# "Hydro+" and search for QCD critical point

Upcoming Heavy-ion collision (HIC) experiments will explore the uncharted regime of the QCD phase diagram with unprecedented precision — discovery potential: QCD critical point.

This in turn calls for the quantitative framework which describes the physical of criticality in realistic HIC environment.

see YY, 1811.06519 for a mini-review; also Chun's talk

In this talk, I will present the formulation of "hydro+" in which the interplay of critical and bulk evolution are incorporated. In addition, I will show preliminary simulation results.

Stephanov-YY, 1712.10305, PRD '18

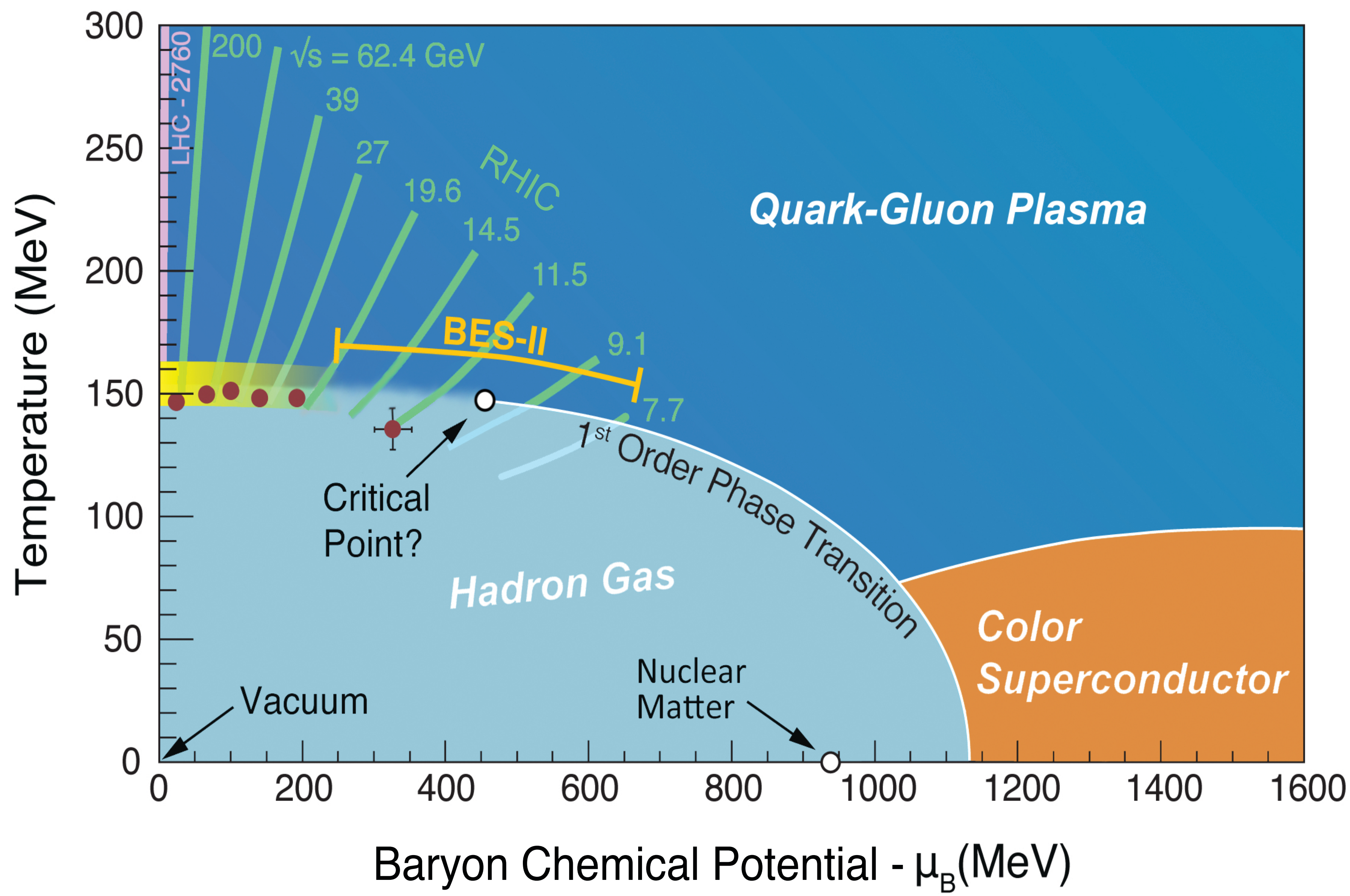
Rajagopal-Ridgway-Weller-YY (in preparation)

