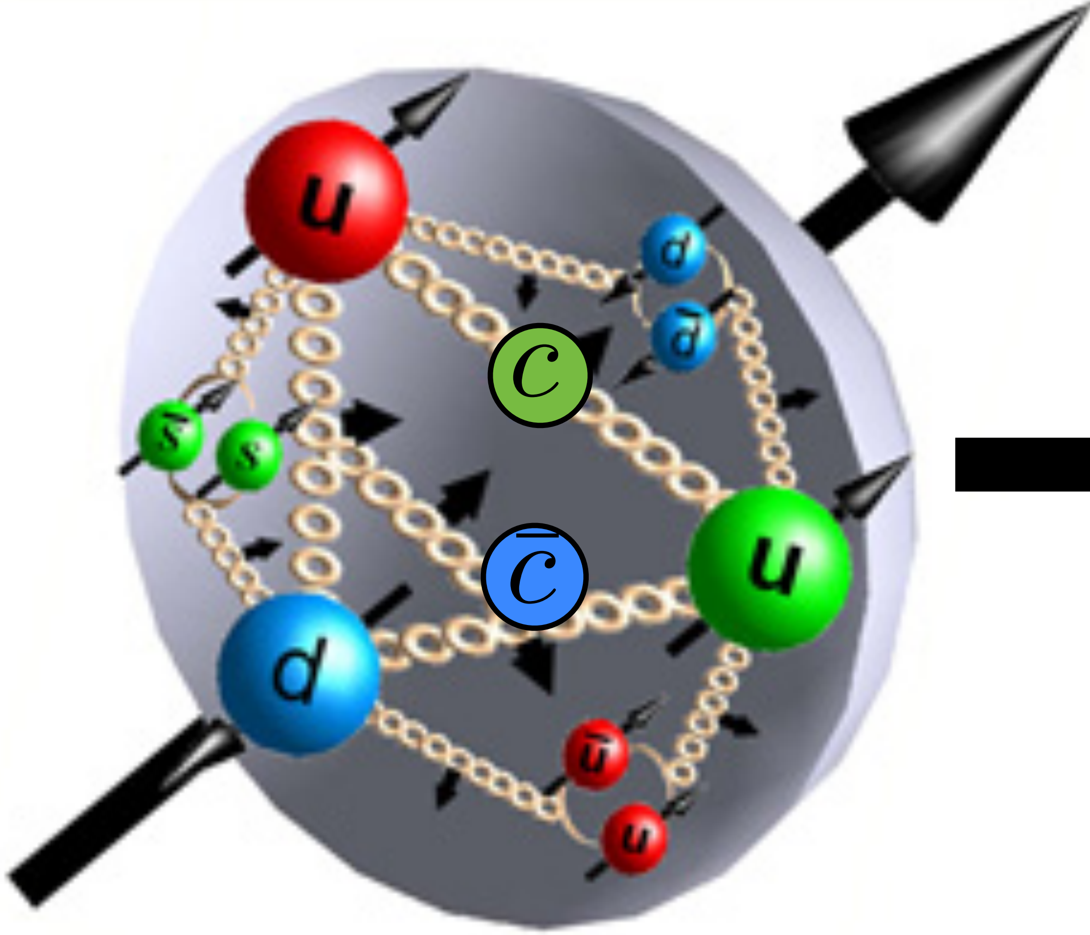
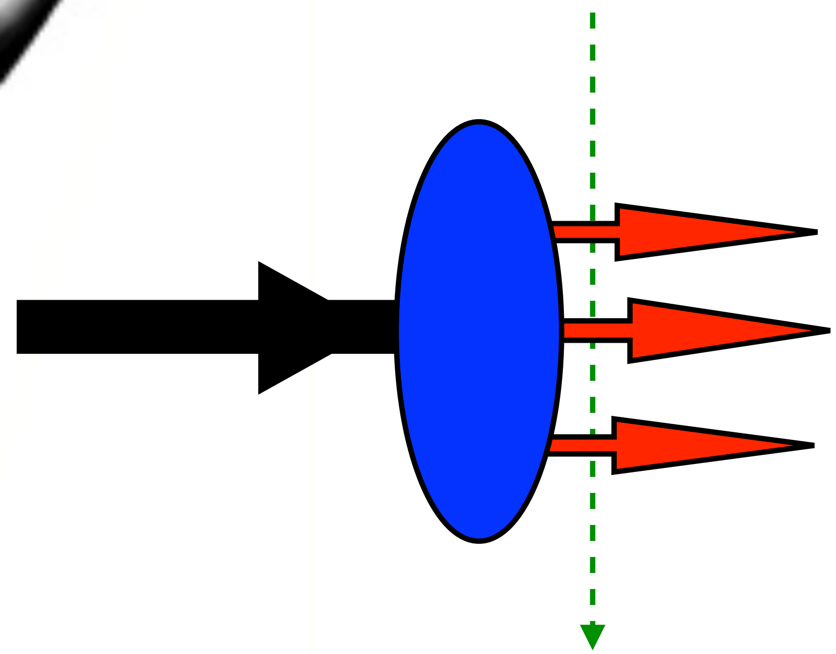
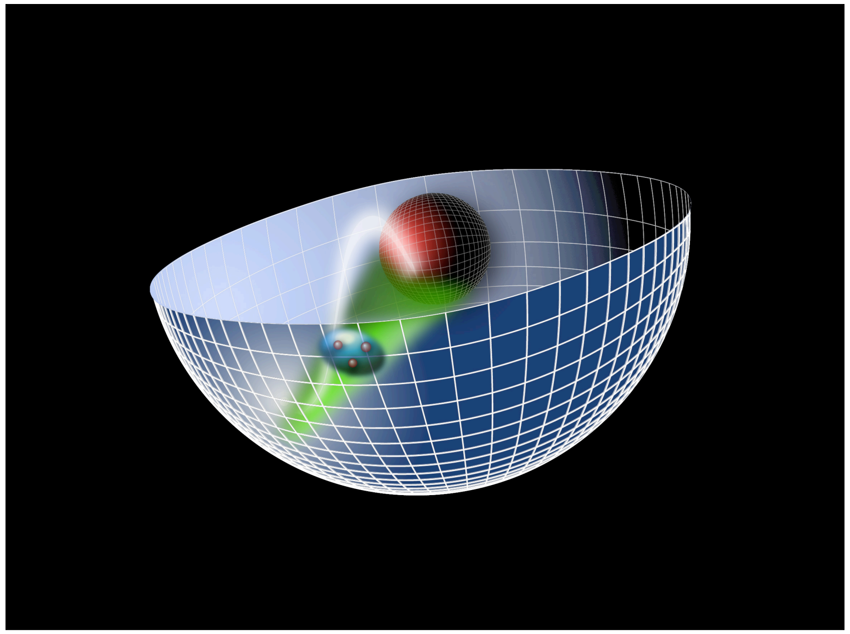
Supersymmetric Features of Hadron Physics and other Novel Properties of Quantum Chromodynamics from Light-Front Holography and Superconformal Algebra



Fixed $\tau = t + z/c$





with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, F. Navarra, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur

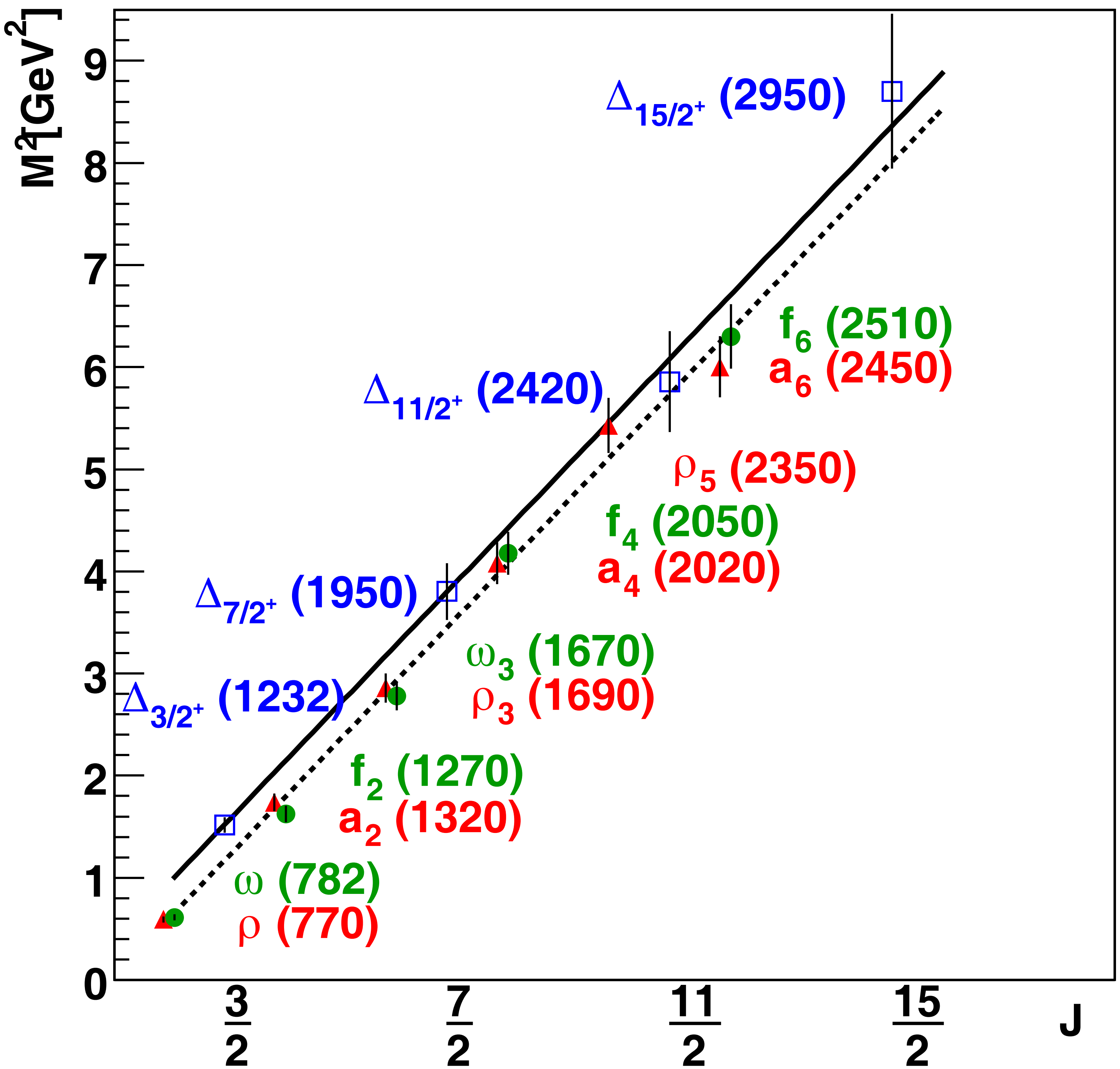
Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

GHP-APS | *8th Workshop of the APS Topical Group on Hadronic Physics* | *Denver Thursday, 11 April 2019*

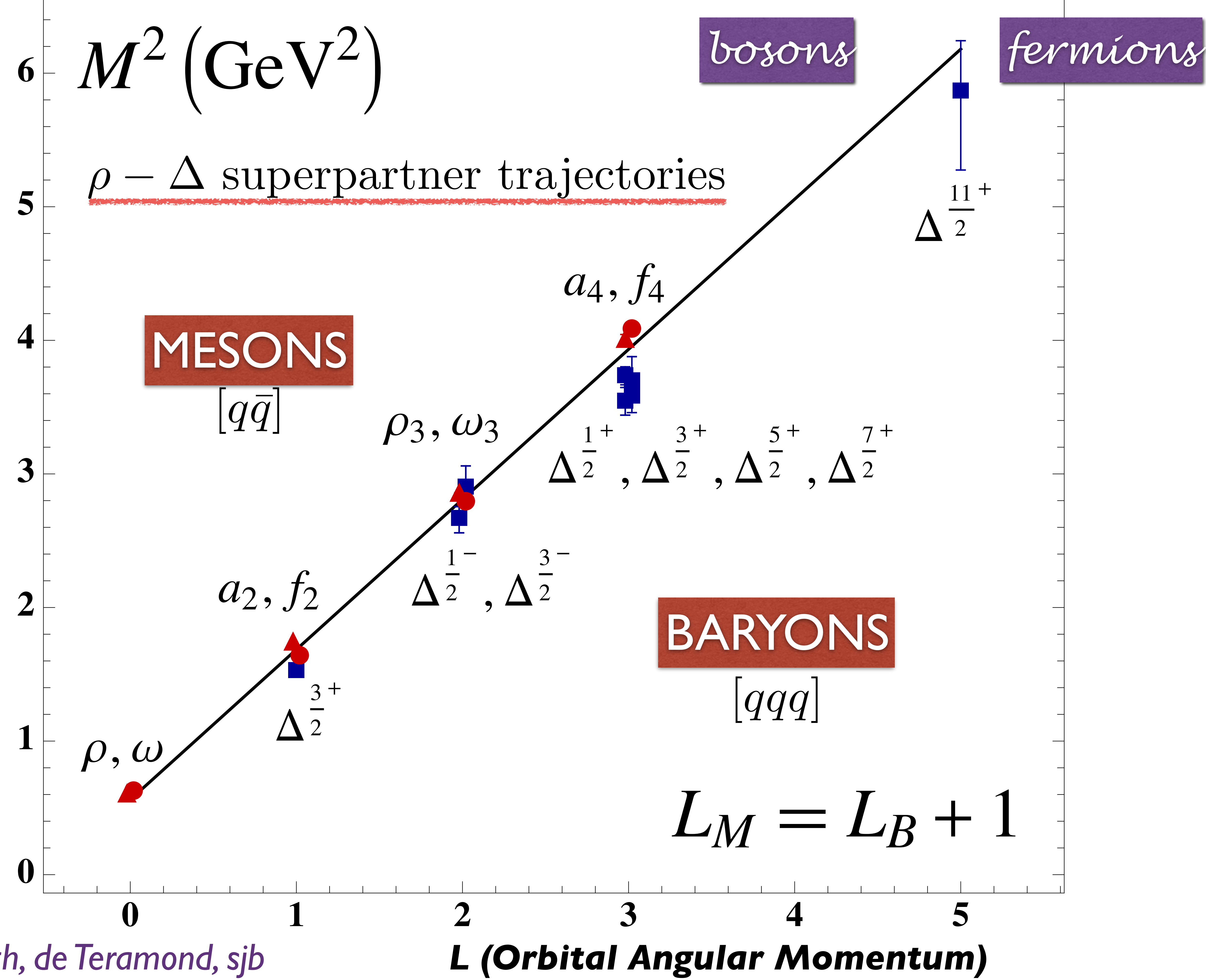
Mesons and Baryons: Same Regge Slope $M^2 \propto J$!

M²[GeV²]



The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with J = L+S.

E. Klempt and B. Ch. Metsch



Dosch, de Teramond, sjb

Profound Questions for Hadron Physics

- *Color Confinement*
- *Origin of QCD Mass Scale*
- *Spectroscopy: Tetraquarks, Pentaquarks, Gluonium, Exotic States*
- *Universal Regge Slopes: n, L, Mesons and Baryons*
- *Massless Pion: Quark-Antiquark Bound State*
- *Dynamics and Spectroscopy*
- *QCD Coupling at all Scales*
- *QCD Vacuum —Do Condensates Exist?*

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

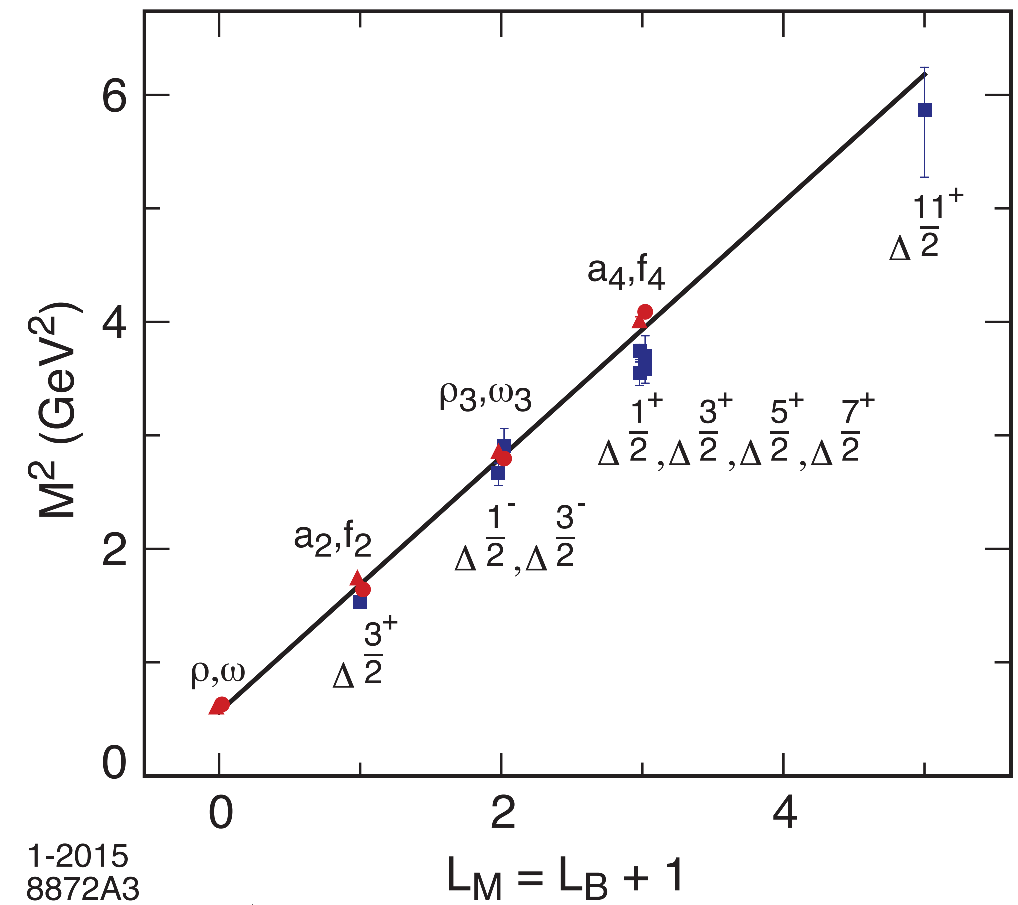
Origin of hadronic mass scale

AdS/QCD
Light-Front Holography
Superconformal Algebra

No parameters except for quark masses

$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024$$

Symmetry for VM first noticed by E. Klempt

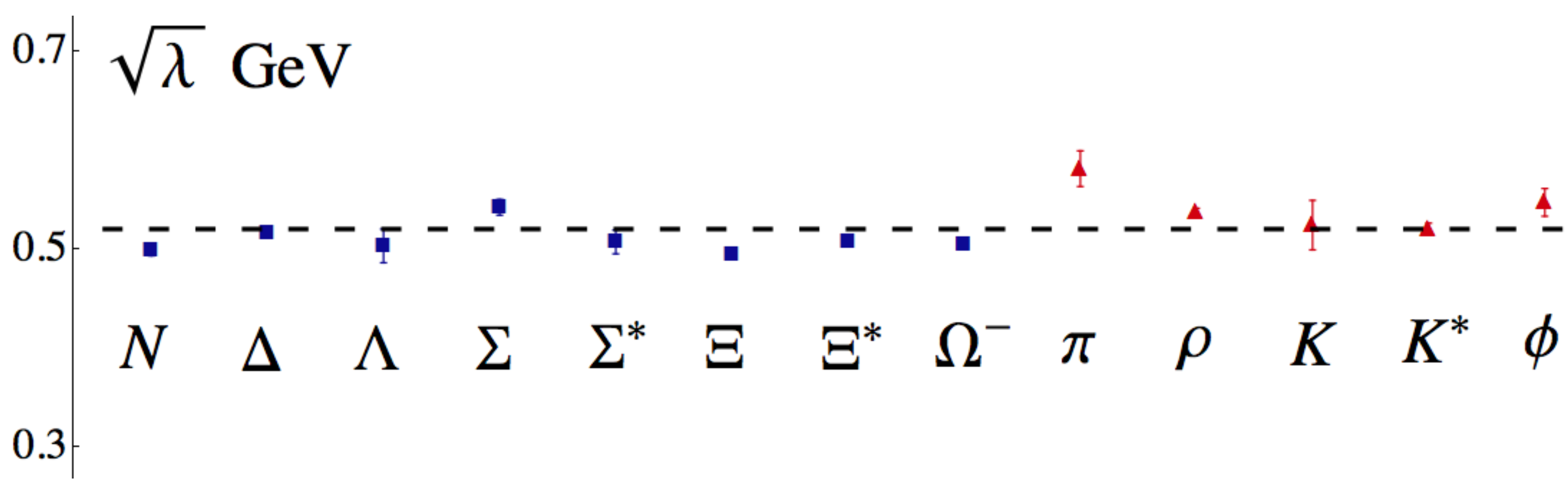


- Quadratic mass correction for light quark masses

$$\Delta M^2[m_1, \cdots, m_n] = \frac{\lambda^2}{F}\frac{\mathrm{d}F}{\mathrm{d}\lambda}$$

with $F[\lambda] = \int_0^1 \cdots \int \mathrm{d}x_1 \cdots \mathrm{d}x_n \, e^{-\frac{1}{\lambda}\left(\sum_{i=1}^n \frac{m_i^2}{x_i}\right)} \delta(\sum_{i=1}^n x_i - 1)$

- How universal is the semiclassical approximation based on superconformal LFHQCD ?



