Thermodynamic properties along the QCD crossover from Lattice

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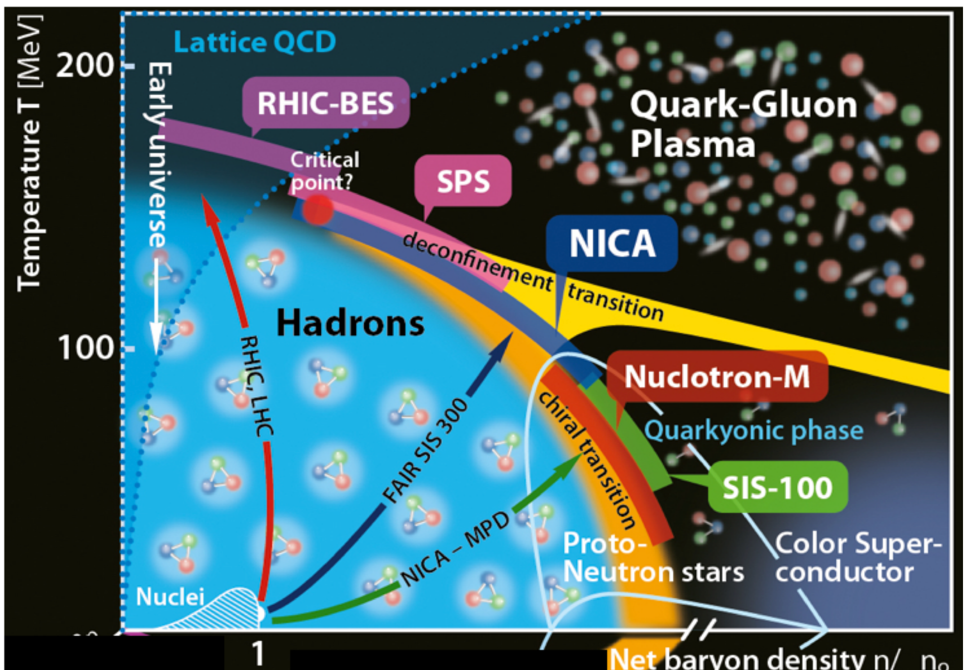
searches for criticality in QCD phase-diagram and thermodynamics along the phase boundary:



phase boundary @ $\mu_B > 0$ & critical point
RHIC, NA61, FAIR, NICA, J-PARC

Remnants of criticality @ $\mu_B = 0$
RHIC, LHC

Thermodynamis along the QCD crossover

GHP19, 8th Workshop of the APS Topical Group on Hadronic Physics, Denver, April 10-12, 2019

Large scale numerical calculations by HotQCD Collaboration

Summit (TOP500: 1)

Titan (TOP500: 9)

Cori (TOP500:12)

Piz Daint (TOP500: 5)

NSC^3 (CCNU, Wuhan, China)

O(N) scaling and the chiral transition temperature

$SU(2)_V \otimes SU(2)_A \sim O(4)$ governed by universal *O(4)* scaling

For sufficiently small m_l and in the vicinity of the transition temperature:

