

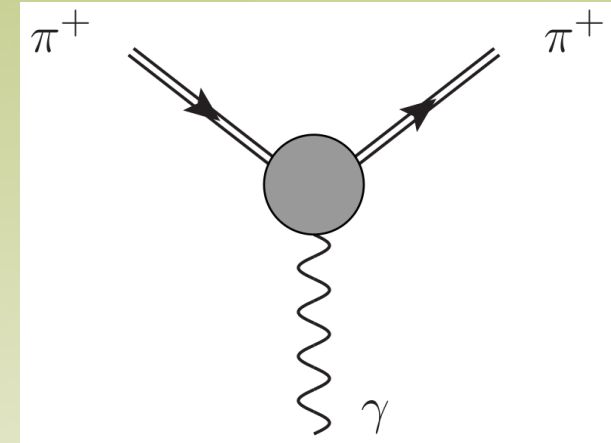
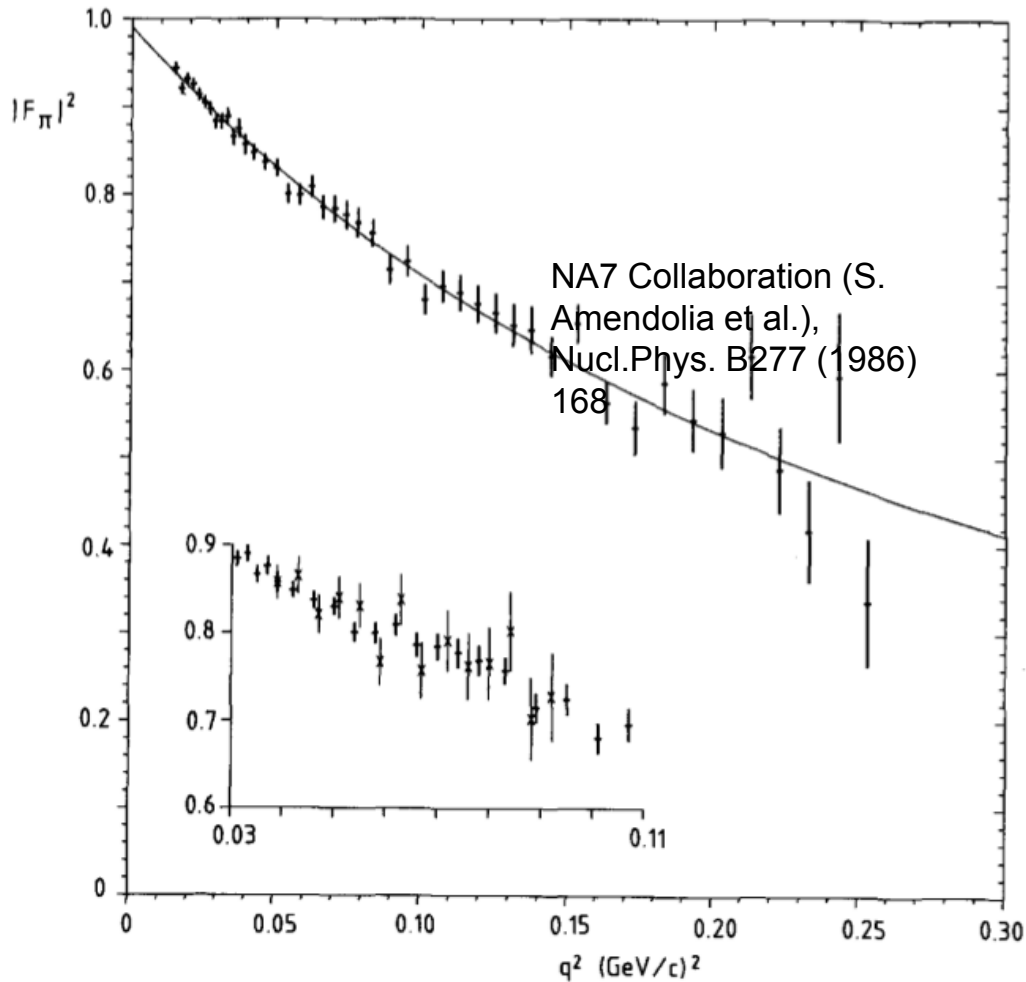
Pion Form Factor with Overlap Fermion on the Lattice