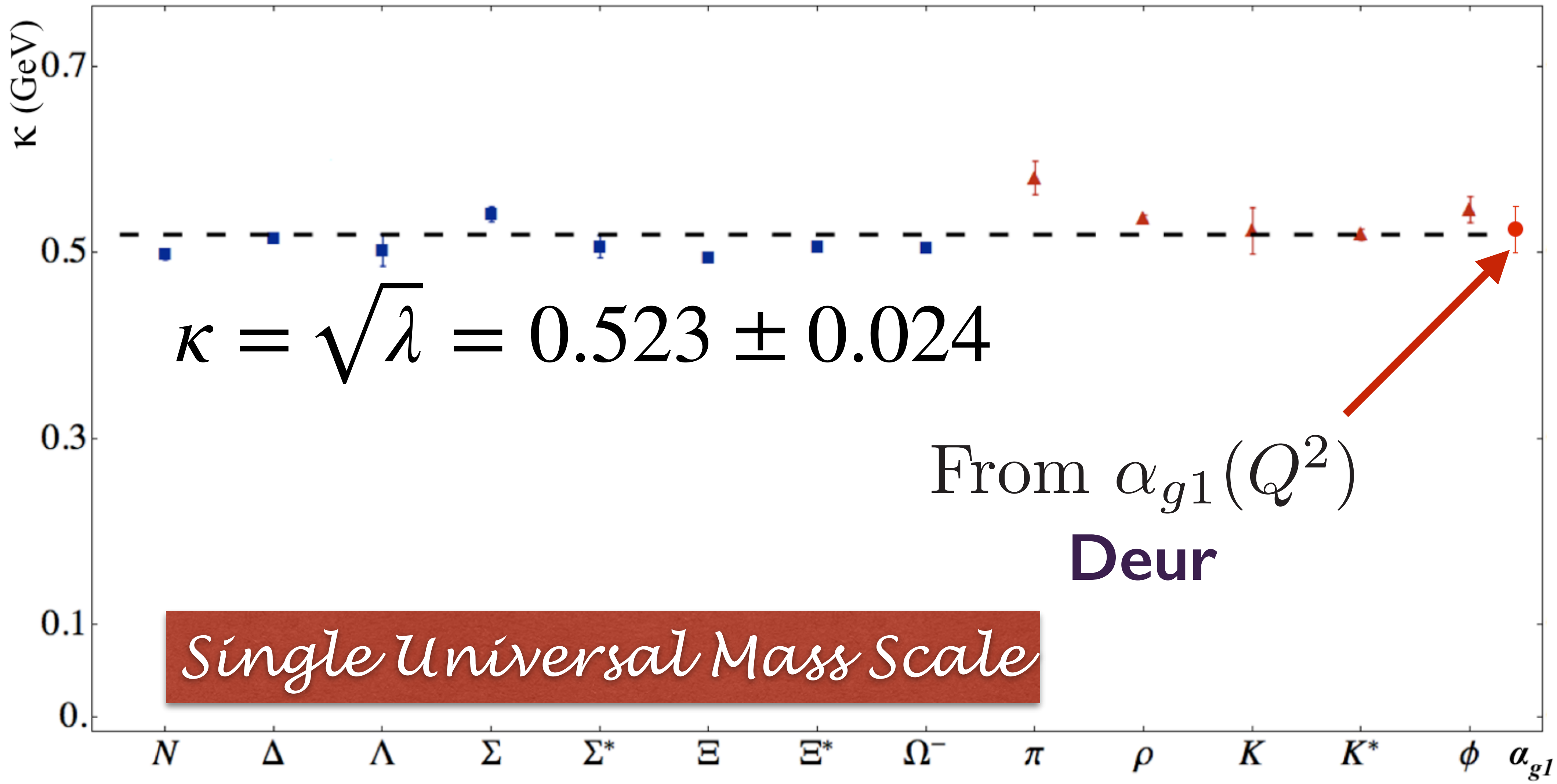
Best fit for hadronic scale $\sqrt{\lambda}$ from different light hadron sectors including radial and orbital excitations

$\lambda = \kappa^2$

de Tèramond, Dosch, Lorce', sjb

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$

κ



Fit to the slope of Regge trajectories, including radial excitations

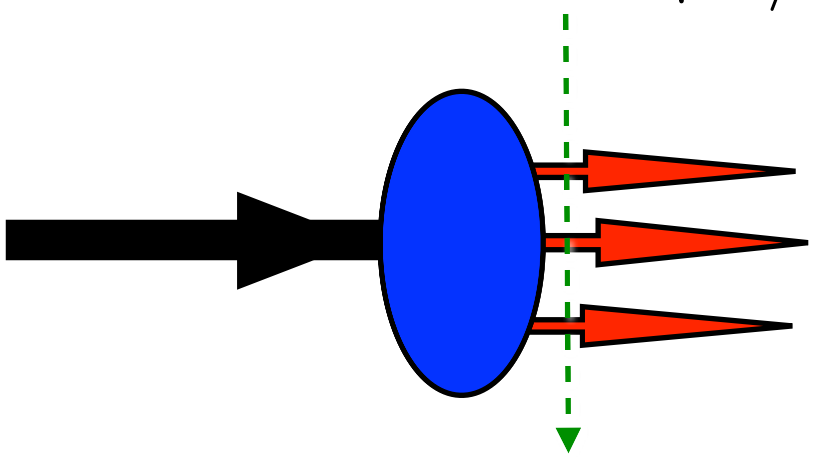
Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\boxed{\mathrm{H}^{QCD}_{LF} |\psi> = M^2 |\psi>}$$

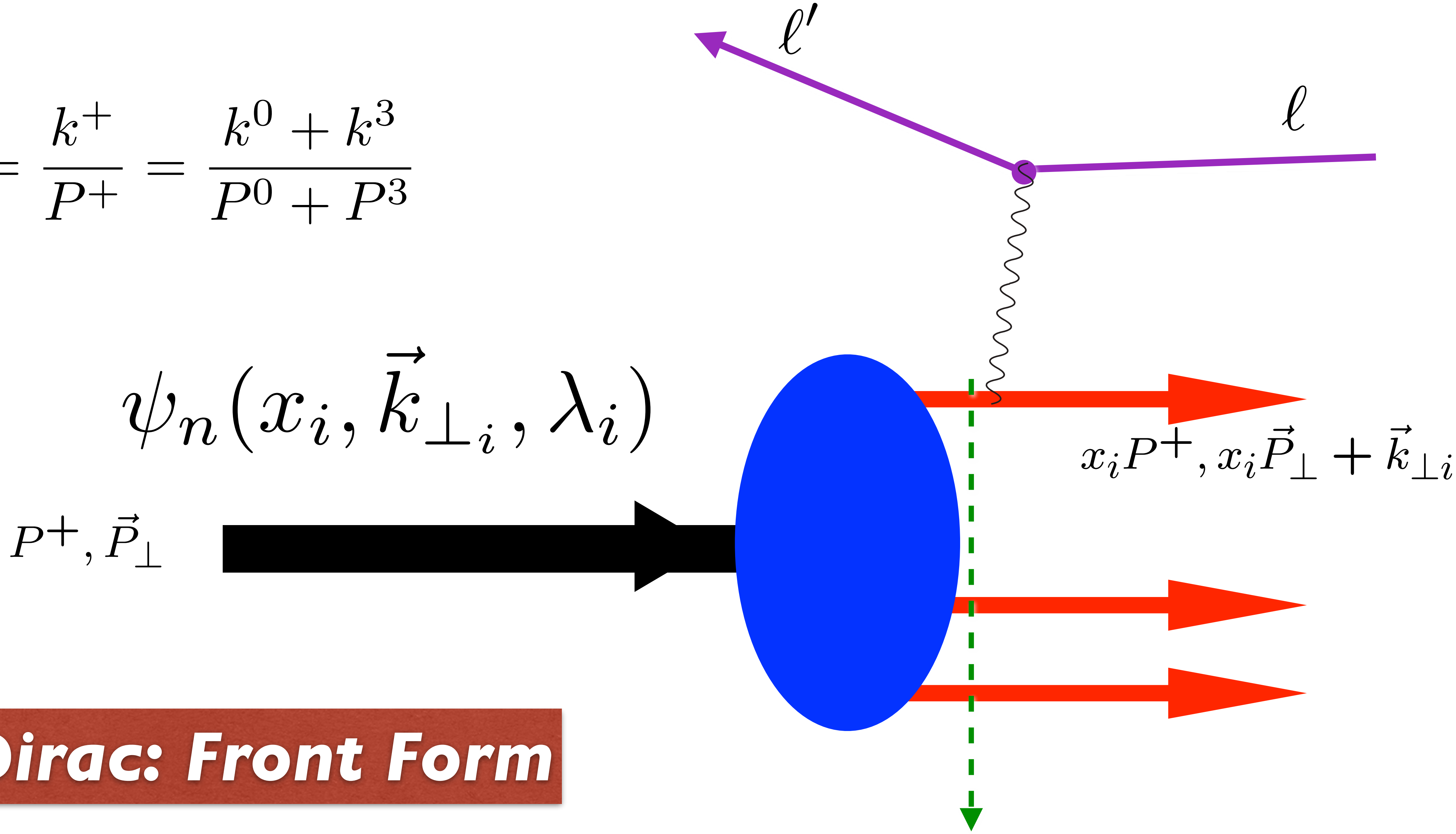
Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$



Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

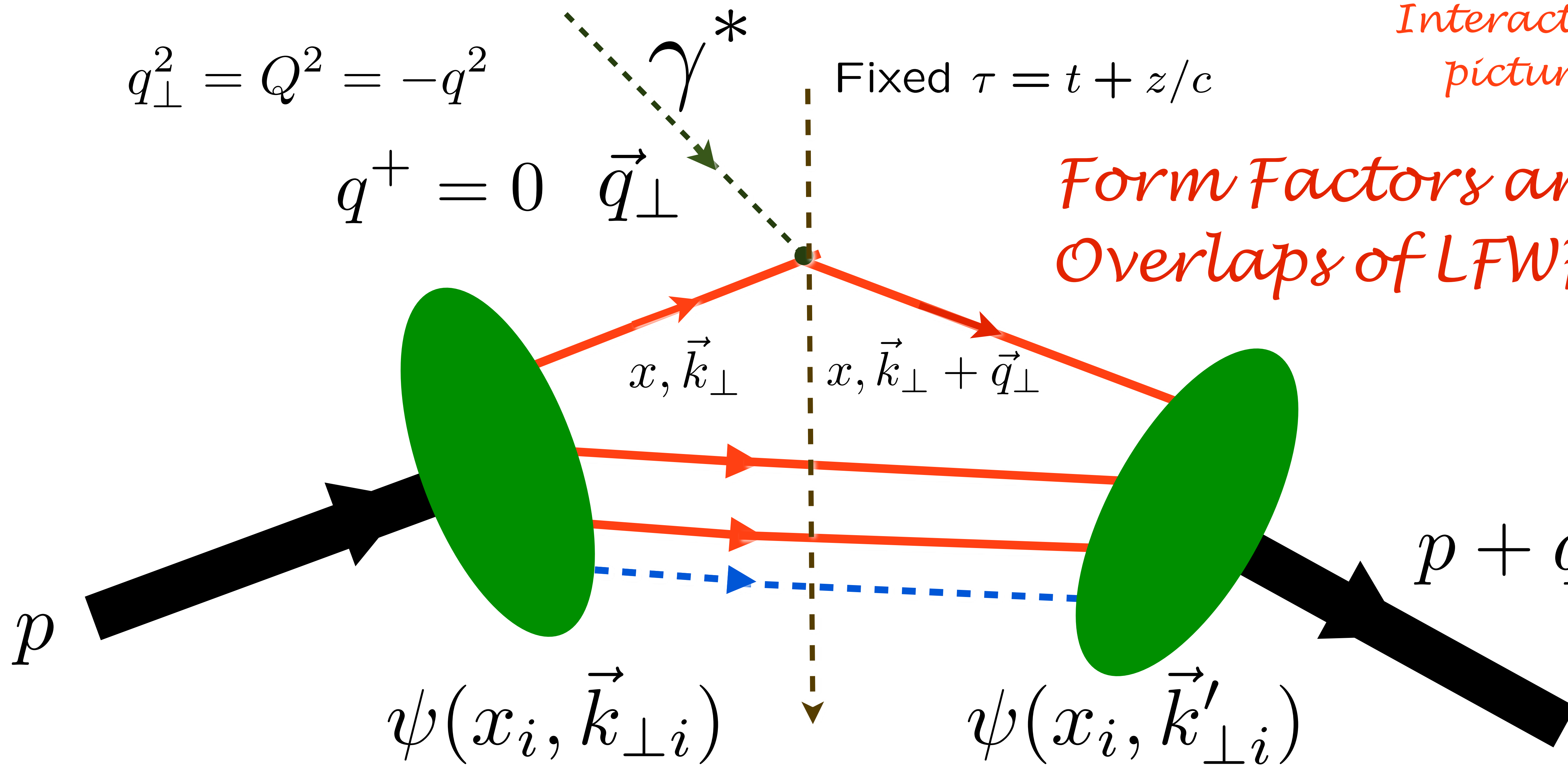
$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Front Form

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

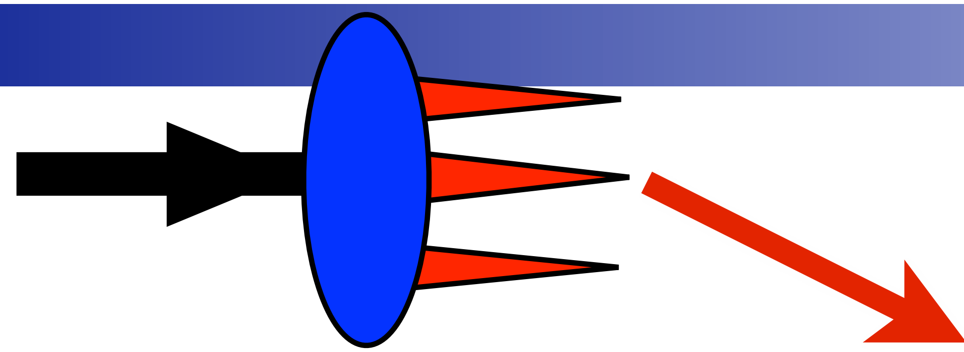
spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Light-Front Wavefunctions underly hadronic observables

Lorce, Pasquini



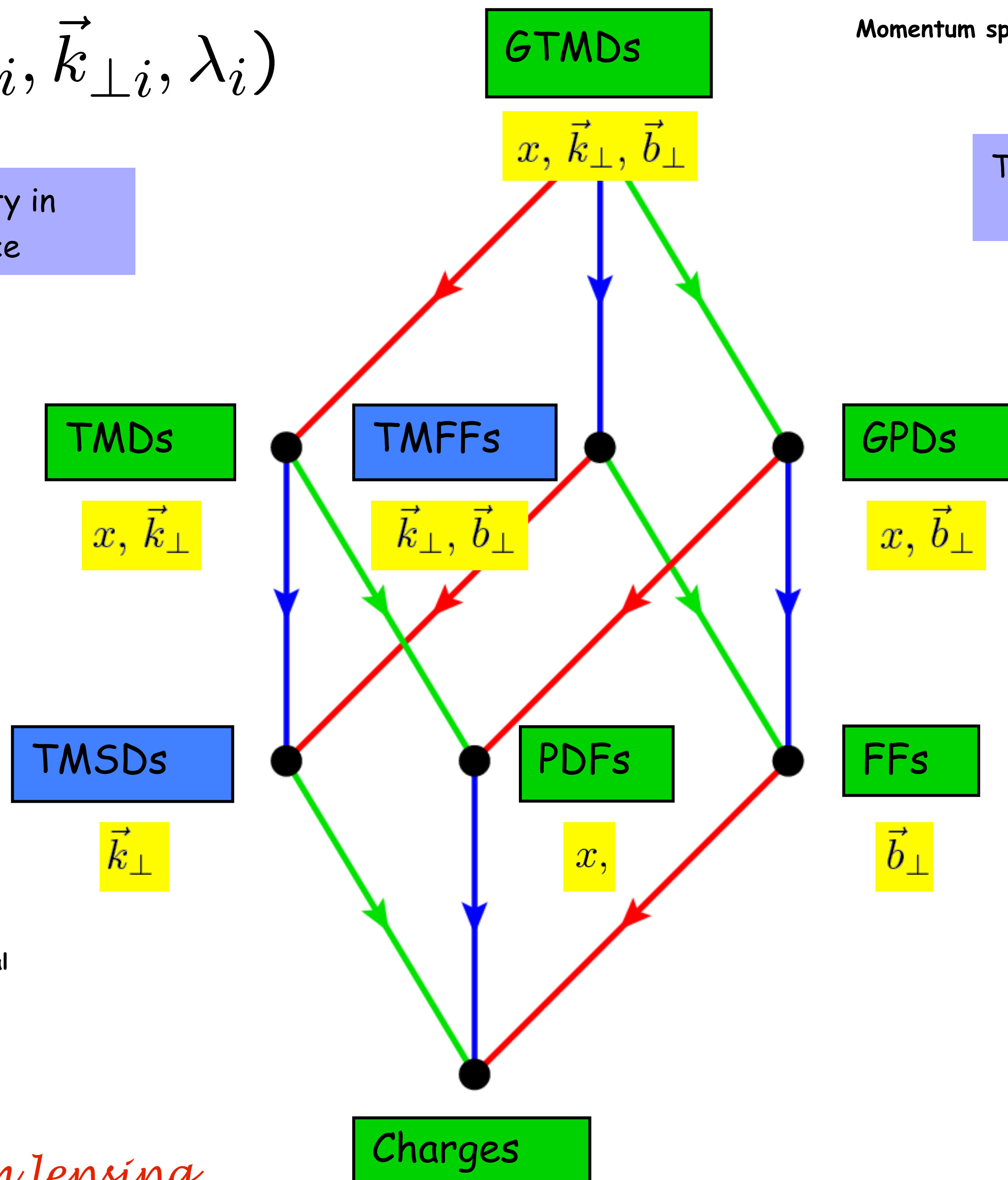
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

Transverse density in position space

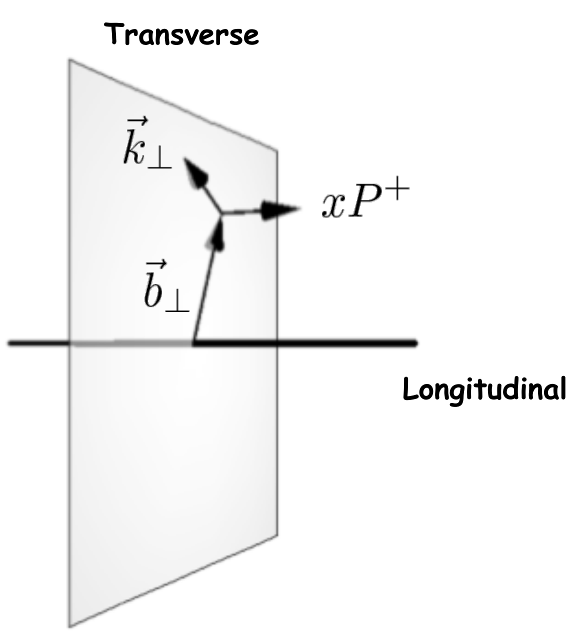
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$



Weak transition form factors

DGLAP, ERBL Evolution Factorization Theorems



Sivers, T-odd from lensing

red arrow: $\int d^2 b_\perp$

blue arrow: $\int dx$

green arrow: $\int d^2 k_\perp$

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

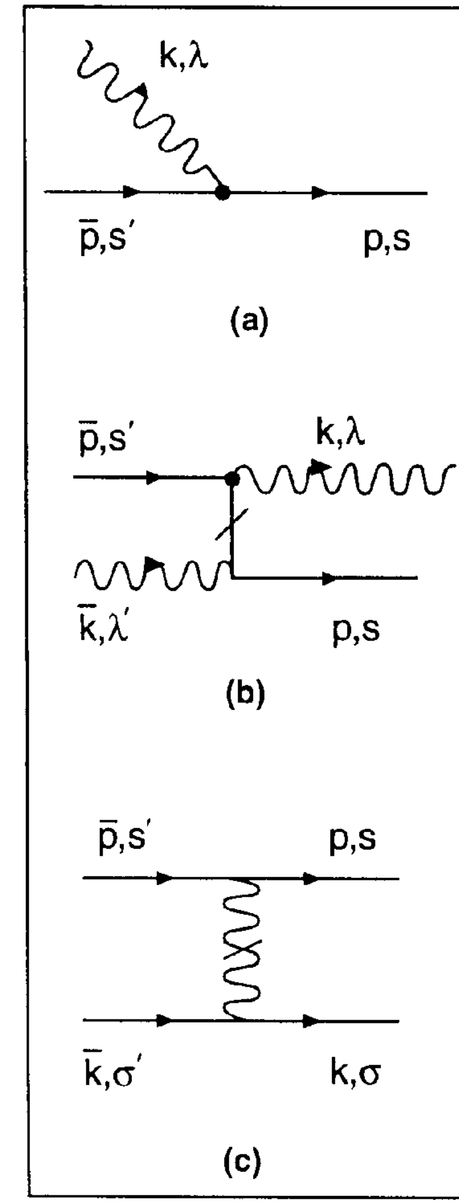
H^{int}_{LF}: Matrix in Fock Space

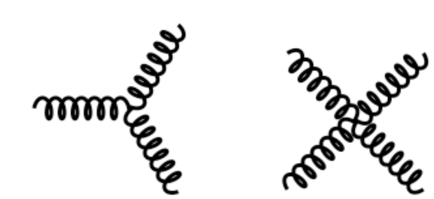
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass



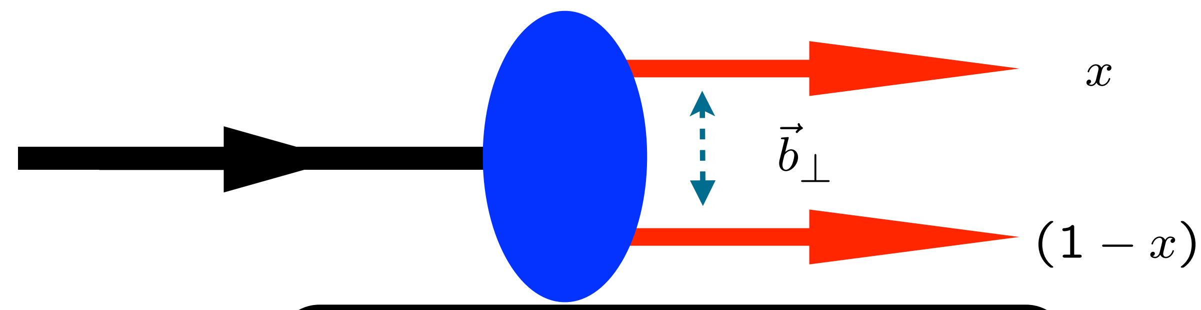


H^{int}_{LF}

Light-Front QCD

$\mathcal{L}_{QCD} \longrightarrow \boxed{H^{LF}_{QCD}}$

Fixed $\tau = t + z/c$



$$\boxed{\zeta^2 = x(1-x)b^2_\perp}$$

$$(H^0_{LF} + H^I_{LF})|\Psi>= M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states
and retarded interactions

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x,\vec{k}_\perp) = M^2\ \psi_{LF}(x,\vec{k}_\perp)$$

Effective two-particle equation

$$\boxed{[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)}$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$m_q = 0$

AdS/QCD:

$$\boxed{U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)}$$

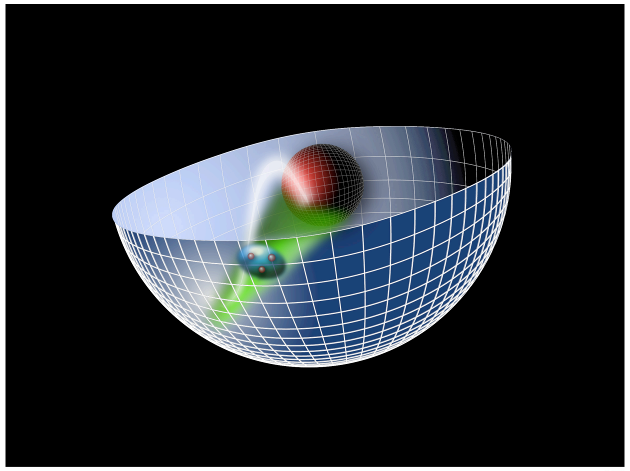
Confining AdS/QCD
potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

$$\kappa \simeq 0.5\ GeV$$

Confinement scale:

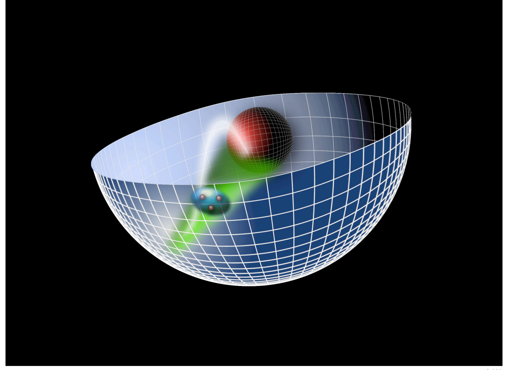
$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Maldacena

AdS$_5$



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

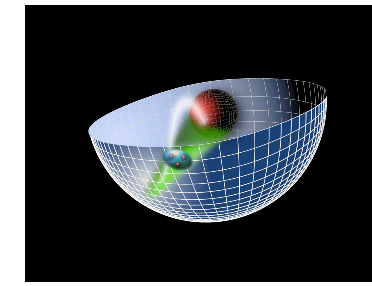
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

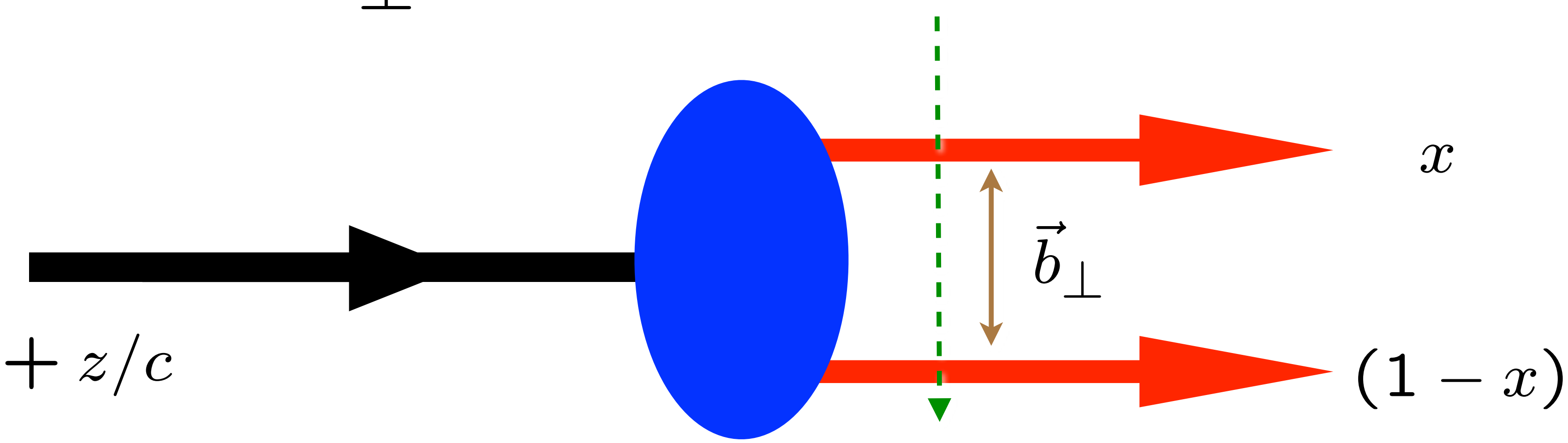
de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ !

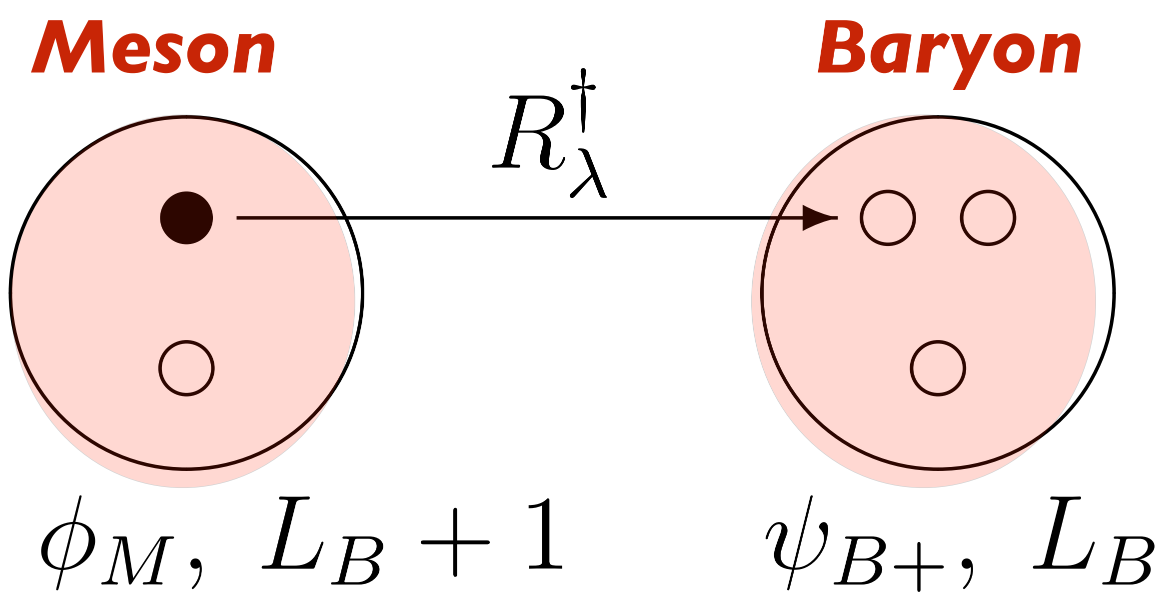
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

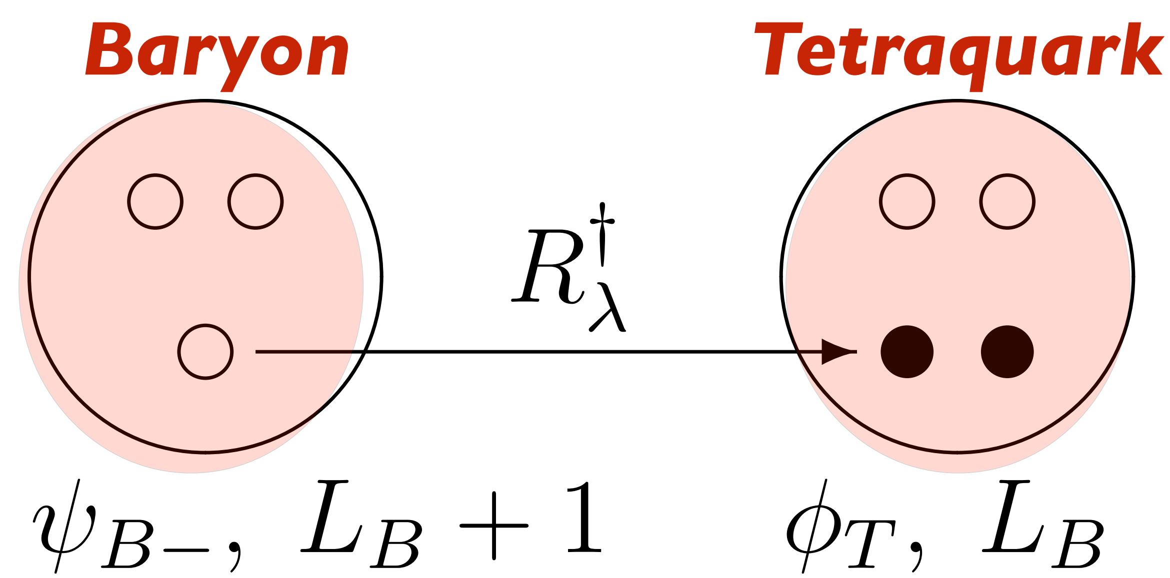
Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!

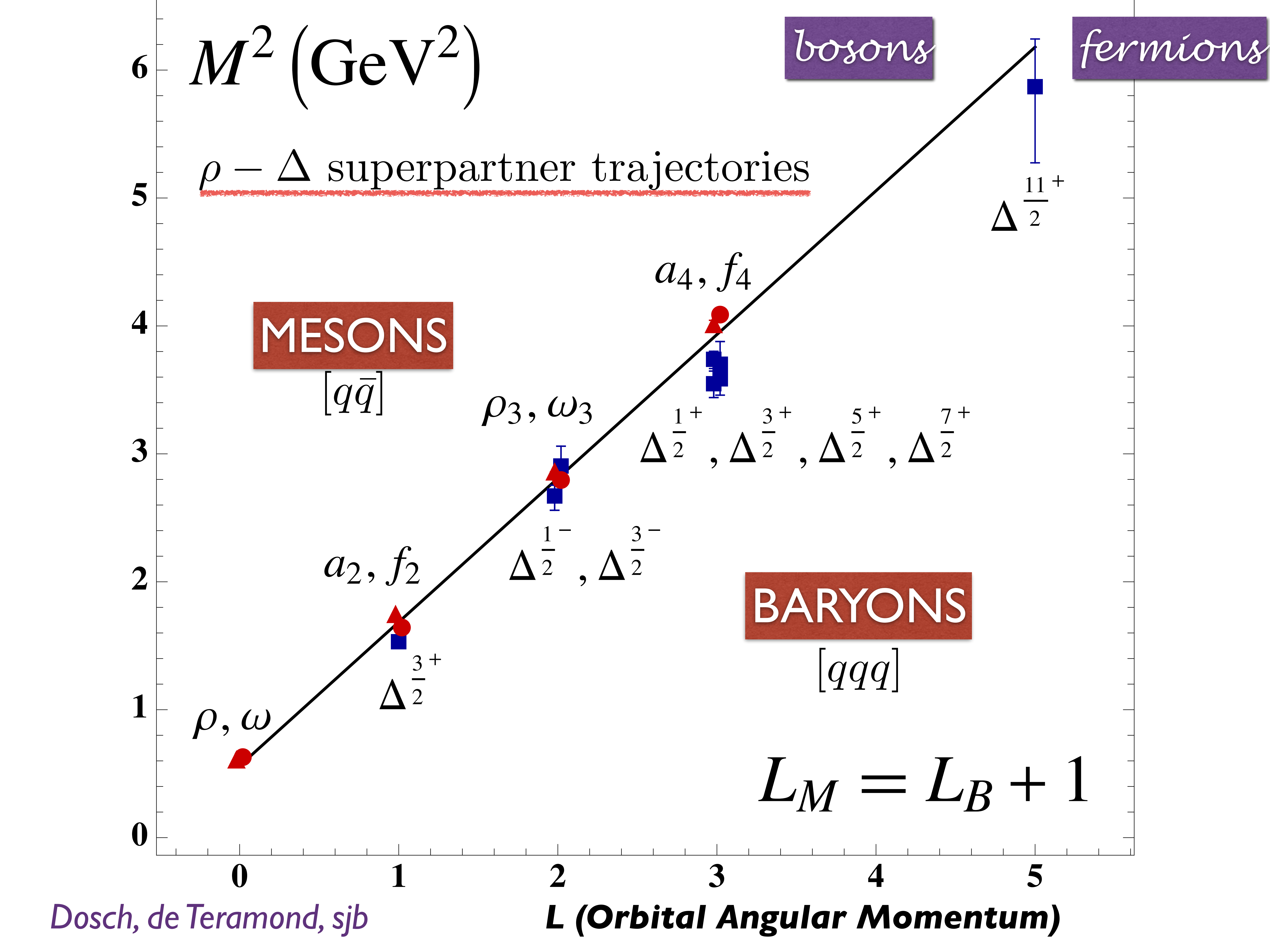


$$R_\lambda^\dagger \; \bar{q} \to [qq]$$
$$\bar{3}_C \to \bar{3}_C$$



$$R_\lambda^\dagger \; q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1



$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta^{\frac{3}{2}^+}$
$\Delta^{\frac{1}{2}^-}, \Delta^{\frac{3}{2}^-}$
$\Delta^{\frac{1}{2}^+}, \Delta^{\frac{3}{2}^+}, \Delta^{\frac{5}{2}^+}, \Delta^{\frac{7}{2}^+}$
$\Delta^{\frac{11}{2}^+}$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

Massless pion!

Meson Spectrum in Soft Wall Model

$$\boxed{m_\pi = 0 \text{ if } m_q = 0}$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\boxed{\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)}$$

$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

G. de Teramond, H. G. Dosch, sjb

Quark separation increases with L

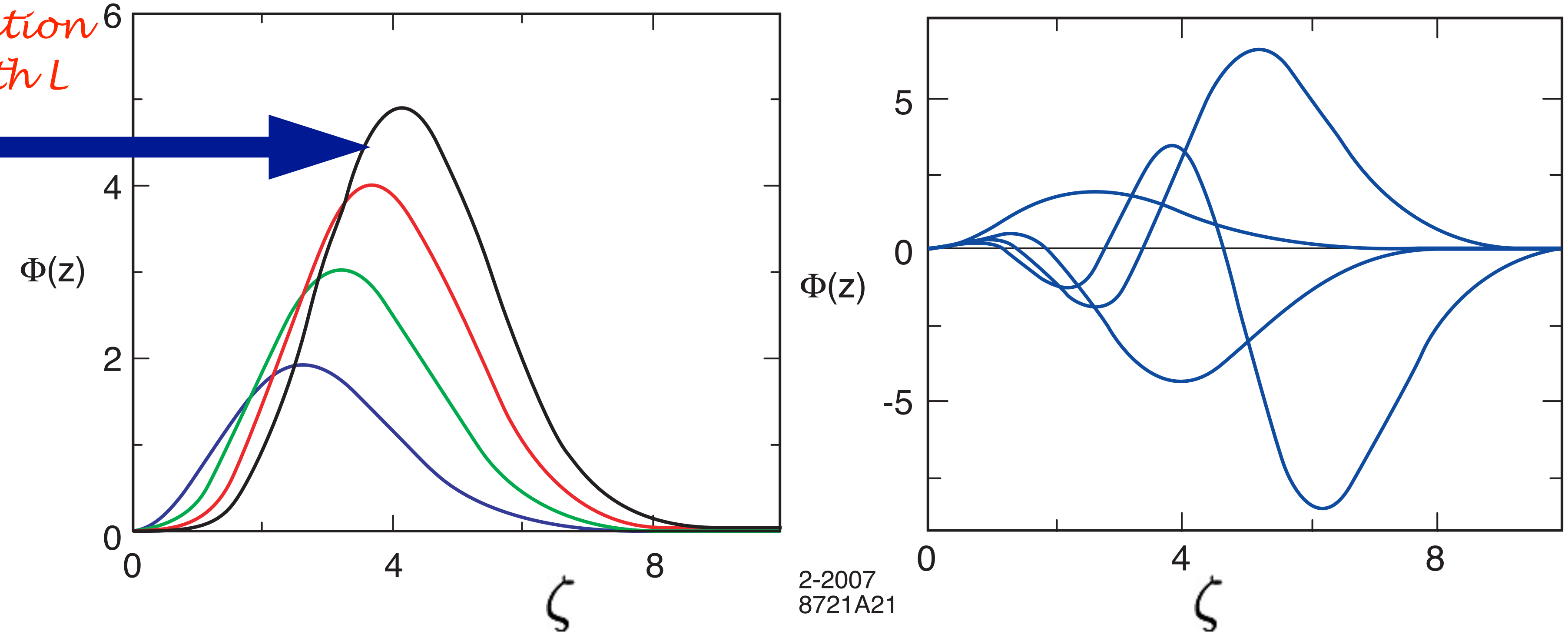
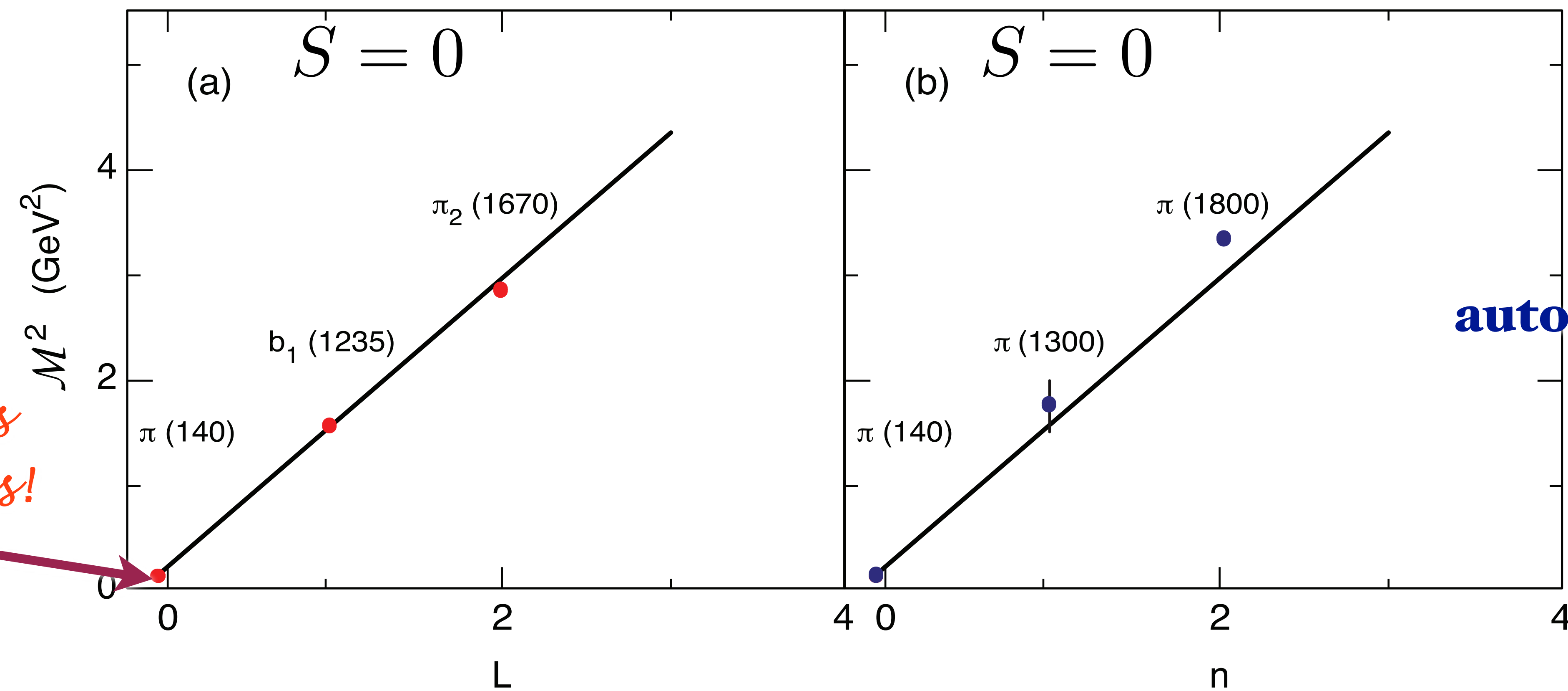


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!

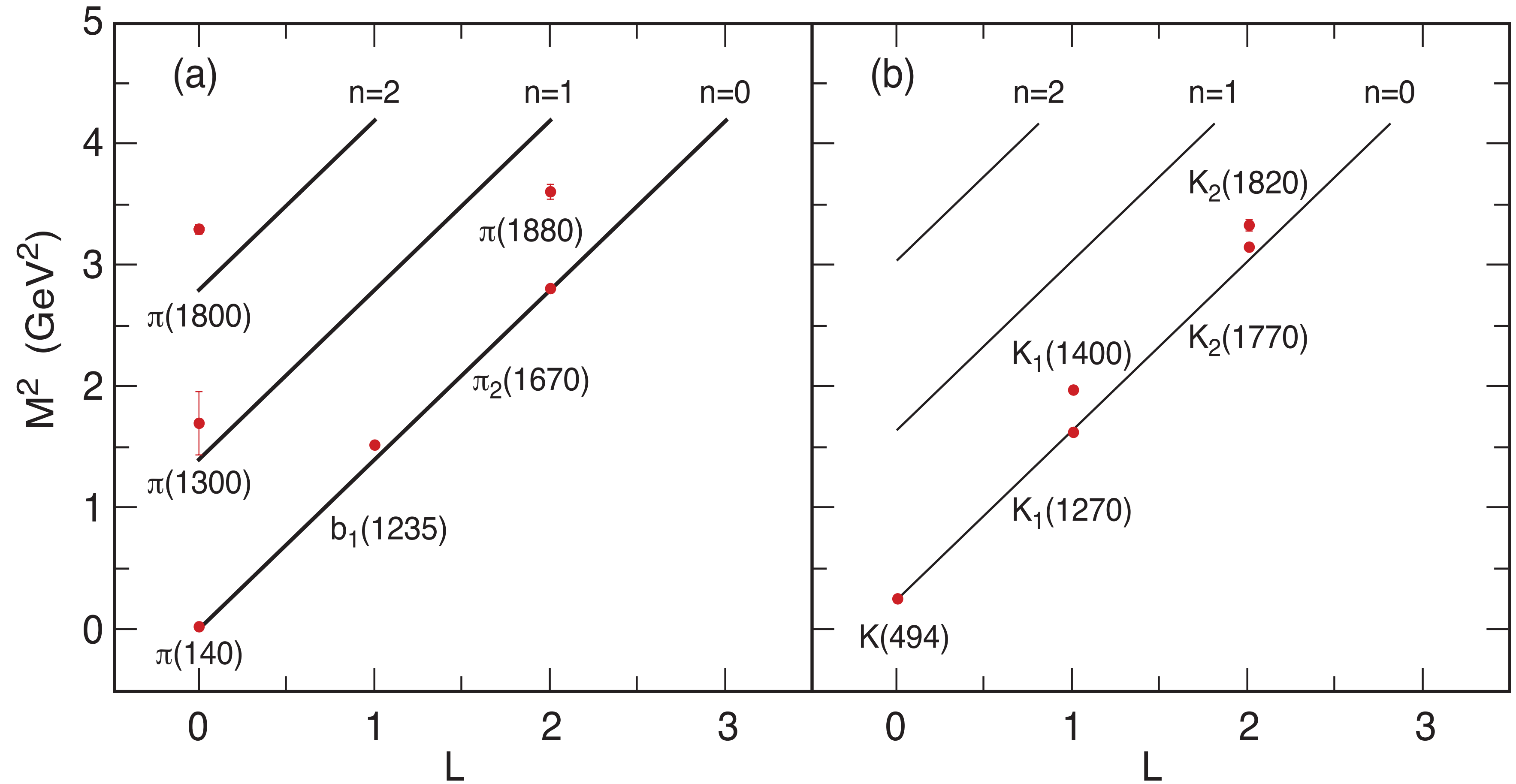


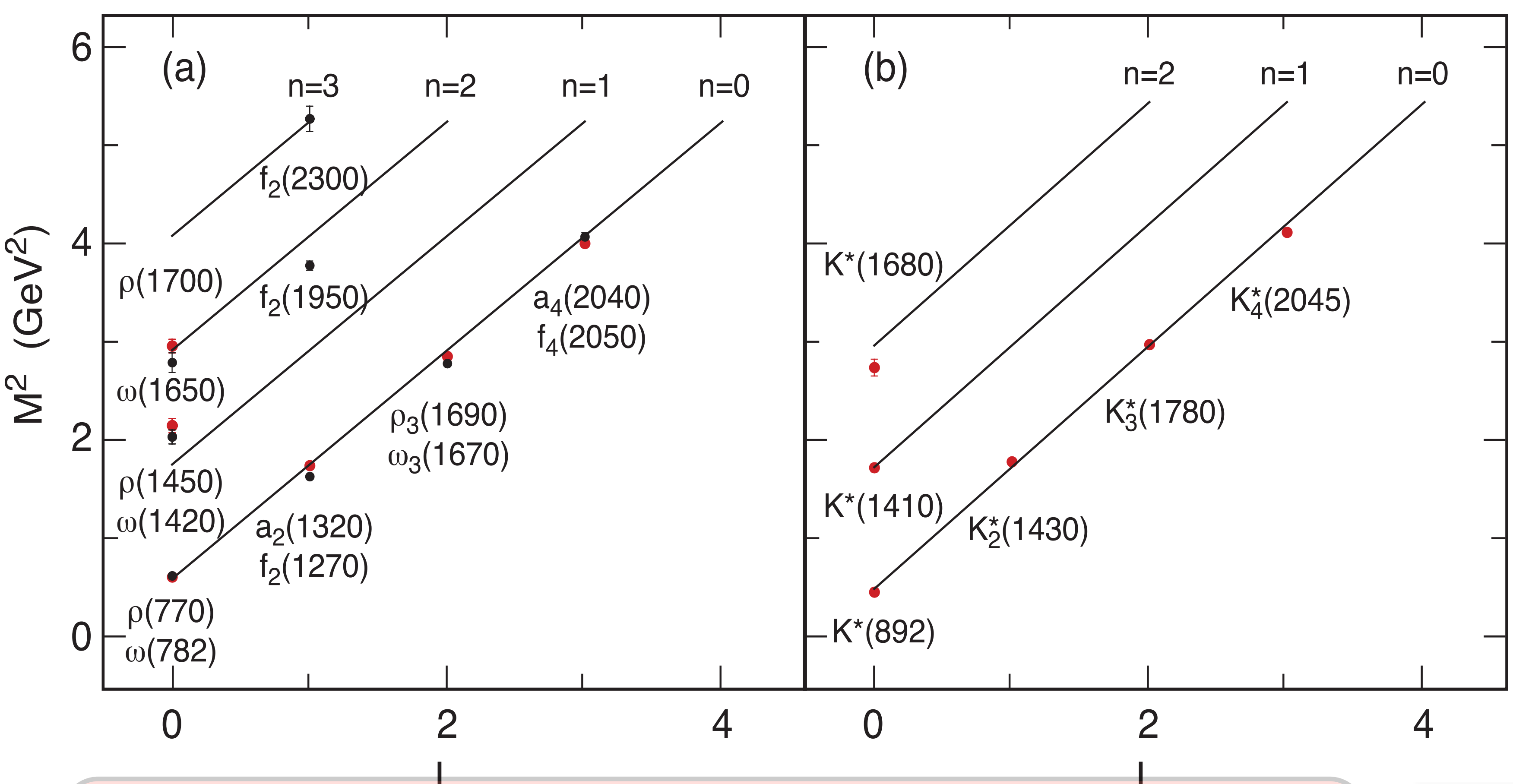
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

