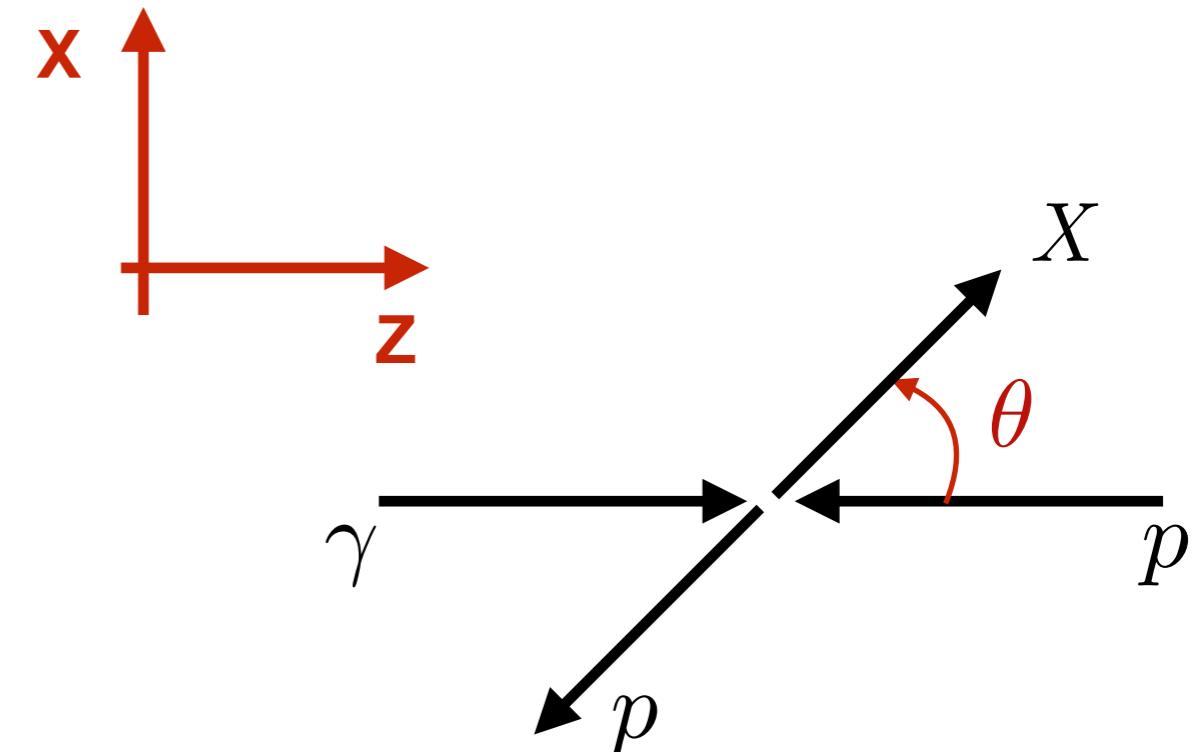
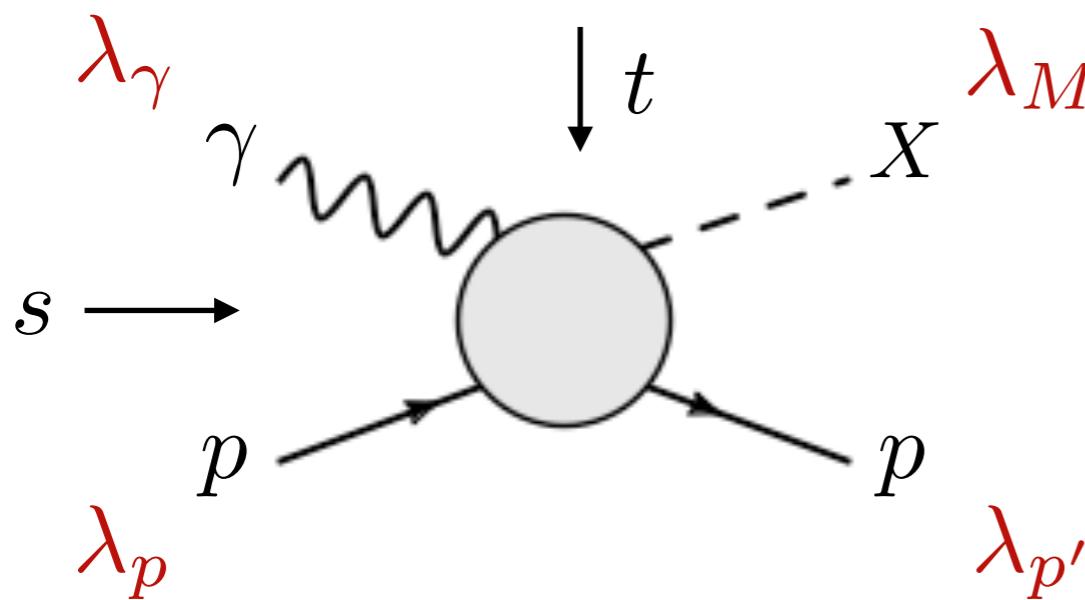
$$\vec{\gamma}p \rightarrow \pi^0 \eta p$$

New observable sensitive to exotic production



Backup Slides

Kinematics



$$A_{\lambda_p \lambda_{p'}}^{\lambda_\gamma \lambda_M}(s, t)$$

λ_i = s-channel helicity of particle i

t = momentum transferred squared

s = center of mass energy squared

High energy approximation

$$\cos \theta \rightarrow 1 + \frac{2t}{s}$$

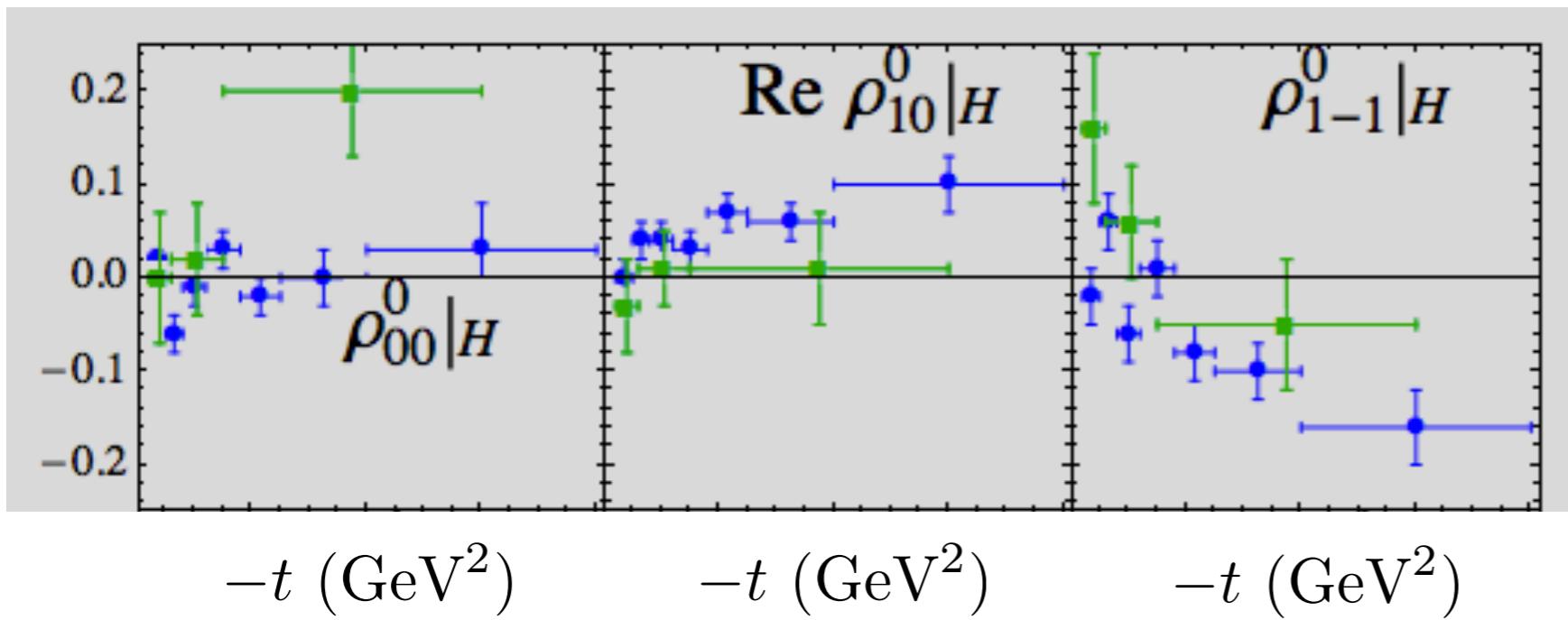
$$\sin \theta \rightarrow 2\sqrt{-t/s}$$

$$\sin \theta/2 \rightarrow \sqrt{-t/s}$$

Spin Density Matrix Elements at 9.3 GeV (SLAC)