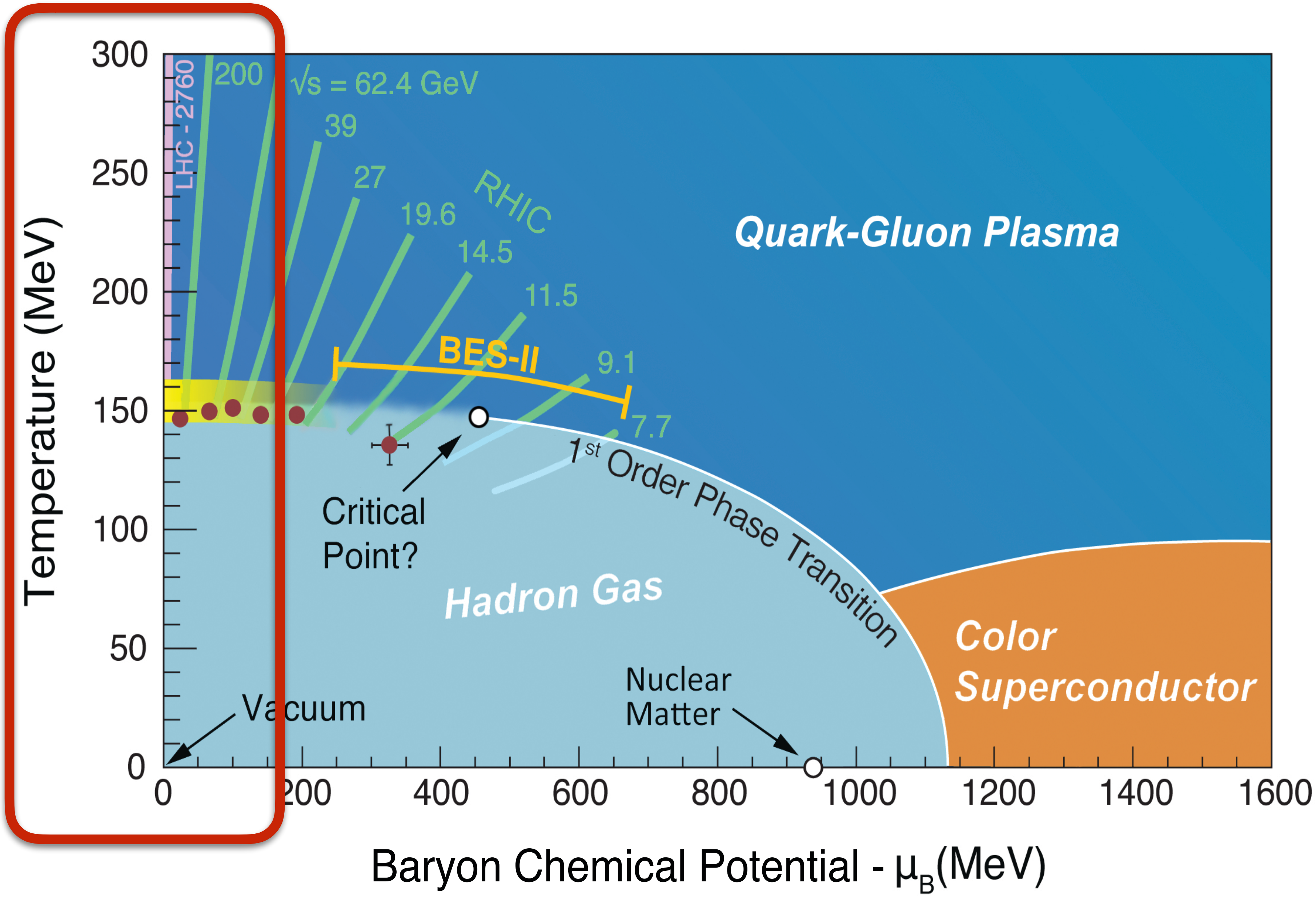
BEST COLLABORATION

Yi Yin

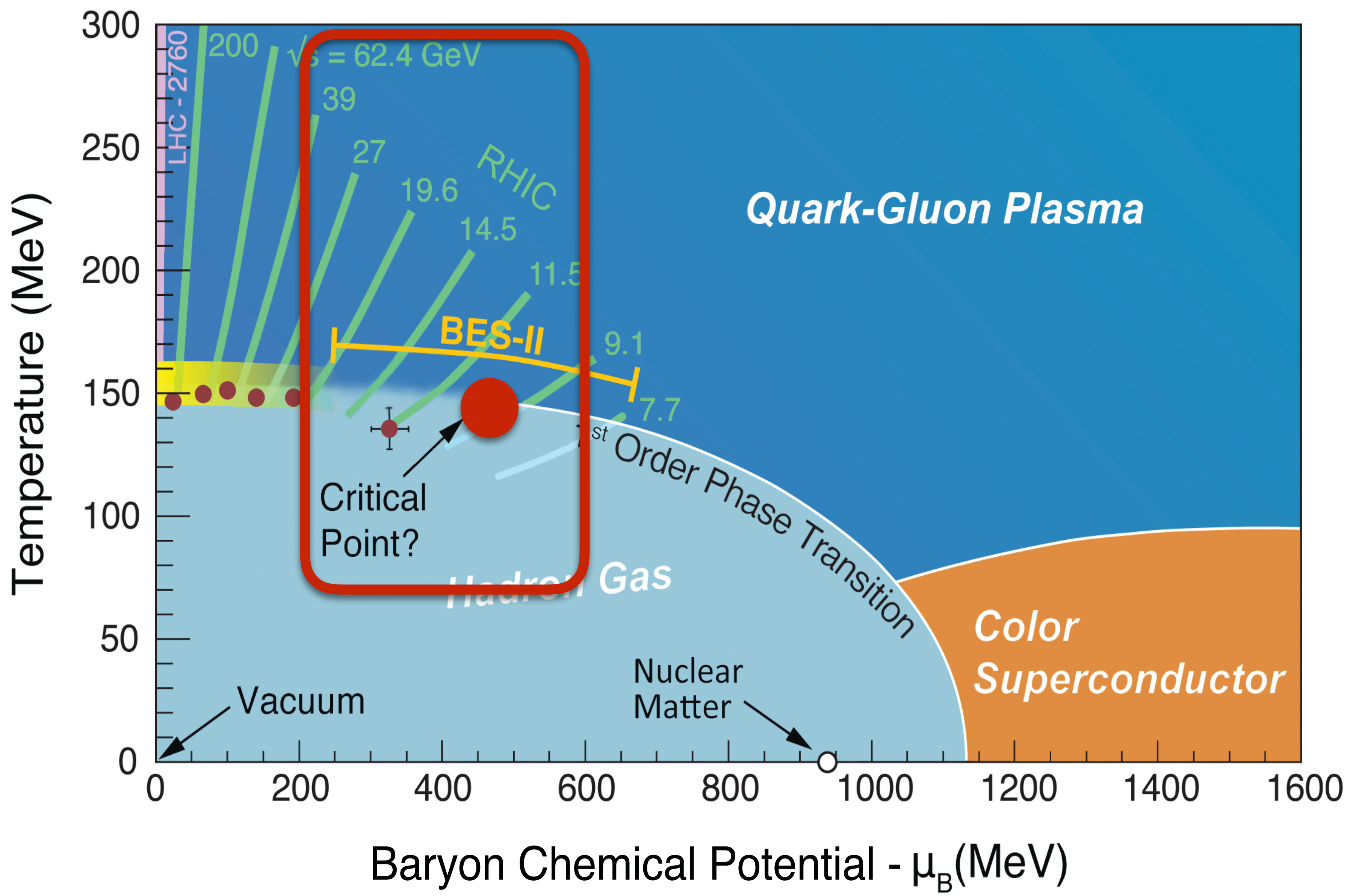
GHP workshop, Denver, Apr.9, 2019

## *QCD phase diagram: "ultimate" phases of (visible) matter in extreme conditions.*






The past decade has seen significant advances on the characterization of the properties of thermal QCD matter at at small $\mu_B$.




The baryon-rich regions in QCD phase diagram: uncharted. But this situation might change dramatically in the near future.

