Recent Theoretical Developments in Small Systems

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8^{th} Workshop of the Topical Group on Hadron Physics

Friday, April 12, 2019

Overview

1. “Dilute,” “Dense,” and Small Systems

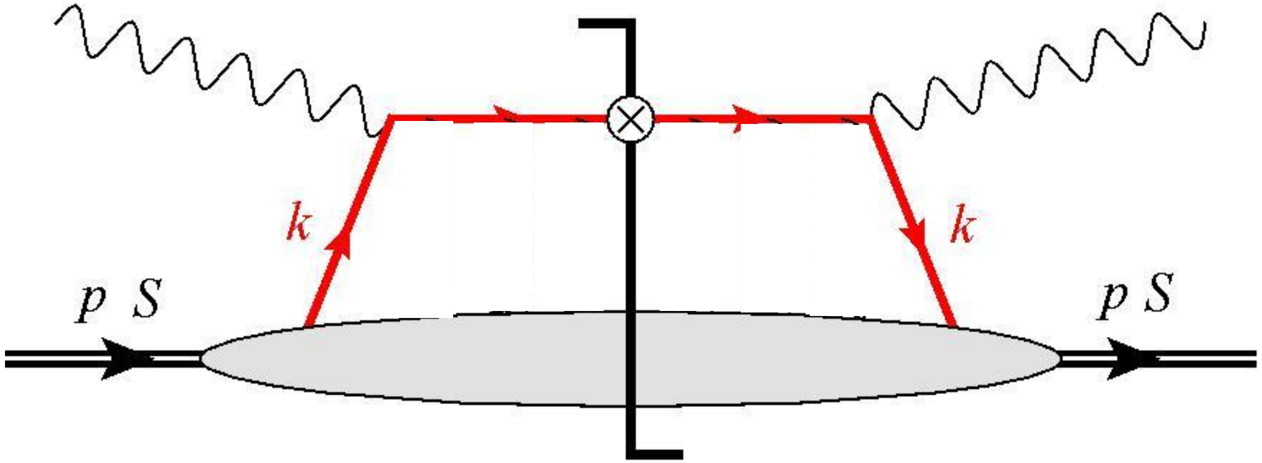
2. “Flow” and “Non-Flow” in Multiparticle Cumulants

3. Non-Flow Correlations in the Literature

1. “Dilute,” “Dense,” and Small Systems
2. “Flow” and “Non-Flow” in Multiparticle Cumulants
3. Non-Flow Correlations in the Literature

"Dilute" Scattering in the Twist Expansion

Inclusive DIS
on the proton



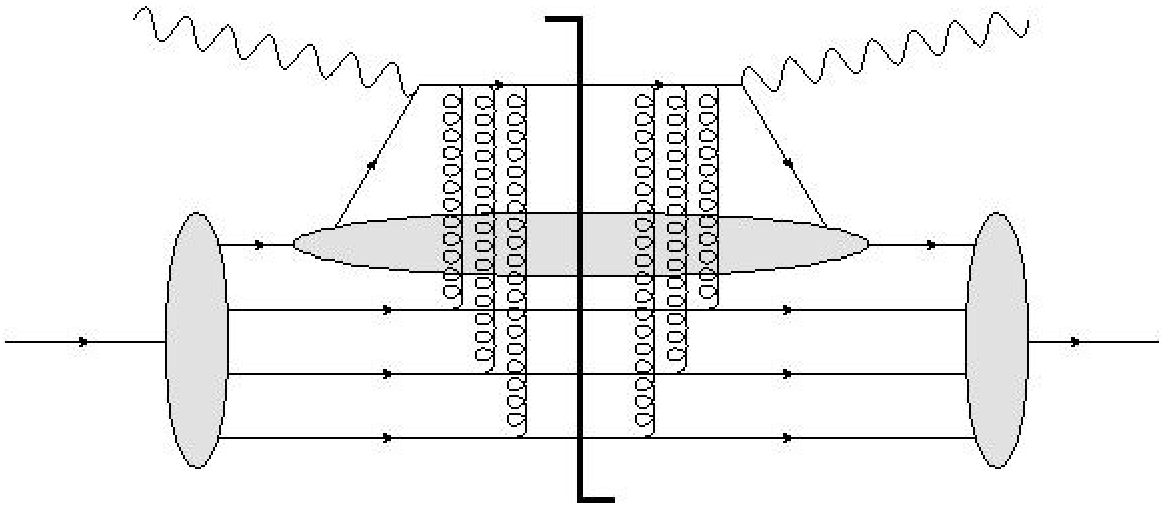
$A^+ = 0$ Gauge

"Dilute" $\equiv$ One hard / active parton

- Collinear factorization:
  - Hard **multi-parton interactions suppressed** by Q^2

“Dense” Scattering in (e.g.) a Heavy Nucleus

$A^{1/3}$ Spectators



“Dense” $\equiv$ Full Resummation

$$\alpha_s A^{1/3} \sim \mathcal{O}(1)$$

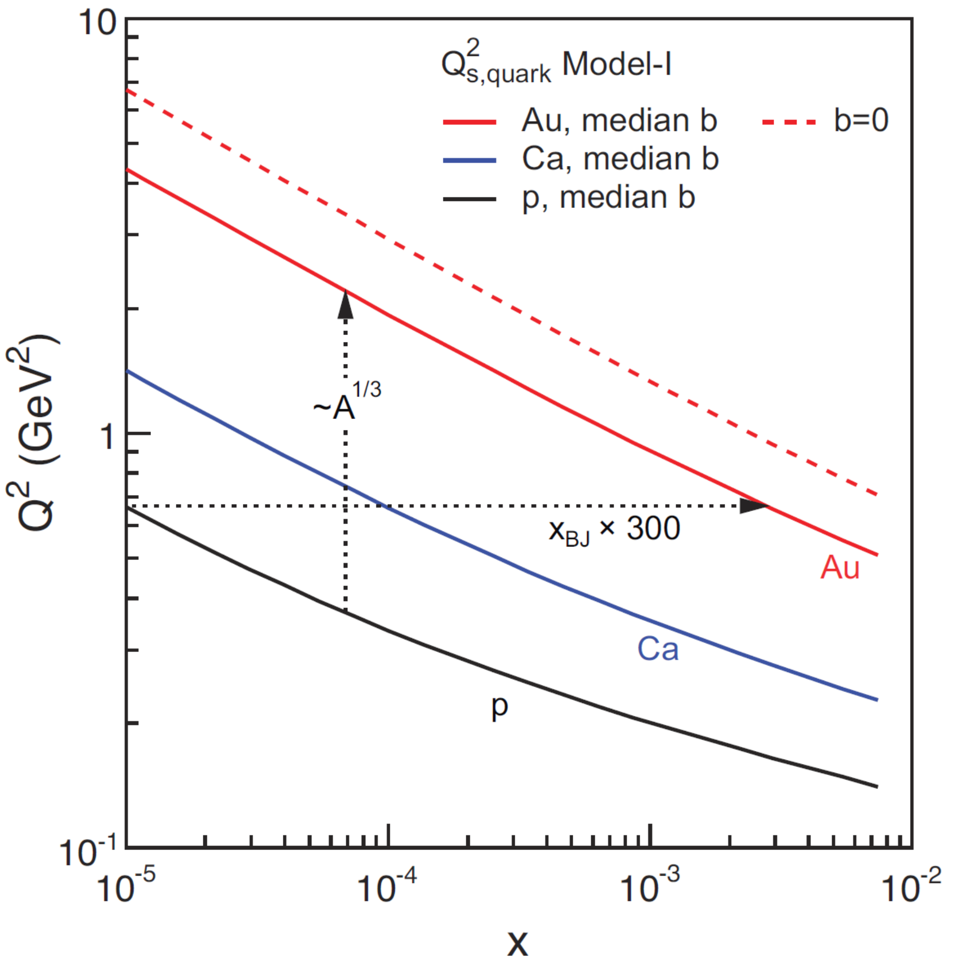
- Requires a parametrically **large charge density**
- Enhancement of **independent** multi-parton interactions

- A parton scattering multiple times on independent sources can be **exponentiated**

$$\frac{d\sigma^{q\bar{q}}}{d^2b} \sim 1 - \exp\left[-\frac{1}{4} r_T^2 Q_s^2(b) \ln \frac{1}{r_T \Lambda}\right]$$

