

# The $g_2$ Structure Function

Recent results from the SANE experiment

8th Workshop of the  
APS Topical Group on  
Hadronic Physics

Whitney R. Armstrong  
Argonne National Laboratory

April 11, 2019



# Outline

- Physics motivation for measuring the  $g_2$  spin structure function
- The twist-3 matrix element  $d_2$
- Recent JLab experimental results

# Deep Inelastic Scattering

$$\sigma_0 = \frac{4\alpha^2 E'^2}{q^4} \left[ \frac{2}{M} F_1 \sin^2(\theta/2) + \frac{1}{\nu} F_2 \cos^2(\theta/2) \right]$$

$$2\sigma_0 A_{||} = -\frac{4\alpha^2}{Q^2} \frac{E'}{E} \left[ \frac{E + E' \cos \theta}{M\nu} g_1 - \frac{Q^2}{M\nu^2} g_2 \right]$$

$$2\sigma_0 A_{\perp} = -\frac{4\alpha^2}{MQ^2} \frac{E'^2}{E} \sin \theta \cos \phi \left[ \frac{1}{M\nu} g_1 + \frac{2E}{M\nu^2} g_2 \right]$$

## Asymmetries

$$A_{||} = \frac{\sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow\downarrow\downarrow}}{\sigma_{\uparrow\downarrow} + \sigma_{\uparrow\uparrow}}$$

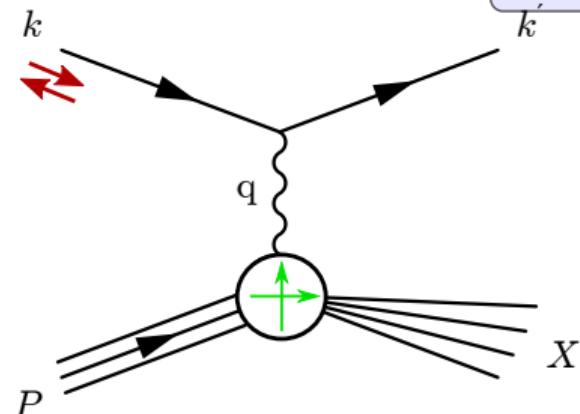
$$A_{\perp} = \frac{\sigma_{\leftarrow\downarrow} - \sigma_{\leftarrow\uparrow\uparrow\downarrow}}{\sigma_{\leftarrow\downarrow} + \sigma_{\leftarrow\uparrow\uparrow}}$$

$$x = Q^2/(2M\nu)$$

$$\nu = E - E'$$

$$W_X^2 = M^2 + 2M\nu - Q^2$$

$$Q^2 = -q^2 = 4EE' \sin^2(\theta/2)$$



## Structure Functions

$$F_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 q_i(x, Q^2)$$

$$F_2(x, Q^2) = 2xF_1(x, Q^2)$$

$$g_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x, Q^2)$$

$$g_2(x, Q^2) = ?$$

Why is a transversely polarized target needed?

$$A_{||} \propto g_1 - \frac{2Mx}{\nu} g_2$$

$\rightarrow g_2$  suppressed by  $1/\nu$

$$A_{\perp} \propto g_1 + g_2$$

$\rightarrow$  In DIS region both contribute.

$\Rightarrow A_{\perp}$  directly sensitive to non-perturbative effects!



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# The dynamical twist-3 matrix element: $d_2$

$$\int_0^1 dx x^{n-1} \{g_1 + \frac{n}{n-1} g_2\} = \frac{1}{2} d_{n-1} E_2^n(Q^2, g)$$

For  $n = 3$

$$\int_0^1 x^2 \{2g_1 + 3g_2\} dx = d_2$$

Interpretations of  $d_2$



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M. Burkardt Phys.Rev.D 88,114502 (2013) and Nucl.Phys.A 735,185 (2004).

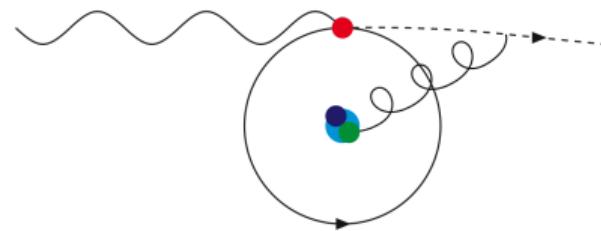
$$d_2 = \frac{1}{2MP^{+2}S_x} \langle P, S | \bar{q}(0)gG^{+y}(0)\gamma^+ q(0) | P, S \rangle$$

but with  $\vec{v} = -c\hat{z}$

$$\sqrt{2}G^{+y} = -E^y + B^x = -(\vec{E} + \vec{v} \times \vec{B})^y$$

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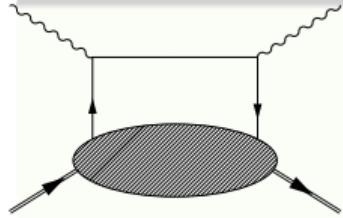


$d_2 \Rightarrow$  average color Lorentz force acting on quark moving backwards (since we are in inf. mom. frame) the instant after being struck by the virtual photon.  $\langle F^y \rangle = -2M^2 d_2$

Quark-gluon Correlations :  $g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$

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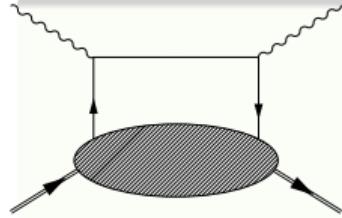
Twist-2 (Wandzura, Wilczek, 1977)



$$\begin{aligned} g_2^{WW}(x, Q^2) &= -g_1^{LT}(x, Q^2) + \int_x^1 g_1^{LT}(y, Q^2) dy/y \\ &\equiv g_2^{tw2}(x, Q^2) \end{aligned}$$

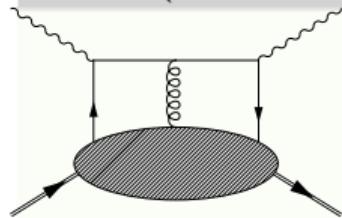
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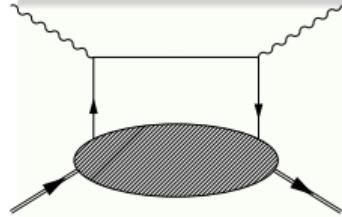
Twist-3 (Cortes, Pire, Ralston, 1992)



$$\bar{g}_2(x, Q^2) = - \int_x^1 \frac{\partial}{\partial y} \left( \frac{m_q}{M} h_T(y, Q^2) + \xi(y, Q^2) \right) \frac{dy}{y}$$
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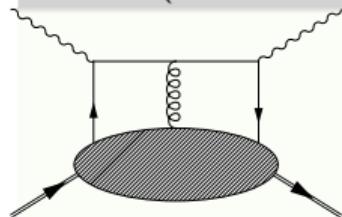
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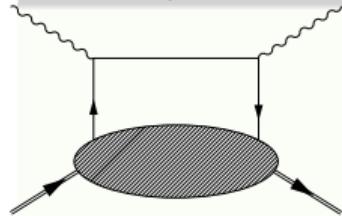
$$d_2(Q^2) = 3 \int_0^1 x^2 \bar{g}_2(x, Q^2) dx$$
$$= \int_0^1 x^2 (2g_1(x, Q^2) + 3g_2(x, Q^2)) dx$$



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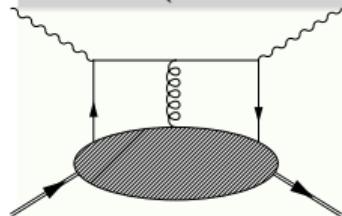
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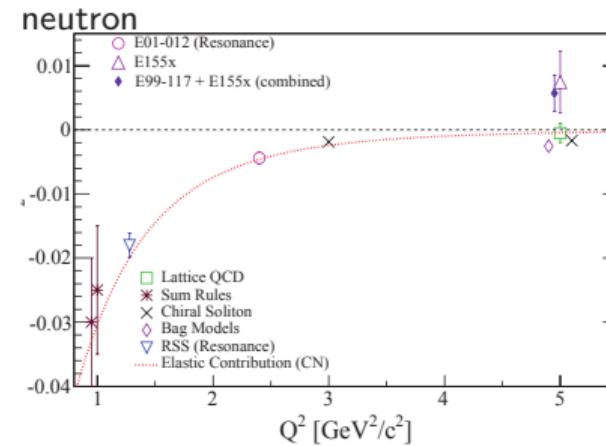
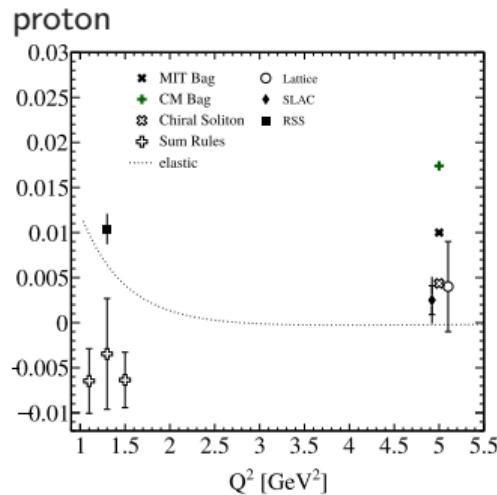


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As  $Q^2$  decreases,  
**when do higher twists begin to matter?**  
**When is the color force non-zero?**

# Predictions and previous measurements of $d_2$

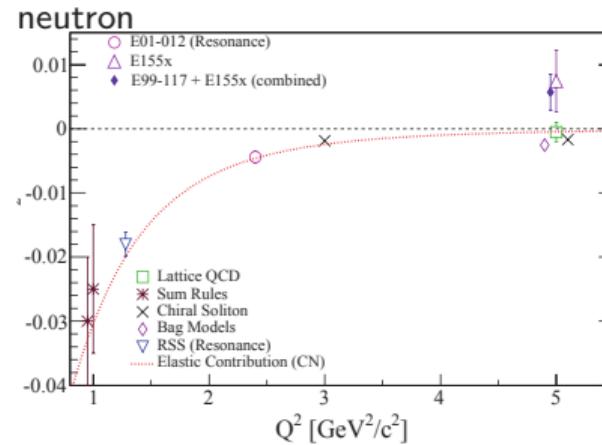
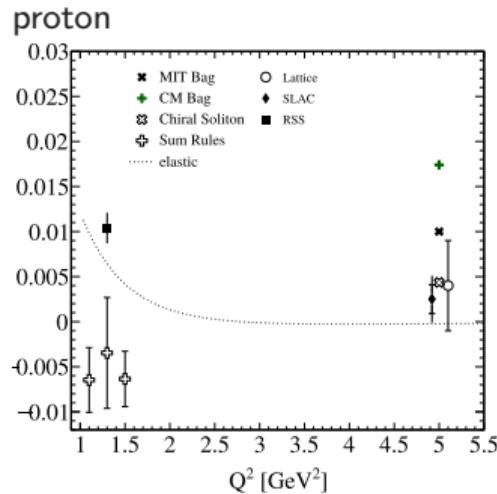


## Lattice QCD

- Ab initio calculations can be done on the lattice

Work in progress: method to extract  $d_2$  from lattice Sivers shift (WA, F.Aslan, M. Engelhardt, S. Liuti, A. Rajan)

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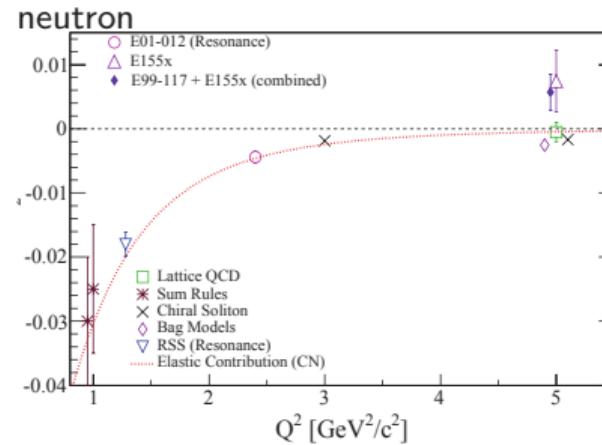
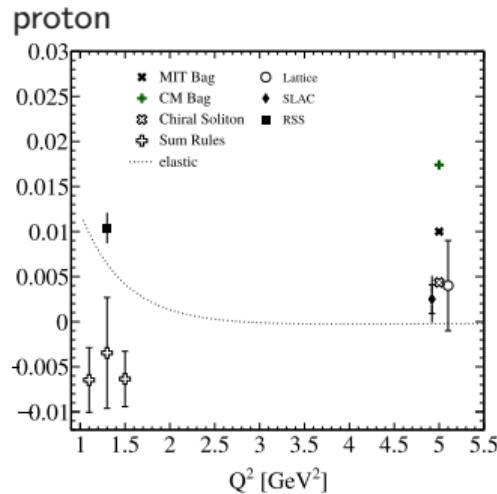


