GHP meeting, Denver, April 11, 2019

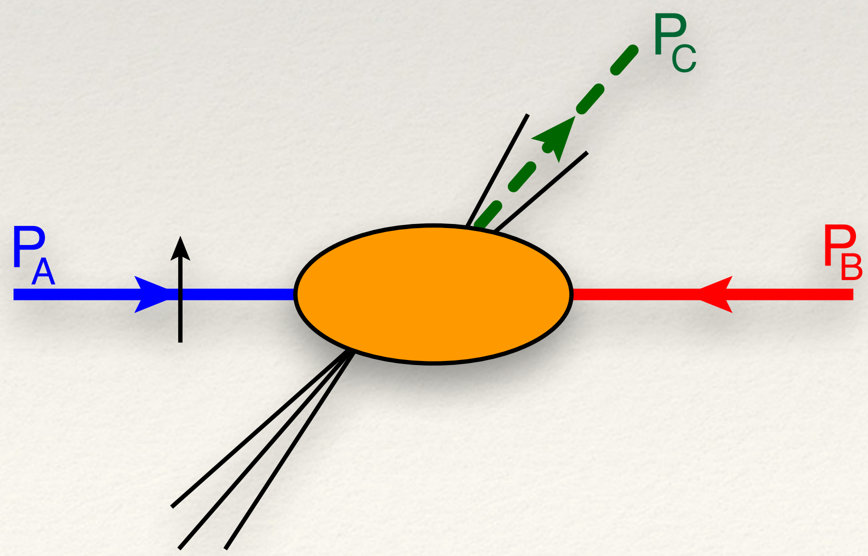
Twist-3 PDFs/FFs

Marc Schlegel
Department of Physics
New Mexico State University

based on
L. Gamberg, Z. Kang, D. Pitonyak, M.S., S. Yoshida, JHEP 1901, 111 (2019)

Transverse Spin Effects in Single-Inclusive Hard Processes

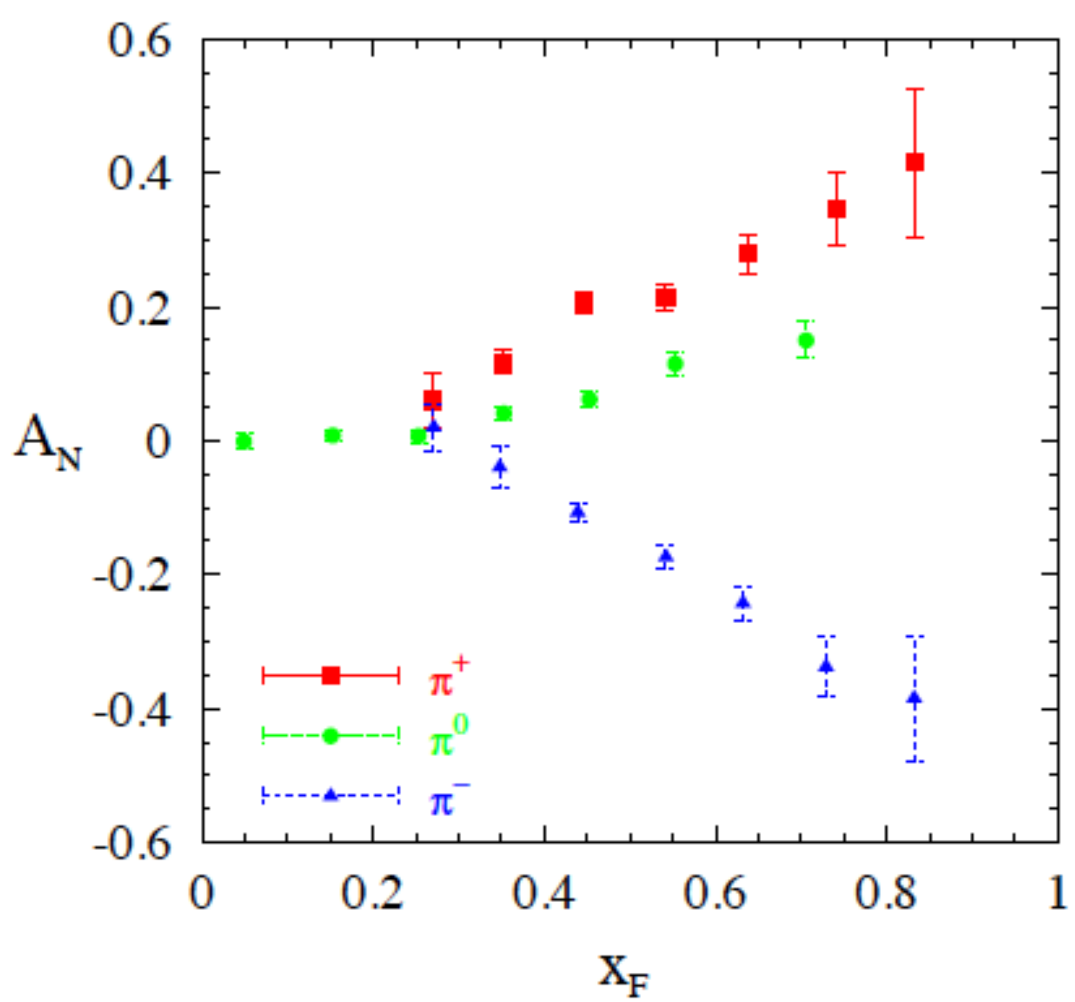
$$P_A^{\uparrow} + P_B \rightarrow P_C + X$$



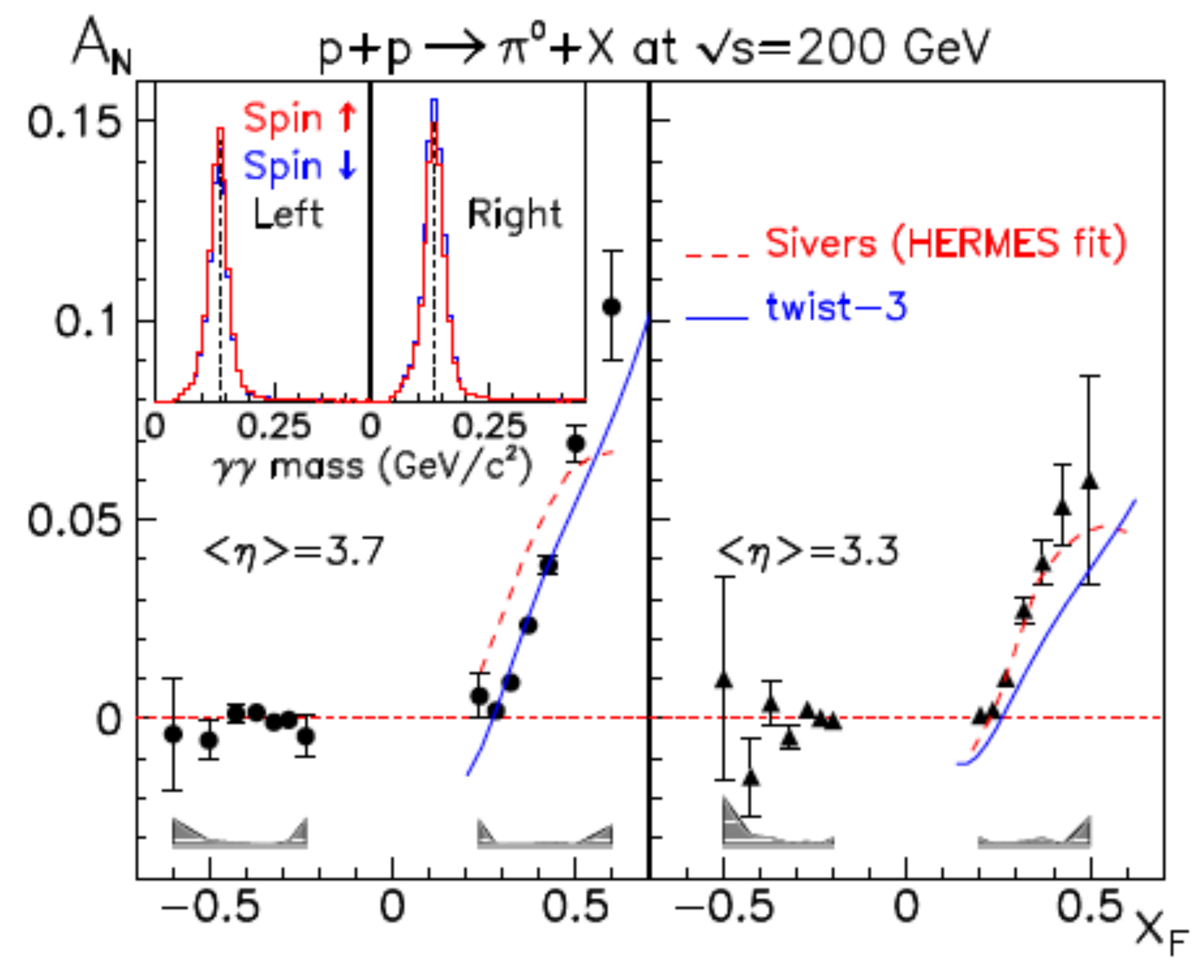
"Show-off" Transverse SSA

$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

$$p + p^{\uparrow} \rightarrow \pi + X$$



$\sqrt{s} = 20$ GeV [*E704 coll. (1991)*]