22

$\vec{\gamma}p \rightarrow pp$



Rho data seems consistent

Omega data more problematic

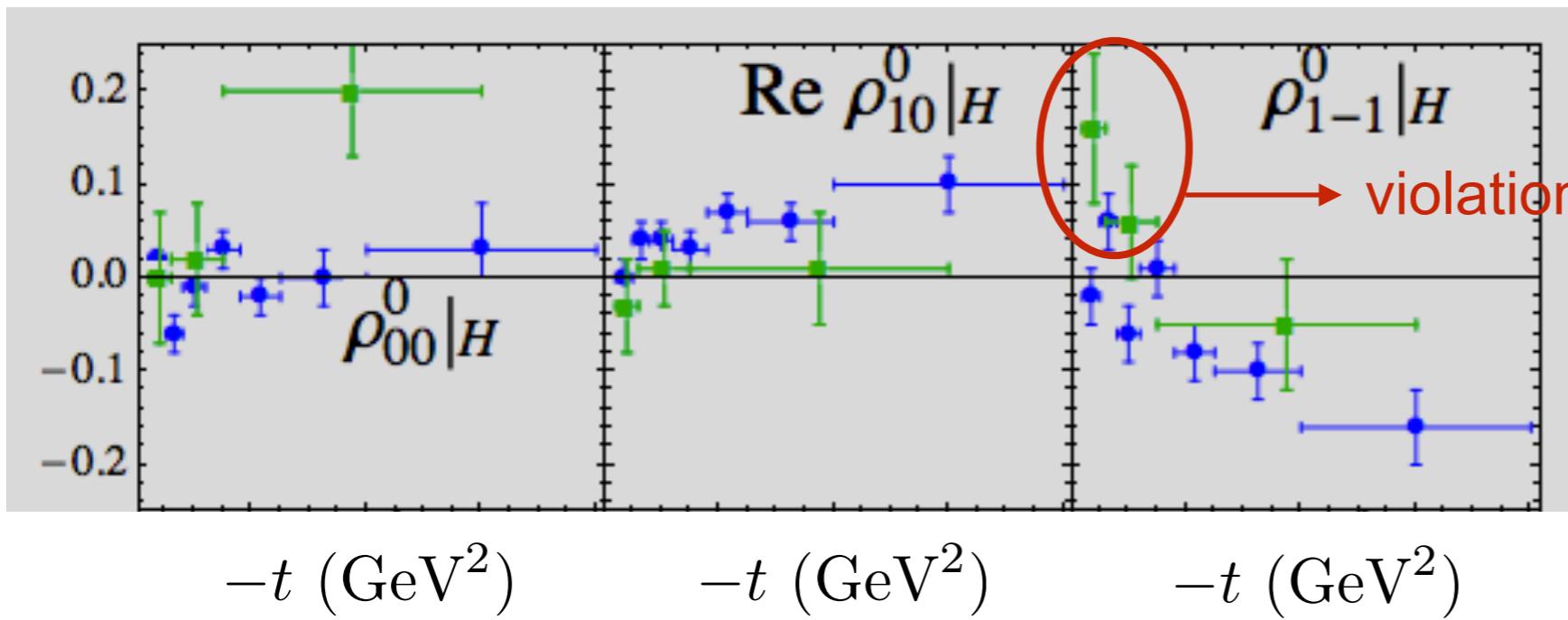
$$\rho_{00}^0 \propto \beta_1^2 \frac{-t}{m_\omega^2}$$

$$\text{Re } \rho_{10}^0 \propto \frac{1}{2} \beta_1 \frac{\sqrt{-t}}{m_\omega}$$

$$\rho_{1-1}^0 \propto \beta_2 \frac{-t}{m_\omega^2}$$

Spin Density Matrix Elements at 9.3 GeV (SLAC)

22

 $\vec{\gamma}p \rightarrow \rho p$ 

$$\rho_{00}^0 \propto \beta_1^2 \frac{-t}{m_\omega^2}$$

$$\text{Re } \rho_{10}^0 \propto \frac{1}{2} \beta_1 \frac{\sqrt{-t}}{m_\omega}$$

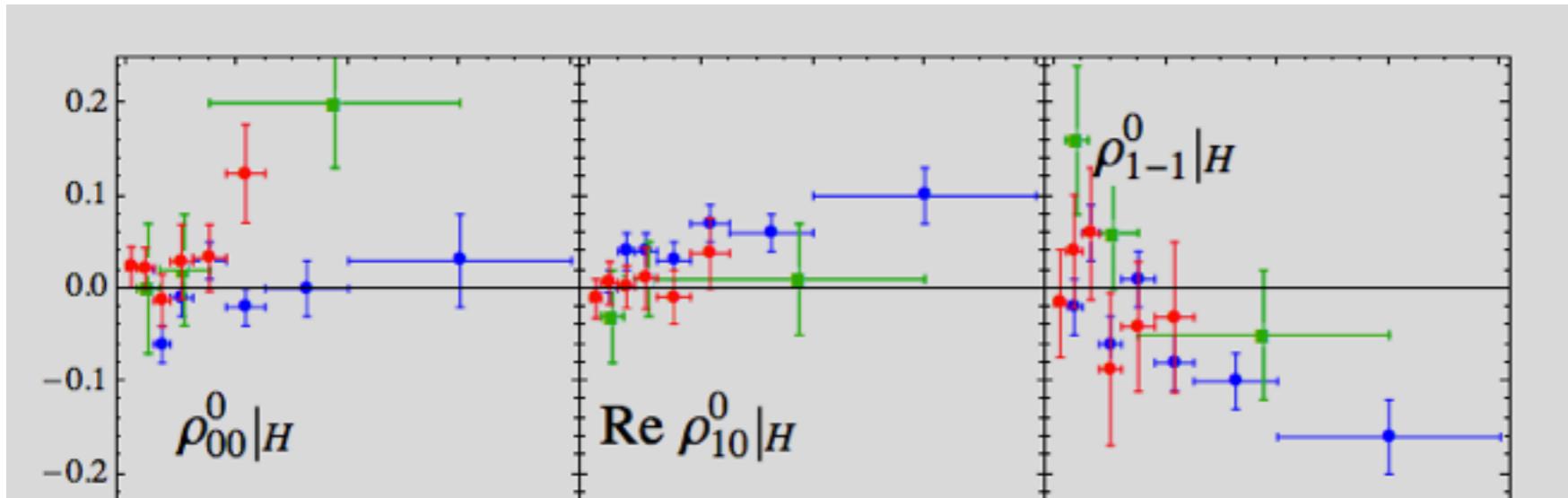
$$\rho_{1-1}^0 \propto \beta_2 \frac{-t}{m_\omega^2}$$

Rho data seems consistent

Omega data more problematic

Spin Density Matrix Elements at 9.3 GeV (SLAC)

22

 $\vec{\gamma}p \rightarrow \rho p$  $-t$ (GeV 2) $-t$ (GeV 2) $-t$ (GeV 2)

$$\rho_{00}^0 \propto \beta_1^2 \frac{-t}{m_\omega^2}$$

$$\text{Re } \rho_{10}^0 \propto \frac{1}{2} \beta_1 \frac{\sqrt{-t}}{m_\omega}$$

$$\rho_{1-1}^0 \propto \beta_2 \frac{-t}{m_\omega^2}$$

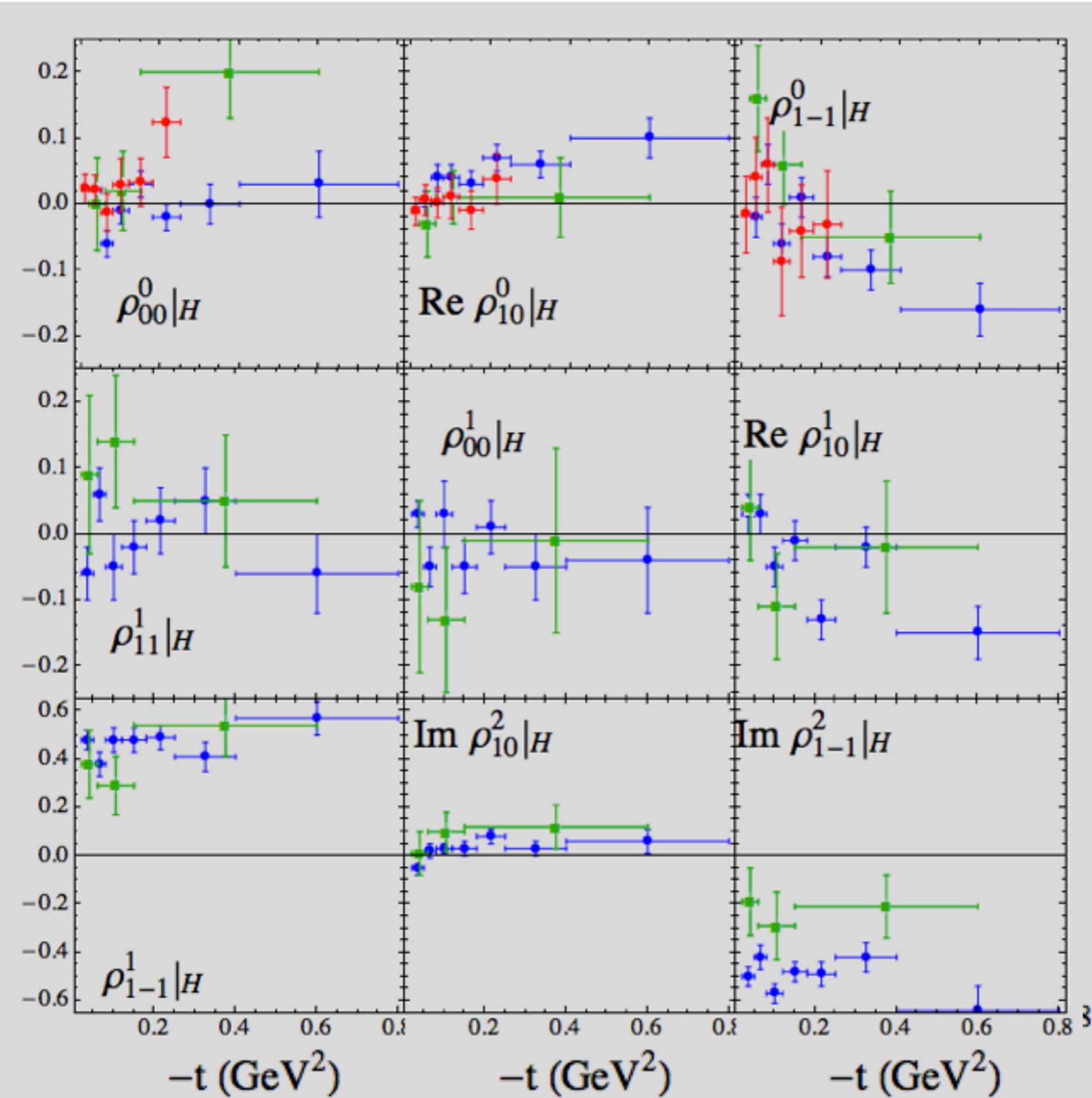
Rho data seems consistent

Omega data more problematic

Spin Density Matrix Elements at 9.3 GeV (SLAC)