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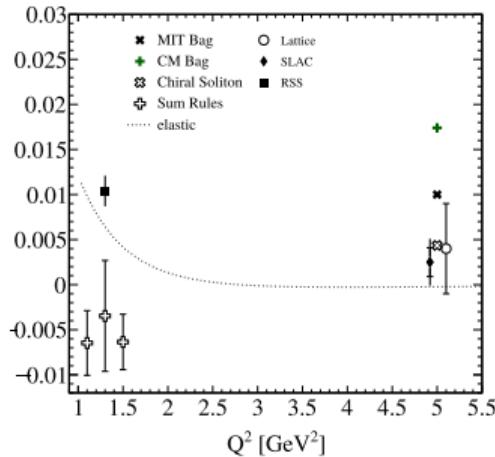
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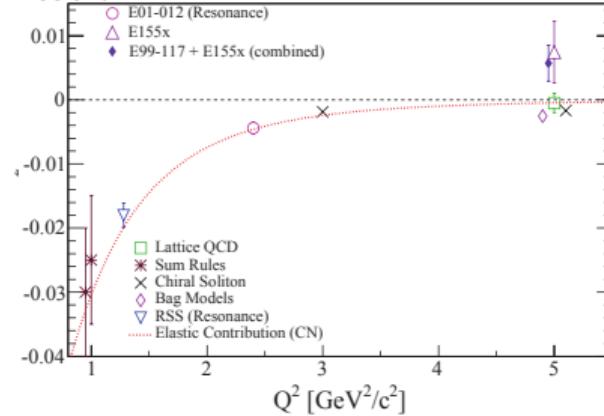
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proton



neutron



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- **Updated and improved lattice results long overdue**

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# Physics with $g_2$

- Polarized DIS is **uniquely** poised to provide insight into **quark-gluon correlations**.
- **Direct access** to higher twist using **transversely polarized target**.
- Twist-3 matrix element  $d_2^p$  proportional to an average **Lorentz color force**.
- Ab initio QCD calculations from the lattice are tested (and modern calculations needed!)
- $\bar{g}_2$  and  $d_2$  connected to quark OAM (PRD 98 (2018) no.7, 074022)
- JLab provides best opportunity to explore valence region

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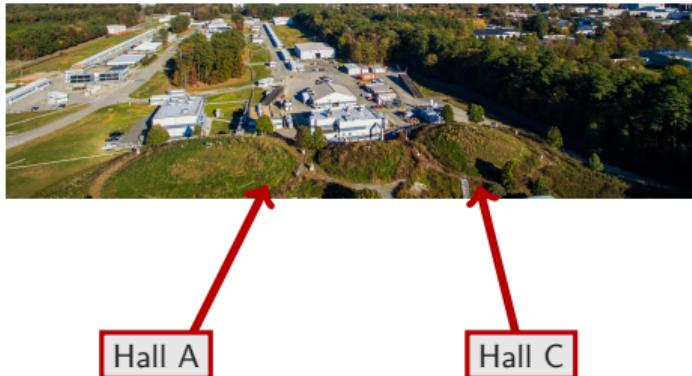
## Important starting point for Nucleon Tomography

- Extraction of  $\bar{g}_2$  is clean (free of non-local effects, fragmentation functions).
- Higher twist distribution  $\bar{g}_2$  provides important boundary condition for HT GPDs
- Quark OAM calculated from Higher twist GPDs
- First point in Qui-Sterman M.E. found in SIDIS



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# Complementary Jlab Experiments

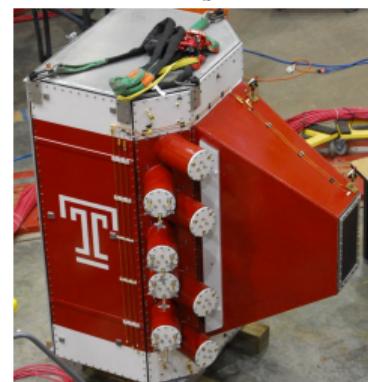
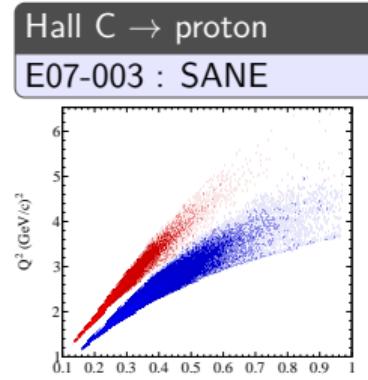
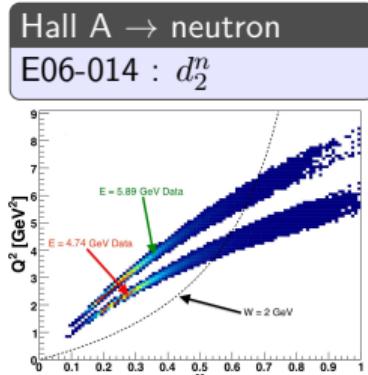


Similar kinematic coverage

Both used 4.7 GeV and 5.9 GeV beams

Both measured  $A_{\parallel}$  and  $A_{\perp}$

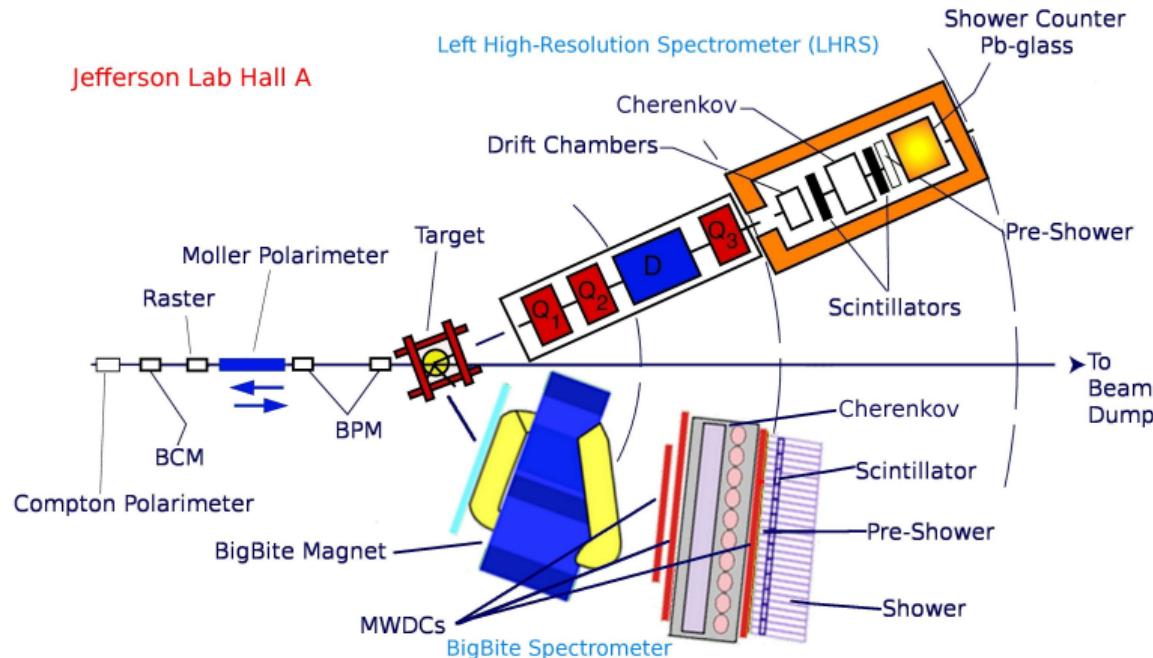
Both required new gas Cherenkov counters with existing detectors



# E06-014 : The $d_2^n$ Experiment

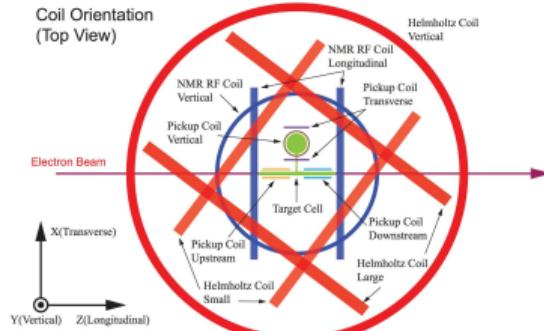
Spokespeople

B. Sawatzky, S. Choi, X. Jiang, and Z.-E. Meziani



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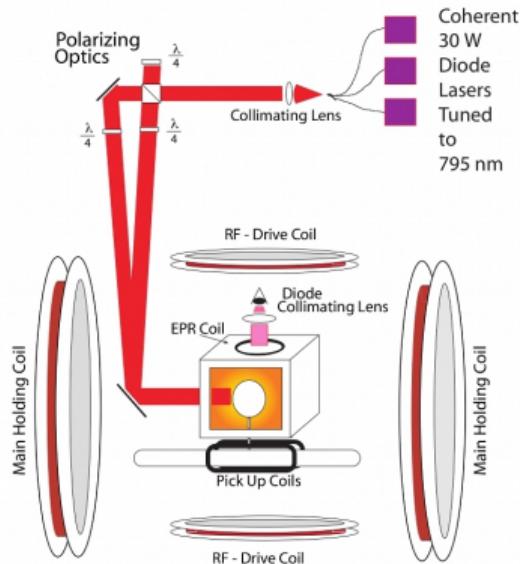
- Polarized  $^3\text{He}$  target



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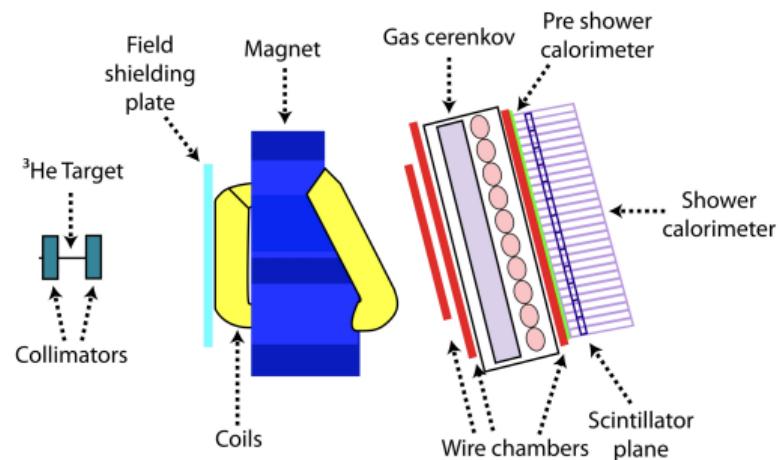


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- Polarized  $^3\text{He}$  target
- BigBite spectrometer



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# E06-014 : The $d_2^n$ Experiment

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- HRS data taken as well.