- Resummation is equivalent to a background of **classical gluon fields**

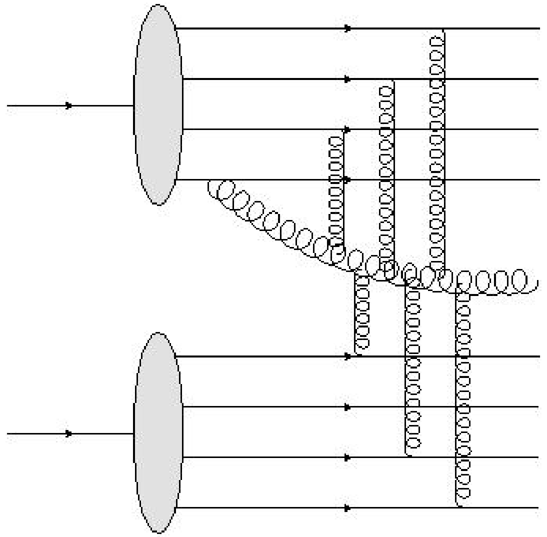
$$\frac{dN_g^{WW}}{d^2k} \sim \int d^2b\, d^2r\, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp} \frac{1}{r_T^2} \frac{d\sigma^{q\bar{q}}}{d^2b}$$



A. Accardi et al., Eur. Phys. J. ***A52*** *(2016)*

"Dense / Dense" – Classical Yang Mills

$A_1^{1/3}$ spectators (projectile)

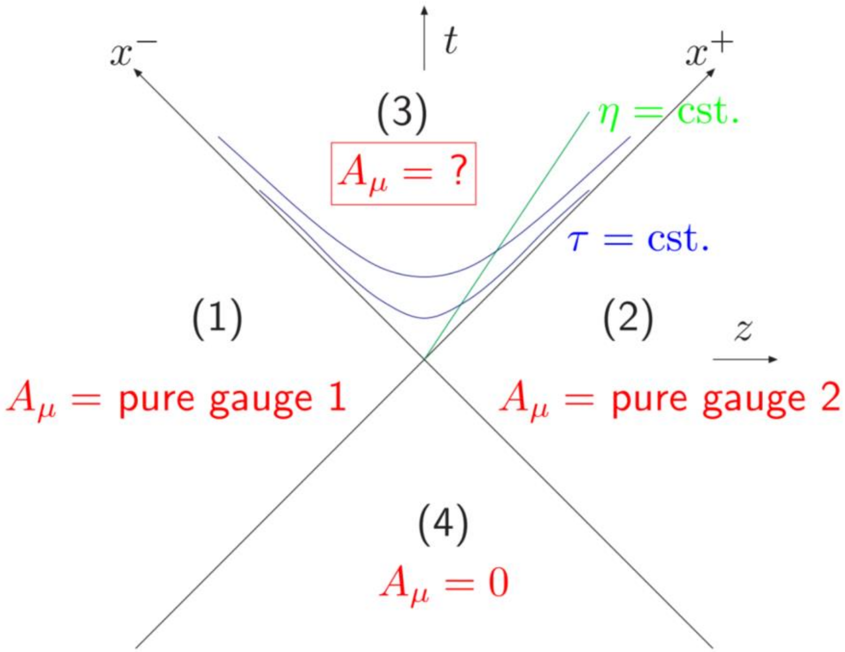


$A_2^{1/3}$ spectators (projectile)

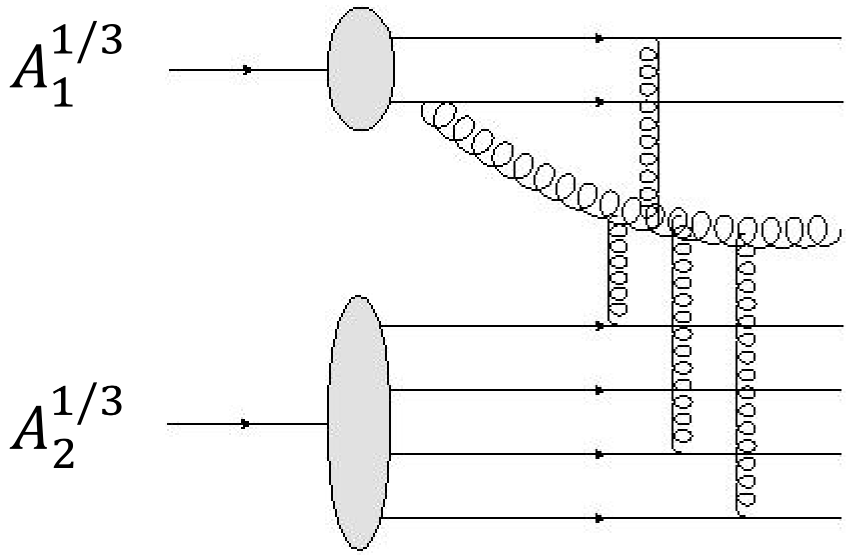
$$\alpha_s\, A_1^{1/3} \sim O(1)$$
$$\alpha_s\, A_2^{1/3} \sim O(1)$$

- A "Dense / Dense" scattering requires **simultaneous resummations**
  - ➢ This task is currently **unsolved in QCD**

- What is done is to carry over the conclusion about **classical fields**
  - Solve the classical Yang-Mills equations for two colliding nuclei



T. Lappi, Phys. Lett. ***B643*** *(2006)*



$$\alpha_s^2 A_1^{1/3} \sim \mathcal{O}(1)$$

$$\alpha_s \ll \alpha_s^2 A_2^{1/3} \ll 1$$

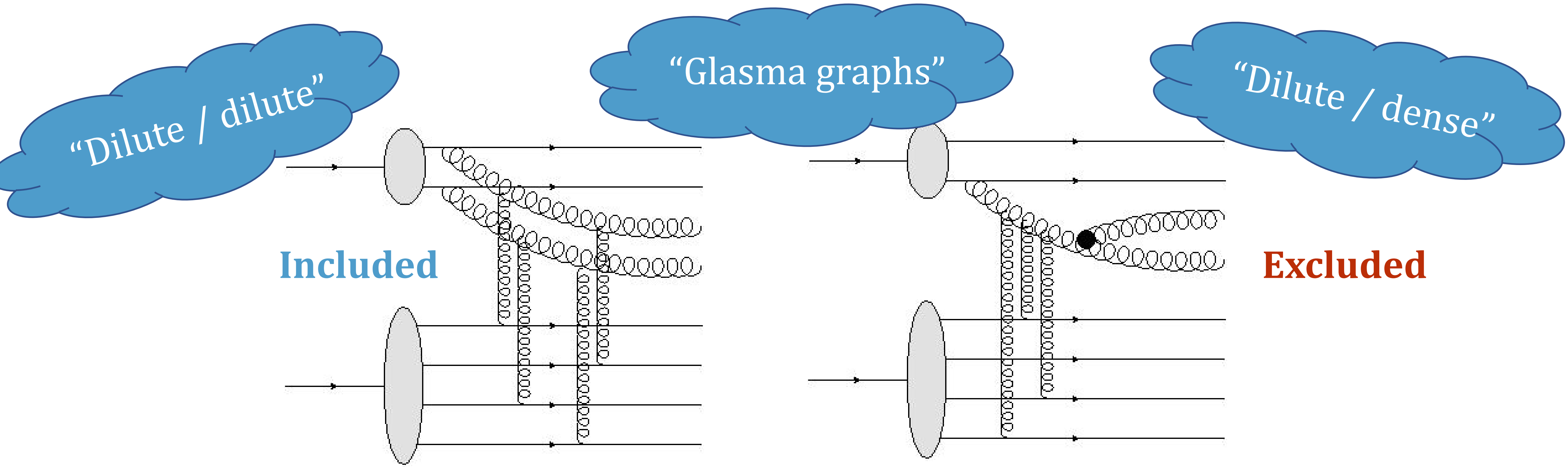
- **Asymmetric** collision: step from dilute/dense **toward dense/dense**
  - High density target: fully resummed multiple rescattering
  - Moderate density projectile: density-enhanced corrections order by order

- Appropriate for "**heavy-light**" ion collisions, i.e.) Cu + Au

Y. Kovchegov, D. Wertepny, Nucl. Phys. ***A906*** *(2013)*

A. Kovner, M. Lublinsky, Int. J. Mod Phys. ***E22*** *(2013)*

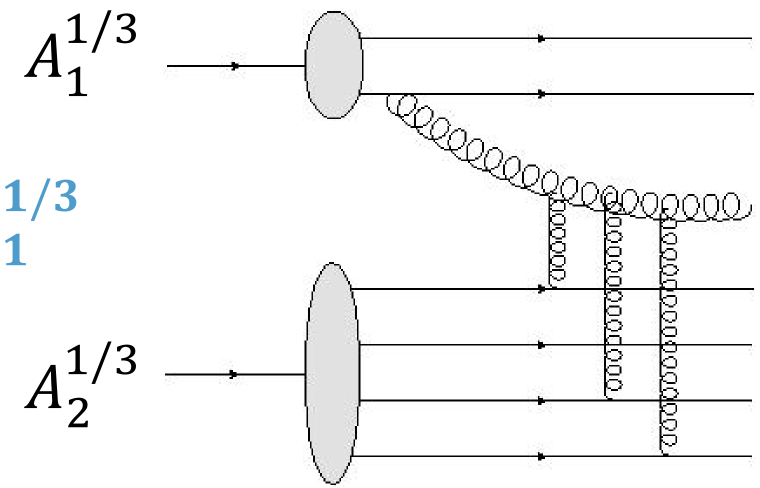
Correlations in Small Systems: Buyer Beware



