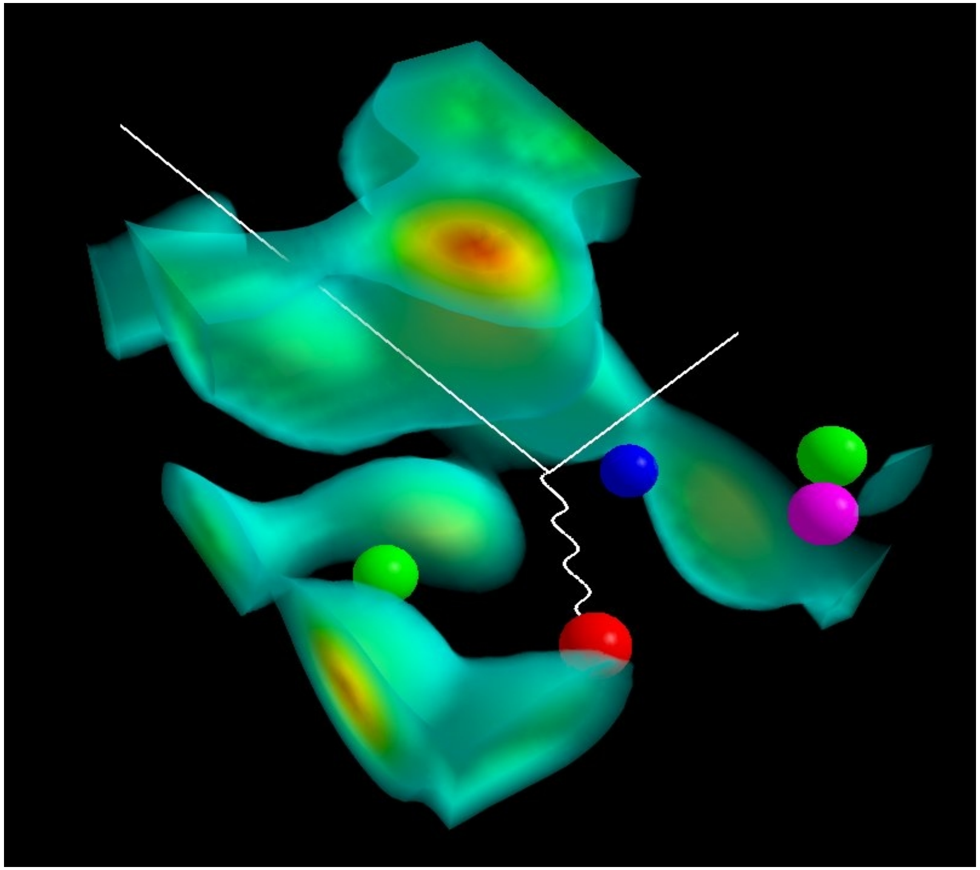
Extraction of the Pion Electromagnetic Form Factor



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Anthony W. Thomas

GHP
Denver : 10th April 2019

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Outline

I. Significance of the pion form factor

II. Traditional approach

III. Improvement

IV. Outlook

Collaborators: Robert Perry and Ayse Kizilersu

I. Importance of the pion in QCD

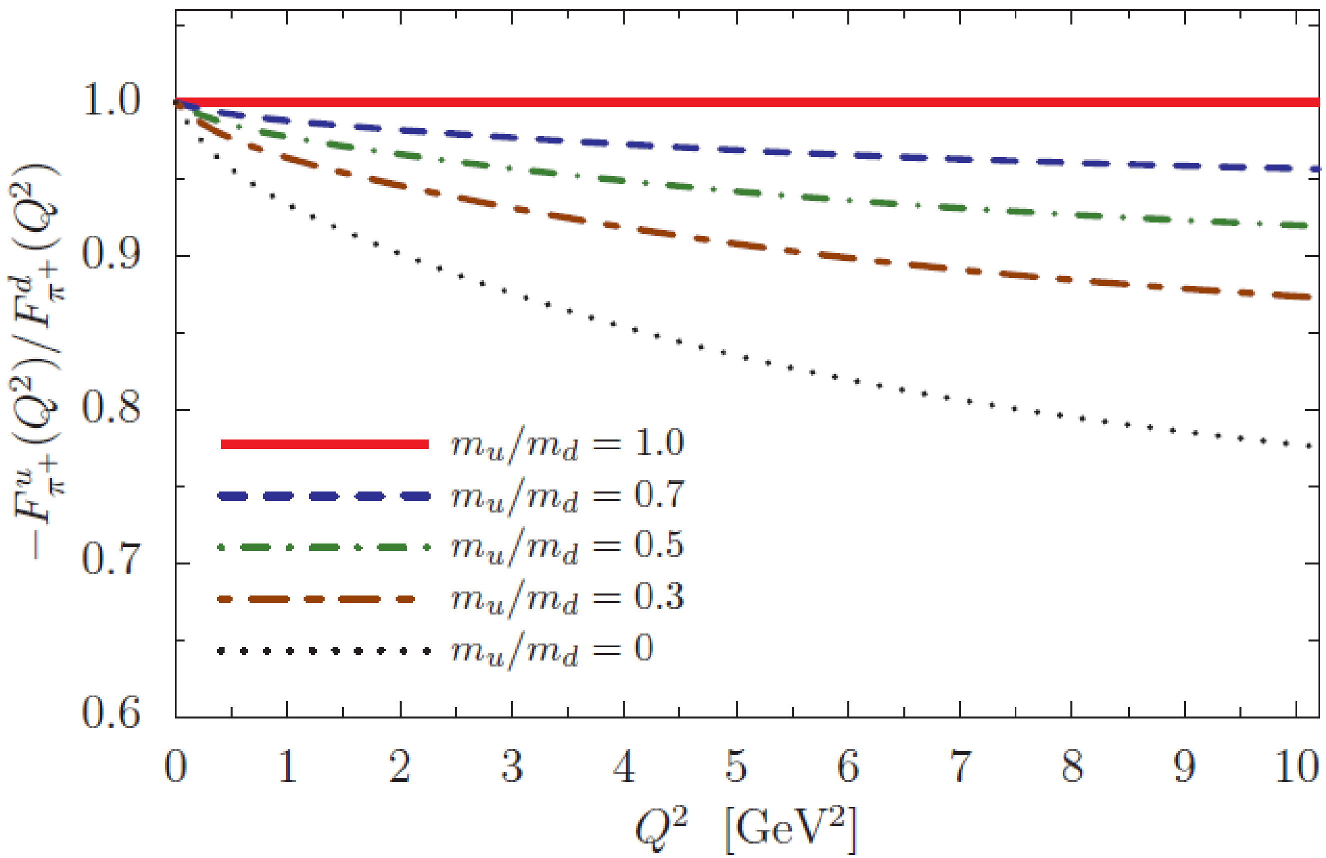
Pion is a pseudo-Goldstone boson in QCD

- Mass vanishes in limit of zero quark mass
- $m_\pi \sim m_q^{1/2}$: and the pion is by far the lightest hadron
- The electromagnetic form factor is a fundamental property, much studied in QCD
- Brodsky and LePage: large Q^2 limit

$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)$$

- Extensive studies in SDE, NJL etc. (Cloët, Roberts….)