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χ QCD Collaboration



Motivation



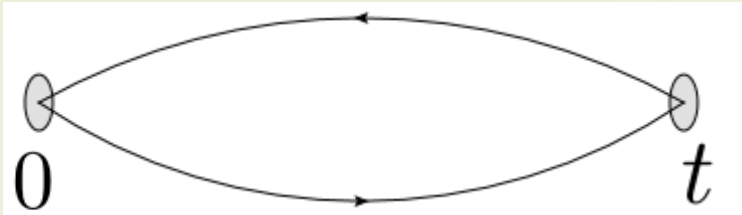
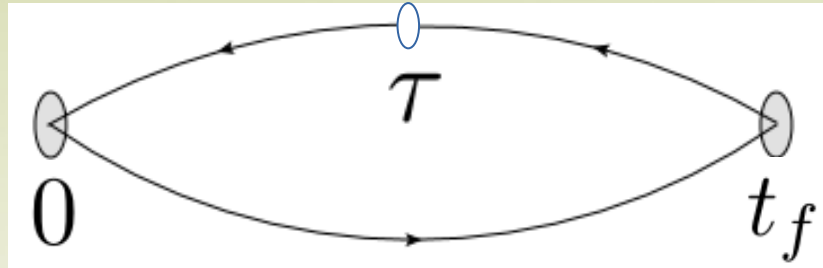
Mean Square Radius	Group
0.42(10)	ESCHRICH
0.439(8)	AMENDOLIA
0.440(30)	DALLY
0.452(11)	PDG

$$\langle \pi^+(\mathbf{p}_f) | V_\mu(x) | \pi^+(\mathbf{p}_i) \rangle = (p_f + p_i)_\mu f_{\pi\pi}(q^2)$$

Pion Form Factor

$$\begin{aligned}
 C_{3pt}(\tau, t_f, \mathbf{p}_i, \mathbf{p}_f) &= \sum_{x_f, z} e^{-i\mathbf{p}_f \cdot (\mathbf{x}_f - z)} e^{i\mathbf{q} \cdot z} \langle T[\chi_{\pi^+}(x_f, t_f) V_\mu(z, \tau) \chi_{\pi^+}^\dagger(0, 0)] \rangle \\
 &\approx \frac{m^2 Z_{p_i} Z_{p_f}}{E(p_i) E(p_f)} e^{-E(p_i)\tau - E(p_f)(t_f - \tau)} \langle \pi(p_f) | V_\mu(0) | \pi(p_i) \rangle \\
 &\quad + C_1 e^{-E^1(p_i)\tau - E(p_f)(t_f - \tau)} + C_2 e^{-E(p_i)\tau - E^1(p_f)(t_f - \tau)}
 \end{aligned}$$

$$\mathbf{q} = \mathbf{p}_f - \mathbf{p}_i$$



$$\begin{aligned}
 C_{2pt}(t, p) &= \sum_x e^{-i\mathbf{p} \cdot \mathbf{x}} \langle T[\chi_{\pi^+}(x, t) \chi_{\pi^+}^\dagger(0, 0)] \rangle \\
 &\approx \frac{m |Z_p|^2}{E(p)} (e^{-E(p)t} + e^{-E(p)(T-t)}) \\
 &\quad + \frac{m |Z_p^1|^2}{E^1(p)} (e^{-E^1(p)t} + e^{-E^1(p)(T-t)})
 \end{aligned}$$

$$\begin{aligned}
 \chi_{\pi^+}(x_f, t_f) &= \bar{u}(x, t) \gamma_5 d(x, t) \\
 \chi_{\pi^+}^\dagger(x_f, t_f) &= -\bar{d}(x, t) \gamma_5 u(x, t) \\
 V_\mu(x) &= \bar{u}(x) \gamma_\mu u(x)
 \end{aligned}$$