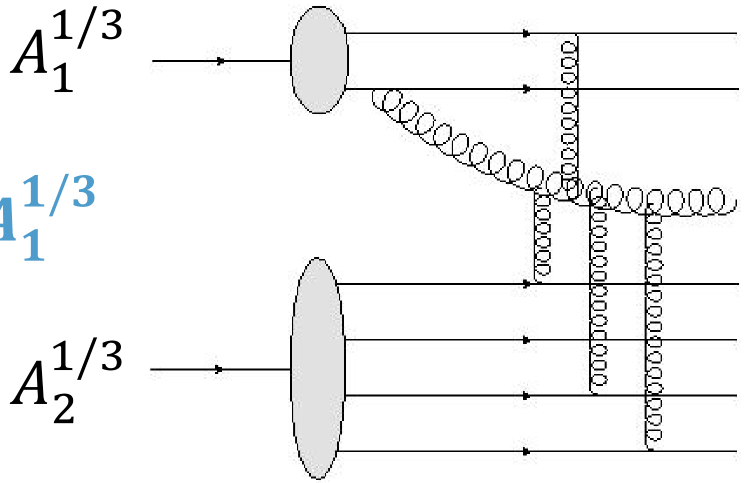
- For high densities, correlations are driven by **independent emission** of particles
  - ➢ The high-pT "dilute" limit **does not reproduce** the truly "dilute" regime

- Caution: **high densities may be assumed** for correlations in **small systems**!

LO in $A_1^{1/3}$



NLO in $A_1^{1/3}$



- Correlations at **mid-rapidity** are dominated by soft **gluons**

$$\Delta\phi \to \Delta\phi + \pi$$

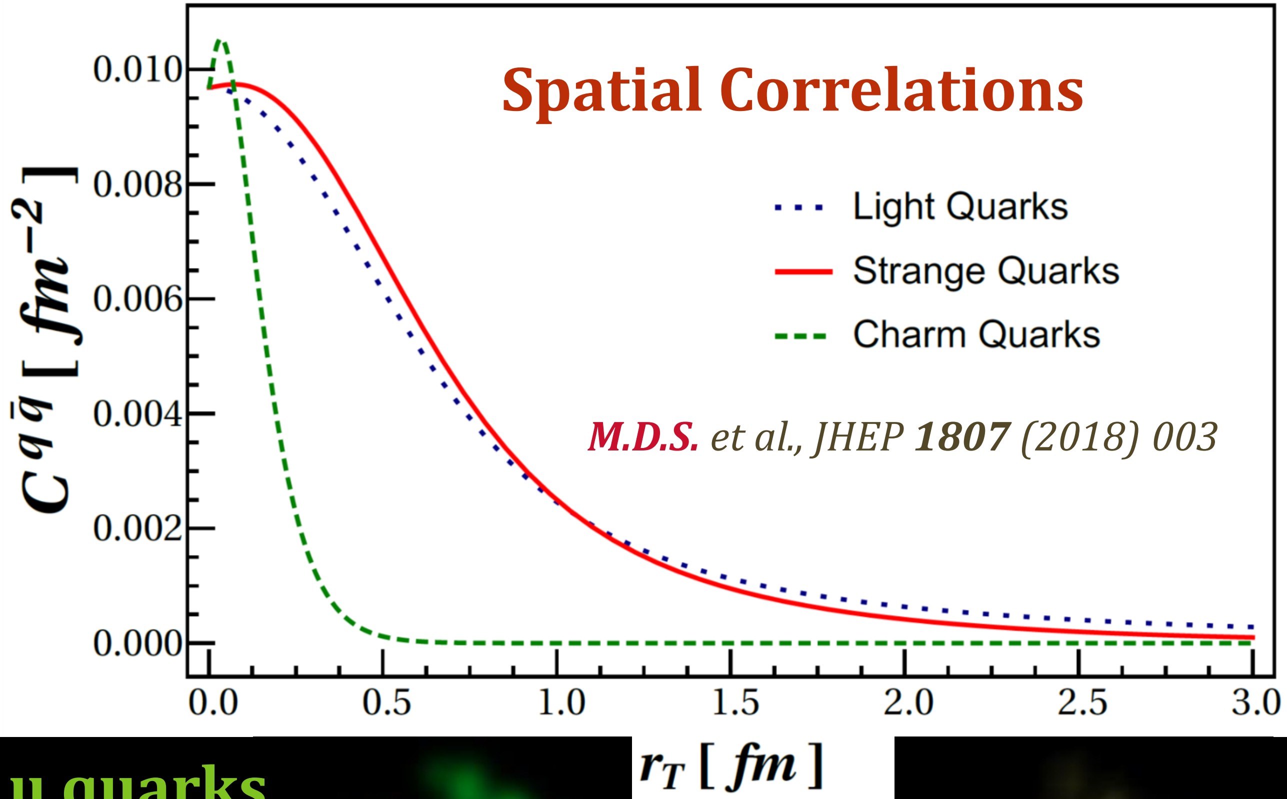
- At leading-density, 2-gluon correlations are symmetric
  - ➢ Reflective of **Bose enhancement**

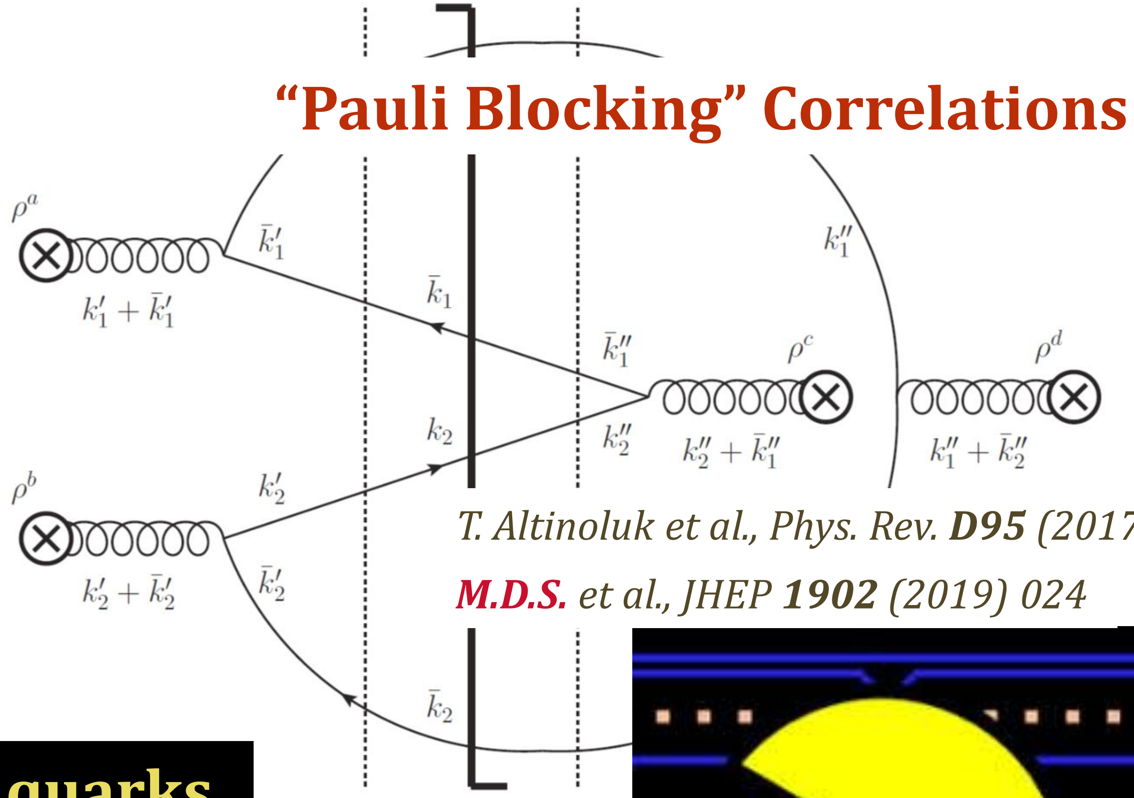
T. Altinoluk et al., Phys. Lett. ***B751*** *(2015), JHEP* ***1805*** *(2018)*
Y. Kovchegov, D. Wertepny, Nucl. Phys. ***A906*** *(2013),* ***A925*** *(2014)*

