Dynamical Nucleon-pion System via Basis Light-front Quantization Approach

Weijie Du, Xingbo Zhao, James P. Vary

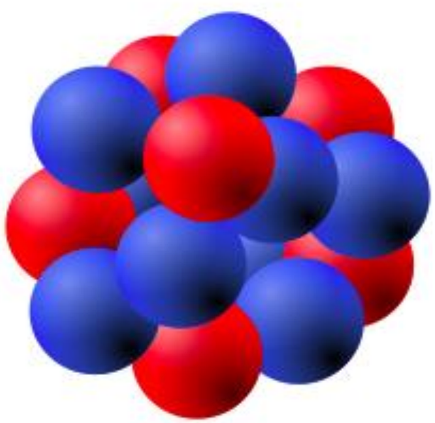
Iowa State University

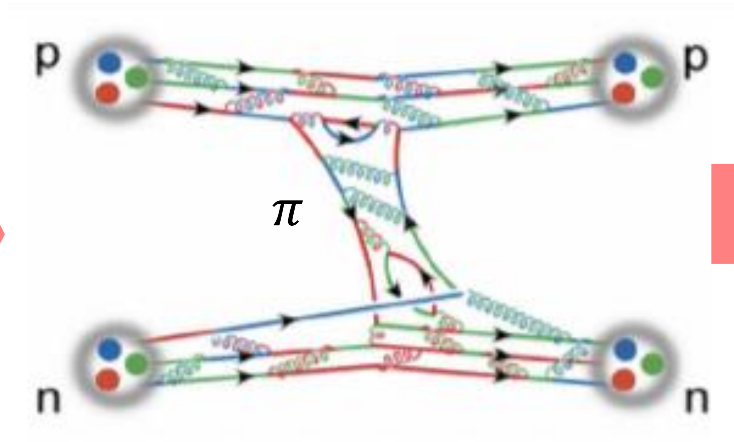
Institute of Modern Physics
Chinese Academy of Sciences
Lanzhou, China

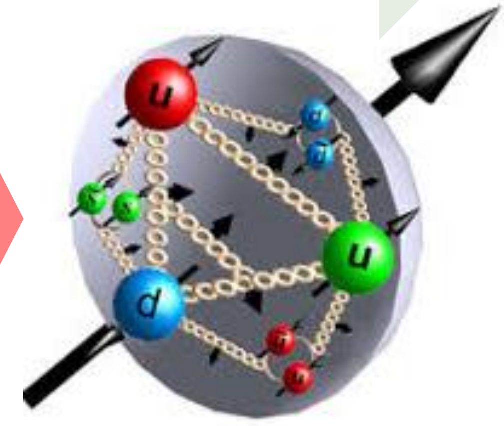
GHP workshop, Apr. 10, 2019

Nuclear physics in the medium energy is challenging

Low energy | High energy







Nucleons & mesons

Challenge-ons

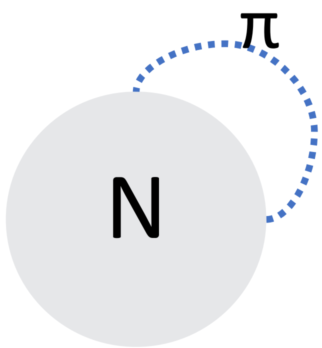
Quarks & gluons

A first question:

What does the proton look like in the medium energy?

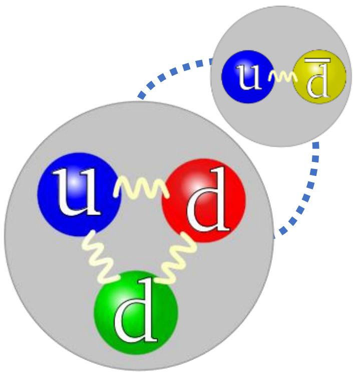
The idea: cloudy-bag model

$$|\text{proton}\rangle = a|N\rangle + b|N\pi\rangle + \text{c}|N\pi\pi\rangle + d|N\bar{N}N\rangle + \ldots.$$



Step: 1

Solve the proton wave function in terms of the nucleon and pion



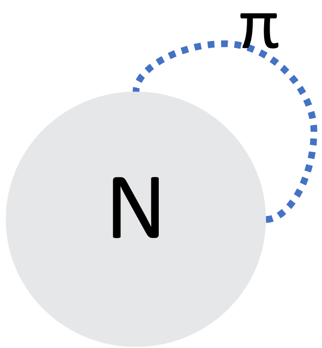
Step: 2

Attach the fundamental degree of freedom: quarks and gluons

$$|\text{proton}\rangle = a|uud\rangle + b|uudg\rangle + \text{c}|uudgg\rangle + d|uudq\bar{q}\rangle + \ldots.$$

Current progress: step 1

$$|\text{proton}\rangle = a|N\rangle + b|N\pi\rangle + \text{c}|N\pi\pi\rangle + d|N\bar{N}N\rangle + \ldots.$$

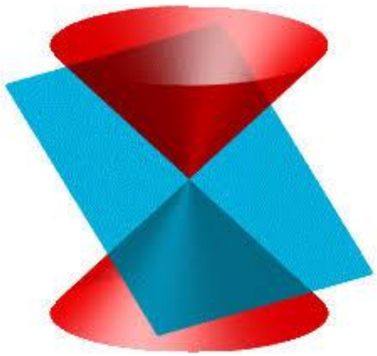


Step: 1

Solve the proton wave function in terms of the nucleon and pion

Developed a non-perturbative, fully relativistic treatment of a chiral nucleon-pion model on the light-front

Why light-front?



✓Wave functions are boost invariant.

✓Vacuum structure is simple (except the zero-mode).

  ✓Fock-sector expansion is convenient.

✓Observables are taken at fixed light-front time (convenient).