Simulation Details

- Lattices

- 24I--Domain Wall Lattice, $24^3 \times 64$, $a = 0.11$ fm, Pion 337 MeV
- 32ID--Domain Wall Lattice, $32^3 \times 64$, $a = 0.143$ fm, Pion 171 MeV
- Overlap Fermion with several valence quark masses

- Sources and Sinks

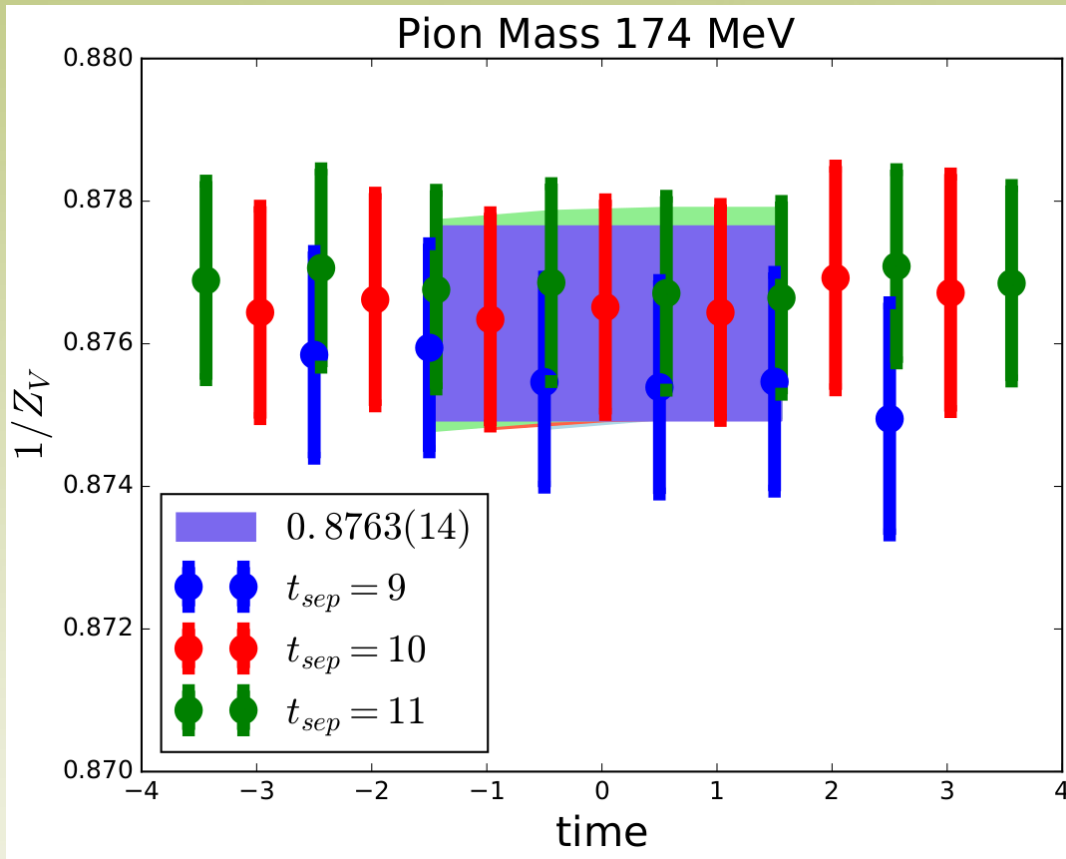
- Grid Source at time 0 with momentum $p_i = q$
- Stochastic Sinks with t_f with momentum $p_f = 0$
- Local vector current at time τ