- **Antisymmetric** correlations enter at **next-to-leading density**

Y. Kovchegov, V. Skokov, Phys. Rev. ***D97*** *(2018)*

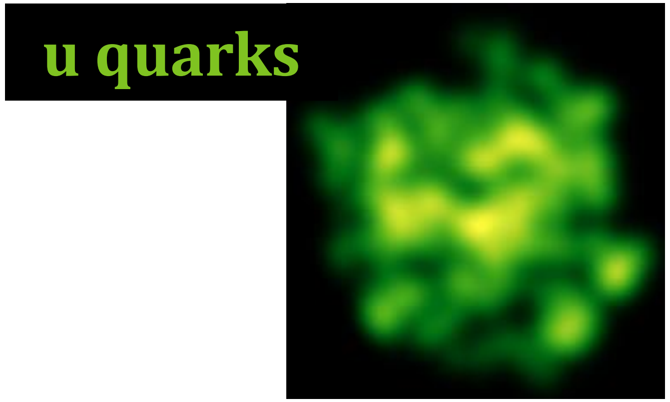
- **Multi-gluon correlations** known up to 3 (Leading / dense) and 4 (Leading / Leading)

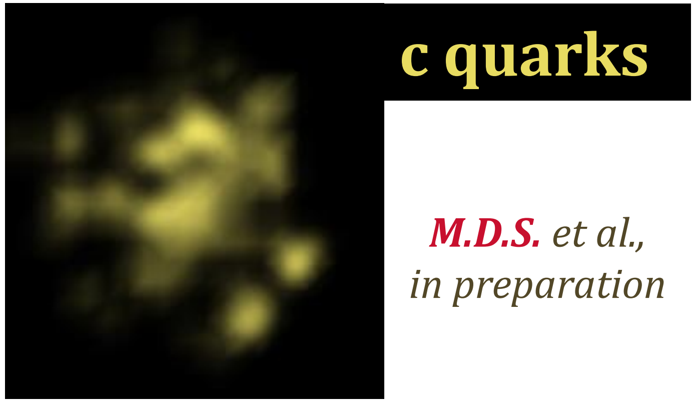




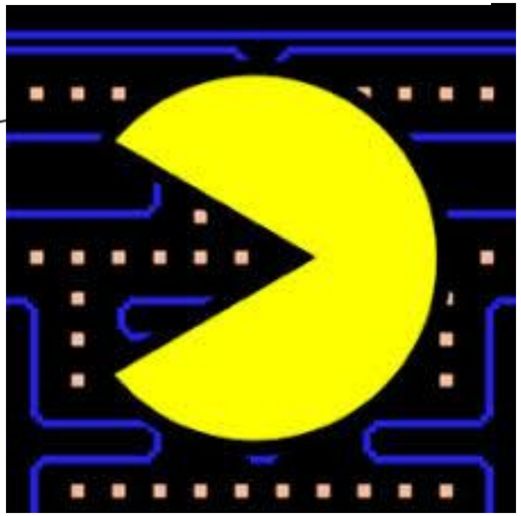
T. Altinoluk et al., Phys. Rev. ***D95*** *(2017)*

M.D.S. *et al., JHEP* ***1902*** *(2019) 024*





M.D.S. *et al.,*
in preparation



Overview

"Flow" versus "Non-Flow"

- **"Flow"** refers to any source of anisotropy in the **single-particle** distribution **in an event**

$$v_n \equiv \frac{1}{N} \left| \int_p e^{in\phi} \frac{dN_1}{d^2p} \right|$$

- Multi-particle distributions arise both from **independent** production (**flow**) and dynamical **correlations** (**non-flow**)

"Flow" "Non-Flow"

$$\frac{dN_2}{d^2p_1\, d^2p_2} \equiv \frac{dN_1}{d^2p_1} \frac{dN_1}{d^2p_2} + \delta_2(p_1, p_2)$$

- **Cumulants** are observables that are **differently sensitive** to **flow** and **non-flow**

M. Luzum, H. Petersen, J. Phys. ***G41*** *(2014)*

$$(v_n\{2\})^2 \equiv \left\langle \frac{1}{N^2} \int_{p_1 p_2} e^{in(\phi_1 - \phi_2)} \frac{dN_2}{d^2p_1 d^2p_2} \right\rangle$$

$$(v_n\{4\})^4 \equiv 2 \left\langle \frac{1}{N^2} \int_{p_1 p_2} e^{in(\phi_1 - \phi_2)} \frac{dN_2}{d^2p_1 d^2p_2} \right\rangle^2 - \left\langle \frac{1}{N^4} \int_{p_1 p_2 p_3 p_4} e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \frac{dN_4}{d^2p_1 \cdots d^2p_4} \right\rangle$$

Multiparticle Cumulants: **Flow Scenario**

- Usually: a **flow-only** scenario
  - **No dynamical correlations** $(\delta_2 = 0)$
  - Multiparticle production **factorizes**
  - All cumulants due to **single-particle anisotropy**
- All cumulants describe the **event-by-event distribution** of the **single-particle anisotropy v_n**
  - Natural **hierarchy**: $v_n\{2\} > v_n\{4\} \approx v_n\{6\} \approx \cdots$

"Flow" only

$$\frac{dN_2}{d^2p_1\, d^2p_2} \equiv \frac{dN_1}{d^2p_1} \frac{dN_1}{d^2p_2}$$

$$(v_n\{2\})^2 \overset{flow}{=} \langle v_n^2 \rangle$$

$$(v_n\{4\})^4 \overset{flow}{=} 2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle$$
$$= (v_n\{2\})^4 - \mathrm{Var}\left(v_n^2\right)$$

M. Luzum, H. Petersen, J. Phys. ***G41*** *(2014)*

Multiparticle Cumulants: **Non-Flow Scenario**

M.D.S. *et al., in preparation*

M. Luzum, H. Petersen, J. Phys. ***G41*** *(2014)*

- Often in initial-state calculations:
  - **Isotropic** single-particle distribution $(v_n = 0)$

$$\frac{dN_1}{d^2p} = \frac{1}{2\pi p_T}\frac{dN}{dp_T}$$

  - Only **dynamical correlations**

"**Non-Flow**" only

$$\frac{dN_2}{d^2p_1\, d^2p_2} \equiv \delta_2(p_1, p_2)$$

- Sequential **hierarchy** of correlations in N_c
  - $\delta_2 \gg \delta_4 \gg \cdots$
  - Usually **imaginary** $v_n\{4\}$

$$(v_n\{2\})^2 \overset{nonflow}{=} \langle \delta_{2,(n)} \rangle$$

$$(v_n\{4\})^4 \overset{nonflow}{=} -2\,\mathrm{Var}\left(\delta_{2,(n)}\right) + \underbrace{\langle \delta_{4,(n)} \rangle}_{\mathcal{O}(10\%)}$$

Comparing Flow vs Non-Flow Correlations

- **MC Glauber** collision geometry sampled using **Trento**

$$s(\vec{x}_\perp) \propto \sqrt{T_A(\vec{x}_\perp)\, T_B(\vec{x}_\perp)}$$

- **Flow correlations**:
  - ➢ Initial entropy density
  - ➢ Evolved using 2+1D viscous hydrodynamics
  - ➢ Approximately linear response

v-USPhydro

$$v_n \propto \epsilon_n = \frac{\int d^2x_\perp\, |\vec{x}_\perp|^n\, e^{in\phi}\, s(\vec{x}_\perp)}{\int d^2x_\perp\, |\vec{x}_\perp|^n\, s(\vec{x}_\perp)}$$

- **Non-flow correlations**:
  - ➢ Two-gluon correlations δ_2
  - ➢ High-pT limit (sometimes called "dilute")
  - ➢ Fixed initial geometry + color averaging
  - ➢ Coefficients cancel in ratios

$$\delta_2(p_1, p_2) \overset{\text{L.O.}}{=} \left(\int d^2x_\perp T_A^2(\vec{x}_\perp)\, T_B^2(\vec{x}_\perp) \right) f(p_1, p_2)$$

