

A theoretical analysis of $\eta' \rightarrow \eta\pi\pi$, and of the doubly radiative $\eta^{(\prime)} \rightarrow \pi^0\gamma\gamma$ and $\eta' \rightarrow \eta\gamma\gamma$ decays

Sergi Gonzàlez-Solís¹

Indiana University

Center for Exploration of Energy and Matter

based on: Gonzàlez-Solís and Passemar EPJC 78 (2018), no.9 758, Escribano,
Gonzàlez-Solís, Jora, Royo 1812.08454 [hep-ph]

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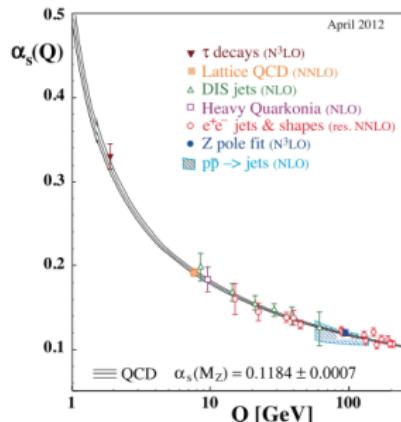


INDIANA UNIVERSITY

¹sgonzal@iu.edu

Approaches to describe the low-energy regime of QCD

- **Lattice** QCD simulations: determination of SM fundamental parameters from first principles (quark masses, α_s , meson decay constants etc.)
- **S-matrix theory** based on analyticity and unitarity arguments, (**dispersion relations**)
- **Chiral Perturbation theory**



Chiral Perturbation Theory

Gasser and Leutwyler, Nucl. Phys. B 250, 465 (1985)

- Low-energy EFT of QCD for light mesons i.e. $\pi^{\pm,0}, K^\pm, K^0, \bar{K}^0, \eta_8$ associated to $SU(3)_L \otimes SU(3)_R \xrightarrow{\text{SCSB}} SU(3)_V$ exhibited by QCD
- Perturbative expansion in terms of p^2 and m_q : $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots$

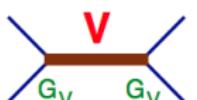
$$\mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle, \quad \phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi_3 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi_3 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix},$$

$$u^2 = e^{i \frac{\sqrt{2}\phi}{F}}, \quad \chi = 2B\mathcal{M}, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad u_\mu = i u^\dagger D_\mu U u^\dagger,$$

$$\mathcal{L}_4 = L_1 \langle u_\mu u^\mu \rangle^2 + L_2 \langle u_\mu u^\nu \rangle \langle u^\mu u_\nu \rangle + L_3 \langle u_\mu u^\mu u_\nu u^\nu \rangle + \dots$$

⚠ η_1 not included due to the axial anomaly

⚠ Valid $\frac{p^2}{M_R^2} < 1$: polynomial cannot reproduce resonance poles



$$q^2 \ll M_{V,A}^2$$



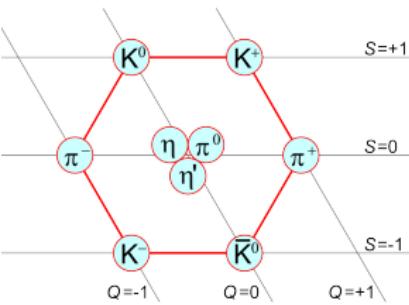
$$\frac{G_V^2}{M_V^2} \propto L_{1,2,3}$$

Large- N_C $U(3)$ ChPT

Kaiser and Leutwyler, EPJC 17, 623 (2000)

- Axial Anomaly is absent; η_1 as the ninth Goldstone boson
- Degrees of freedom: $\pi^{\pm,0}$, K^\pm , K^0 , \bar{K}^0 and the η and η'

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$



- Simultaneous triple expansion in terms of $p^2 \sim m_q \sim 1/N_C$

$$\begin{aligned} \mathcal{L}_\chi &= \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{3} m_1^2 \ln^2 \det u, \\ \Phi &= \begin{pmatrix} \frac{1}{\sqrt{2}} \pi_3 + \frac{1}{\sqrt{6}} \eta_8 + \frac{1}{\sqrt{3}} \eta_1 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi_3 + \frac{1}{\sqrt{6}} \eta_8 + \frac{1}{\sqrt{3}} \eta_1 & K^0 \\ K^- & K^0 & -\frac{2}{\sqrt{6}} \eta_8 + \frac{1}{\sqrt{3}} \eta_1 \end{pmatrix}. \end{aligned}$$

⚠ η' heavier than some resonances $\frac{M_{\eta'}^2}{M_R^2} > 1$

Resonance Chiral Theory

Ecker, Gasser, Pich and de Rafael, Nucl. Phys. B 321, 311 (1989)

- Resonance as explicit degrees of freedom

$$\mathcal{L}_{R\chi T} = \mathcal{L}^{p^2} + \mathcal{L}_S + \mathcal{L}_{\text{kin}}^S$$

$$\mathcal{L}_S = \textcolor{blue}{c_d} \langle S_8 u_\mu u^\mu \rangle + \textcolor{red}{c_m} \langle S_8 \chi_+ \rangle + \tilde{\textcolor{blue}{c_d}} S_1 \langle u_\mu u^\mu \rangle + \tilde{\textcolor{red}{c_m}} S_1 \langle \chi_+ \rangle,$$

$$S_8 = \begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 & a_0^+ & \kappa^+ \\ a_0^- & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & -\frac{2}{\sqrt{6}}\sigma_8 \end{pmatrix}, \quad S_1 = \sigma_1$$

$$\begin{aligned} \mathcal{L}_S = & \frac{2\textcolor{blue}{c_d}}{f^2} \langle S_8 (\partial_\mu \Phi) (\partial^\mu \Phi) \rangle + 4B_0 \textcolor{red}{c_m} [\langle S_8 \mathcal{M} \rangle - \frac{1}{4f^2} \langle S_8 (\Phi^2 \mathcal{M} + \mathcal{M} \Phi^2 + 2\Phi \mathcal{M} \Phi) \rangle] \\ & + \frac{2\tilde{\textcolor{blue}{c_d}}}{f^2} S_1 \langle (\partial_\mu \Phi) (\partial^\mu \Phi) \rangle + 4B_0 \tilde{\textcolor{red}{c_m}} S_1 [\langle \mathcal{M} \rangle - \frac{1}{4f^2} \langle (\phi^2 \mathcal{M} + \mathcal{M} \Phi^2 + 2\Phi \mathcal{M} \Phi) \rangle] \end{aligned}$$

⚠ Resonances spoils power counting, no systematic EFT but a model based on the Large- N_C limit as a guideline

Recent experimental activity on η and η' physics

- Phenomenology of η and η' among their main objectives

A2	CERN NA6	BESIII	LHCb
$\eta \rightarrow e^+ e^- \gamma$ [1]	$\eta \rightarrow e^+ e^- \gamma$ [4]	$\eta' \rightarrow e^+ e^- \gamma$ [5]	$\eta^{(\prime)} \rightarrow \pi^+ \pi^-$ [12]
$\eta \rightarrow \pi^0 \gamma \gamma$ [2]	$\eta \rightarrow \pi^+ \pi^- \gamma$ [4]	$\eta' \rightarrow \pi^0 \gamma \gamma$ [6]	
$\eta' \rightarrow \eta \pi^0 \pi^0$ [3]	$\eta \rightarrow e^+ e^- e^+ e^-$ [4]	$\eta' \rightarrow \omega e^+ e^-$ [7]	
	$\eta \rightarrow \pi^+ \pi^- e^+ e^-$ [4]	$\eta' \rightarrow 4\pi$ [8]	
		$\eta' \rightarrow 3\pi$ [9]	
		$\eta' \rightarrow \pi^+ \pi^- e^+ e^-$ [10]	
		$\eta' \rightarrow \eta \pi \pi$ [11]	

- Experimental precision for η - η' observables is increasing
- Better theoretical predictions are demanded
- To have a better and more complete knowledge of QCD at low-energies

[1] Adlarson et.al. Phys.Rev. C 94 6 065206 (2016) ; [2] Nefknes et.al. Phys.Rev. C 90 2 025206 (2014)

[3] Adlarson et.al. ArXiv: 1709.04230

[4] Adlarson et.al. Phys.Rev. C 94 6 065206 (2016) ; [5] Ablikim et.al. Phys.Rev. D 92 1 012001 (2015)

[6] Ablikim et.al. Phys.Rev. D96 (2017) ; [7] Ablikim et.al. Phys.Rev. D 92 5 051101 (2015)

[8] Ablikim et.al. Phys.Rev.Lett. 112 251801 (2014) ; [9] Ablikim et.al. Phys.Rev.Lett. 118 1 012001 (2017)

[10] Ablikim et.al. Phys.Rev. D 87 9 092011 (2013) ; [11] Ablikim et.al. ArXiv: 1709.04627

[12] Aaij et.al. Phys.Lett. B 764 233 (2017)

$\eta' \rightarrow \eta\pi\pi$ decays

• Motivation

- Main decay channel of the η' : $\text{BR}(\eta' \rightarrow \eta\pi^0\pi^0) = 22.3(8)\%$,
 $\text{BR}(\eta' \rightarrow \eta\pi^+\pi^-) = 42.9(7)\%$ PDG [2017]
- New data recently released the A2 and BESIII collaborations

• Objectives

- To test any of the extensions of ChPT e.g. resonance chiral theory, Large- N_C $U(3)$ ChPT
- To study the effects of the $\pi\pi$ and $\pi\eta$ final-state interactions
- To extract the associated **Dalitz-plot parameters** from fits to data

$$|M(X, Y)|^2 = |N|^2 (1 + \textcolor{red}{a}Y + \textcolor{red}{b}Y^2 + \textcolor{red}{c}X + \textcolor{red}{d}X^2 + \dots)$$

• Theoretical framework

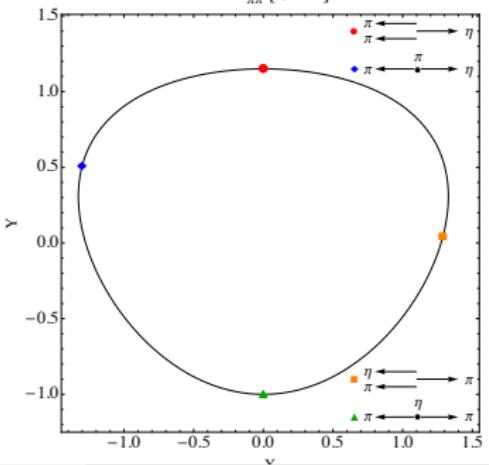
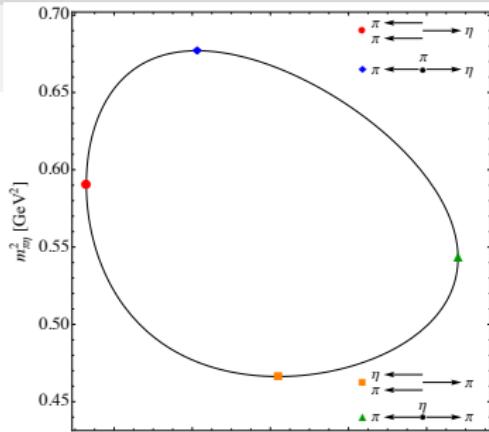
- $U(3)$ ChPT with resonances at one-loop
- Final-state interaction through the N/D unitarization method