Recent study of charge symmetry violation



Charge symmetry breaking effects in pion and kaon structure

Parada T. P. Hutauruk,[1] Wolfgang Bentz,[2,3] Ian C. Cloët,[4] and Anthony W. Thomas[5]

PHYSICAL REVIEW C **97**, 055210 (2018)

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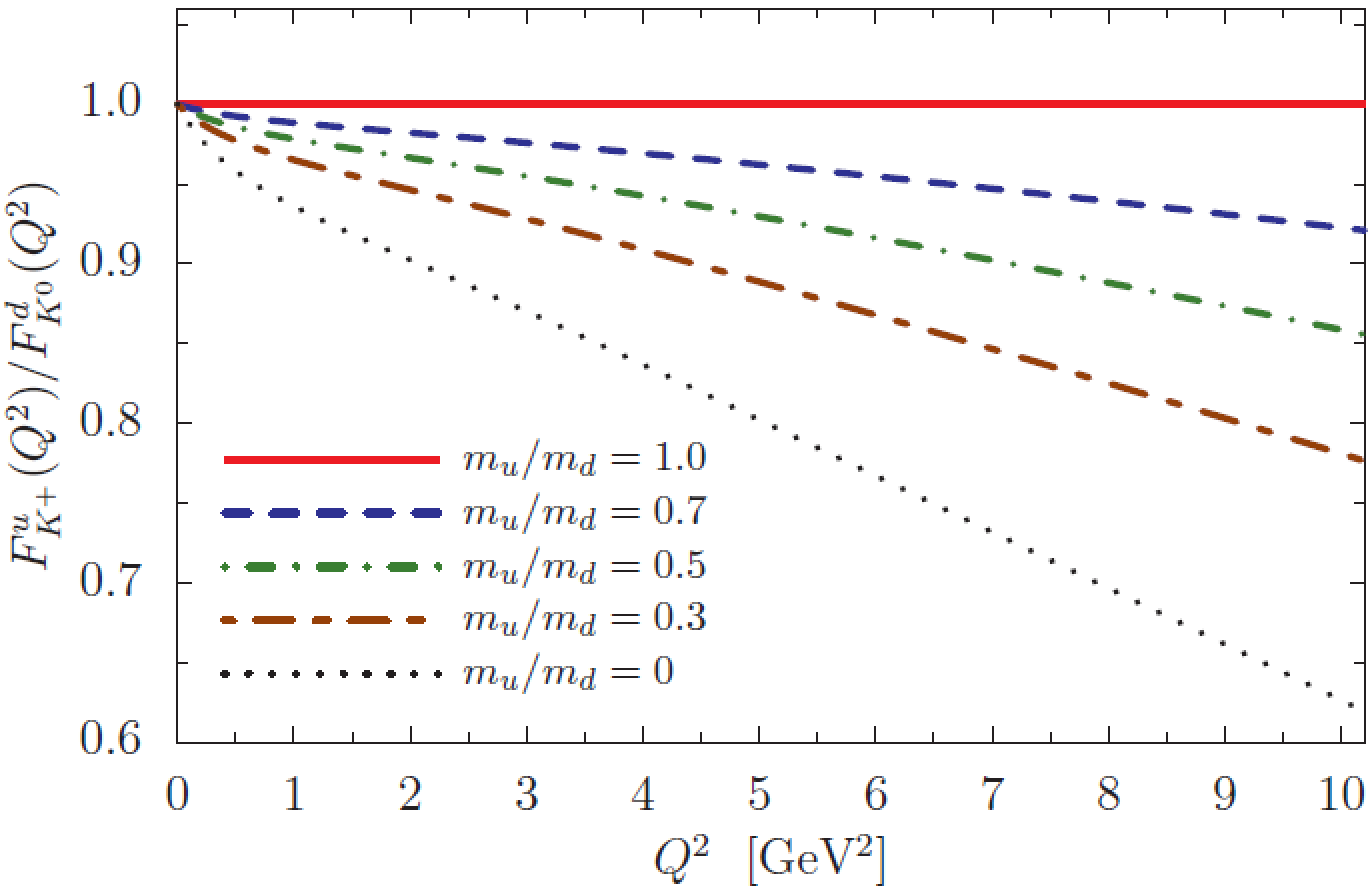
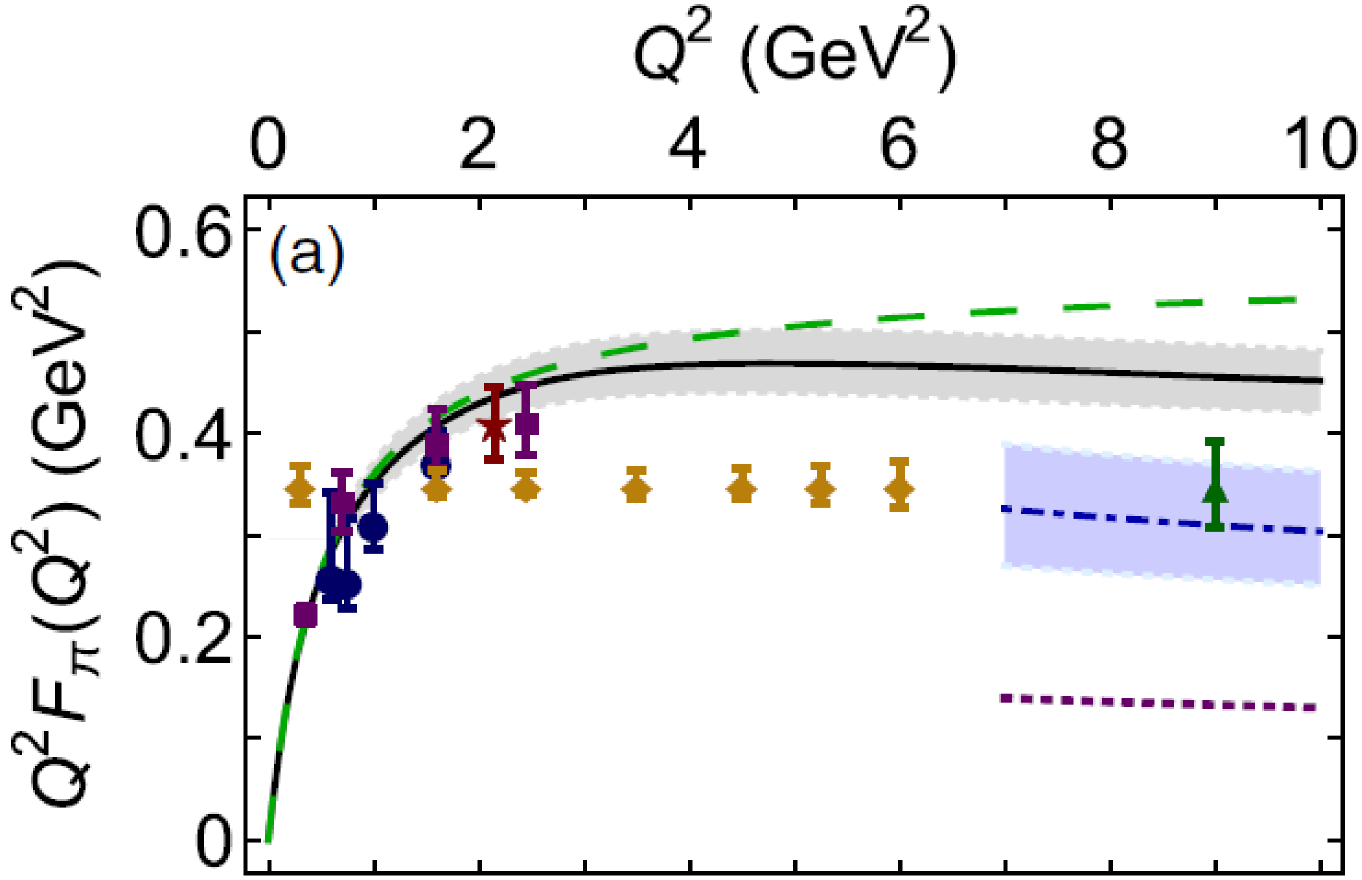
and in the Kaon form factor