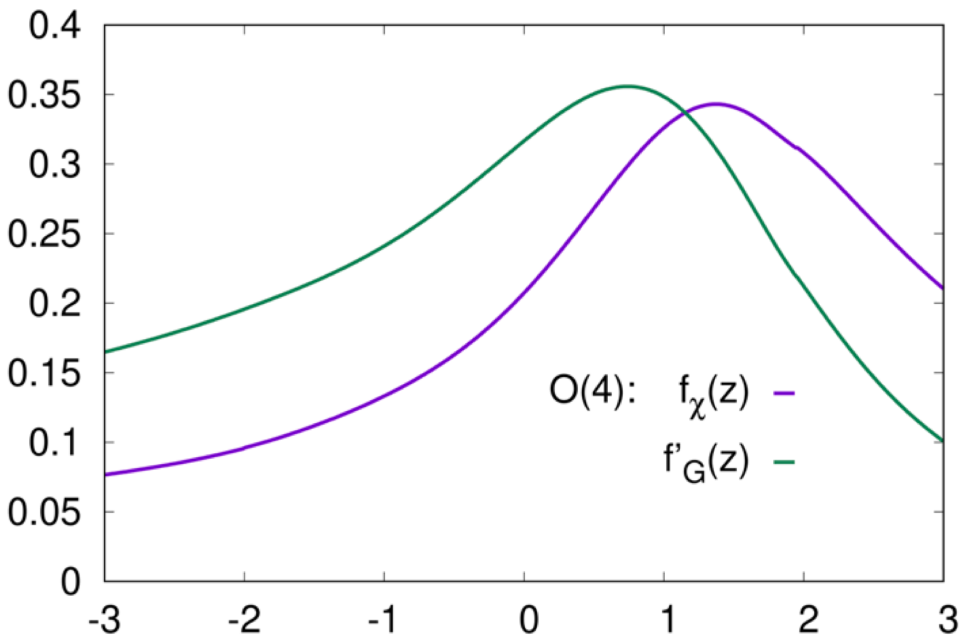
$$f(T,m_l) = -\frac{T}{V}\ln Z = f_{reg}(T,m_l) + f_s(t,h),\ t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0} + \kappa\frac{\mu_q^2}{T^2}\right),\ H = \frac{m_l}{m_s}, h = \frac{H}{h_0}$$

$$\langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f \qquad M = -\frac{\partial f_s(t,h)}{\partial H} = h^{1/\delta} f_G(z),\ z = t/h^{1/\beta\delta}$$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the susceptibilities

$$\chi_{m,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} \sim m_l^{1/\delta - 1} \longrightarrow T_{m,l} \qquad \chi_{t,l} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t} \sim m_l^{\frac{\beta-1}{\beta\delta}} \longrightarrow T_{t,l}$$



$= T_c^0$ in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1} f_\chi(z) + reg.\right)$$

universal scaling function has a peak at $z=z_p$

$$T_c(H) = T_{m,l} = T_c^0 + T_c^0 \frac{z_p}{z_0} H^{1/(\beta\delta)} + \dots$$

The chiral cross-over temperature for physical masses

Chiral order parameter:

$$\Sigma = \frac{1}{f_K^4}\left[m_s\langle \bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle \bar{s}s\rangle\right] \qquad \langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f$$

and the corresponding susceptibilities:

$$\chi^\Sigma = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma \qquad \chi = \frac{m_s^2}{f_K^4}\left[\langle(\bar{u}u + \bar{d}d)^2\rangle - (\langle \bar{u}u\rangle + \langle \bar{d}d\rangle)^2\right]$$

For non-zero chemical potential we use Taylor expansion

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \quad \chi(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n}$$

$$C_0^{\Sigma} = \Sigma \qquad C_0^{\chi} = \chi$$

Derivatives in μ_X^2 are similar to derivatives in T $\qquad$ $e.g.\ \partial_T C_0^{\chi} \sim C_2^{\chi}$

$\Rightarrow$ the following quantities will peak at T_c

$\chi^\Sigma,\ C_0^{\chi}(T) \sim \chi_{l,m} \qquad \partial_T C_0^{\Sigma},\ C_2^{\Sigma}(T) \sim \chi_{t,m}$