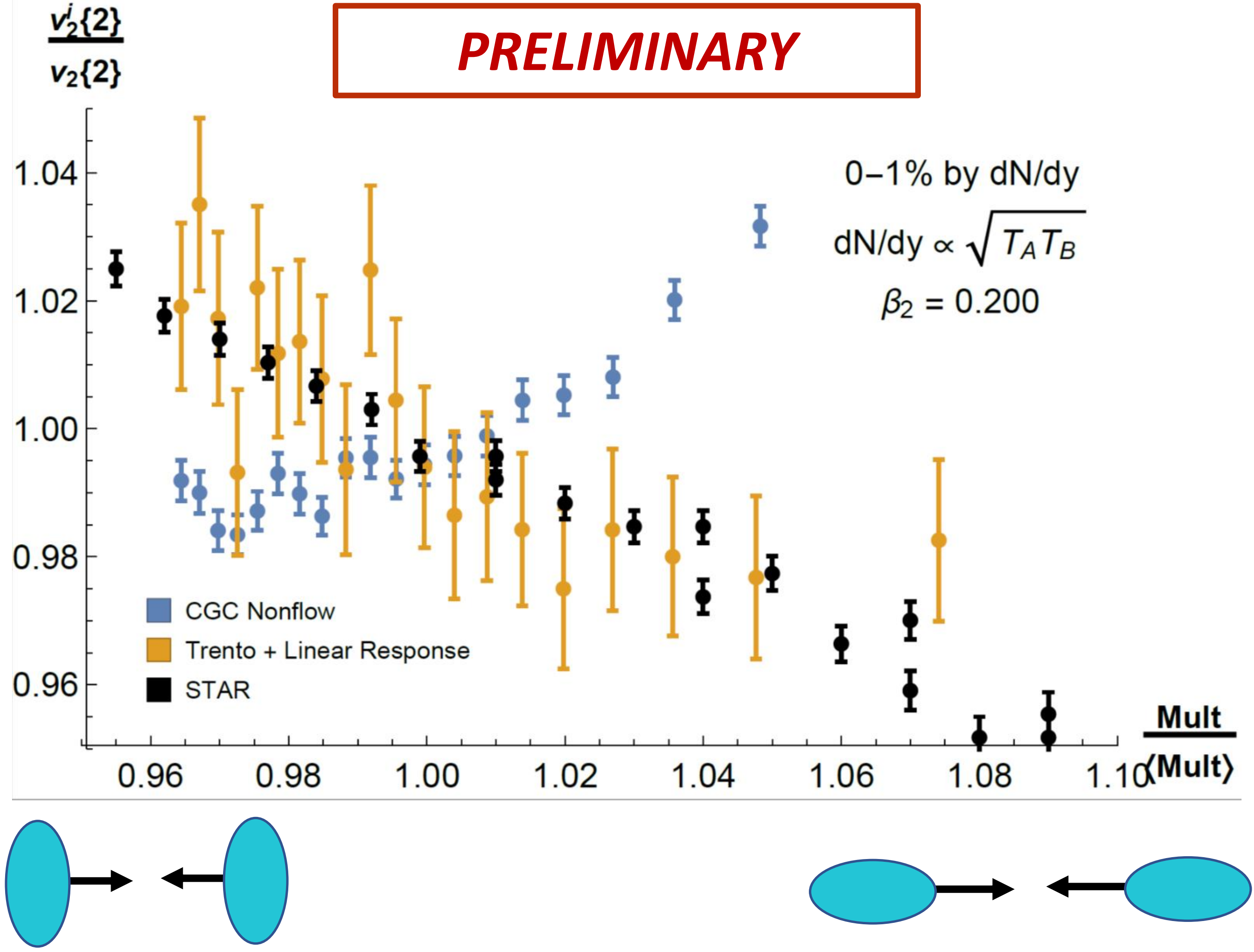
$$\delta_2(p_1, p_1) \overset{\text{N.L.O.}}{=} \left(\int d^2x_\perp T_A^3(\vec{x}_\perp)\, T_B^3(\vec{x}_\perp) \right) g(p_1, p_2)$$

***M.D.S.** et al., in preparation*

Results: Flow vs Non-Flow in UU

- **Opposite behavior** for $v_2\{2\}$:
- For **flow**, v_2 comes from **geometry**
  - ➢ Greatest for **side-on-side** collisions
  - ➢ Smaller multiplicities
- For **non-flow**, $v_2\{2\}$ comes from **multiplicity** via Q_s
- Disfavors a non-flow explanation