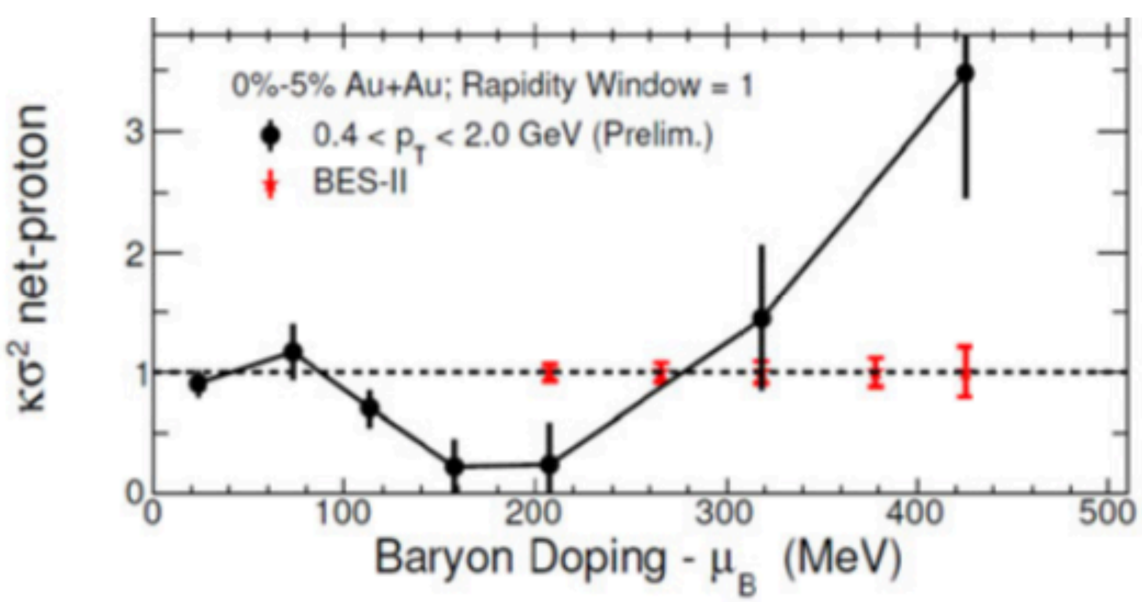
Discovery potential: the missing landmark, namely QCD critical point (C.P.).

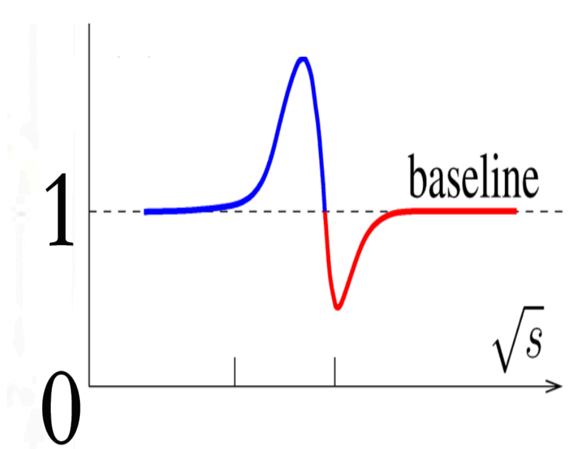
## *Experimental status: interesting and intriguing*

Hadrons (in particular protons) multiplicity fluctuations are expected to be enhanced near C.P. .

$$K_2 \sim \sum_{\text{event}} \left(N_{\text{proton}} - \bar{N}_{\text{proton}}\right)^2 , \; K_4 \sim \sum_{\text{event}} \left(N_{\text{proton}} - \bar{N}_{\text{proton}}\right)^4 - \ldots .$$

Hints: non-monotonicity and sign change of fourth cumulant (e.g. $K_4$) as a function of beam energy within line of theory expectation

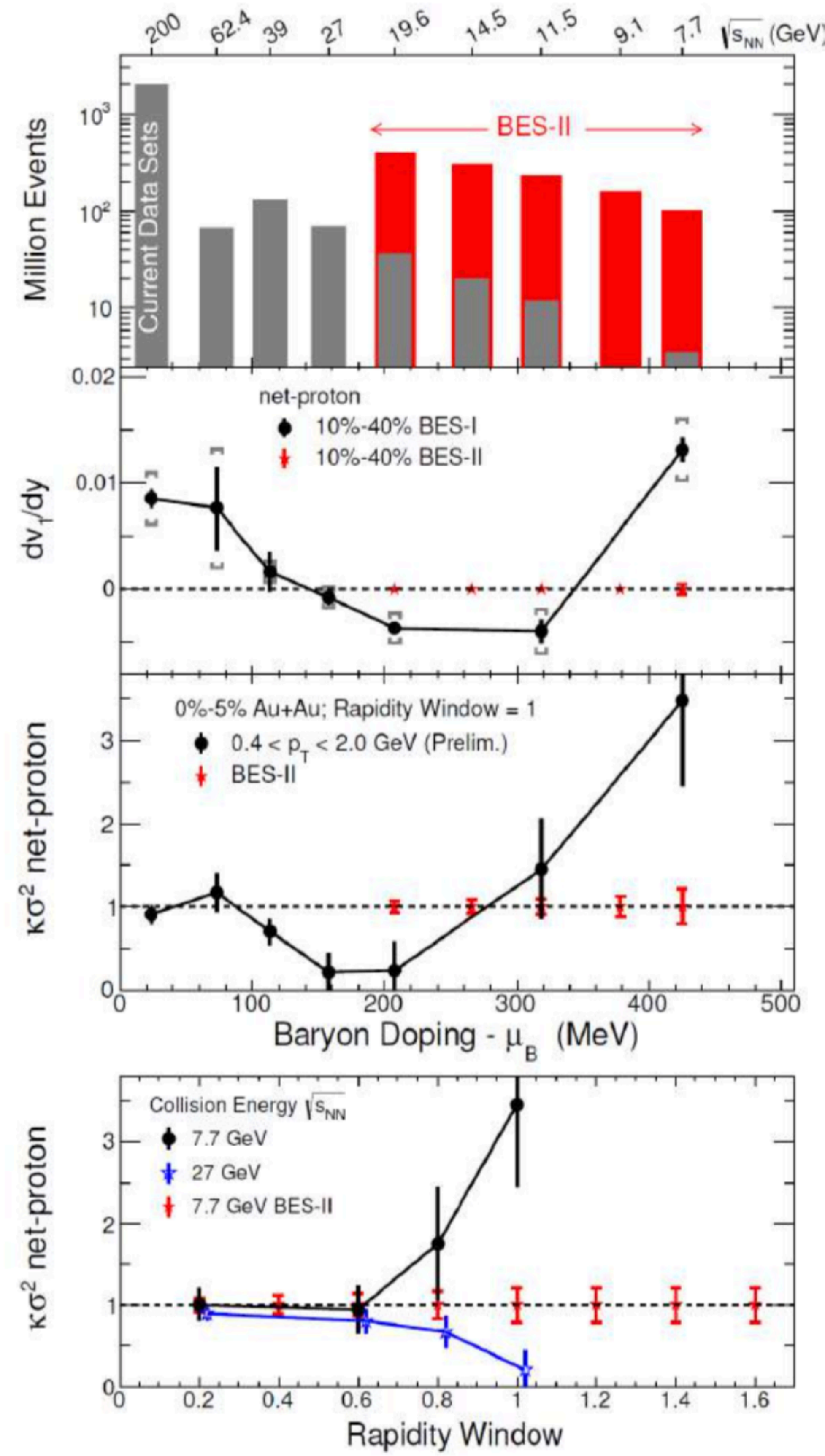



Stephanov, PRL 11

BESII at RHIC will kick off this year(2019) with unprecedented precision and kinematic coverage.