Methodology: Basis Light-front Quantization

[Vary *et al.*, 2008]

- Relativistic eigenvalue problem for light-front Hamiltonian

$$P^{-}|b\rangle = P^{-}_{b}|b\rangle$$

Non-perturbative

Fully relativistic

- P^{-} : light-front Hamiltonian
- $|\beta\rangle$: eigenvector → light-front wave function
  - $|\beta\rangle$ boost invariant
  - $|\beta\rangle$ encodes the hadronic properties
- P^{-}_{β} : eigenvalue → hadron mass spectrum

- Observables

$$O \equiv \langle\beta'|\hat{O}|\beta\rangle$$

Starting point: chiral model of nucleon and pion

Relativistic $N\pi$ chiral Lagrangian density

$$\mathcal{L} = \underbrace{\frac{1}{4} f^2 \mathrm{Tr}\Big(\partial_\mu U \; \partial^\mu U^\dagger\Big) + \frac{1}{4} M_\pi^2 f^2 \mathrm{Tr}\Big(U + U^\dagger - 2\Big)}_{\text{pion field}}$$

$$+ \underbrace{\bar{N}\Big\{\gamma_\mu i\partial^\mu - M_N + \frac{1}{1+(\frac{\pi}{2f})^2}\Big[\frac{1}{2f}\gamma_\mu\gamma_5 \vec{\tau}\cdot\partial^\mu\vec{\pi} - (\frac{1}{2f})^2 \gamma_\mu \vec{\tau}\cdot\vec{\pi}\times\partial^\mu\vec{\pi}\Big]\Big\}N}_{\text{N}\pi\text{ interaction}}$$

$$U = \frac{1 + i\gamma_5 \vec{\tau}\cdot\vec{\pi}/(2f)}{1 - i\gamma_5 \vec{\tau}\cdot\vec{\pi}/(2f)} = 1 + i\gamma_5 \frac{\vec{\tau}\cdot\vec{\pi}}{f} - \frac{1}{2f^2}\pi^2 + O\Big(\frac{1}{f^3}\Big)$$

f = 93 MeV: pion decay constant
M_π = 137 MeV: pion mass
M_N = 938 MeV: nucleon mass

[Miller, 1997]

Basis construction

1. Fock-space expansion:

$$|\text{proton}\rangle = a|N\rangle + b|N\pi\rangle + \mathrm{c}|N\pi\pi\rangle + d|N\bar{N}N\rangle + \ldots.$$

2. For each Fock particle:

Transverse: 2D harmonic oscillator basis: $\Phi_{n,m}^{b}(\vec{p}_\perp)$

with radial (angular) quantum number *n* (*m*); scale parameter *b*

Longitudinal: plane-wave basis, labeled by *k*

- Helicity: labeled by λ
- Isospin: labeled by t

3. E.g.,

$$|N\pi\rangle = |n^N, m^N, k^N, \lambda^N, t^N, n^\pi, m^\pi, k^\pi, \lambda^\pi, t^\pi\rangle$$

Basis truncation

Symmetries and conserved quantities:

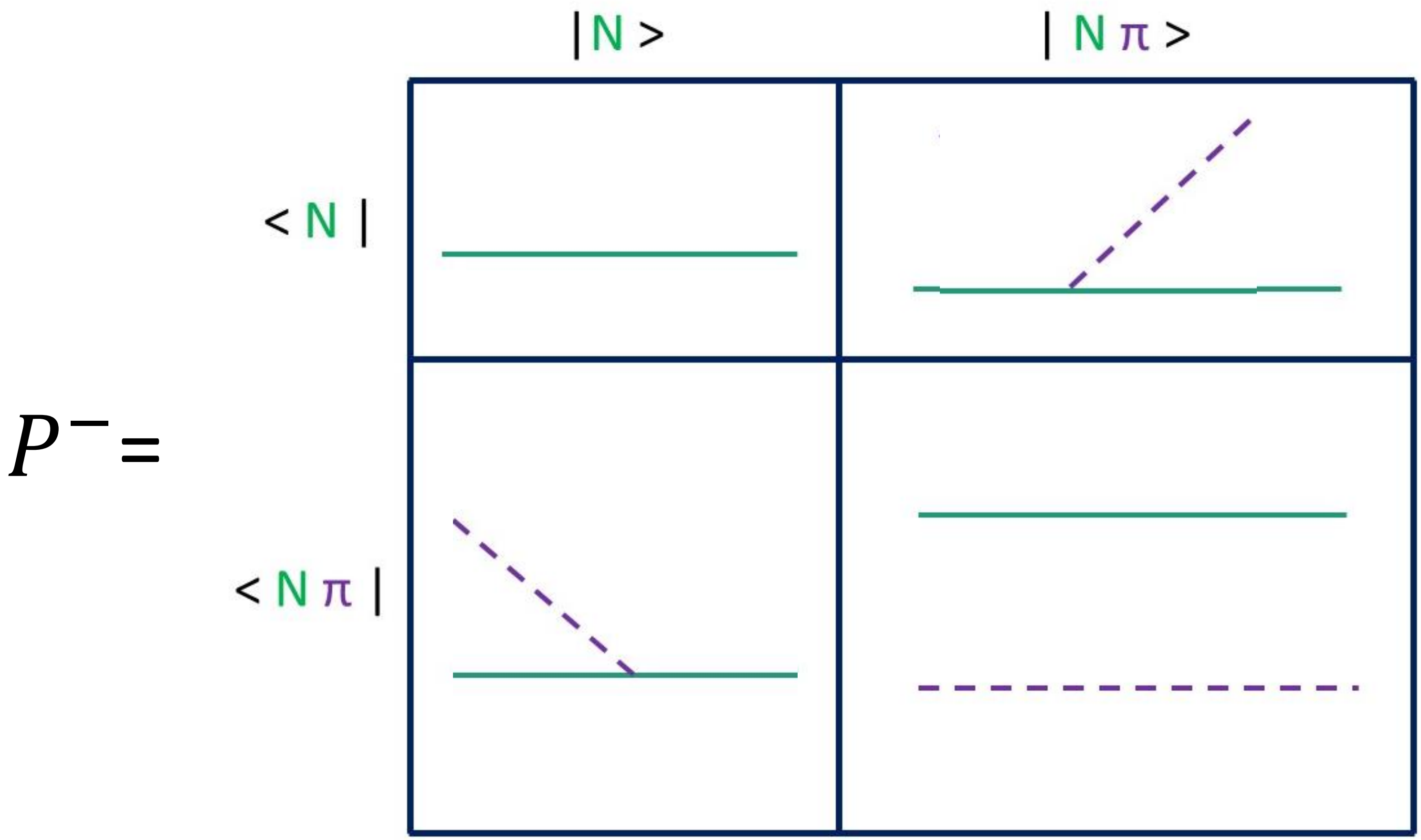
- Longitudinal momentum: $\sum_i k_i = K$
- Total angular momentum projection: $\sum_i (m_i + \lambda_i) = J_z$
- Total isospin projection: $\sum_i t_i = T_z$