Kinematics and Dalitz plot variables

- $s = (p_{\eta'} - p_\eta)^2$
- $t = (p_{\eta'} - p_{\pi^+})^2$
- $u = (p_{\eta'} - p_{\pi^-})^2$
- $s + t + u = m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2$

⇒ only two independent variables,
e.g. s and $t - u \propto \cos \theta_s$

- $X = \frac{\sqrt{3}}{Q} (T_{\pi_1} - T_{\pi_2}) = \frac{\sqrt{3}}{Q} (\textcolor{brown}{u} - \textcolor{blue}{t})$
- $Y = \frac{m_\eta + 2m_\pi}{m_\pi} \frac{T_\eta}{Q} - 1$
 $= \frac{m_\eta + 2m_\pi}{m_\pi} \frac{(m_{\eta'} - m_\eta)^2 - \textcolor{red}{s}}{2m_{\eta'} Q} - 1$
- $Q = m_{\eta'} - m_\eta - 2m_\pi$



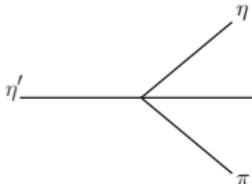
$\eta' \rightarrow \eta\pi\pi$: Leading order

- ChPT Lagrangian at $\mathcal{O}(p^2)$

$$\mathcal{L}^{p^2} = \frac{F_\pi^2}{4} \langle u_\mu u^\mu \rangle + \frac{F_\pi^2}{4} \langle \chi_+ \rangle + \frac{F_\pi^2}{3} m_1^2 \ln^2 \det u$$

- Expanding in powers of Φ

$$\begin{aligned} \mathcal{L}^{p^2} &= \frac{1}{2} \langle \partial_\mu \Phi \partial^\mu \Phi \rangle + \frac{1}{12f^2} \underbrace{\langle (\Phi(\partial_\mu \Phi) - (\partial_\mu \Phi)\Phi)(\Phi(\partial^\mu \Phi) - (\partial^\mu \Phi)\Phi) \rangle}_{+B_0 \left\{ -\langle \mathcal{M}\Phi^2 \rangle + \frac{1}{6f^2} \langle \mathcal{M}\Phi^4 \rangle \right\} + \mathcal{O}\left(\frac{\Phi^6}{f^4}\right)} \\ &\quad \downarrow \text{no contribution} \end{aligned}$$

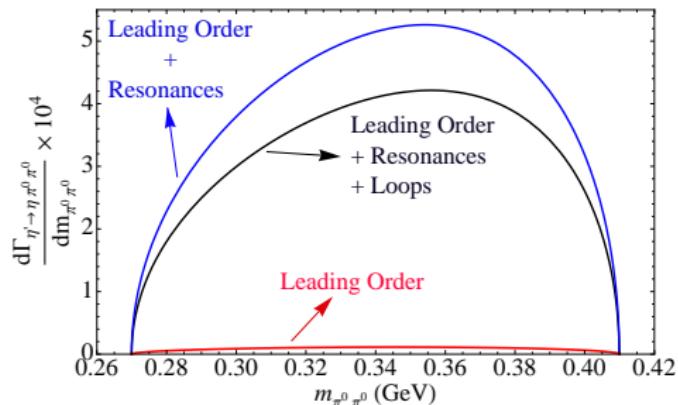
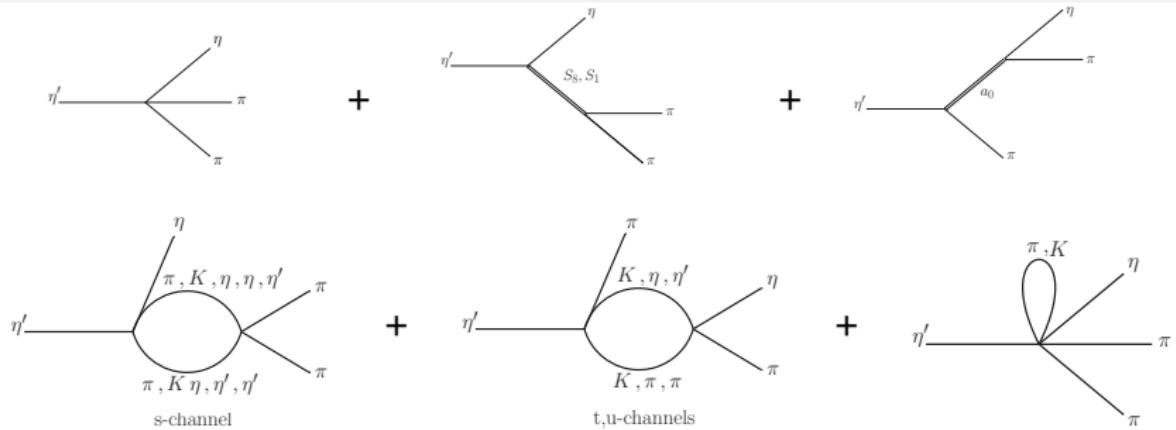
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$$= \boxed{\mathcal{M}_{\eta' \rightarrow \eta\pi\pi}^{\text{LO}} = \frac{M_\pi^2}{6F_\pi^2} (2\sqrt{2} \cos 2\theta - \sin 2\theta)}$$

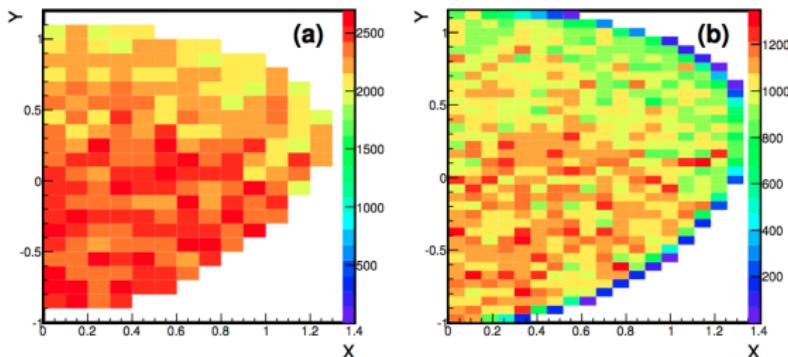
	$BR(\eta' \rightarrow \eta\pi^+\pi^-)$	$BR(\eta' \rightarrow \eta\pi^0\pi^0)$
Leading Order	1.1%	0.6%
PDG 2018	42.6(7)	22.8(8)%

- Reason for this difference: amplitude is chirally suppressed (vanishes when $M_\pi^2 \rightarrow 0$)
- Higher order effects? • Resonances exchanges (a_0, f_0, σ) • $\pi\pi, \pi\eta$ final state interactions

$\eta' \rightarrow \eta\pi\pi$: Scalar Resonance and loop contributions



Fits to experimental data



- We relate the experimental Dalitz plot data with the differential decay distribution from theory through

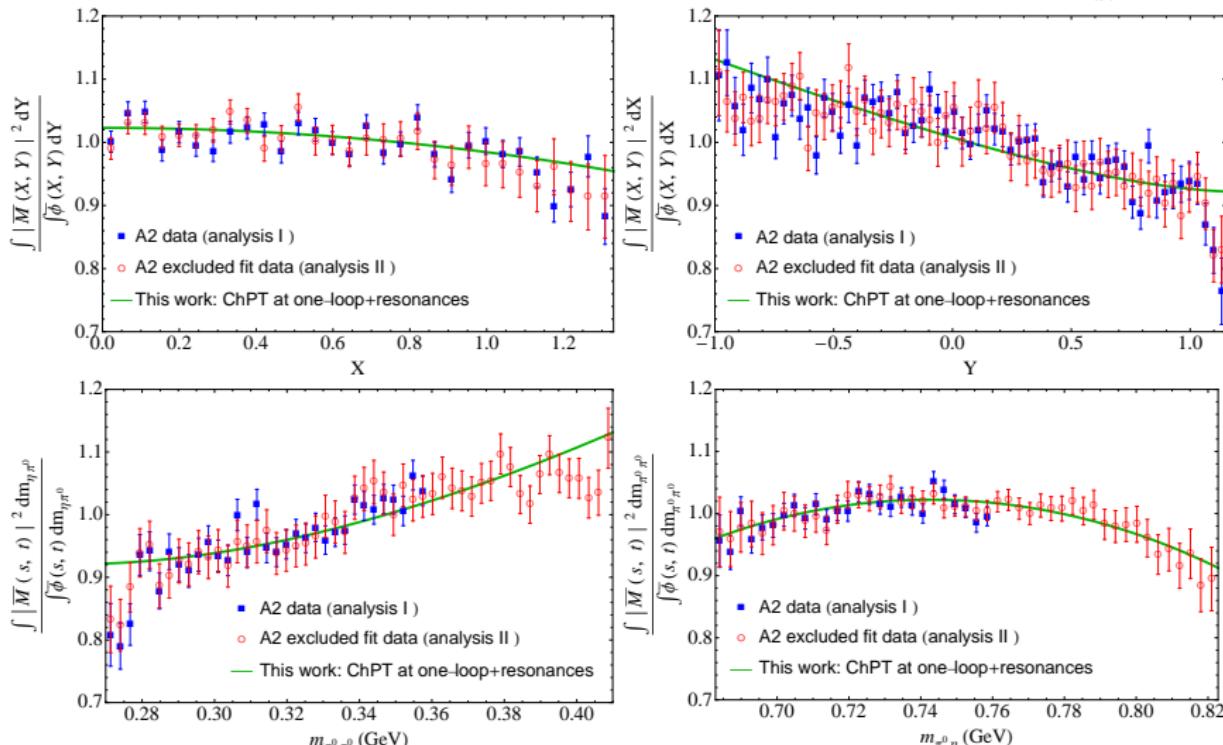
$$\frac{d^2N_{\text{events}}}{dX dY} = \frac{d\Gamma(\eta' \rightarrow \eta\pi^0\pi^0)}{dX dY} \frac{N_{\text{events}}}{\Gamma_{\eta'} \bar{B}(\eta' \rightarrow \eta\pi^0\pi^0)} \Delta X \Delta Y,$$

- N_{events} = 463066 (analysis I) and 473044 (analysis II)
- $\Delta X = \Delta Y = 0.10$