Quantitative framework: we need to understand the dynamics of fluctuations.



# *Critical fluctuations are offequilibrium in expanding fireballs*

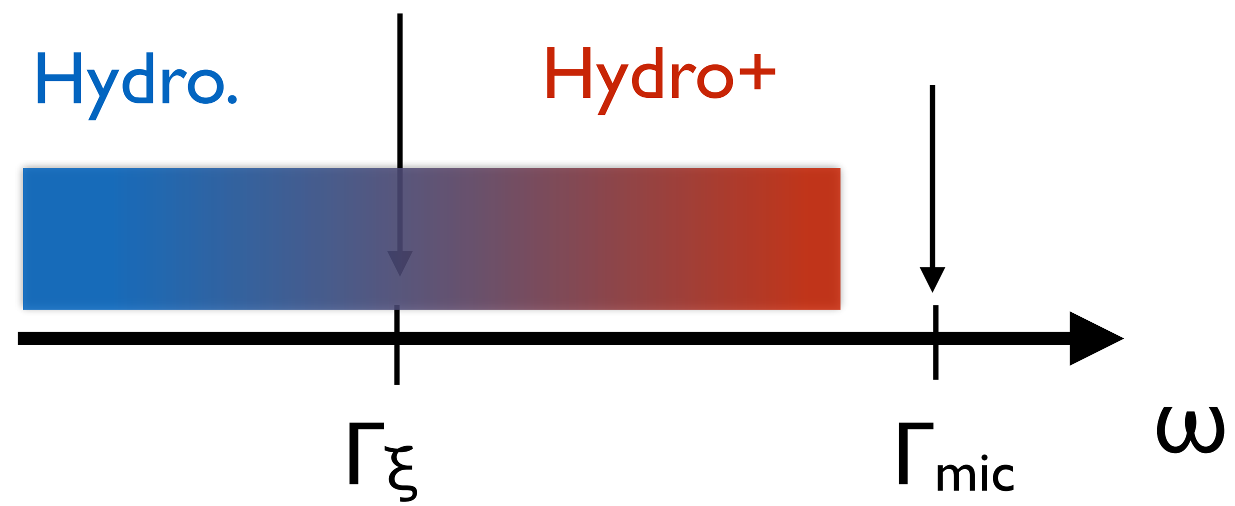
“Critical slowing down”: the larger fluctuations are, the more prominent offequilibrium effects are.

$$\Gamma(Q) = D\, Q^2 = \frac{\sigma_B}{c_p} Q^2 \qquad\qquad \Gamma_\xi = \Gamma(Q = \xi^{-1}) \propto \xi^{-3}$$

As a consequence, critical fluctuation can be different from the equilibrium expectation *qualitatively* !

**S. Mukherjee, R. Venugopalan and YY, PRC15**

Further, hydro ceases to work when fluctuations are offequilibrium. (typical $1/\Gamma_\xi$ can be 3-5 fm )

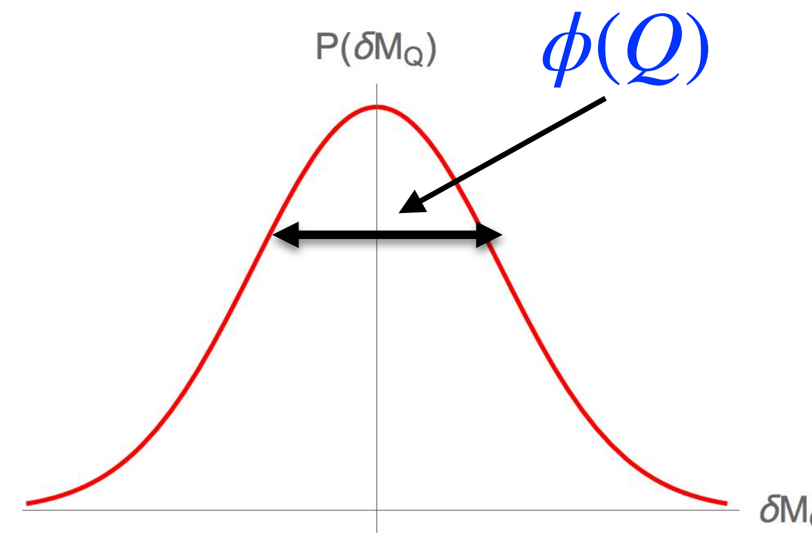



“Hydro+” aims at formulating a hydro-like theory describing intertwined dynamics of hydro. d.o.f and critical fluctuations.

# The construction of "Hydro+"

Stephanov-YY, 1712.10305, PRD '18

## *Fluctuations as dynamical d.o.f.s*




The "+" of "hydro+" is (Winger transform of) the two point function of the fluctuating order parameter field δM:

$$\phi(t,x;Q) = \int d\Delta x\, e^{-i\Delta x\, Q} \langle\, \delta M(t, x + \Delta x/2)\, \delta M(t, x - \Delta x/2)\, \rangle$$

(In future: extension to higher p.t. functions)

The fluctuations depend non-trivially on momentum Q (or wavelength) near C.P. E.g, for a homogeneous and equilibrate system.

$$\phi_{eq}(Q) \sim \frac{1}{\xi^{-2} + Q^2}$$

$$\begin{cases} \phi_{eq}(Q \gg \xi^{-1}) \sim Q^{-2} \\ \phi_{eq}(Q \sim \xi^{-1}) \sim \xi^{2} \end{cases}$$

In an expanding and inhomogeneous fluid, ϕ(t,x;Q) describes the "occupation" of critical fluctuations at momentum Q at time t at each fluid cell labeled by coordinate x .

# *Dynamics of* $\phi$

Stephanov-YY, 1712.10305, PRD '18;
Akamatsu-Teaney-Yan-YY, 1811.05081.

For QCD critical point and for description of the dynamics of $\phi$, we will consider M ~ s/n. (Therefore $\phi_{eq}(Q=0)$ is related to $c_p$ .)