Truncations:

- Fock-sector truncation $|\mathrm{p}\rangle = a|N\rangle + b|N\pi\rangle$
- "$K_{\max}$" truncation in longitudinal direction $K = K_{max}$
- "$N_{\max}$" truncation in transverse direction $\sum_i \left[2n_i + |m_i| + 1\right] \le N_{\max}$

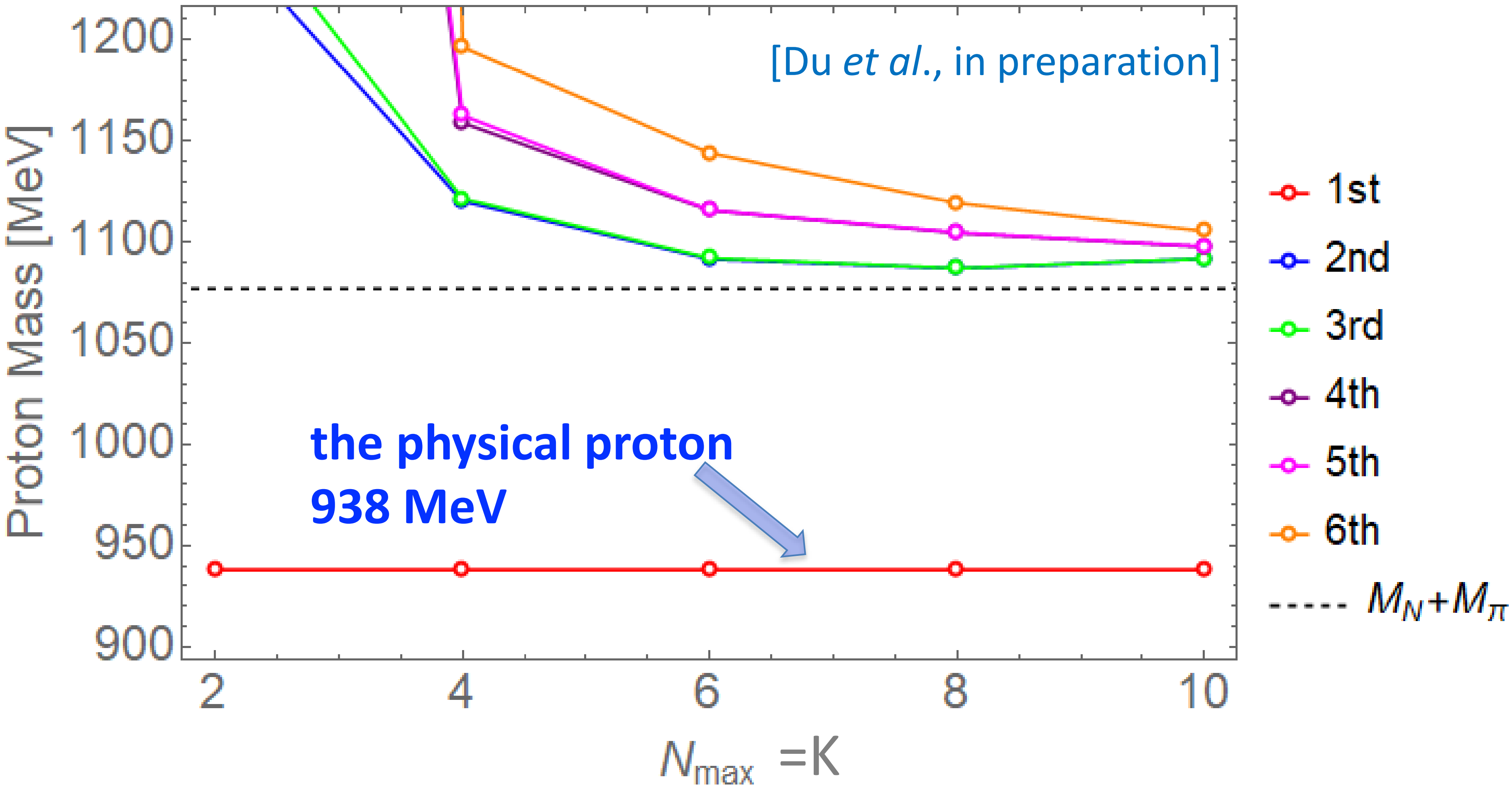
UV cutoff $\Lambda \sim b\sqrt{N_{\max}}$ IR cutoff $\lambda \sim b/\sqrt{N_{\max}}$

Light-front Hamiltonian in the basis representation



- Legendre transformation to obtain P^{-}
- Only one-pion processes: up to the order of $1/f$
- Eigenvalue problem of relativistic bound state

