On the Lepton Angular Distribution in Drell-Yan and Vector Boson Production

Jen-Chieh Peng

University of Illinois at Urbana-Champaign

8th Workshop of the APS Topical
Group on Hadron Physics
April 10-12, 2019

Based on the papers of Wen-Chen Chang, Evan McClellan, Oleg Teryaev and JCP
(and Daniel Boer for one paper)

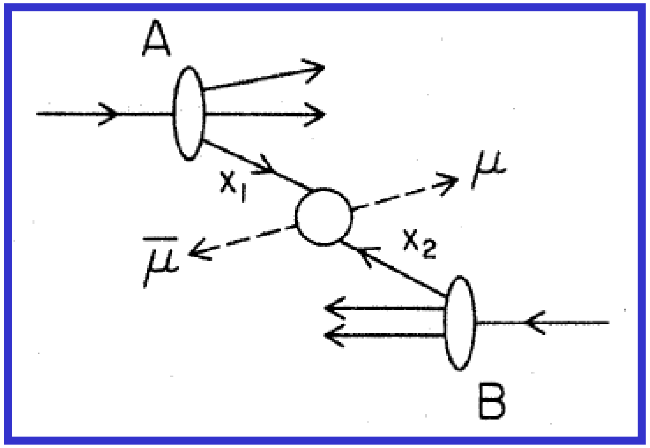
Phys. Lett. B758 (2016) 384;
Phys. Rev. D 96 (2017) 054020;
Phys. Lett. B789 (2019) 356;
Phys. Rev. D 99 (2019) 014032

The Drell-Yan Process

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

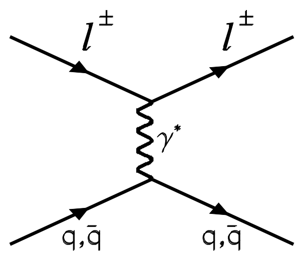
On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function νW_2 near threshold.

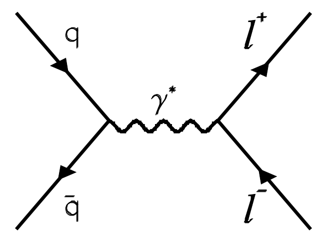


$$\left(\frac{d^2\sigma}{dx_1 dx_2}\right)_{D.Y.} = \frac{4\pi\alpha^2}{9 s x_1 x_2} \sum_a e_a^2 \left[q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1) q_a(x_2) \right]$$

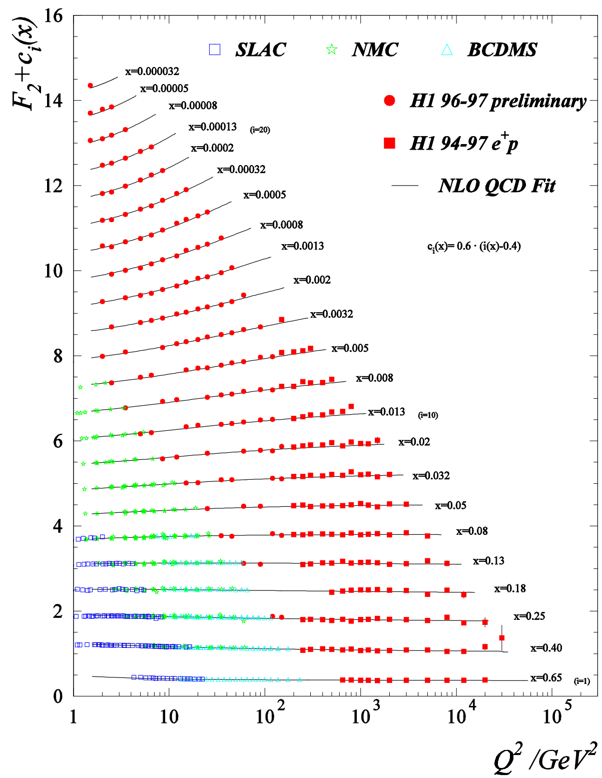
Complementarity between DIS and Drell-Yan

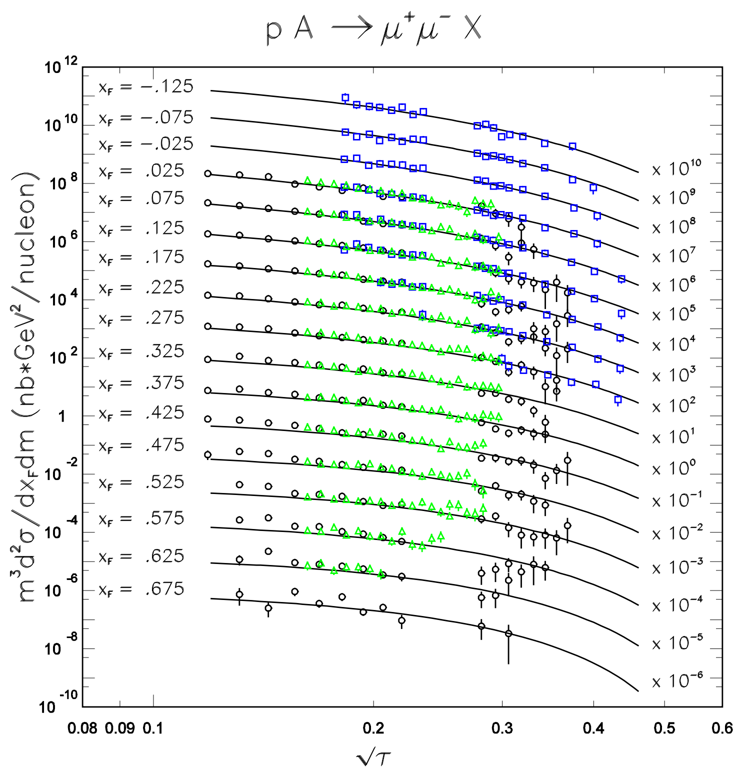
DIS





Drell-Yan





Ann.Rev.Nucl. Part. Sci. 49 (1999) 217;

Peng and Qiu, Prog. Part. Nucl. Phys. 76 (2014)43

Both DIS and Drell-Yan process are tools to probe the quark and antiquark structure in hadrons (factorization, universality)

Angular Distribution in the "Naïve" Drell-Yan

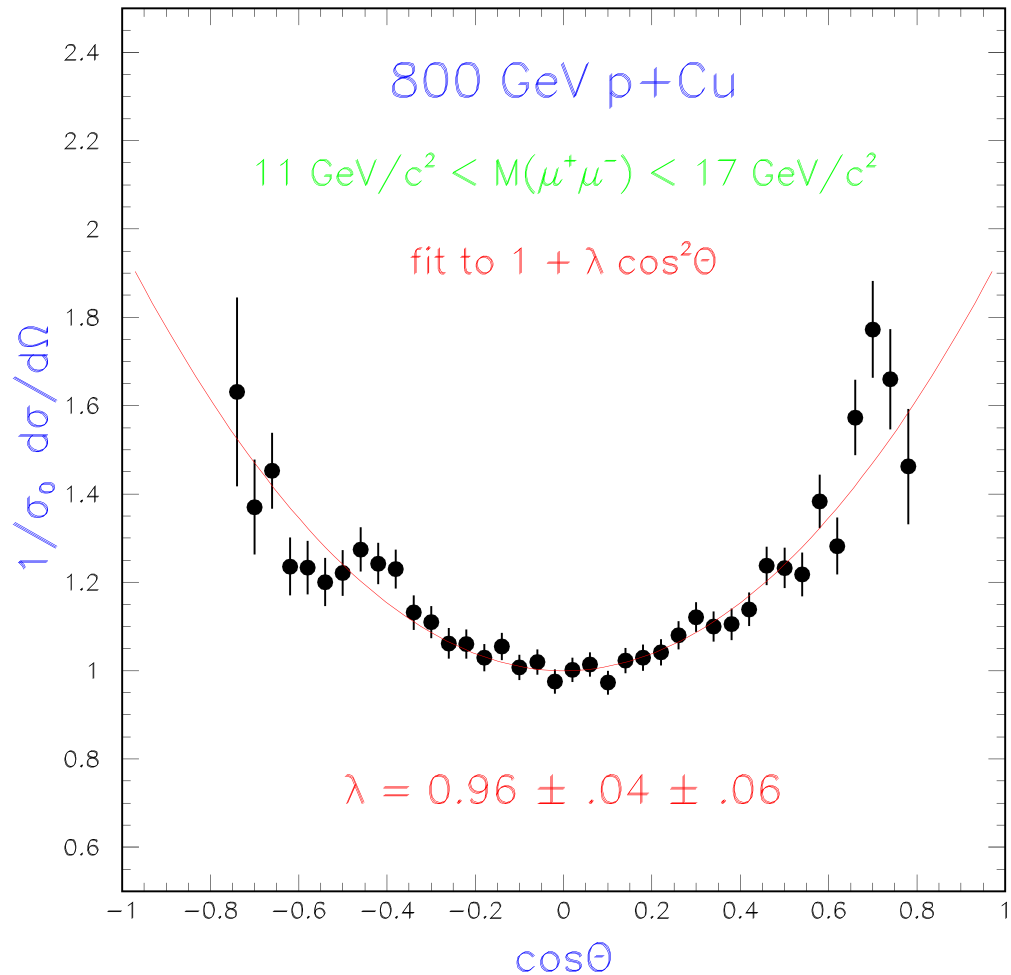
VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1+\cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's[10] vector-dominance model, where θ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