22

$\vec{\gamma}p \rightarrow pp$



$$\rho_{00}^0 \propto \beta_1^2 \frac{-t}{m_\omega^2}$$

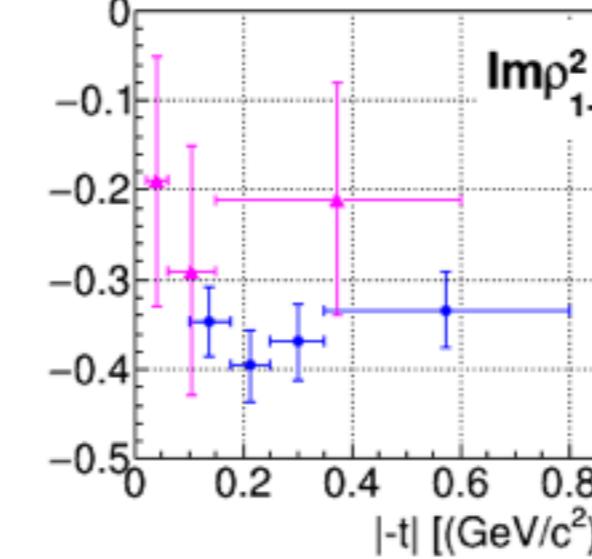
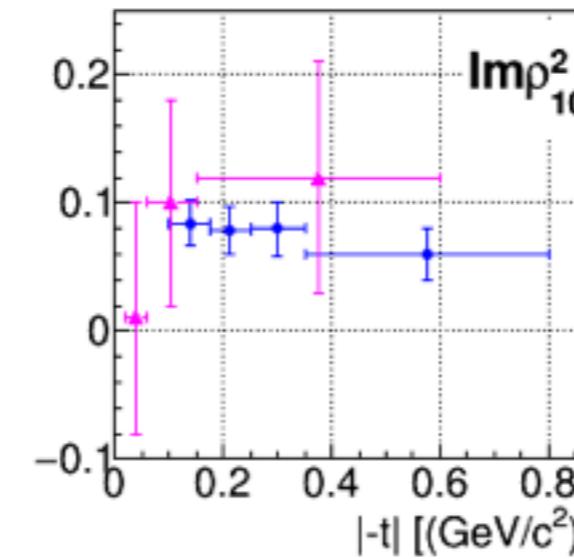
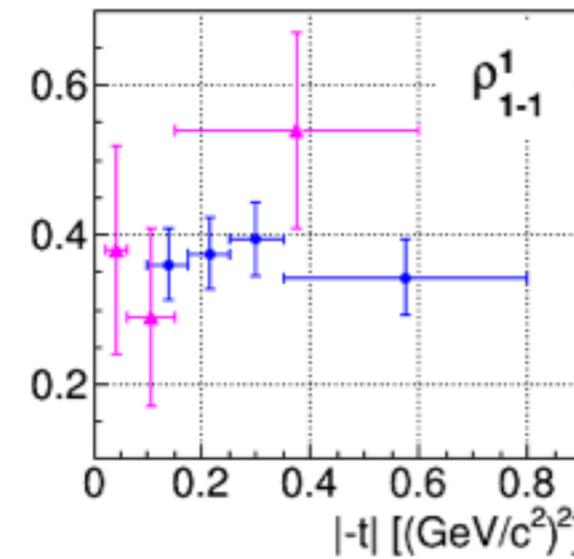
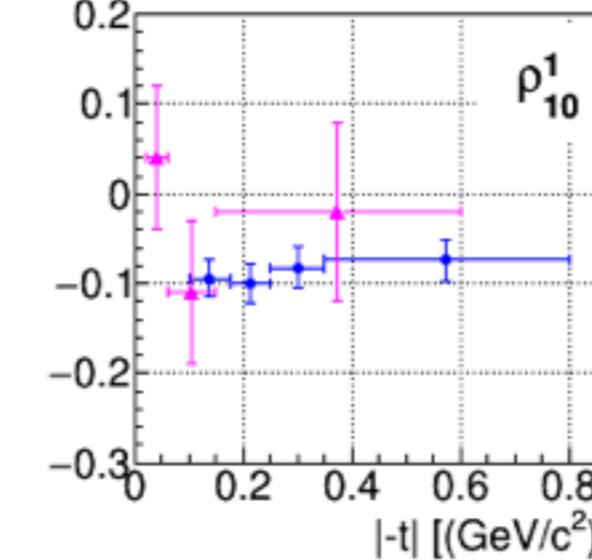
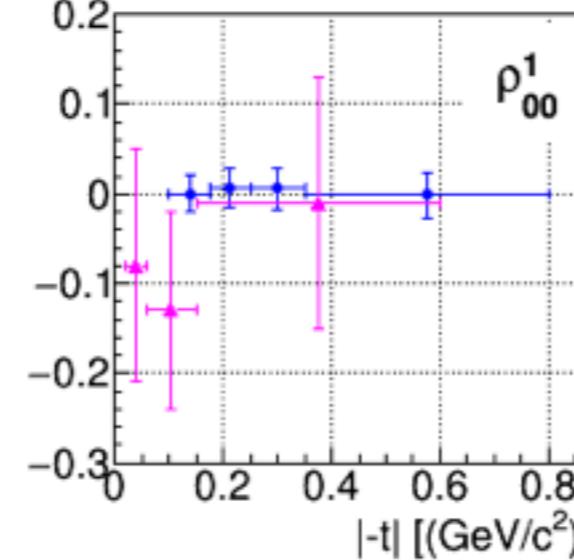
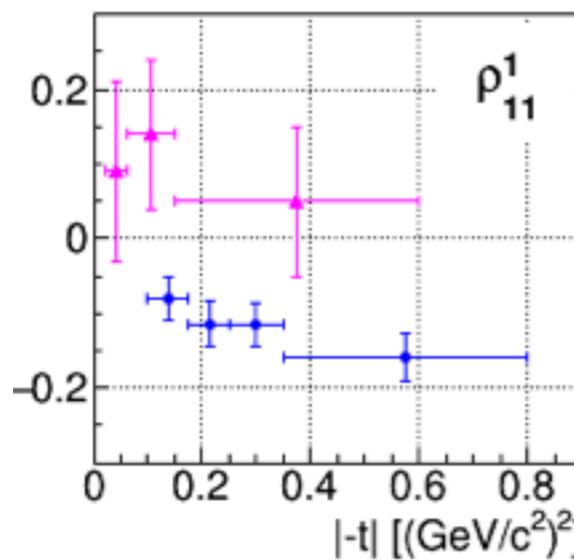
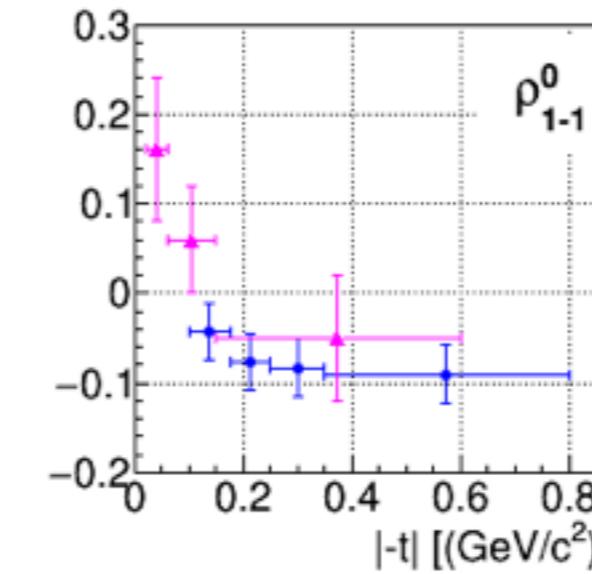
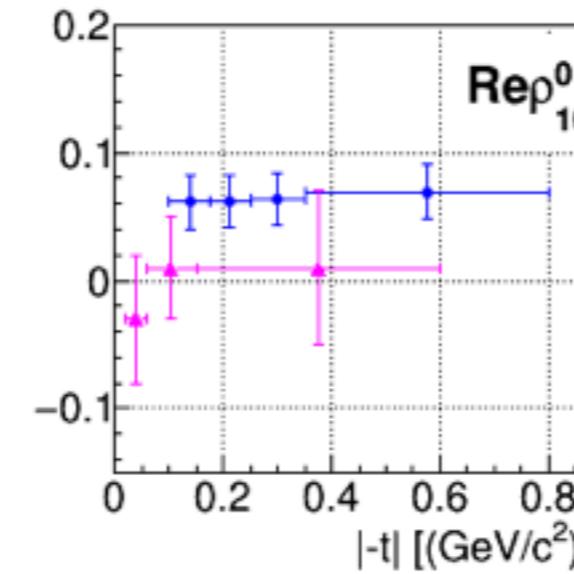
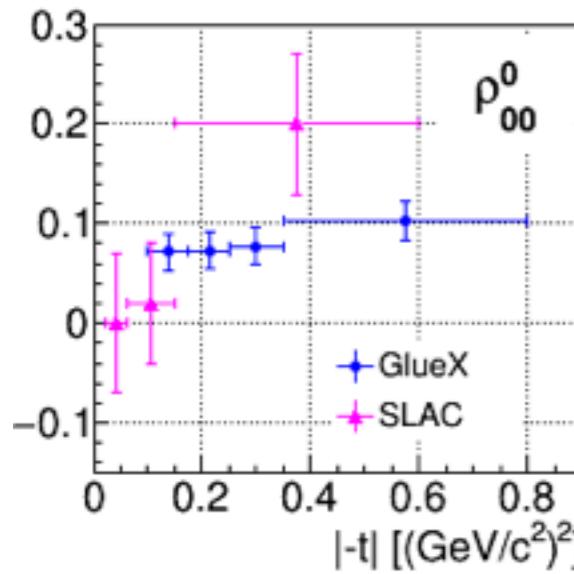
$$\text{Re } \rho_{10}^0 \propto \frac{1}{2} \beta_1 \frac{\sqrt{-t}}{m_\omega}$$

$$\rho_{1-1}^0 \propto \beta_2 \frac{-t}{m_\omega^2}$$

$$\rho_{1-1}^1 = \pm \frac{1}{2} + \mathcal{O}(t^2)$$

$$\text{Im } \rho_{1-1}^2 = \mp \frac{1}{2} + \mathcal{O}(t^2)$$

top sign for natural exchange
bottom sign for unnatural exch.