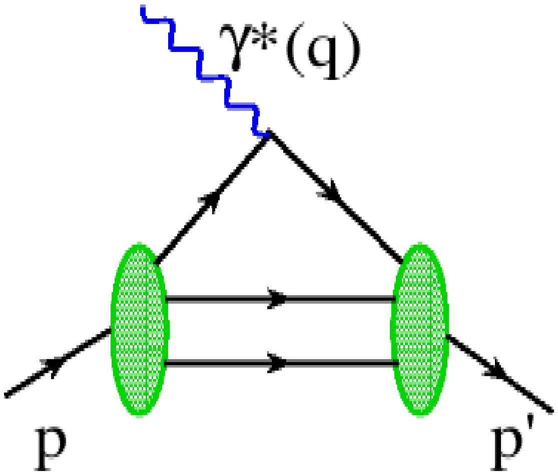
Mass spectrum of the proton system



- Fock sector-dependent renormalization applied
- Mass counter-term applied to $|N\rangle$ sector only

[Karmanov et al, 2008, 2012]

Observable: proton's Dirac form factor

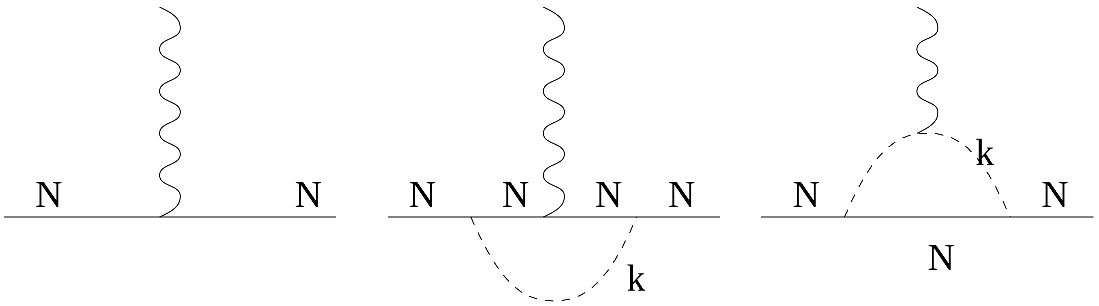


$$\langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;\Lambda\rangle = F_1(q^2)$$

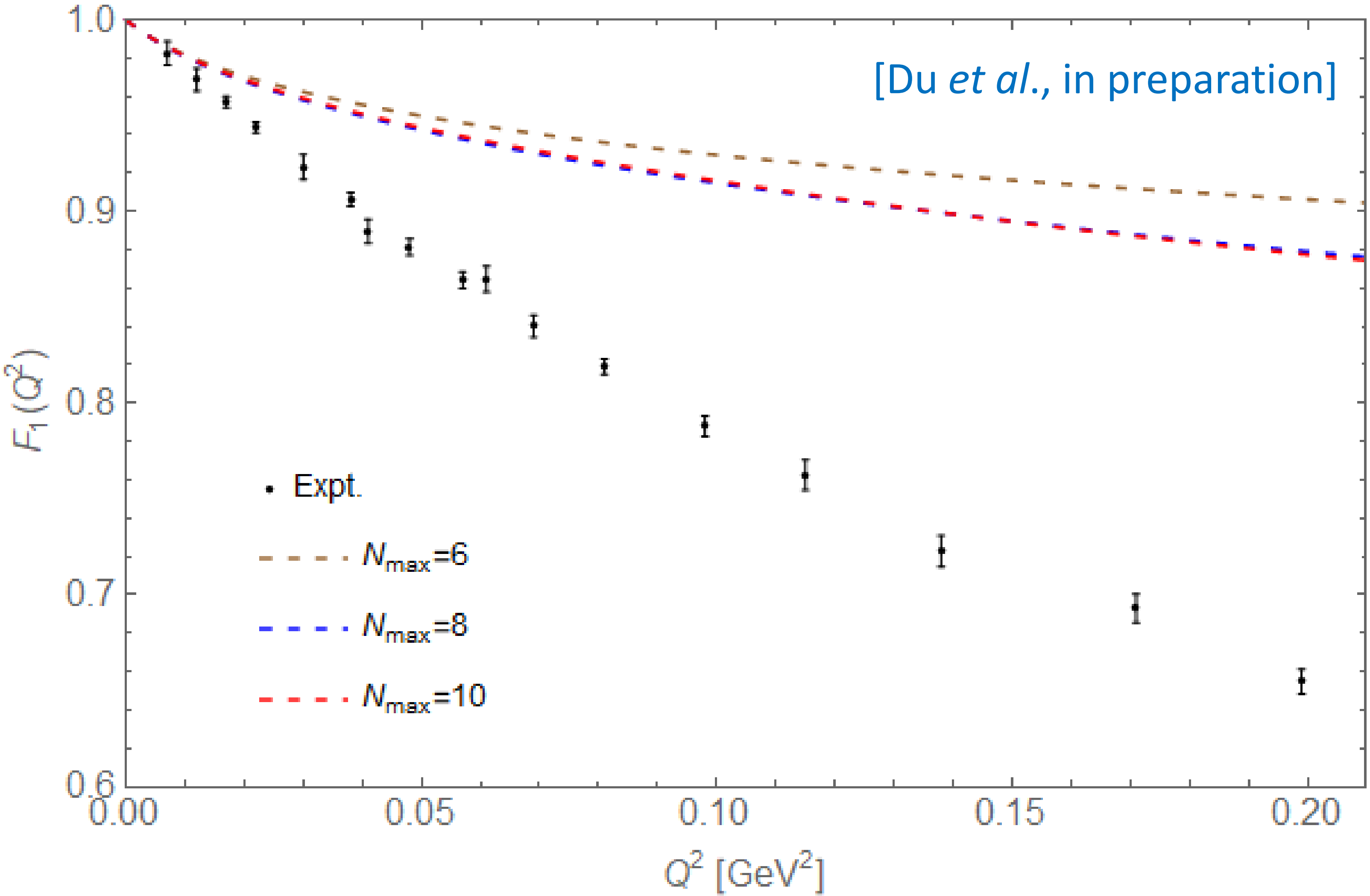
$$Q^2 = -q^2$$

$$F_1(q^2) = F^p_{1,f}(q^2) + F^{p\pi^0}_{1,f}(q^2) + F^{n\pi^+}_{1,b}(q^2)$$

Schematically:

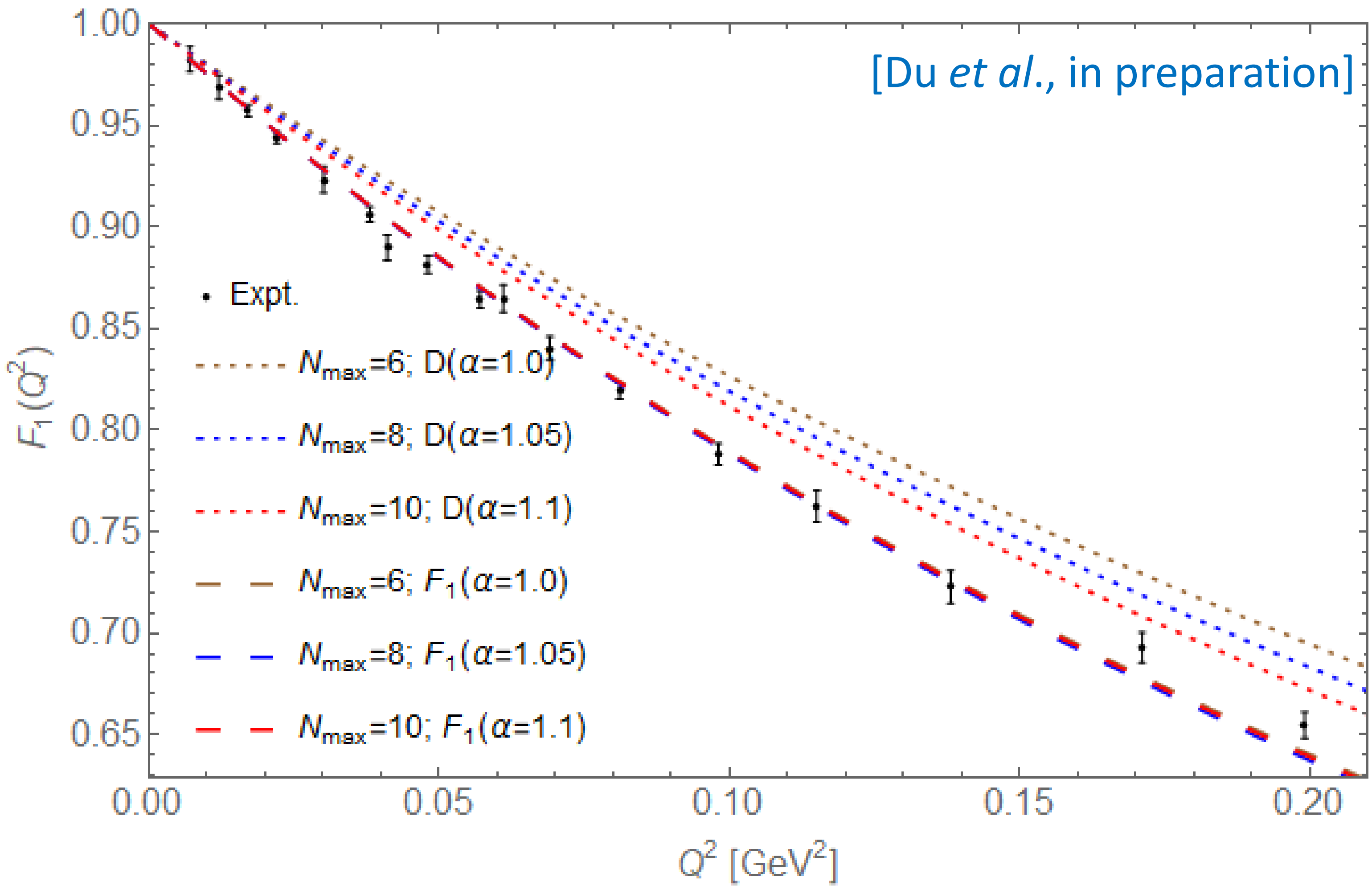


Proton’s Dirac form factor



- Constituent nucleon and pion are treated as point particles
- Higher Fock-sectors could be important

Proton's Dirac form factor



$$F_1(\alpha_1, \alpha_2, q^2) = D_1(\alpha_1, q^2)\left[F^{p}_{1,f}(q^2) + F^{p\pi^0}_{1,f}(q^2)\right] + D_2(\alpha_2, q^2) F^{n\pi^+}_{1,b}(q^2)$$

Dipole form for constituents' internal structures

$$D_1(\alpha_1, q^2) = D_2(\alpha_2, q^2) \equiv D(\alpha, q^2) = \frac{1}{(1+\alpha q^2)^2}$$

Summary and outlook

- Chiral model of nucleon-pion is treated by Basis Light-front Quantization method:
  - fully relativistic, non-perturbative treatment
  - close connection with observables of hadron structure
- Initial calculations are performed & results (mass spectrum, Dirac form factor) are promising

---

- Convergence study
- Higher order interactions in the chiral Lagrangian
- More observables: GPD, TMD, GTMD…
- Inclusion of quarks and gluons and comparison with parton picture
- Pion cloud for studying light flavor sea-quark asymmetry

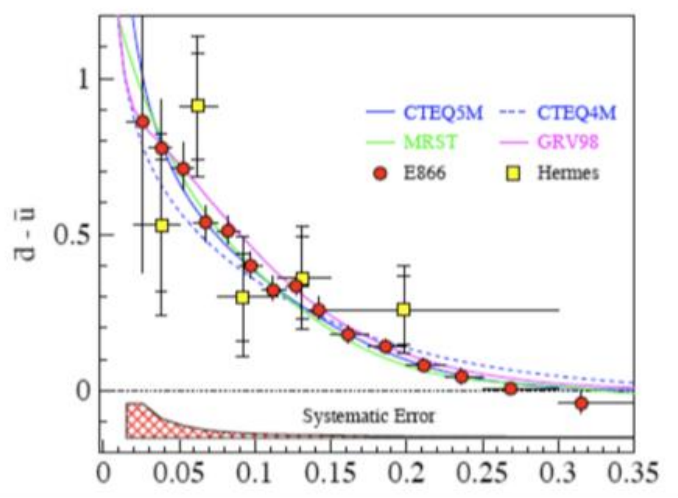
Thank you!