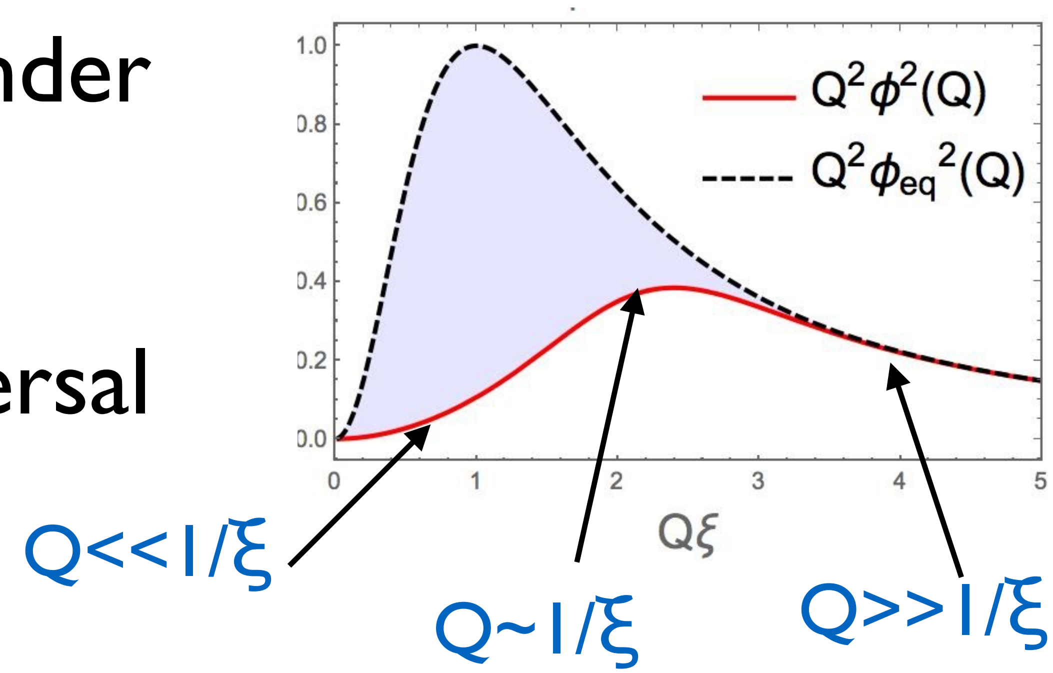
We consider relaxation rate equation

$$u^\mu \partial_\mu \phi = \Gamma_\phi(Q)\,(\phi(Q) - \phi_{\text{eq}}(e,n;Q))$$

$$\begin{cases} \Gamma_\phi(Q \ll \xi^{-1}) \sim Q^2 \\ \Gamma_\phi(Q \sim \xi^{-1}) \sim \xi^3 \\ \Gamma_\phi(Q \gg \xi^{-1}) \sim Q^3 \end{cases}$$

This form of relaxation rate equation can be derived from stochastic hydro. under certain simplifications.

The relaxation rate $\Gamma_\phi(Q)$ is a universal function (model H).




The Q-dependence of $\Gamma_\phi(Q)$ induces interesting Q-dependence of $\phi(Q)$.

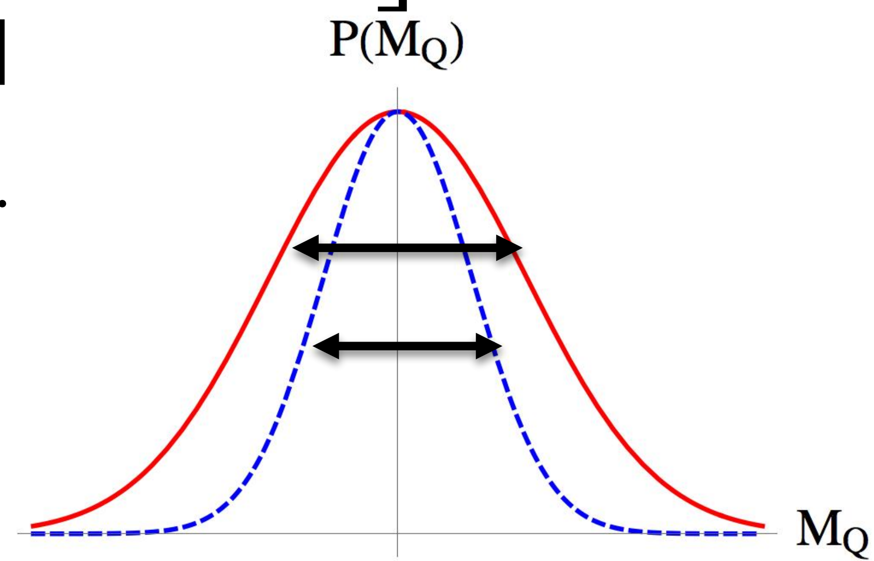
## *Coupling ϕ to hydro.*

Stephanov-YY, 1712.10305, PRD '18

Generalized entropy $s_{(+)}$: log of the number of microscopic states with given e, n, ϕ.

$$s_{(+)} = s(\epsilon, n) + \Delta s\,, \qquad \Delta s = \frac{1}{2}\int_Q \left[\log(\frac{\phi}{\phi_{eq}}) - \frac{\phi}{\phi_{eq}} + 1\right] + \ldots$$

From $s_{(+)}$, one could define other generalized thermodynamic functions such as $\beta_{(+)}$ and $p_{(+)}$.




E.o.M for hydro. variables remain the same:

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu = 0\,.$$

The stress-energy tensor now depends on ϕ

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p_{(+)}\left(g^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial) \qquad p(\epsilon, n) \to p_{(+)}(\epsilon, n, \phi)$$

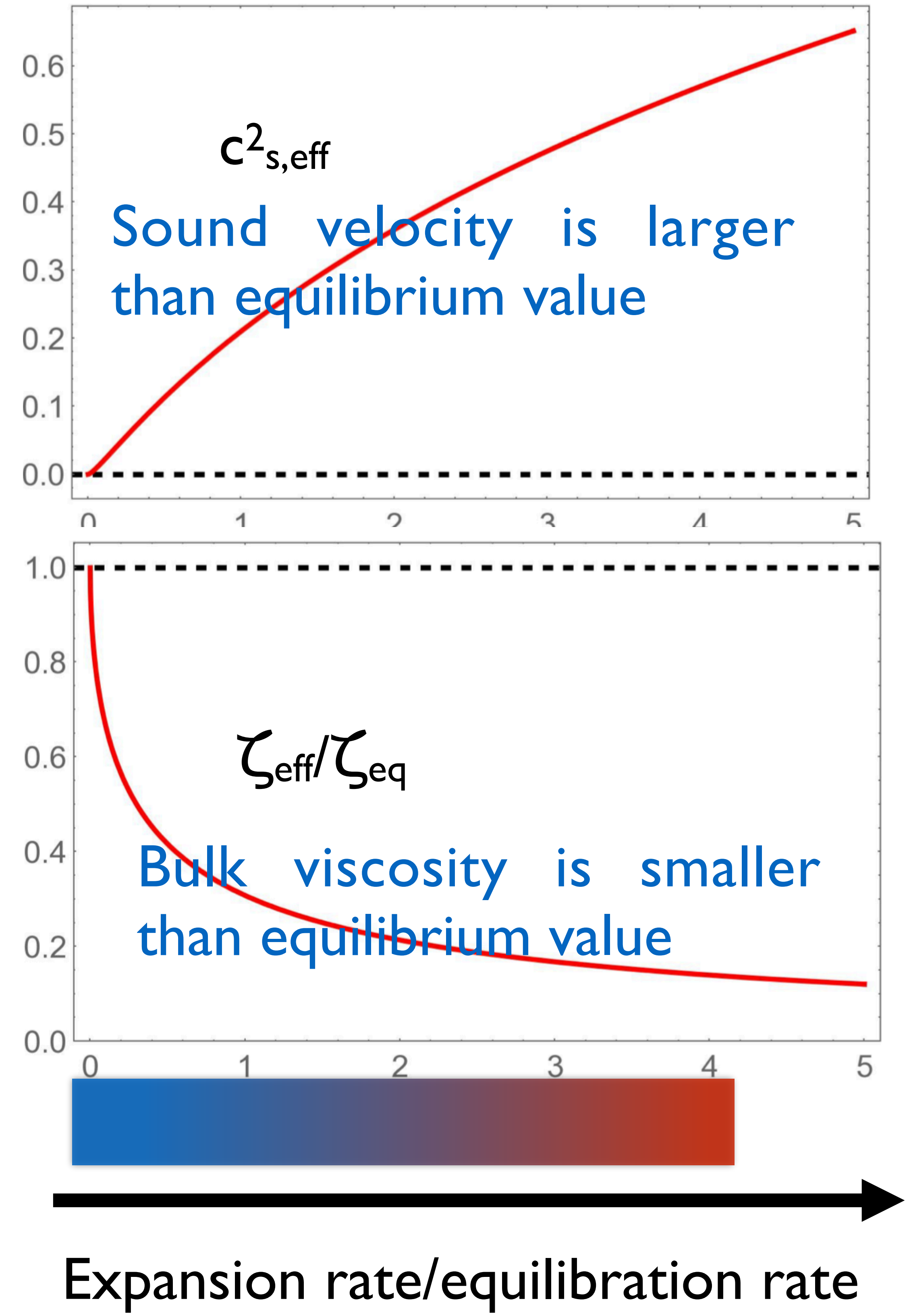
Similar for the transport coefficients $\quad \zeta \to \zeta_{(+)}\,, \qquad \eta \to \eta_{(+)}$