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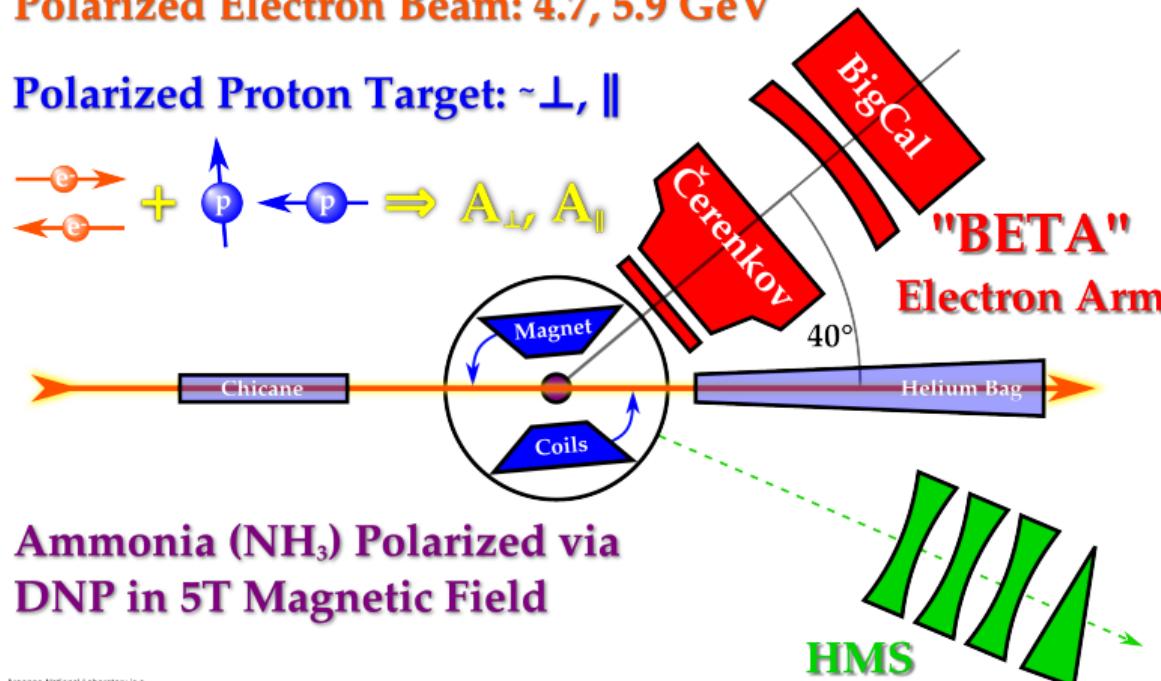
# E07-003 : Spin Asymmetries of the Nucleon Experiment

Spokespeople

S. Choi, M. Jones, Z.-E. Meziani, O.A. Rondon

Polarized Electron Beam: 4.7, 5.9 GeV

Polarized Proton Target:  $\sim \perp, \parallel$

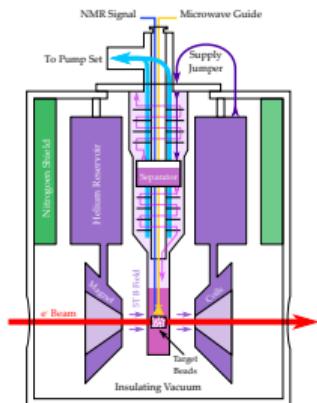
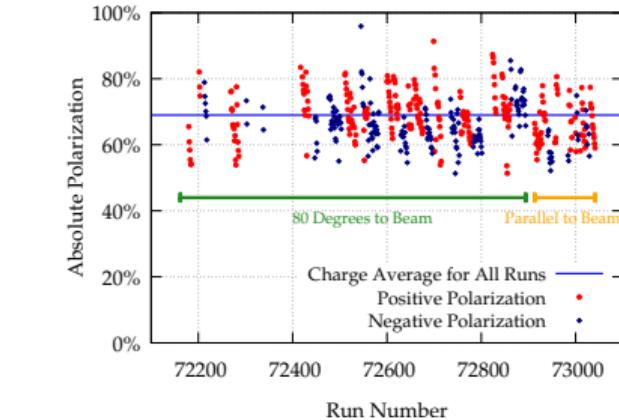


Ammonia ( $\text{NH}_3$ ) Polarized via DNP in 5T Magnetic Field



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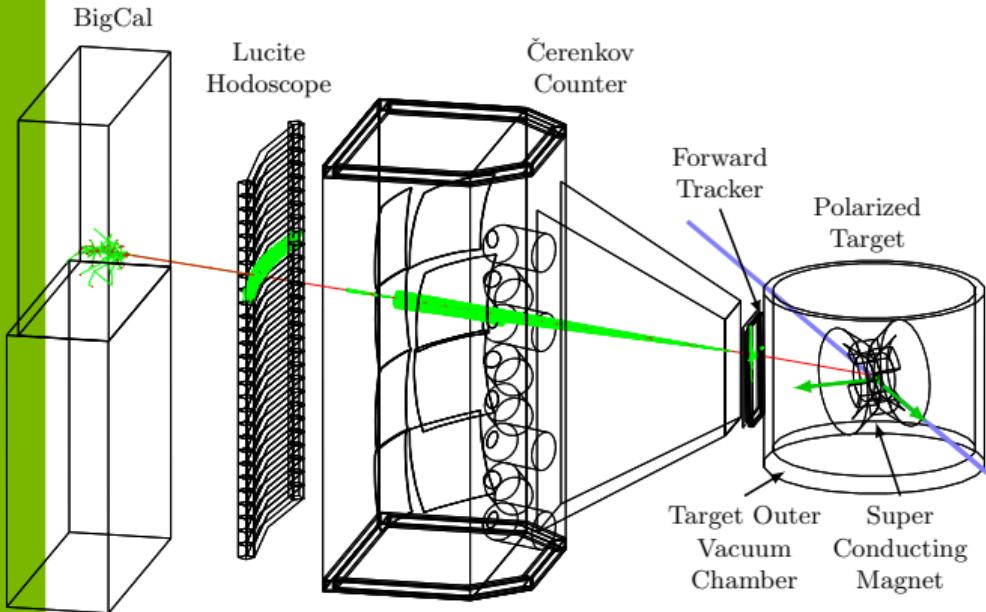
# E07-003 : Polarized Ammonia Target



- 5.1 T magnetic field
- Ammonia beads held by a cup, placed in LHe
- Average polarization was about 69%

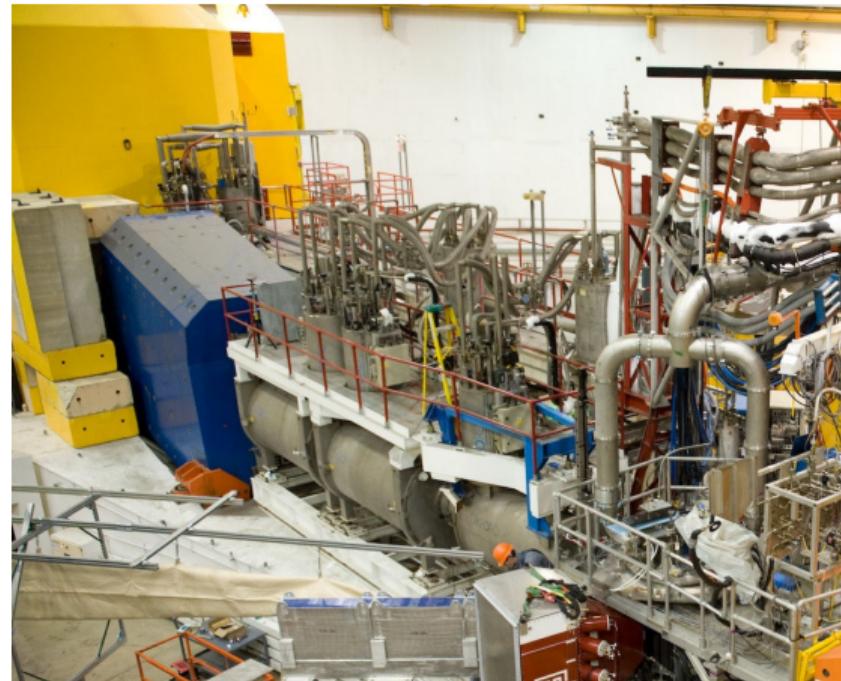
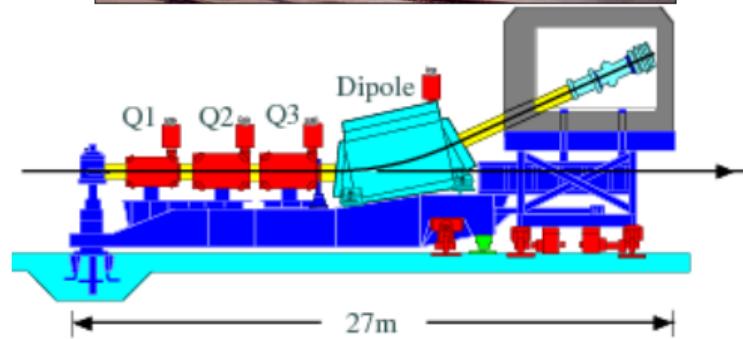


# E07-003 : Big Electron Telescope Array

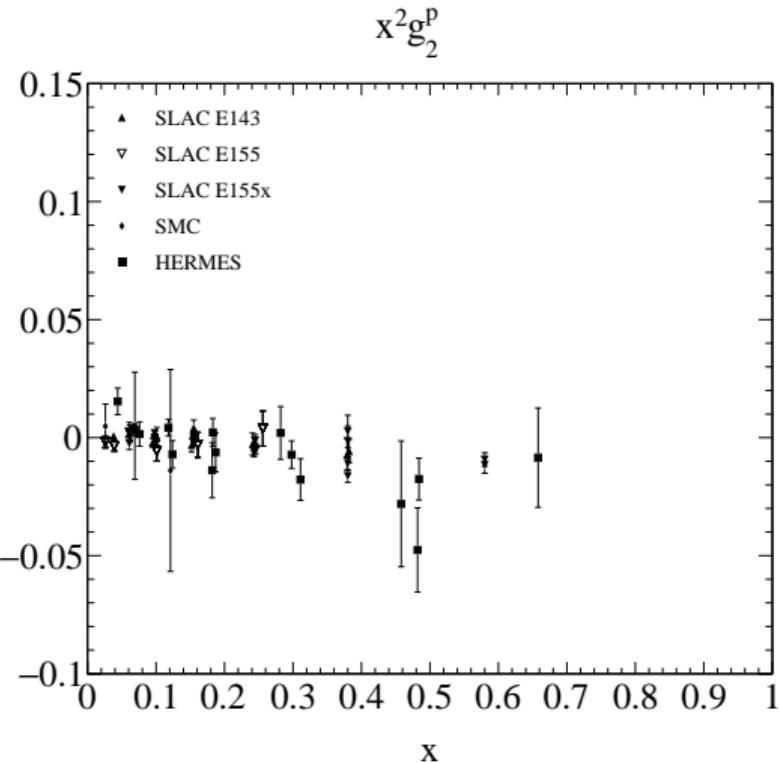
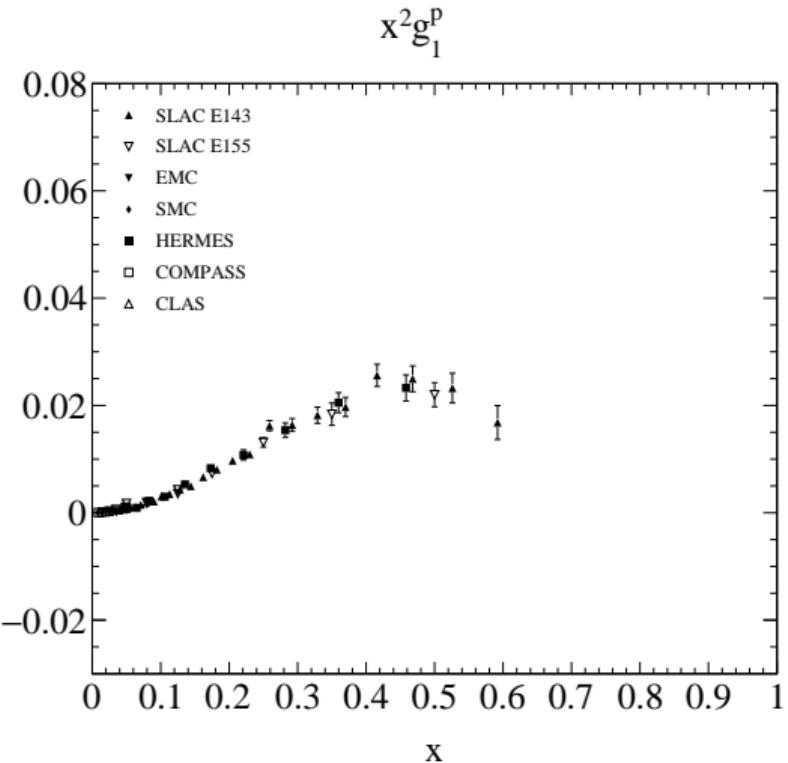