Comparison of data with predictions



But: what are the "experimental" numbers?

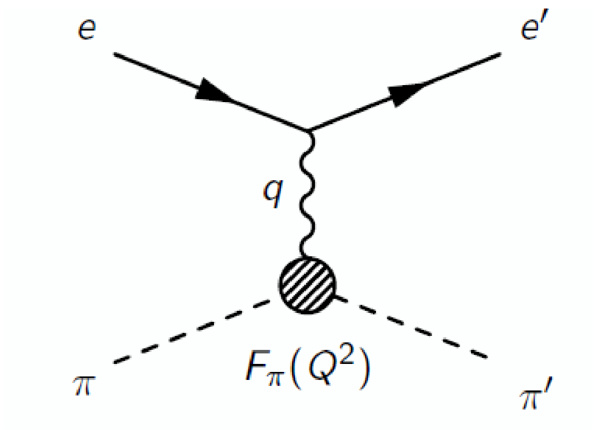
Chen et al., Phys. Rev. D98 (2018) 091505

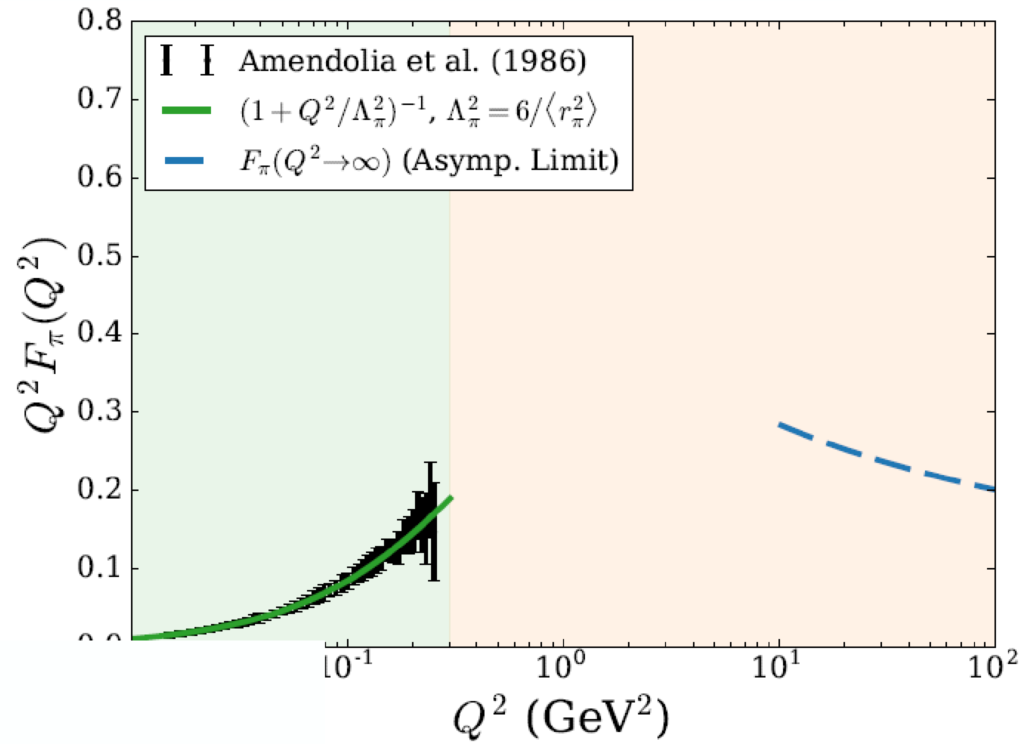
II. Current approach to data analysis

Direct measurement limited to very low Q^2

At low energy, scatter pion beam from electrons in liquid hydrogen target.

Measure recoiling pion and electron.

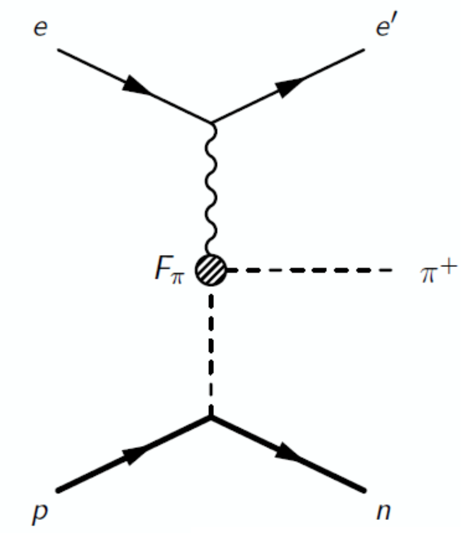




Differential cross section is

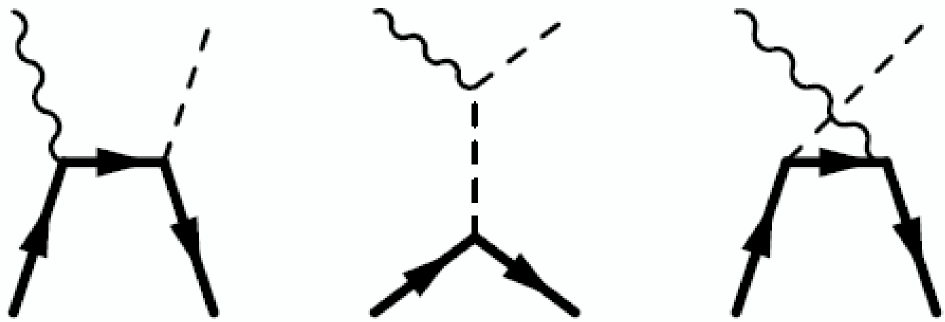
$$\frac{d\sigma}{dq^2} = |F_\pi|^2 \frac{k}{q^4} \left(1 - \frac{q^2}{q^2_{\text{max}}}\right)$$