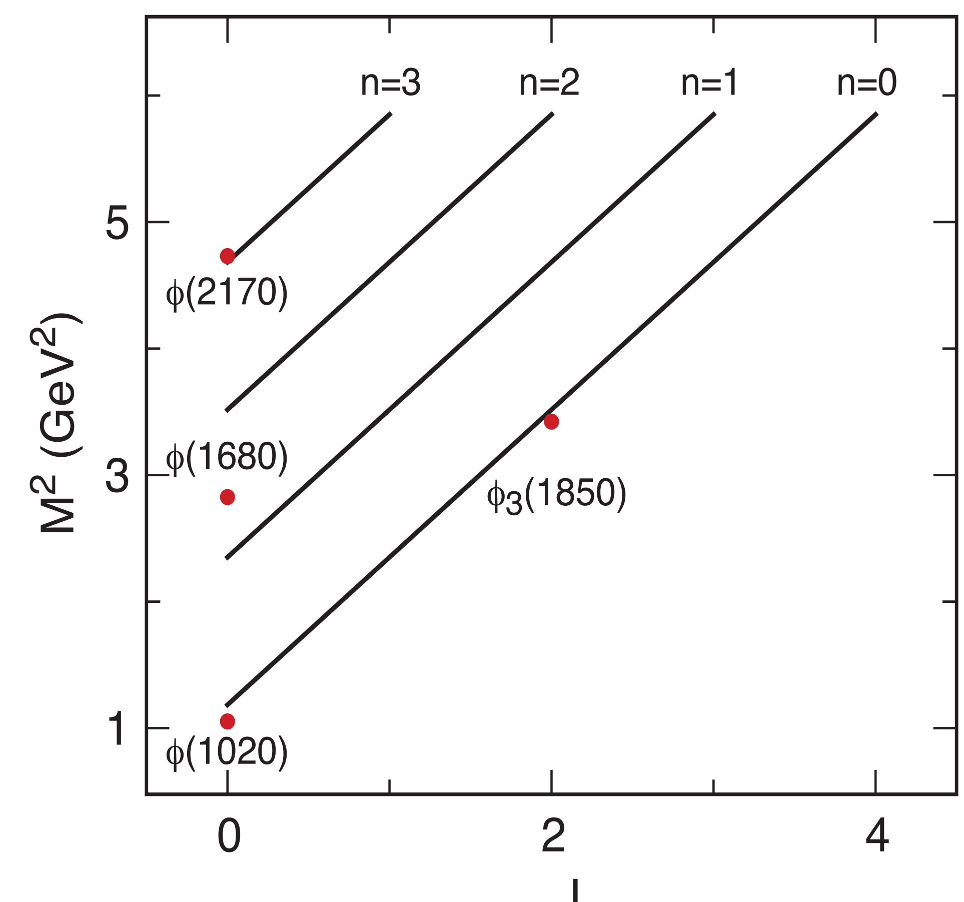
Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$







$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

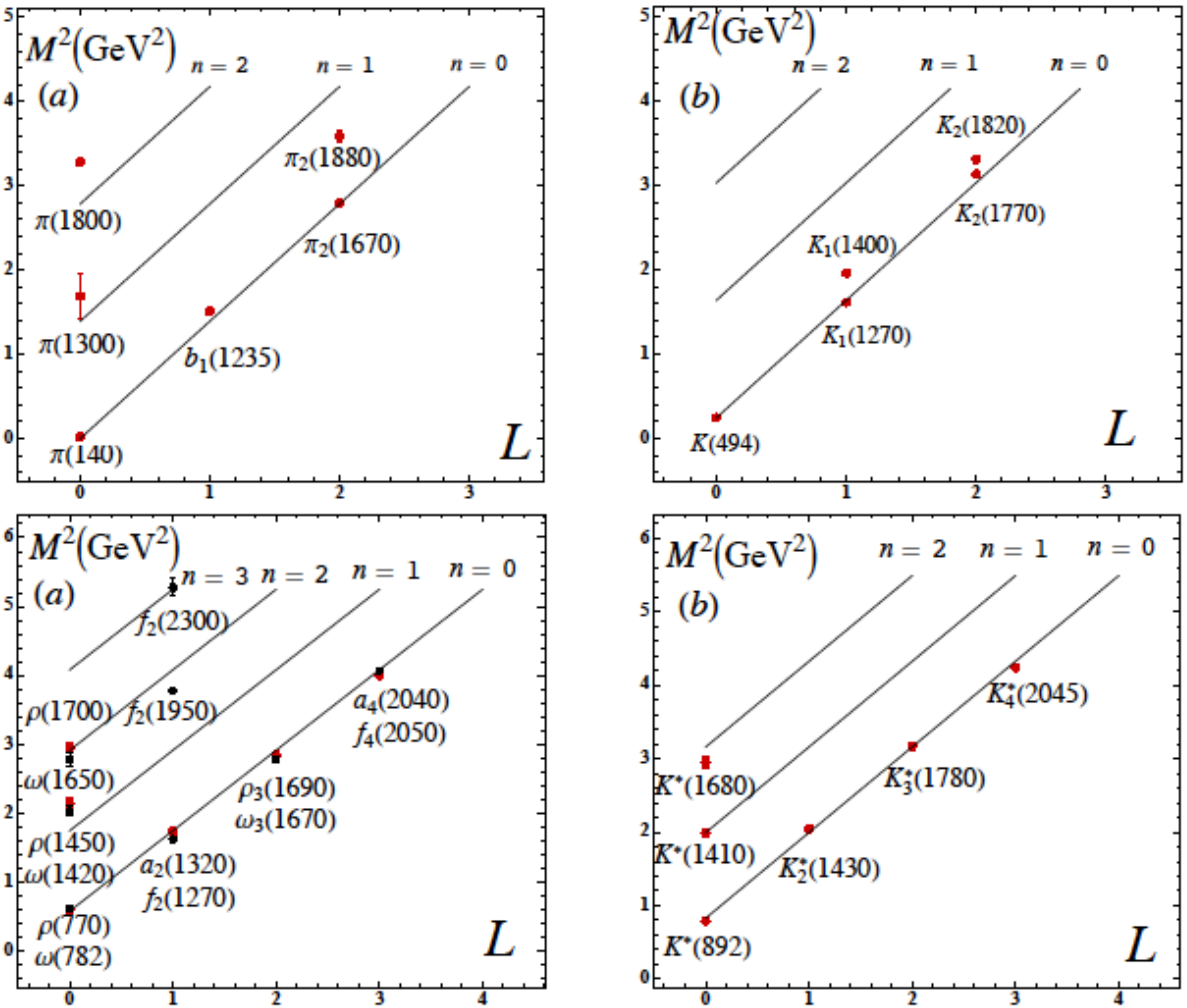
Equal Slope in n and L

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar q}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism

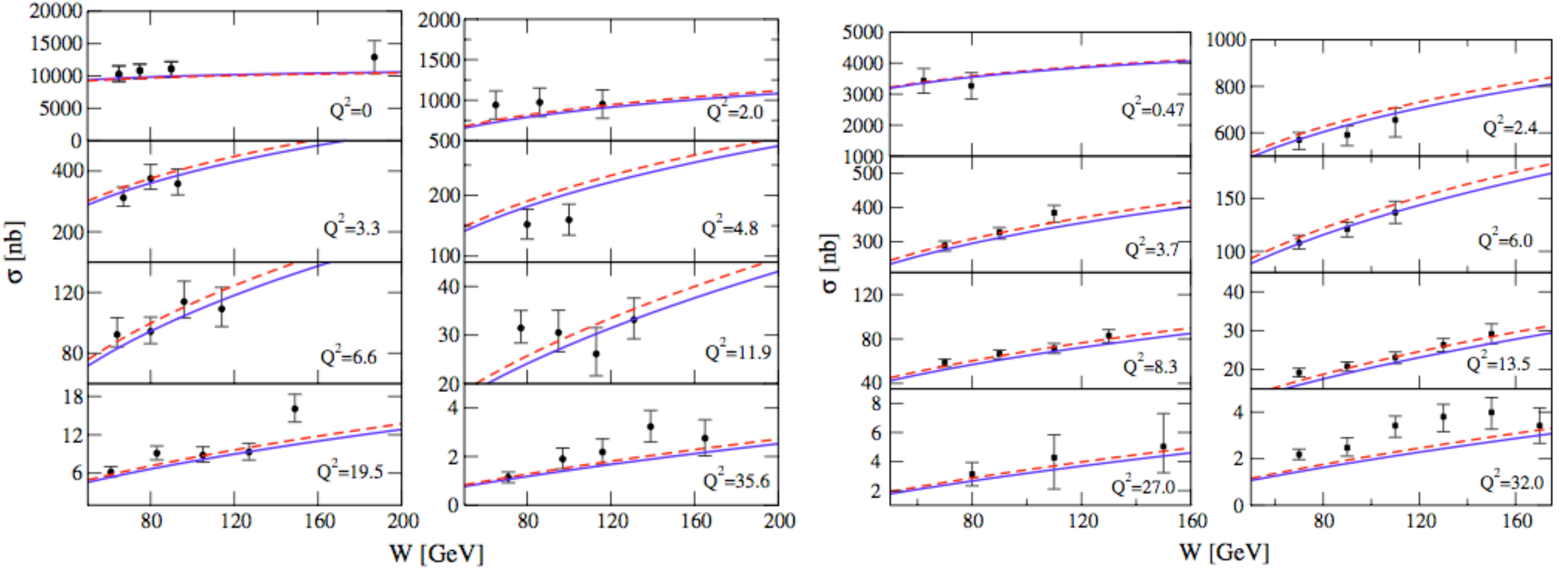


Effective mass from $m(p^2)$

Roberts, et al.

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

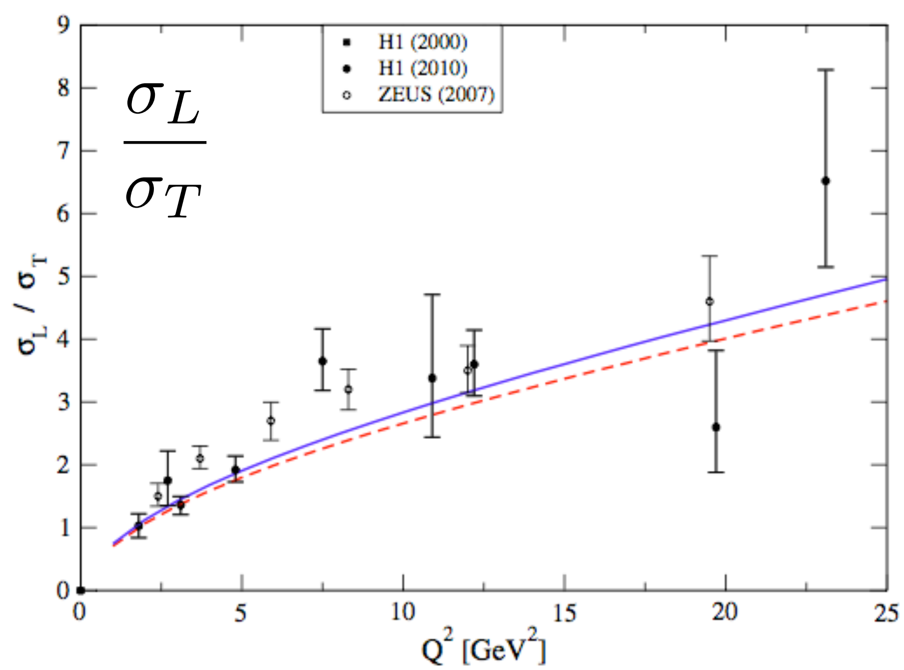
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H1 (b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

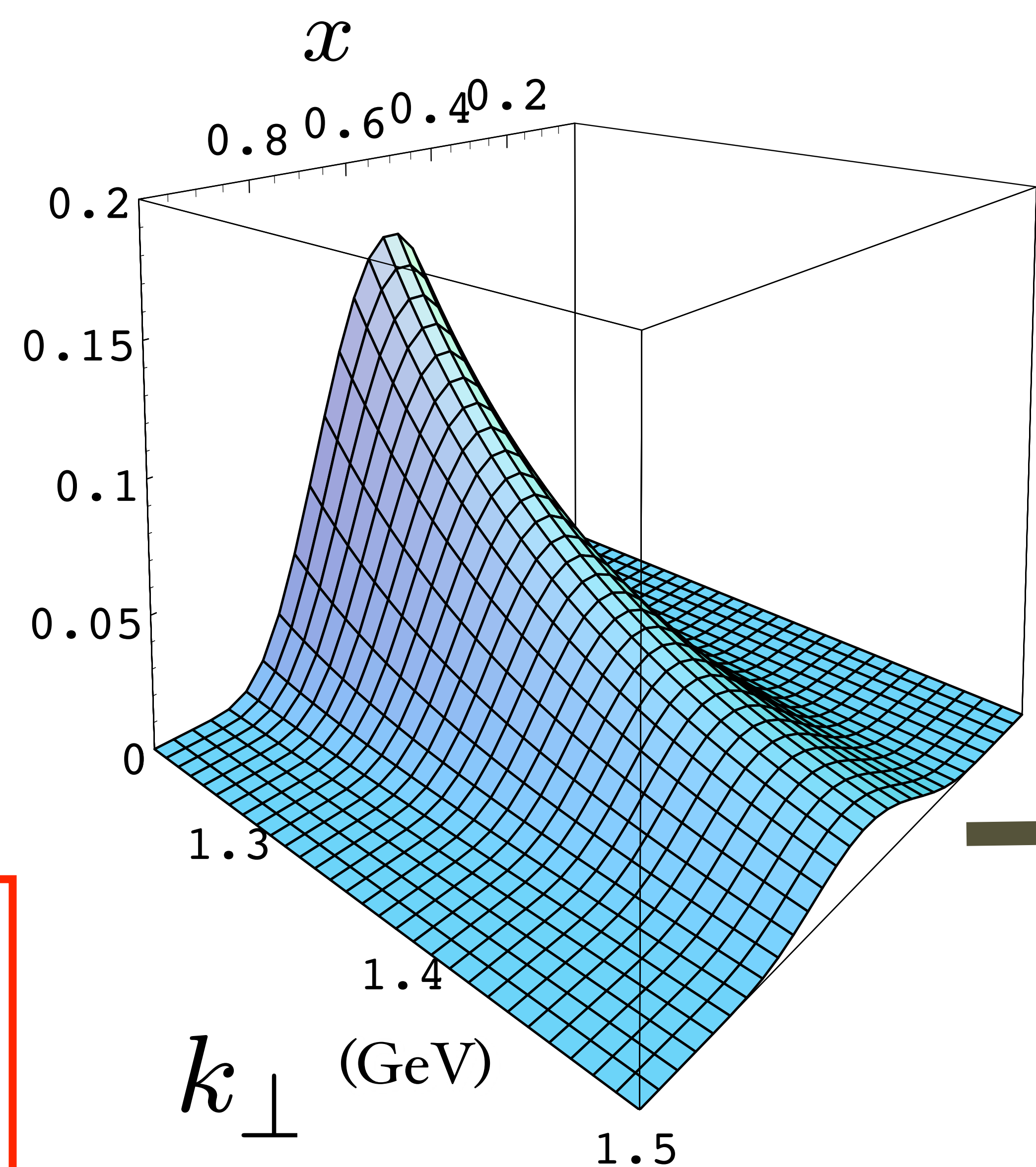


$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Prediction from AdS/QCD: Meson LFWF

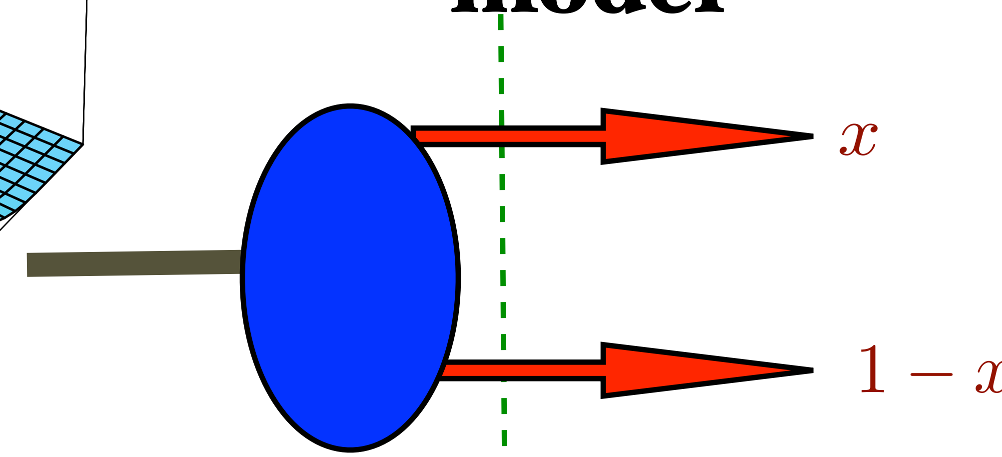
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

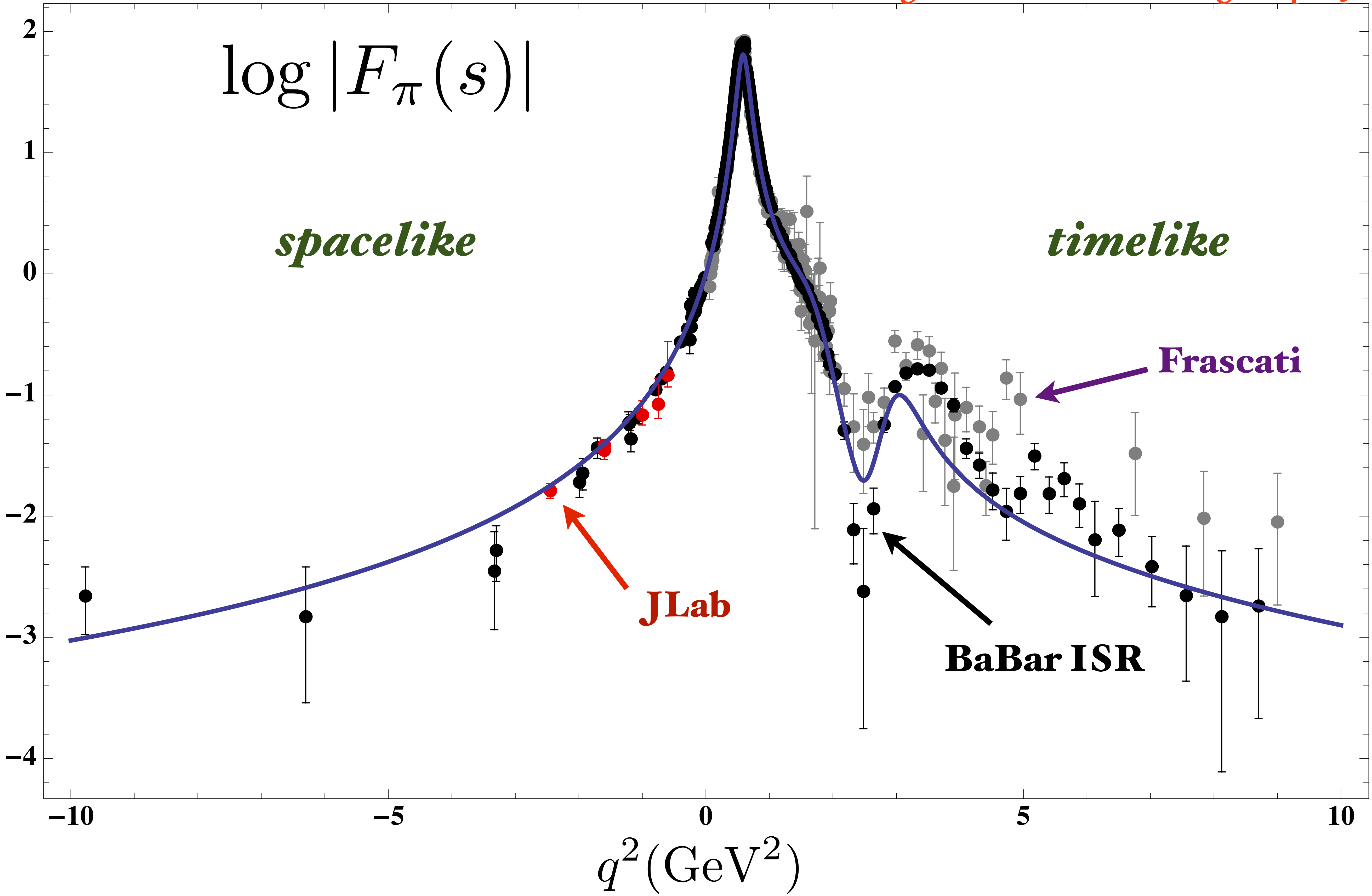
$$\boxed{\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