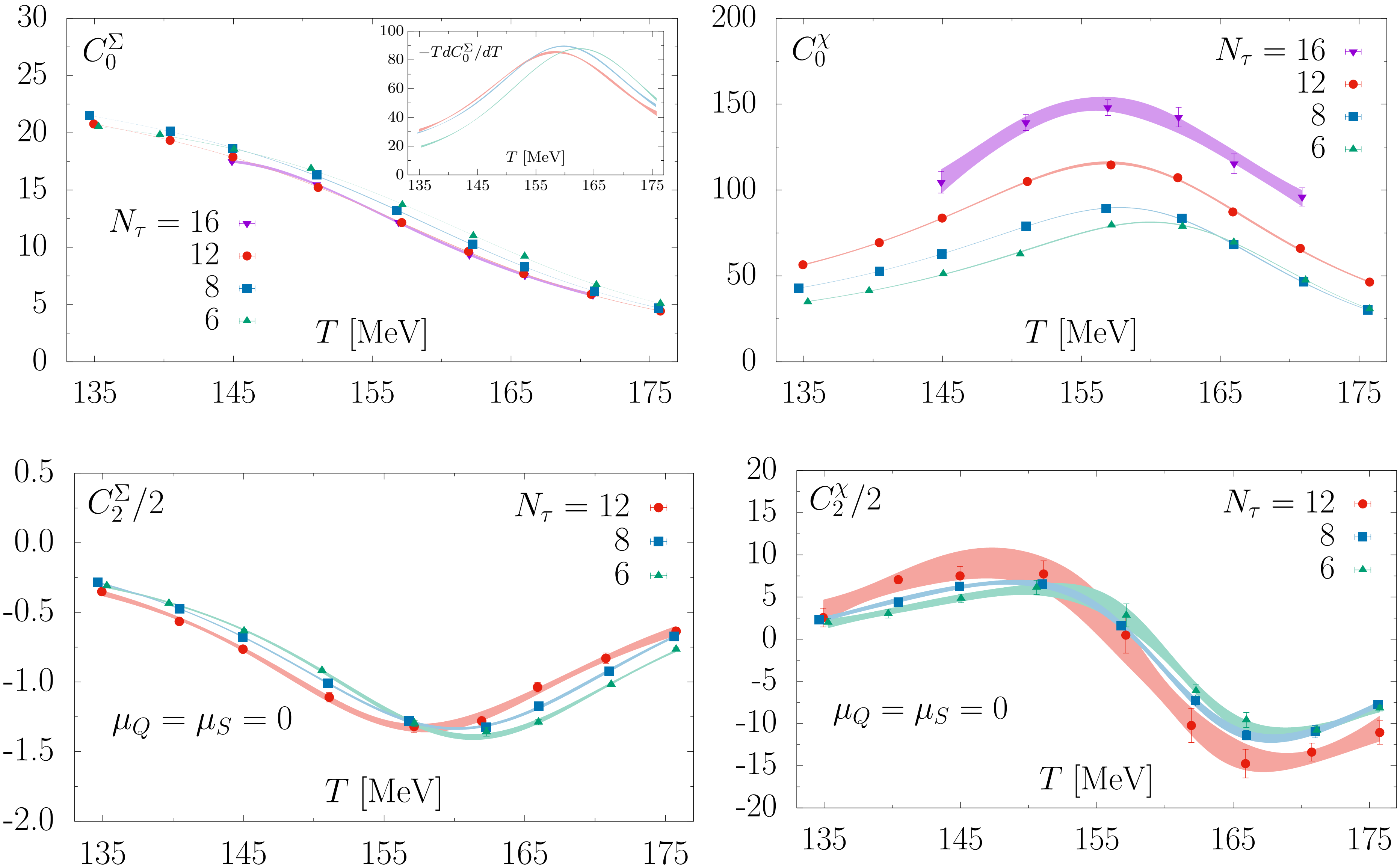
HotQCD, arXiv:1812.08235

5 different definitions of T^{pc}:

$$\partial_T C_0^{\chi} = 0,\ \partial_T C_0^{\Sigma} = 0\ ,C_2^{\chi} = 0 \qquad \partial_T^2 C_0^{\Sigma} = 0,\ \partial_T C_2^{\Sigma} = 0$$