- Fitting Strategy

- Joint fit of C_{2pt} and C_{3pt} with excited state



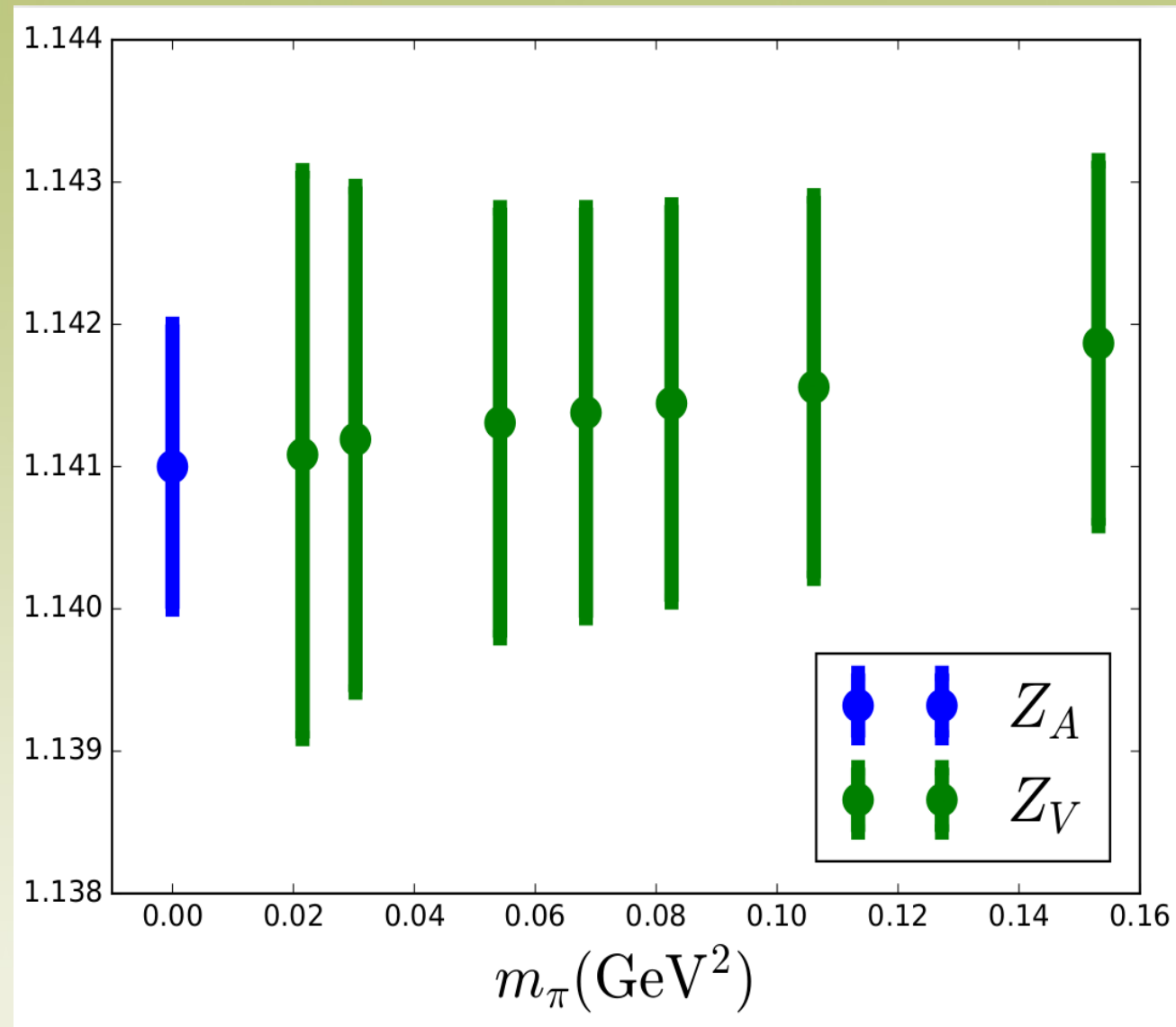
Zero Momentum Transfer



$$\frac{1}{Z_V} = \frac{\langle \pi(p_f) | V_\mu(0) | \pi(p_f) \rangle}{2E_f}$$

By adding excited state to two point and three point, joint fit has been done with χ^2 to be around 0.6