Pion Form Factor from AdS/QCD and Light-Front Holography



Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Light-Front Holography: First Approximation to QCD

- Color Confinement, Analytic form of confinement potential
- Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)
- Massless quark-antiquark pion bound state in chiral limit, GMOR
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincarè Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- OPE: Constituent Counting Rules
- Hadronization at the Amplitude Level: Many Phenomenological Tests

QCD Lagrangian

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau) >= i\frac{\partial}{\partial \tau}|\psi(\tau) >$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

- **Dosch, de Teramond, sjb**

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$\boxed{U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)}$$

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

$$\boxed{Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

$\boxed{\text{Consider } R_w = Q + wS;}$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \big(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \big)$$

$$G_{22} = \big(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \big)$$

$\boxed{\text{Identify } f - \frac{1}{2} = L_B \ , \quad w = \kappa^2}$ $\lambda = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+ = M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J = M^2\phi_J$$

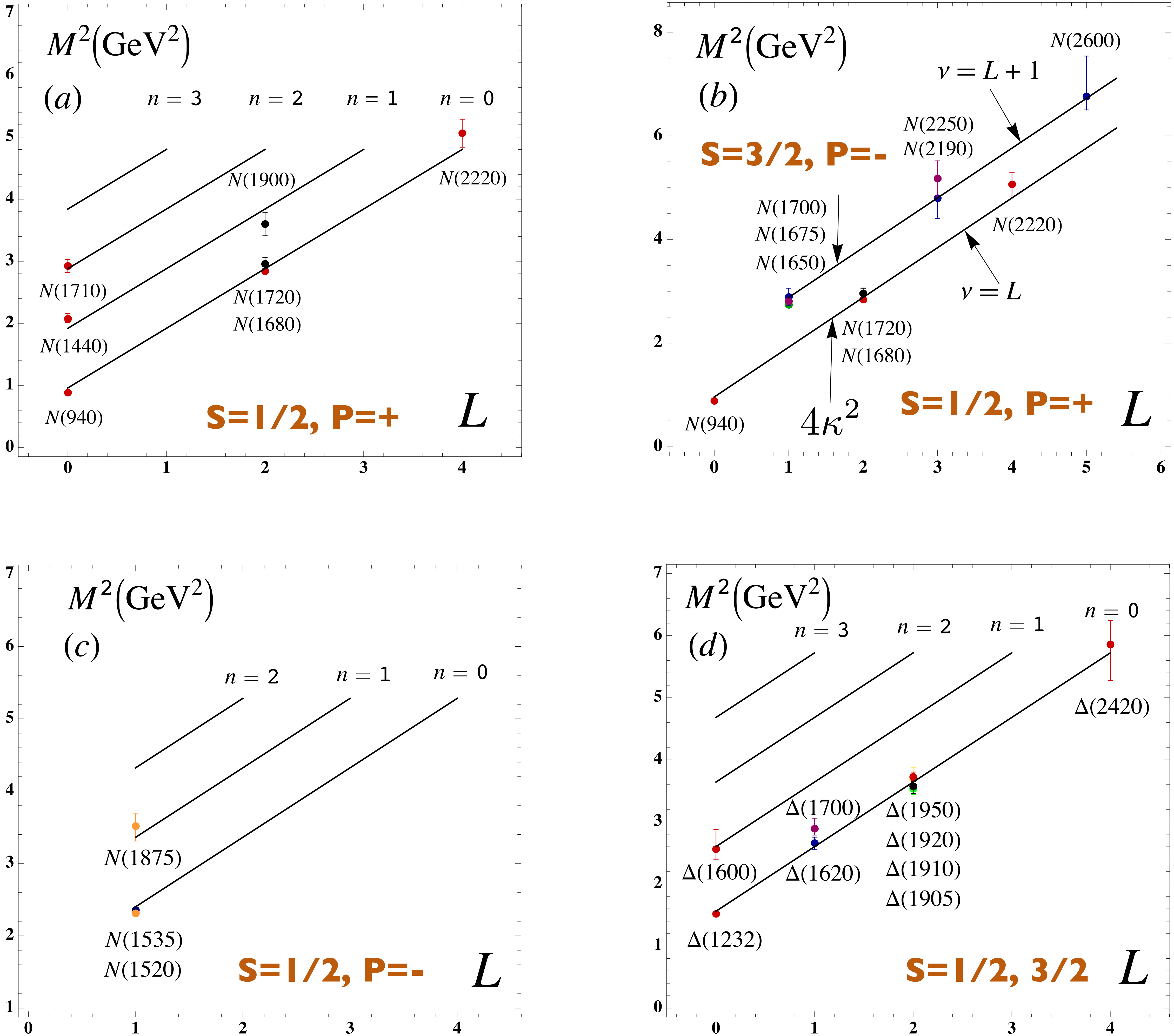
S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ *!*

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$



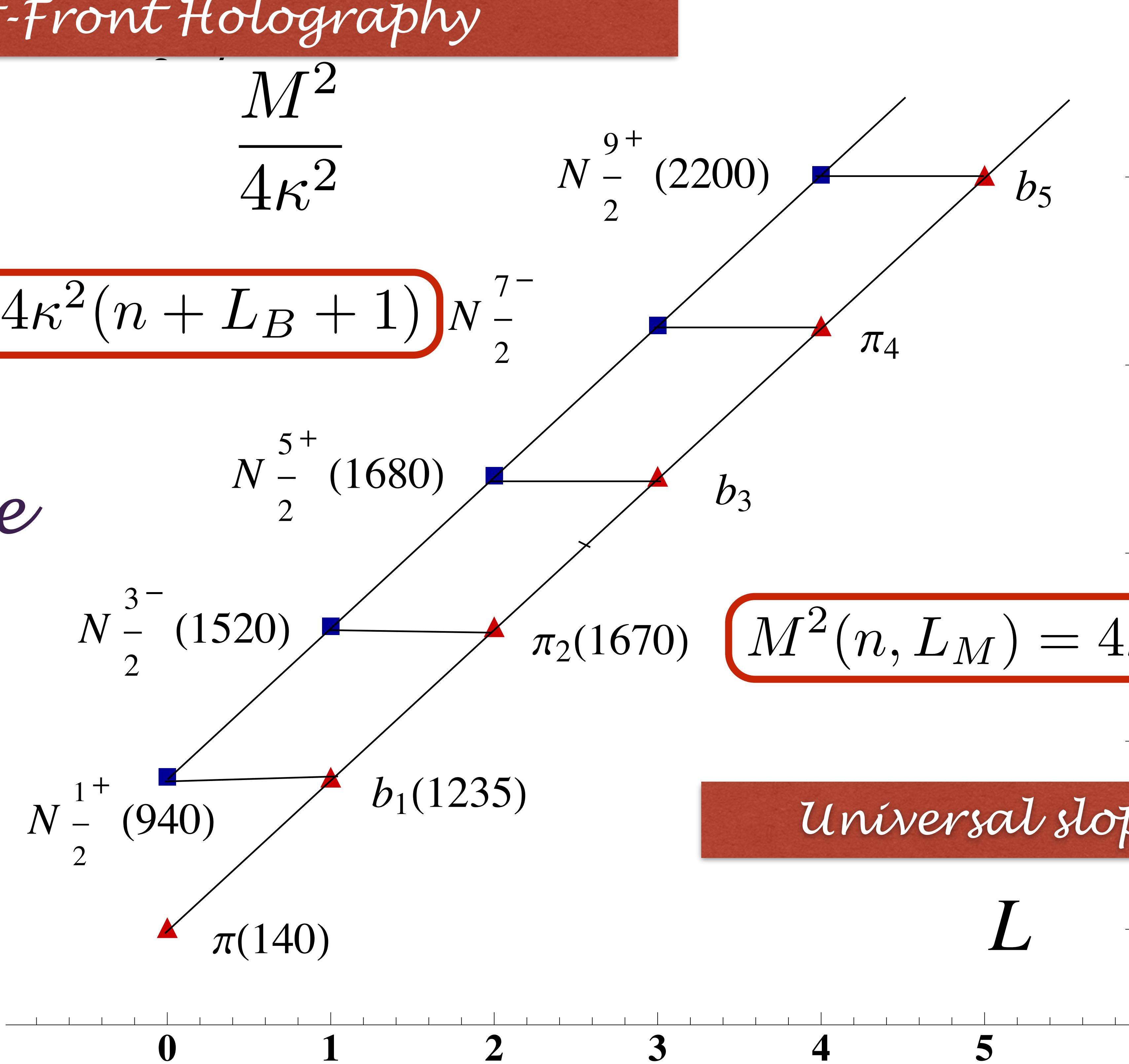
$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
$\nu = L + 1$
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

Superconformal Quantum Mechanics Light-Front Holography

de Tèramond, Dosch, Lorcè, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

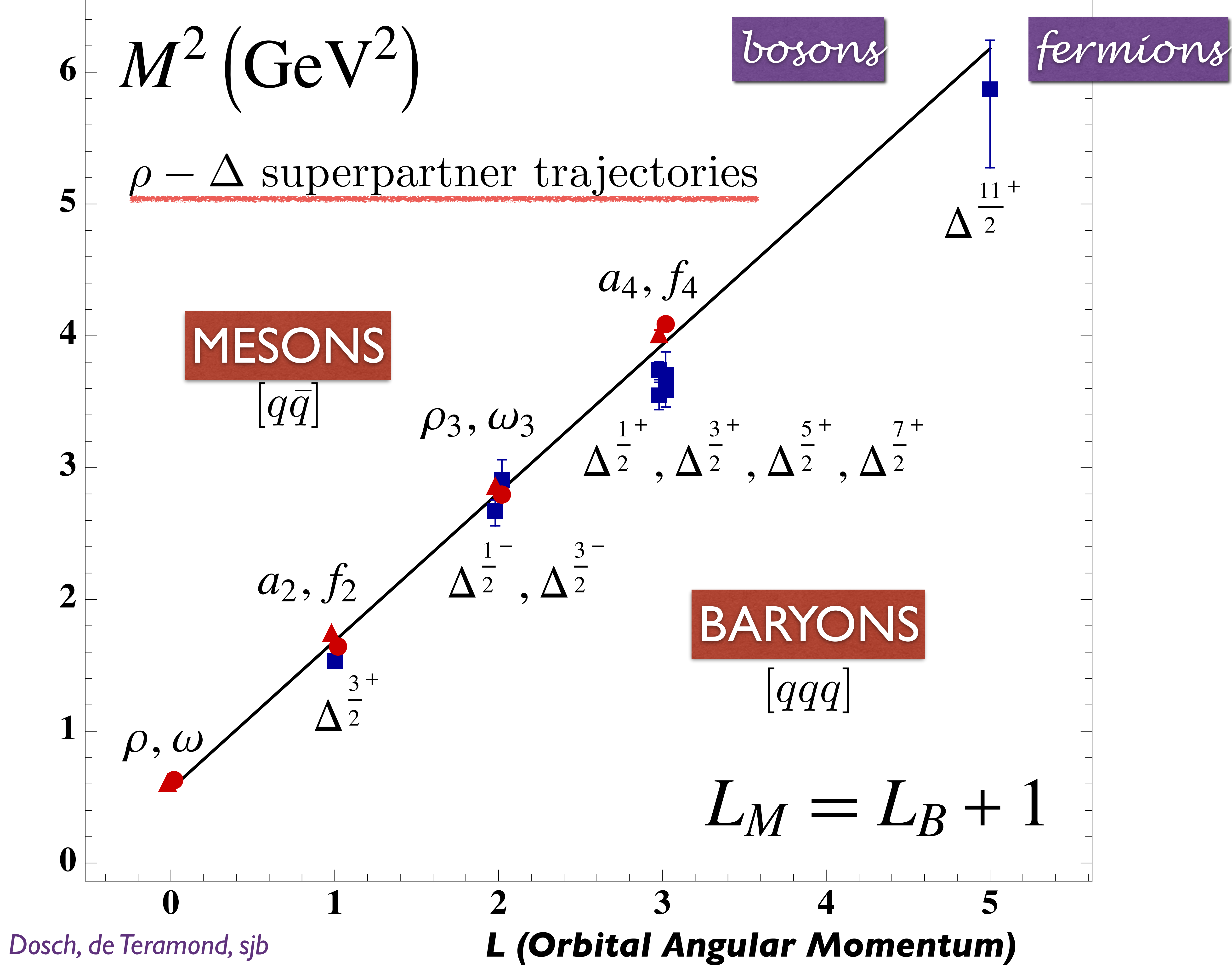


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
ρ, ω
a_2, f_2
ρ_3, ω_3
a_4, f_4
$\Delta \frac{3}{2}^+$
$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$
$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$
$\Delta \frac{11}{2}^+$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

LF Holography

Baryon LFWFs

Superconformal Quantum Mechanics

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\int_0^\infty d\zeta \int_0^1 dx \psi_+^2(\zeta^2, x) = \int_0^\infty d\zeta \int_0^1 dx \psi_-^2(\zeta^2, x) = \frac{1}{2}$$

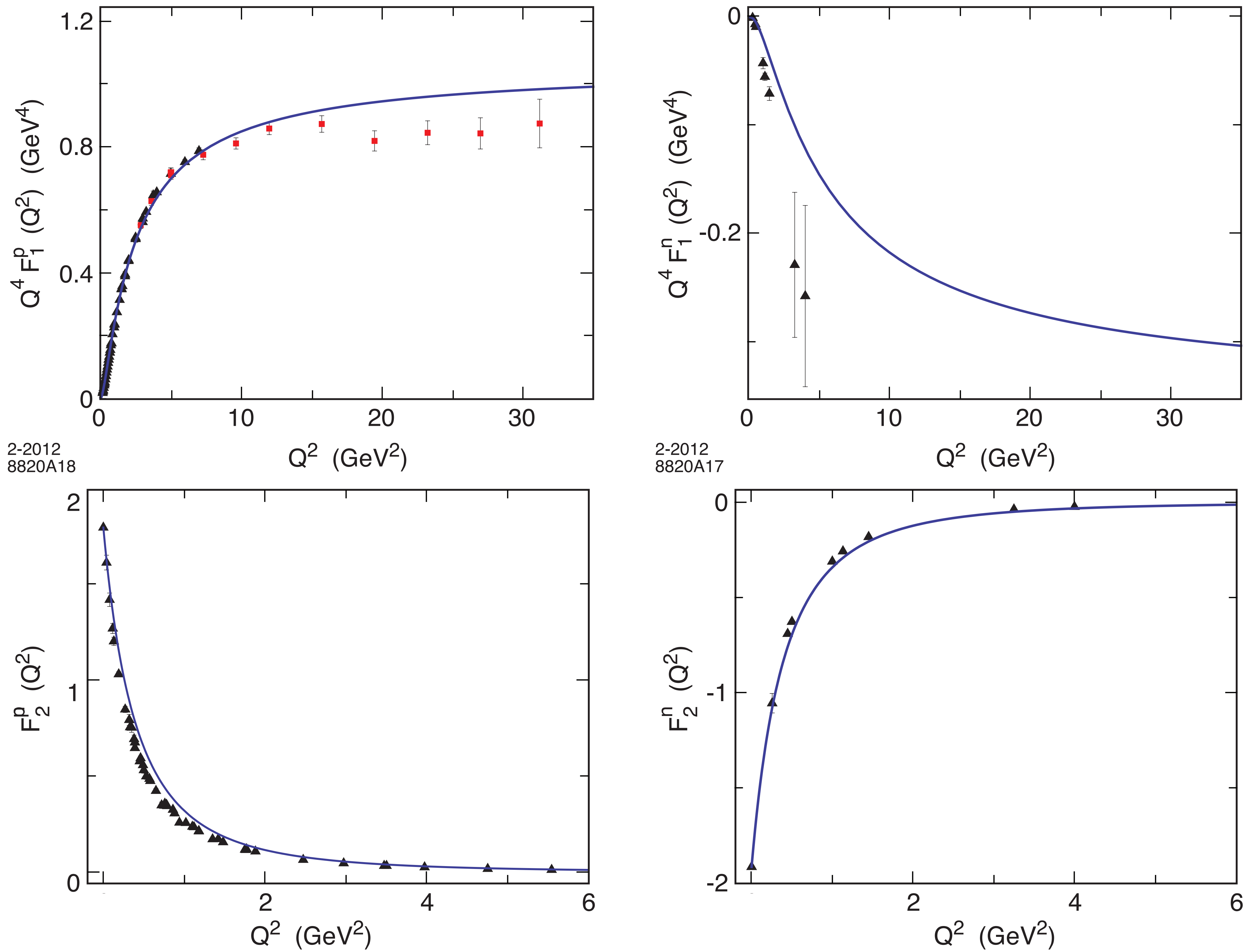
Quark Chiral Symmetry of Eigenstate!

Nucleon: Equal Probability for L=0,1

$$J^z = +1/2: \quad \frac{1}{\sqrt{2}}[\,|\,S_q^z = +1/2,\, L^z = 0 > + \,|\,S_q^z = -1/2,\, L^z = +1 >\,]$$

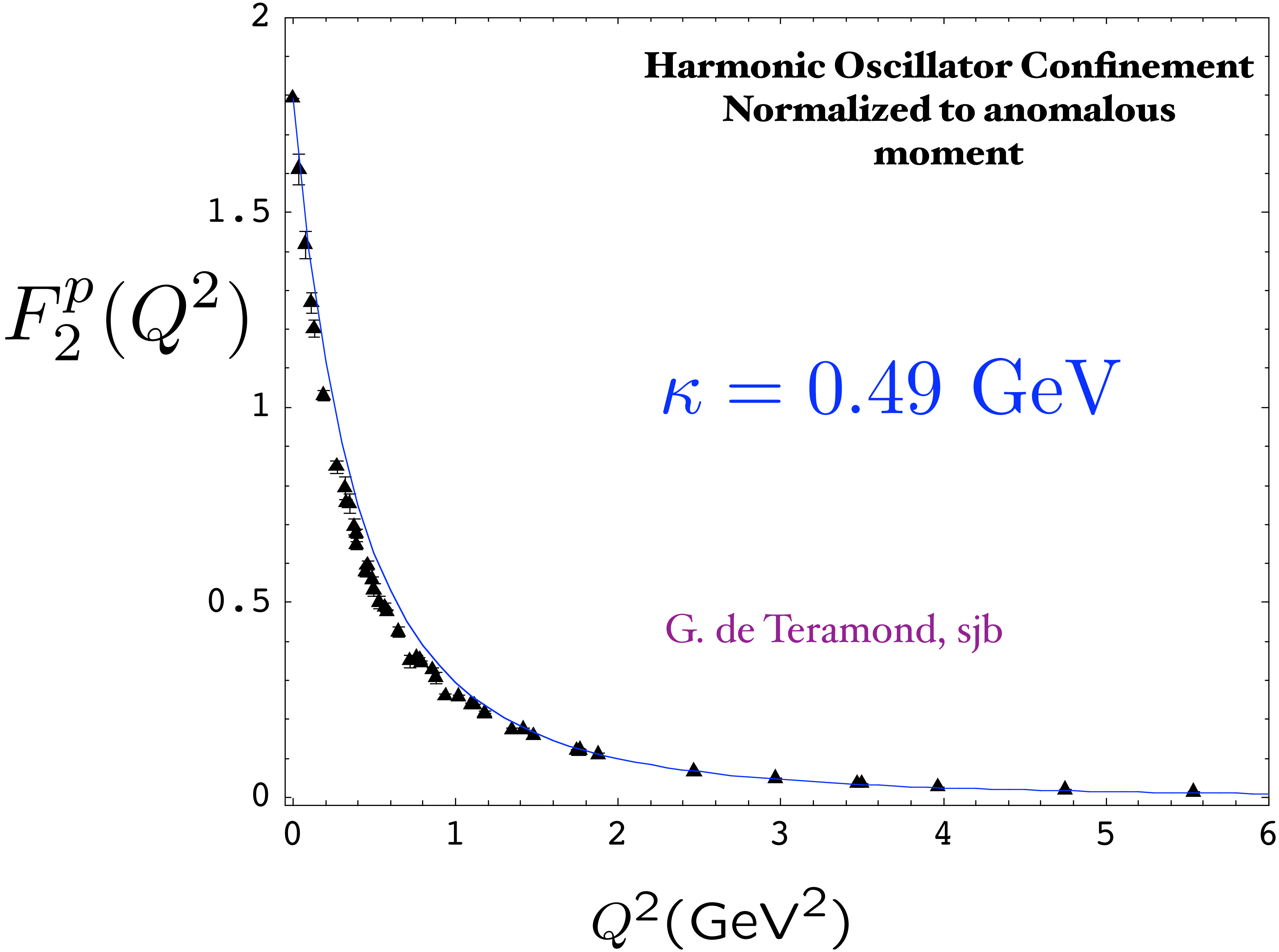
Nucleon spin carried by quark orbital angular momentum