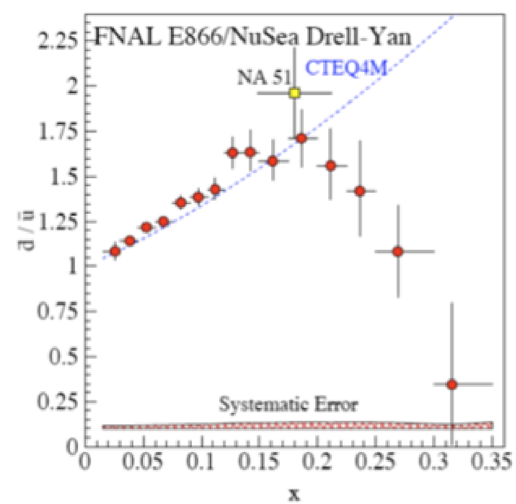
Backups

Cloudy-bag model for sea quark asymmetry

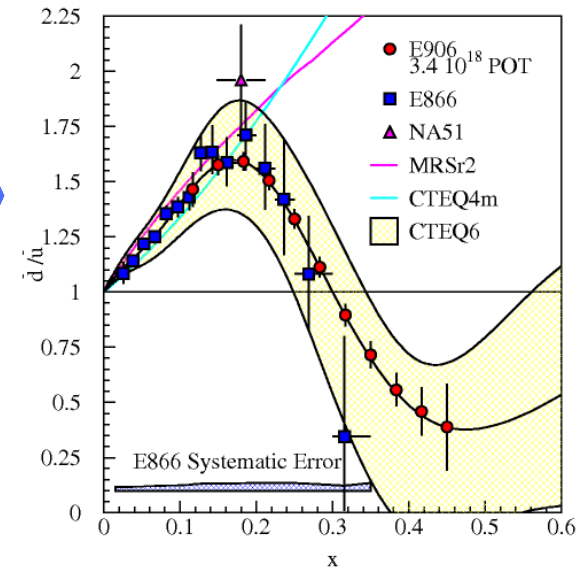
[Theberge, Thomas and Miller, 1980]

E866 Drell-Yan Experiment





E906 proposal



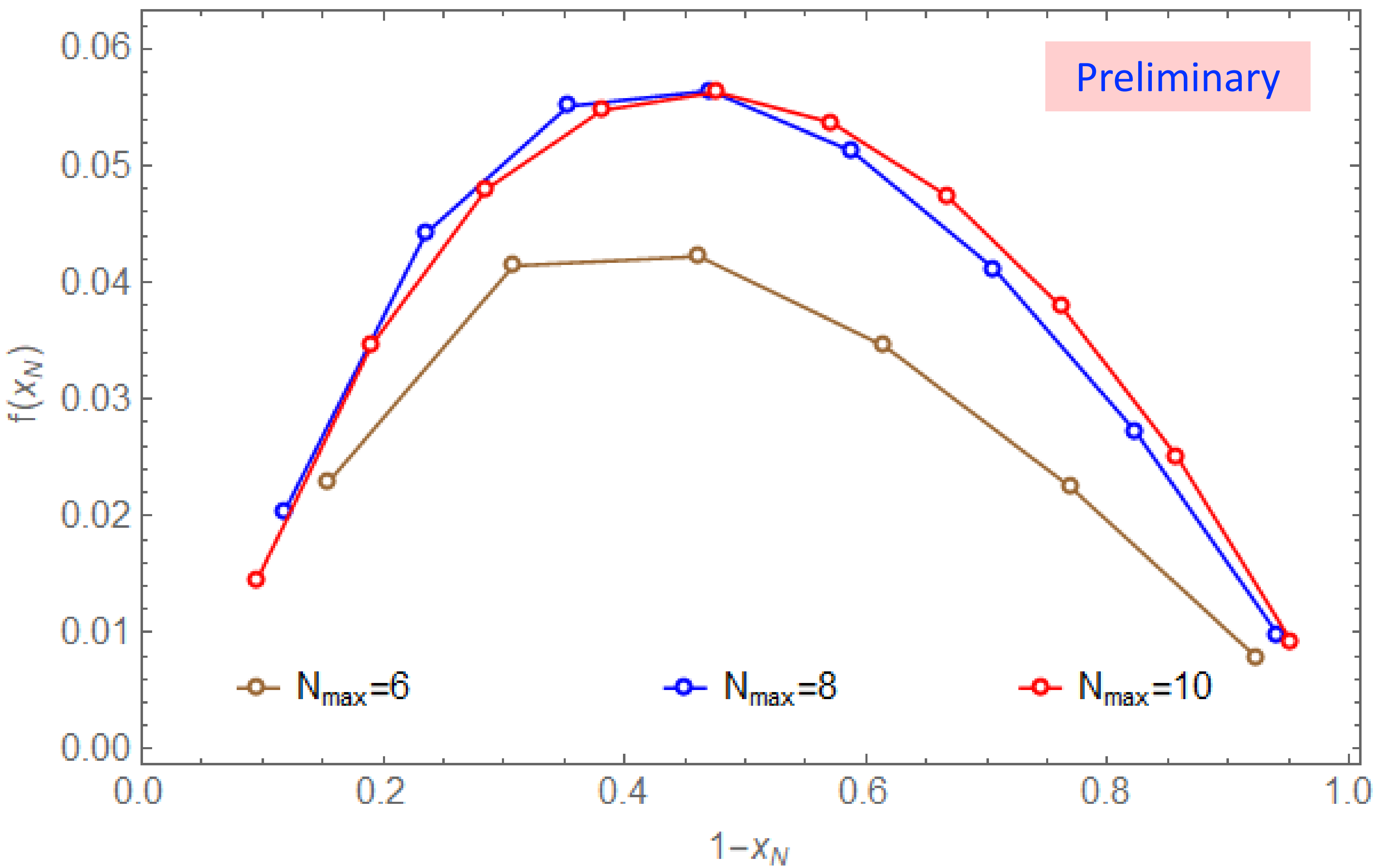
Adapted from Alberg and Miller, APS April Meeting 2018