Importantly, the gradient of $p_{(+)}$ accelerate the hydro. flow.

# *Effective sound velocity and bulk viscosity from "hydro+"*

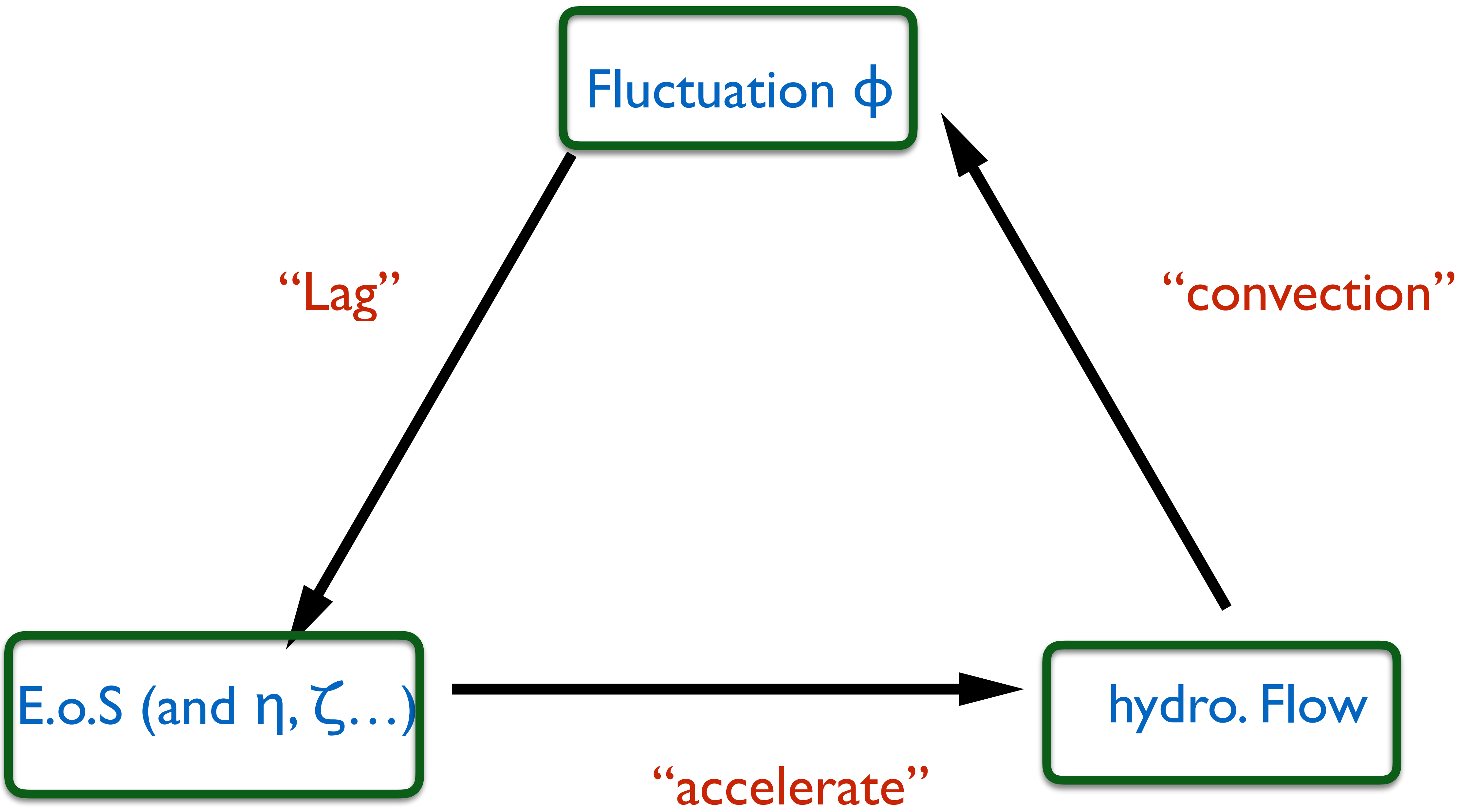
By solving linearized "hydro+", we could determine frequency-dependent "effective sound velocity" and "effective bulk viscosity".

At linearized level, "hydro+"="one loop" calculation of hydro. fluctuations, e.g. by Onuki, PRA, 1997.




Importantly, "hydro+" is local, and can be applied for numerical simulation!