# E07-003 : Spin Asymmetries of the Nucleon Experiment

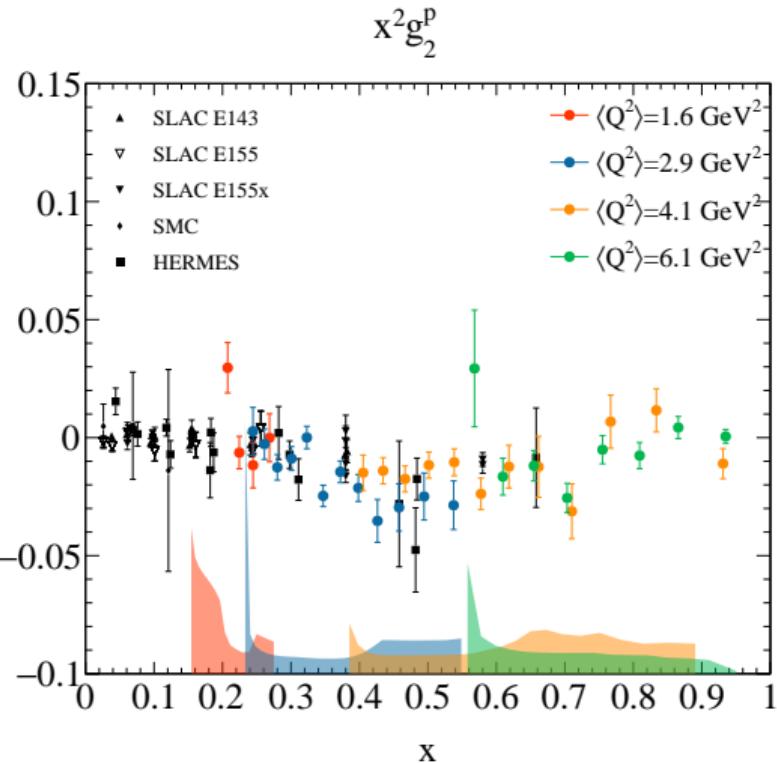
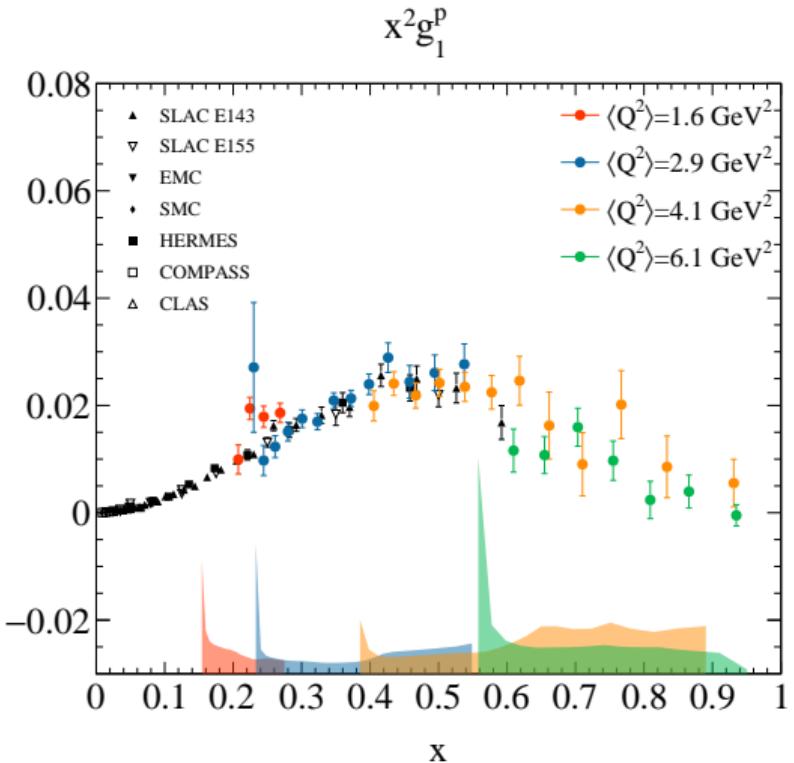
HMS data taken as well for resonance spin structure (Hoyoung Kang) and  $G_E/G_M$  (Anusha Liyanage)



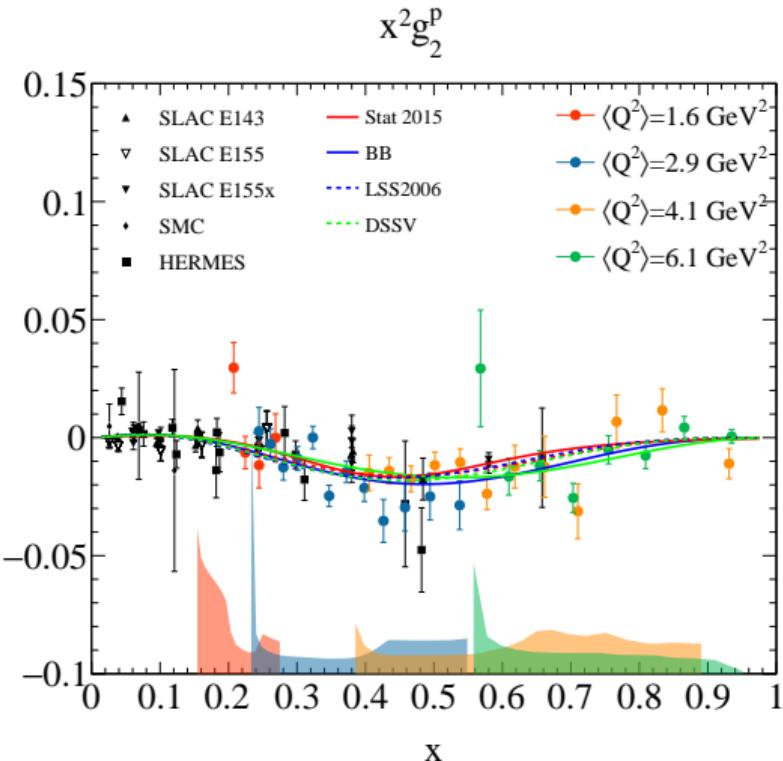
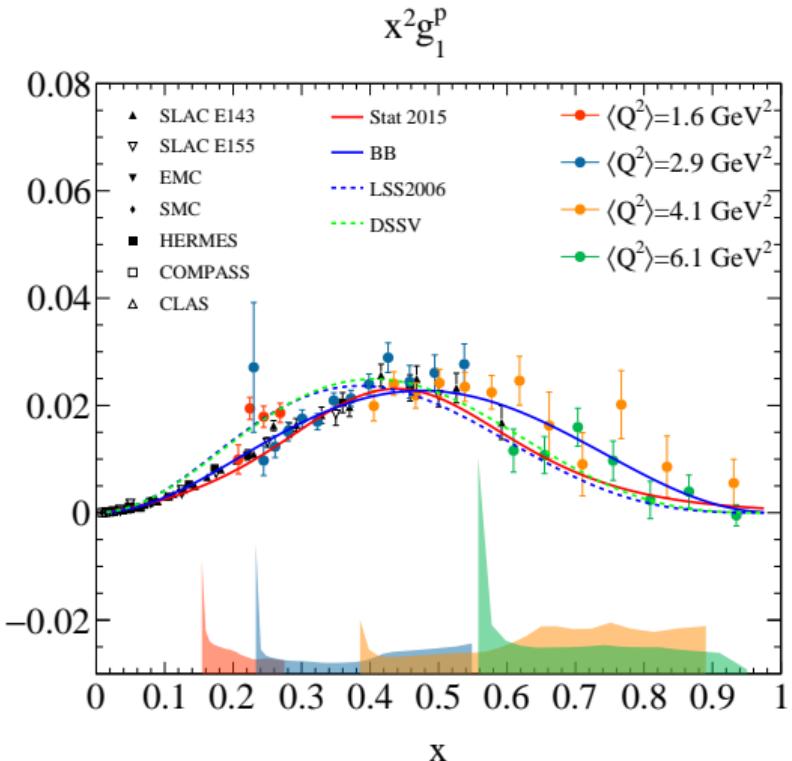
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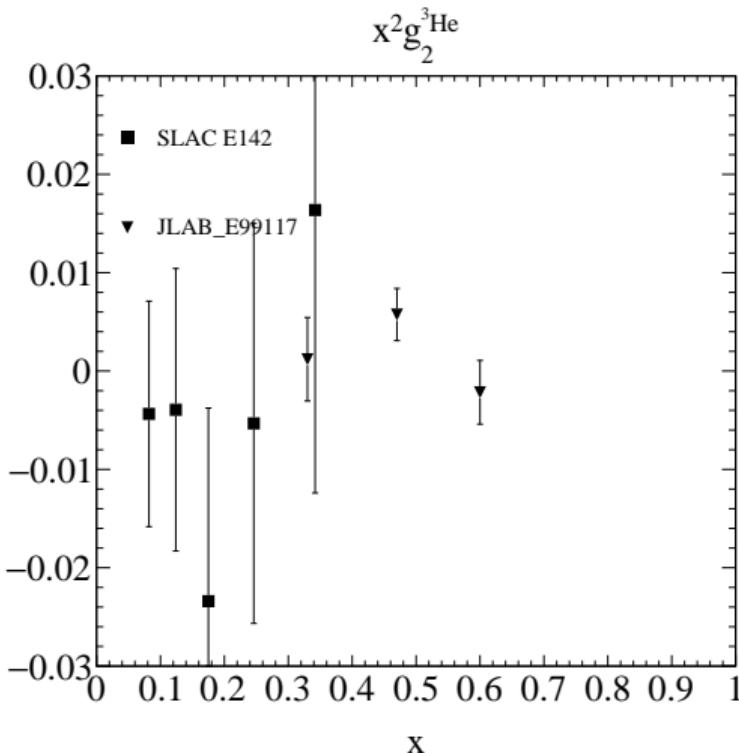
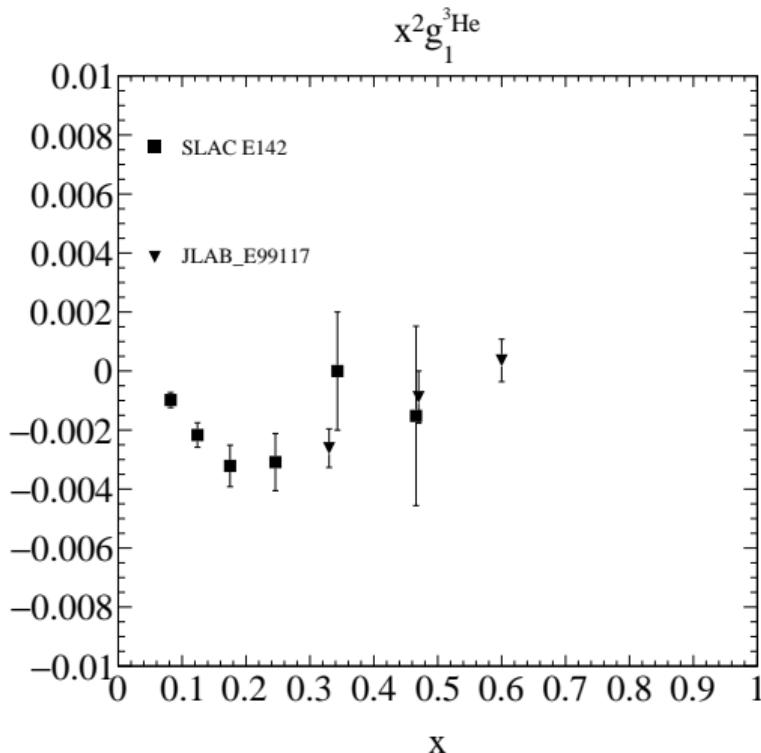
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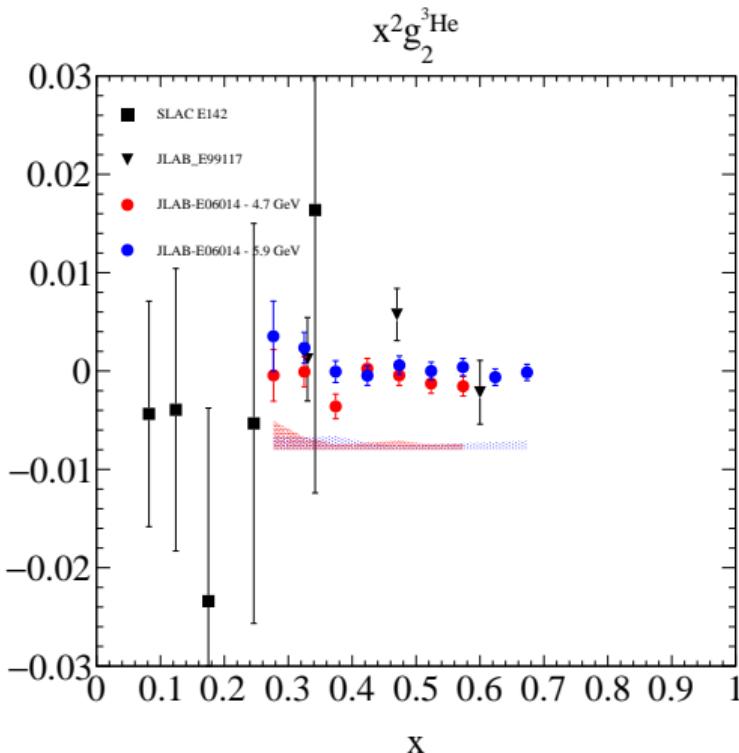
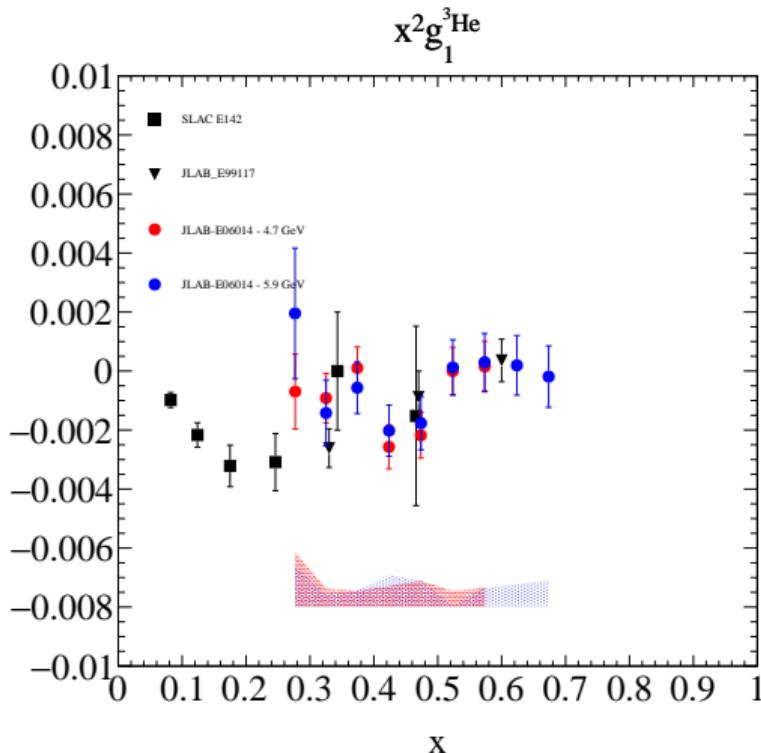