$\sqrt{s} = 200$ GeV [*STAR coll. (2008)*]

large effects

cannot be explained in the standard parton model

(using transversity)

➙ collinear Twist-3 Formalism

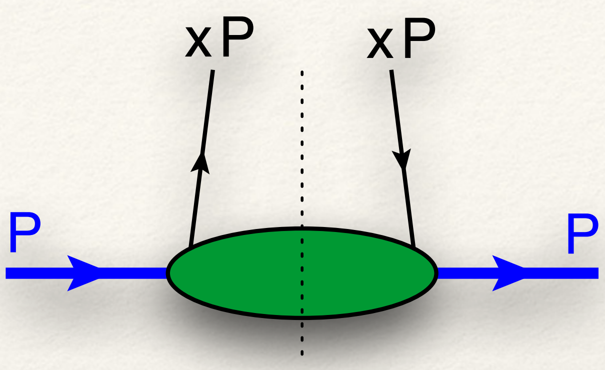
(Efremov, Teryaev, Qiu, Sterman)

Collinear twist-3 formalism: several types of (*chiral-even&odd*) matrix elements compete

Collinear twist-3 formalism: several types of (*chiral-even&odd*) matrix elements compete

intrinsic twist-3 PDF

$$g_T^q(x) = -\frac{1}{M}\int \frac{d\lambda}{4\pi}\, \mathrm{e}^{i\lambda x} \langle P, S_T |\, \bar{q}(0)\, \not{S}_T \gamma_5\, q(\lambda n)\, | P, S_T \rangle$$

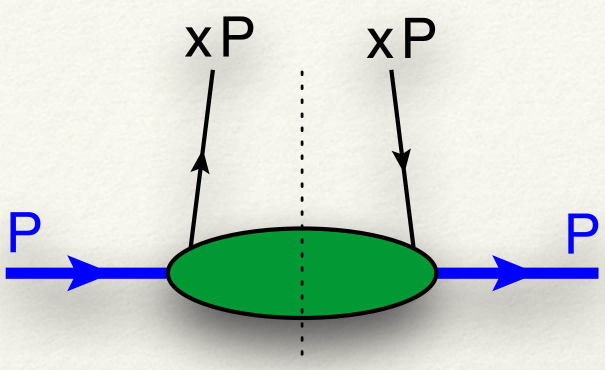


- sensitive to 'bad quark field components',
- twist-3 characteristics hidden in Dirac structure
- generates the g_2 structure function in DIS
- No probabilistic density interpretation

Collinear twist-3 formalism: several types of (*chiral-even&odd*) matrix elements compete

intrinsic twist-3 PDF

$$g_T^q(x) = -\frac{1}{M}\int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\langle P, S_T|\, \bar{q}(0)\, \not{S}_T\gamma_5\, q(\lambda n)\,|P, S_T\rangle$$

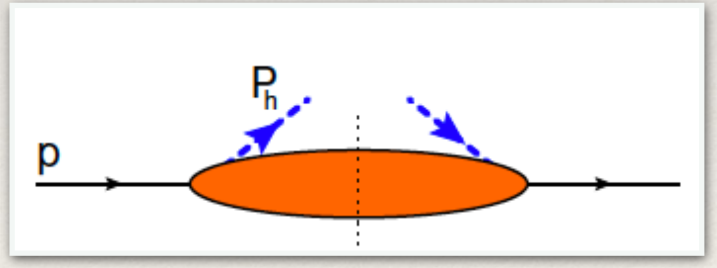


- sensitive to 'bad quark field components',
- twist-3 characteristics hidden in Dirac structure
- generates the g_2 structure function in DIS
- No probabilistic density interpretation

intrinsic twist-3 FF

$$G_T^q(z) = -\frac{1}{M_h}\frac{z}{4N_c}\sum_X\int \frac{d\lambda}{2\pi}\, e^{-i\frac{\lambda}{z}}\langle 0|\, q(0)\,|P_h S_{hT}, X\rangle\langle P_h S_{hT}, X|\bar{q}(\lambda n)\, \not{S}_{hT}\gamma_5|0\rangle$$

- Final state hadron spin may be reconstructed, e.g., for a Λ (later)

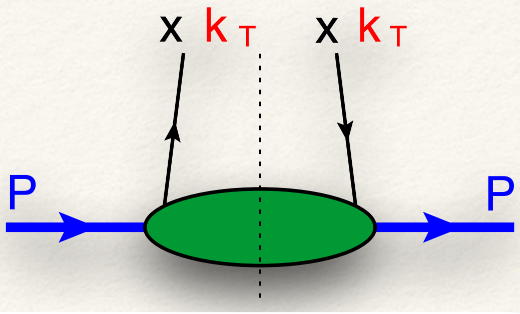


$$D_T^q(z) = \frac{1}{M_h}\frac{z}{4N_c}\sum_X\int \frac{d\lambda}{2\pi}\, e^{-i\frac{\lambda}{z}}\langle 0|\, q(0)\,|P_h S_{hT}, X\rangle\langle P_h S_{hT}, X|\bar{q}(\lambda n)\, \epsilon^{P_h n \alpha S_h}\gamma_\alpha|0\rangle$$

- yet another function, time-reversal violated in fragmentation process (!)

kinematical twist-3 PDFs:

Small transverse quark/gluon momenta k_T:



$$(k_T \times S_T)\, f_{1T}^{\perp,q}(x,k_T^2) \propto \int \frac{d\lambda\, d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{i\lambda x + i k_T\cdot z_T} \langle P,S_T|\bar{q}(0)\, \not{n} \mathcal{W}\, q(\lambda n + z_T)\,|P,S_T\rangle$$

Sivers function

$$(k_T \cdot S_T)\, g_{1T}^{q}(x,k_T^2) \propto \int \frac{d\lambda\, d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{i\lambda x + i k_T\cdot z_T} \langle P,S_T|\bar{q}(0)\, \not{n}\gamma_5 \mathcal{W}\, q(\lambda n + z_T)\,|P,S_T\rangle$$

'transhelicity'

Collinear twist-3 formalism: TMD moments are needed

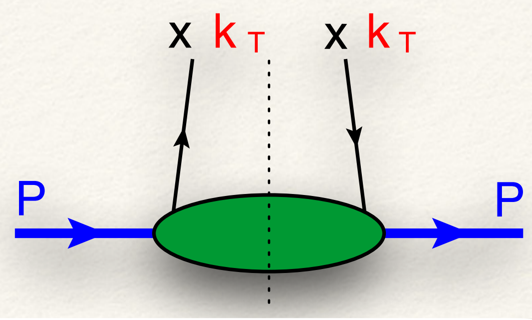
$$f_{1T}^{\perp,(1)}(x) = \int d^2k_T\, \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x,k_T^2)$$

$$g_{1T}^{(1)}(x) = \int d^2k_T\, \frac{k_T^2}{2M^2} g_{1T}(x,k_T^2)$$

→ twist-3 characteristics through small transverse parton momentum k_T

kinematical twist-3 PDFs:

Small transverse quark/gluon momenta k_T:



$$(k_T \times S_T)\, f_{1T}^{\perp,q}(x,k_T^2) \propto \int \frac{d\lambda\, d^2 z_T}{(2\pi)^3}\, e^{i\lambda x + i k_T\cdot z_T} \langle P,S_T|\bar{q}(0)\, \not{n}\, \mathcal{W}\, q(\lambda n + z_T)\,|P,S_T\rangle$$