Electro-production at higher-Q^2



Also have interference from s- and u- channel terms.

- Proportional to Nucleon Form Factors



(a) s-channel (b) t-channel (c) u-channel

Pion is initially off-shell. What does this mean for the extraction of F_π?

- Theoretically?
- Empirically?

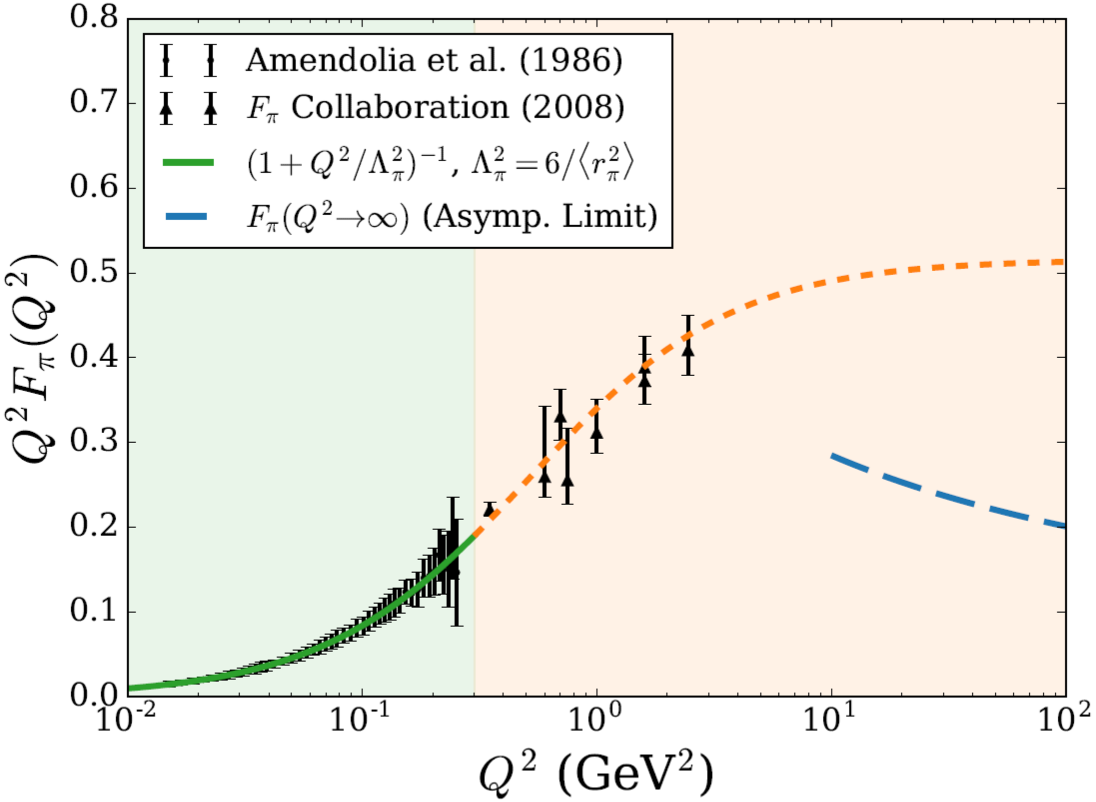
Jefferson Lab data from F_π Collaboration

PHYSICAL REVIEW C 78, 045202 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. I. Measurements of the cross section for the $^1\text{H}(e, e'\pi^+)n$ reaction

PHYSICAL REVIEW C 78, 045203 (2008)

Charged pion form factor between $Q^2 = 0.60$ and 2.45 GeV^2. II. Determination of, and results for, the pion form factor



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Theoretical Framework for Analysis

Cross section may be decomposed into 4 structure functions.

$$(2\pi)\frac{d^2\sigma}{dtd\phi} = \frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{LT}}{dt}\cos\phi + \epsilon\frac{d\sigma_{TT}}{dt}\cos 2\phi$$

ϵ is a measure of the virtual photon polarization

Important, as is known that Longitudinal cross section dominated by t-channel pion exchange.

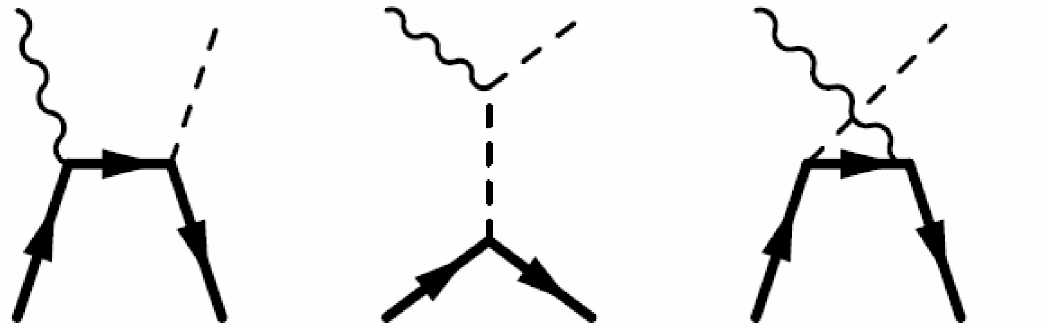
- A good reconstruction of this structure function gives us a good change of extracting the pion form factor.

Modern extractions of the form factor use the VGL Model

Vanderhaeghen, Guidal, Laget (VGL) Model

- **Calculate Born diagrams:**

$i\mathcal{M}^{\mu}_{\mathrm{BTM}} =$



(a) s-channel (b) t-channel (c) u-channel

- **Sum needed to ensure gauge invariance**

- **Reggeize pion exchange:** $i\mathcal{M}^{\mu}_{\mathrm{R}} = S_F^{\pi\,-1} S_{\pi}^{\mathrm{R}}(t) \times [i\mathcal{M}^{\mu}_{\mathrm{BTM}}]$

- **Multiply by ONE overall form factor – different form factors would destroy gauge invariance**

VGL Model (cont.)