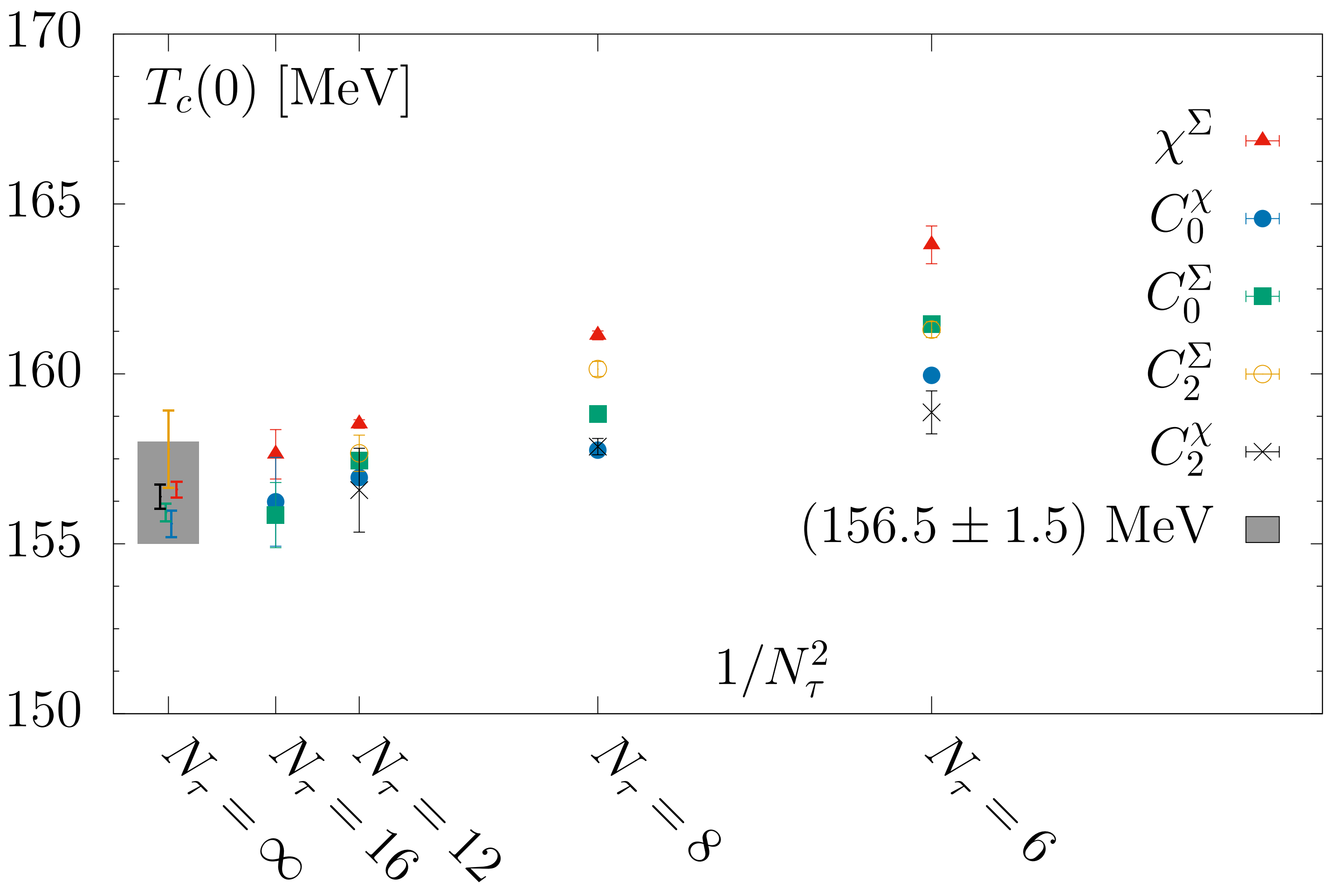
The 5 different T_c values reduce to $T_{l,m}$ and $T_{l,t}$ if regular part is zero

Lattice calculations based on 100K - 500 K configurations, $N_\tau = 6 - 12$, and 4K configurations for $N_\tau = 16$