Sivers function

$$(k_T \cdot S_T)\, g_{1T}^{q}(x,k_T^2) \propto \int \frac{d\lambda\, d^2 z_T}{(2\pi)^3}\, e^{i\lambda x + i k_T\cdot z_T} \langle P,S_T|\bar{q}(0)\, \not{n}\gamma_5\, \mathcal{W}\, q(\lambda n + z_T)\,|P,S_T\rangle$$

'transhelicity'

Collinear twist-3 formalism: TMD moments are needed

$$f_{1T}^{\perp,(1)}(x) = \int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x,k_T^2)$$

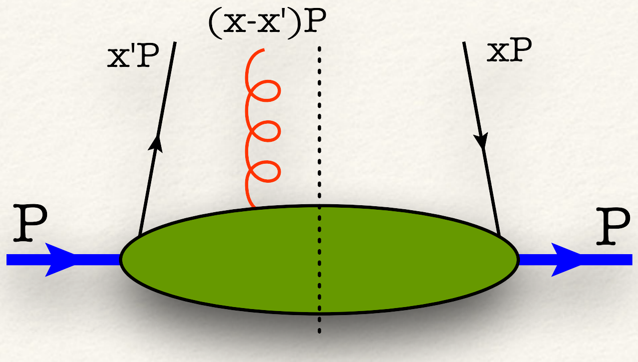
$$g_{1T}^{(1)}(x) = \int d^2k_T\, \frac{k_T^2}{2M^2}\, g_{1T}(x,k_T^2)$$

→ twist-3 characteristics through small transverse parton momentum k_T

Same for fragmentation: *kinematical* twist-3 FF with transverse spin:

$$\Delta_\partial^\alpha(z) = \int d^2p_T\, p_T^\alpha\, \Delta(z, z p_T) \longrightarrow \quad G_{1T}^{\perp(1),\Lambda/q}(z) \quad D_{1T}^{\perp(1),\Lambda/q}(z)$$

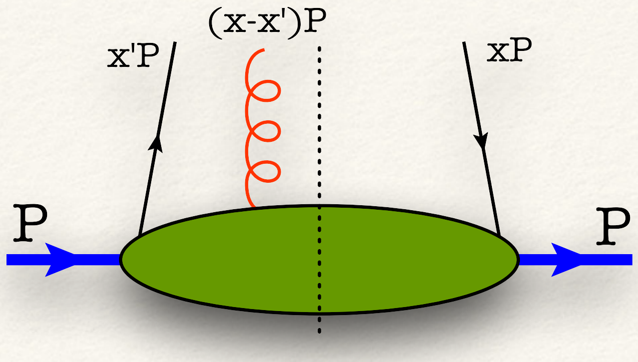
Dynamical twist-3: Quark - Gluon - Quark Correlations
(ETQS-matrix elements)



$$2M\, i\epsilon^{Pn\rho S}\, F_{FT}^{q}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} \mathrm{e}^{i\lambda x'} \mathrm{e}^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

$$2M\, S_T^{\rho} G_{FT}^{q}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} \mathrm{e}^{i\lambda x'} \mathrm{e}^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\gamma_5\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

Dynamical twist-3: Quark - Gluon - Quark Correlations (ETQS-matrix elements)



$$2M\, i\epsilon^{Pn\rho S} F^q_{FT}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} \mathrm{e}^{i\lambda x'} \mathrm{e}^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

$$2M\, S^\rho_T G^q_{FT}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} \mathrm{e}^{i\lambda x'} \mathrm{e}^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\gamma_5\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

‘dynamical twist - 3’

→ 3 - parton correlator: suppression by additional propagator

→ dependence on two parton momenta x, x’:
2-dimensional support, richer parton dynamics

→ so far: only “diagonal support” $\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$ constraint by data

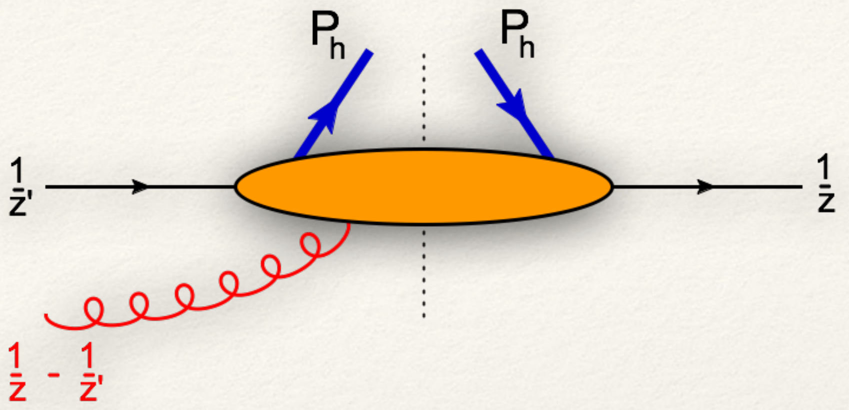
→ ‘integrated’ $F_{FT}(x,x')$: average transverse color Lorentz force on struck quark
[Burkardt, PRD88, 114502]

$$F^{n\rho} = [\vec{E} + \vec{n} \times \vec{B}]^\rho \propto \int dx \int dx'\, F_{FT}(x,x') \propto \int dx\, x^2\, g_T(x)$$

'dynamical' twist-3 FF with transverse spin:

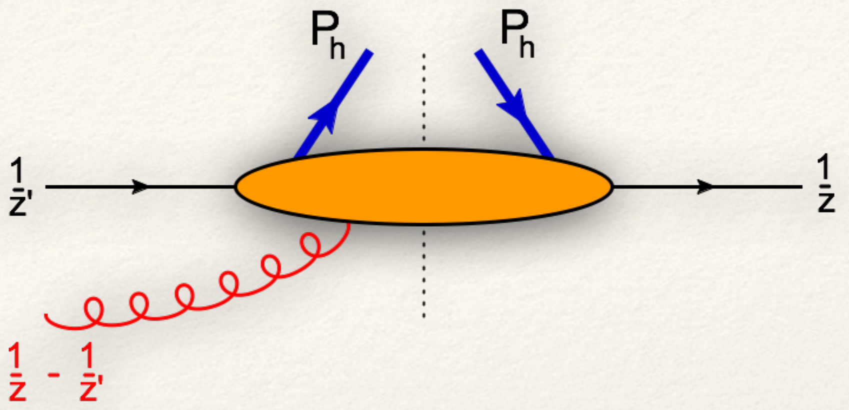
$$\Delta_F^{\alpha}(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m) |P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\,|0\rangle$$

$$\Longrightarrow\ \hat{D}_{FT}^{\Lambda/q}(z,z')\,,\ \hat{G}_{FT}^{\Lambda/q}(z,z')$$



'dynamical' twist-3 FF with transverse spin:

$$\Delta_F^\alpha(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m) |P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$
$$\Longrightarrow \hat{D}_{FT}^{\Lambda/q}(z,z')\,,\ \hat{G}_{FT}^{\Lambda/q}(z,z')$$



Same operator as for dynamical twist-3 PDFs, but:

- No gluonic or fermonic poles

$$FF(z,z) = 0 \qquad FF(z,0) = 0 \qquad \frac{\partial}{\partial z'} FF(z,z')\Big|_{z'=z} = 0$$

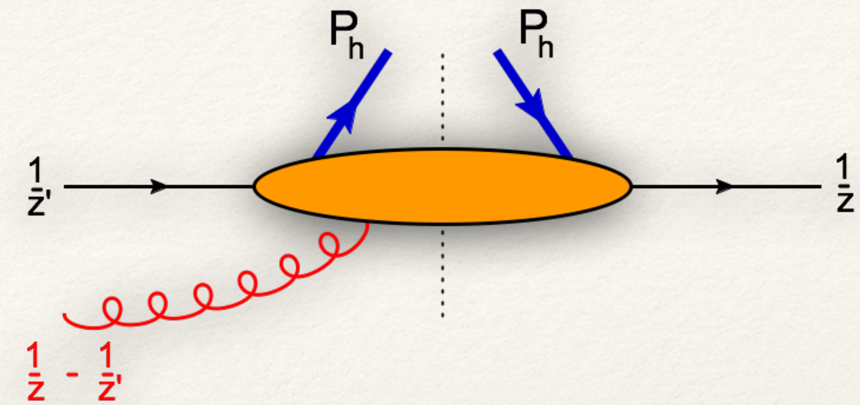
- Support properties different

$$z \leq z' < \infty$$

- No time reversal: dynamical twist-3 FFs are complex

'dynamical' twist-3 FF with transverse spin:

$$\Delta_F^{\alpha}(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m) |P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$
$$\Longrightarrow \hat{D}_{FT}^{\Lambda/q}(z,z')\,,\ \hat{G}_{FT}^{\Lambda/q}(z,z')$$