Z_V and Z_A Compare



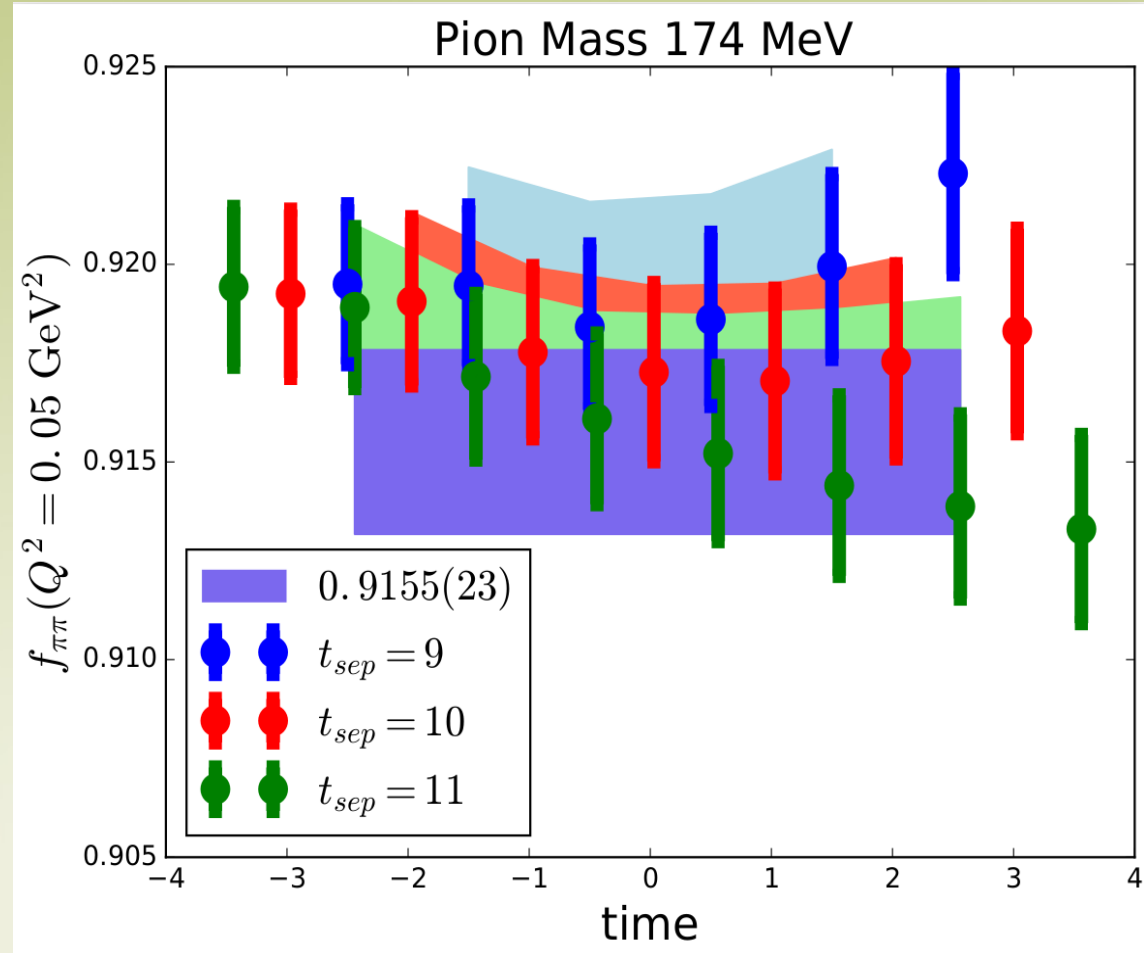
$$Z_A = \frac{2m_q \langle 0 | P | \pi \rangle}{m_\pi \langle 0 | A_4 | \pi \rangle}$$