Drell-Yan angular distribution

Lepton Angular Distribution of "naïve" Drell-Yan:

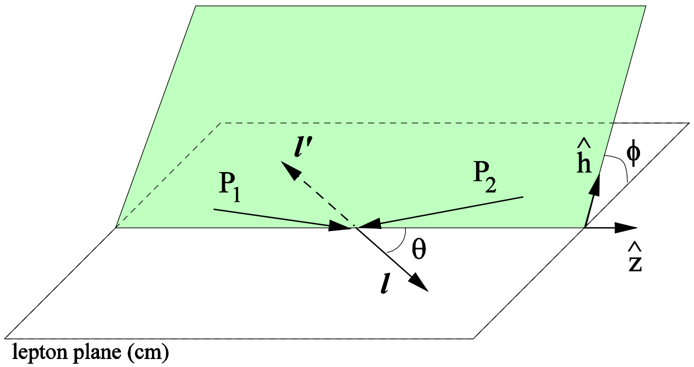
$$\frac{d\sigma}{d\Omega} = \sigma_0 (1 + \lambda \cos^2 \theta); \quad \lambda = 1$$



Data from Fermilab E772

(Ann. Rev. Nucl. Part. Sci. 49 (1999) 217-253)

Drell-Yan lepton angular distributions



Θ and Φ are the decay polar and azimuthal angles of the μ^- in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

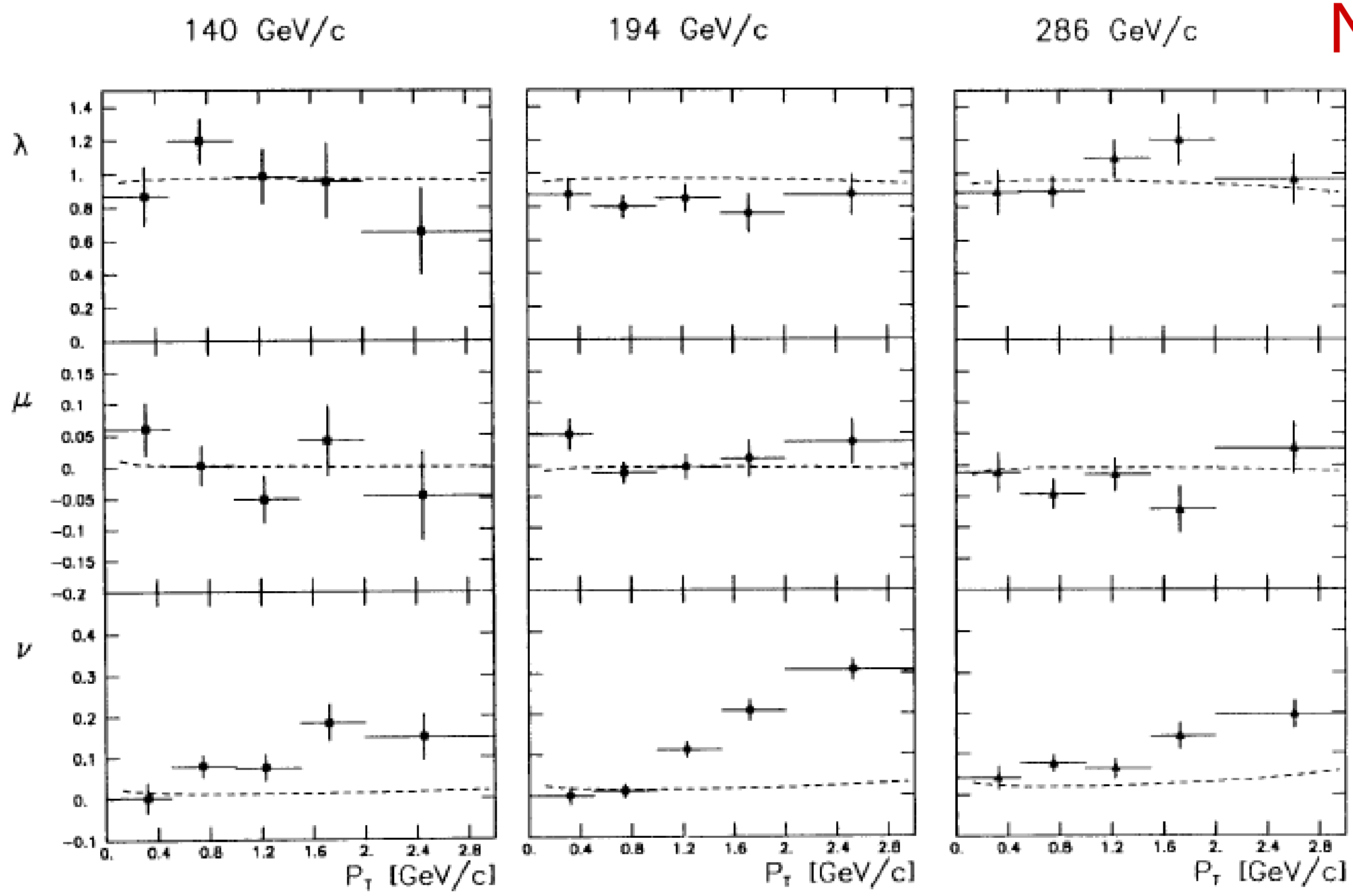
Lam-Tung relation: $1-\lambda=2\nu$

- Reflect the spin-1/2 nature of quarks (analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