HotQCD, arXiv:1812.08235

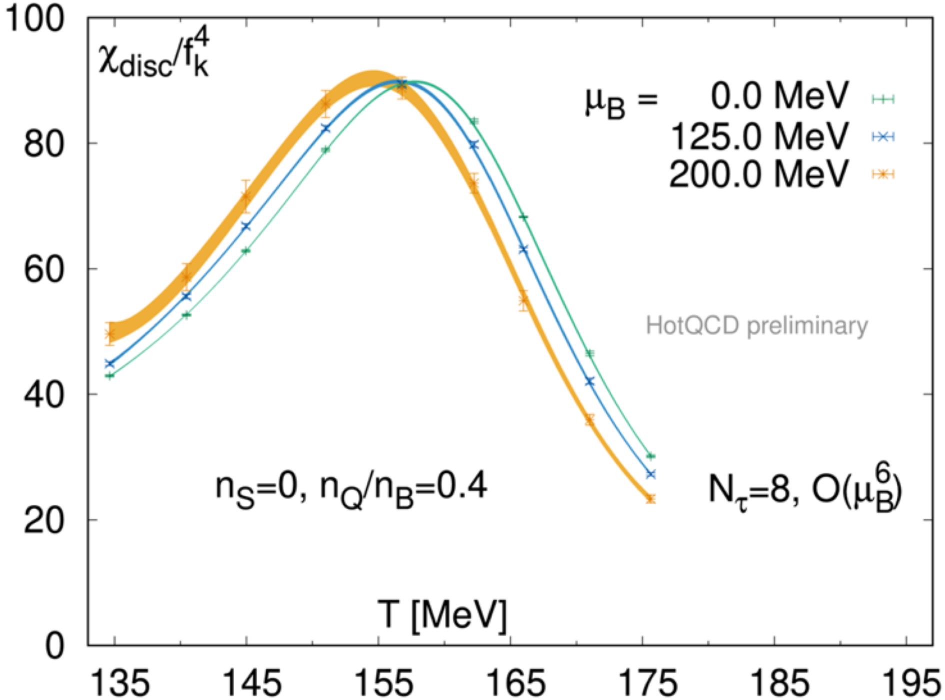
Different definitions of T_c surprisingly agree in the continuum limit and we for zero chemical potential we get $T_c = 156 \pm 1.5$ MeV

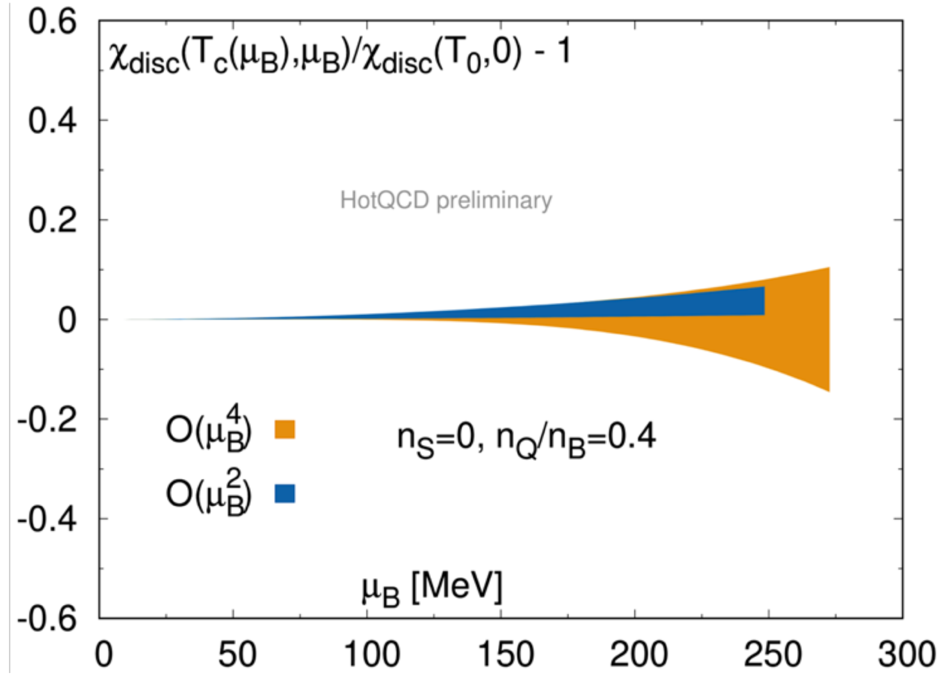


HotQCD, arXiv:1812.08235

The chiral susceptibility at baryon density non-zero density

Conditions in heavy ion collisions: $n_B > 0$, $n_S = 0$, $n_Q = 0.4 n_B$ (for Au, Pb)