$$Z_V = \frac{2E_f}{\langle \pi(p_f) | V_\mu(0) | \pi(p_f) \rangle}$$

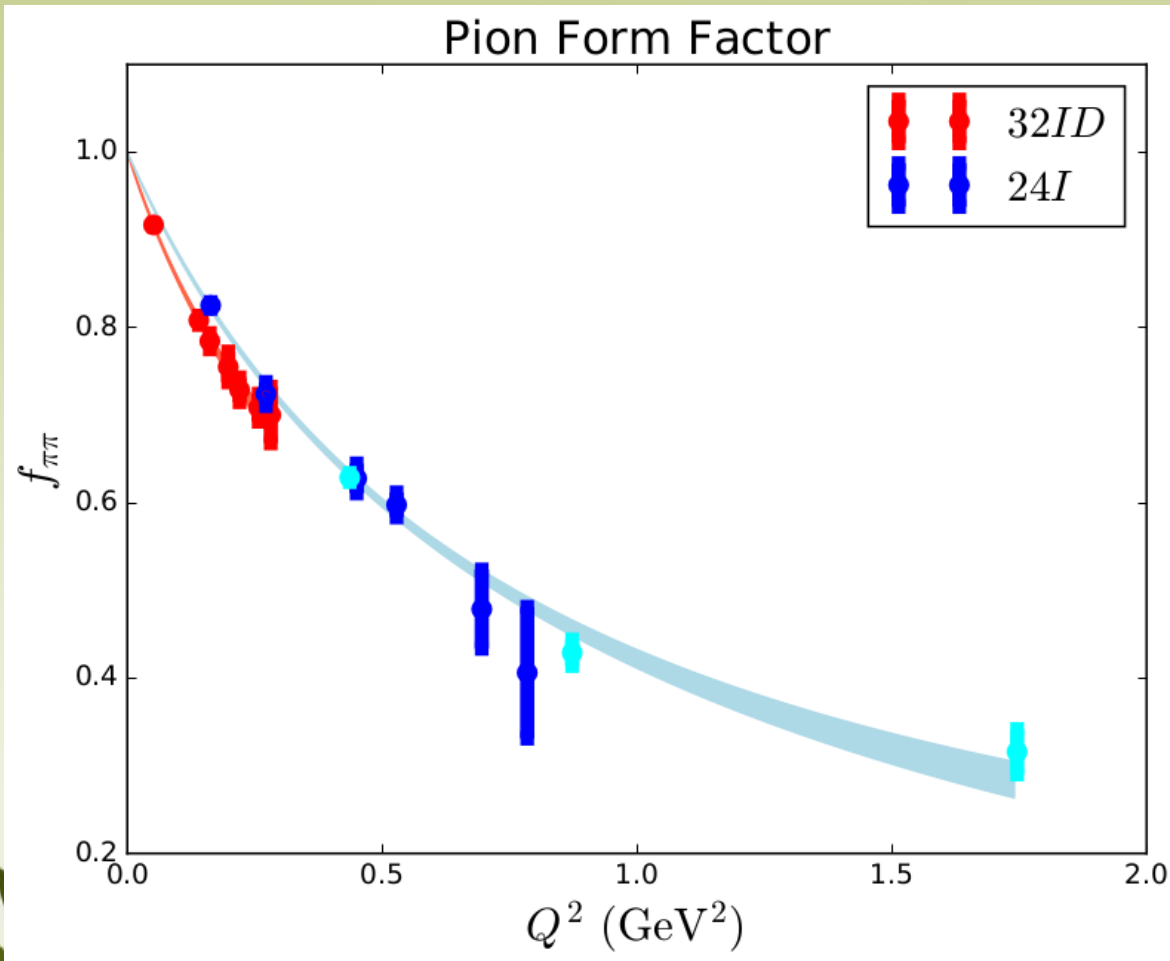
321D $f_{\pi\pi}(Q^2 = 0.05 \text{ GeV}^2)$

$$f_{\pi\pi}(Q^2) = \frac{2E_f}{E_i + E_f} \frac{\langle \pi(p_f) | V_\mu(0) | \pi(p_i) \rangle}{\langle \pi(p_f) | V_\mu(0) | \pi(p_f) \rangle}$$

At small momentum transfer, we get stable results even with small valence pion mass