# $d_2^n$ results for $x^2 g_1^{^3He}$ and $x^2 g_2^{^3He}$



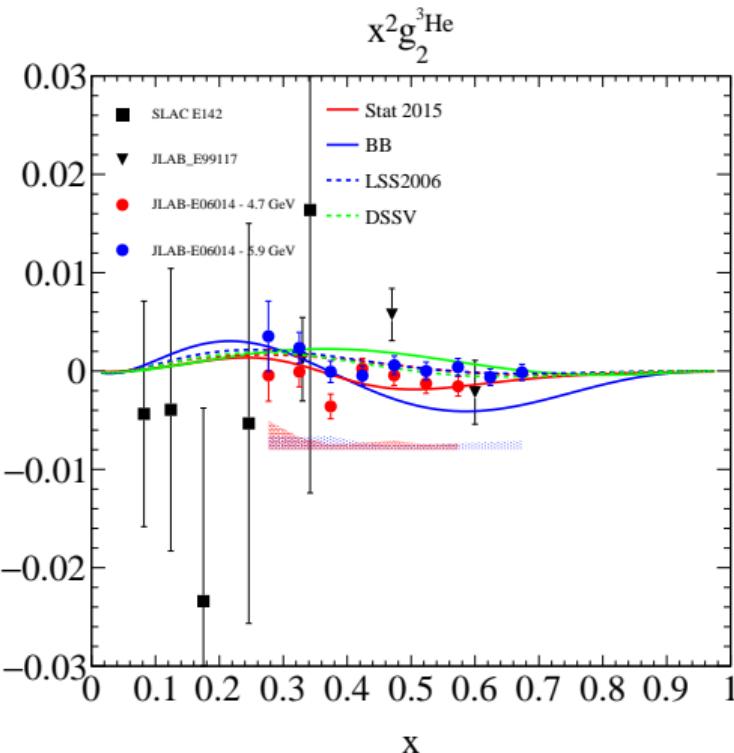
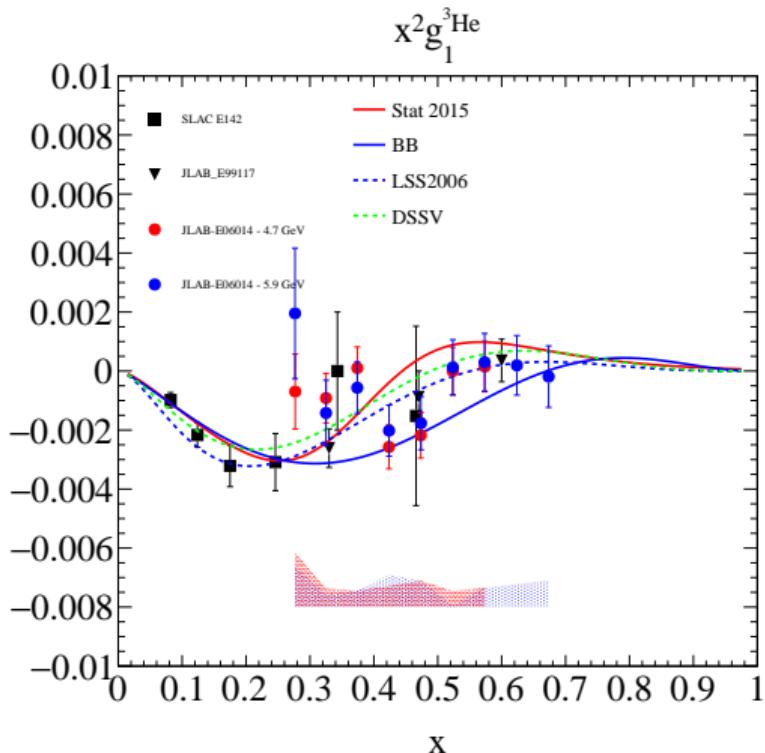
Models are showing  $g_{2R}^{WW}$

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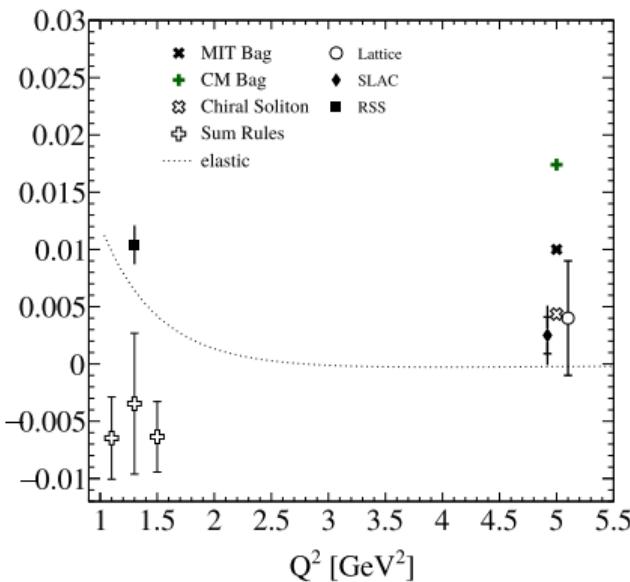
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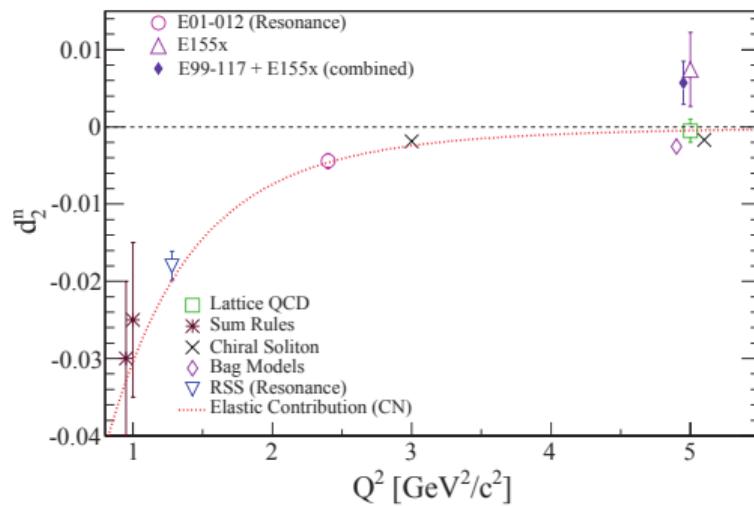