Fits to experimental data: ChPT at one loop with resonances

- Fit 1: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d = c_m$

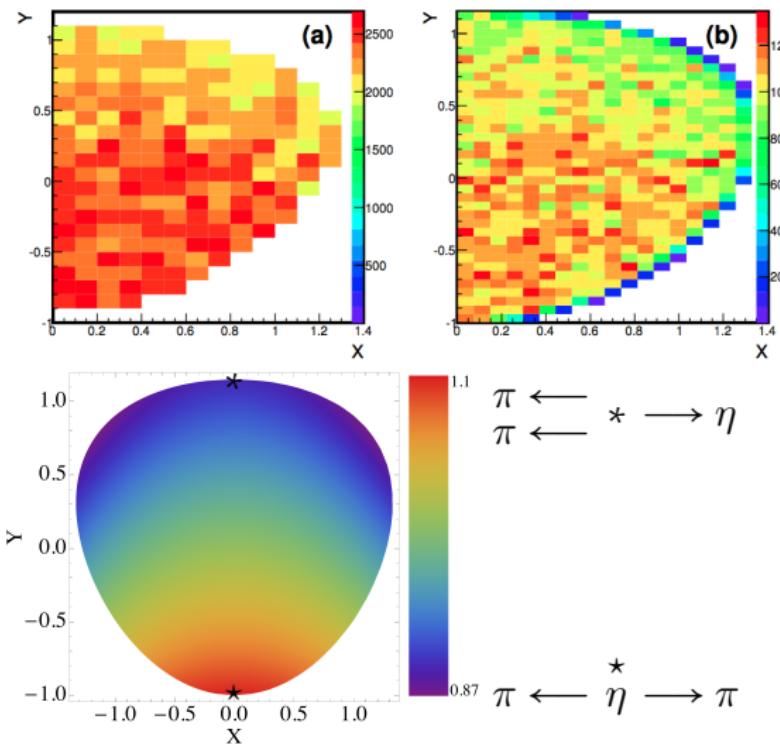
Fit parameters : $M_S = 973(5)$ MeV , $c_d = c_m = 30.1(4)$ MeV , $\chi^2_{\text{dof}} = 1.22$



Fits to experimental data: ChPT at one loop with resonances

- Dalitz plot slope parameters

A2 Coll. 1709.04230



Dalitz Parameters (A2):

$$a[Y] = -0.074(8)(6)$$

$$b[Y^2] = -0.063(14)(5)$$

$$d[X^2] = -0.050(9)(5)$$

Dalitz Parameters (this work):

$$a[Y] = -0.095(6)$$

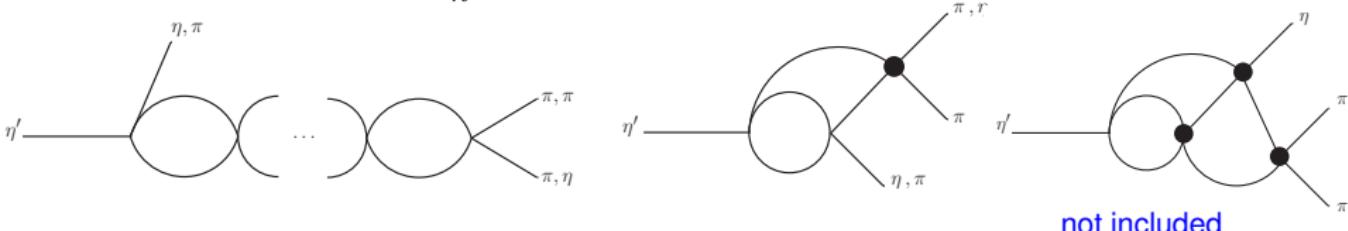
$$b[Y^2] = 0.005(1)$$

$$d[X^2] = -0.037(5)$$

Unitarity

- Unitarity relation

$$\text{Im} \mathcal{M}_{\eta' \rightarrow \eta \pi \pi} = \frac{1}{2} \sum_n (2\pi)^4 \delta^4(p_\eta + p_1 + p_2 - p_n) \mathcal{T}_{n \rightarrow \eta \pi \pi}^* \mathcal{M}_{\eta' \rightarrow n}$$



- Two-particle unitarity relation for the partial-wave decay amplitude

$$\text{Im} (m_{\eta' \rightarrow \eta \pi \pi}^{00}(s)) = \sigma_\pi(s) (t_{\pi \pi \rightarrow \pi \pi}^{00}(s))^* m_{\eta' \rightarrow \eta \pi \pi}^{00}(s) \times \theta(s - 4m_\pi^2),$$

$$\text{Im} (m_{\eta' \rightarrow \eta \pi \pi}^{02}(s)) = \sigma_\pi(s) (t_{\pi \pi \rightarrow \pi \pi}^{02}(s))^* m_{\eta' \rightarrow \eta \pi \pi}^{02}(s) \times \theta(s - 4m_\pi^2),$$

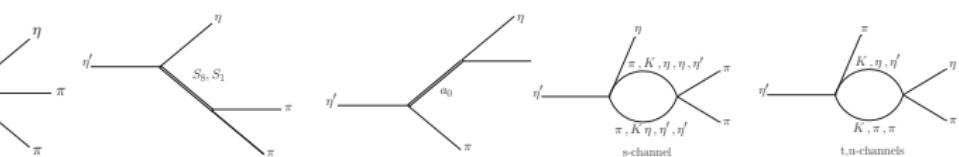
$$\text{Im} (m_{\eta' \rightarrow \eta \pi \pi}^{10}(t)) = \frac{\lambda^{1/2}(t, m_\pi^2, m_\eta^2)}{t} (t_{\pi \eta \rightarrow \pi \eta}^{10}(t))^* \times m_{\eta' \rightarrow \eta \pi \pi}^{10}(t) \theta(t - (m_\pi + m_\eta)^2),$$

$$\text{Im} (m_{\eta' \rightarrow \eta \pi \pi}^{10}(u)) = \frac{\lambda^{1/2}(u, m_\pi^2, m_\eta^2)}{u} (t_{\pi \eta \rightarrow \pi \eta}^{10}(u))^* \times m_{\eta' \rightarrow \eta \pi \pi}^{10}(u) \theta(u - (m_\pi + m_\eta)^2),$$

N/D unitarisation method applied to $\eta' \rightarrow \eta\pi\pi$

- Amplitude at one-loop in Large- N_C $U(3)$ ChPT with resonances

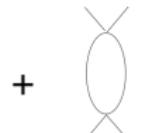
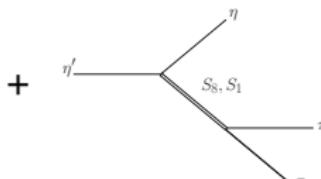
$$\mathcal{M}^{\eta' \rightarrow \eta\pi\pi}(s) =$$



- N/D representation of $\mathcal{M}^{\eta' \rightarrow \eta\pi\pi}(s)$

$$m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s) = [1 + N_{\pi\pi}^{IJ}(s)g_{\pi\pi}(s)]^{-1} R_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s)$$

$$N_{\pi\pi \rightarrow \pi\pi}^{IJ}(s) =$$



$$R_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s) =$$

$$g_{\pi\pi}(s) = a_{\pi\pi}(\mu) - \bullet = \frac{1}{16\pi^2} \left(a_{\pi\pi}(\mu) + \log \frac{m_\pi^2}{\mu^2} - \sigma(s) \log \frac{\sigma(s)-1}{\sigma(s)+1} \right)$$

Partial waves

- Unitarized amplitude in terms of the *S*-and-*D*-waves

$$\begin{aligned} \mathcal{M}_{\eta' \rightarrow \eta \pi \pi}^{I=0}(s, \cos \theta_s) &= \sum_J 32\pi(2J+1)P_J(\cos \theta_s)m^{IJ}(s) \\ &= 32\pi P_0(\cos \theta_s) \frac{m^{00}(s)}{1 + g_{\pi\pi}(s)t_{\pi\pi}^{00}(s)} + 160\pi P_2(\cos \theta_s) \frac{m^{02}(s)}{1 + g_{\pi\pi}(s)t_{\pi\pi}^{02}(s)} \end{aligned}$$

$$m^{IJ}(s) = \frac{1}{32\pi} \frac{s}{\lambda(s, m_{\eta'}^2, m_\eta^2)^{1/2} \lambda(s, m_\pi^2, m_\pi^2)^{1/2}} \int_{t_{\min}}^{t_{\max}} dt P_J(\cos \theta_s) \mathcal{M}^I(s, t, u)$$

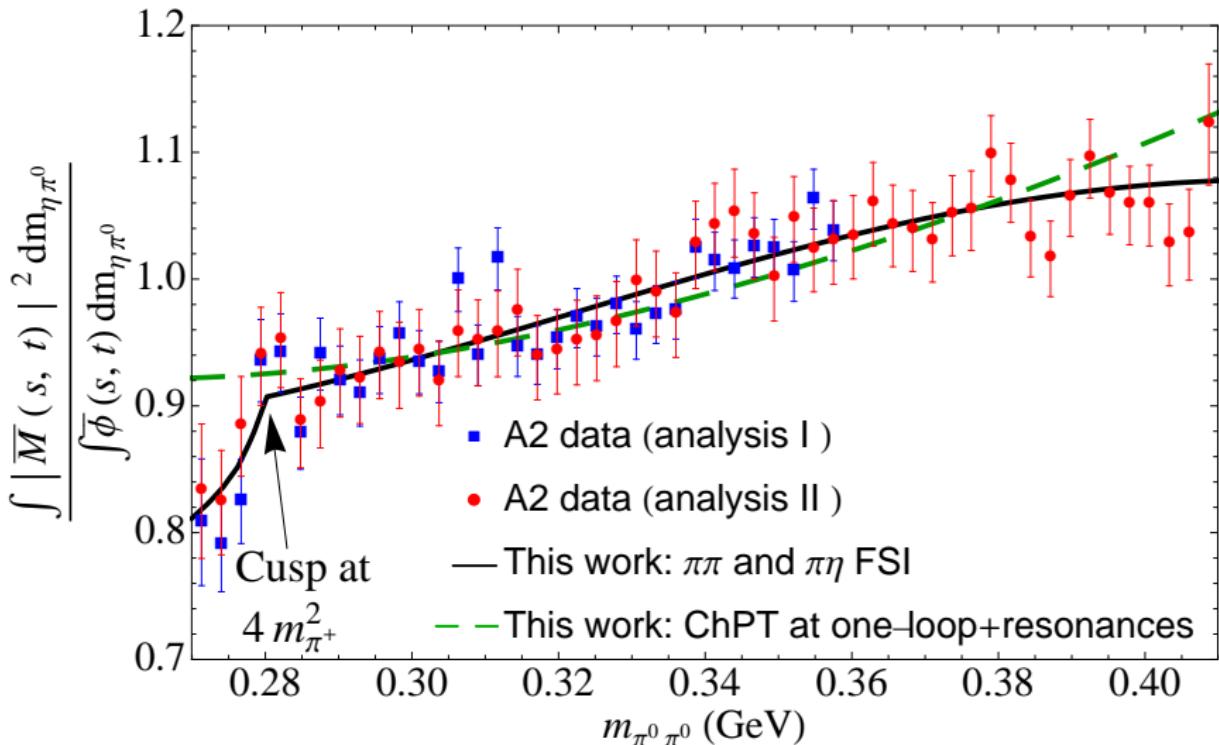
$$\cos \theta_s = -\frac{s(m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2 - s - 2t)}{\lambda(s, m_{\eta'}^2, m_\eta^2)^{1/2} \lambda(s, m_\pi^2, m_\pi^2)^{1/2}},$$

$$P_0(\cos \theta_s) = 1, \quad P_2(\cos \theta_s) = \frac{1}{2} \left[-1 + 3(\cos \theta_s)^2 \right] \propto X^2$$