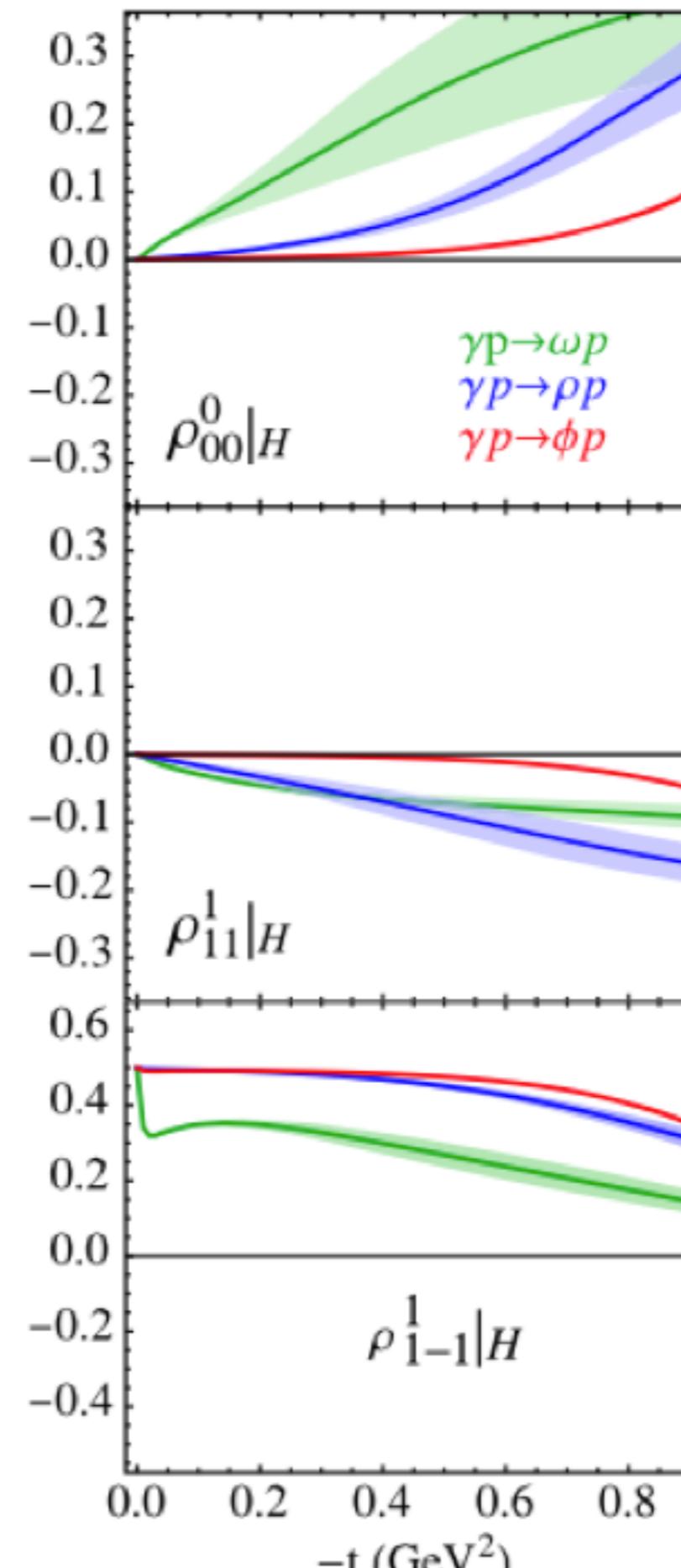
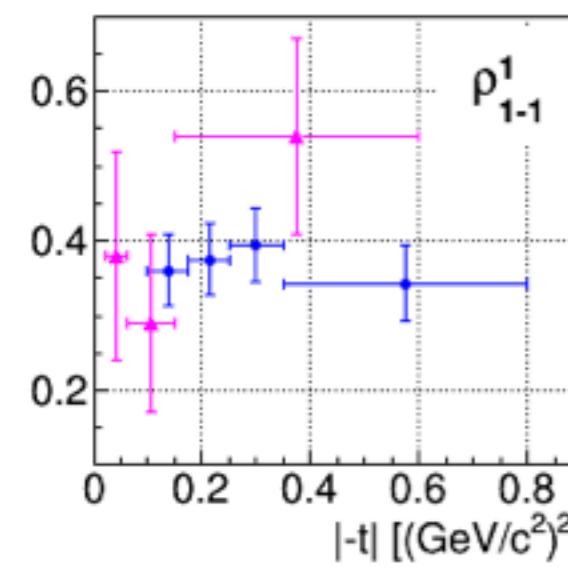
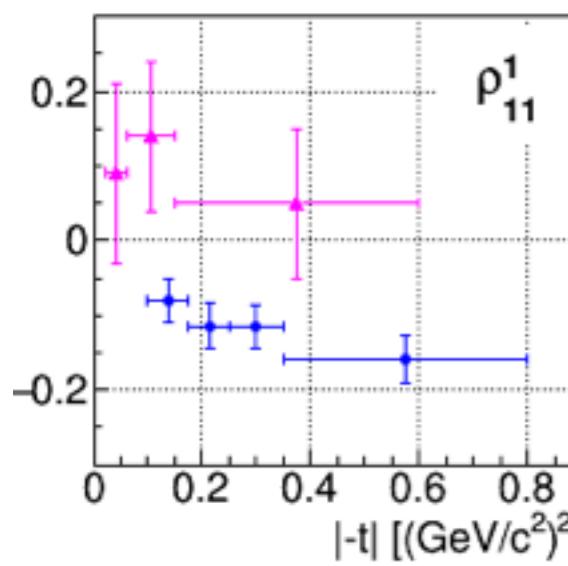
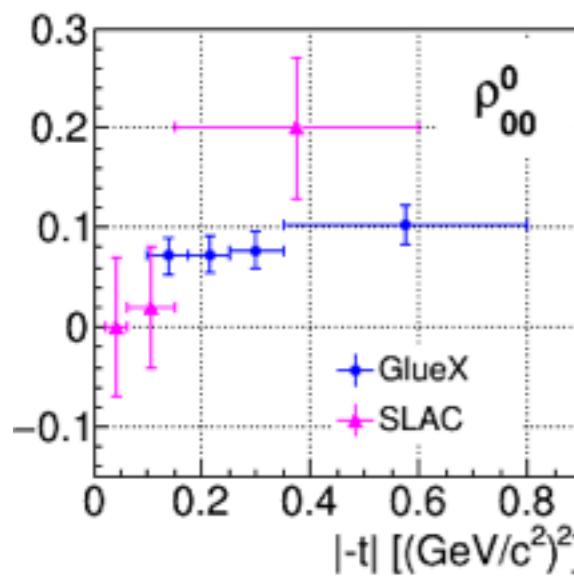
$\vec{\gamma}p \rightarrow \omega p$