Same operator as for dynamical twist-3 PDFs, but:

- No gluonic or fermonic poles

$$FF(z,z) = 0 \qquad FF(z,0) = 0 \qquad \frac{\partial}{\partial z'} FF(z,z')\Big|_{z'=z} = 0$$

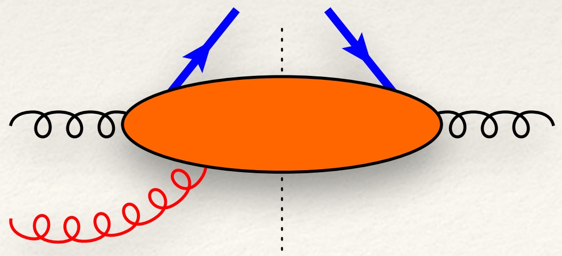
- Support properties different

$$z \leq z' < \infty$$

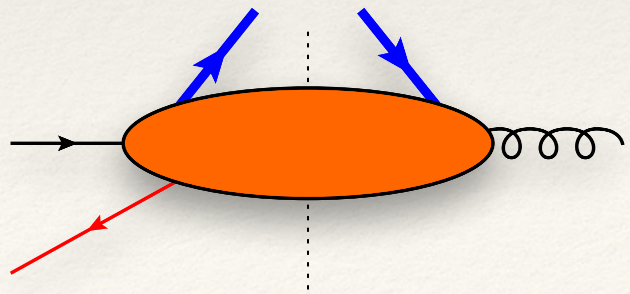
- No time reversal: dynamical twist-3 FFs are complex

More 'dynamical' twist-3 functions (in pp at LO, in ep/e+e- at NLO)

triple-gluon correlations



qq-gluon correlation



QCD EoM relation & Lorentz-Invariance Relations

[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 2016]

QCD EoM for Twist-3 PDFs

$$g_{1T}^{(1)}(x) = x\, g_T(x) - \frac{m_q}{M}\, h_1(x) + \int_{-1}^{1} dx' \, \frac{F_{FT}(x,x') - G_{FT}(x,x')}{x - x'}$$

QCD EoM for Twist-3 FFs

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)] - \Im[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

$$G_{1T}^{\perp(1)}(z) - \frac{G_T(z)}{z} = \int_0^1 d\beta \frac{\Re[\hat{D}_{FT}(z,z/\beta)] - \Re[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

QCD EoM relation & Lorentz-Invariance Relations

[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 2016]

QCD EoM for Twist-3 PDFs

$$g_{1T}^{(1)}(x) = x\, g_T(x) - \frac{m_q}{M} h_1(x) + \int_{-1}^{1} dx' \, \frac{F_{FT}(x,x') - G_{FT}(x,x')}{x - x'}$$

QCD EoM for Twist-3 FFs

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)] - \Im[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

$$G_{1T}^{\perp(1)}(z) - \frac{G_T(z)}{z} = \int_0^1 d\beta \frac{\Re[\hat{D}_{FT}(z,z/\beta)] - \Re[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

LIR for Twist-3 PDFs

based on translation invariance

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) - 2\int_{-1}^{1} dx' \, \frac{G_{FT}(x,x')}{(x-x')^2}$$

LIR for Twist-3 FFs

based on translation invariance

$$\frac{D_T(z)}{z} = -\left(1 - z\frac{d}{dz}\right) D_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

$$\frac{G_T(z)}{z} = \frac{G_1(z)}{z} + \left(1 - z\frac{d}{dz}\right) G_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Re[\hat{G}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

QCD EoM relation & Lorentz-Invariance Relations

[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 2016]

QCD EoM for Twist-3 PDFs

$$g_{1T}^{(1)}(x) = x\, g_T(x) - \frac{m_q}{M}\, h_1(x) + \int_{-1}^{1} dx' \, \frac{F_{FT}(x,x') - G_{FT}(x,x')}{x - x'}$$

QCD EoM for Twist-3 FFs

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)] - \Im[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

$$G_{1T}^{\perp(1)}(z) - \frac{G_T(z)}{z} = \int_0^1 d\beta \frac{\Re[\hat{D}_{FT}(z,z/\beta)] - \Re[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

LIR for Twist-3 PDFs

based on translation invariance

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) - 2\int_{-1}^{1} dx' \, \frac{G_{FT}(x,x')}{(x-x')^2}$$

LIR for Twist-3 FFs

based on translation invariance

$$\frac{D_T(z)}{z} = -\left(1 - z\frac{d}{dz}\right) D_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

$$\frac{G_T(z)}{z} = \frac{G_1(z)}{z} + \left(1 - z\frac{d}{dz}\right) G_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Re[\hat{G}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

Two equations, three functions → eliminate 'intrinsic & kinematical twist-3'

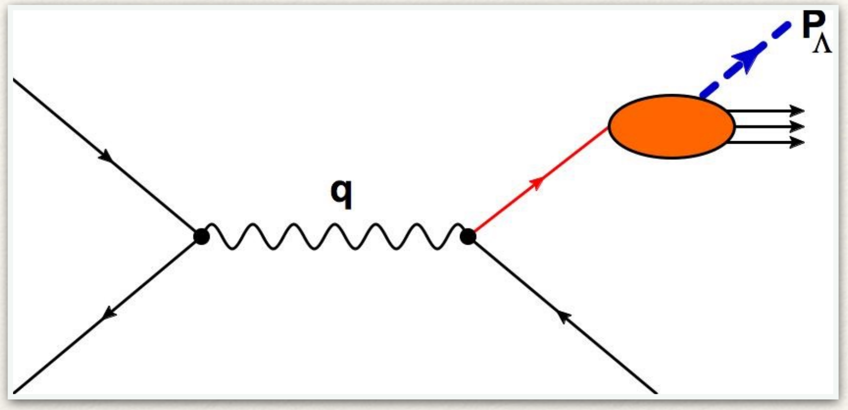
$$g_T(x) = \int_x^1 \frac{dy}{y}\, g_1(y) + \frac{m_q}{M}\left(\frac{1}{x}\, h_1(x) - \int_x^1 \frac{dy}{y^2}\, h_1(y)\right)$$
$$+ \int_x^1 \frac{dy}{y^2} \int_{-1}^1 dz \left[\frac{(1-y\delta(y-x))\, F_{FT}(y,z)}{y-z} - \frac{(3y-z-y(y-z)\delta(y-x))\, G_{FT}(y,z)}{(y-z)^2}\right]$$

EoM & LIR relation crucial for gauge invariance, invariance of LC vector n

$\Lambda^{\uparrow}$ - production in electron-positron annihilation at NLO

Unpolarized $e^+ e^- \longrightarrow \Lambda X$ cross section

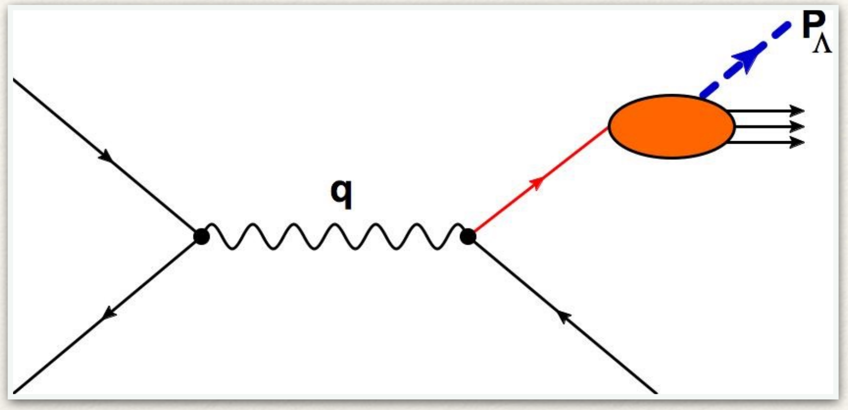
"Parton Model like" at LO



$$E_\Lambda \frac{d\sigma}{d^3 \vec{P}_\Lambda} \propto \sum_q e_q^2 \; D_1^{\Lambda/q}(z_h)$$