Research achievements: $\eta' \rightarrow \eta\pi\pi$ decays

Eur. Phys. J. C78 (2018) no.9, 758

• Results



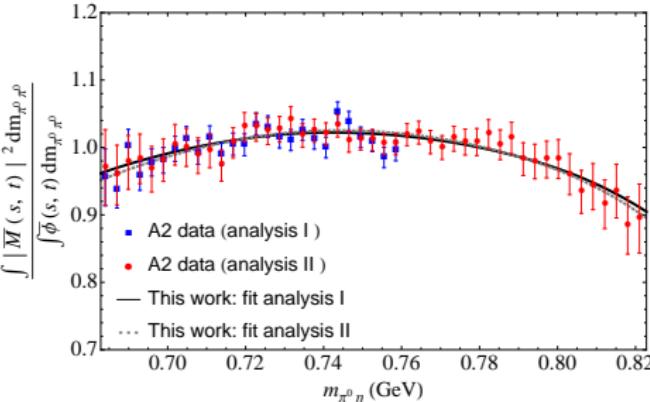
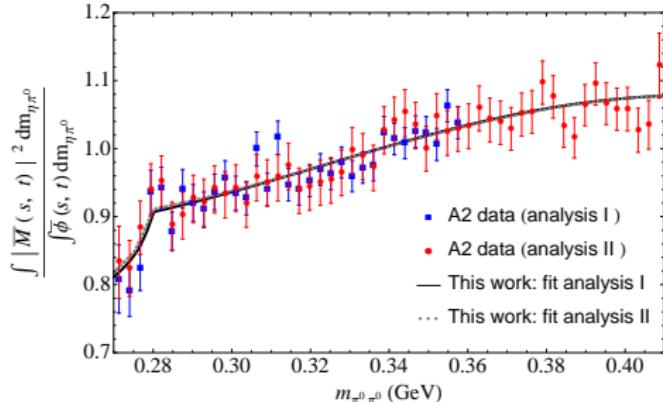
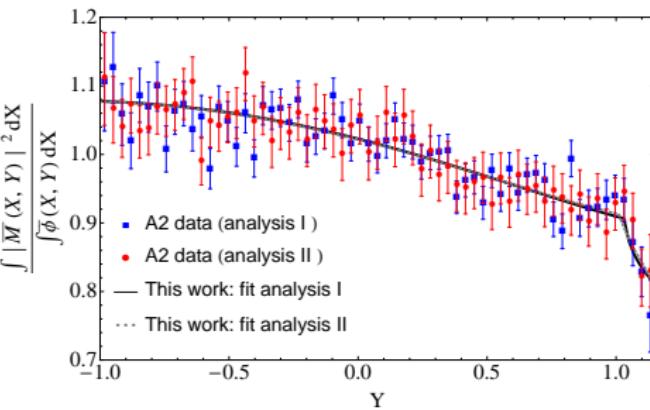
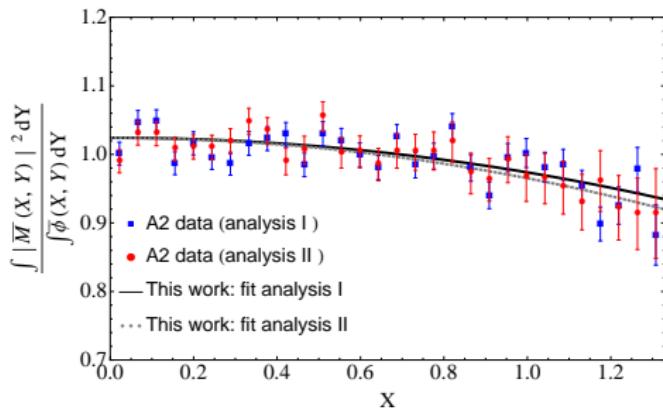
Fits to experimental data: $\pi\pi$ final-state interactions effects

Parameter	Fit A	Fit B	Fit C	A2 Collaboration
M_S	1001(24)	988(10)	930(7)(39)	
c_d	29.5(1.8)	28.6(7)	23.5(6)(2.4)	
c_m	= c_d	= 41.1(1)	= 80(20)	
\tilde{c}_d	17.0(1.0)	16.5(4)	13.6(4)(1.4)	
\tilde{c}_m	= \tilde{c}_d	= 18.9(9)	46.2(11.5)	
$a_{\pi\pi}(\mu)$	0.73(25)	0.24(12)	0.41(11)(19)	
χ^2_{dof}	220.4/197 ~ 1.12	220.1/197 ~ 1.12	220.1/197 ~ 1.12	

$a[Y]$	-0.075(9)	-0.075(7)	-0.074(7)(1)	-0.074(8)(6)
$b[Y^2]$	-0.051(1)	-0.056(1)	-0.053(1)(1)	-0.063(14)(5)
$c[X]$	0	0	0	0
$d[X^2]$	-0.049(14)	-0.050(4)	-0.049(4)(1)	-0.050(9)(5)

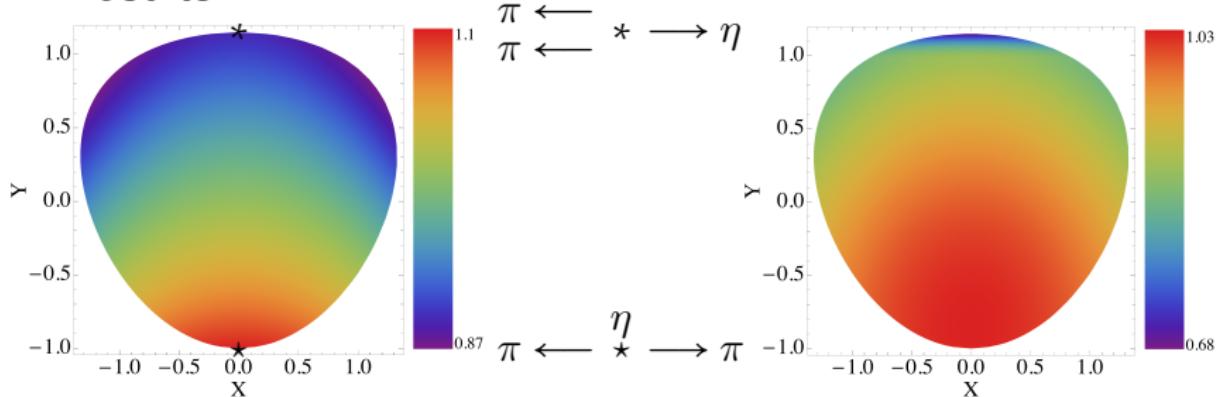
Parameter	Fit A (ChPT)	Fit B (ChPT)	Fit C (ChPT)	A2 Collaboration
M_S	973(5)	992(7)	926(5)(25)	
c_d	30.1(4)	32.9(5)	25.7(4)(1.9)	
c_m	= c_d	= 41.1(1)	= 80(20)	
\tilde{c}_d	17.4(2)	18.4(3)	14.8(2)(1.1)	
\tilde{c}_m	= \tilde{c}_d	= 18.9(9)	46.2(11.5)	
χ^2_{dof}	242.2/198 ~ 1.22	246.4/198 ~ 1.24	242.3/198 ~ 1.22	
$a[Y]$	-0.095(6)	-0.083(6)	-0.090(7)(2)	-0.074(8)(6)
$b[Y^2]$	0.005(1)	-0.001(1)	0.004(1)(0)	-0.063(14)(5)
$c[X]$	0	0	0	0
$d[X^2]$	-0.037(5)	-0.057(5)	-0.041(7)(1)	-0.050(9)(5)

Fits to experimental data: $\pi\pi$ and $\pi\eta$ final-state interactions



Fits to $\eta' \rightarrow \eta\pi\pi$ data

• Results



ChPT

$$a[Y] = -0.095(6)$$

$$b[Y^2] = 0.005(1)$$

$$d[X^2] = -0.037(5)$$

Dalitz slope parameters

$$\Rightarrow$$

Final-state interactions

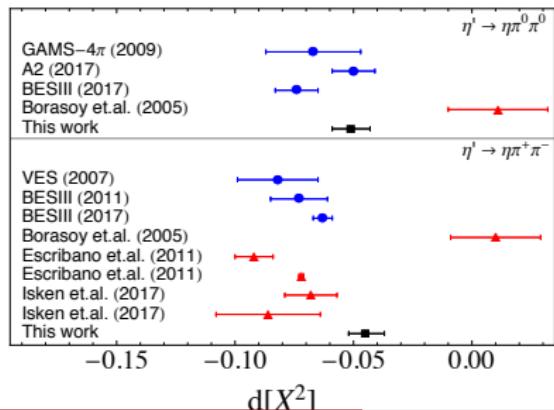
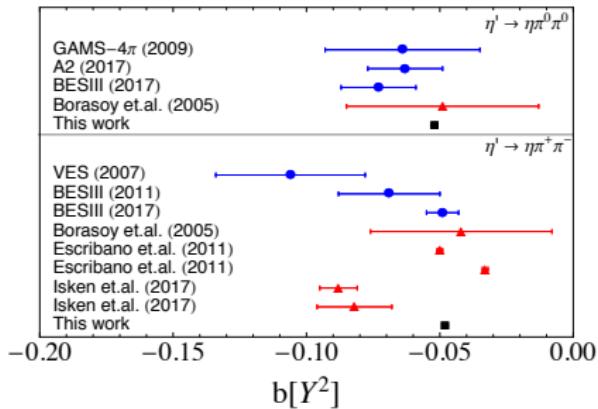
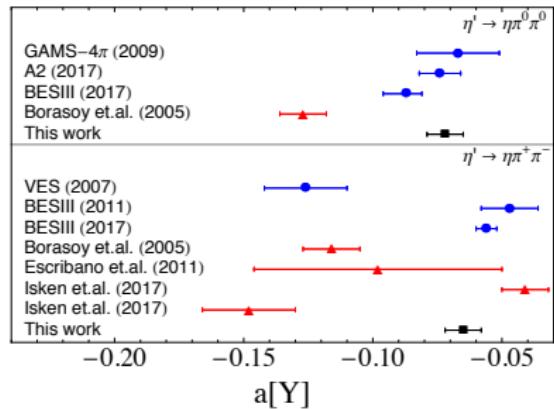
$$a[Y] = -0.073(7)(5)$$

$$b[Y^2] = -0.052(1)(2)$$

$$d[X^2] = -0.052(8)(5)$$

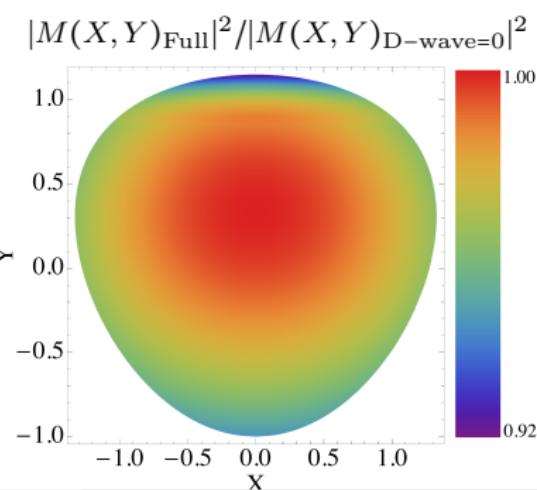
- We are willing to provide our parametrization to the interested experimental groups e.g. GlueX, BESIII etc.