Decay angular distributions in pion-induced Drell-Yan

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

NA10 $\pi^- + W$



Z. Phys.

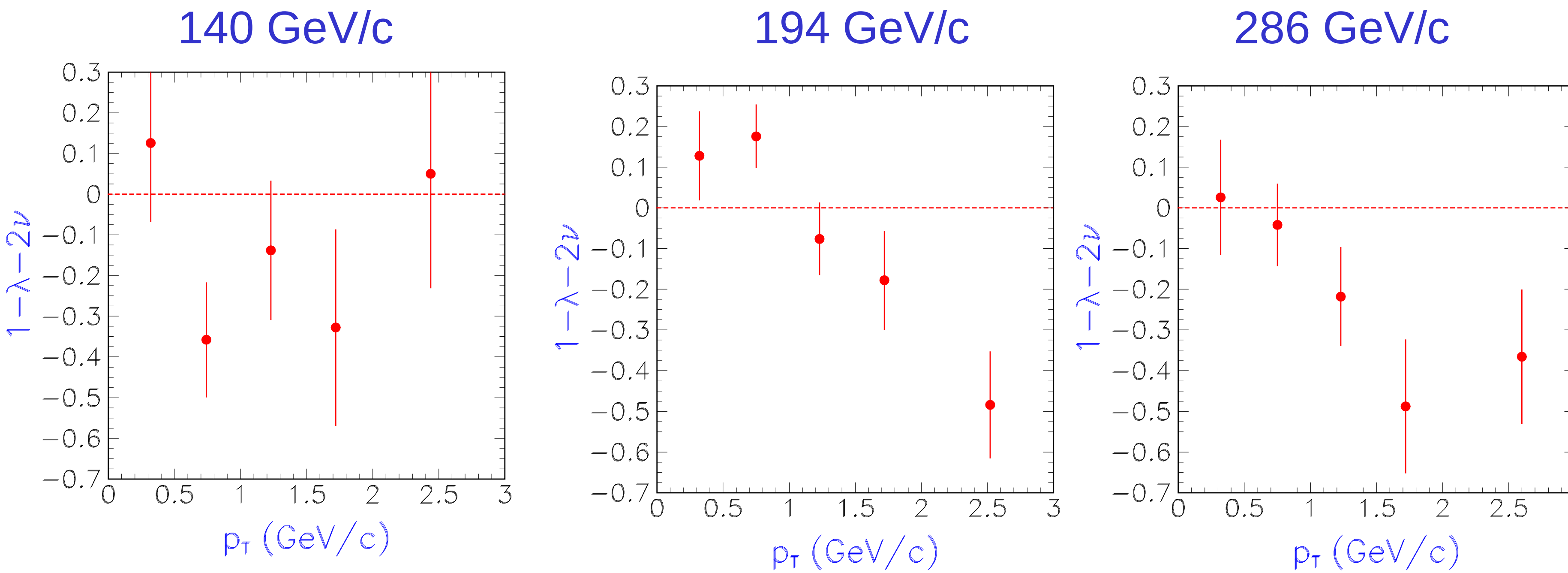
37 (1988) 545

Dashed curves are from pQCD calculations

$\nu \neq 0$ and ν increases with p_T

Decay angular distributions in pion-induced Drell-Yan

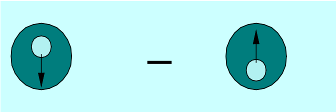
Is the Lam-Tung relation (1-λ-2ν=0) violated?



Data from NA10 (Z. Phys. 37 (1988) 545)

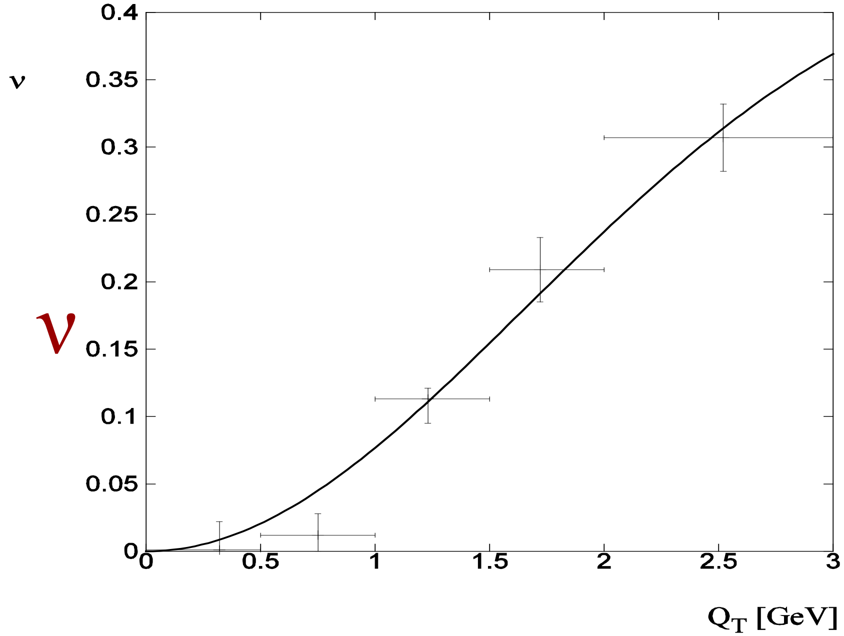
Violation of the Lam-Tung relation suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer,Väntinnen, Vogt, etc.)

Boer-Mulders function $h_1^{\perp}$



- Boer pointed out that the cos2ϕ dependence can be caused by the presence of the Boer-Mulders function.

- $h_1^{\perp}$ can lead to an azimuthal dependence with $\nu \propto \left(\frac{h_1^{\perp}}{f_1}\right)\left(\frac{\bar{h}_1^{\perp}}{\bar{f}_1}\right)$



Boer, PRD 60 (1999) 014012

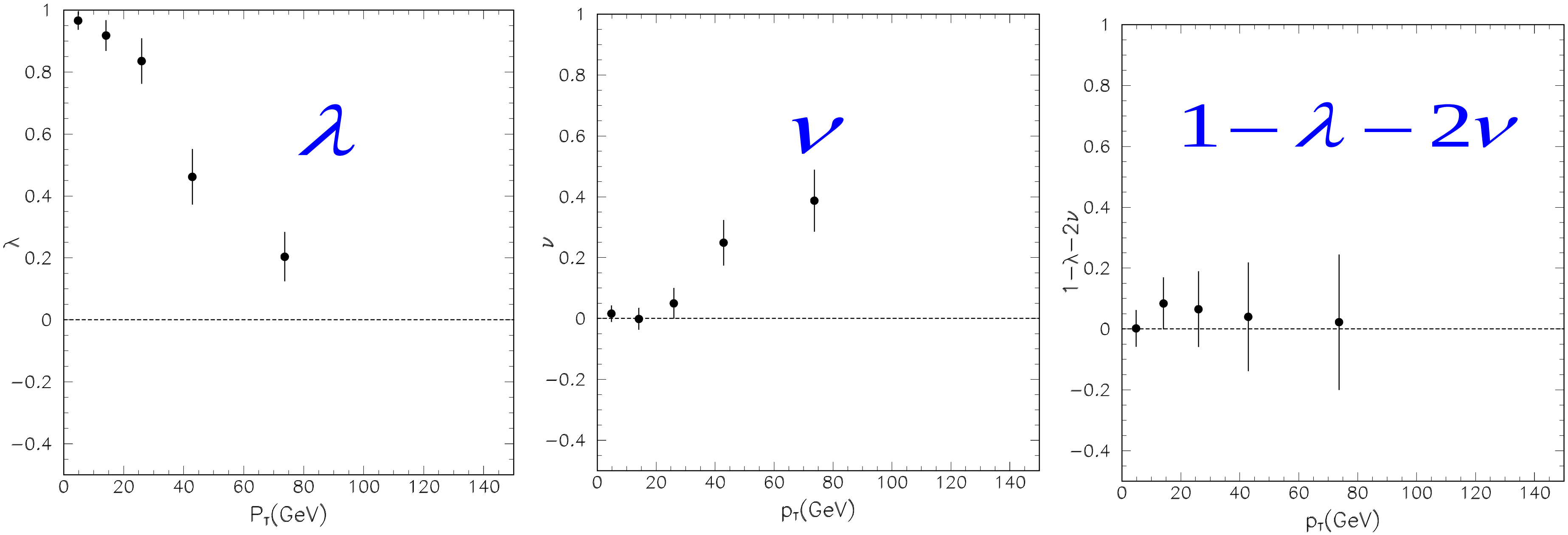
The violation of the Lam-Tung relation is due to the presence of the Boer-Mulders TMD function

The puzzle is resolved. It also leads to the first extraction of the Boer-Mulders function

Angular distribution data from CDF Z-production

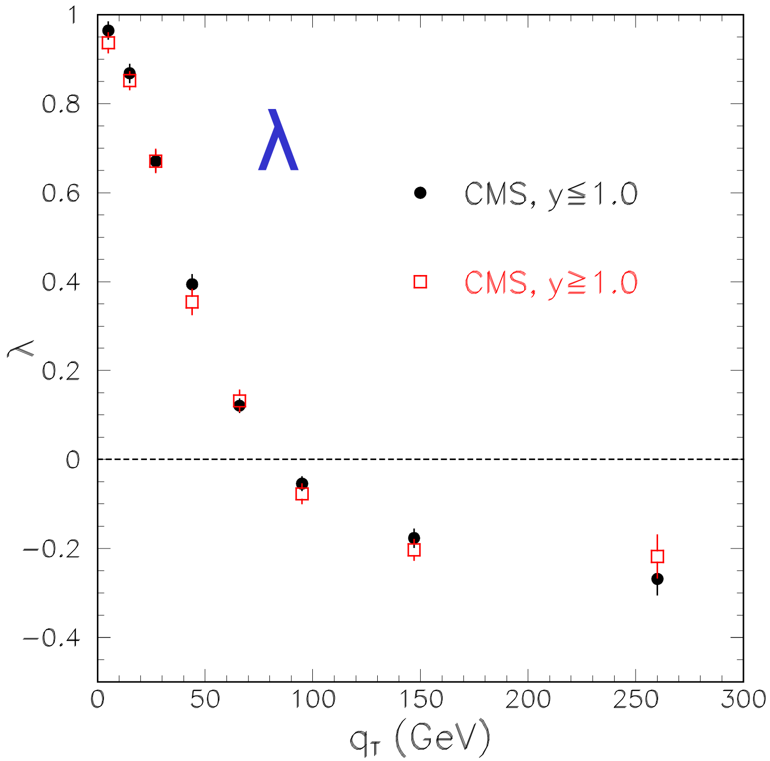
$$p + \overline{p} \rightarrow e^{+} + e^{-} + X \quad \text{at} \sqrt{s} = 1.96\,\text{TeV}$$