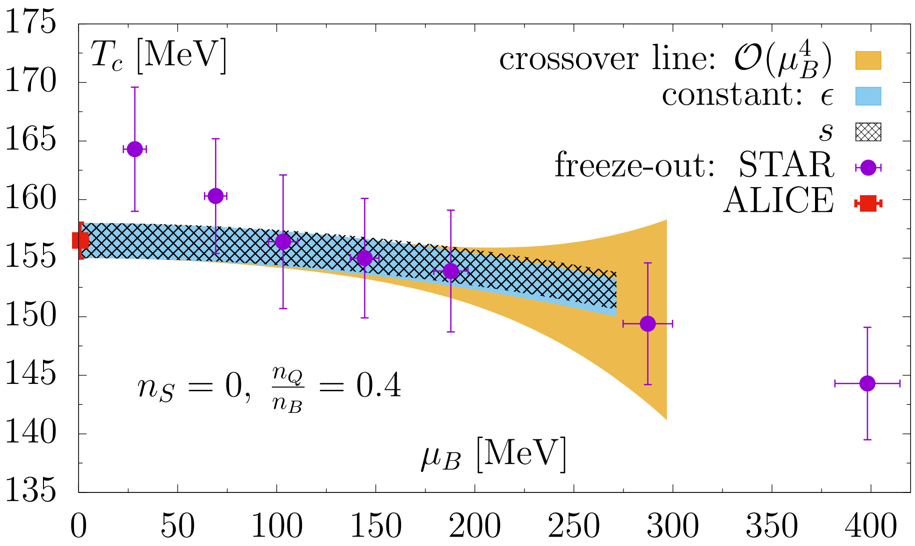




little change in peak-height & width with increasing baryon chemical potential: no indication of a stronger transition becoming stronger

The chiral cross-over temperature at non-zero density

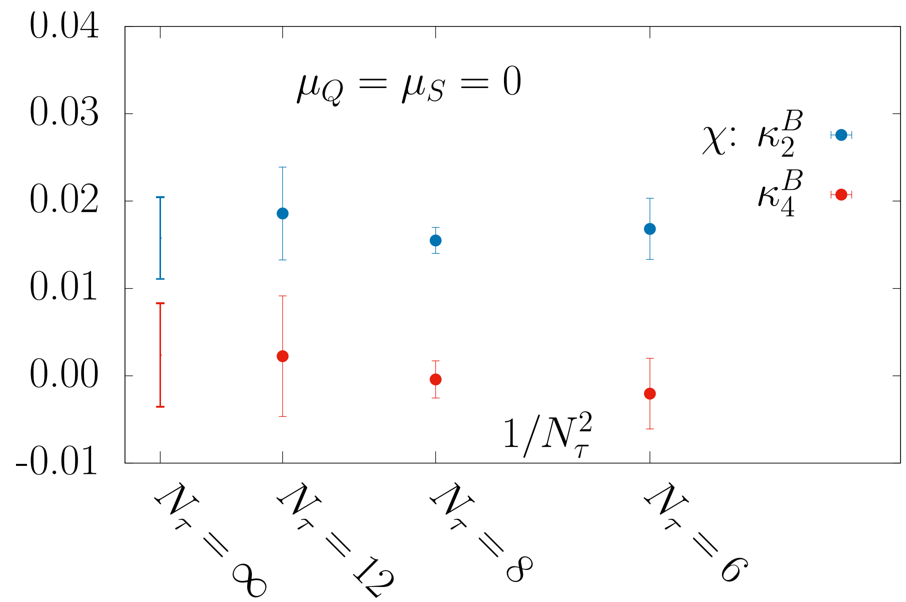
$$T_c(\mu_B) = T_c(0)\left[1 - \kappa_2^B\left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B\left(\frac{\mu_B}{T_c(0)}\right)^4\right]$$