Models are showing  $g_{2R}^{WW}$

## proton

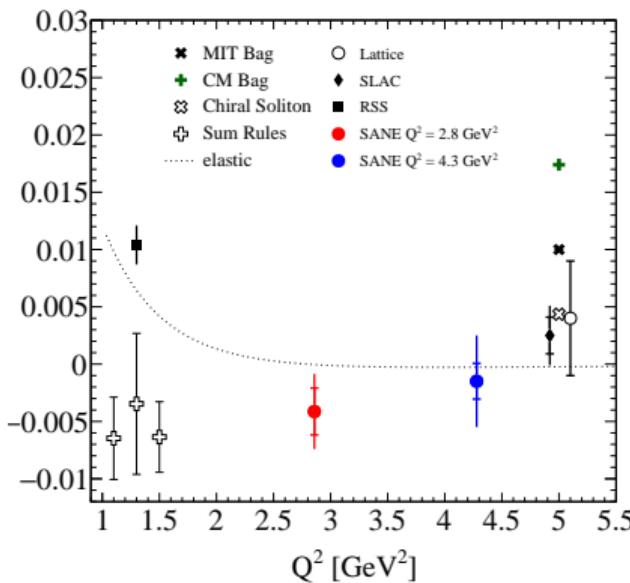


## neutron

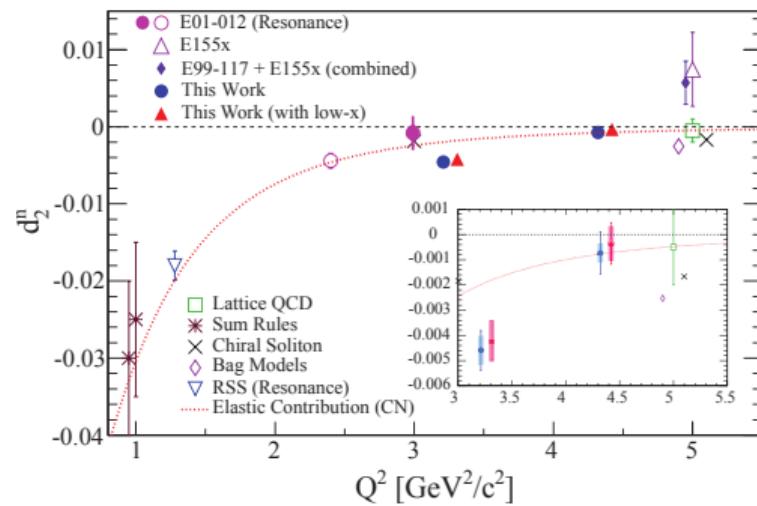


## Existing data

# proton



# neutron

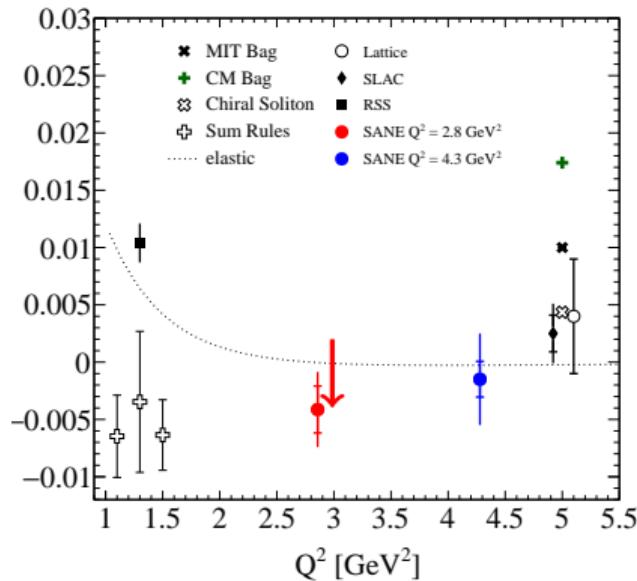


Neutron from  $d_2^n$  experiment: D.Flay, et.al.  
PRD.94(2016)no.5,052003

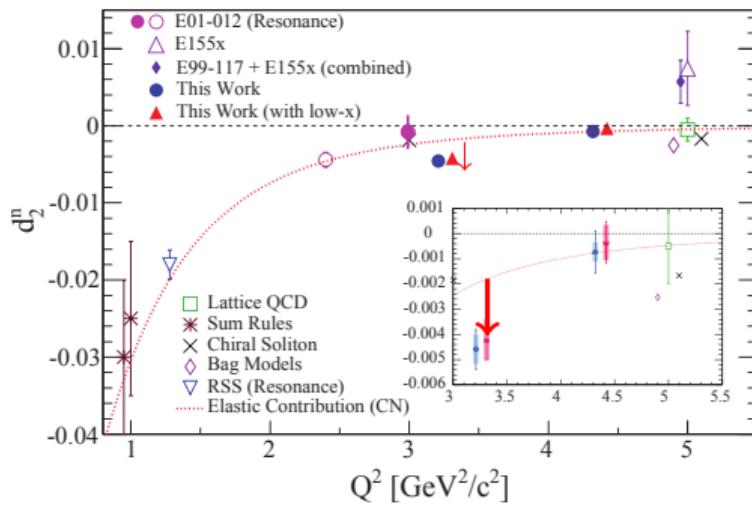
## SANE and $d_2^n$ Result

- $d_2$  dips around  $Q^2 \sim 3$  GeV $^2$  for proton and neutron

# proton



# neutron

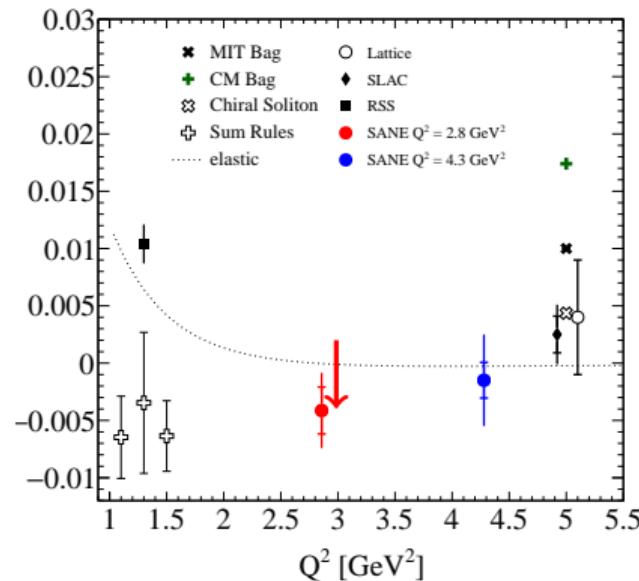


Neutron from  $d_2^n$  experiment: D.Flay, et.al.  
PRD.94(2016)no.5,052003

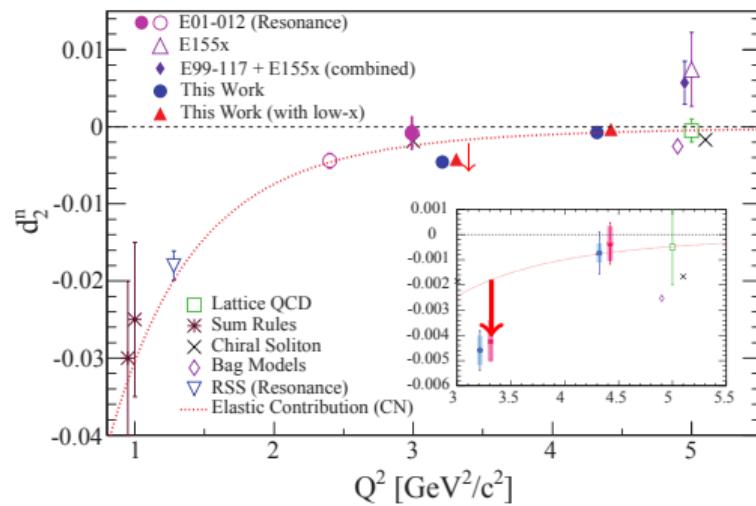
## SANE and $d_2^n$ Result

- $d_2$  dips around  $Q^2 \sim 3$  GeV $^2$  for proton and neutron
- Is this an isospin independent average color force?

## proton



## neutron



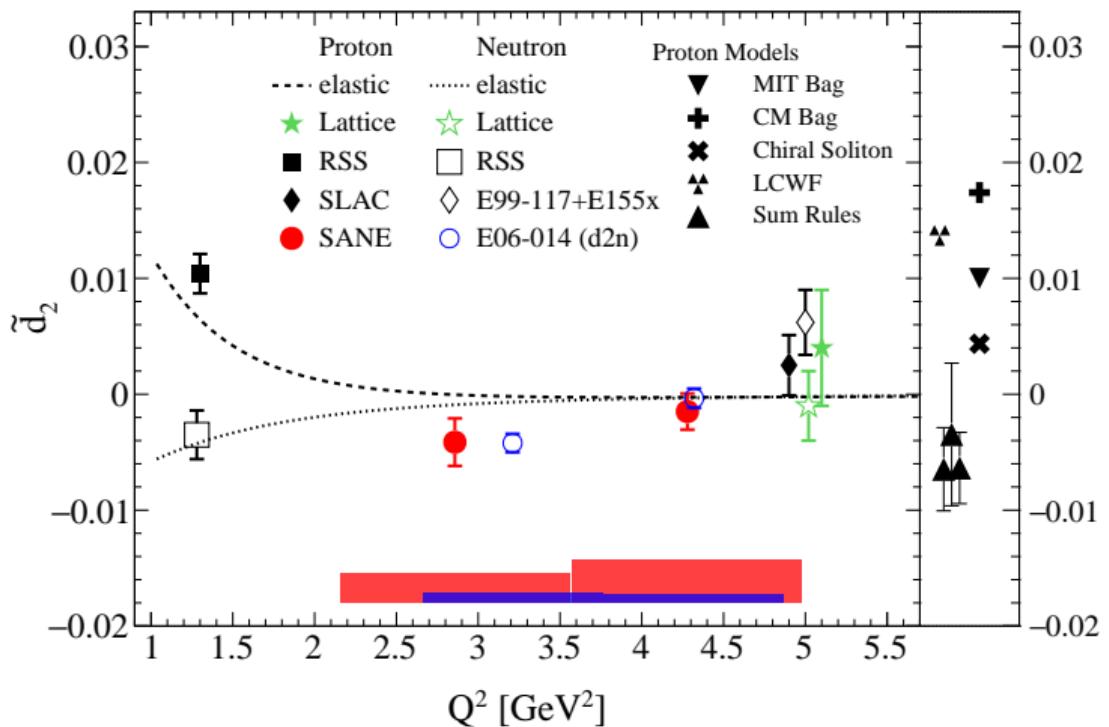
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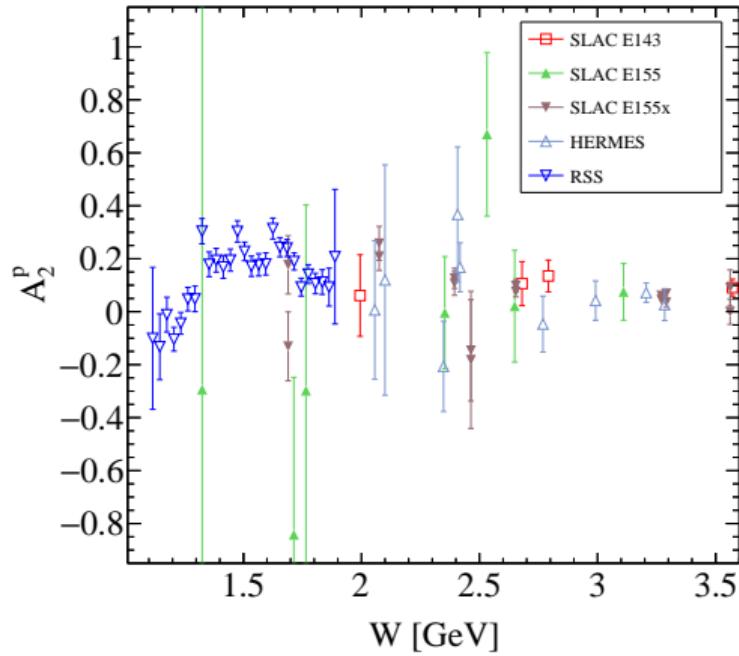
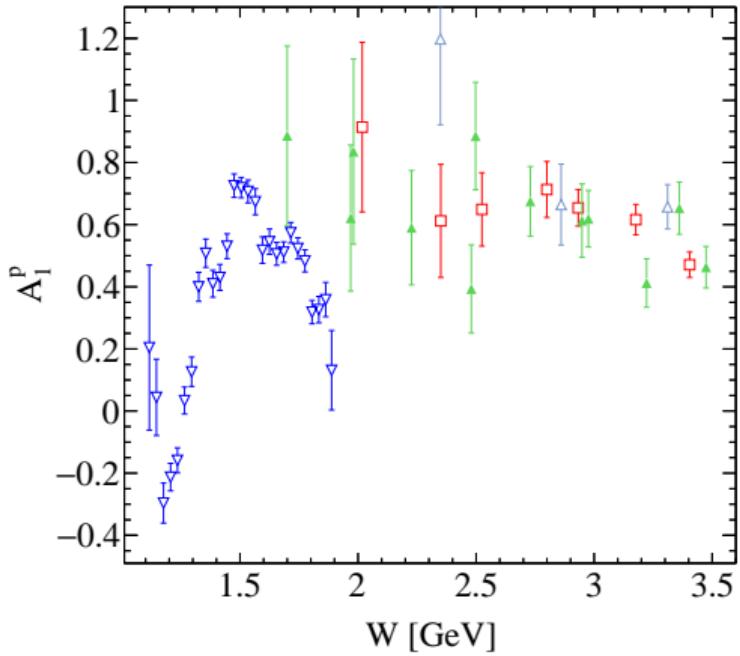
- $d_2$  dips around  $Q^2 \sim 3$  GeV $^2$  for proton and neutron
- Is this an isospin independent average color force?
- Updated Lattice calculations are long over due!