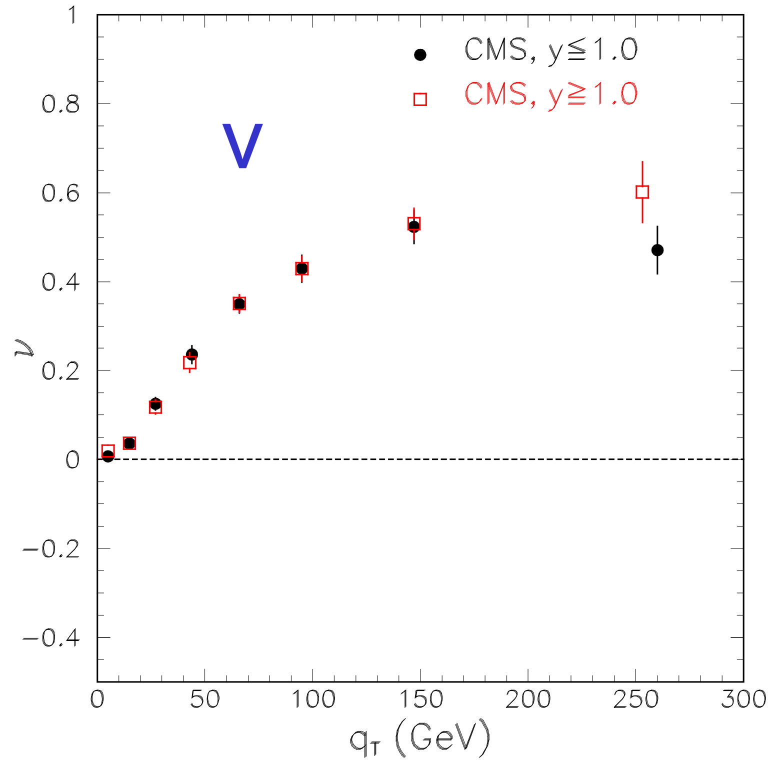
arXiv:1103.5699 (PRL 106 (2011) 241801)



- Strong p_T (q_T) dependence of λ and ν
- Lam-Tung relation (1-λ = 2ν) is satisfied within experimental uncertainties (TMD is not expected to be important at large p_T)

Recent CMS (ATLAS) data for *Z*-boson production in *p*+*p* collision at 8 TeV

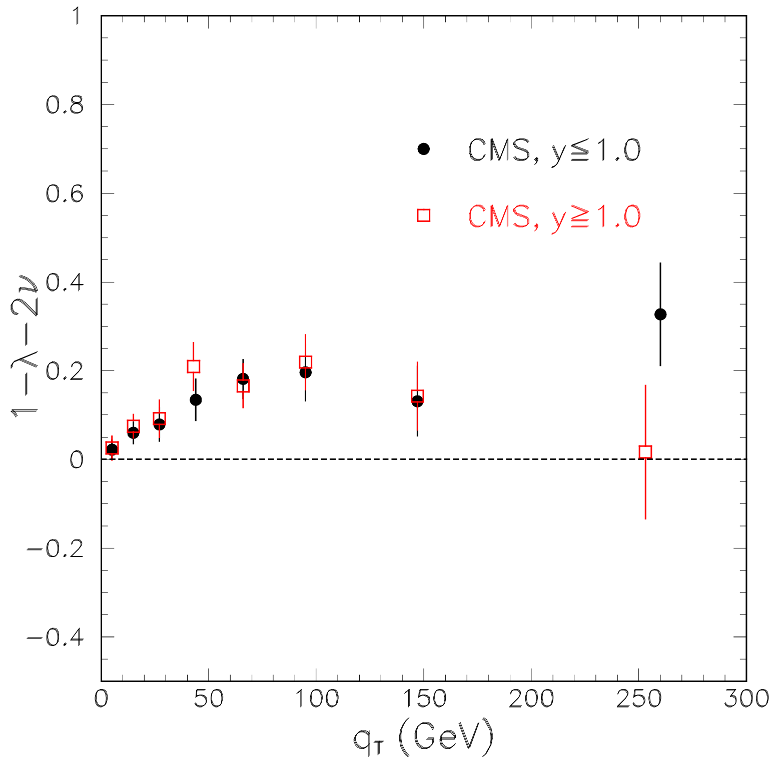




(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking q_T (p_T) dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

Recent data from CMS for *Z*-boson production in *p*+*p* collision at 8 TeV



- Yes, the Lam-Tung relation is violated (1-λ > 2ν)!
- Can one understand the origin of the violation of the Lam-Tung relation (It cannot be due to the Boer-Mulders function)?

Interpretation of the CMS Z-production results

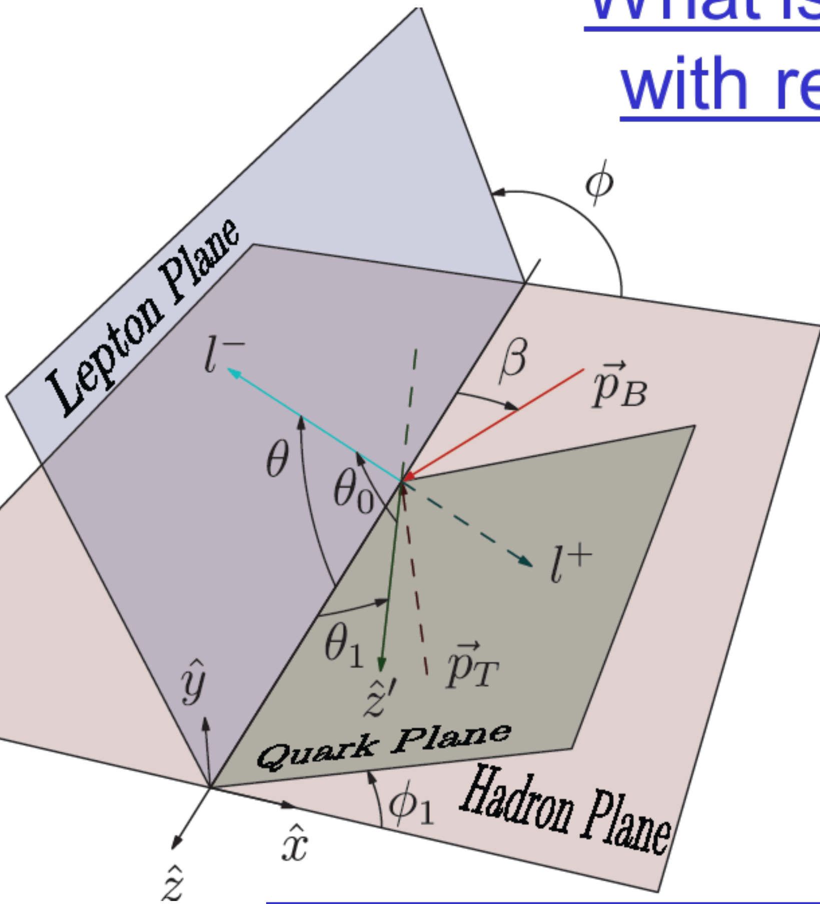
$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta \cos 2\phi + A_3 \sin\theta \cos\phi + A_4 \cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi$$

Questions:

- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the q_T dependence of $A_0, A_1, A_2,$ etc?
- Can one understand the origin of the violation of Lam-Tung relation?

$\lambda = \frac{2-3A_0}{2+A_0}$; $\nu = \frac{2A_2}{2+A_0}$; L-T relation, $1-\lambda = 2\nu$, becomes $A_0 = A_2$

How is the angular distribution expression derived?



What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

Azimuthally symmetric !

How to express the angular distribution in terms of θ and φ?