# *A brief summary: the workflow of "Hydro+"*



*We are now ready to see preliminary simulation results.*

# "Hydro+ in action"

Rajagopal-Ridgway-Weller-YY (in preparation)

Greg Ridgway

Ryan Weller

## *Simulating "hydro+" in a simplified set-up*

Rajagopal-Ridgway-Weller-YY (in preparation)

We wish to see "hydro+" in action in a Bjorken and radial expanding ($v_r \neq 0$) and inhomogeneous fluid:

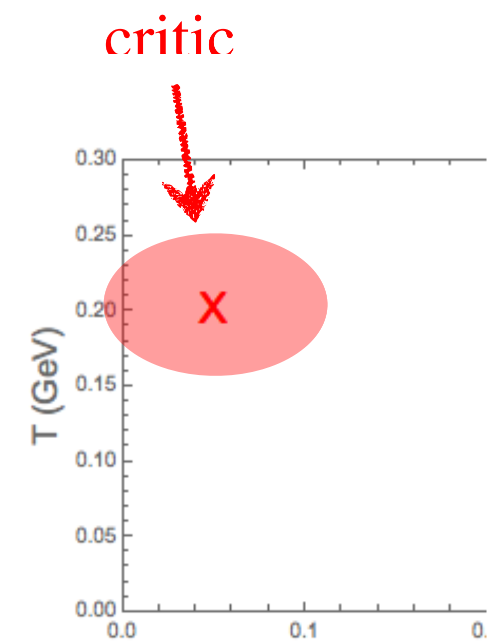
$v_r \neq 0$ is necessary to see the effects of convection.

Inhomegeneity is needed to have the nonzero gradient of $p_{(+)}$ .

The simplified set-up with the essence of "hydro+":

1+1 Hydro: boost invariant and azimuthally symmetric flow (i.e. functions of $\tau$ and r ).

Placing a C.P. near =0 (no eq for baryon density.)

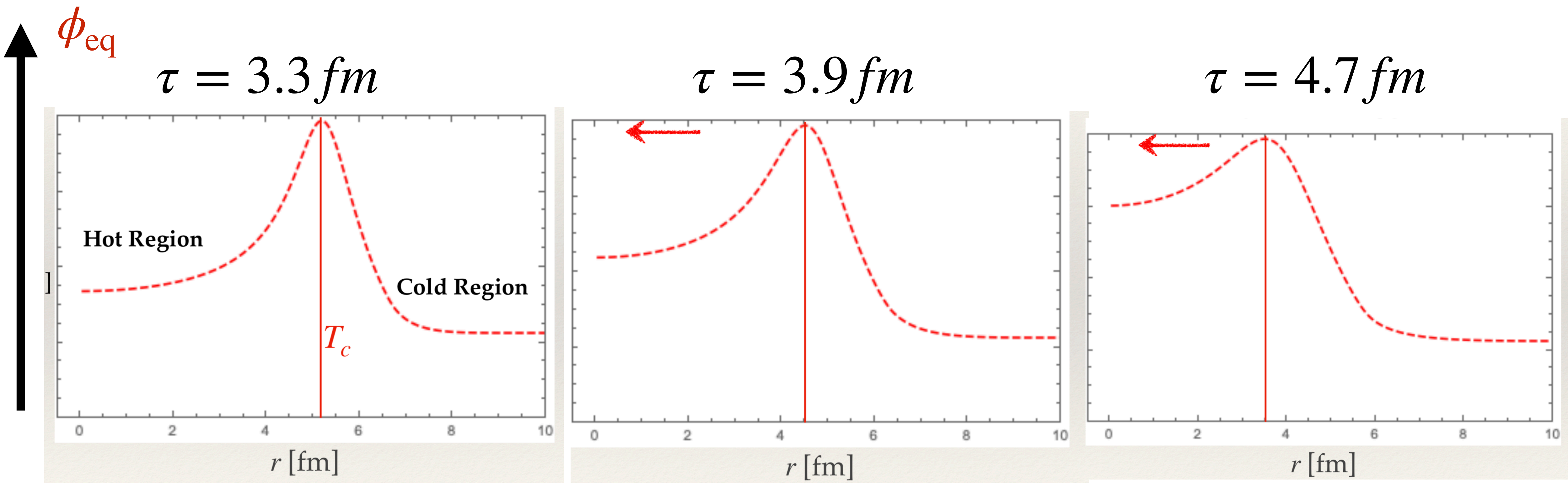



Disclaimer: we are not doing phenomenology here. This is an exercise to understand how to implement Hydro+ in practice, and to prepare for future quantitative studies.

## *The snapshot of $\phi_{eq}$ vs r*

(We consider a representative mode $Q \sim 1/\xi_{max}$ in this talk.)

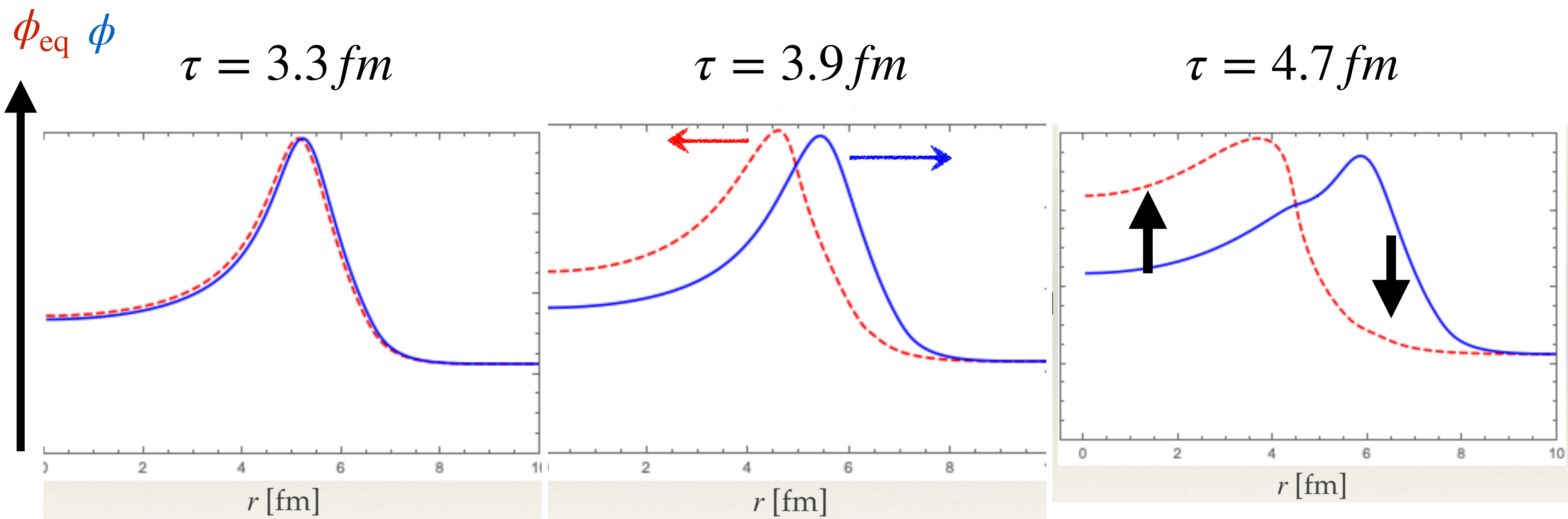
Equilibrium $\phi_{eq}$ in red.



# *The snapshot of ϕ vs r*

(We consider a representative mode $Q \sim 1/\xi_{max}$ in this talk.)