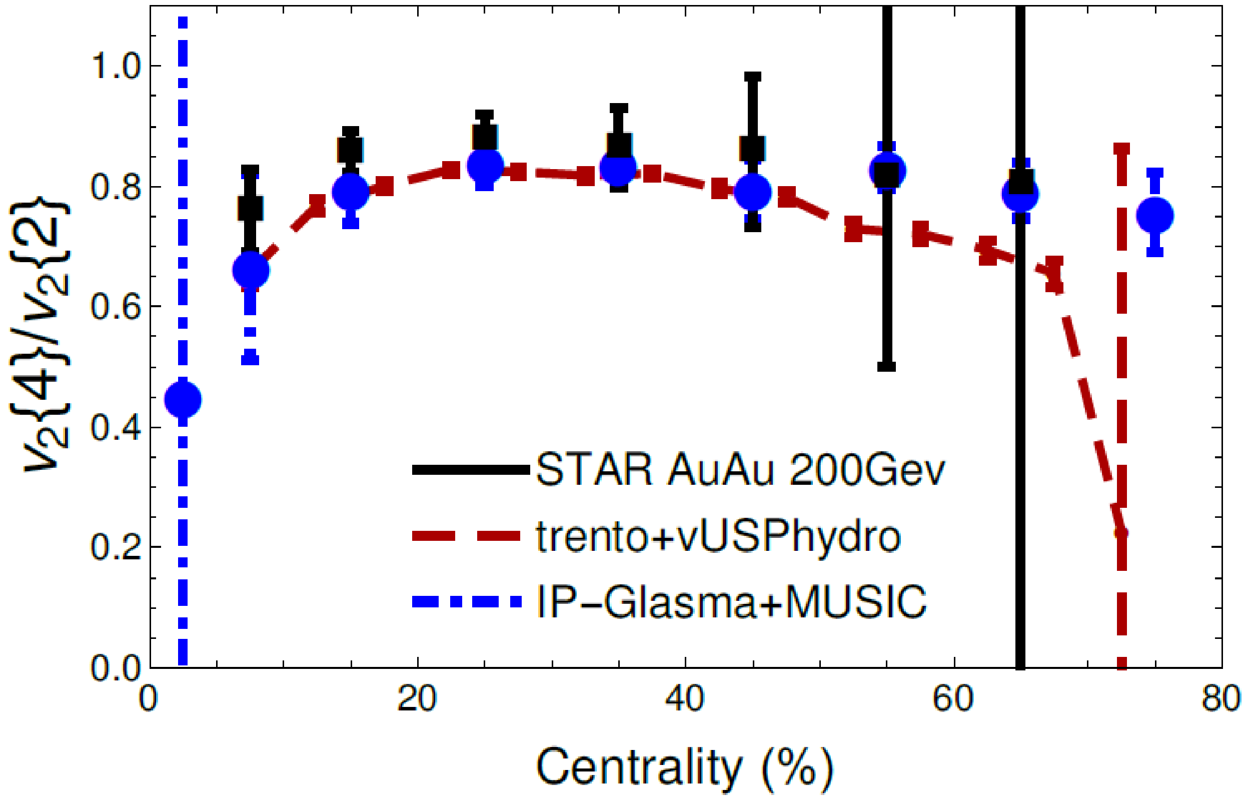
M.D.S. et al., in preparation



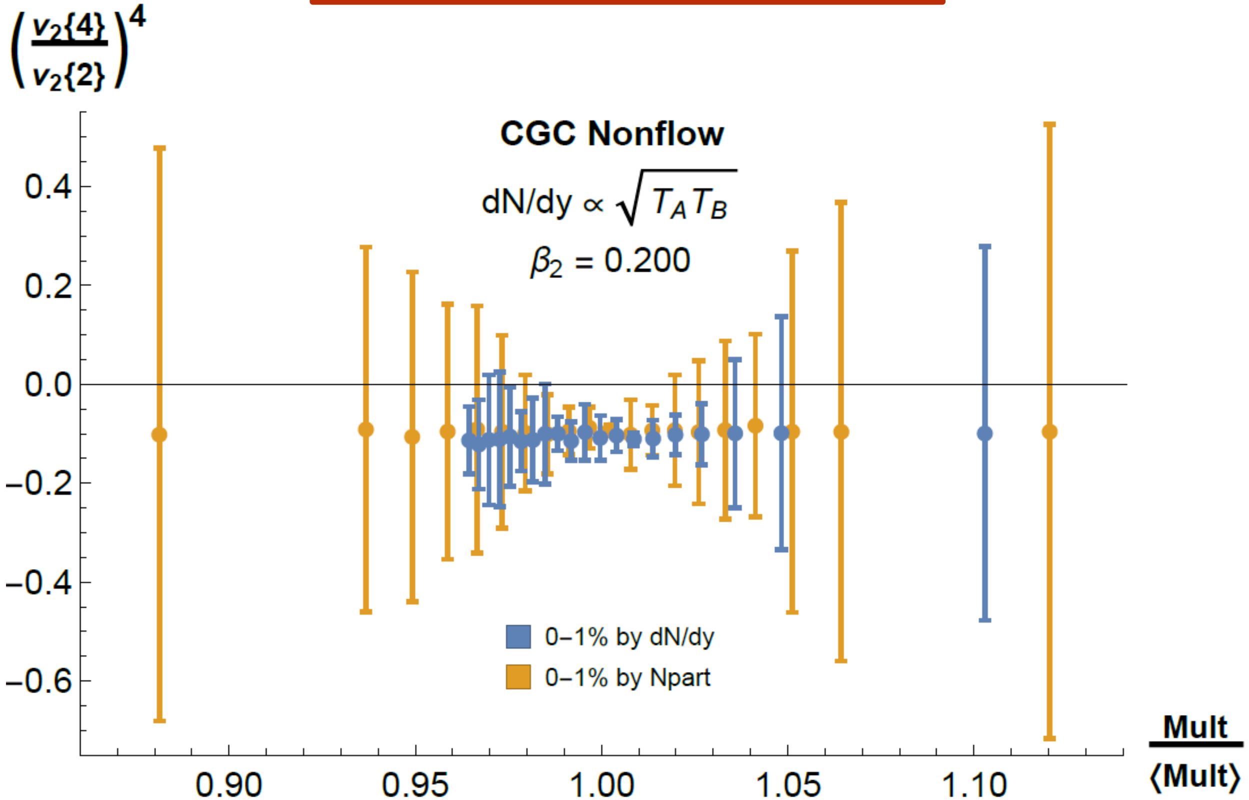
Results: Flow vs Non-Flow in UU

***M.D.S.** et al., in preparation*

PRELIMINARY



PRELIMINARY



- **Hydro** describes data with $v_2\{4\} > 0$
- For **CGC non-flow**, $v_2\{4\}$ **is imaginary**

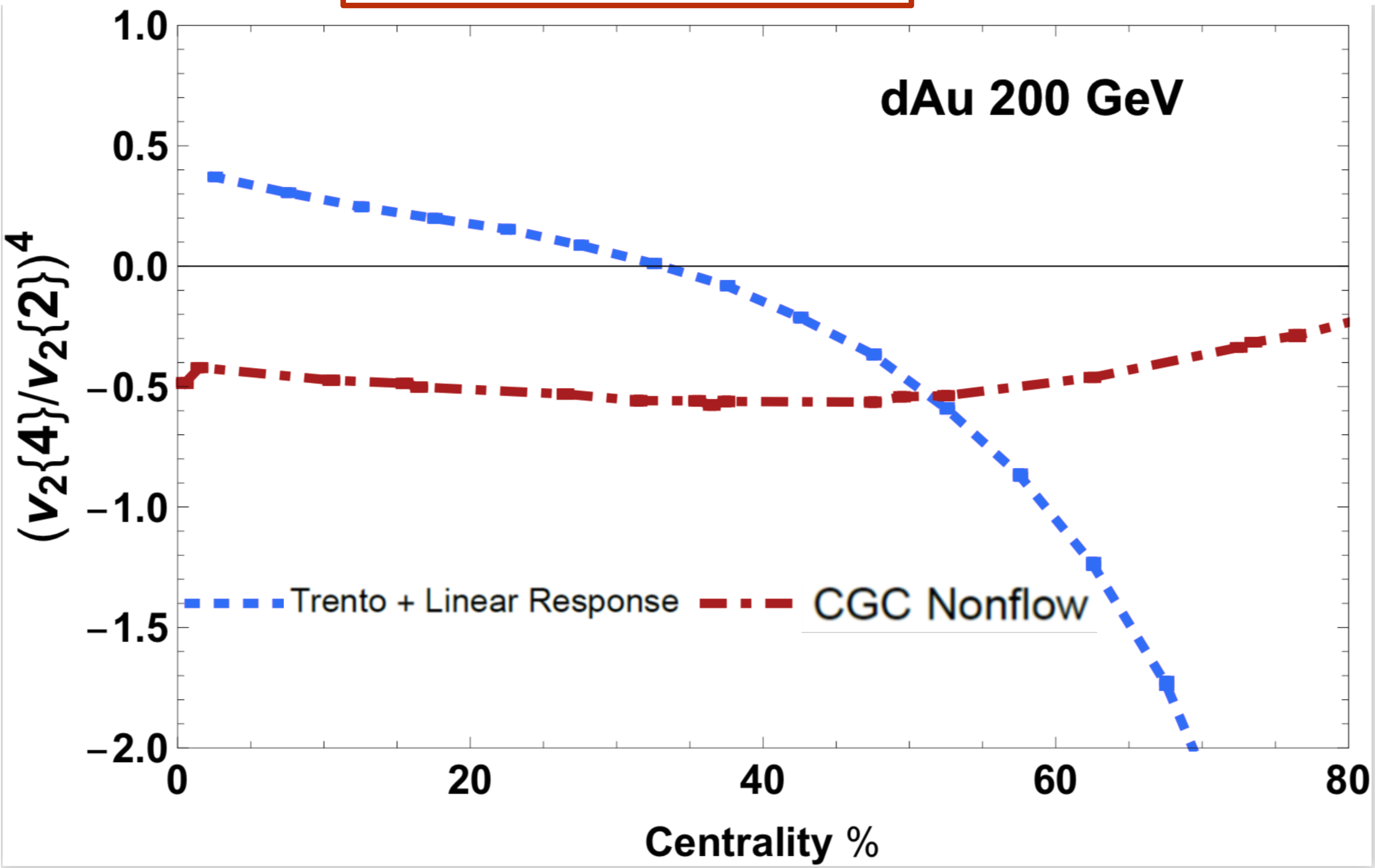
Results: Flow vs Non-Flow in dAu

- For $v_2\{2\}$ in dAu:
  - The **flow** signal is **flattened** due to round collision geometry
  - The **nonflow** signal is qualitatively the **same**

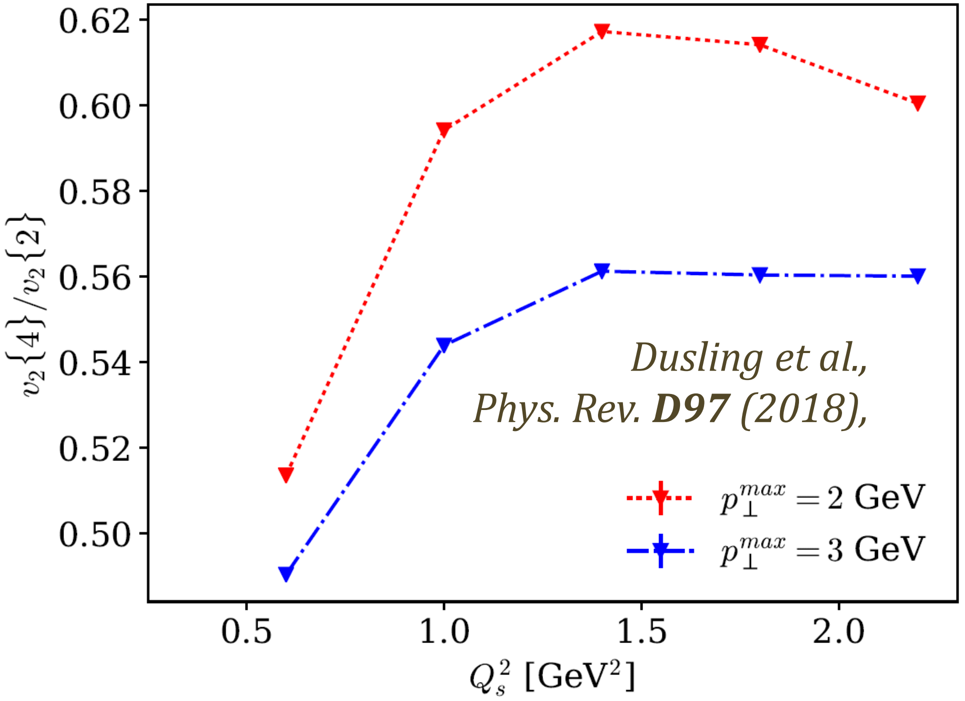
- For $v_2\{4\}$ in dAu:
  - The **flow** signal yields **real values** in central collisions
  - The **nonflow** signal is always **imaginary**