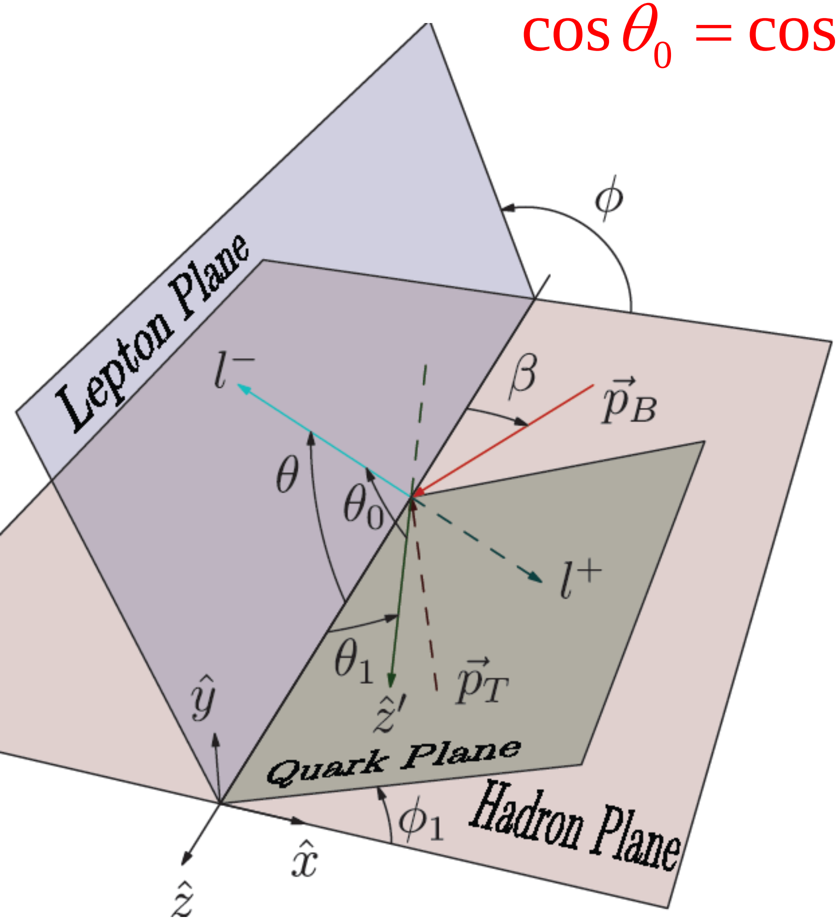
Use the following relation:

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

How is the angular distribution expression derived?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$



$$\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta) \\
& + (\frac{1}{2}\sin 2\theta_1\cos\phi_1)\sin 2\theta\cos\phi \\
& + (\frac{1}{2}\sin^2\theta_1\cos 2\phi_1)\sin^2\theta\cos 2\phi \\
& + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\
& + (\frac{1}{2}\sin^2\theta_1\sin 2\phi_1)\sin^2\theta\sin 2\phi \\
& + (\frac{1}{2}\sin 2\theta_1\sin\phi_1)\sin 2\theta\sin\phi \\
& + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi .
\end{aligned}$$

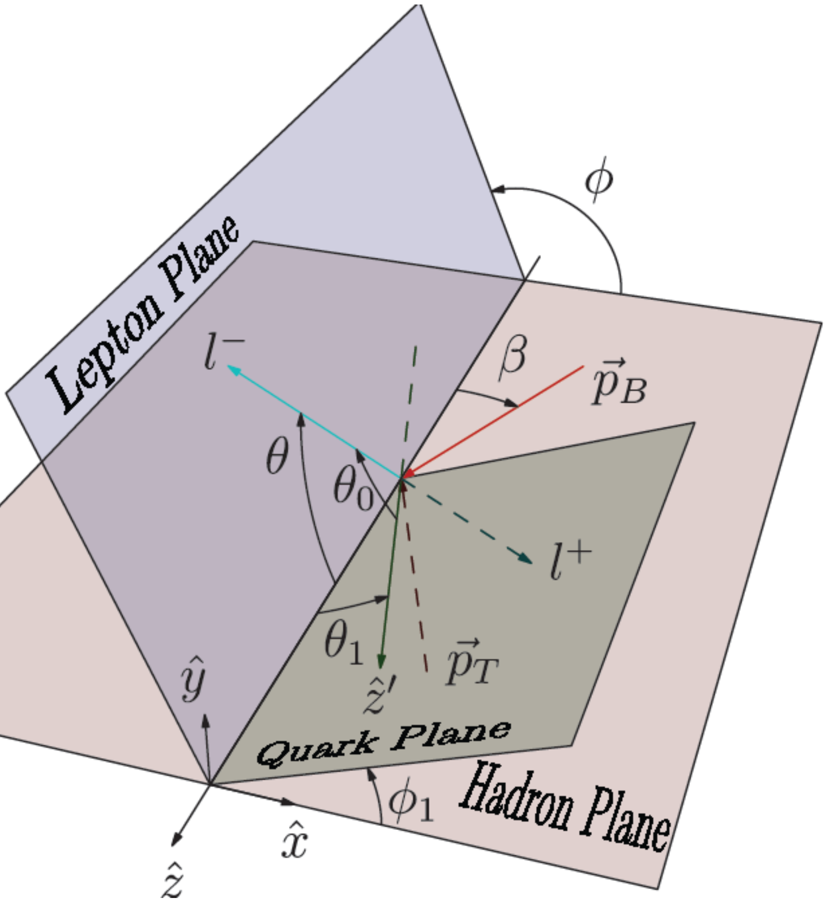
All eight angular distribution terms are obtained!

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta)$$
$$+ (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta \cos\phi$$
$$+ (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta \cos 2\phi$$
$$+ (a\sin\theta_1 \cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta$$
$$+ (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta \sin 2\phi$$
$$+ (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta \sin\phi$$
$$+ (a\sin\theta_1 \sin\phi_1)\sin\theta\sin\phi.$$

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta)$$
$$+ A_1 \sin 2\theta \cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta \cos 2\phi$$
$$+ A_3 \sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi$$
$$+ A_6 \sin 2\theta \sin\phi$$
$$+ A_7 \sin\theta \sin\phi$$

$A_0 - A_7$ are entirely described by θ_1, ϕ_1 and a

Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

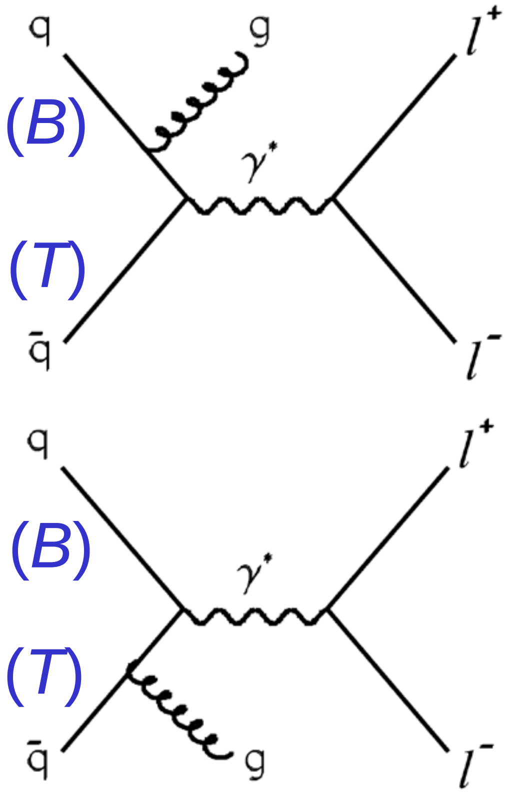
$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

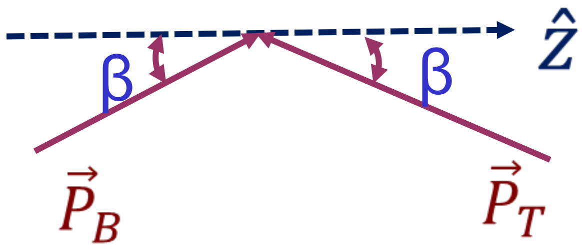
$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

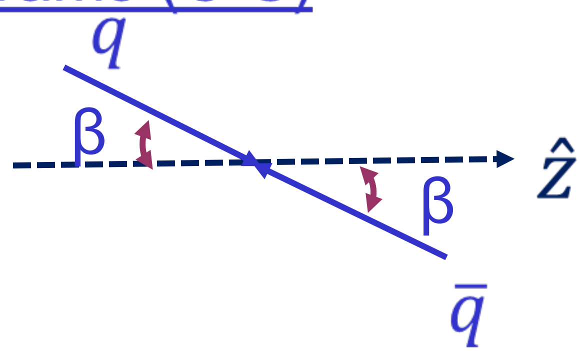
What are the values of θ_1 and ϕ_1 at order α_s?