- **Final amplitude with which experimental data is fit:**

$$\begin{aligned} i\mathcal{M}^{\mu}_{\mathrm{VGL}} &= F_{\pi}(Q^2) \times [i\mathcal{M}^{\mu}_{\mathrm{R}}] \\ &= F_{\pi}(Q^2) \times S_F^{\pi\,-1} S_{\pi}^{\mathrm{R}}(t) \times [i\mathcal{M}^{\mu}_{\mathrm{BTM}}] \end{aligned}$$

- **This is, of course, fine at the pion pole ($t = + m_{\pi}^2$) but data lies well away – typically $t < -2\, m_{\pi}^2$**

Current procedure

Measure cross section at a range of t values for fixed Q^2 and W.

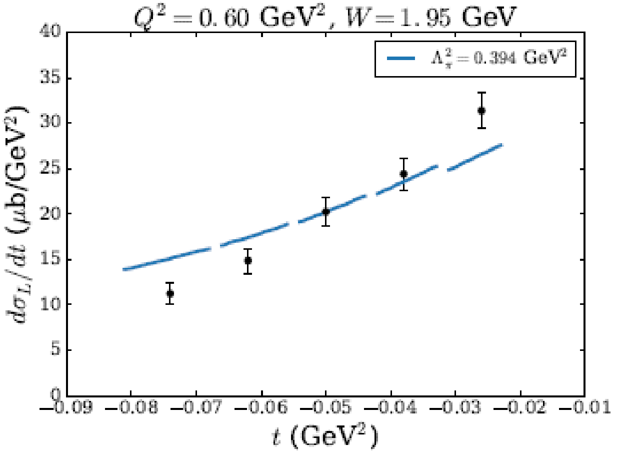
Longitudinal cross section is

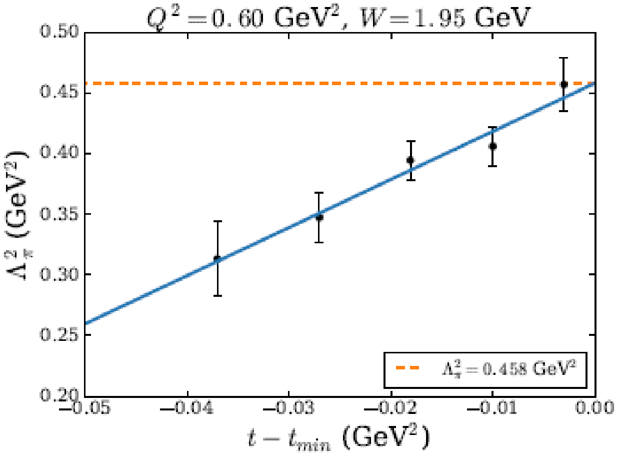
$$\frac{d\sigma_L}{dt} \propto |F_\pi|^2$$

$$F_\pi(Q^2) = (1 + Q^2/\Lambda_\pi^2)^{-1}$$

Fit each data point individually.

Extrapolate these points to $t = t_{\text{min}}$, where there is least contamination from interfering backgrounds not included in the VGL model.

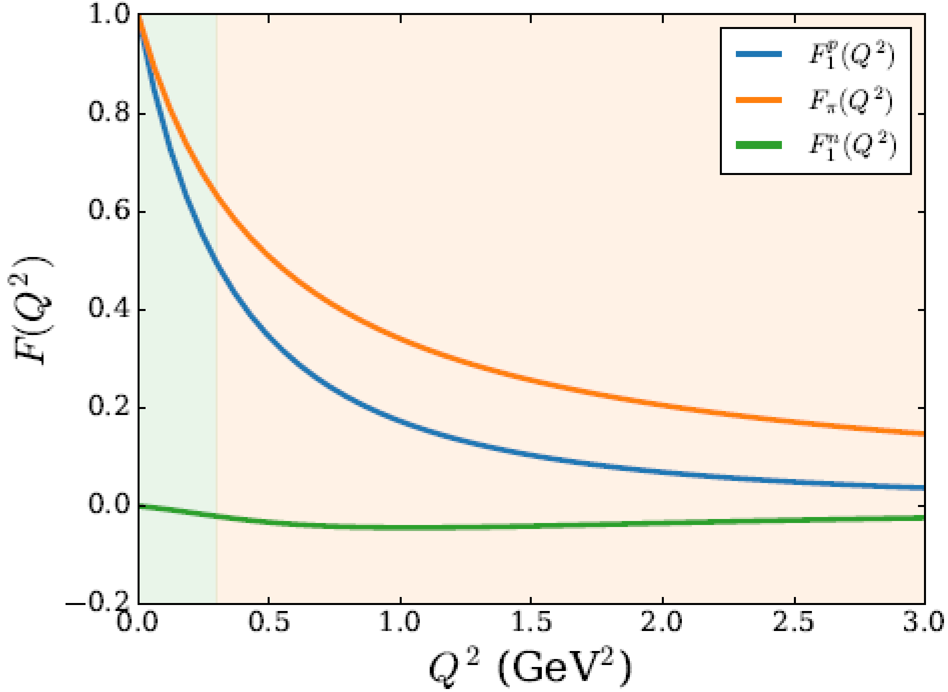




- **Extracted form factor at given Q^2 depends on t**

Physical form factors are rather different

Currently, form factors are all the same: can we do better?



How can we incorporate the 'off-shellness' of particles.

III. New approach

Follow similar path to Miller

PHYSICAL REVIEW C **80**, 045210 (2009)

Electromagnetic form factors and charge densities from hadrons to nuclei

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(Received 18 August 2009; published 22 October 2009)

A simple exact covariant model in which a scalar particle Ψ is modeled as a bound state of two different particles is used to elucidate relativistic aspects of electromagnetic form factors $F(Q^2)$. The model form factor is computed using an exact covariant calculation of the lowest order triangle diagram. The light-front

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Psi_N)^2 - \frac{1}{2} m_N^2 \Psi_N^2 + \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 - g_{\pi N} \Psi_N^\dagger \boldsymbol{\tau} \cdot \boldsymbol{\pi} \Psi_N$$

Include electromagnetic interactions via $\partial_\mu \to \partial_\mu + ieA_\mu$

Construct a Gauge Invariant Bosonic Model – very much a toy model

- We wish to determined the model dependence of the currently extracted form factor for the pion.
- Our strategy will be to calculate the cross section and pion form factor in a simple model.
- Examine how well we can extract the pion form factor from the cross section