Z-expansion Fitting

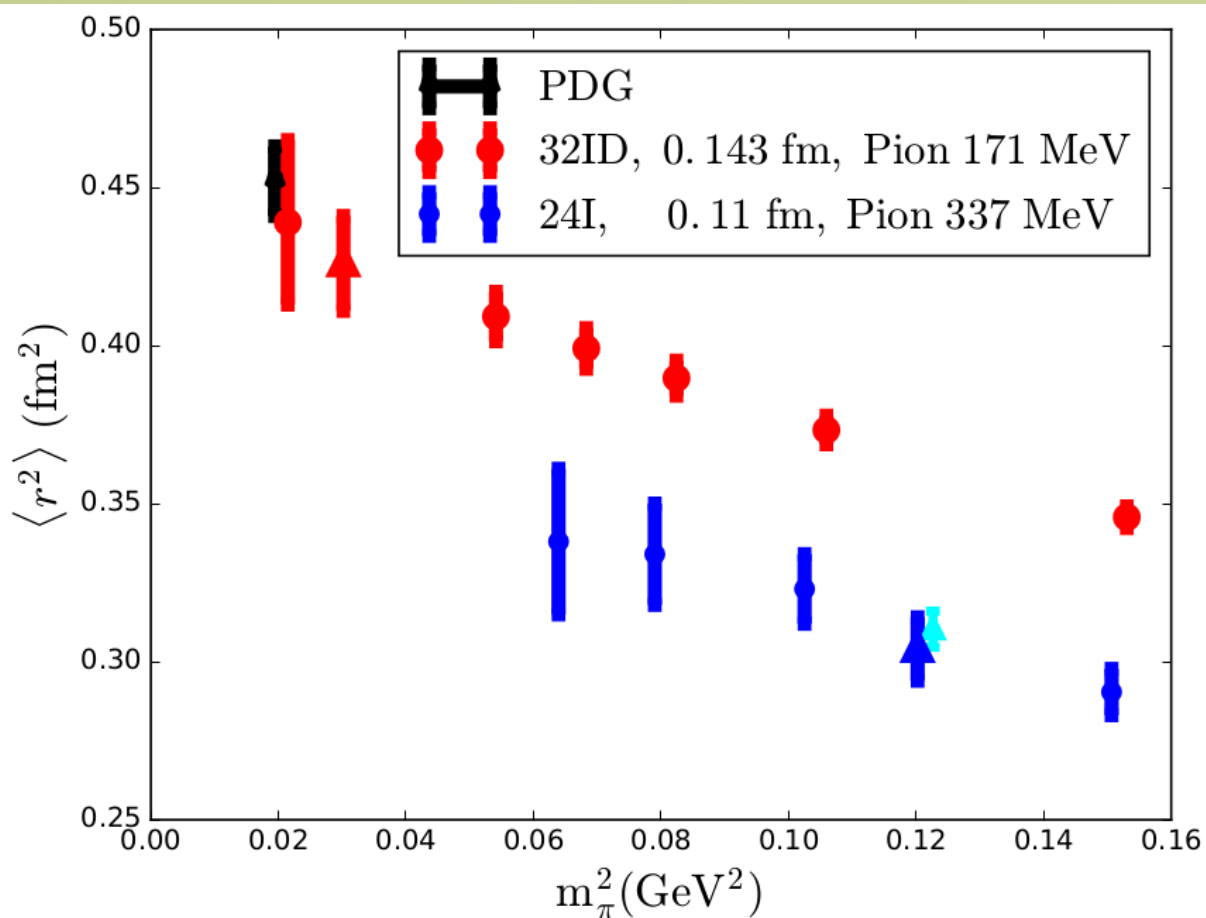


$$f_{\pi\pi}(Q^2) = 1 + \sum_{k=1}^{k_{max}} a_k z^k$$

$$z(t, t_{cut}, t_0) = \frac{\sqrt{t_{cut} - t_0} - \sqrt{t_{cut} - t}}{\sqrt{t_{cut} - t_0} + \sqrt{t_{cut} - t}}$$