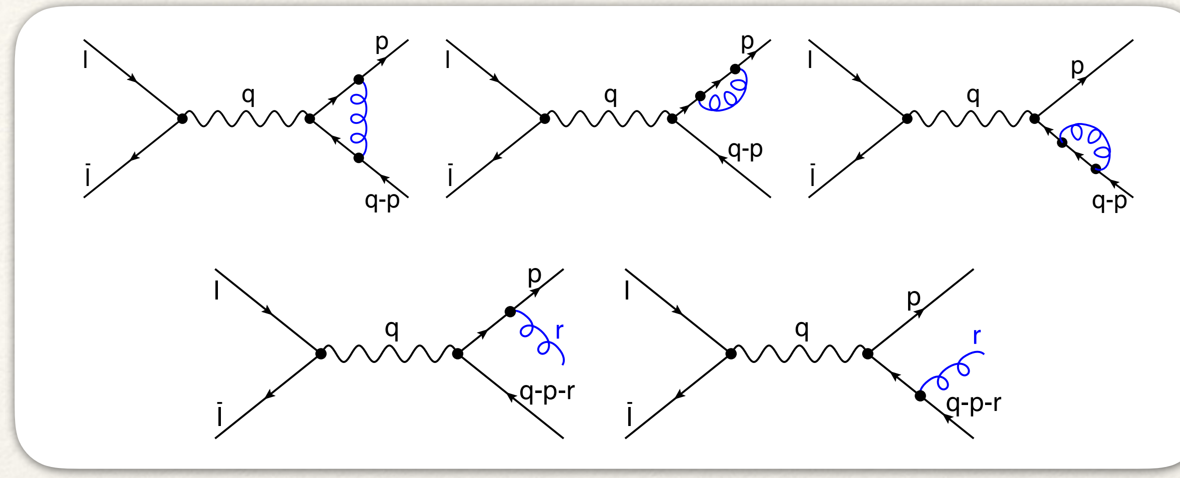
Unpolarized $e^+ e^- \longrightarrow \Lambda X$ cross section

"Parton Model like" at LO



$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2 \; D_1^{\Lambda/q}(z_h)$$

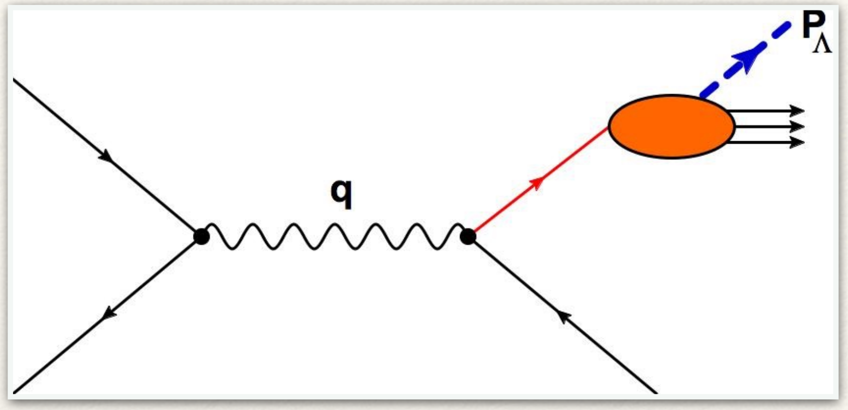
NLO



$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\rm NLO} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w}\Big[\hat{\sigma}^{\overline{\rm MS},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w, \mu) + \hat{\sigma}^{\overline{\rm MS},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w, \mu)\Big]$$

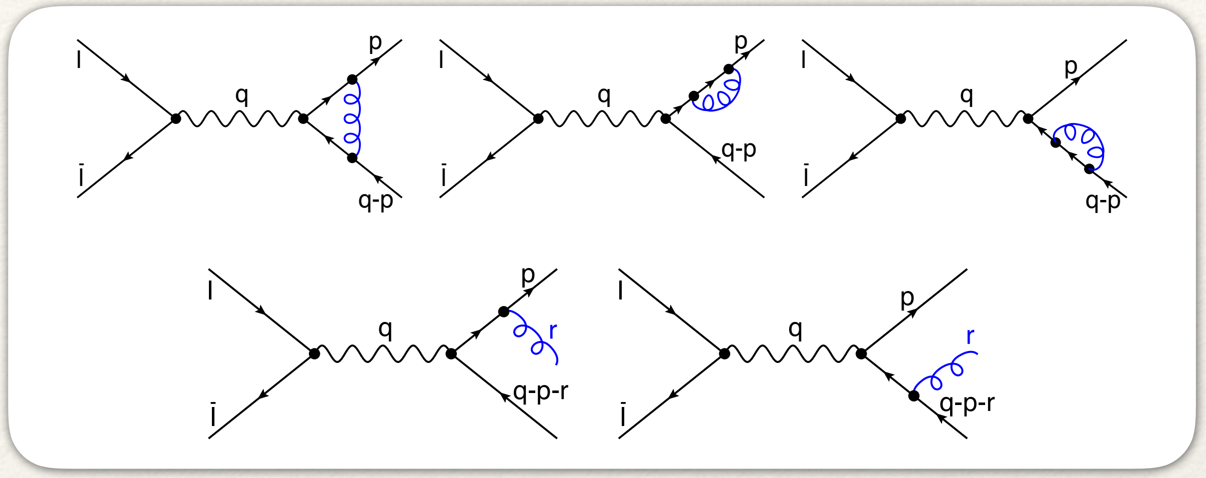
Unpolarized $e^+ e^- \longrightarrow \Lambda X$ cross section

"Parton Model like" at LO



$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2 \; D_1^{\Lambda/q}(z_h)$$

NLO



$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\rm NLO} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w} \Big[\hat{\sigma}^{\overline{\rm MS},q}(w,s/\mu^2) D_1^{\Lambda/q}(z_h/w,\mu) + \hat{\sigma}^{\overline{\rm MS},g}(w,s/\mu^2) D_1^{\Lambda/g}(z_h/w,\mu)\Big]$$

Typical NLO features:

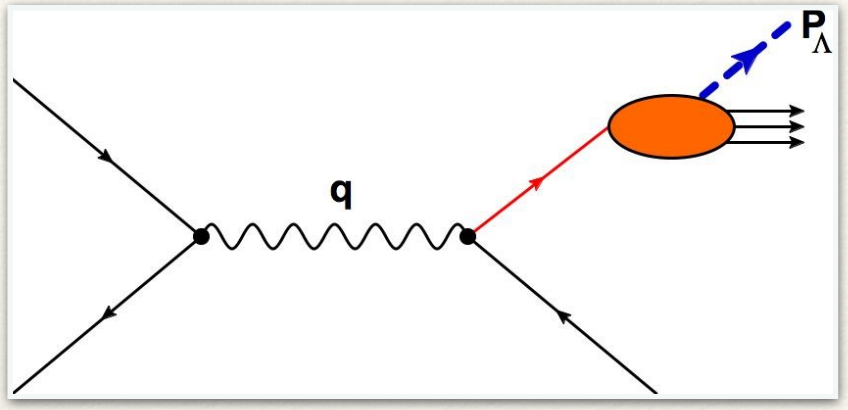
- infrared safe (cancellation of $1/\varepsilon^2$ - poles in dim. reg.)

$$\hat{\sigma}_{\rm virt} + \hat{\sigma}_{\rm real} = \mathcal{O}(1/\varepsilon)$$

$$\hat{\sigma}^{q/g} \propto -\frac{1}{\varepsilon} P_{q/g\,q}(w) + \mathcal{O}(\varepsilon^0)$$