	Current Extraction	This Analysis
Model	$i\mathcal{M}^{\mu}_{\text{VGL}} = F_{\pi}(Q^2) S_F^{\pi\,-1} S_{\pi}^{\text{R}}(t)[i\mathcal{M}^{\mu}_{\text{BTM}}]$	$i\mathcal{M}^{\mu} = F_{\pi}(Q^2)[i\mathcal{M}^{\mu}_{\text{BTM}}]$
	$\downarrow$ fit to... $\downarrow$	$\downarrow$ fit to... $\downarrow$
Data	${}^1\text{H}(e, e'\pi^+)n$	$i\mathcal{M}^{\mu}_{\text{1-Loop}}$

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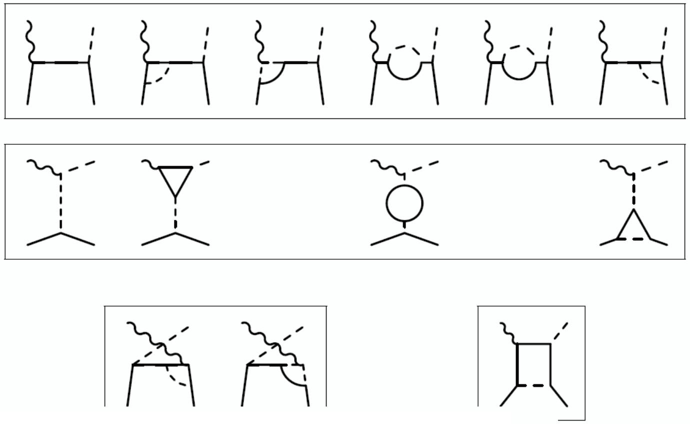
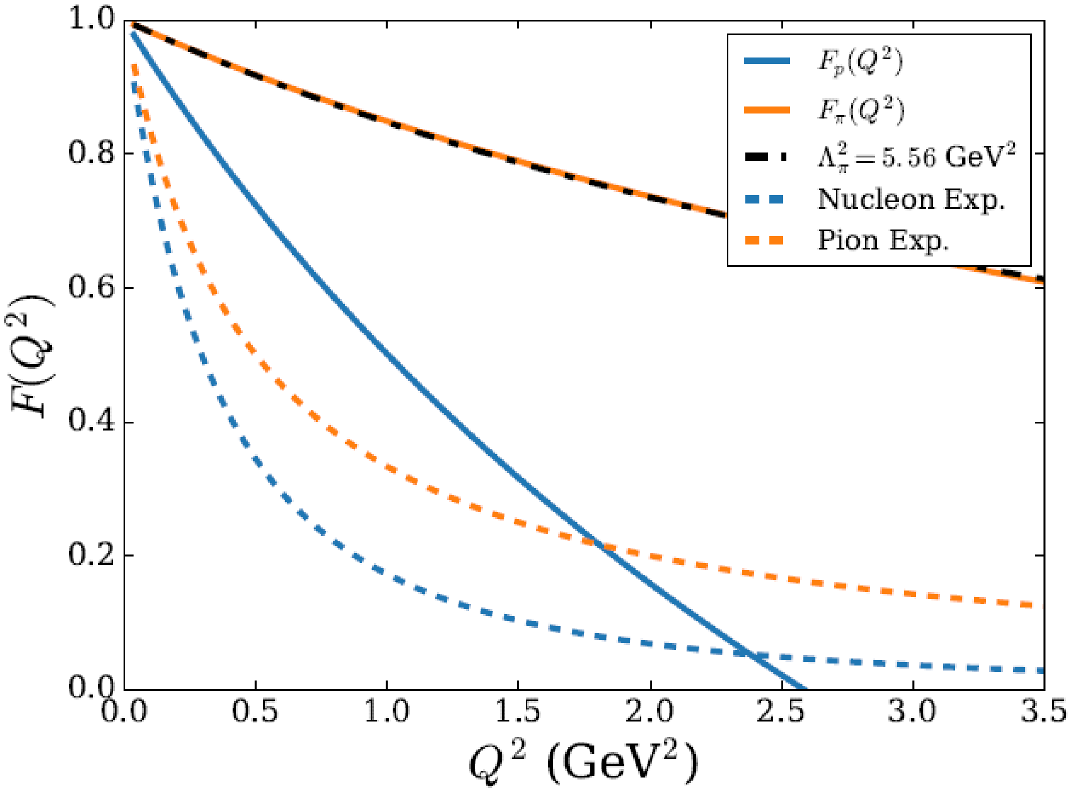
Simple bosonic model

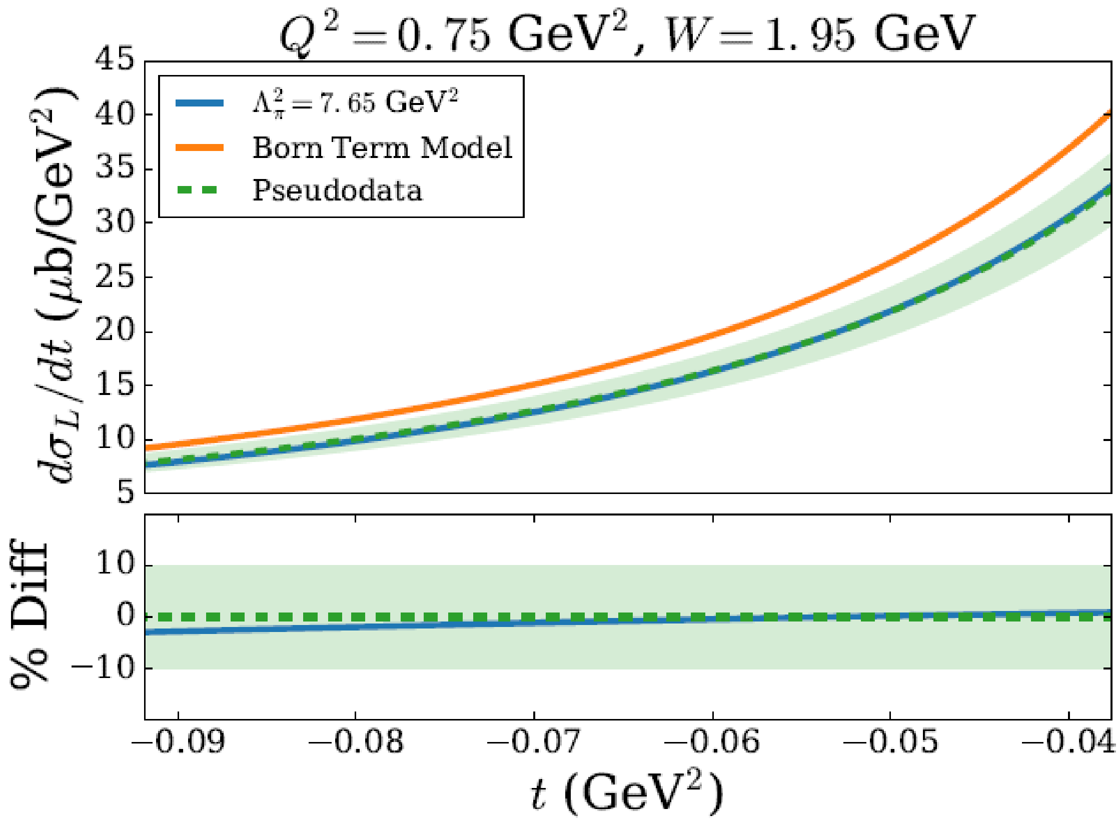
- Simple bosonic model of nucleon and pion electromagnetic form factors and πNN vertex