1) $q\bar{q} \to \gamma^*(Z^0)g$

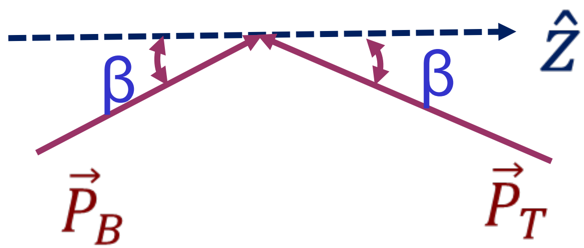
In γ^* rest frame (C-S)

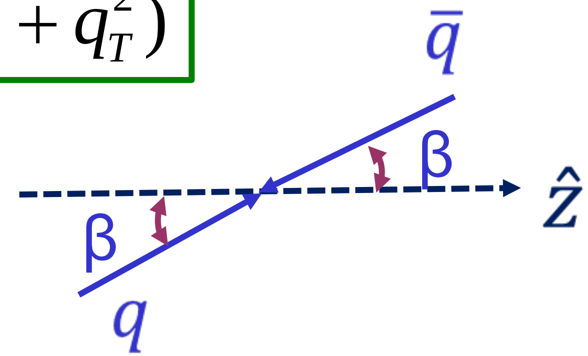






$$\sin^2\beta = q_T^2/(Q^2 + q_T^2)$$





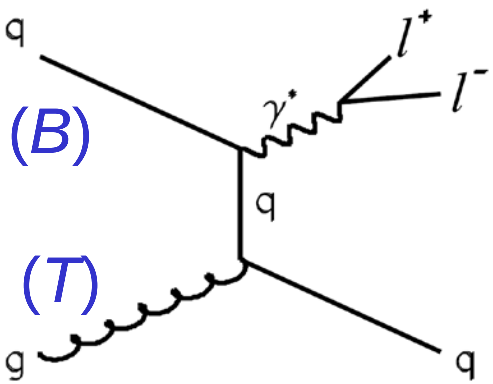
$\theta_1 = \beta$ and $\phi_1 = 0$; $A_0 = A_2 = \sin^2\beta$

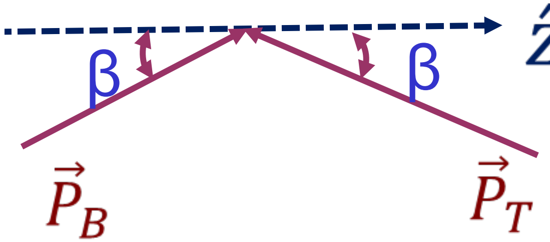
$$\lambda = \frac{2-3A_0}{2+A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \qquad \nu = \frac{2A_2}{2+A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

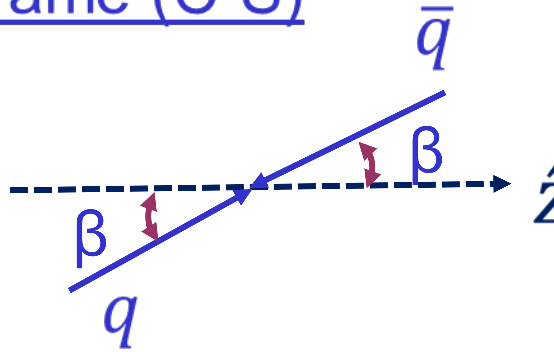
What are the values of θ_1 and ϕ_1 at order α_s?

2) $qg \rightarrow \gamma^*(Z^0)q$

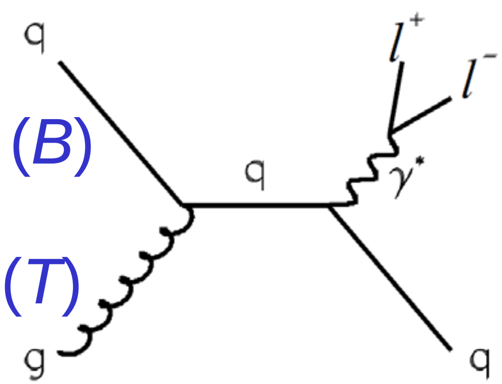
In γ^* rest frame (C-S)

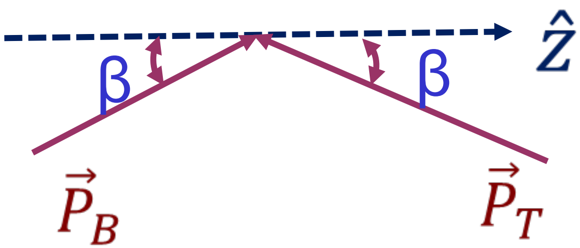


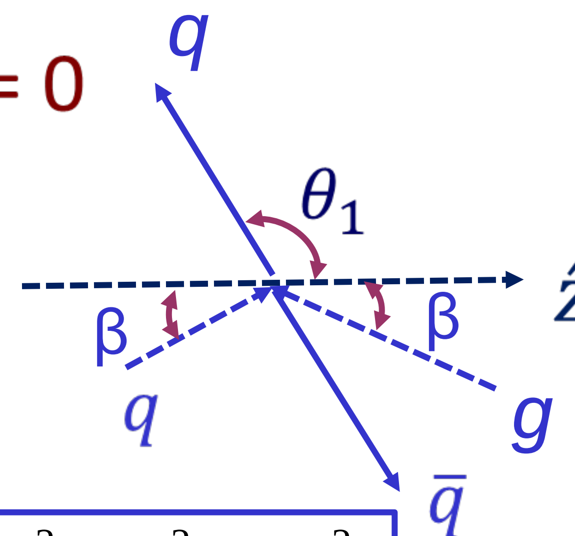




$\theta_1 = \beta$ and $\phi_1 = 0$





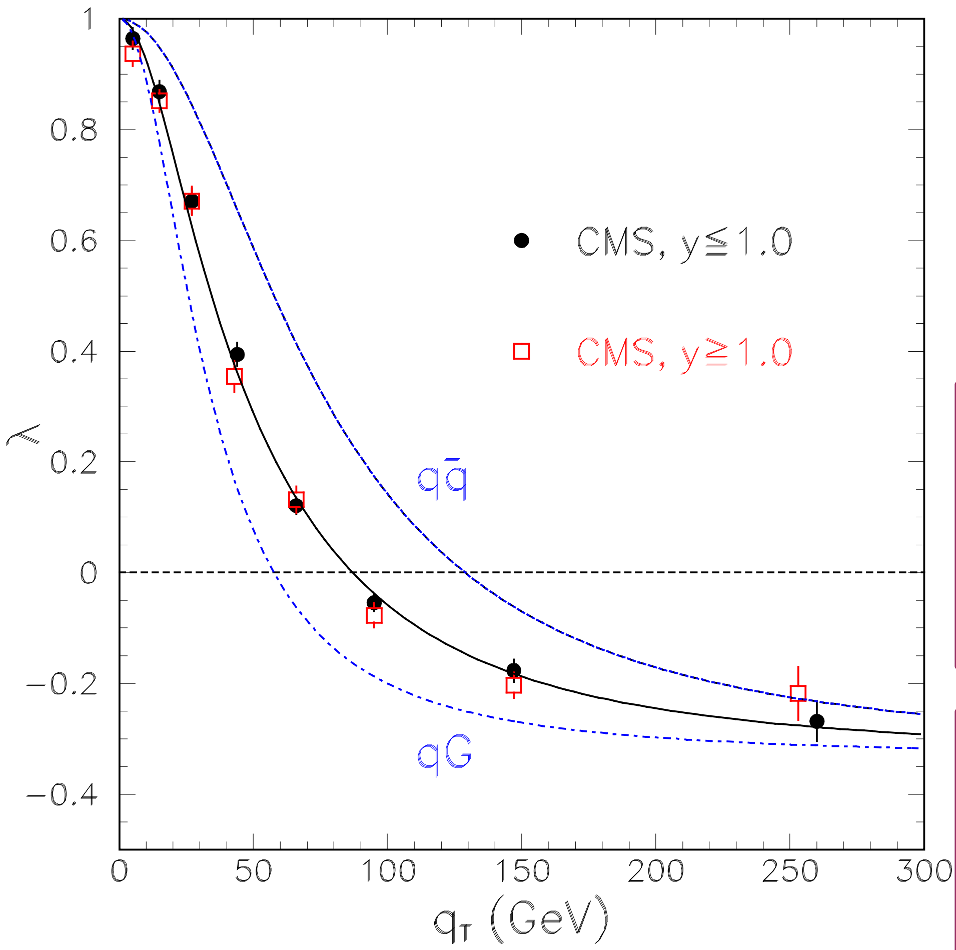


$\theta_1 > \beta$ and $\phi_1 = 0$; $A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