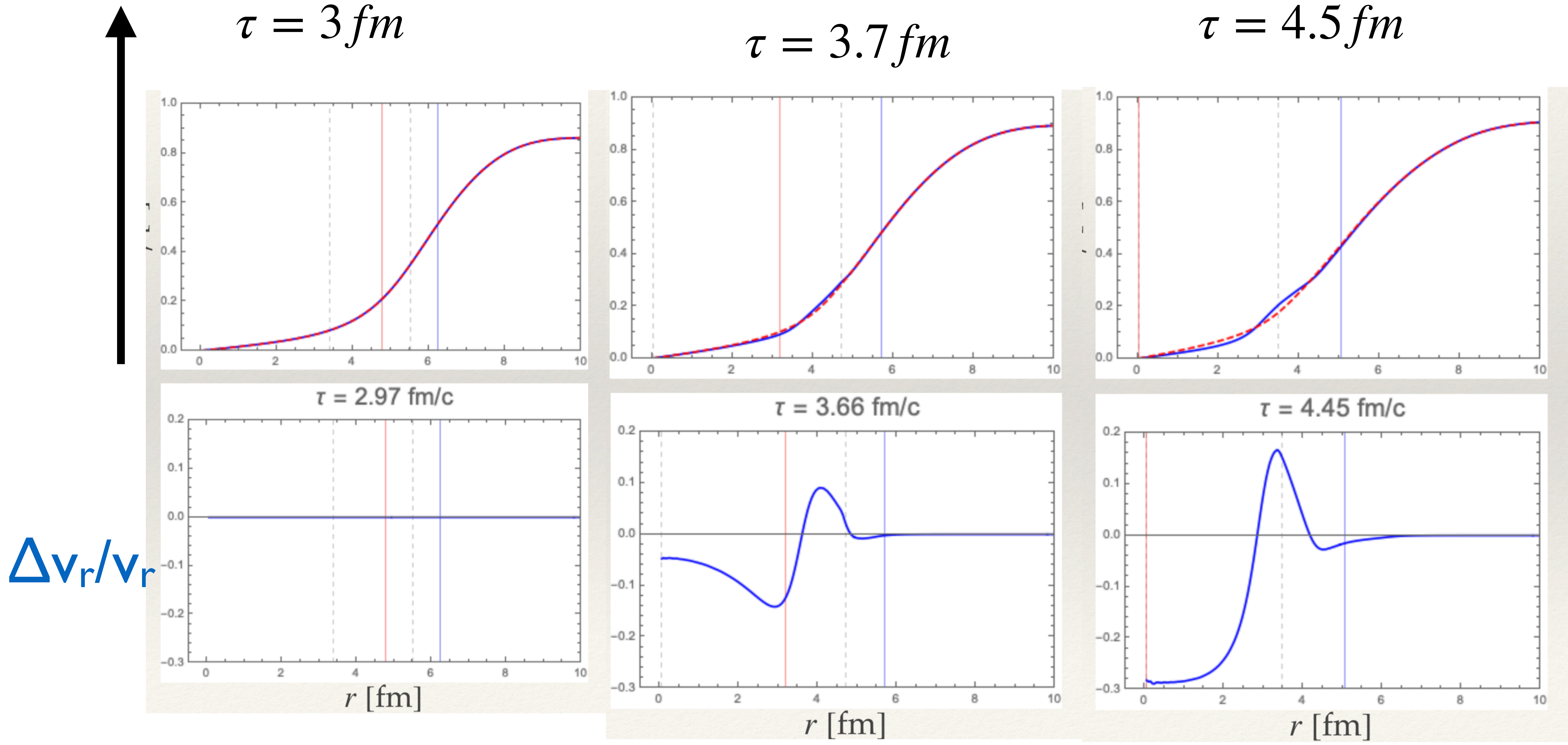
Equilibrium $\phi_{eq}$ in red. Offequilibrium $\phi$ in blue.




The evolution of ϕ is driven by critical slowing down effect and convection of the flow.

# *The snapshot of radial flow vs r*

Red: $v_r$ from hydro; Blue: $v_r$ from hydro+



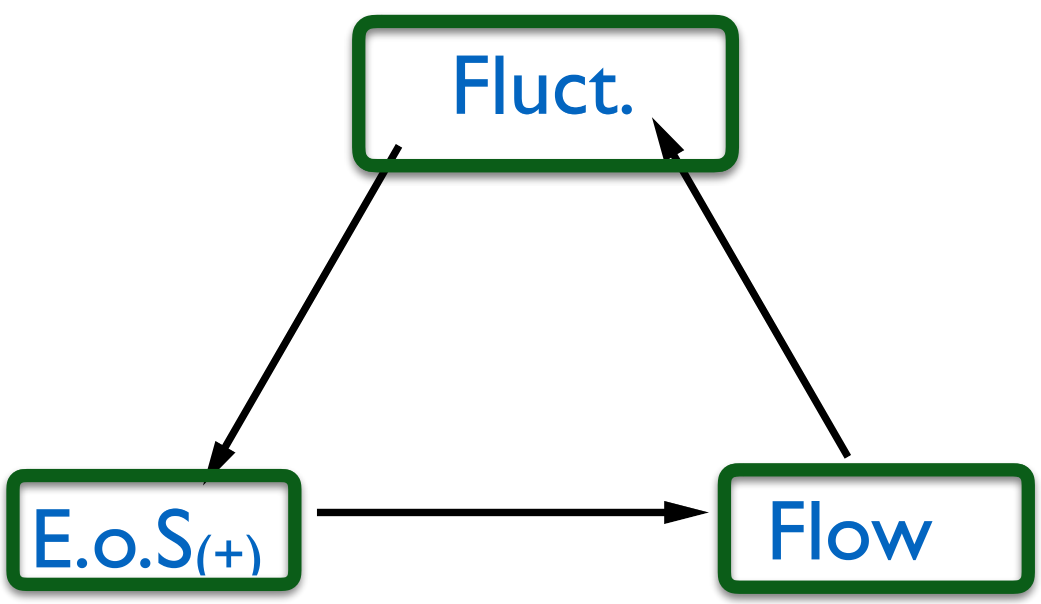

The relative difference in radial flow between hydro and hydro+ becomes sizable, and is comparable to that between hydro with E.o.S with and without a critical point.

# Conclusion and outlook

## *Conclusion and outlook*

Understanding critical dynamics are crucial to maximize the discovery potential of upcoming HIC experiments — we are working to build the needed theoretical tools.

"Hydro+" is formulated and good progress is made on the numerical implication of "hydro+" based on different hydro codes (VH1+1, OSU hydro. and MUSIC).




Rajagopal-Ridgway-Weller-YY, in preparation; see also Lipei Du-Heinz; Chun Shen.

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Stay tuned for new results from both experiments and quantitative theoretical studies.

# Back-up