purple points: SLAC

blue points: GlueX



Preliminary

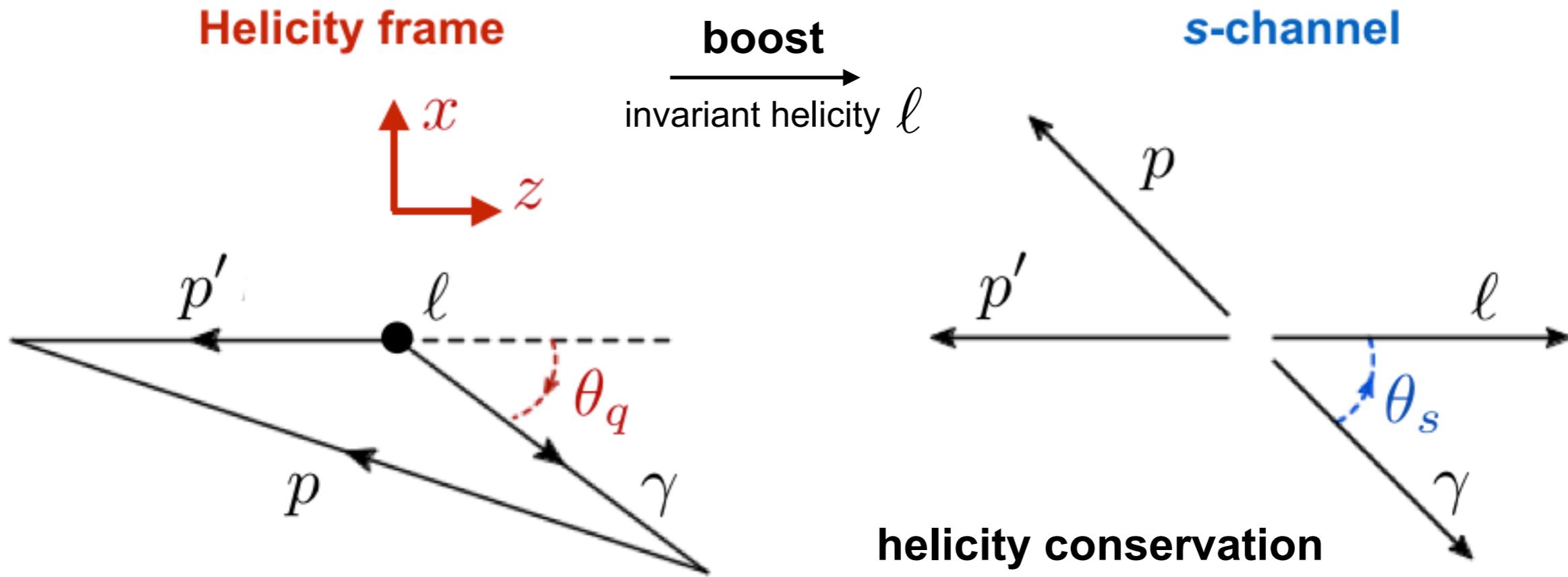
$\vec{\gamma}p \rightarrow \omega p$


purple points: SLAC

blue points: GlueX

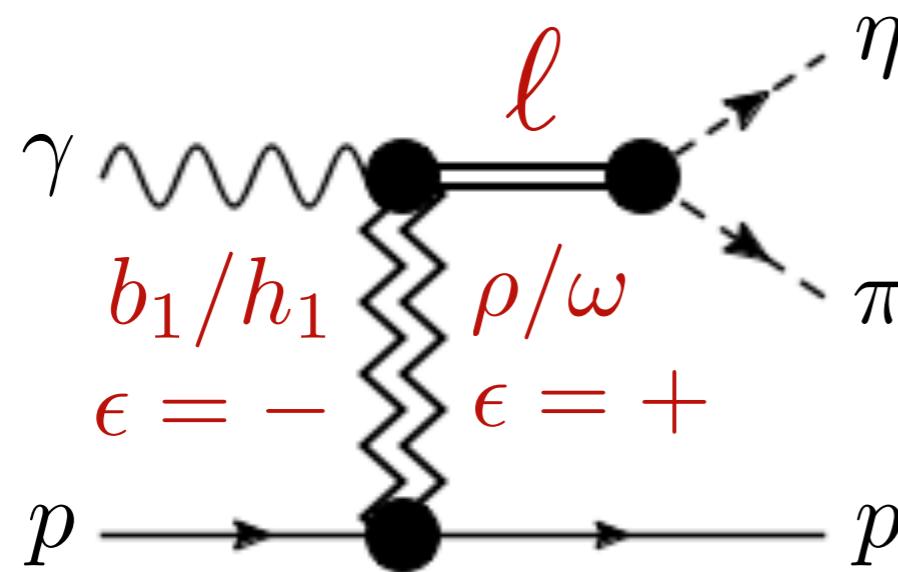


Preliminary



between γ and ℓ

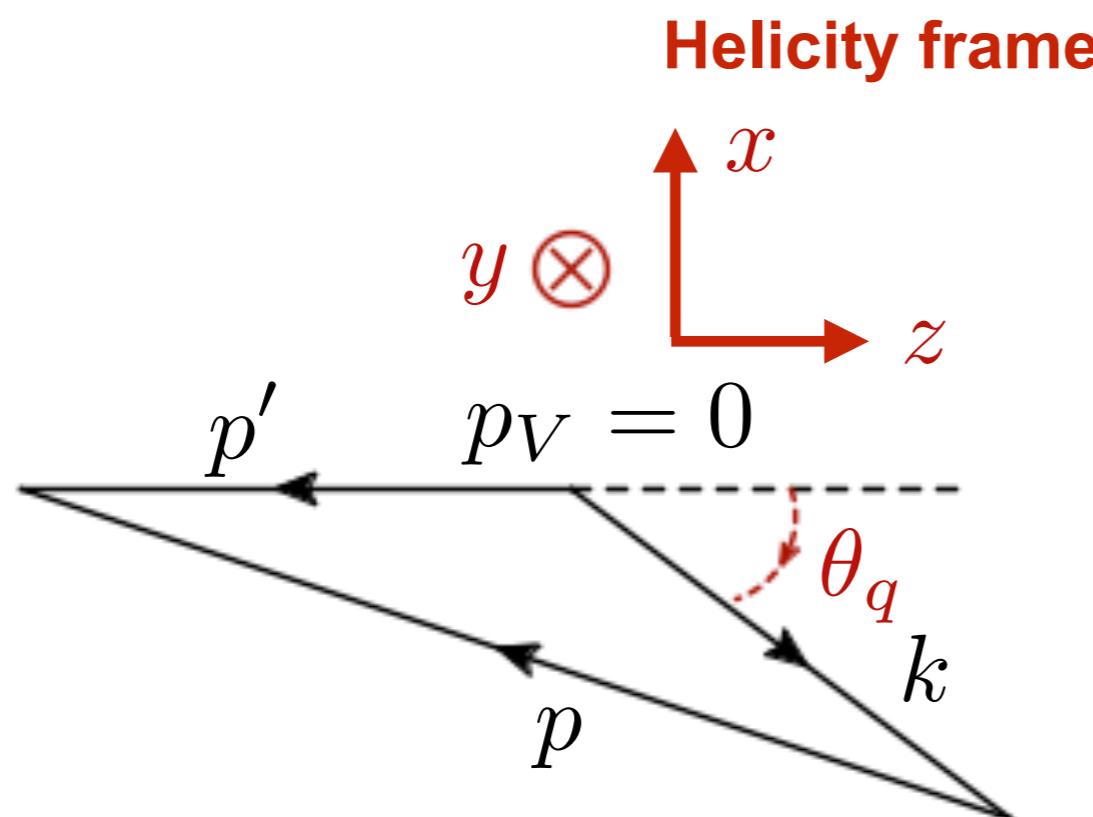
$$T_{\lambda_\gamma m} \simeq \delta_{\lambda_\gamma, m} T_{\lambda_\gamma m} + \dots$$



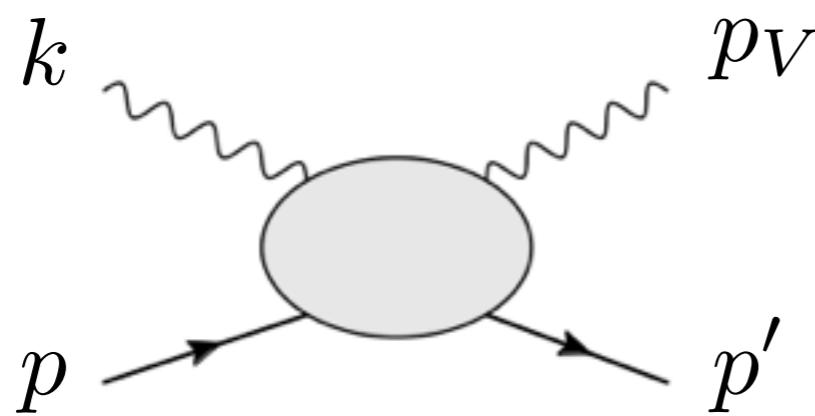
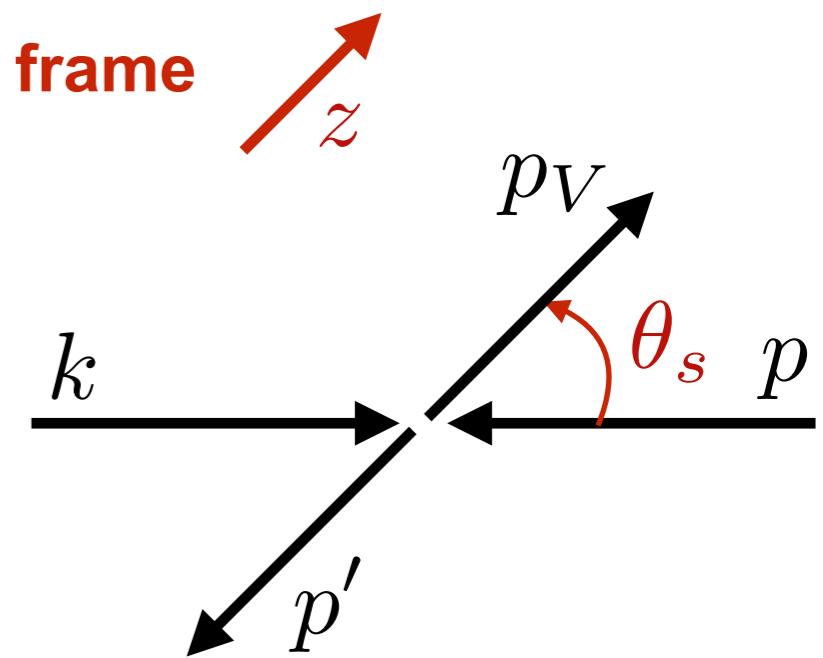
Reflectivity basis:

$$[\ell]_m^{(\epsilon)} = T_{1m} - \epsilon T_{-1-m}$$

Dominant: $(\epsilon = +, m = 1)$

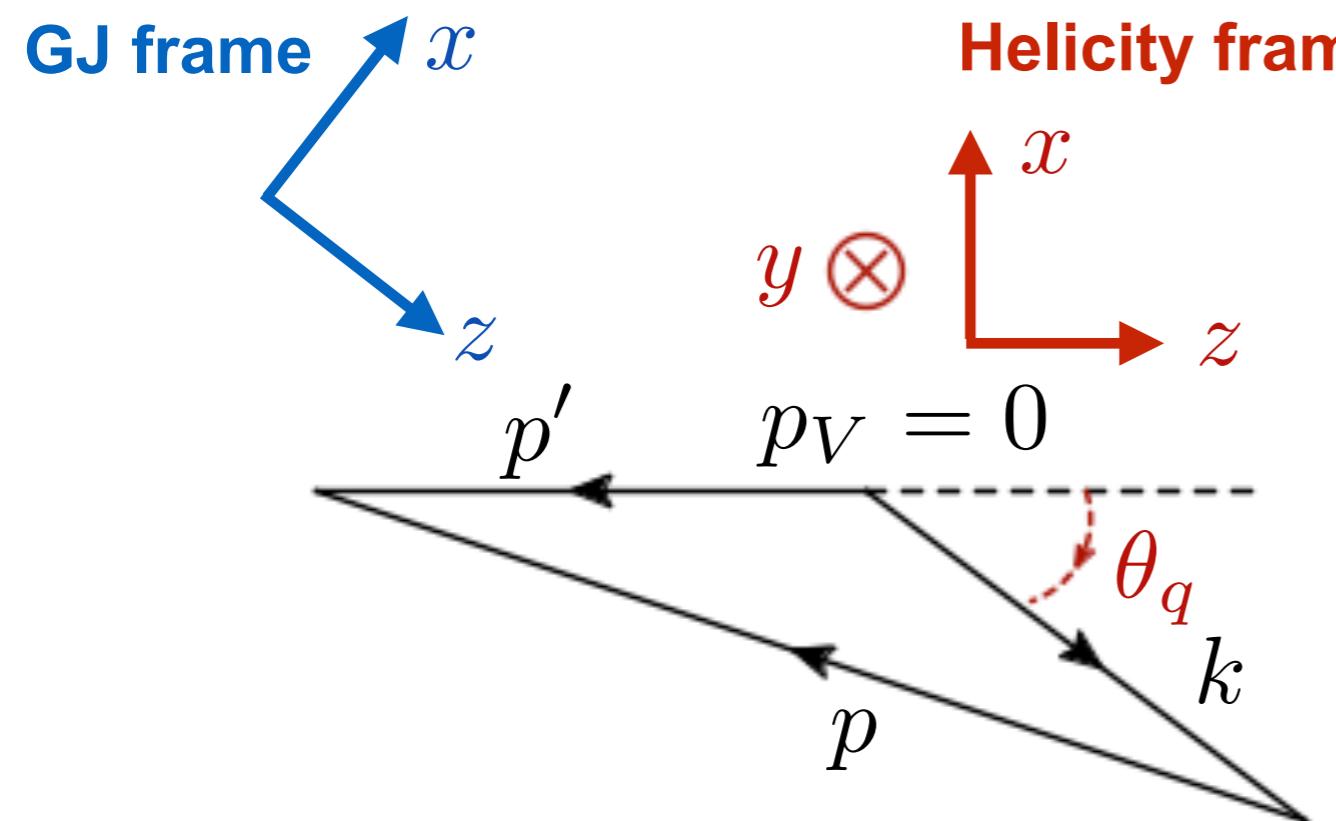


s-channel frame

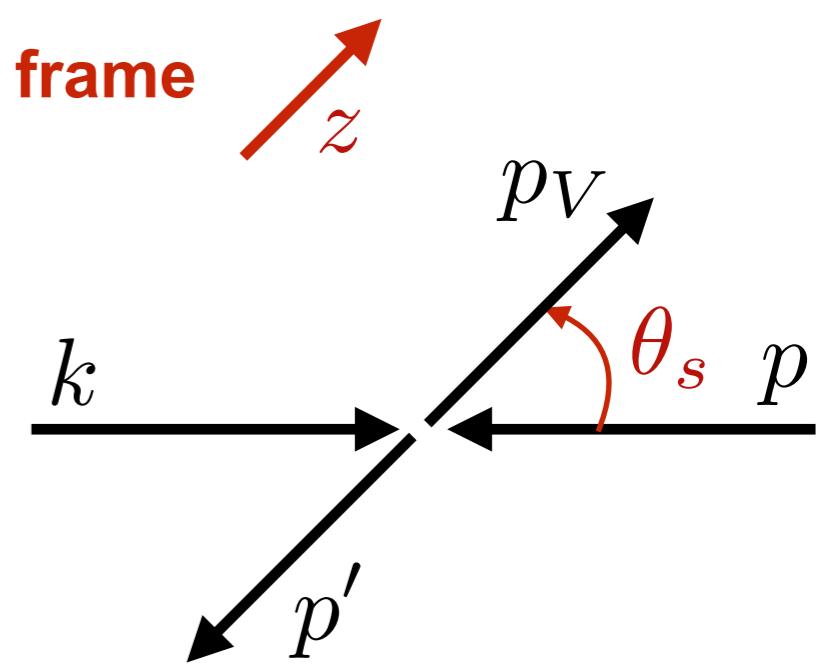


Frames

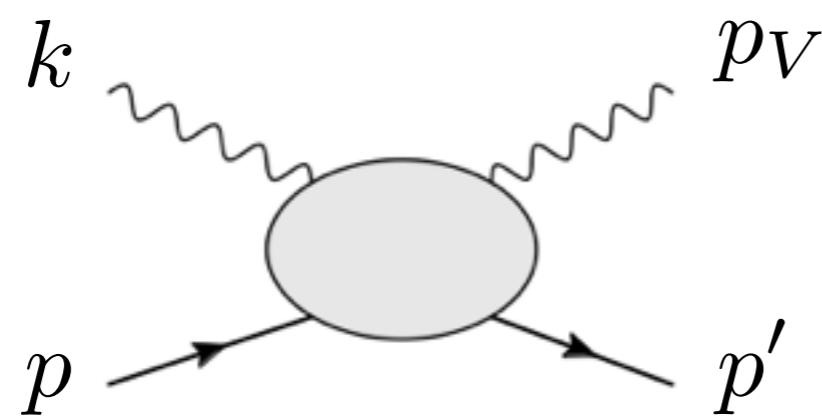
25



s-channel frame



t-channel frame



z



k

p

p'

p

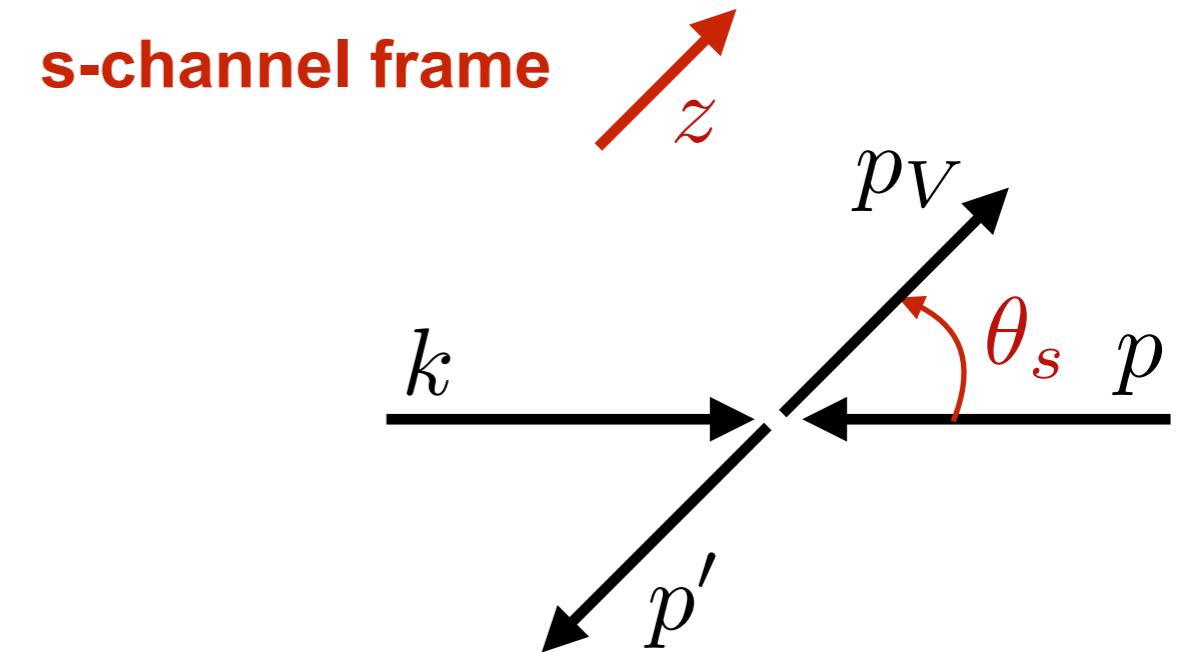
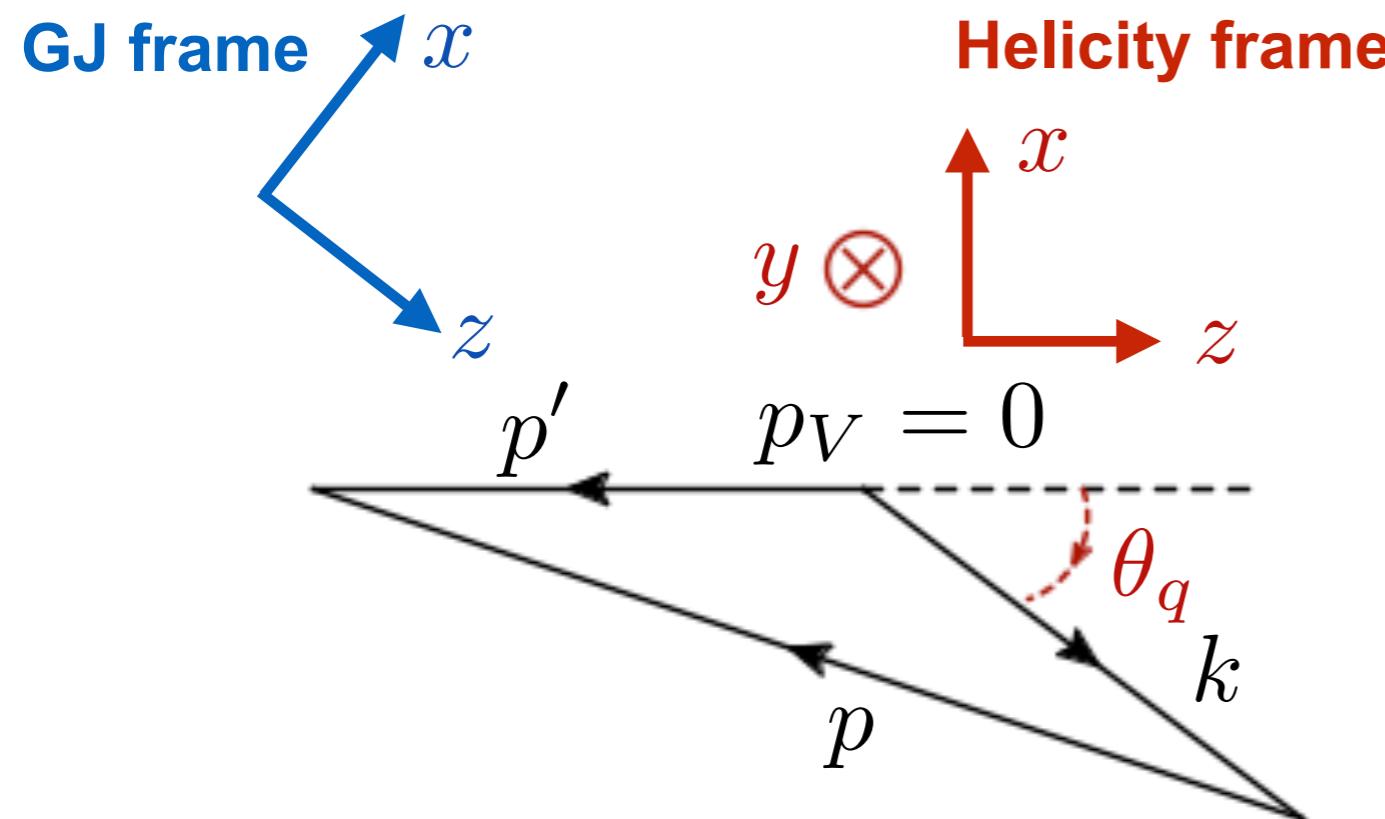
p'