Compare with CMS data on λ

(*Z* production in *p*+*p* collision at 8 TeV)



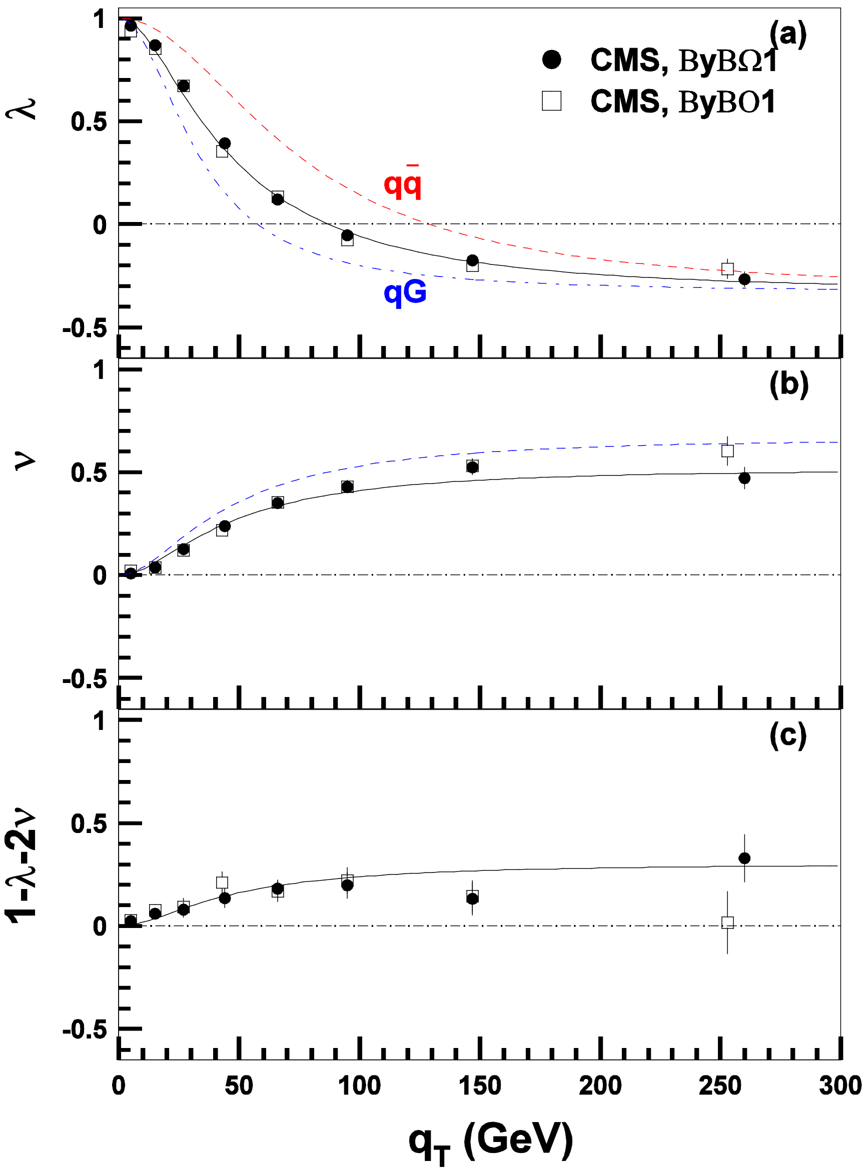
$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \rightarrow Zg$$

$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \rightarrow Zq$$

For both processes
$\lambda = 1$ at $q_T = 0$ ($\theta_1 = 0^o$)
$\lambda = -1/3$ at $q_T = \infty$ ($\theta_1 = 90^o$)

Data can be well described with a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

Compare with CMS data on Lam-Tung relation

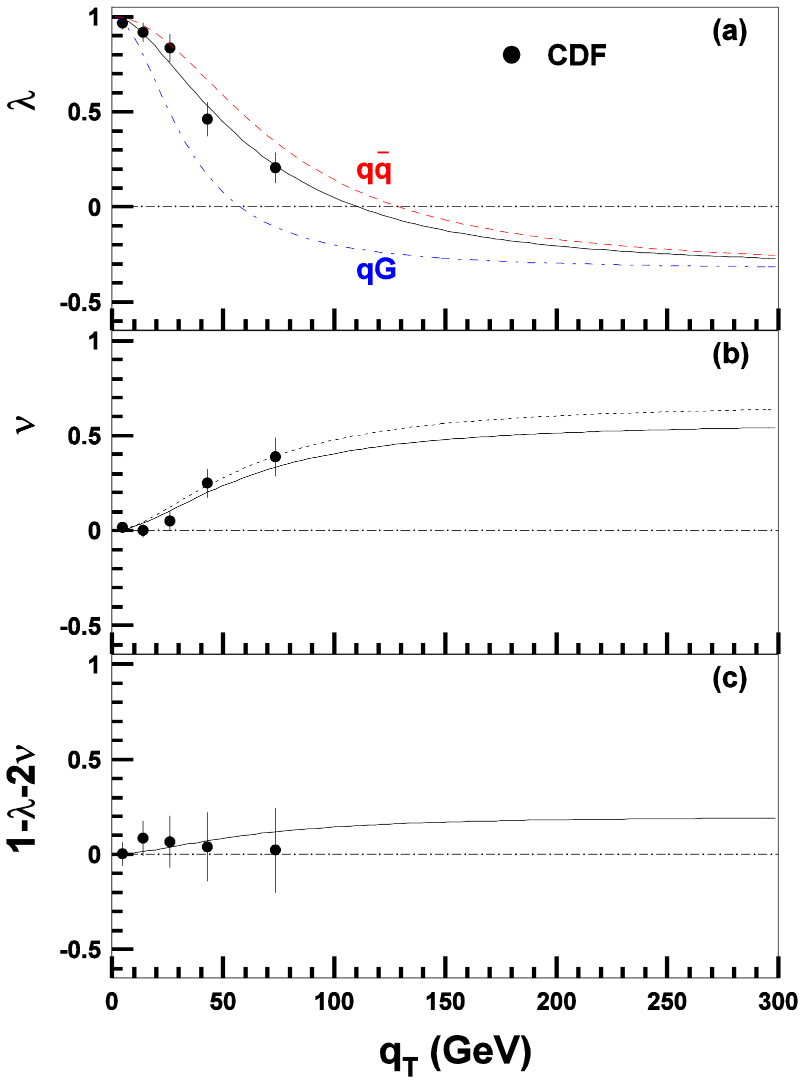


Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes, and $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$

Violation of Lam-Tung relation is well described

Compare with CDF data

(Z production in $p + \bar{p}$ collision at 1.96 TeV)