Dalitz Plot Parameters

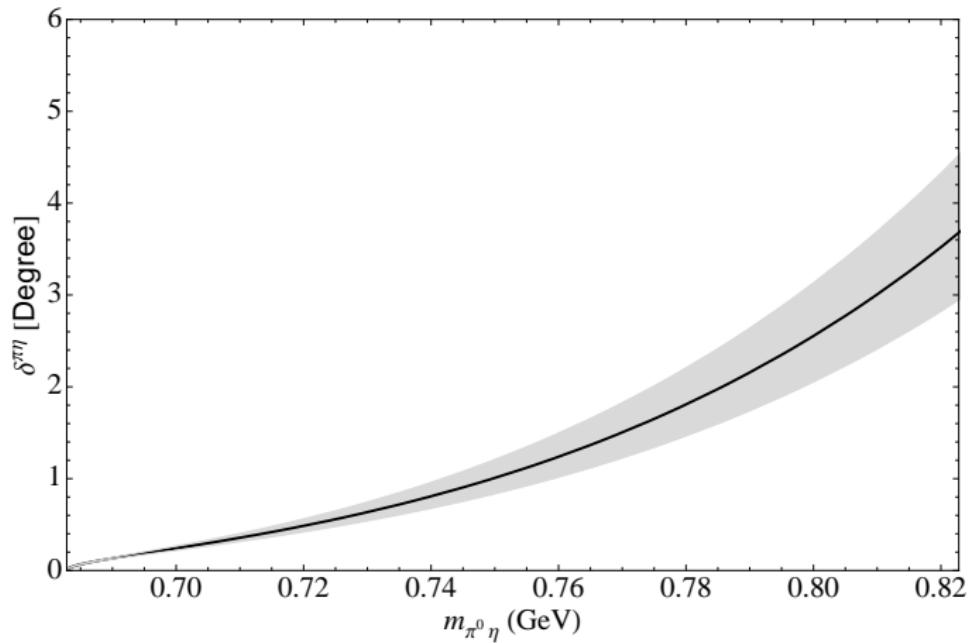


The role of the *D*-wave $\pi\pi$ FSI

Parameter	Analysis I	
	Fit 1 (with <i>D</i> -wave)	Fit 1 (w/o <i>D</i> -wave)
M_S	1017(68)(24)	996(66)(25)
c_d	30.4(4.8)(9)	23.3(3.5)(1.5)
c_m	= c_d	= c_d
\tilde{c}_d	17.6(2.8)(5)	13.5(2.0)(9)
\tilde{c}_m	= \tilde{c}_d	= \tilde{c}_d
$a_{\pi\pi}$	0.76(61)(6)	2.01(1.61)(71)
χ^2_{dof}	1.12	1.24
$a[Y]$	-0.074(7)(8)	-0.091(9)(4)
$b[Y^2]$	-0.049(1)(2)	-0.013(1)(5)
$c[X]$	0	0
$d[X^2]$	-0.047(8)(4)	-0.031(6)(3)
$\kappa_{03}[Y^3]$	0.001	0.001
$\kappa_{21}[YX^2]$	-0.004	-0.001
$\kappa_{22}[Y^2X^2]$	0.001	0.0004

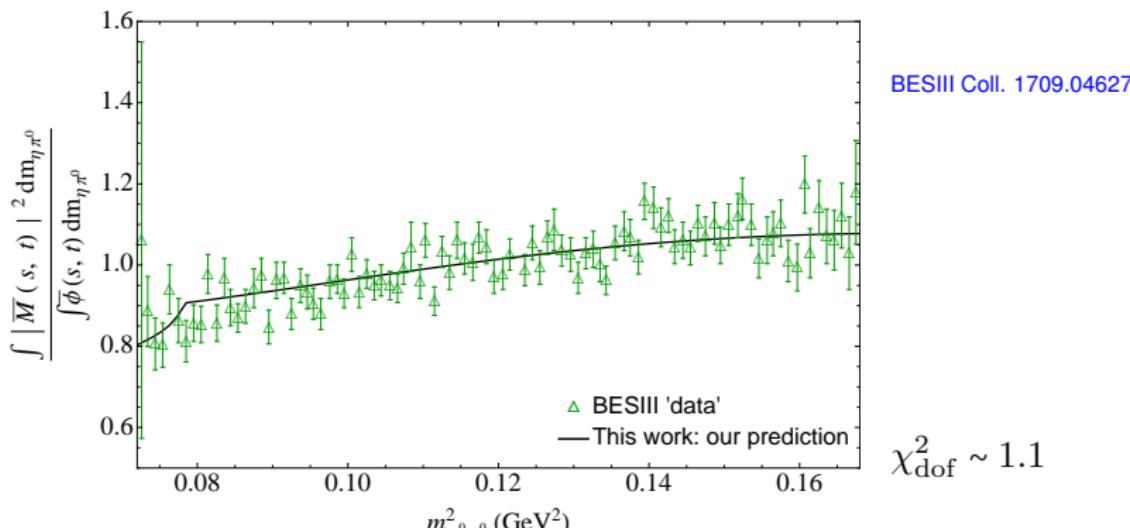


$\pi\eta$ phase shift within the phase space



Comparison with BESIII 2017 experimental data

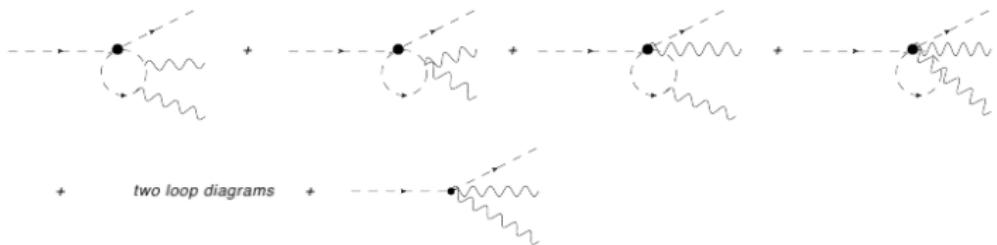
- Data is not publicly available
- Less events than A2 (351016 vs $1.2 \cdot 10^5$)
- No cusp structure seen
- Contrary to A2 both $\eta' \rightarrow \eta\pi^0\pi^0$ and $\eta' \rightarrow \eta\pi^+\pi^-$ are measured



$\eta^{(\prime)} \rightarrow \pi^0 \gamma\gamma$ and $\eta' \rightarrow \eta \gamma\gamma$ decays

Motivation

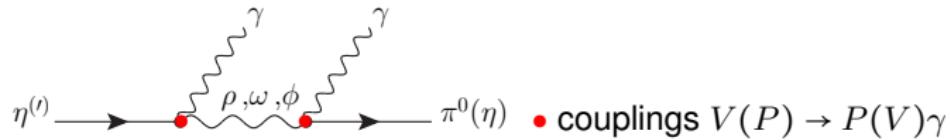
- The tree level contributions are $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ vanish
- First non-vanishing contribution comes from $\mathcal{O}(p^4)$ loops
- The first sizable contribution comes at $\mathcal{O}(p^6)$: LEC's not well known
- To test ChPT and a wide range of chiral models e.g. VMD or $\text{L}\sigma\text{M}$



Objectives

- To perform a first analysis of the recent $\eta' \rightarrow \pi^0 \gamma\gamma$ BESIII data and provide a first prediction for the non-yet-measured decay $\eta' \rightarrow \eta \gamma\gamma$

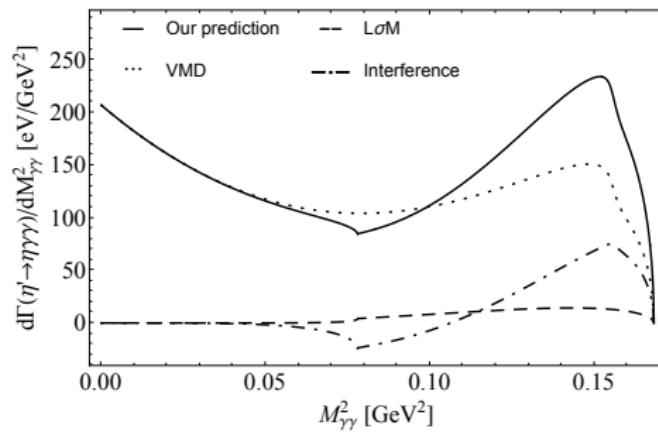
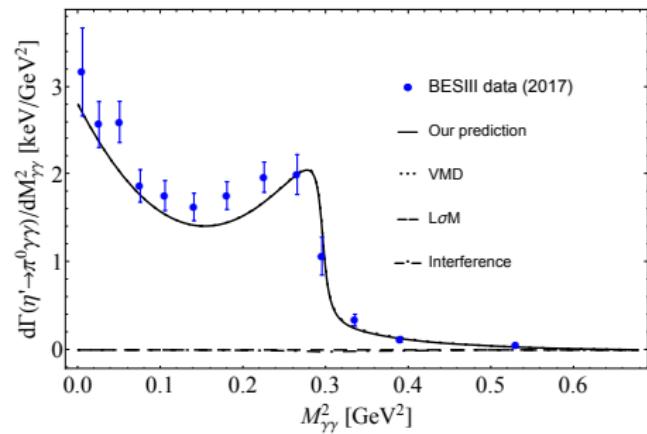
Theoretical framework: Vector Meson Dominance and $\text{L}\sigma\text{M}$



Research achievements: $\eta^{(')} \rightarrow \pi^0 \gamma\gamma$ and $\eta' \rightarrow \eta \gamma\gamma$ decays