# $d_2^p$ Results

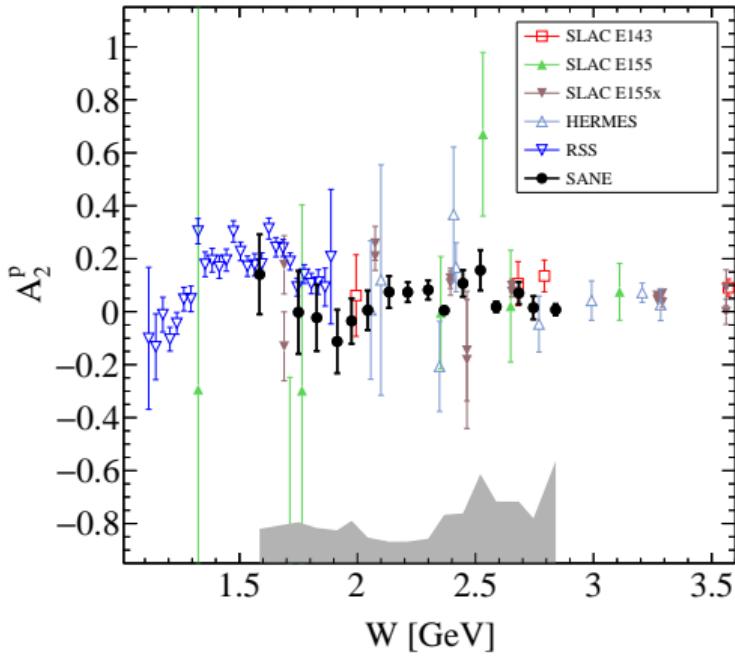
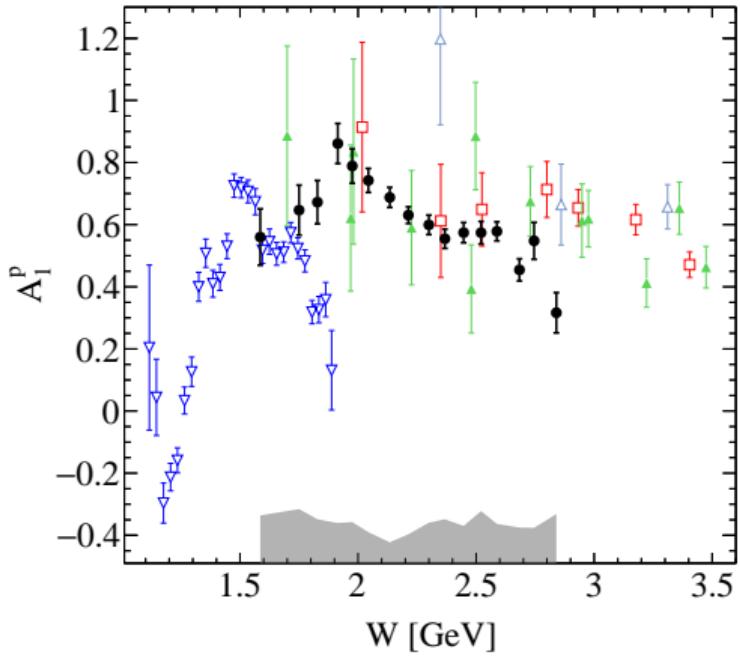
PRL 122, 022002 (2019)



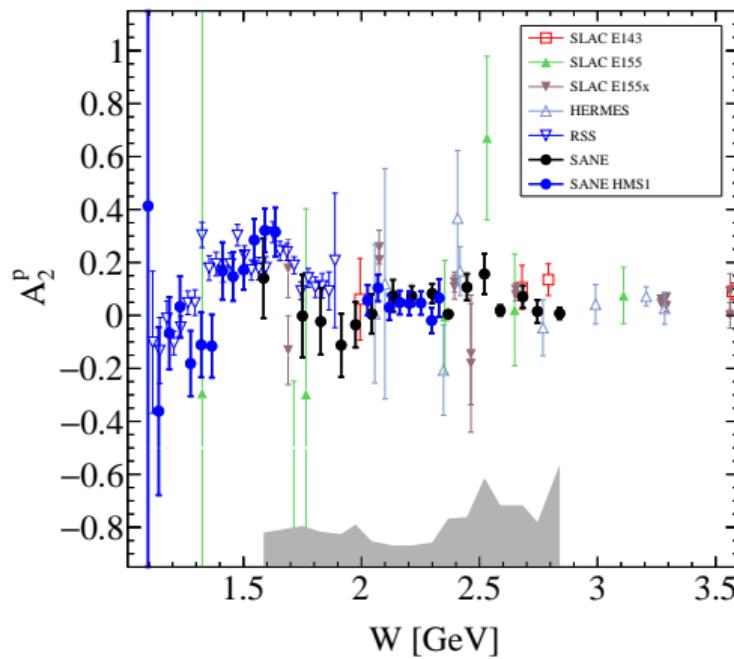
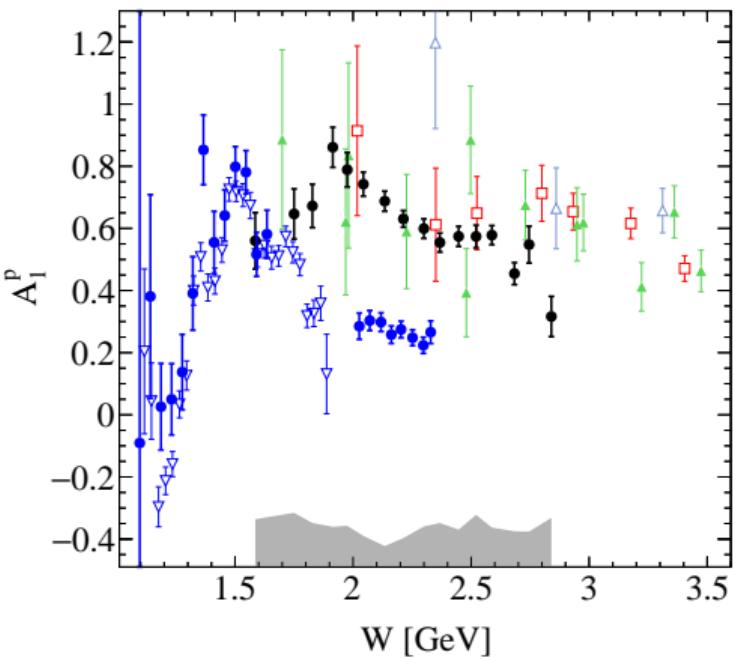
# Virtual Compton Scattering Asymmetries



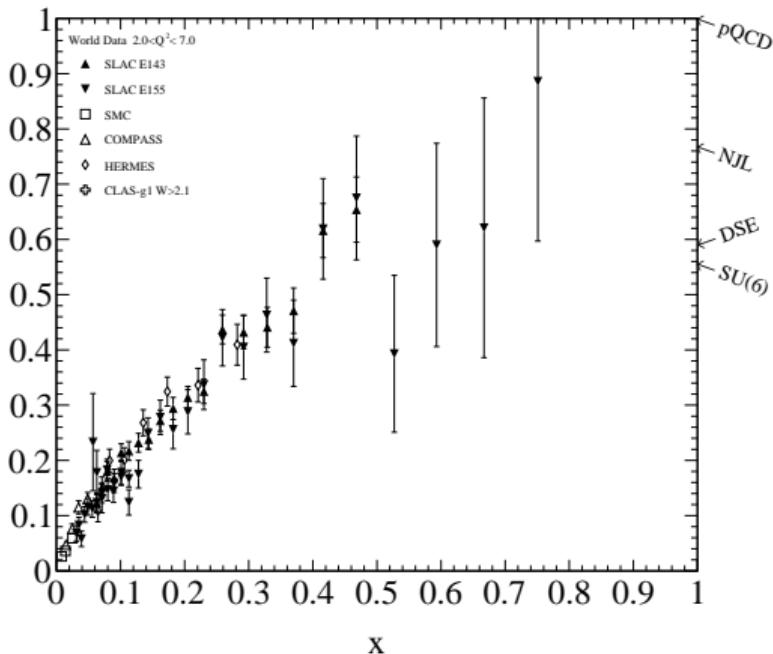
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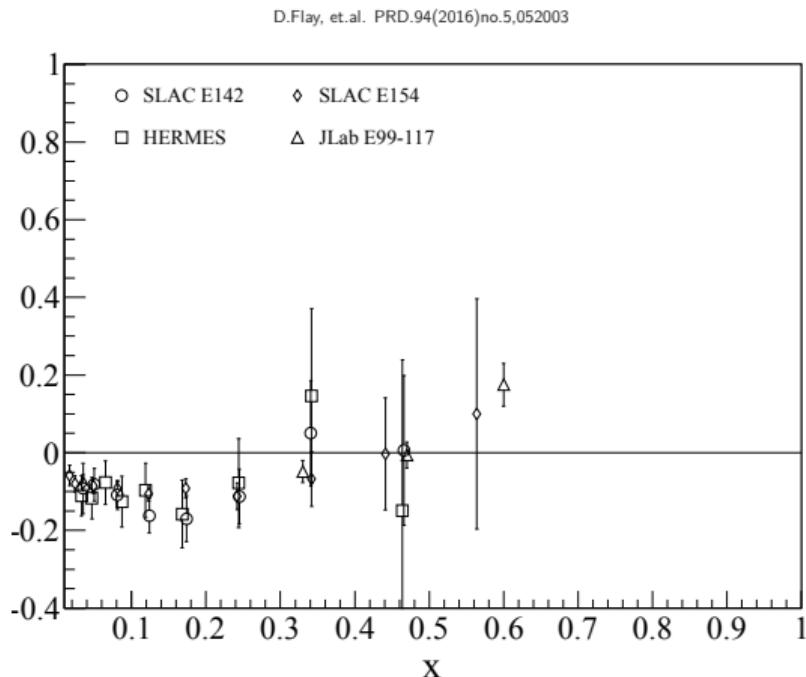
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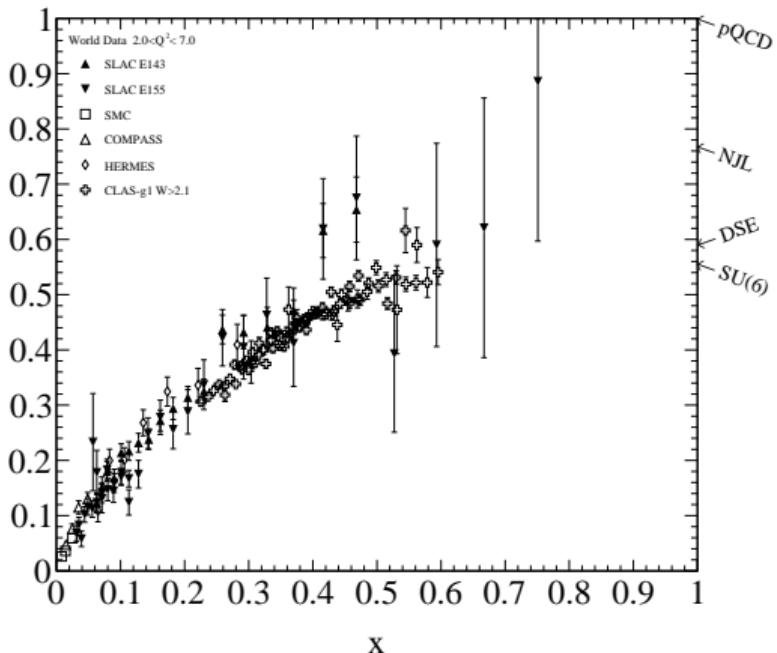
# Valence domain: $A_1$ at high $x$