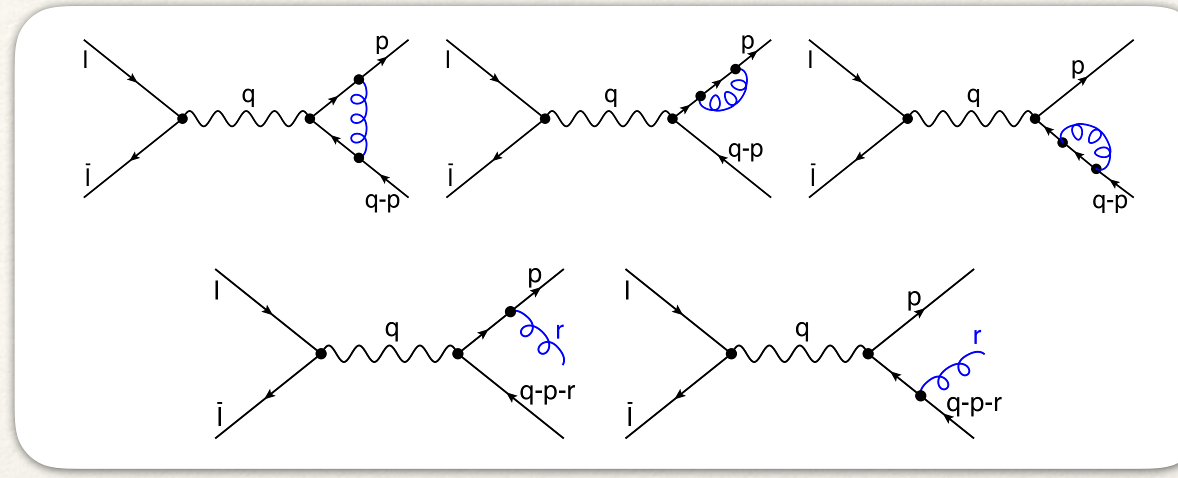
Unpolarized $e^+ e^- \longrightarrow \Lambda X$ cross section

“Parton Model like” at LO



$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2 \; D_1^{\Lambda/q}(z_h)$$

NLO



$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\mathrm{NLO}} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w} \Big[\hat{\sigma}^{\bar{\mathrm{MS}},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w,\mu) + \hat{\sigma}^{\bar{\mathrm{MS}},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w,\mu)\Big]$$

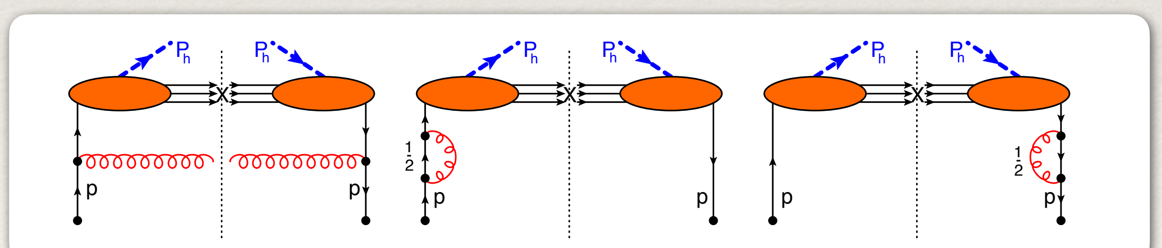
Typical NLO features:

- infrared safe (cancellation of $1/\varepsilon^2$ - poles in dim. reg.)

$$\hat{\sigma}_{\mathrm{virt}} + \hat{\sigma}_{\mathrm{real}} = \mathcal{O}(1/\varepsilon)$$

$$\hat{\sigma}^{q/g} \propto -\frac{1}{\varepsilon} P_{q/g\,q}(w) + \mathcal{O}(\varepsilon^0)$$

- MSbar renormalization of fragmentation functions → DGLAP evolution

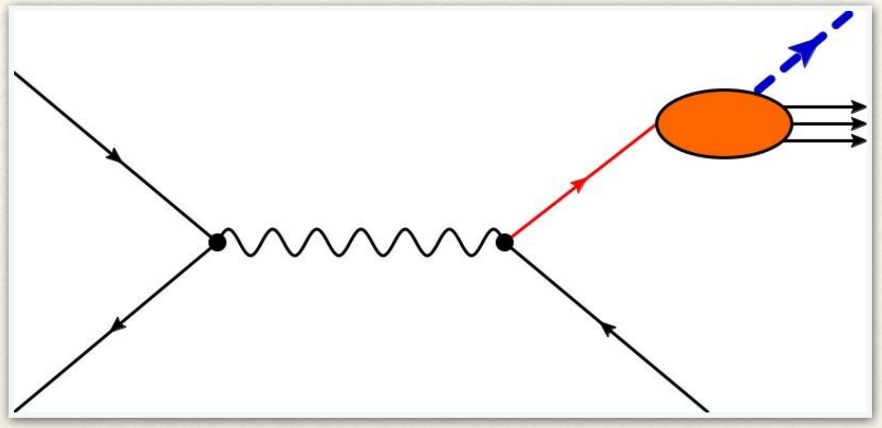


$$D_{1,\mathrm{bare}}^{\Lambda/q}(z) = D_{1,\mathrm{ren}}^{\Lambda/q}(z) + \frac{\alpha_s}{2\pi}\frac{S_\varepsilon^{\bar{\mathrm{MS}}}}{\varepsilon} \sum_{i=q,g} \int_z^1 \frac{dw}{w} P_{iq}(w) D_{1,\mathrm{ren}}^{\Lambda/i}\left(\frac{z}{w}\right) + \mathcal{O}(\alpha_s^2)$$

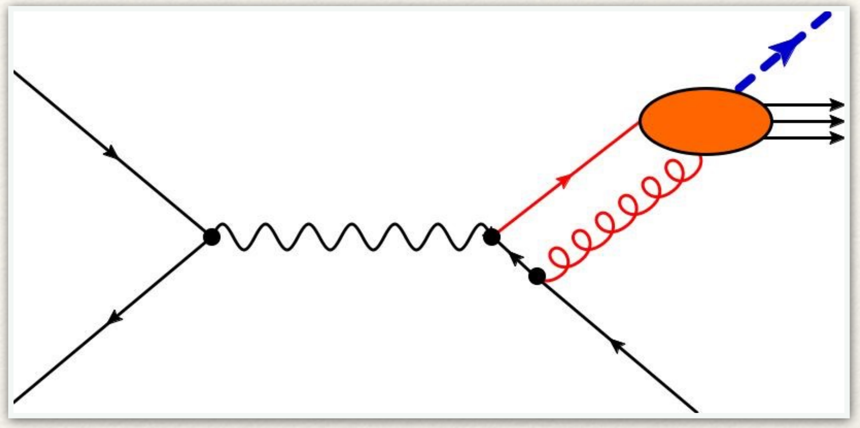
O(1/ε) cancels, necessary condition for one-loop factorization!

Transverse Λ polarization at LO

'intrinsic' & 'kinematical' twist-3 FF:



'dynamical' twist-3 FF:



$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[\frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta\, \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

→ Equation of Motion:

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[2\, \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]$$

or:

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[-2\, D_{1T}^{\perp(1)\Lambda/q}(z_h) + 2 \int_0^1 d\beta\, \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

Single-Transverse Λ spin asymmetry

- Unique effect driven by a single fragmentation function D_T → absent in DIS (1γ)
- EoM needed at LO to preserve e.m. current conservation of hadronic tensor ($q_\mu W^{\mu\nu} = 0$) (EoM not optional!)