1812.08454

• Results



$$\Gamma(\eta' \rightarrow \pi^0 \gamma\gamma) = 0.57(5) \text{ eV}$$

$$\Gamma_{\text{VMD}}(\eta' \rightarrow \pi^0 \gamma\gamma) = 0.57(5) \text{ eV}$$

$$BR(\eta' \rightarrow \pi^0 \gamma\gamma) = 2.90(25) \times 10^{-4}$$

$$BR_{\text{exp}}(\eta' \rightarrow \pi^0 \gamma\gamma) = 3.20(7)(23) \times 10^{-4}$$

$$\Gamma(\eta' \rightarrow \eta \gamma\gamma) = 24.5(6) \text{ eV}$$

$$\Gamma_{\text{VMD}}(\eta' \rightarrow \eta \gamma\gamma) = 21.6(5) \text{ eV}$$

$$BR(\eta' \rightarrow \eta \gamma\gamma) = 1.3(1) \times 10^{-4}$$

$$BR_{\text{exp}}(\eta' \rightarrow \eta \gamma\gamma) = ?$$

Summary

- $\eta' \rightarrow \eta\pi\pi$ analyzed within $U(3)$ ChPT at one-loop with resonances
- We have illustrated a method (N/D) to resumme two-particle FSI
- Dalitz plot parameters:
 - Y -variable is linear in s : Importance of $\pi\pi$ FSI
 - X -variable appear in the form $\cos\theta_s = Xf(Y)$: Importance of the D -wave
 - $\pi\eta$ FSI effects are small
- We are in position to provide our parameterization to experimental groups
- Same method can be applied for $\eta^{(\prime)} \rightarrow 3\pi$ decays
- Predictions for $\eta' \rightarrow \pi^0\gamma\gamma$ and $\eta' \rightarrow \eta\gamma\gamma$

Dalitz plot parameters: current state-of-the-art

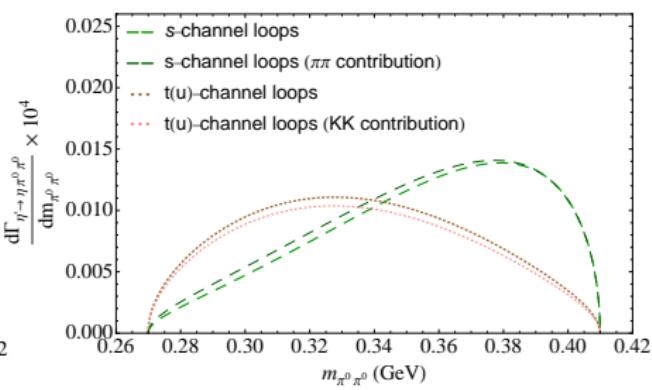
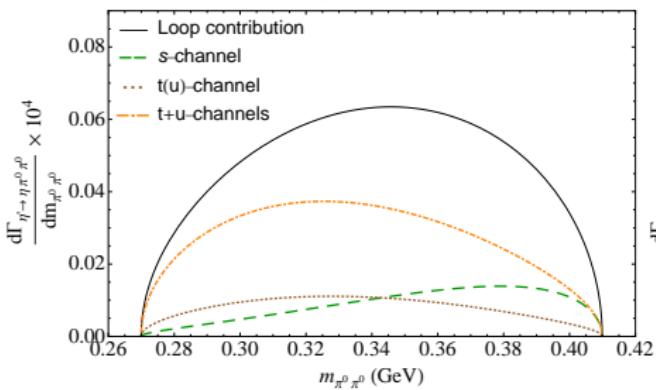
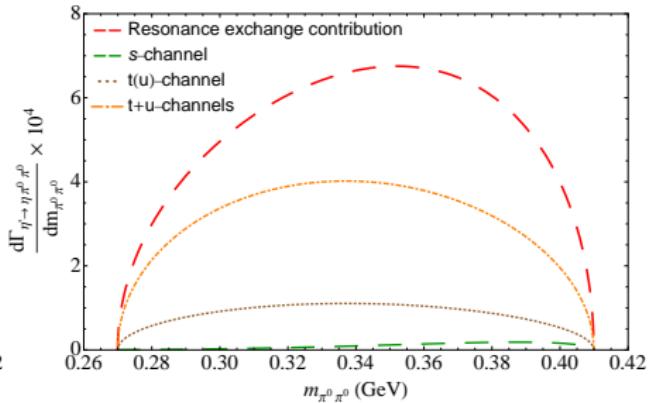
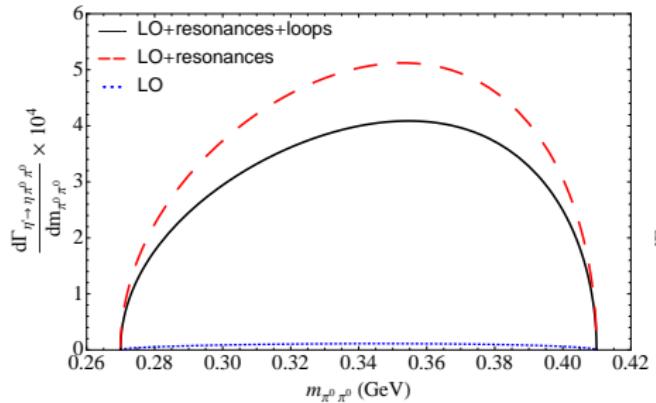
- Dalitz plot to compare experiment and theory

$$|M(X, Y)|^2 = |N|^2 (1 + \textcolor{blue}{a}Y + \textcolor{red}{b}Y^2 + \textcolor{violet}{c}X + \textcolor{brown}{d}X^2 + \dots)$$

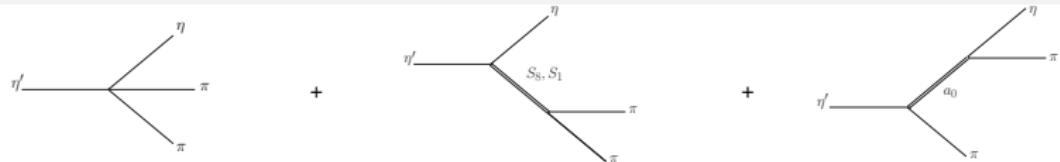
- $\textcolor{blue}{a}, \textcolor{red}{b}, \textcolor{violet}{c}, \textcolor{brown}{d}$ are the Dalitz plot parameters

$\eta' \rightarrow \eta\pi^0\pi^0$	$\textcolor{blue}{a}[Y]$	$\textcolor{red}{b}[Y^2]$	$\textcolor{violet}{c}[X]$	$\textcolor{brown}{d}[X^2]$
GAMS-4 (2009)	-0.067(16)(4)	-0.064(29)(5)	= 0	-0.067(20)(3)
GAMS-4 (2009)	-0.066(16)(4)	-0.063(28)(4)	-0.107(96)(3)	0.018(78)(6)
A2 (2017)	-0.074(8)(6)	-0.063(14)(5)	—	-0.050(9)(5)
BESIII (2017)	-0.087(9)(6)	-0.073(14)(5)	0	-0.074(9)(4)
Borasoy <i>et.al.</i> '05	-0.127(9)	-0.049(36)	0	0.011(21)
Fariborz <i>et.al.</i> '14	-0.024	0.0001	0	-0.029
$\eta' \rightarrow \eta\pi^+\pi^-$	$\textcolor{blue}{a}[Y]$	$\textcolor{red}{b}[Y^2]$	$\textcolor{violet}{c}[X]$	$\textcolor{brown}{d}[X^2]$
VES (2007)	-0.127(16)(8)	-0.106(28)(14)	0.015(11)(14)	-0.082(17)(8)
BESIII (2011)	-0.047(11)(3)	-0.069(19)(9)	0.019(11)(3)	-0.073(12)(3)
BESIII (2017)	-0.056(4)(3)	-0.049(6)(6)	$2.7(2.4)(1.8) \cdot 10^{-3}$	-0.063(4)(4)
Borasoy <i>et.al.</i> '05	-0.116(11)	-0.042(34)	0	0.010(19)
Escribano <i>et.al.</i> '10	-0.098(48)	-0.050(1)	0	-0.092(8)
Escribano <i>et.al.</i> '10	-0.098(48)	-0.033(1)	0	-0.072(1)
Kubis <i>et.al.</i> '17	-0.148(18)	-0.082(7)	0	-0.068(11)
Kubis <i>et.al.</i> '17	-0.041(9)	-0.088(7)	0	-0.086(22)

Hierarchy of the contributions

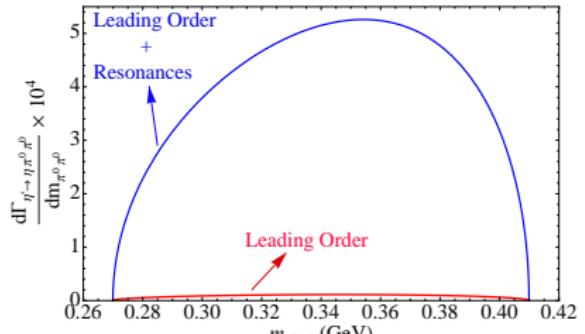


$\eta' \rightarrow \eta\pi\pi$: Scalar Resonance contributions

 c_d : dominant c_m : suppressed ($\propto m_\pi^2$)

$$\begin{aligned} \mathcal{M}^{\text{Res}}(s, t, u) = & \frac{(2\sqrt{2}\cos 2\theta - \sin 2\theta)}{9F_\pi^4} \times \left\{ -\frac{24\tilde{c}_m(m_K^2 - m_\pi^2)(2\tilde{c}_m m_\pi^2 + \tilde{c}_d(s - 2m_\pi^2))}{M_{S_1}^2 - s} \right. \\ & - \frac{(2c_m(m_\pi^2 - 4m_K^2) + 3c_d(m_\eta^2 + m_{\eta'}^2 - s))(2c_m m_\pi^2 + c_d(s - 2m_\pi^2))}{M_{S_8}^2 - s} + \frac{12c_d c_m m_\pi^2}{M_{S_8}^2} (m_\pi^2 - m_K^2) \\ & \left. + \frac{3(4c_m^2 m_\pi^4 - 2c_d c_m m_\pi^2(m_\eta^2 + m_{\eta'}^2 + 2m_\pi^2 - 2t) + c_d^2(m_\eta^2 + m_\pi^2 - t)(m_{\eta'}^2 + m_\pi^2 - t))}{M_{a_0}^2 - t} + (t \leftrightarrow u) \right\}. \end{aligned}$$

Source	c_m	c_d	BR (%)
$a_0 \rightarrow \eta\pi$ (Guo'09)	80	26	~ 16
Res. saturation (Ecker'88)	42	32	~ 35
$K\pi$ scattering Jamin et.al. '00	43 76.7 85	30 24.8 13	~ 27 ~ 13 ~ 1
$PP \rightarrow PP$ Guo et.al '11 Guo et.al '12 Pich et.al. '14	31.5 41.9 41.1	15.6 19.8 39.8	~ 1 ~ 4 ~ 85
PDG			22.8(8)



Fits to experimental data: ChPT at one loop with resonances

- Fit 2: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d \neq c_m$

Fit parameters : $M_S = 954(47)$ MeV , $c_d = 28.0(4.6)$ MeV , $c_m = 53.4(52.0)$ MeV , $\chi^2_{\text{dof}} = 1.23$

Dalitz parameters : $a = -0.093(45)$, $b = 0.004(3)$, $d = -0.039(18)$

- Fit 3: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $c_m = 41.1(1)$ $\tilde{c}_m = 18.9(9)$ (Pich'14)