$$t = -Q^2, t_{cut} = 4\pi^2$$

Pion Radius from 24I and 32ID

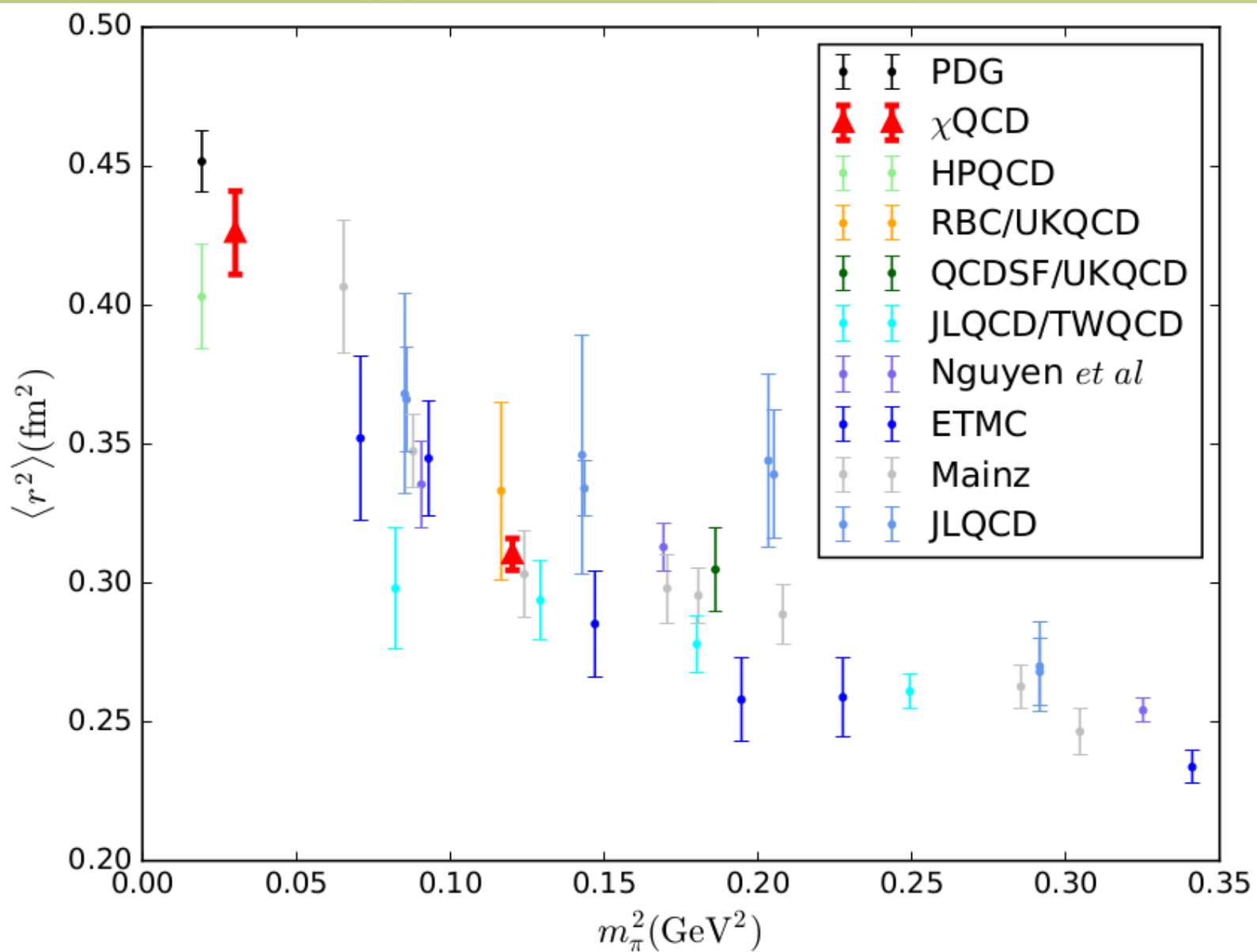


$$\langle r^2 \rangle = 6 \frac{d}{dQ^2} f_{\pi\pi}(Q^2)$$

Strong valence pion mass dependence observed here

Need more lattices to explain the difference between 24I and 32ID

Pion Radius



J. Koponen, et. al., Phys. Rev. D 93, 054503 (2016), arXiv:1511.07382

JLQCD, Phys. Rev. D 93, 034504 (2016), arXiv:1510.06470

Bastian B. Brandt, International Journal of Modern Physics E, Vol. 22 (2013) 1330030, arXiv: 1310.6389

Summary

- We have a preliminary result of pion form factor with sea pion mass 171 MeV and 337 MeV
- 32ID result agrees with experiment after extracted to physical pion mass point
- More detailed analysis will be needed for the FFT production
- Production with different lattice spacing and sea pion masses are needed to control and understand various systematics