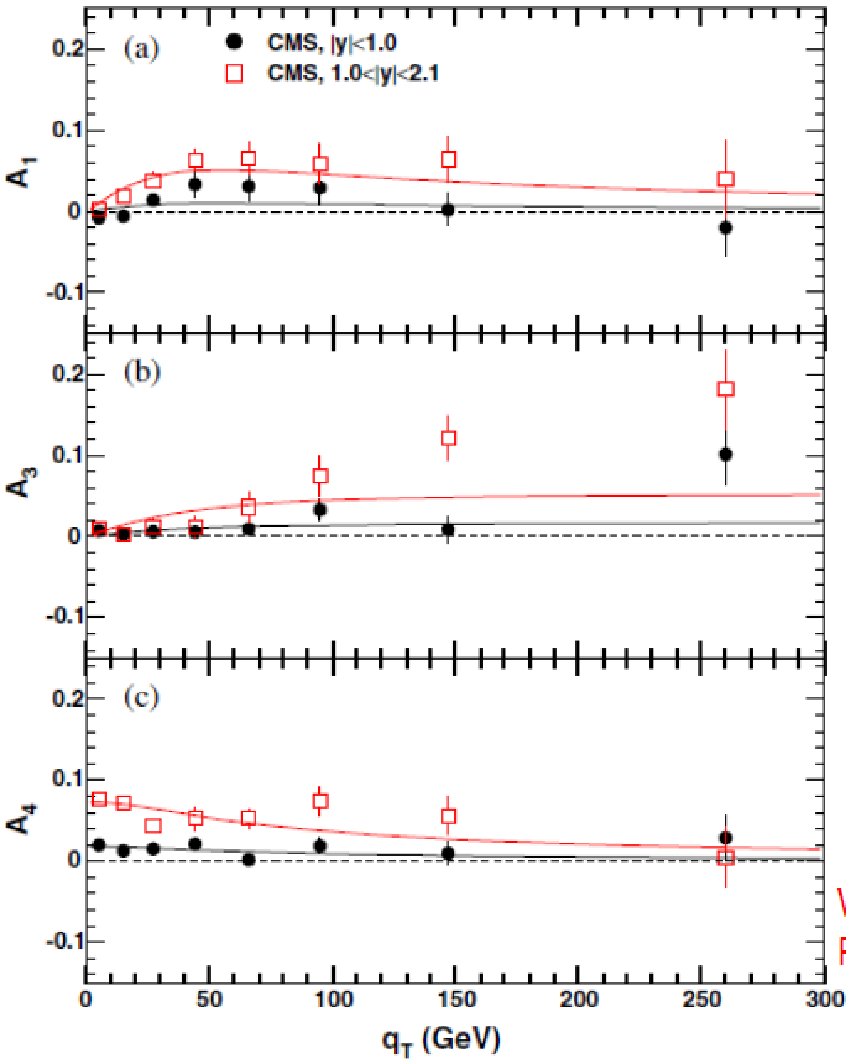


Solid curves correspond to a mixture of 27.5% qG and 72.5% $q\bar{q}$ processes, and

$$\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.85$$

Violation of Lam-Tung relation is not ruled out

Compare CMS data on A_1, A_3 and A_4 with calculations



$$A_1 = r_1 \left[f \frac{q_T Q}{Q^2 + q_T^2} + (1 - f) \frac{\sqrt{5} q_T Q}{Q^2 + 5 q_T^2} \right]$$

$$A_3 = r_3 \left[f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

$$A_4 = r_4 \left[f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{Q}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

Rapidity of A_1, A_3 and A_4 are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Phys. Rev. D 96, 054020 (2017)

Future prospects

- Extend this study to semi-inclusive DIS at high p_T (involving two hadrons and two leptons)
  - Relevant for EIC measurements
- Rotational invariance, equality, and inequality relations formed by various angular distribution coefficients
  - See preprint arXiv: 1808.04398 (Phys Lett B789 (2019) 352)
- Comparison with pQCD calculations
  - See preprint arXiv: 1811.03256 (PRD 99 (2019) 014032)
  - Lambertson and Vogelsang, PRD 93 (2016) 114013

Future prospects

On the Rotational Invariance and Non-Invariance of Lepton Angular Distributions in Drell-Yan and Quarkonium Production

Jen-Chieh Peng[a], Daniël Boer[b], Wen-Chen Chang[c], Randall Evan McClellan[a,d], Oleg Teryaev[e]

arXiv:1808.04398 (Phys Lett B789 (2019) 352)

Quantities invariant under rotations along the y-axis (Faccioli et al.)

$$\mathcal{F} = \frac{1+\lambda+\nu}{3+\lambda}$$

$$\mathcal{F} = \frac{1+\lambda_0 - 2\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{3+\lambda_0} = \frac{1+\lambda_0 - 2\lambda_0 y_1^2}{3+\lambda_0}$$

$$\tilde{\lambda} = \frac{2\lambda+3\nu}{2-\nu}$$

$$\tilde{\lambda} = \frac{\lambda_0 - 3\lambda_0 \sin^2\theta_1 \sin^2\phi_1}{1+\lambda_0 \sin^2\theta_1 \sin^2\phi_1} = \frac{\lambda_0 - 3\lambda_0 y_1^2}{1+\lambda_0 y_1^2}$$

$$\tilde{\lambda}' = \frac{(\lambda-\nu/2)^2+4\mu^2}{(3+\lambda)^2}$$

$$\tilde{\lambda}' = \frac{\lambda_0^2(z_1^2+x_1^2)^2}{(3+\lambda_0)^2} = \frac{\lambda_0^2(1-y_1^2)^2}{(3+\lambda_0)^2}$$

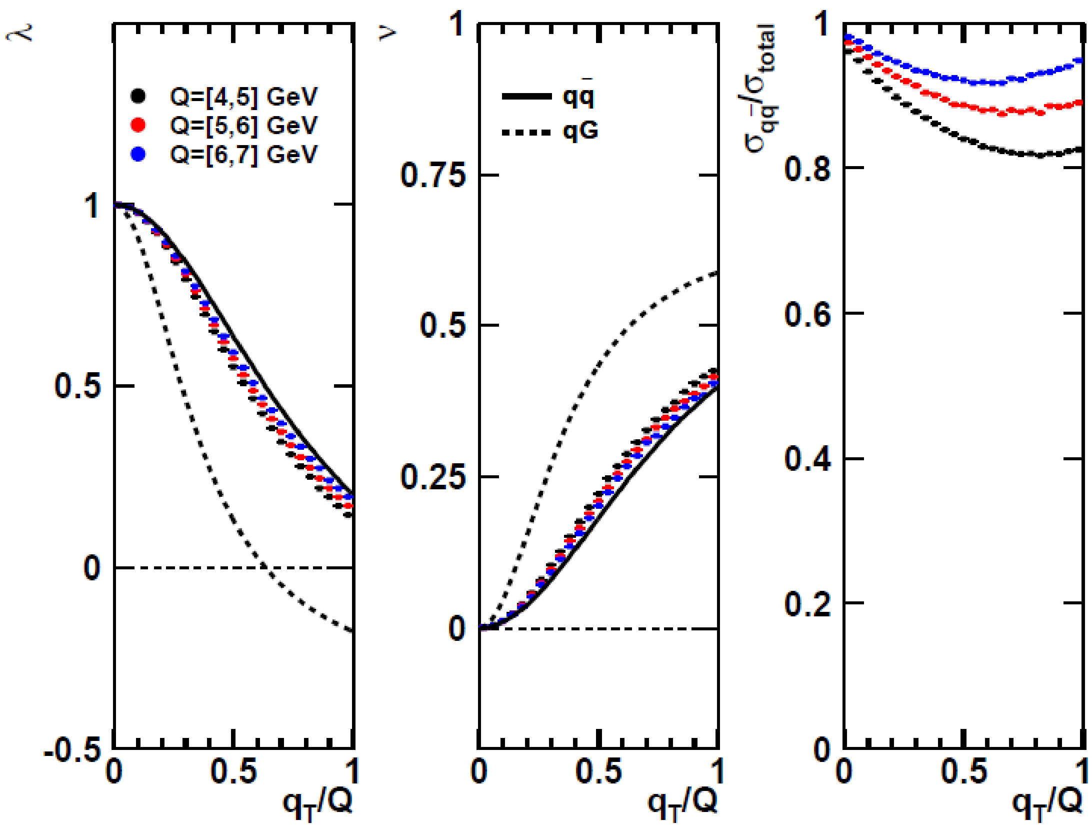
Future prospects

- Extend this study to fixed-target Drell-Yan data

Lepton Angular Distributions of Fixed-target Drell-Yan Experiments in Perturbative QCD and a Geometric Approach

Wen-Chen Chang,[1] Randall Evan McClellan,[2,3] Jen-Chieh Peng,[3] and Oleg Teryaev[4]

COMPASS π^{-}+W at 190 GeV



arXiv:1811.03256v1

PRD 99 (2019) 014032

Summary

- The lepton angular distribution coefficients $A_0 - A_7$ can be described in terms of the polar and azimuthal angles of the $q - \bar{q}$ axis
- Violation of the Lam-Tung relation is due to the acoplanarity of the $q - \bar{q}$ axis and the hadron plane. This can come from order α_s^2 or higher processes or from intrinsic k_T
- This approach can be extended to fixed-target Drell-Yan and many other hard-processes
- Extraction of the Boer-Mulders function in the Drell-Yan process must take into account of the pQCD effects