Fit parameters : $M_S = 968(11)$ MeV , $c_d = 29.8(9)$ MeV , $\tilde{c}_d = 21.2(1.2)$ MeV , $\chi^2_{\text{dof}} = 1.24$

Dalitz parameters : $a = -0.092(5)$, $b = 0.004(2)$, $d = -0.041(11)$

- Fit 4: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ with $c_m = 80(20)$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ (Guo'09)

Fit parameters : $M_S = 926(5)(25)$ MeV , $c_d = 25.7(4)(1.9)$ MeV , $\chi^2_{\text{dof}} = 1.22$

Dalitz parameters : $a = -0.090(7)(3)$, $b = 0.004(1)(0)$, $d = -0.041(7)(1)$

- The **Dalitz plot parameters remain stable** independently of the allowed parameters to fit
- We also have tried a fit letting **all couplings to float** i.e. c_m, \tilde{c}_m, c_d and \tilde{c}_d , but in this case the **fit becomes unstable** since there are too many parameters to fit

N/D applied to $\eta' \rightarrow \eta\pi\pi$

- Amplitude at one-loop in Large- N_C $U(3)$ ChPT with resonances

$$\mathcal{M}^{\eta' \rightarrow \eta\pi\pi}(s) = \mathcal{M}(s)^{(2)} + \mathcal{M}(s)^{\text{Res(s,t,u)}} + \mathcal{M}(s)^{\text{Loop(s,t,u)}} \quad (2)$$

- N/D representation of Eq. (2)

$$m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s) = [1 + N_{\pi\pi}^{IJ}(s)g_{\pi\pi}(s)]^{-1} R_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s) \quad (3)$$

where

$$N_{\pi\pi}^{IJ}(s) = t_{\pi\pi}^{IJ}(s)^{(2)+\text{Res+Loop}},$$

$$R_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s) = m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s)^{(2)+\text{Res+Loop}},$$

- Chiral expansion of Eq. (3) leads

$$m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s) = m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s)^{(2)+\text{Res+Loop}} - t_{\pi\pi}^{IJ}(s)^{(2)}g_{\pi\pi}(s)m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s)^{(2)} + \dots$$

$$\boxed{\text{Im} \left(m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s) \right) = t_{\pi\pi}^{IJ}(s)\sigma_{\pi}(s)m_{\eta' \rightarrow \eta\pi\pi}^{IJ}(s)}$$

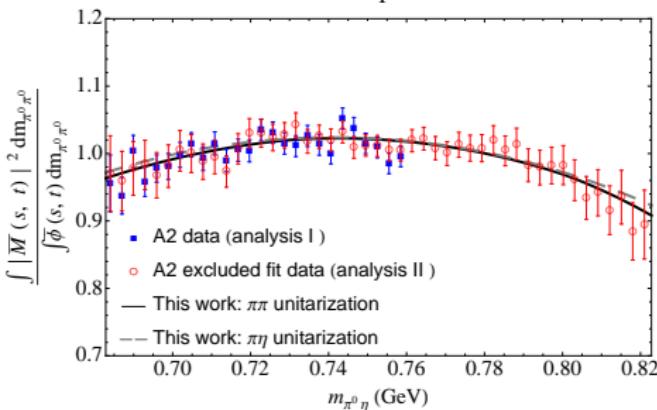
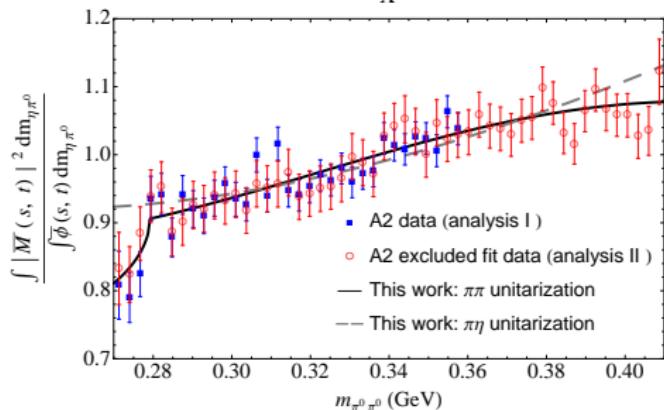
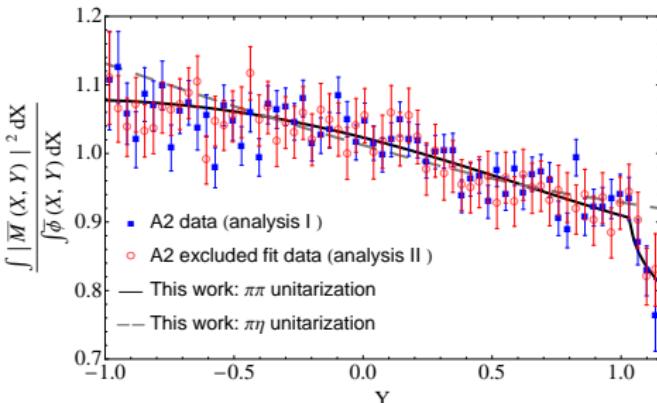
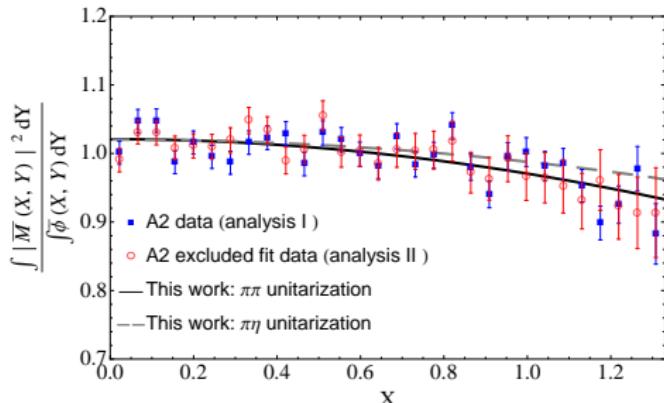
Fits to experimental data: $\pi\eta$ final-state interactions effects

- Fit restrictions: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $a_{\pi\eta} = 2.0^{+3.1}_{-3.4}$ (Guo'11)

Parameter	Fit A	Fit B	Fit C	Fit D
M_S	985(7)(20)	913(12)(32)	999(9)(14)	940(9)(47)
c_d	30.6(5)(7)	24.4(1.0)(4.6)	32.3(6)(9)	26.3(5)(2.5)
c_m	$= c_d$	100.5(3.0)(50.0)	$= 41.1(1)$	$= 80(21)$
\tilde{c}_d	17.7(2)(2)	14.1(6)(2.7)	18.7(4)(5)	15.2(3)(1.4)
\tilde{c}_m	$= \tilde{c}_d$	58.0(1.7)(28.9)	$= 18.9(9)$	46.2(12.1)
χ^2_{dof}	243.2/198 ~ 1.23	242.9/197 ~ 1.23	244.5/198 ~ 1.24	242.7/197 ~ 1.23
$a[Y]$	-0.094(6)(9)	-0.086(9)(14)	-0.083(7)(7)	-0.089(8)(15)
$b[Y^2]$	0.005(1)(1)	0.004(1)(1)	0.001(1)(1)	0.004(1)(1)
$d[X^2]$	-0.031(5)(4)	-0.035(8)(9)	-0.049(5)(13)	-0.035(7)(9)

Parameter	Fit A (ChPT)	Fit B (ChPT)	Fit C (ChPT)	Fit D (ChPT)
M_S	973(5)	954(47)	992(7)	926(5)(25)
c_d	30.1(4)	28.0(4.6)	32.9(5)	25.7(4)(1.9)
c_m	$= c_d$	53.4(52.0)	$= 41.1(1)$	$= 80(20)$
\tilde{c}_d	17.4(2)	16.1	18.4(3)	14.8(2)(1.1)
\tilde{c}_m	$= \tilde{c}_d$	30.8	$= 18.9(9)$	46.2(11.5)
χ^2_{dof}	242.2/198 ~ 1.22	242.0/197 ~ 1.23	246.4/198 ~ 1.24	242.3/198 ~ 1.22
$a[Y]$	-0.095(6)	-0.093(45)	-0.083(6)	-0.090(7)(2)
$b[Y^2]$	0.005(1)	0.004(3)	-0.001(1)	0.004(1)(0)
$c[X]$	0	0	0	0
$d[X^2]$	-0.037(5)	-0.039(18)	-0.057(5)	-0.041(7)(1)

Fits to experimental data: $\pi\eta$ final-state interactions



Fits to experimental data: $\pi\pi$ and $\pi\eta$ final-state interactions

- Fit restrictions: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $a_{\pi\eta} = 2.0^{+3.1}_{-3.4}$ (Guo'11)

Parameter	Analysis I		Analysis II	
	Fit 1	Fit 2	Fit 1	Fit 2
M_S	1017(68)(24)	999(33)(23)	1040(79)(28)	1020(48)(28)
c_d	30.4(4.8)(9)	29.1(2.4)(1.6)	32.0(5.3)(9)	30.9(3.4)(2.2)
c_m	= c_d	= 41.1(1)	= c_d	= 41.1(1)
\tilde{c}_d	17.6(2.8)(5)	16.8(1.4)(9)	18.5(2.8)(5)	17.8(2.0)(1.3)
\tilde{c}_m	= \tilde{c}_d	= 18.9(9)	= \tilde{c}_d	= 18.9(9)
$a_{\pi\pi}$	0.76(61)(6)	0.34(22)(19)	0.98(58)(9)	0.57(38)(20)
χ^2_{dof}	1.12	1.12	1.23	1.23
$a[Y]$	-0.074(7)(8)	-0.073(6)(9)	-0.071(6)(8)	-0.070(6)(9)
$b[Y^2]$	-0.049(1)(2)	-0.054(1)(2)	-0.050(2)(1)	-0.054(1)(1)
$d[X^2]$	-0.047(8)(4)	-0.047(2)(4)	-0.055(6)(4)	-0.055(6)(4)
$\kappa_{03}[Y^3]$	0.001	0.003	0.001	0.002
$\kappa_{21}[YX^2]$	-0.004	-0.005	-0.005	-0.005
$\kappa_{22}[Y^2X^2]$	0.001	0.002	0.002	0.002