- Light flavor sea quark asymmetry in the proton suggests pion cloud

$$p \to n\pi^-, p \to p\pi^0$$

- We can use our treatment and experimental constraints on the parameters of a chiral Lagrangian to make predictions

Parton distribution function



Chiral rotation of nucleon field

Introduce χ such that $N = U^{1/2}\chi$ [Miller, 1997]

$$\mathcal{L} = \underbrace{\frac{1}{4}f^2\mathrm{Tr}\Big(\partial_\mu U\ \partial^\mu U^\dagger\Big) + \frac{1}{4}M_\pi^2 f^2 \mathrm{Tr}\Big(U + U^\dagger - 2\Big)}_{\text{pion field}}$$

$$+ \underbrace{\bar{\chi}\Big\{\gamma_\mu i\partial^\mu - M_N - M_N(U-1)\Big\}\chi}_{\text{N}\pi\text{ interaction}}$$

Constraint equation for nucleon field χ_-

$$\chi_- = \frac{1}{p^+}\gamma^0\Big[\gamma^\perp \cdot p^\perp + M_N U\Big]\chi_+$$

$$\begin{cases} \chi_+ = \Lambda_+ \chi \\ \chi_- = \Lambda_- \chi \end{cases}$$

Procedure

1. Derive light-front Hamiltonian P^{-} from Lagrangian
2. Construct basis states $|\alpha\rangle$
3. Calculate matrix elements of P^{-} : $\langle\alpha'|P^{-}|\alpha\rangle$
4. Diagonalize P^{-} to obtain its eigen-energies and LFWFs
5. Renormalization – iteratively fix bare parameters in P^{-}
6. Evaluate observables: $O \equiv \langle\beta'\big|\hat{O}\big|\beta\rangle$

Common variables in light-front dynamics

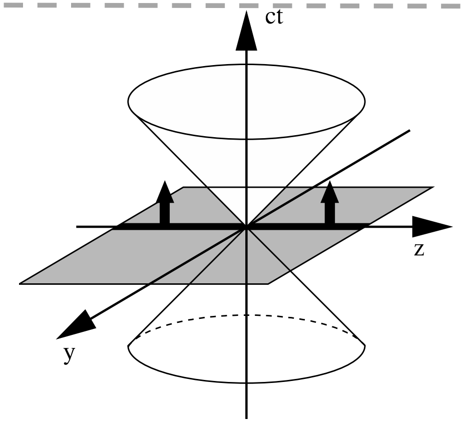
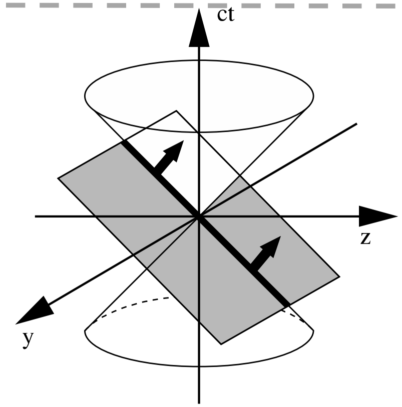
- Light-front time $x^+ = x^0 + x^3$
- Light-front Hamiltonian $P^- = P^0 - P^3$
- Longitudinal coordinate $x^- = x^0 - x^3$
- Longitudinal momentum $P^+ = P^0 + P^3$
- Transverse coordinate $x^\wedge = x^{1,2}$
- Transverse momentum $P^\wedge = P^{1,2}$