Transverse Λ polarization at NLO

[Gamberg, Kang, Pitonyak, M.S., Yoshida, JHEP 2019]

- Study the NLO dynamics for twist-3 fragmentation in the simplest process
- Different compared to twist-3 distributions (no pole contributions)

Transverse Λ polarization at NLO

[Gamberg, Kang, Pitonyak, M.S., Yoshida, JHEP 2019]

- Study the NLO dynamics for twist-3 fragmentation in the simplest process
- Different compared to twist-3 distributions (no pole contributions)

Virtual & Real diagrams (qg/q - channel here, gg/g, qqb/g not shown)

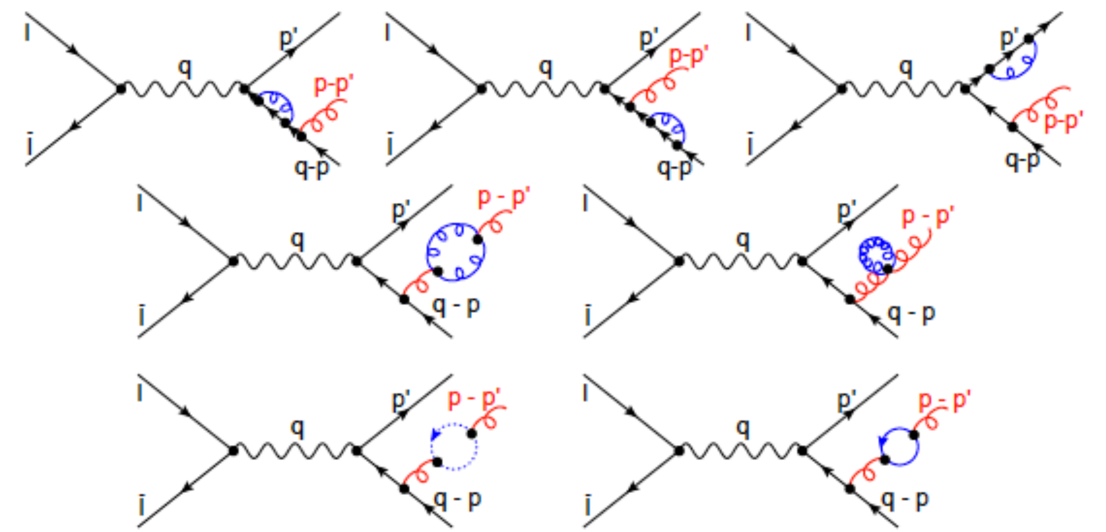


Figure 6: Self-energy corrections

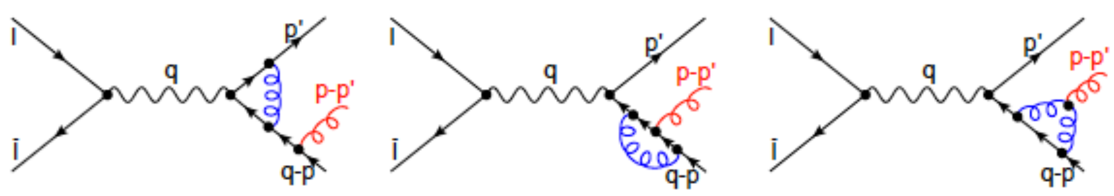


Figure 7: Vertex corrections

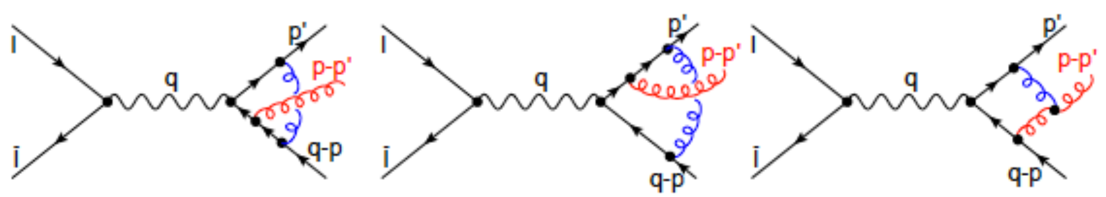


Figure 8: Box corrections

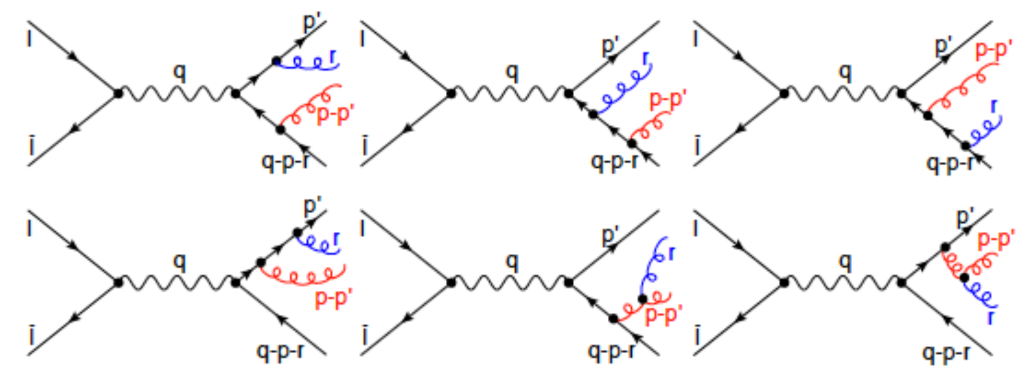


Figure 9: Real corrections

- E.o.M. - relations are crucial:
  Eliminate 'intrinsic' twist-3 contributions:
  only then color gauge invariance at NLO! ✔
- Imaginary parts: In the dynamical fragmentation process & loop diagrams
- Infrared $1/\varepsilon^2$ - poles cancel ✔
- $1/\varepsilon$ - poles of imaginary parts of loops cancel through E.o.M. ✔
- $1/\varepsilon$ - collinear poles of real parts of loops through MSbar - renormalization (?)

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\overline{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \sum_{q=u,\overline{u},\ldots} e_q^2 \Big[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

LO

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{d\sigma_U^{\mathrm{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

$$+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})$$

LO

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

$$+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})$$

LO

NLO

2-quark correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\bar{u},\ldots} e_q^2 \Big[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

$$+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})$$

$$+ \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})$$

LO

NLO

2-quark correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})
\end{aligned}
$$

LO

NLO

2-quark correlation w/ EoM

NLO

2-gluon correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times
$$

$$
\sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}
$$

$$
+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})
$$

$$
+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}_{H_1^{(1)}}^{g;\mathrm{EoM}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})
$$

$$
+ \hat{\sigma}_{\Im D_{FT}}^{qg;\mathrm{EoM}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\mathrm{EoM}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta}
$$

LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^\varepsilon \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta}
\end{aligned}
$$

LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times
$$

$$
\sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}
$$

$$
+\frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})
$$

$$
+\hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}_{H_1^{(1)}}^{g;\mathrm{EoM}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w})
$$

$$
+\hat{\sigma}_{\Im D_{FT}}^{qg;\mathrm{EoM}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\mathrm{EoM}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta}
$$

$$
+\hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta})
$$

$$
+\hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta})
$$

LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta})
\end{aligned}
$$

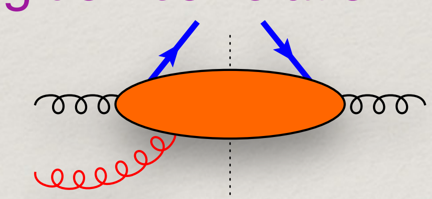
LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{d\sigma_U^{\mathrm{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = {}& (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\dots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \right)
\end{aligned}
$$

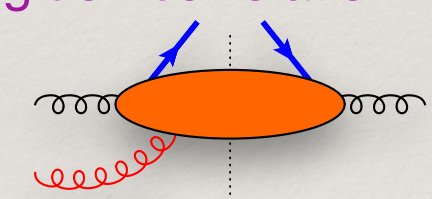
LO

NLO
2-quark correlation w/ EoM

NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {}& (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^{q} - \hat{G}_{FT}^{q}](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \left(\sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \right)
\end{aligned}
$$

[redacted]

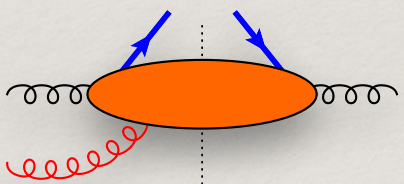
LO

NLO
2-quark correlation w/ EoM

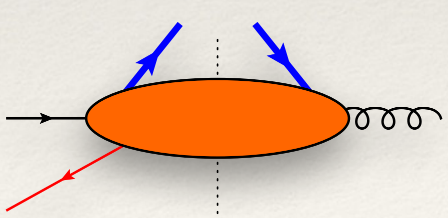
NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



NLO
qq-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta) \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta) \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \Bigg(\sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \Bigg) \\
& + \hat{\sigma}^q_{\Re}(w,\beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \Bigg\} \Bigg] + \mathcal{O}(\Lambda^2/s),
\end{aligned}
$$

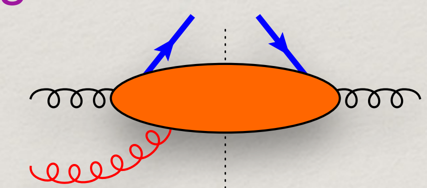
LO

NLO
2-quark correlation w/ EoM

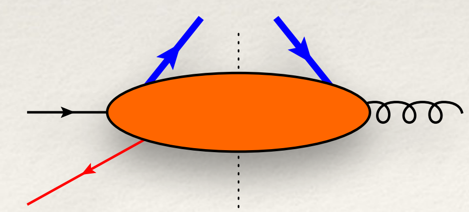
NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