$\pi\eta$ final state interactions effects

- Perturbative expansion supplemented by *S-wave* $\pi\eta$ FSI

$$\mathcal{M}_{\eta' \rightarrow \eta\pi\pi}^{I=1}(s, t, u, \cos\theta_t, \cos\theta_u) = \mathcal{M}(s, t, u)^{(2)+\text{Res+Loop}}$$

$$+ 32\pi P_0(\cos\theta_t) \frac{m_{\eta' \rightarrow \eta\pi\pi}^{10}(t)^{(2)+\text{Res+Loop}}}{1 + g_{\pi\eta}(t)t_{\pi\eta}^{10}(t)^{(2)+\text{Res+Loop}}}$$

$$+ 32\pi P_0(\cos\theta_u) \frac{m_{\eta' \rightarrow \eta\pi\pi}^{10}(u)^{(2)+\text{Res+Loop}}}{1 + g_{\pi\eta}(u)t_{\pi\eta}^{10}(u)^{(2)+\text{Res+Loop}}}$$

$$- 32\pi P_0(\cos\theta_t)m_{\eta' \rightarrow \eta\pi\pi}^{10}(t)^{(2)+\text{Res+Loop}} - 32\pi P_0(\cos\theta_u)m_{\eta' \rightarrow \eta\pi\pi}^{10}(u)^{(2)+\text{Res+Loop}}$$

Inclusion of $\pi\pi$ and $\pi\eta$ final-state interactions

- Perturbative expansion + S -and D -wave $\pi\pi$ and S -wave $\pi\eta$ FSI

$$\mathcal{M}(s, t, u, \cos \theta_{s,t,u}) = \mathcal{M}(s, t, u)^{(2)+\text{Res+Loop}}$$

$$+ 32\pi P_0(\cos \theta_s) \frac{m_{\eta' \rightarrow \eta\pi\pi}^{00}(s)^{(2)+\text{Res+Loop}}}{1 + g_{\pi\pi}(s)t_{\pi\pi}^{00}(s)^{(2)+\text{Res+Loop}}}$$

$$+ 160\pi P_2(\cos \theta_s) \frac{m_{\eta' \rightarrow \eta\pi\pi}^{02}(s)^{(2)+\text{Res+Loop}}}{1 + g_{\pi\pi}(s)t_{\pi\pi}^{02}(s)^{(2)+\text{Res+Loop}}}$$

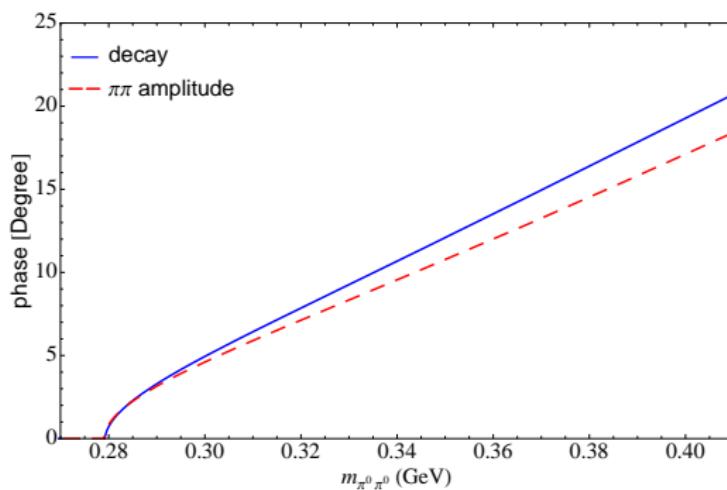
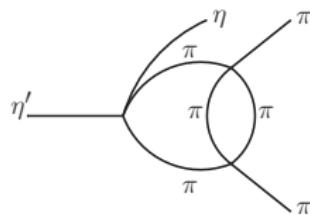
$$- 32\pi P_0(\cos \theta_s) m_{\eta' \rightarrow \eta\pi\pi}^{00}(t)^{(2)+\text{Res+Loop}} - 160\pi P_2(\cos \theta_s) m_{\eta' \rightarrow \eta\pi\pi}^{02}(u)^{(2)+\text{Res+Loop}}$$

$$+ 32\pi P_0(\cos \theta_t) \frac{m_{\eta' \rightarrow \eta\pi\pi}^{10}(t)^{(2)+\text{Res+Loop}}}{1 + g_{\pi\eta}(t)t_{\pi\eta}^{10}(t)^{(2)+\text{Res+Loop}}}$$

$$+ 32\pi P_0(\cos \theta_u) \frac{m_{\eta' \rightarrow \eta\pi\pi}^{10}(u)^{(2)+\text{Res+Loop}}}{1 + g_{\pi\eta}(u)t_{\pi\eta}^{10}(u)^{(2)+\text{Res+Loop}}}$$

$$- 32\pi P_0(\cos \theta_t) m_{\eta' \rightarrow \eta\pi\pi}^{10}(t)^{(2)+\text{Res+Loop}} - 32\pi P_0(\cos \theta_u) m_{\eta' \rightarrow \eta\pi\pi}^{10}(u)^{(2)+\text{Res+Loop}} .$$

Unitarity Violations

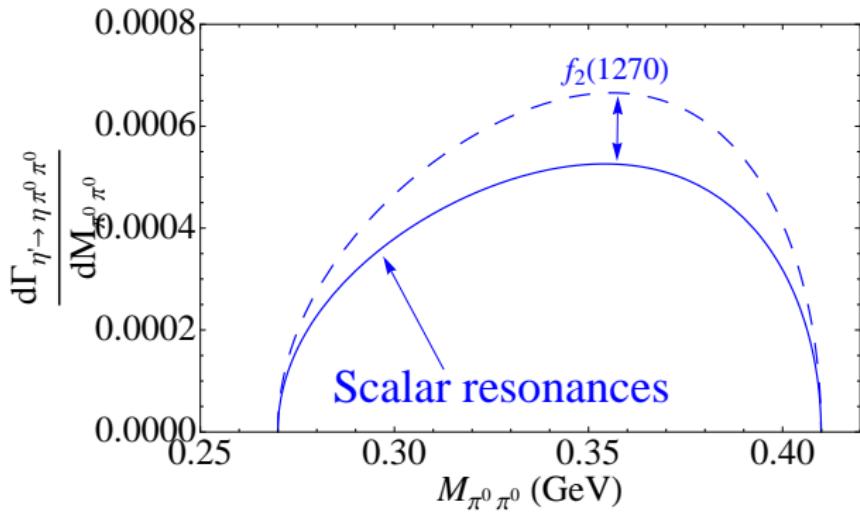


Tensor Resonance contributions

Ecker and Zauner, EPJC 52, 315 (2007)

$$\mathcal{L}_T = -\frac{1}{2} \langle T_{\mu\nu} D_T^{\mu\nu,\rho\sigma} T_{\rho\sigma} \rangle + g_T \langle T_{\mu\nu} \{u^\mu, u^\nu\} \rangle + \beta \langle T_\mu^\mu u_\nu u^\nu \rangle$$

$$g_T = 28 \text{ MeV}, \quad \beta = -g_T, \quad M_T = 1300 \text{ MeV}$$



Masses and couplings

Source	M_{S_8}	M_{S_1}	M_{a_0}	c_d	c_m
$a_0 \rightarrow \eta\pi$ (Guo et.al. '09)	980	$= M_{S_8}$	$= M_{S_8}$	26	80
res. saturation (Ecker et.al. '88)	983	$= M_{S_8}$	$= M_{S_8}$	32	42
$K\pi$ scattering (Jamin et.al. '00)					
	1400	$= M_{S_8}$	$= M_{S_8}$	30	43
	1190	$= M_{S_8}$	$= M_{S_8}$	45.4	$= c_d$
	1260	$= M_{S_8}$	$= M_{S_8}$	24.8	76.7
	1360	$= M_{S_8}$	$= M_{S_8}$	13	85
<hr/> $PP \rightarrow PP$ ($P = \pi, K, \eta$)					
Guo et.al '11	1370^{+132}_{-57}	1063^{+53}_{-31}	$= M_{S_8}$	$15.6^{+4.2}_{-3.4}$	$31.5^{+19.5}_{-22.5}$
Guo et.al '12	1397^{+73}_{-61}	1100^{+30}_{-63}	$= M_{S_8}$	$19.8^{+2.0}_{-5.2}$	$41.9^{+3.9}_{-9.2}$
Ledwig et.al. '14	1279(9)	808.9(4)	$= M_{S_8}$	39.8(1)	41.1(1)
This work					
Resonances+loops	972(6)	$= M_{S_8}$	$= M_{S_8}$	29.9(4)	$= c_d$
Resonances+loops	953	$= M_{S_8}$	$= M_{S_8}$	27.8	53.2
$\pi\pi$ final state interactions	998(19)	$= M_{S_8}$	$= M_{S_8}$	29.3(1.2)	$= c_d$
	1021	$= M_{S_8}$	$= M_{S_8}$	32.5	$4c_m c_d = f^2$
$\pi\eta$ final state interactions	978(7)	$= M_{S_8}$	$= M_{S_8}$	29.8(7)	$= c_d$
$\pi\pi + \pi\eta$ final state interactions	990(95)	$= M_{S_8}$	$= M_{S_8}$	32.0(6.8)	$= c_d$

Summary

- $\eta' \rightarrow \eta\pi\pi$ analyzed within $U(3)$ ChPT at one-loop with resonances
- We have illustrated a method (N/D) to resum two-particle FSI
- Dalitz plot parameters:
 - Y -variable is linear in s : Importance of $\pi\pi$ FSI
 - X -variable appear in the form $\cos\theta_s = X f(Y)$: Importance of the D -wave
 - $\pi\eta$ FSI effects are small

Experiment	a [Y]	b [Y^2]	c [X]	d [X^2]
GAMS4 π (c=0) '09	-0.067(16)(4)	-0.064(29)(5)	0	-0.067(20)(3)
VES '07	-0.127(16)(8)	-0.106(28)(14)	0.015(11)(14)	-0.082(17)(8)
BESIII '11	-0.047(11)(3)	-0.069(19)(9)	0.019(11)(3)	-0.073(12)(3)
A2'17	-0.074(8)(6)	-0.063(14)(5)	—	-0.050(9)(5)
BESIII'17	-0.087(9)(6)	-0.073(14)(5)	0	-0.074(9)(4)
BESIII'17	-0.056(4)(3)	-0.049(6)(6)	2.7(2.4)(1.8) · 10 ⁻³	-0.063(4)(4)
Previous Estimates				
Borasoy et.al.'05	-0.127(9)	-0.049(36)	0	0.011(21)
Borasoy et.al.'05	-0.116(11)	-0.042(34)	0	0.010(19)
Escribano et.al.'10	-0.098(48)	-0.050(1)	0	-0.092(8)
Escribano et.al.'10	-0.098(48)	-0.033(1)	0	-0.072(1)
Fariborz et.al.'14	-0.024	0.0001	0	-0.029
Kubis et.al.'17	-0.148(18)	-0.082(7)	0	-0.068(11)
Kubis et.al.'17	-0.041(9)	-0.088(7)	0	-0.086(22)
This talk				
Resonances	-0.096(9)	0.002(1)	0	-0.036(6)
Resonances+loops	-0.095(6)	0.005(1)	0	-0.037(5)
$\pi\pi$ FSI	-0.075(9)	-0.051(1)	0	-0.049(14)
$\pi\eta$ FSI	-0.095(6)(5)	0.005(1)(1)	0	-0.034(5)(3)
$\pi\pi + \pi\eta$	-0.073(7)(5)	-0.052(1)(2)	0	-0.052(8)(5)