p_V

θ_t

Frames

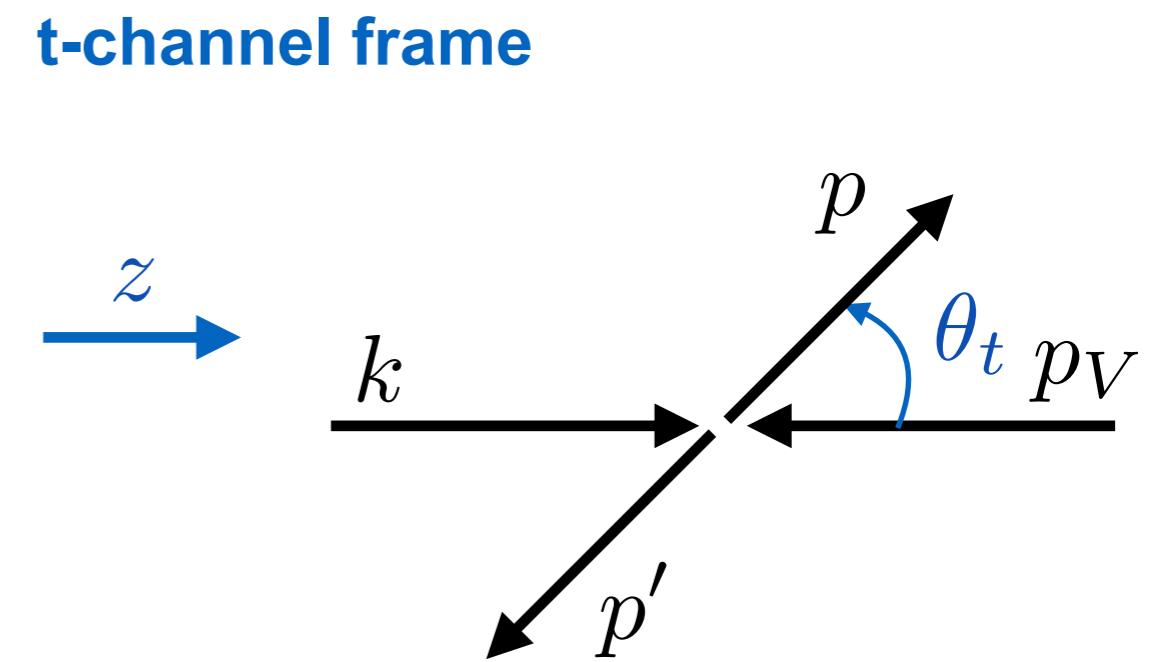
25



$\rho_{MM'}|_H = \rho_{MM'}|_{s\text{-chan}}$

$\rho_{MM'}|_{GJ} = \rho_{MM'}|_{t\text{-chan}}$

rotation



Measured Intensities

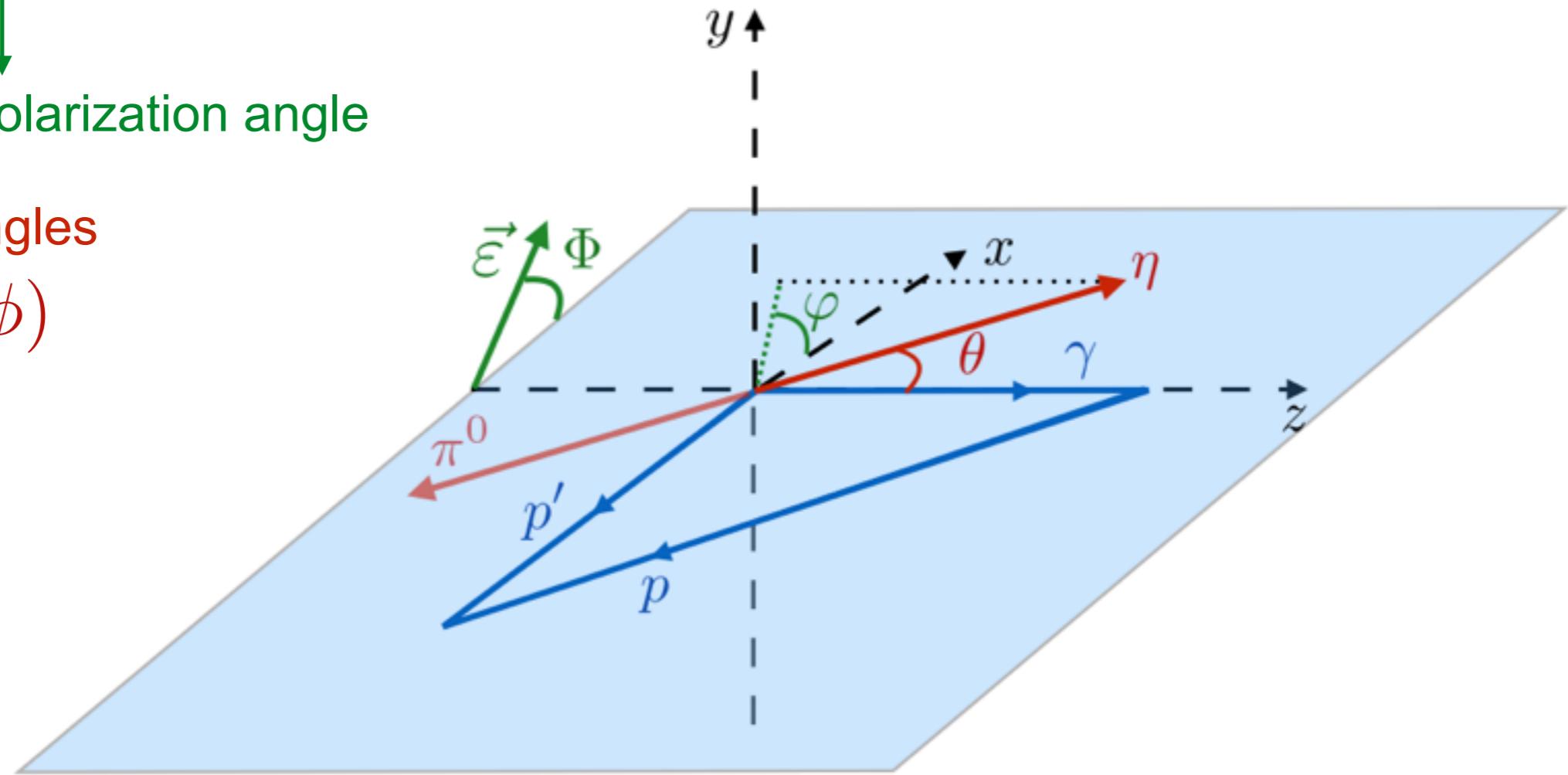
$$I(\Omega, \Phi) = I^0(\Omega) - P_\gamma I^1(\Omega) \cos 2\Phi - P_\gamma I^2(\Omega) \sin 2\Phi$$



 polarization angle



 η decay angles
 $\Omega = (\theta, \phi)$

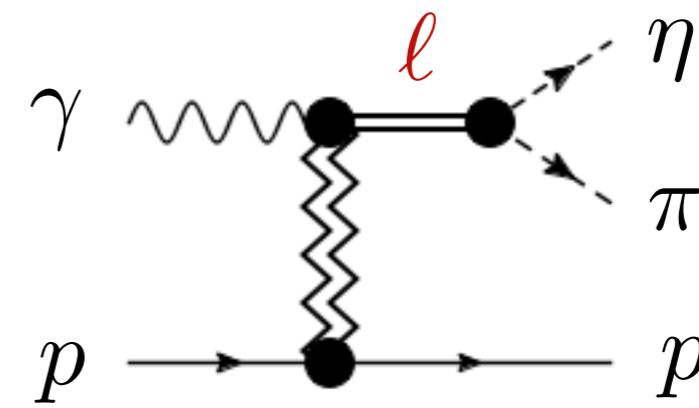


Implicit variables

Beam energy (fixed)

momentum transfer (integrated)

$\eta\pi$ invariant mass (binned)



Spin Density Matrix Elements

27

$$\rho_{00}^0 = \frac{2}{N} \sum_{\lambda, \lambda'} \left| T_{\lambda, \lambda'}^{1,0} \right|^2$$

$$N = \sum_{\lambda, \lambda', \lambda_\gamma, \lambda_V} \left| T_{\lambda_\gamma, \lambda_V}^{\lambda, \lambda'} \right|^2$$

$$\text{Re } \rho_{10}^0 = \frac{1}{N} \text{Re} \sum_{\lambda, \lambda'} \left(T_{\lambda, \lambda'}^{1,1} - T_{\lambda, \lambda'}^{-1,-1} \right) T_{\lambda, \lambda'}^{*,1,0}$$

$$\rho_{11}^1 = \frac{2}{N} \text{Re} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{-1,1} T_{\lambda, \lambda'}^{1,1*}$$

$$\rho_{1-1}^0 = \frac{2}{N} \text{Re} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{1,1} T_{\lambda, \lambda'}^{*,1,-1}$$

$$\rho_{00}^1 = \frac{2}{N} \text{Re} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{-1,0} T_{\lambda, \lambda'}^{1,0*}$$

$$\rho_{1-1}^1 + \text{Im } \rho_{1-1}^2 = \frac{2}{N} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{-1,1} T_{\lambda, \lambda'}^{*,1,-1}$$

$$\text{Re } \rho_{10}^1 + \text{Im } \rho_{10}^2 = \frac{1}{N} \text{Re} \sum_{\lambda, \lambda'} \mathcal{M}_{\lambda, \lambda'}^{-1,1} \mathcal{M}_{\lambda, \lambda'}^{1,0*}$$

$$\rho_{1-1}^1 - \text{Im } \rho_{1-1}^2 = \frac{2}{N} \sum_{\lambda, \lambda'} T_{\lambda, \lambda'}^{1,1} T_{\lambda, \lambda'}^{*,1,-1}$$

$$\text{Re } \rho_{10}^1 - \text{Im } \rho_{10}^2 = \frac{1}{N} \text{Re} \sum_{\lambda_\gamma, \lambda, \lambda'} \mathcal{M}_{\lambda, \lambda'}^{1,1} \mathcal{M}_{\lambda, \lambda'}^{-1,0*}$$