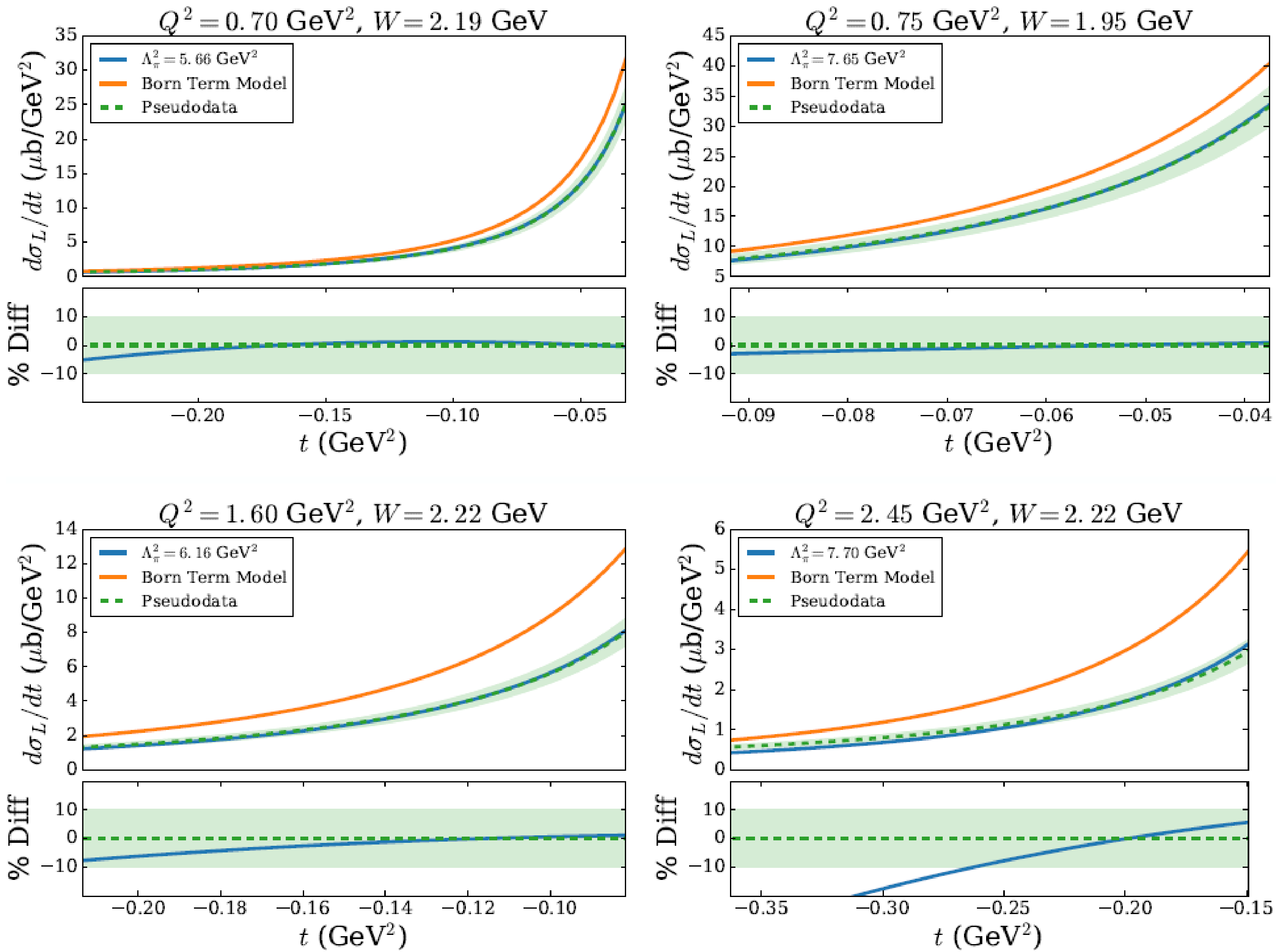
Example fit 1

Q^2 (GeV^2)	W (GeV)	$\lvert t_{\text{low}}\rvert$ (GeV^2)	$\lvert t_{\text{high}}\rvert$ (GeV^2)
0.35	2.10	0.010	0.040
0.60	1.95	0.025	0.074
0.70	2.19	0.030	0.250
0.75	1.95	0.037	0.093
1.00	1.95	0.060	0.140
1.60	1.95	0.135	0.255
1.60	2.22	0.079	0.215
2.45	2.22	0.145	0.365



t range chosen to be same as experiment.

More examples….