Dispersion relation

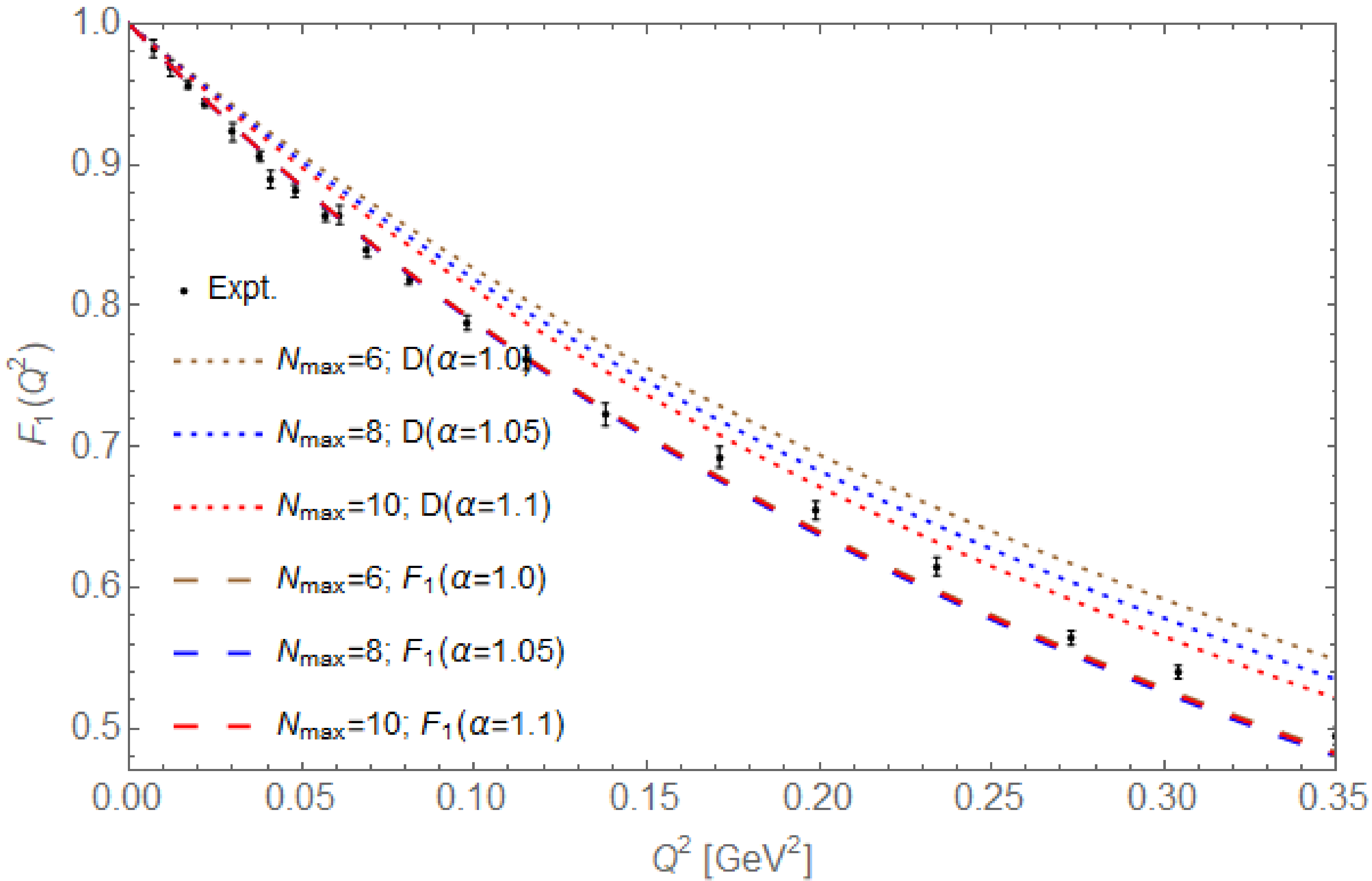
$$P^- = \frac{m^2 + P_\wedge^2}{P^+}$$

Light-front vs equal-time quantization

[Dirac 1949]

equal-time dynamics	V.S.	light-front dynamics
$t \equiv x^0$		$t \equiv x^+ = x^0 + x^3$
		
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$		$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$H = P^0$		$P^- = P^0 - P^3$
$\vec{P}, \vec{J}$		$\vec{P}^{\perp}, P^+, \vec{E}^{\perp}, E^+, J_z$

Proton's Dirac form factor



For larger Q^2, our prediction is consistently smaller than the experimental results.

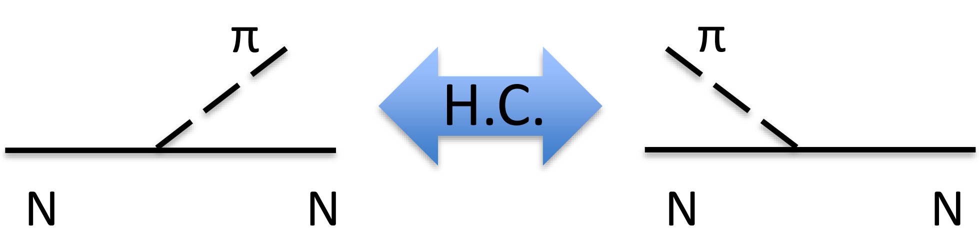
Light-front Hamiltonian density

Legendre transformation

Expand U to the order of $1/f$

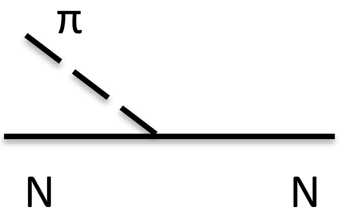
$$\mathcal{P}^- = \underbrace{\frac{1}{2}\partial^\perp\pi_a\cdot\partial^\perp\pi_a + \frac{1}{2}M_\pi^2\pi_a\pi_a + \chi_+^\dagger\frac{(p^\perp)^2+M_N^2}{p^+}\chi_+}_{\text{kinetic energy for free pion and nucleon}}$$

$$+\underbrace{\chi_+^\dagger\Big[-\gamma^\perp\cdot i\partial^\perp + M_N\Big]\frac{1}{p^+}M_N\Big[i\gamma_5\frac{\vec{\tau}\cdot\vec{\pi}}{f}\Big]\chi_+ + \chi_+^\dagger M_N\Big[-i\gamma_5\frac{\vec{\tau}\cdot\vec{\pi}}{f}\Big]\frac{1}{p^+}\Big[\gamma^\perp\cdot i\partial^\perp + M_N\Big]\chi_+}_{\text{one-pion emission and absorption}}$$



Light-front Hamiltonian in basis representation

$$P^-_{\text{int}} = \quad P^-_{\text{int;abs}} + P^-_{\text{int;em}}$$



$$P^-_{\text{int;abs}} = i\frac{M}{f}\sum_{p_1^+}\sum_{p_2^+}\sum_{k^+}\frac{1}{2\pi\sqrt{2Lk^+}}\delta(p_1^+|k^+ + p_2^+)$$

$$\sum_{s_1,s_2}\sum_{t_1,t_2}\sum_{\lambda}\int\frac{d^2p_1^\perp}{\sqrt{(2\pi)^2}}\frac{d^2k^\perp}{\sqrt{(2\pi)^2}}\frac{d^2p_2^\perp}{\sqrt{(2\pi)^2}}\delta^{(2)}(p_1^\perp - k^\perp - p_2^\perp)$$

$$\times\, b^\dagger(p_1,s_1,t_1)a(k,\lambda)b(p_2,s_2,t_2)$$

$$\times \underbrace{\zeta^\dagger(s_1)\left\{\frac{\gamma^\perp\cdot p_1^\perp + M_N}{p_1^+}\gamma_5 - \gamma_5\frac{-\gamma^\perp\cdot p_2^\perp + M_N}{p_2^+}\right\}\zeta(s_2)}_{\text{spinor structure function}} \quad \underbrace{T^\dagger(t_1)\Big[\sum_a \tau_a\epsilon_a(\lambda)\Big]T(t_2)}_{\text{isospinor structure function}}$$

Nuclear physics in the medium energy is challenging