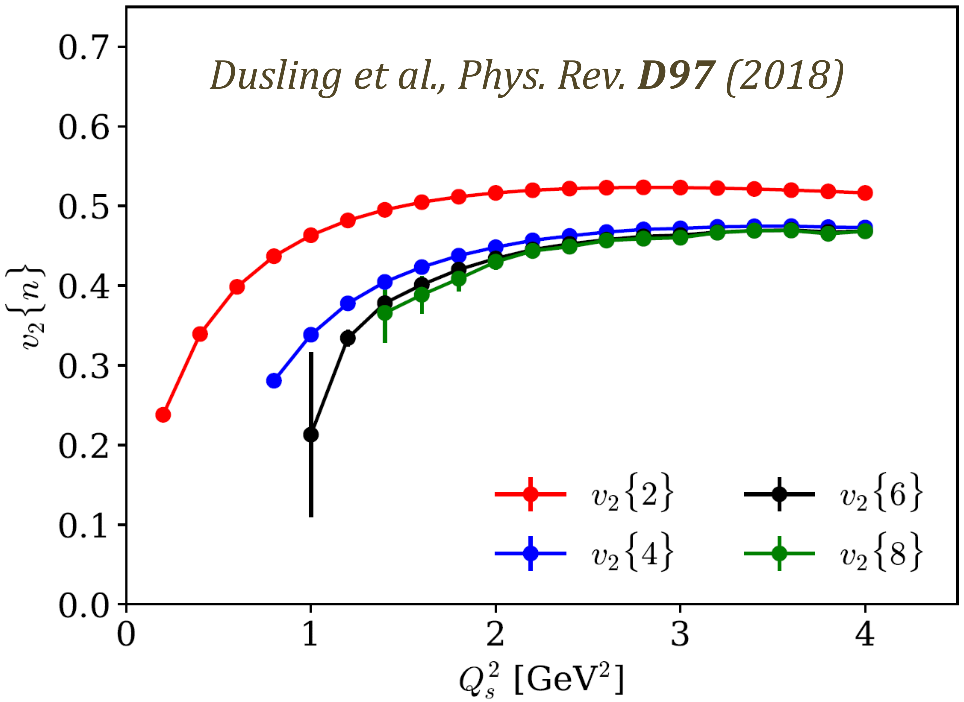
PRELIMINARY

M.D.S. et al., in preparation



Overview



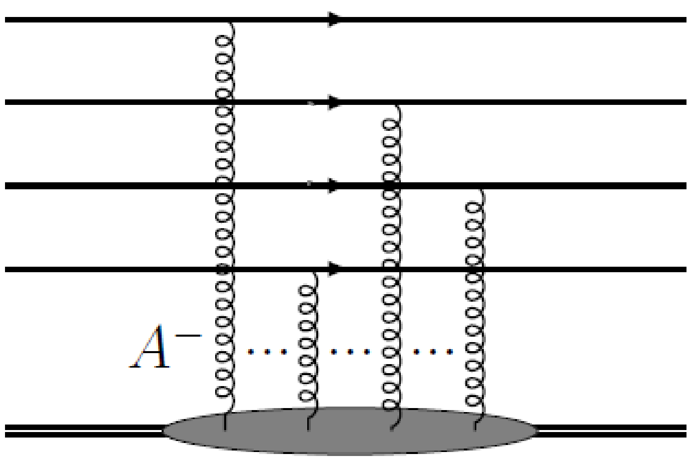


- It seems like **imaginary $v_2\{4\}$** is inevitable in **any non-flow mechanism**
  - ➢ How have **non-flow** calculations in the **literature** gotten the **systematics right**?

$$(v_n\{4\})^4 \overset{nonflow}{=} -2\,\mathrm{Var}\left(\delta_{2,\,(n)}\right) + \underbrace{\langle \delta_{4,\,(n)} \rangle}_{\mathcal{O}(10\%)}$$

A Seminal Benchmark: The "Parton Model"

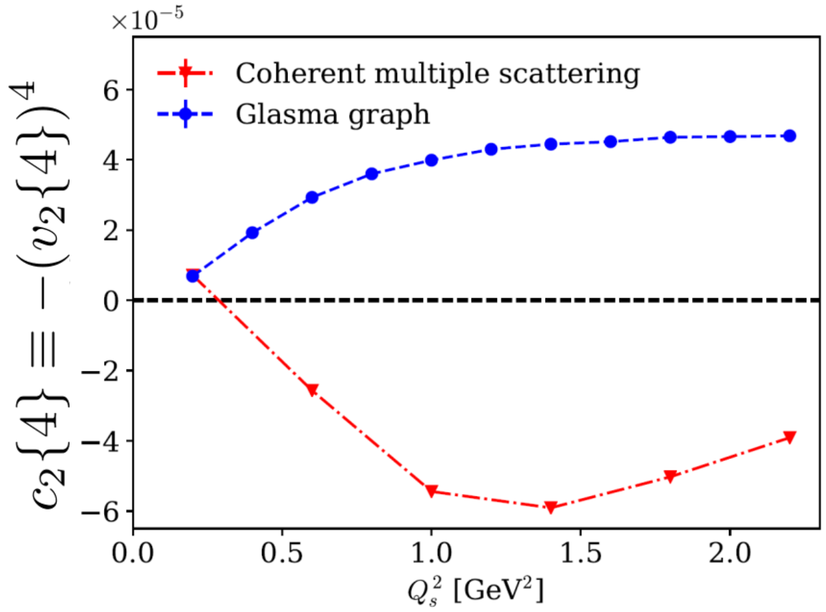
Dusling et al., Phys. Rev. ***D97*** *(2018), Phys. Rev. Lett.* ***120*** *(2018)*



- **Valence quarks** from projectile + correlated **color fields**
  - **Independently** sampled from Gaussian geometry
  - **Non-flow** correlations due to color averaging

$$\left\langle \frac{d^m N}{d^2\mathbf{p_1}\cdots d^2\mathbf{p_m}} \right\rangle = \frac{1}{(4\pi^3 B_p)^m} \prod_{i=1}^{m} \int d^2\mathbf{b_i} \int d^2\mathbf{r_i}\; e^{-|\mathbf{b_i}|^2/B_p} e^{-|\mathbf{r_i}|^2/4B_p} e^{i\mathbf{p_i}\cdot\mathbf{r_i}} \left\langle \prod_{j=1}^{m} D\left(\mathbf{b_j} + \frac{\mathbf{r_j}}{2}, \mathbf{b_j} - \frac{\mathbf{r_j}}{2}\right) \right\rangle$$

M.D.S. *et al., in preparation*

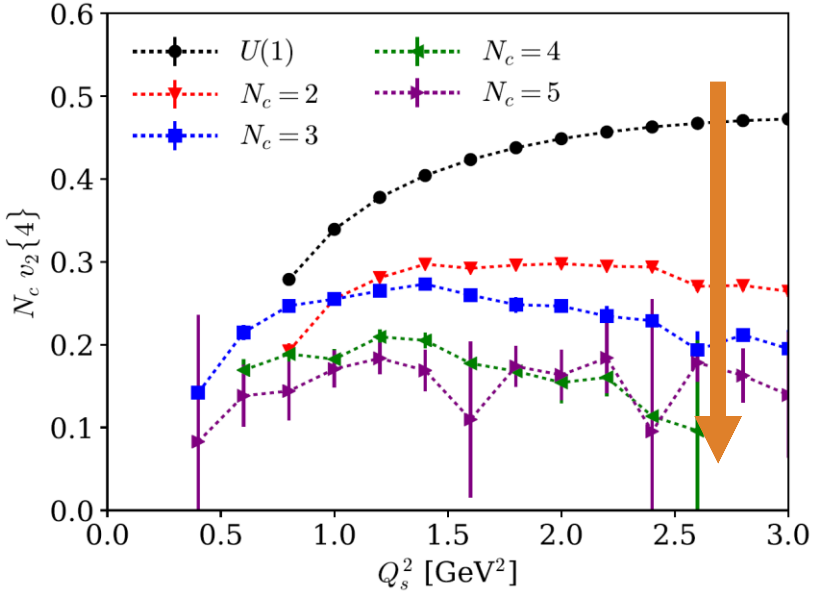


- Obtains $v_2\{4\}$ which can be **real** or **imaginary**…
  - **Static geometry** does not fluctuate
  - Driven entirely by **dynamical 4-particle correlations**

$$(v_n\{4\})^4 \overset{nonflow}{=} \underbrace{-2\,\mathrm{Var}\left(\delta_{2,\,(n)}\right)}_{0} + \underbrace{\langle \delta_{4,\,(n)} \rangle}_{\text{any sign}}$$

- **Real** values of $v_2\{4\}$ is an **accident** of the over-simplified **parton model**
  - Will be **broken** in more realistic models with **fluctuating geometry**
  - Apparent from **different N_c scaling** of $v_2\{4\}$

Dusling et al., Phys. Rev. ***D97*** *(2018)*



This should then give $v_2\{4\} \sim 1/N_c$. We see from Fig. (14) that $N_c\, v_2\{4\}$ begins to converge for $N_c \geq 3$; however, due to limited statistics, the error bars are large.