Using $SU(6)$ flavor symmetry and normalization to static quantities



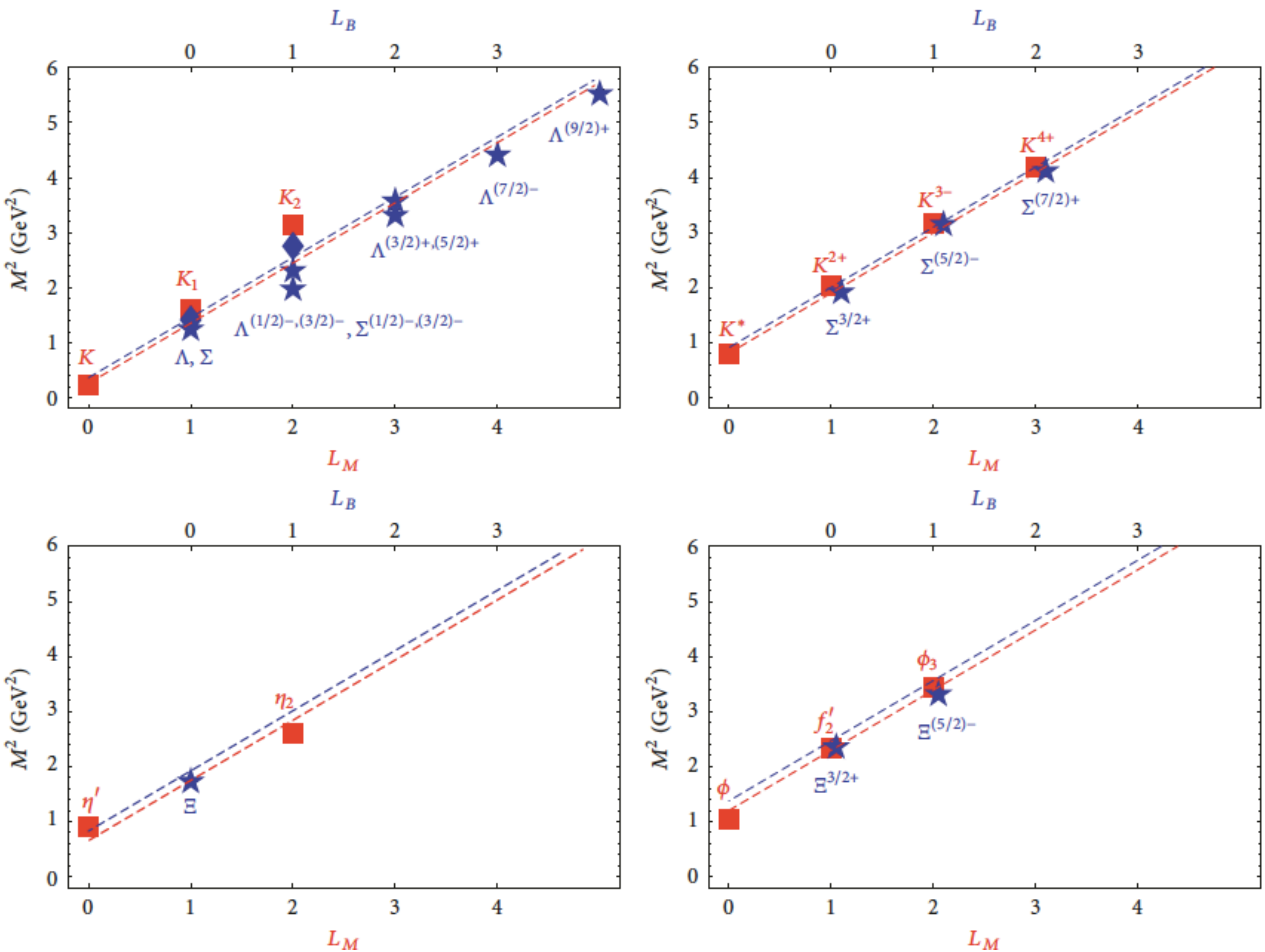
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



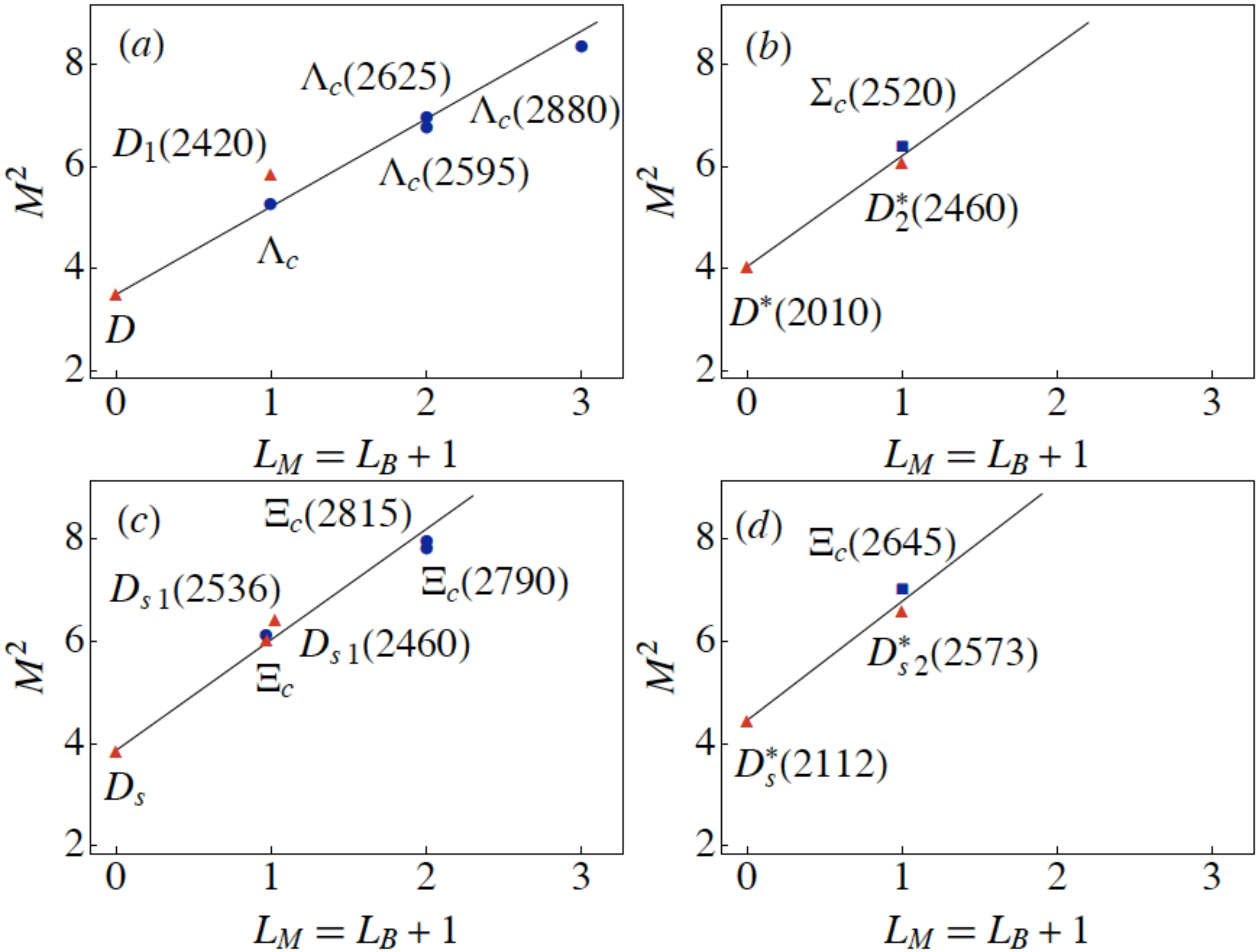
Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Dosch, de Teramond, sjb

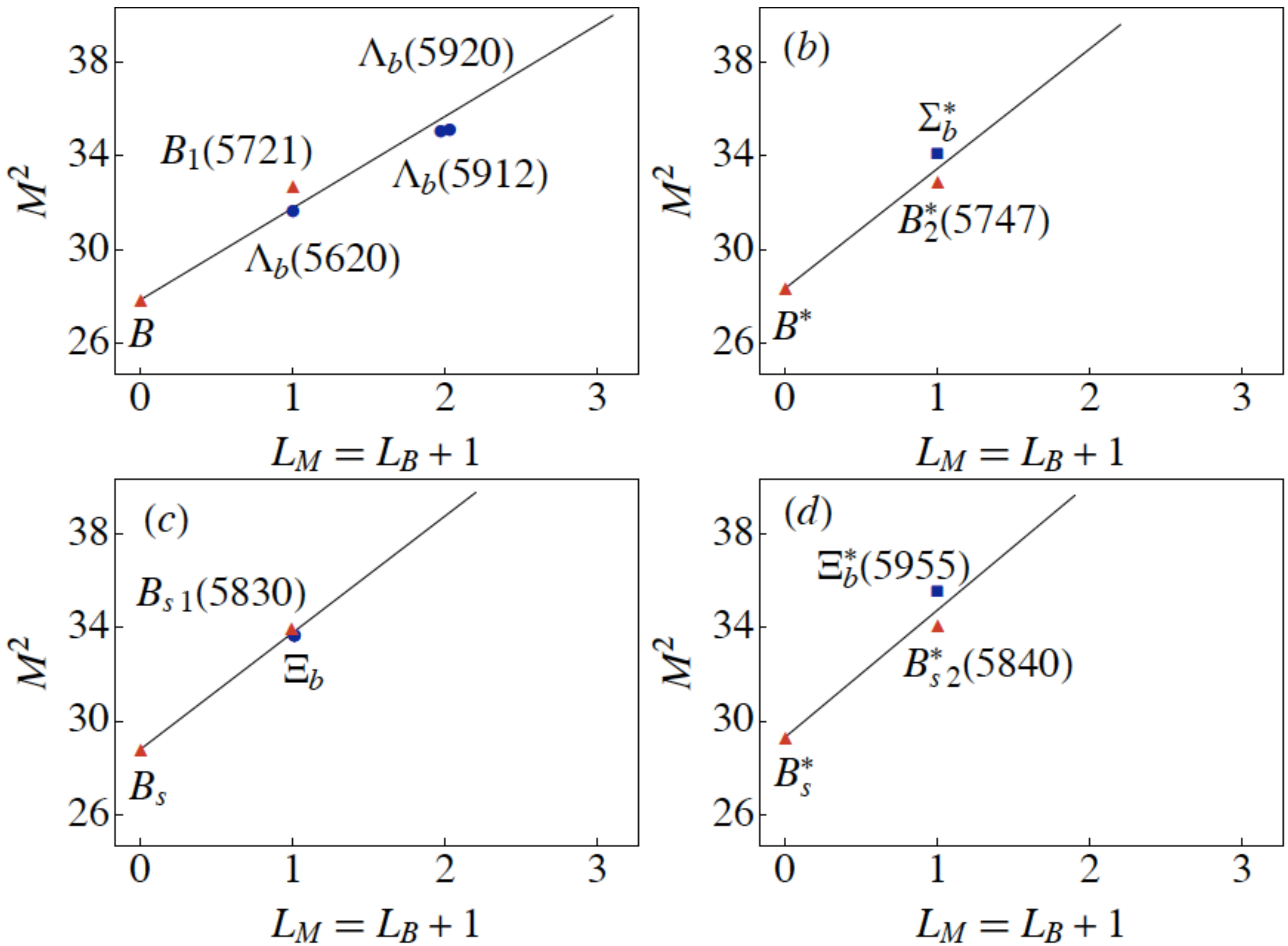
Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

Heavy-light and heavy-heavy hadronic sectors

- Extension to the heavy-light hadronic sector

  [H. G. Dosch, GdT, S. J. Brodsky, PRD **92**, 074010 (2015), PRD **95**, 034016 (2017)]

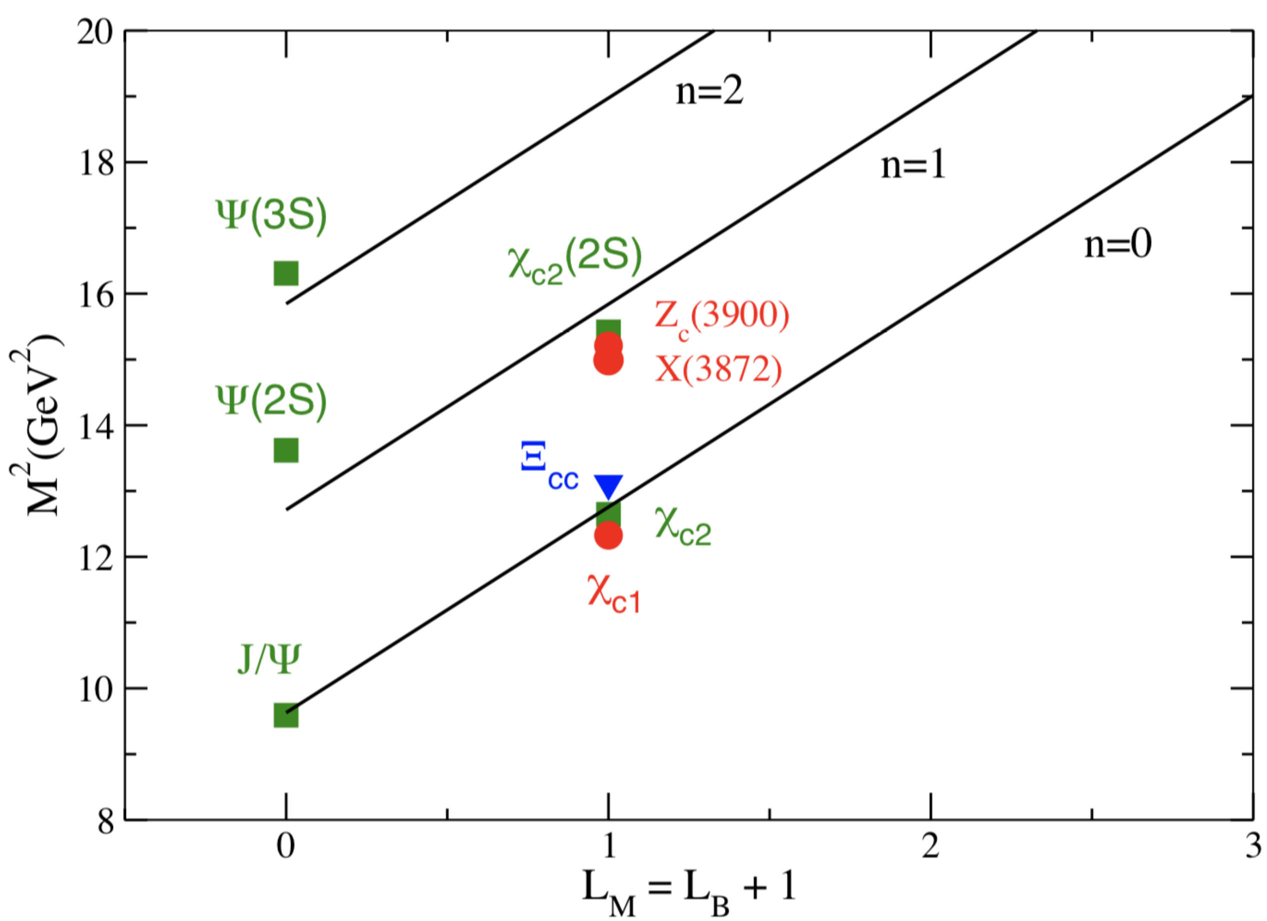
- Extension to the double-heavy hadronic sector

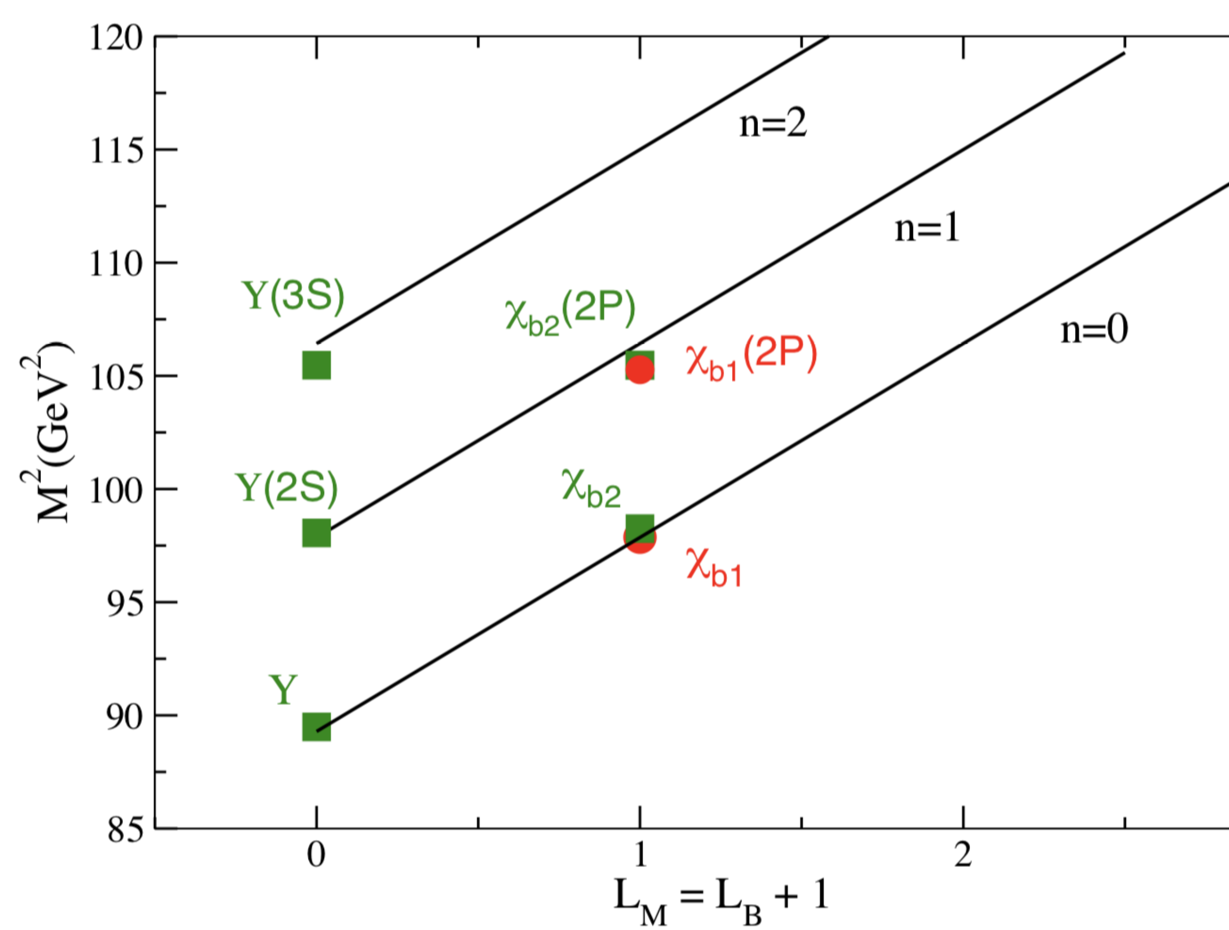
  [M. Nielsen and S. J. Brodsky, PRD, 114001 (2018)]

  [M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch, F. S. Navarra, L. Zou, PRD **98**, 034002 (2018)]

- Extension to the isoscalar hadronic sector

  [L. Zou, H. G. Dosch, GdT,S. J. Brodsky, arXiv:1901.11205 [hep-ph]]





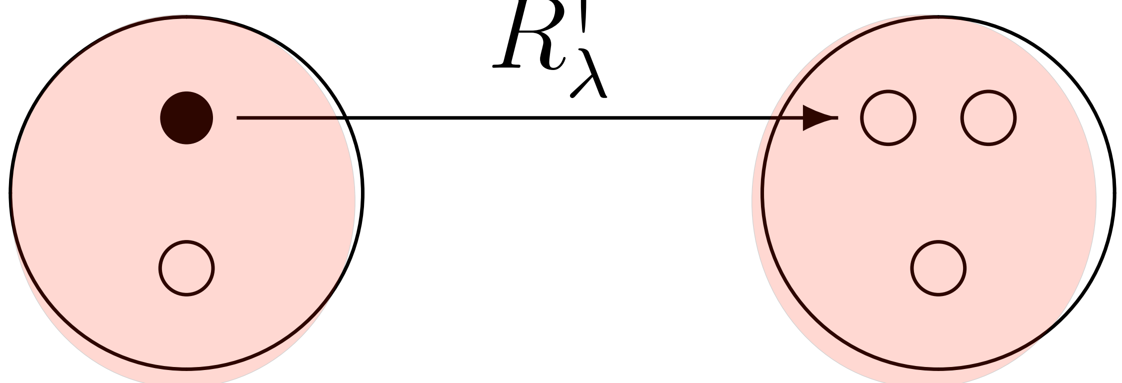
Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!