Observables: Moments of Angular distribution

28

$$H^0(LM) = \frac{1}{2\pi} \int I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$H^1(LM) = \frac{-1}{\pi P_\gamma} \int I(\Omega, \Phi) \cos 2\Phi d_{M0}^L(\theta) \cos M\phi \, d\Omega d\Phi$$

$$\text{Im } H^2(LM) = \frac{1}{\pi P_\gamma} \int I(\Omega, \Phi) \sin 2\Phi d_{M0}^L(\theta) \sin M\phi \, d\Omega d\Phi$$

$$H^1(LM) + \text{Im } H^2(LM) \propto \sum_{\epsilon, \ell\ell', mm'} \left(\frac{2\ell' + 1}{2\ell + 1} \right)^{1/2} \epsilon (-1)^m C_{\ell' 0 L 0}^{\ell 0} C_{\ell' m' L M}^{\ell m} [\ell]_{-m}^{(\epsilon)} [\ell']_{m'}^{(\epsilon)*}$$

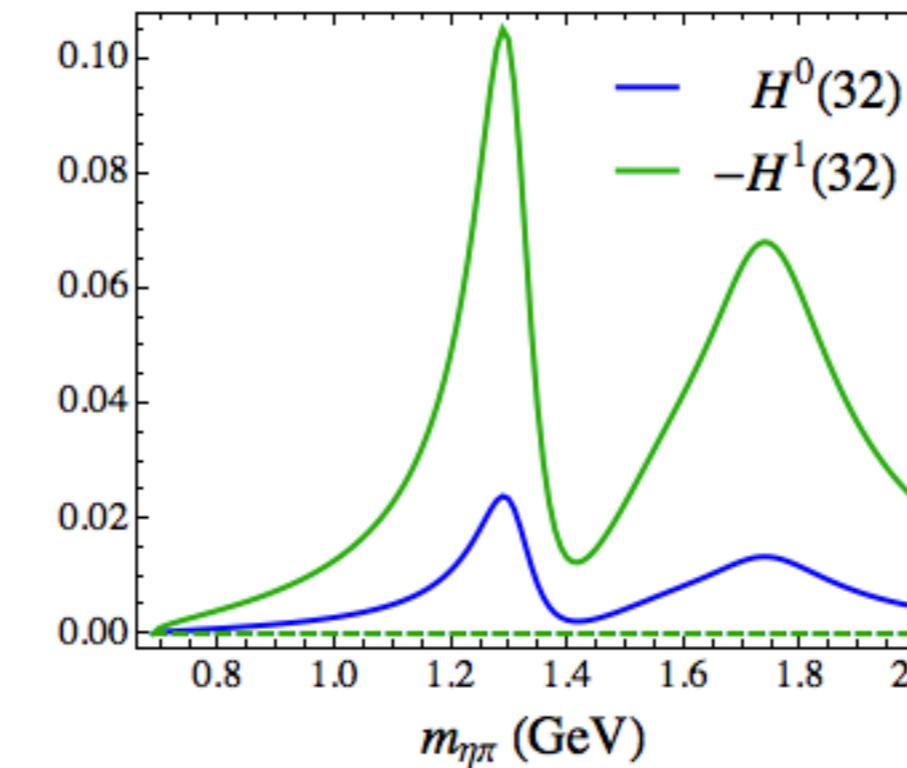
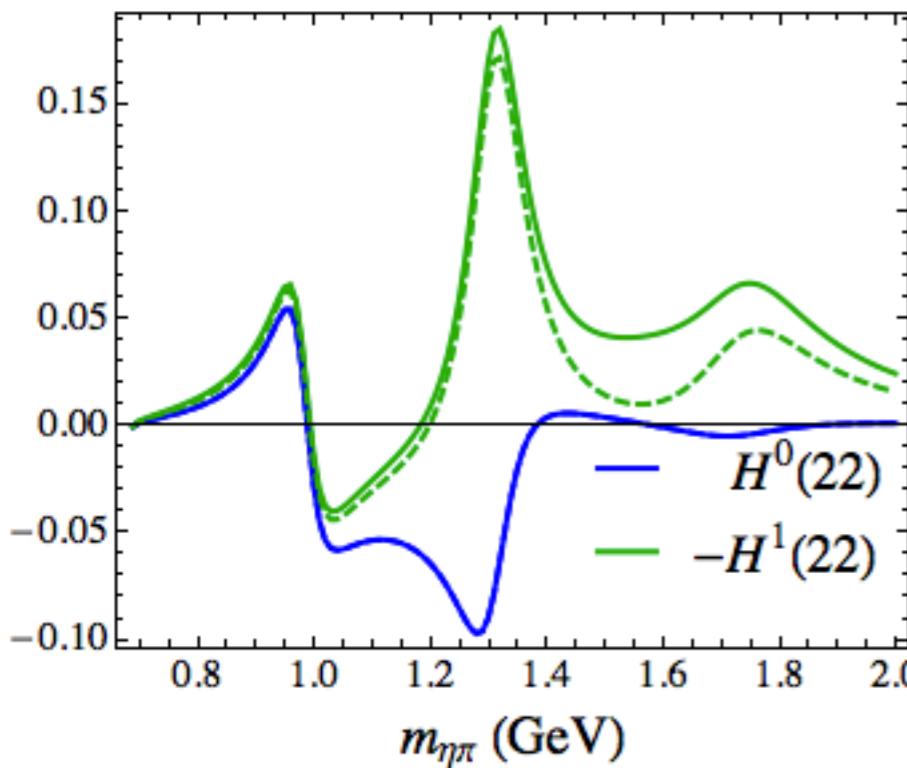
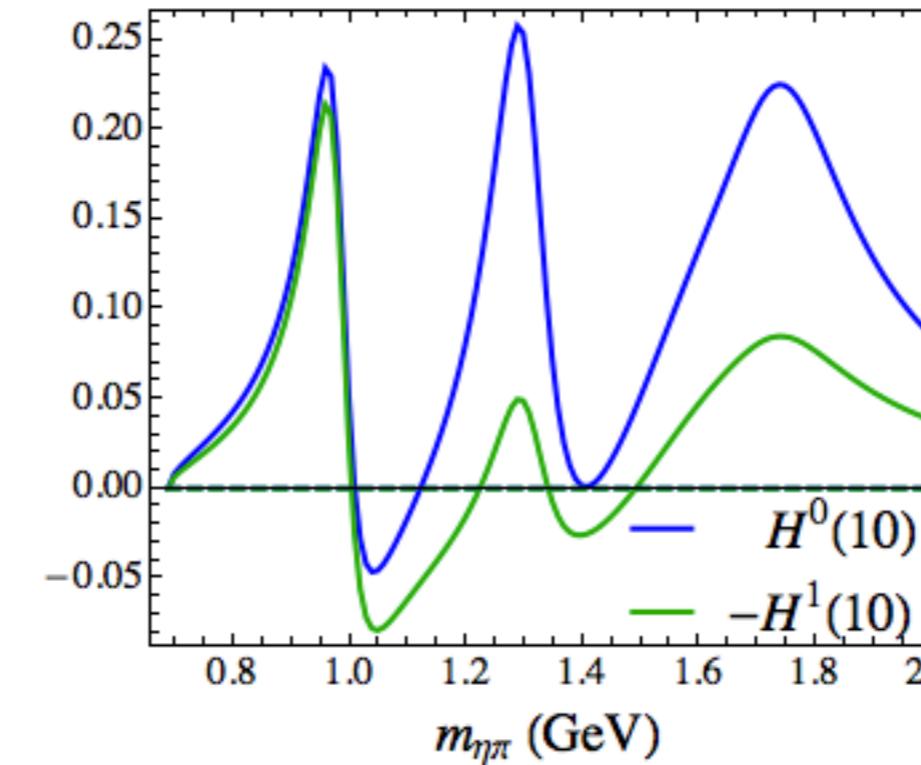
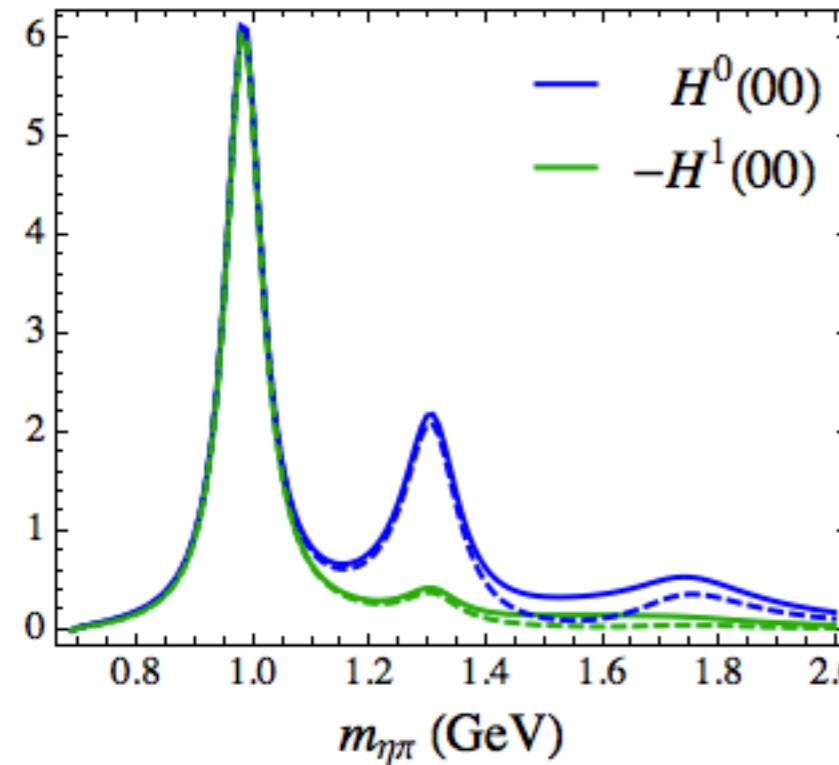
$m' = m - M$
 \uparrow
 $0 \leq -m ; 0 \leq m'$
 \downarrow
 \downarrow

The model features
only positive projections

$$H^1(LM) + \text{Im } H^2(LM) = 0 \quad M \geq 1$$

Moments

29



P-wave apparent
in odd moments but
not in even moments

solid lines: S + P + D waves
dashed lines: S + D waves