Dusling et al., Phys. Rev. ***D97*** *(2018)*

even in the full MV model beyond the glasma graph approximation, no connected cumulant remains for $c_n\{4\}$ at the N_c^{-4} order. thus, we need to go to go to the N_c^{-6} order

K. Fukushima, Y. Hidaka., JHEP ***1711*** *(2017)*

$v_2\{4\}$ does **not** scale like $\frac{1}{N_c^4}$

Rather: $\frac{1}{N_c^6}$ because $Var = 0$

Cumulant Hierarchies… And Lack Thereof

- Significant **misattribution** to the origin of $v_2\{4\}$
  - Write that **2-particle correlations** drive higher cumulants
  - Plot the **flow-like hierarchy** of the **Abelian** theory for $N_c = 1$

$$v_n\{m\} \sim N_c^{-2+2/m}$$

K. Fukushima, Y. Hidaka., JHEP ***1711*** *(2017)*

- Actually due to **successively higher** multiparticle correlations
  - **Successive suppression** of higher cumulants
  - $v_2\{2\} > v_2\{4\} > v_2\{6\} > \cdots$

Dusling et al., Phys. Rev. Lett. ***120*** *(2018)*

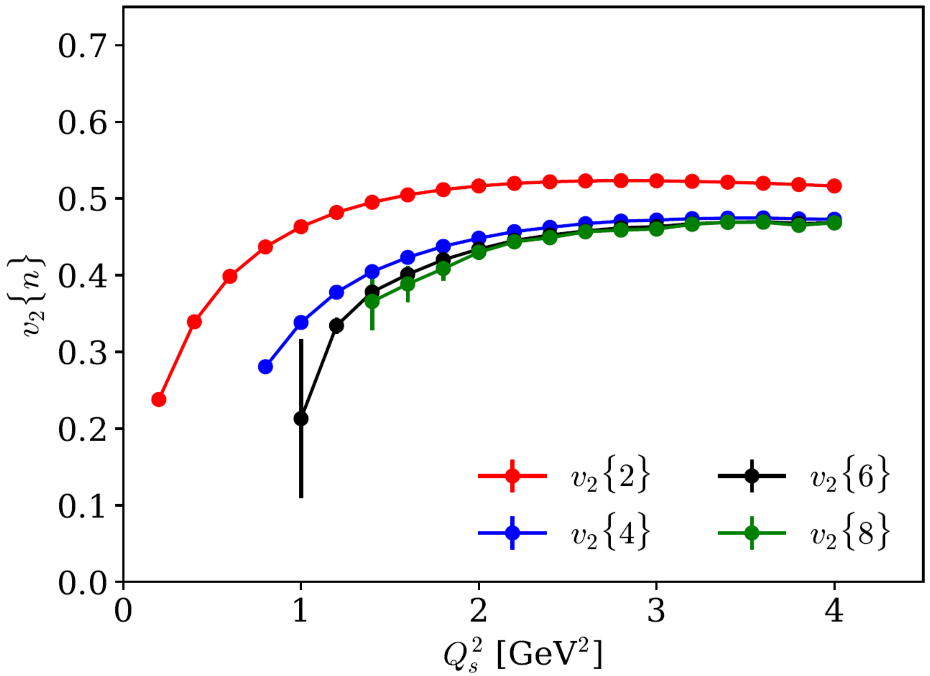


FIG. 5. Two-, four-, six- and eight-particle Fourier harmonics for coherent multiple scattering off Abelian fields plotted as a function of $Q_{s,T}^2$.

$N_c = 1$!!!

The Next Benchmark: MSTV

*Mace et al., Phys. Rev. Lett. **121** (2018), and arXiv:1901.10506*

$$\frac{dN}{d^2k_\perp}\left[\rho_p,\rho_t\right] \overset{L.O.}{=} \frac{2}{(2\pi)^3}\frac{\delta^{ij}\delta^{\ell m}+\epsilon_T^{ij}\epsilon_T^{\ell m}}{k_T^2}\,\Omega^{a\,ij}_{(\vec{k}_\perp)}\left[\rho_p,\rho_t\right]\left(\Omega^{a\,\ell m}_{(\vec{k}_\perp)}\left[\rho_p,\rho_t\right]\right)^*$$

$$\frac{dN}{d^2k_{\perp 1}\cdots d^2k_{\perp n}} = \int [\mathcal{D}\rho_p]\,[\mathcal{D}\rho_t]\,W[\rho_p]\,W[\rho_t]\;\frac{dN}{d^2k_{\perp 1}}[\rho_p,\rho_t]\cdots\frac{dN}{d^2k_{\perp n}}[\rho_p,\rho_t]$$

- Event-by-event fluctuations:
  - ➢ MC Glauber **geometry**
  - ➢ Event-by-event color fields: **operators** in ρ_p, ρ_t

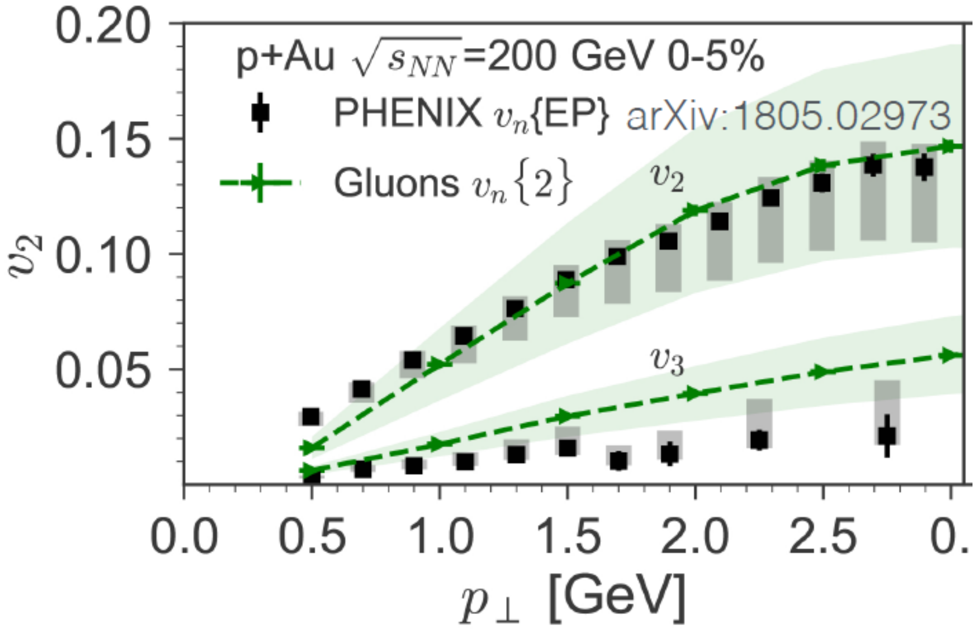
- Two-gluon correlations from **semi-dilute / dense** expressions
  - ➢ Inherently **factorized** at operator level (density-enhanced)

Thoughts on the Physics of MSTV

$$\mathcal{V}_n\Big[\rho_p, \rho_t\Big] = \frac{\int d^2k_\perp \, e^{in\phi} \, \frac{dN}{d^2k_\perp}\Big[\rho_p, \rho_t\Big]}{\int d^2k_\perp \, \frac{dN}{d^2k_\perp}\Big[\rho_p, \rho_t\Big]}$$

$$(v_n\{2\})^2 = \int [\mathcal{D}\rho_p] \, [\mathcal{D}\rho_t] \, W[\rho_p] \, W[\rho_t] \, \mathcal{V}_n[\rho_p, \rho_t] \Big(\mathcal{V}_n[\rho_p, \rho_t]\Big)^*$$

- Cumulants arise from **single-particle anisotropies** of **color fields**
  - ➢ **Flow!**
  - ➢ Naturally gets 2-particle systematics



- **Open questions** (for me):
  - What do they get for $v_2\{4\}$?
  - Interpretation of the **"distribution"** $\frac{dN}{d^2k}$?
  - What does the **averaging** really do to the flow?

Outlook

- **In Preparation:**
  - A systematic analysis of **flow vs. non-flow** contributions **in general**
  - Quantitative flow vs. non-flow results for **UU** and **small systems**

- **Next steps:**
  - The **parton model, revisited** – a useful toy to **turn on / off** flow and non-flow
  - Detailed analysis of color-flow mechanisms in MSTV
  - Flow from **density gradients** in the CGC…?

see D. Wertepny talk: DIS2019