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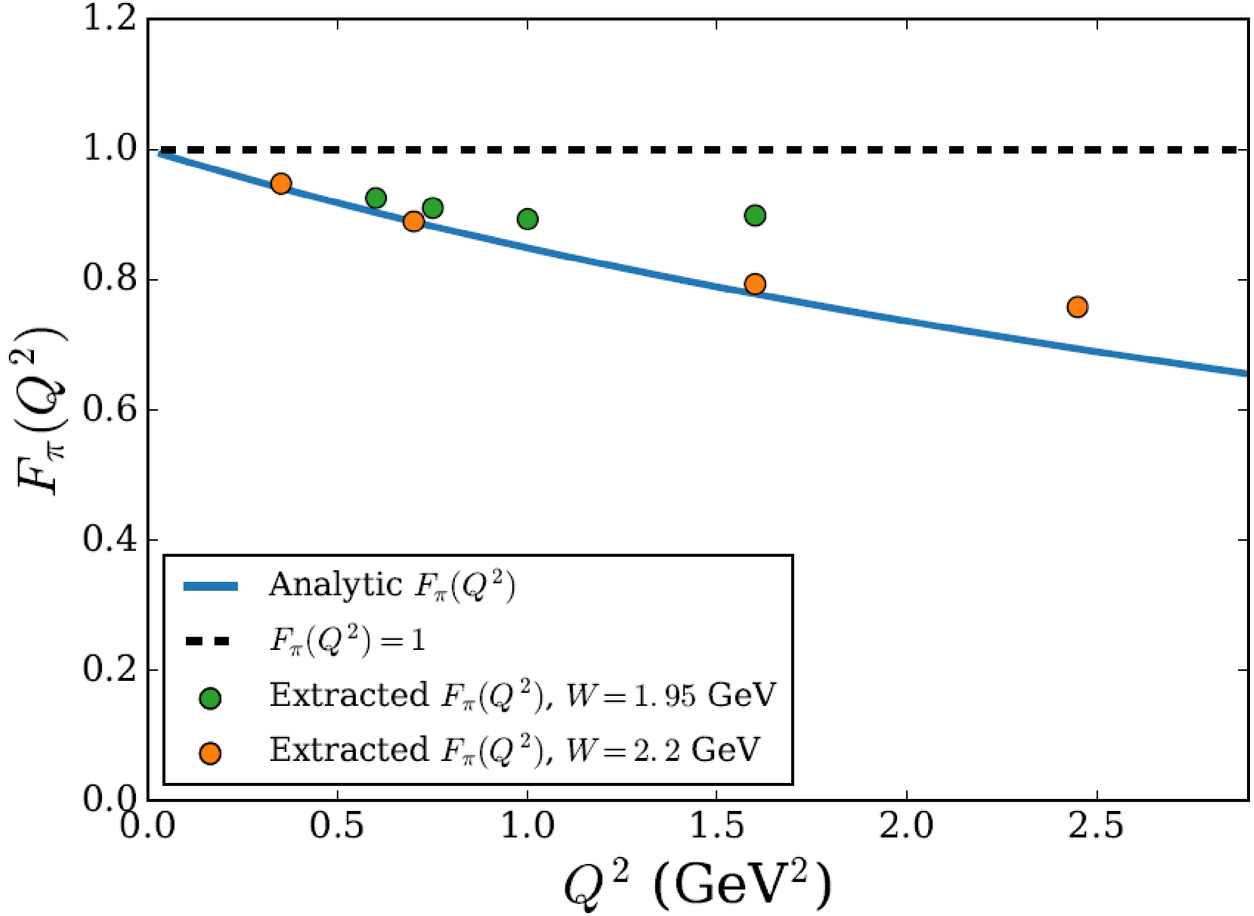
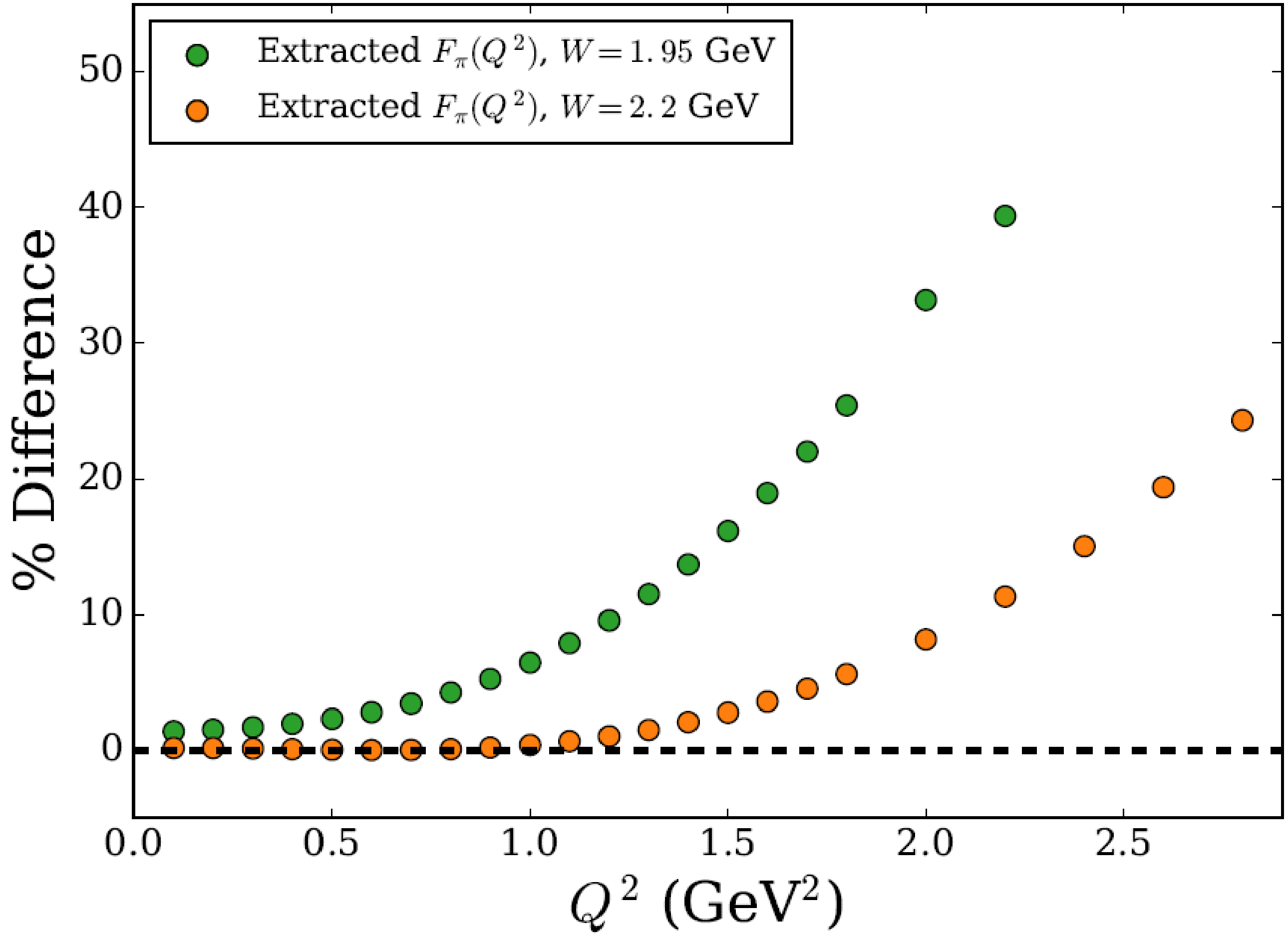
Overview 1

- **Error can be 10-20% at larger momentum transfer**
- **As Q^2 rises we need to go to higher W to reduce the error in the extracted value**



Overview 2

- **Error can be 10-20% at larger momentum transfer**



Summary and Outlook

- A precise extraction of the pion form factor requires serious consideration of gauge invariance

- The model used so far is really a toy model
  - purely bosonic

- Nevertheless, it suggests that at higher Q^2 one needs to go to higher energy to reduce the errors

- Current errors at large Q^2 could be as large as 20%

- Fermion implementation underway

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