Meson **Baryon**

$R^\dagger_\lambda$



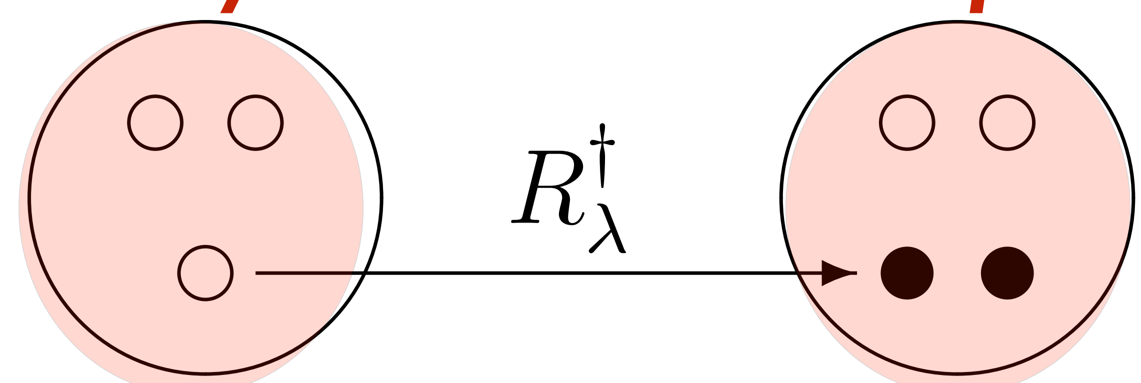
$\phi_M, \ L_B + 1$ $\psi_{B+}, \ L_B$

$R^\dagger_\lambda \ \bar{q} \rightarrow [qq]$

$\bar{3}_C \rightarrow \bar{3}_C$

Baryon **Tetraquark**

$R^\dagger_\lambda$



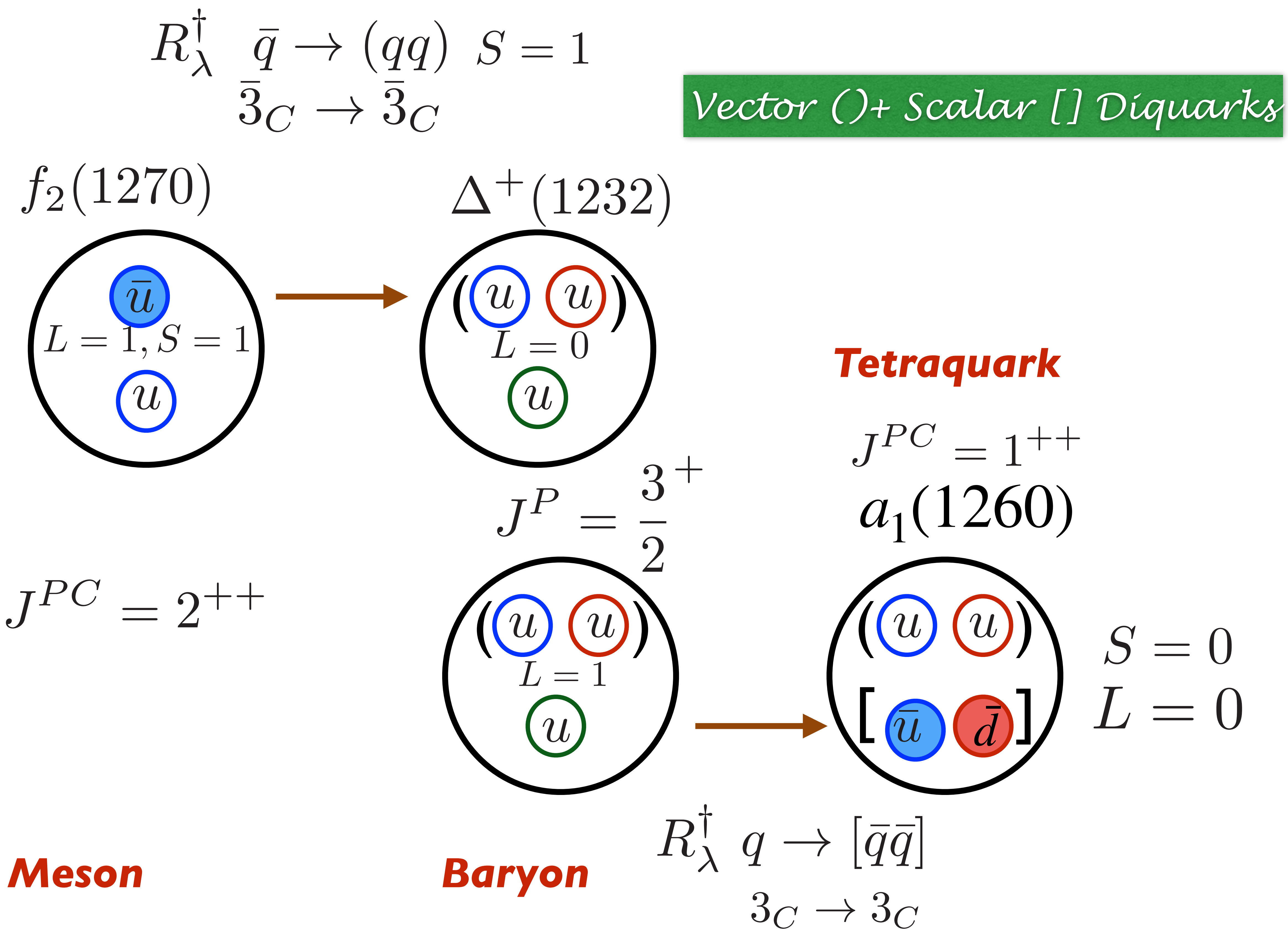
$R^\dagger_\lambda \ q \rightarrow [\bar{q}\bar{q}]$

$3_C \rightarrow 3_C$

$\psi_{B-}, \ L_B + 1$ $\phi_T, \ L_B$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Superconformal Algebra 4-Plet



Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar q q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar q q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar u \bar d]$	0^{++}	$f_0(980)$
$\bar q q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar u \bar d]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar q q$	1^{--}	$\rho(770)$, $\omega(780)$	—	—	—	—	—	—
$\bar q q$	2^{++}	$a_2(1320)$, $f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar u \bar d]$	1^{++}	$a_1(1260)$
$\bar q q$	3^{--}	$\rho_3(1690)$, $\omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar u \bar d]$	2^{--}	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar q q$	4^{++}	$a_4(2040)$, $f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar u \bar d]$	3^{++}	$a_3(\sim 2070)?$
$\bar q s$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar q s$	$1^{+(-)}$	$\bar K_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar s \bar q]$	$0^{+(+)}$	$K_0^*(1430)$
$\bar q s$	$2^{-(+)}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar s \bar q]$	$1^{-(+)}$	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar s q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar s q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar s \bar q]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar s q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar s q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar q \bar q]$	$1^{+(+)}$	$K_1(1400)$
$\bar s q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar q \bar q]$	$2^{-(-)}$	$K_2(\sim 1700)?$
$\bar s q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar q \bar q]$	$3^{+(+)}$	$K_3(\sim 2070)?$
$\bar s s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar s s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar s \bar q]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar s s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar s \bar q]$	1^{-+}	$\Phi'(1750)?$
$\bar s s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar s s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar s \bar q]$	1^{++}	$f_1(1420)$
$\bar s s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar s \bar q]$	2^{--}	$\Phi_2(\sim 1800)?$
$\bar s s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar s \bar q]$	$1^{+(+)}$	$K_1(\sim 1700)?$

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

Complete Regge spectrum in n, L

$$3_C \times 3_C = \bar{3}_C + 6_C$$

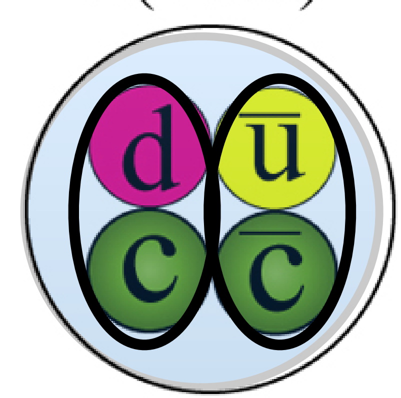
Bound!

- Diquark Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states
- Confinement Force Similar to quark-antiquark mesons $\bar{3}_C \times 3_C = 1_C$
- Isospin $I = 0, \pm 1, \pm 2$ Charge $Q = 0, \pm 1, \pm 2$

$uu\bar{d}\bar{d}$ $uu\bar{s}\bar{d}$ $uu\bar{s}\bar{s}$

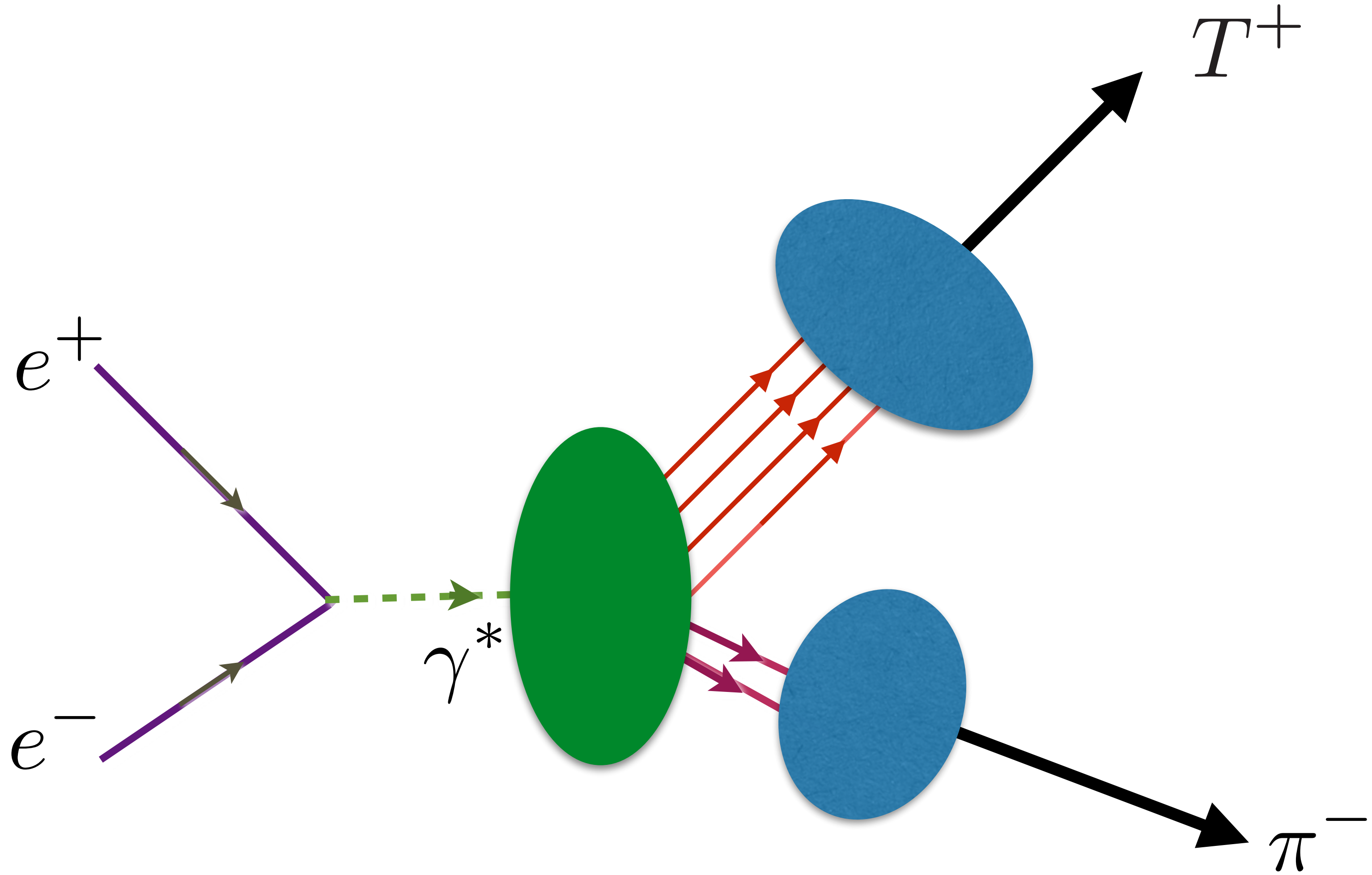
Q= +2

Z(4430)



Q= - 1

$$\sigma(e^+e^- \to MT) \propto \frac{1}{s^{N-1}} \quad N = 6$$

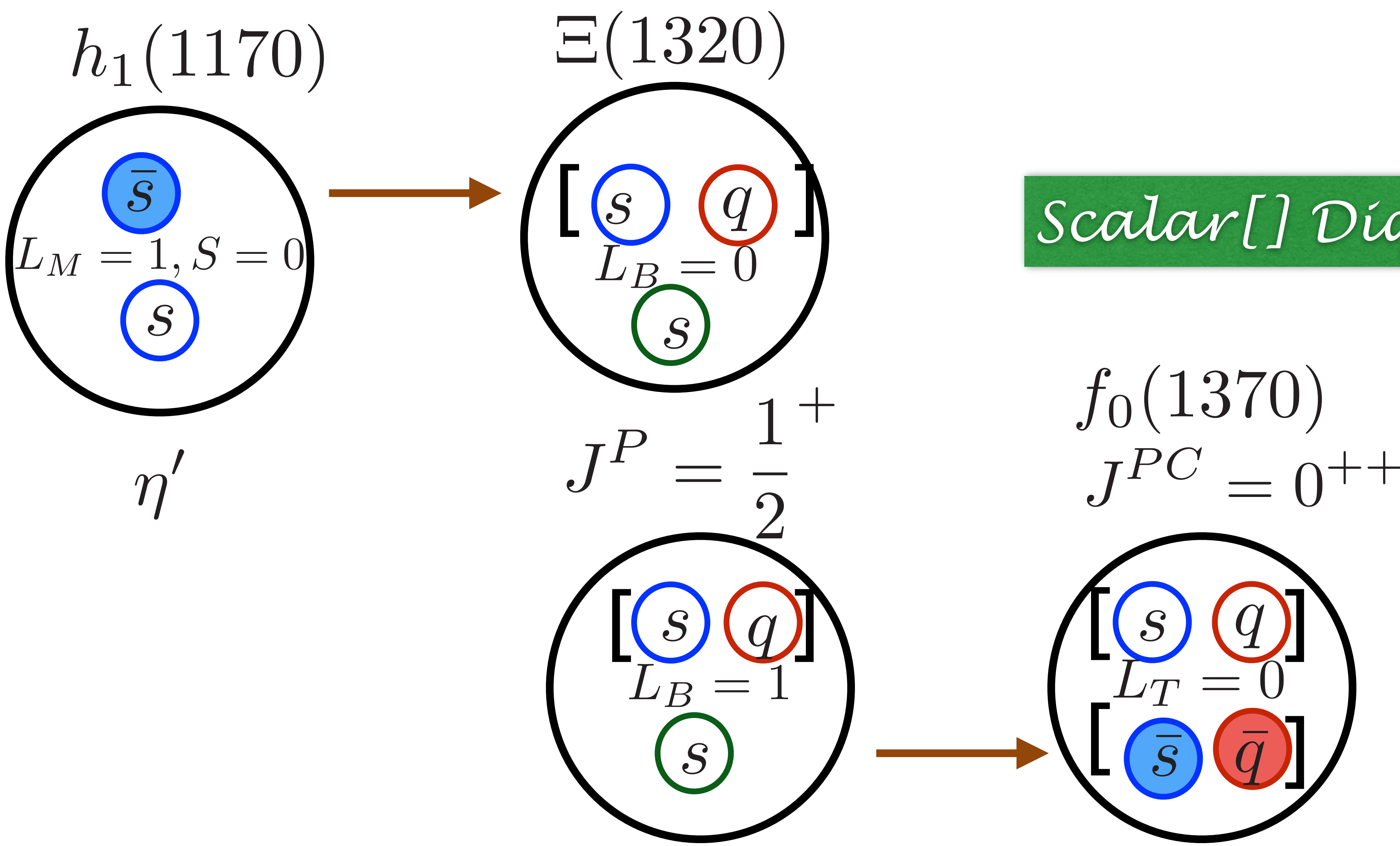


Use counting rules to identify composite structure

Lebed, sjb

Double-Strange Baryon

$$\overset{S=0}{R_\lambda^\dagger\ \bar{q} \to [qq]}$$
$$\bar{3}_C \to \bar{3}_C$$



$$R_\lambda^\dagger\ q \to [\bar{q}\bar{q}] \quad S = 0$$
$$3_C \to 3_C$$

Scalar[] Diquarks

Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

M. Nielsen, sjb

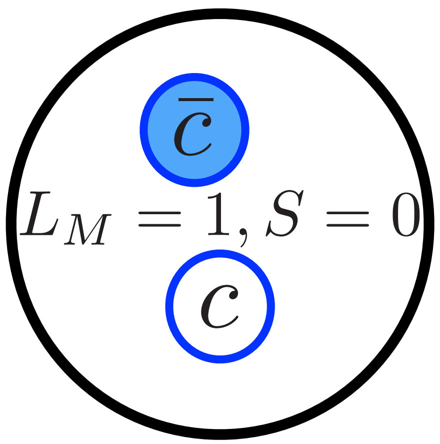
predictions

beautiful agreement!

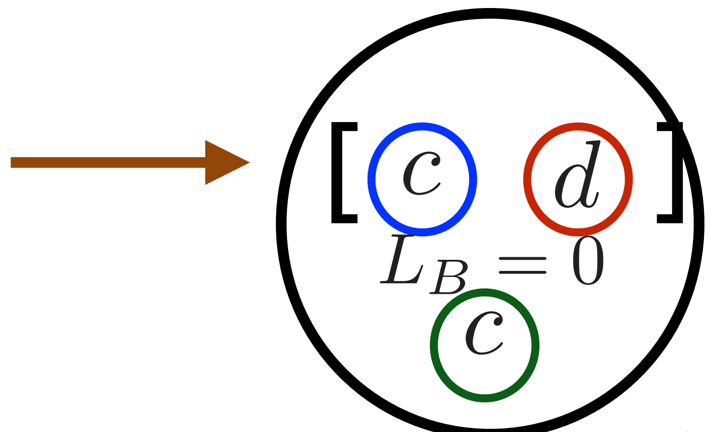
Double-Charm Baryon (SELEX)

$$S = 0$$
$$R_\lambda^\dagger \; \bar{q} \to [qq]$$
$$\bar{3}_C \to \bar{3}_C$$

$h_c(3525)$



$\Xi^+_{CC}(3520)$



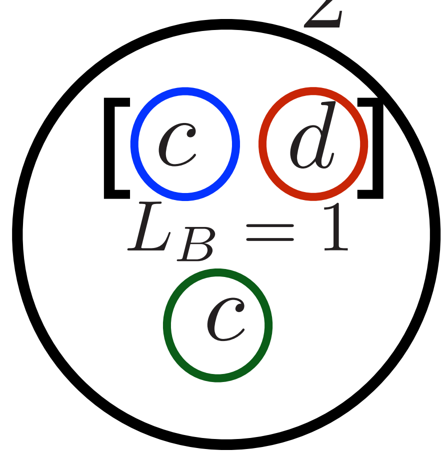
Predict Tetraquark $T_{c\bar{c}q\bar{q}}$
$M_T \sim 3520 \; MeV$

Scalar Diquarks

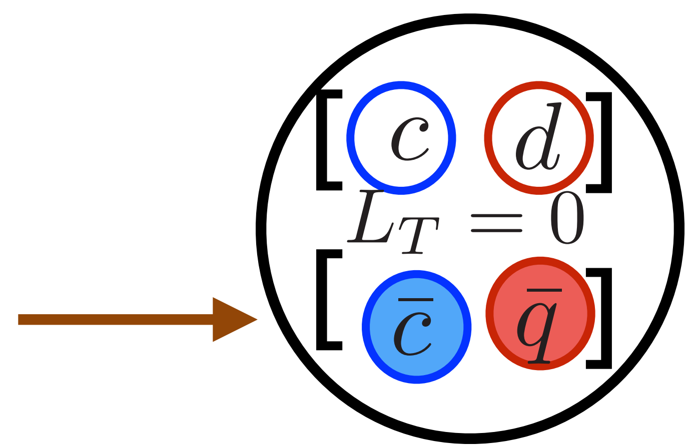
$$J^P = \frac{1}{2}^+$$

η_c'

$$J^{PC} = 1^{+-}$$



$$J^{PC} = 0^{++}$$



$$R_\lambda^\dagger \; q \to [\bar{q}\bar{q}] \qquad S = 0$$
$$3_C \to 3_C$$

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad\qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.}$$

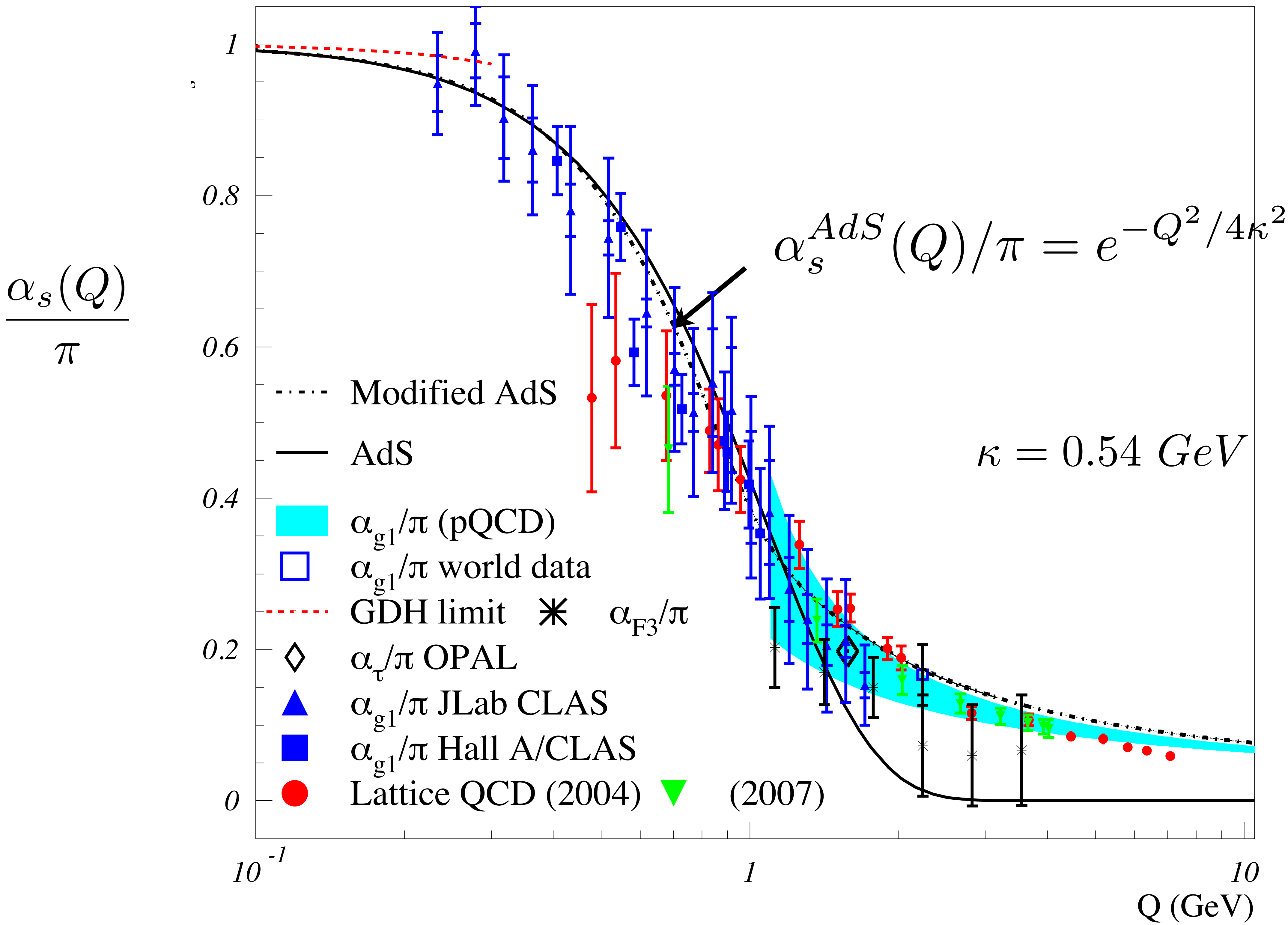
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

All-Scale QCD Coupling

Deur, de Tèramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:

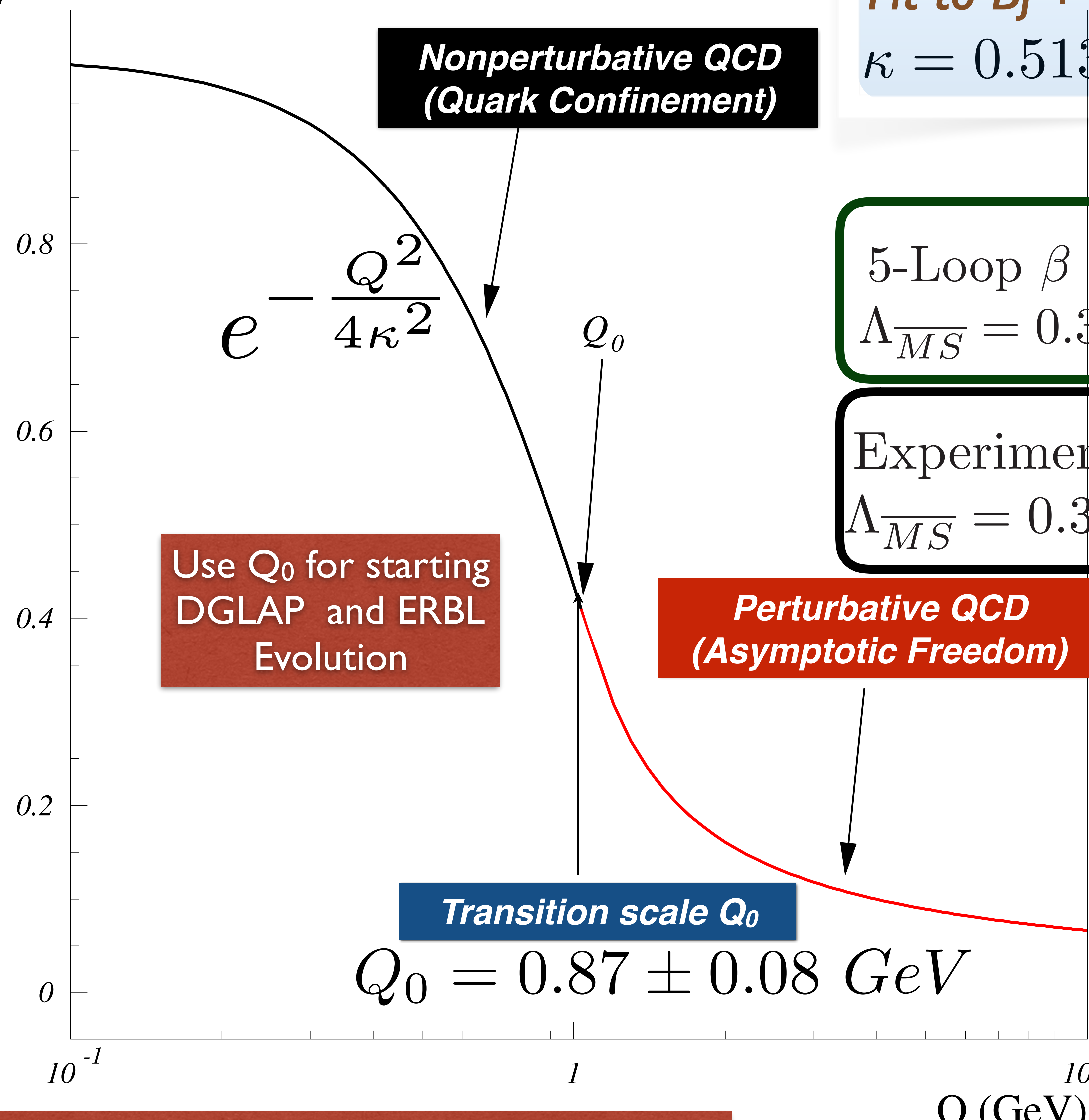
$$\kappa = 0.513 \pm 0.007\ GeV$$

5-Loop β Prediction:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$$

Experiment:

$$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$$



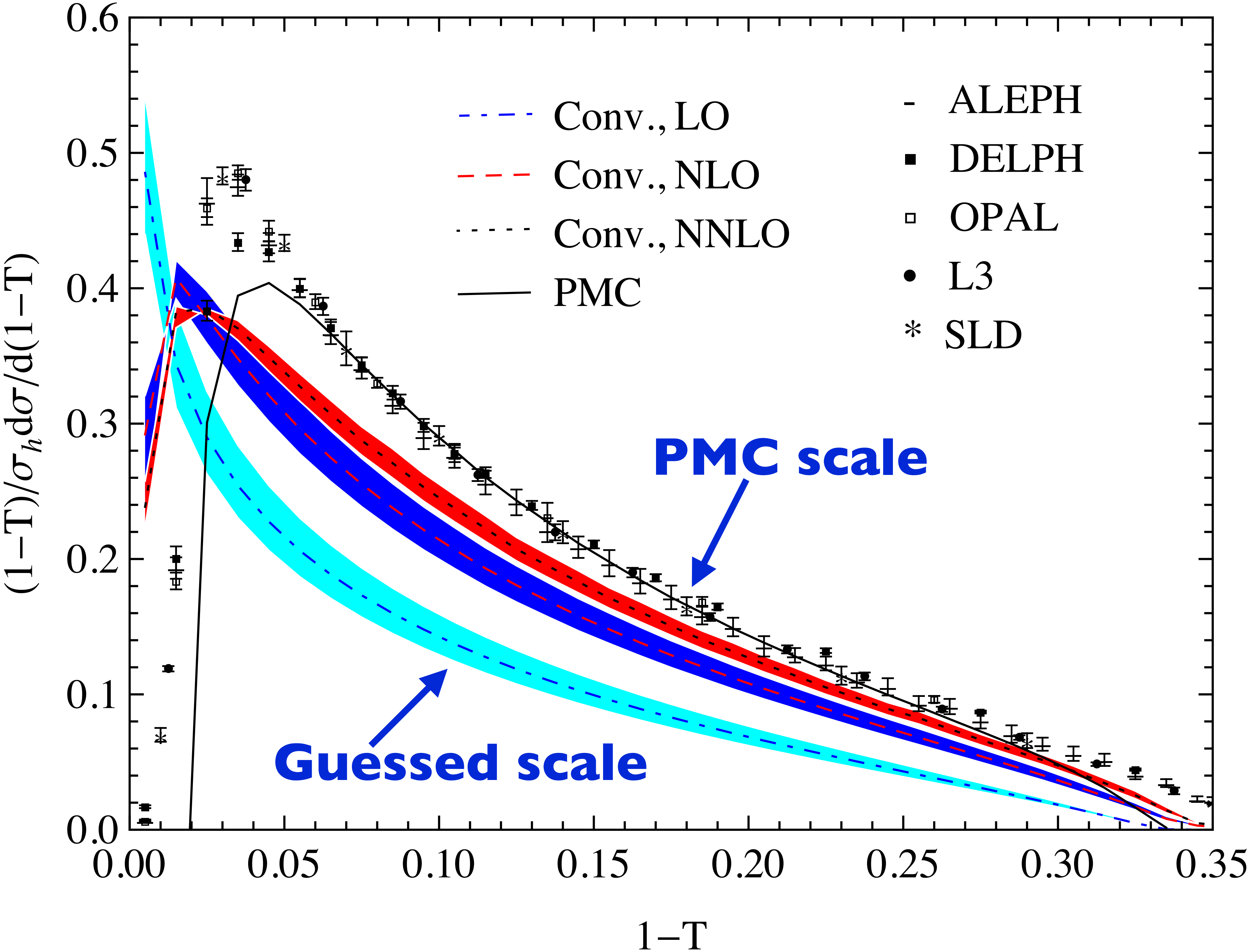
$$\lambda \equiv \kappa^2$$

$\overline{MS}$ *scheme*

Reverse Dimensional Transmutation!

T. Gehrmann, N. H¨afliger, P. F. Monni

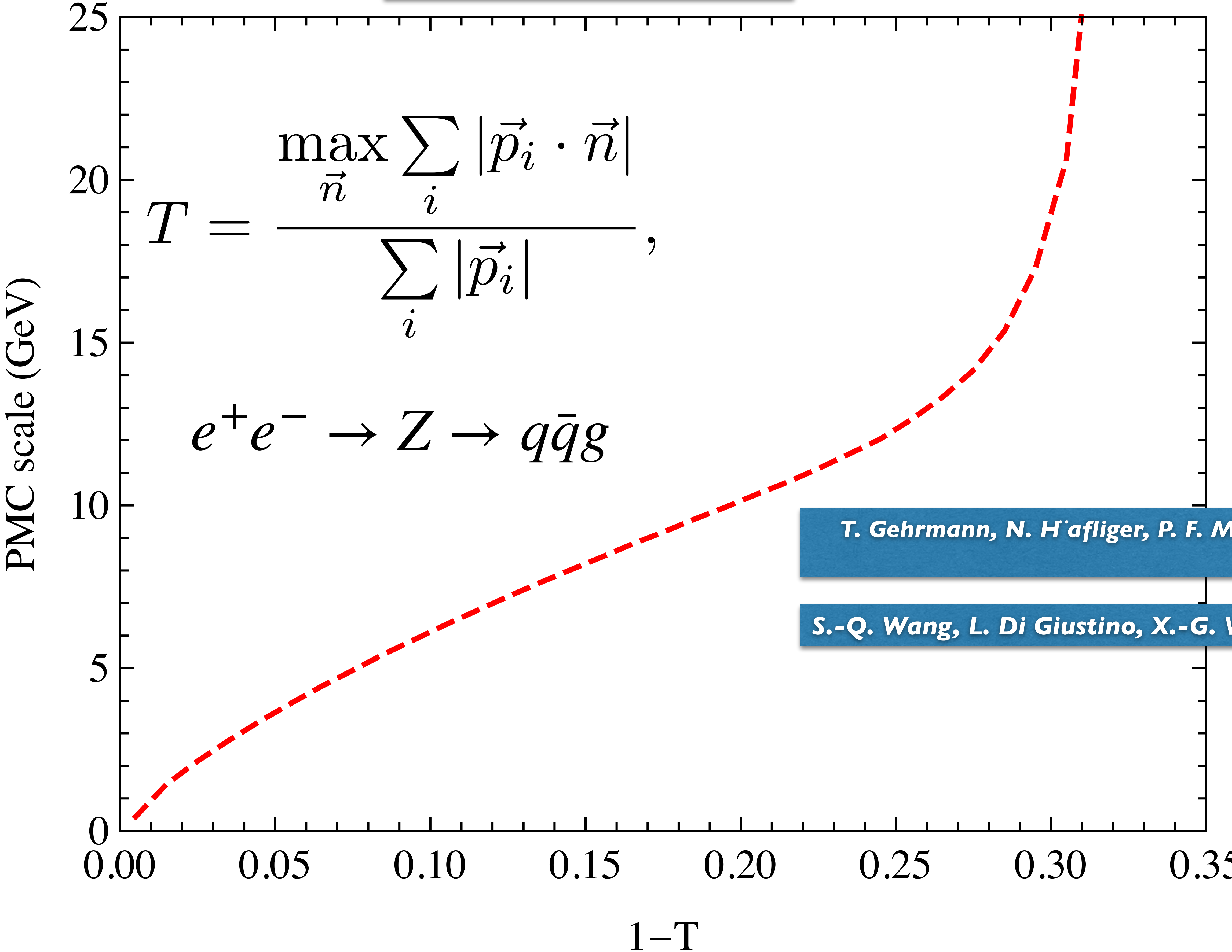
S.-Q. Wang, L. Di Giustino, X.-G. Wu, sjb



Principle of Maximum Conformality (PMC)

Renormalization scale depends on the thrust

Not constant



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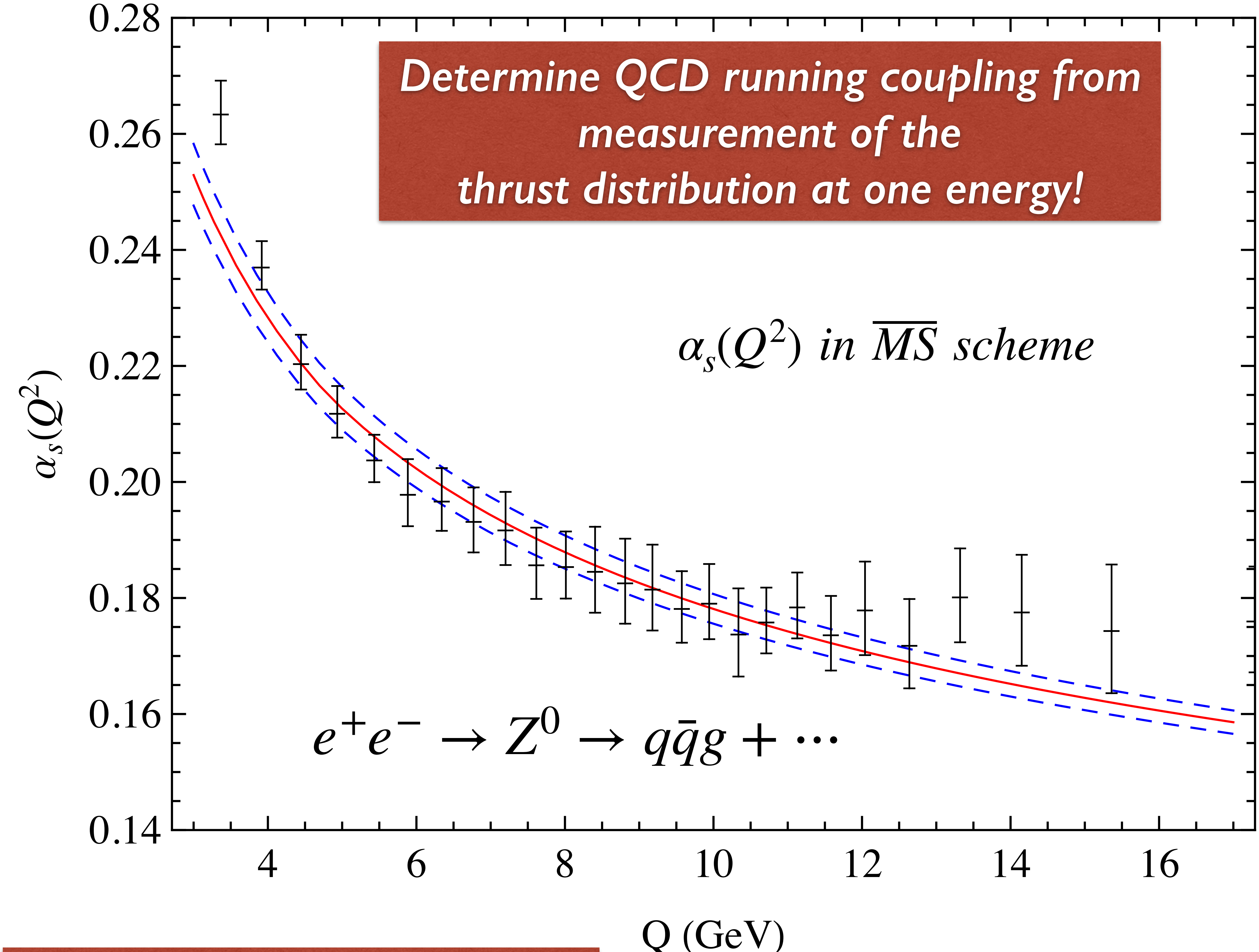

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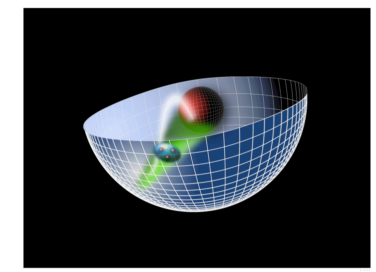
Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time τ*

- *Causality: Information within causal horizon: Light-Front*

- *Light-Front Holography: AdS₅ = LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$



- *Introduce Mass Scale κ while retaining the Conformal Invariance of the Action (dAFF)*

- *Unique Dilaton in AdS₅:* $e^{+\kappa^2 z^2}$

- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$

- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\boxed{\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]}$$

Light-Front Holography: First Approximation to QCD

- Color Confinement, Analytic form of confinement potential
- Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)
- Massless quark-antiquark pion bound state in chiral limit, GMOR
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincarè Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- OPE: Constituent Counting Rules
- Hadronization at the Amplitude Level: Many Phenomenological Tests
- Systematically improvable: Basis LF Quantization (BLFQ)

Invariance Principles of Quantum Field Theory

- *PoIncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*

- *Causality: Information within causal horizon: Front Form*

- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*

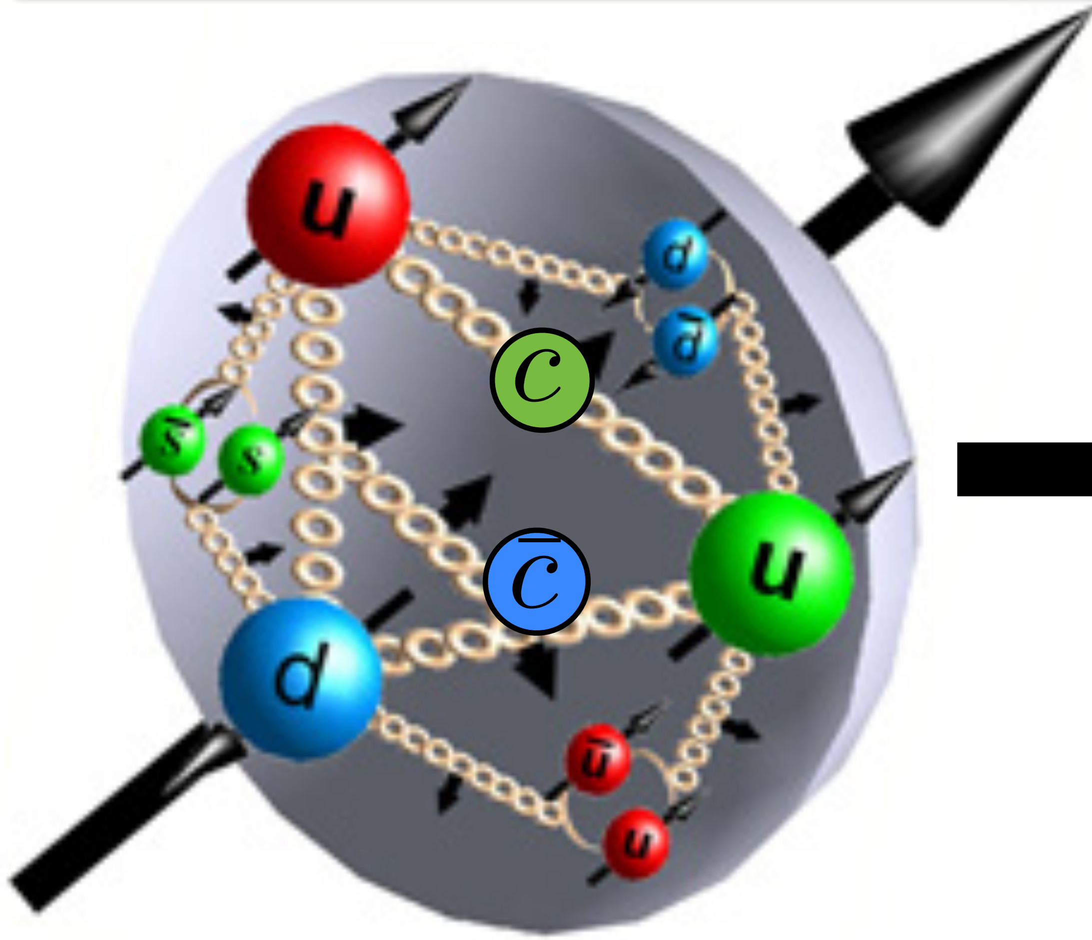
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)*

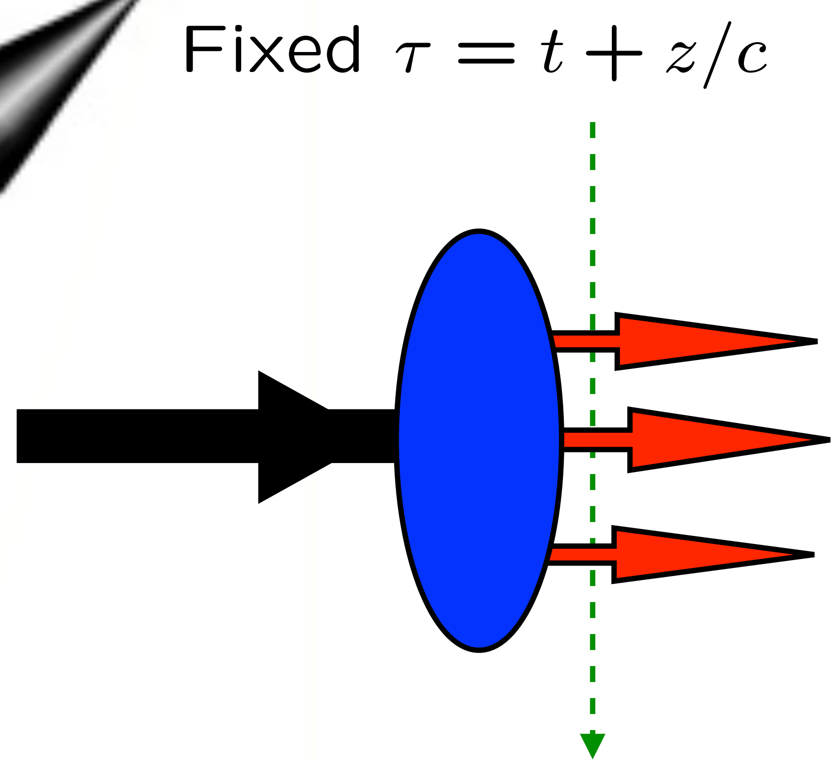
- *Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)*

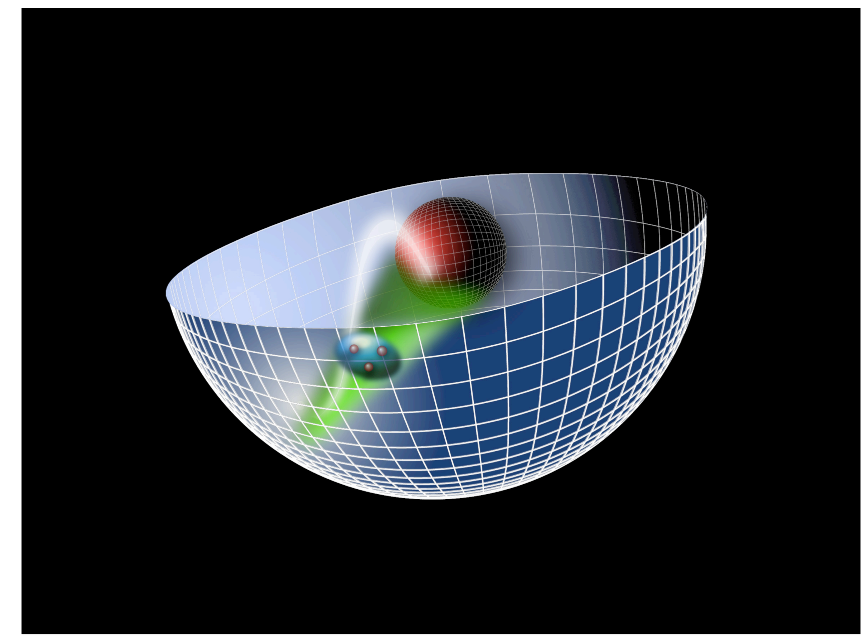
Manifestation of QCD symmetries

QCD symmetries	LFHQ
Lagrangian QCD conformal invariant	Action conformal QM invariant under conf transf $\rightarrow$ LF potential is harmonic oscillator
$SU(3)_C$ local invariance $\rightarrow$ $\overline{3} \sim 3 \times 3$	Connection between mesons, baryons and tetraquarks Emerging hadronic SUSY (Hadronic SUSY introduced by Miyazawa 1967)
$SU(3)_C$ and conformal invariant Lagrangian.	SUSY + Conformal = Superconformal Fixes uniquely the interaction and modification of AdS
$SU(2)_A$ spontaneously broken $\rightarrow I = 1$ pseudoscalar Goldstone boson $\equiv$ pion	Goldstone boson not required: zero mass bound state protected by superconformal algebra
$U(1)_A$ <u>not</u> spontaneosuly broken (no massless I = 0 particle)	Nonperturbative hard breaking encoded by additional term in LF Hamiltonian $\sim \lambda$ for isoscalar sector

Supersymmetric Features of Hadron Physics and other Novel Properties of Quantum Chromodynamics from Light-Front Holography and Superconformal Algebra







with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen,
F. Navarra, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur

Stan Brodsky

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Thursday, 11 April 2019