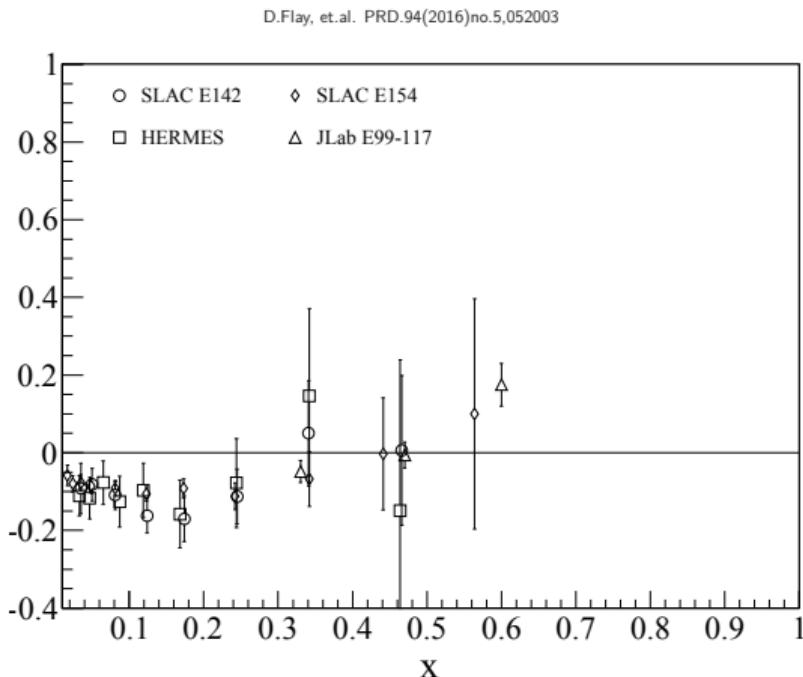
- $A_1$  as  $x \rightarrow 1$



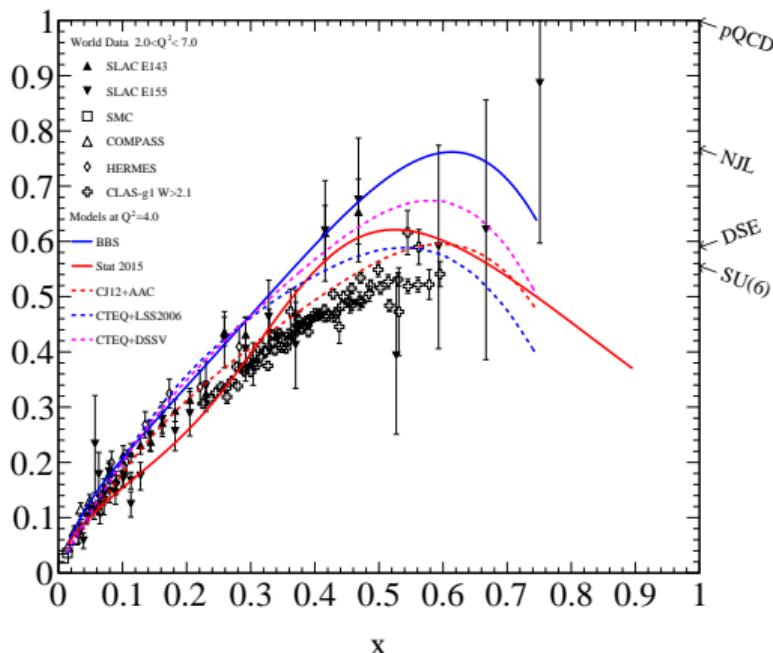
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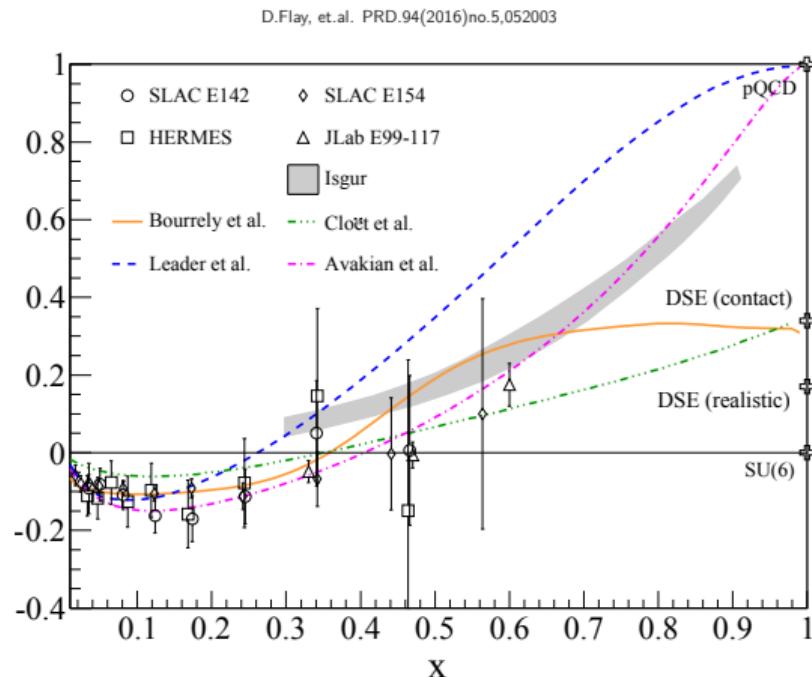
- $A_1$  as  $x \rightarrow 1$
- CLAS data. Note: only the combination  $A_1 + \eta A_2$  is measured by CLAS.



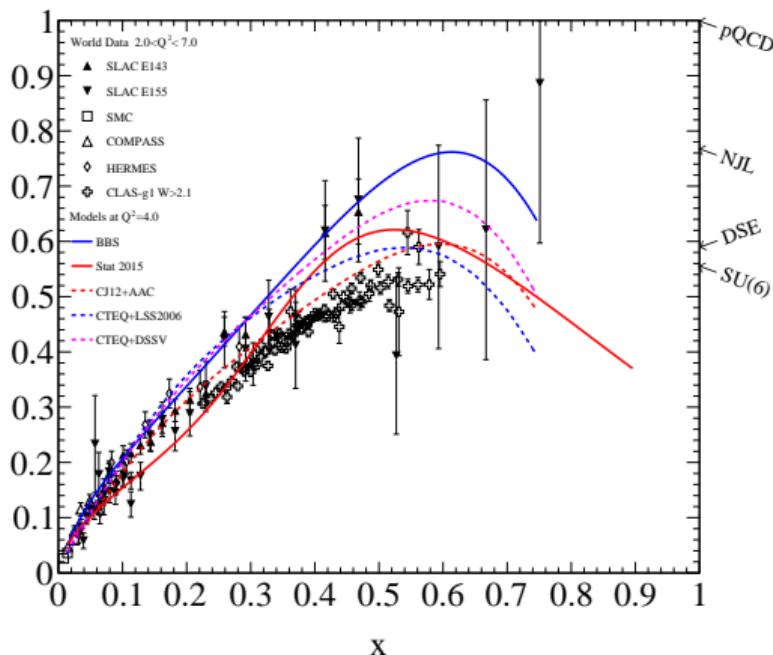
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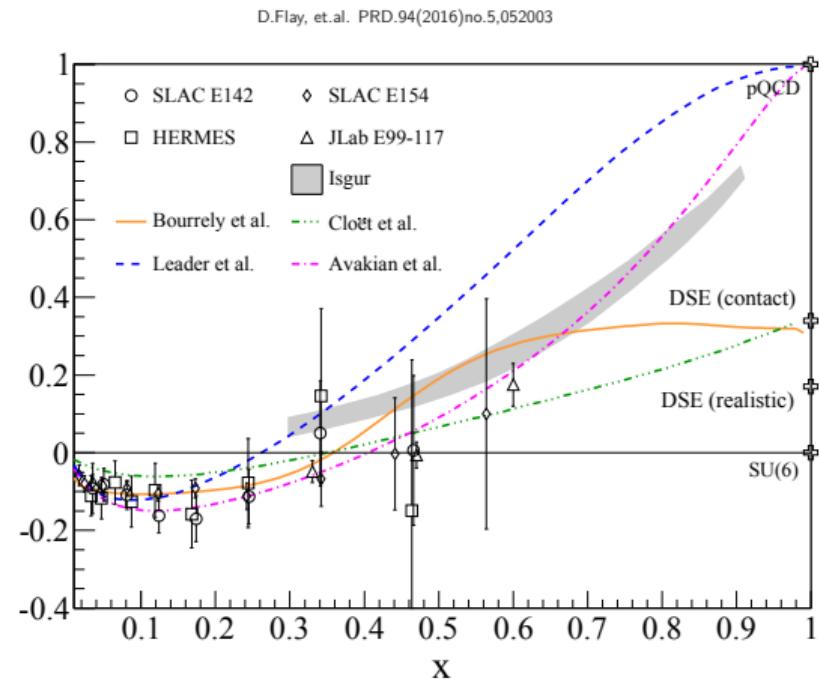
- $A_1$  as  $x \rightarrow 1$
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- Many predictions from models and fits



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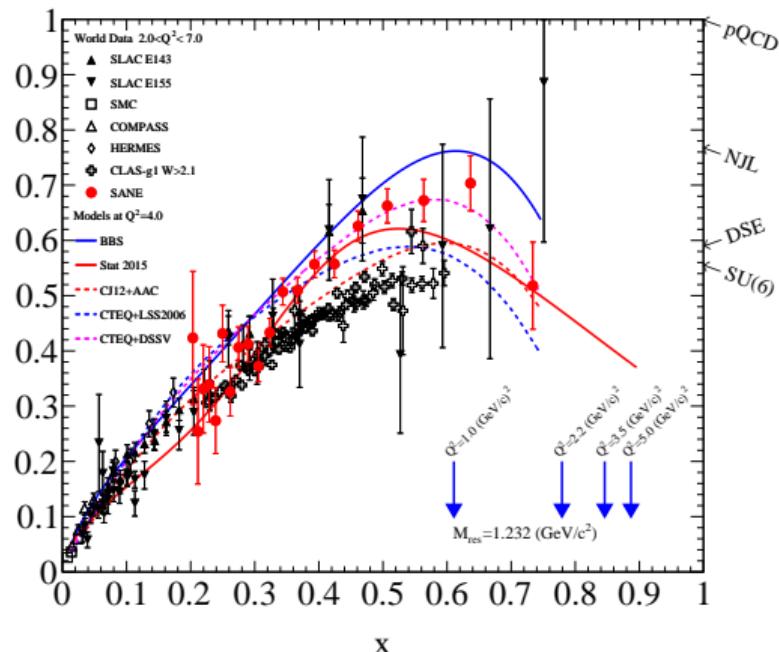


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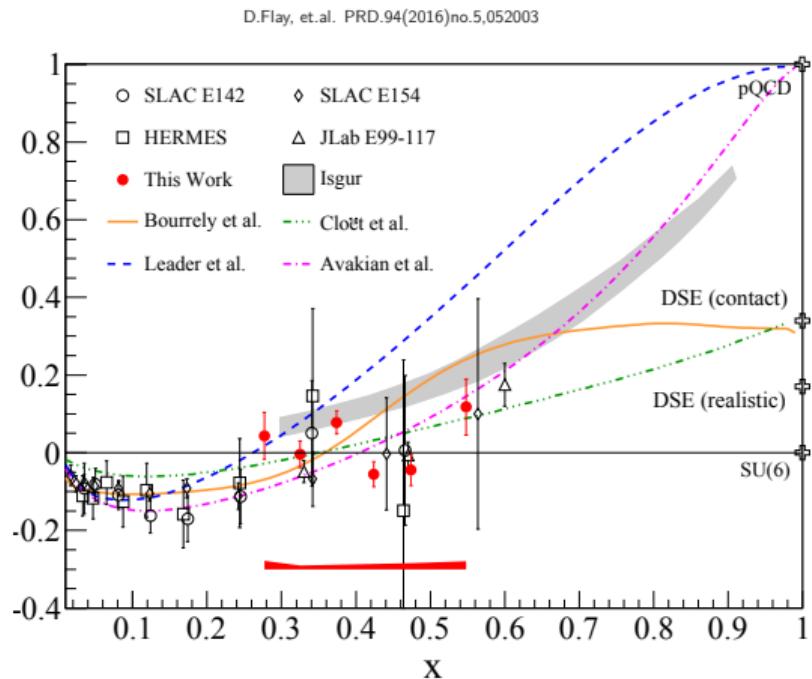


- Dyson-Schwinger Equations (DSE)  $x = 1$  predictions (Roberts, Holt, Schmidt)

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- Dyson-Schwinger Equations (DSE)  $x = 1$  predictions (Roberts, Holt, Schmidt)
- SANE data goes out to  $x \simeq 0.8 \rightarrow$  use duality to check limit

# Summary

- SANE results *significantly* improve world data on  $g_2^p$  and  $g_1^p$  (archival paper in the works)
- $d_2^p$  and  $d_2^n$  **scale dependence is puzzling** and should be compared with **modern Lattice calculations**
- $d_2^p$  and  $d_2^n$  results suggest some interesting QCD physics
- **Precision**  $g_2$  measurements at varying  $Q^2$  are needed to verify apparent scale dependence

Thank You!