NLO
qq-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{d\sigma_U^{\mathrm{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon}\, \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2}\, \frac{2M_h \epsilon^{P_h m l S_h}}{s}\,(2v-1)\times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2\int_0^1 d\beta\, \frac{\Im[\hat{D}^q_{FT}-\hat{G}^q_{FT}](z_h,\frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta)\, \frac{\Im[\hat{D}^q_{FT}](\frac{z_h}{w},\frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta)\, \frac{\Im[\hat{G}^q_{FT}](\frac{z_h}{w},\frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}^{gg}_{FT}-\hat{G}^{gg}_{FT}+(1-\varepsilon)\hat{H}^{gg}_{FT}](\tfrac{z_h}{w},\tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\hat{D}^{gg}_{FT}+\hat{G}^{gg}_{FT}+\frac{\varepsilon}{2}\hat{H}^{gg}_{FT}](\tfrac{z_h}{w},\tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \left(\sum_{q=u,d,\ldots} \Im[\hat{D}^{q\bar{q}}_{FT}](\tfrac{z_h}{w},\tfrac{z_h}{w\beta}) \right) \\
& + \hat{\sigma}^q_{\Re}(w,\beta)\, \frac{\Re[\hat{D}^q_{FT}-\hat{G}^q_{FT}](\frac{z_h}{w},\frac{z_h}{w\beta})}{1-\beta} \Bigg\} \Bigg] + \mathcal{O}(\Lambda^2/s),
\end{aligned}
$$

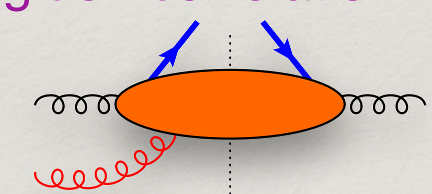
LO

NLO
2-quark correlation w/ EoM

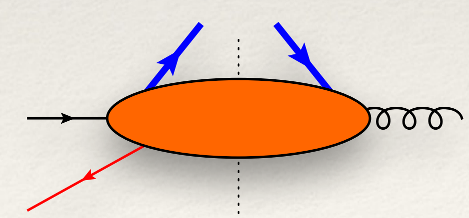
NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



NLO
qq-gluon correlation w/ EoM



NLO
imaginary parts of loops

Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon}\, \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2}\, \frac{2M_h \epsilon^{P_h m l S_h}}{s}\, (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[-2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta\, \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta\, \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta)\, \frac{\Im[\hat{D}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta)\, \frac{\Im[\hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \left(\sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \right) \\
& + \hat{\sigma}^q_{\Re}(w,\beta)\, \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \Bigg\} \Bigg] + \mathcal{O}(\Lambda^2/s),
\end{aligned}
$$

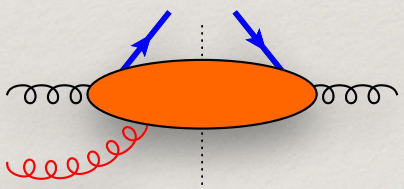
LO

NLO
2-quark correlation w/ EoM

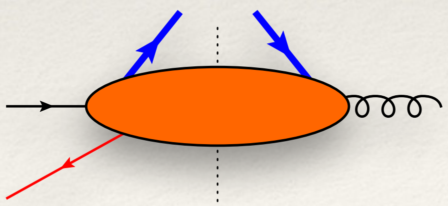
NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



NLO
qq-gluon correlation w/ EoM



NLO
imaginary parts of loops

All partonic factors calculated in Feynman gauge & Light-cone gauge, both calculations *agree*!

If we assume that twist-3 factorization holds…

If we assume that twist-3 factorization holds…

read off evolution equations from collinear divergences for quark twist-3 FF $D_T(z)$

$$\frac{\partial}{\partial \ln \mu^2}\Big(D_T^f(z;\mu)\Big) = \frac{z}{2}\int_z^1 \frac{dw}{w^2}\int_0^1 d\beta \Bigg[P^{[1]}_{1,f\to f}(w)\, D_{1T}^{\perp(1),f}(\tfrac{z}{w};\mu) + P^{[1]}_{1,f\to g}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z}{w};\mu)$$
$$+ P^{[1]}_{2,f\to fg}(w,\beta)\,\frac{\Im[\hat{D}^{fg}_{FT} - \hat{G}^{fg}_{FT}](\tfrac{z}{w},\beta;\mu)}{1-\beta} + P^{[1]}_{3,f\to fg}(w,\beta)\,\frac{2\,\Im[\hat{D}^{fg}_{FT}](\tfrac{z}{w},\beta;\mu)}{(1-\beta)^2}$$
$$+ \sum_{f'=q',\bar{q}'} P^{[1]}_{4,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{D}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)] + \sum_{f'=q',\bar{q}'} P^{[1]}_{5,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{G}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)]$$
$$+ P^{[1]}_{6,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_2^s(\tfrac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{7,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_2^a(\tfrac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{8,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_1(\tfrac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2}\Bigg]$$

ordinary DGLAP splitting functions

$$P^{[1]}_{1,f\to f}(w) = -2\frac{C_F\alpha_s}{2\pi}\left(\frac{1+w^2}{(1-w)_+} + \frac{3}{2}\delta(1-w)\right)$$

$$P^{[1]}_{1,f\to g}(w) = 4\frac{C_F\alpha_s}{2\pi}\left(\frac{1+(1-w)^2}{w}\right)$$

Others: more complicated

If we assume that twist-3 factorization holds…

read off evolution equations from collinear divergences for quark twist-3 FF $D_T(z)$

$$\frac{\partial}{\partial \ln\mu^2}\Big(D_T^f(z;\mu)\Big) = \frac{z}{2}\int_z^1 \frac{\mathrm{d}w}{w^2}\int_0^1 \mathrm{d}\beta \Bigg[P^{[1]}_{1,f\to f}(w)\, D_{1T}^{\perp(1),f}(\tfrac{z}{w};\mu) + P^{[1]}_{1,f\to g}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z}{w};\mu)$$
$$+ P^{[1]}_{2,f\to fg}(w,\beta)\,\frac{\Im[\hat{D}^{fg}_{FT} - \hat{G}^{fg}_{FT}](\frac{z}{w},\beta;\mu)}{1-\beta} + P^{[1]}_{3,f\to fg}(w,\beta)\,\frac{2\,\Im[\hat{D}^{fg}_{FT}](\frac{z}{w},\beta;\mu)}{(1-\beta)^2}$$
$$+ \sum_{f'=q',\bar{q}'} P^{[1]}_{4,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{D}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)] + \sum_{f'=q',\bar{q}'} P^{[1]}_{5,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{G}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)]$$
$$+ P^{[1]}_{6,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}^{s}_{2}(\frac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{7,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}^{a}_{2}(\frac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{8,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_{1}(\frac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2}\Bigg]$$

ordinary DGLAP splitting functions

$$P^{[1]}_{1,f\to f}(w) = -2\frac{C_F\alpha_s}{2\pi}\left(\frac{1+w^2}{(1-w)_+} + \frac{3}{2}\,\delta(1-w)\right)$$

$$P^{[1]}_{1,f\to g}(w) = 4\frac{C_F\alpha_s}{2\pi}\left(\frac{1+(1-w)^2}{w}\right)$$

Others: more complicated

Final proof of one-loop factorization:

Need to derive evolution equation directly from correlator!

Previous work on *unpolarized* chiral-odd twist-3 fragmentation:

[Belitsky, Kuraev, NPB 1996; Ma, Zhang, PLB 2017]

“The Gribov-Lipatov reciprocity fulfilled for two-particle cut-vertices only!”

Summary

- Transverse Spin Polarization: Long history, measured in ep/pp-collisions, theoretical treatment more complicated
- We can learn about the parton dynamics in the nucleon and fragmentation process, non-perturbative QCD EoM and LIR are crucial
- Λ - production in e^+e^-: NLO completed,
  → calculate 'splitting functions' for polarized Λ fragmentation function
  → Double Spin Asymmetry A_{LT} equally important…
- A lot more work to do…