The freeze-out condition corresponding to constant energy density or constant entropy density agrees with the crossover line within errors

The μ_B dependence of T_c is small

$$\kappa_2^{B,\chi} \simeq \kappa_2^{B,\Sigma}$$

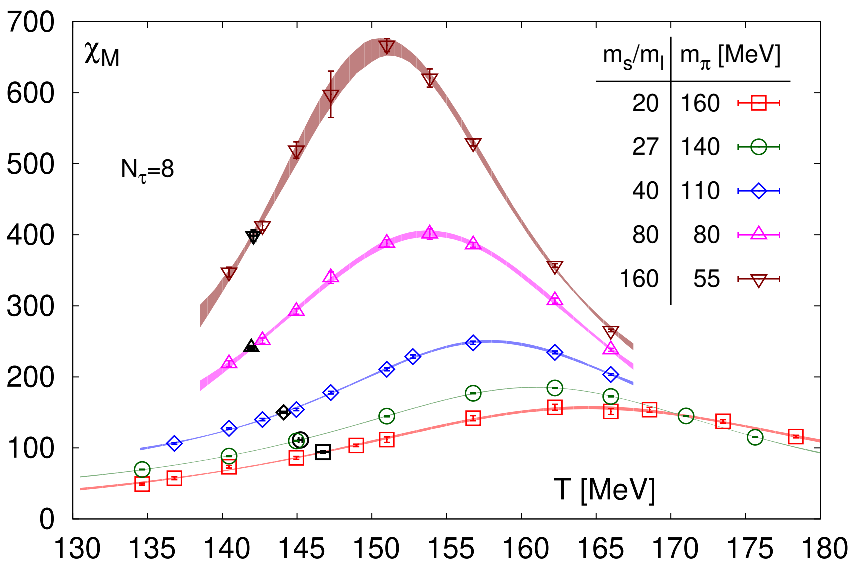


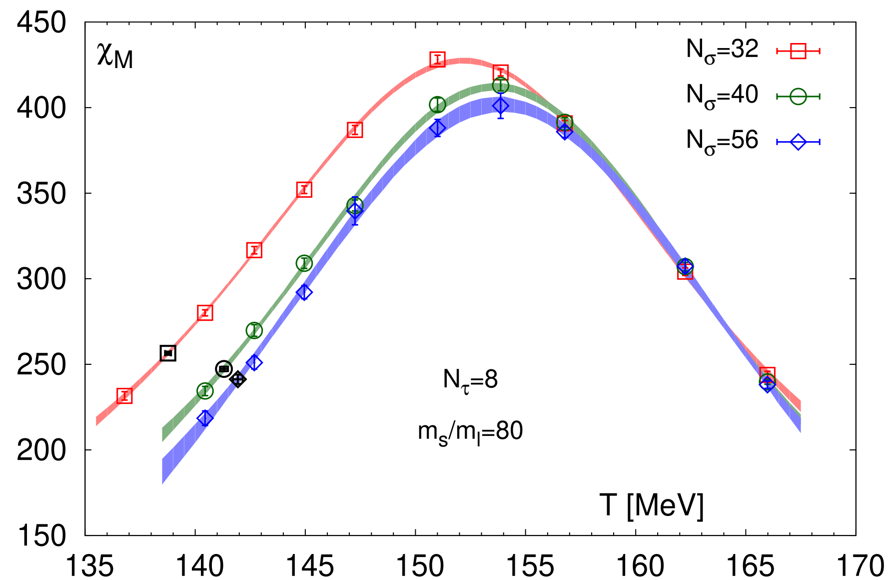
HotQCD, arXiv:1812.08235

Chiral phase transition in 2+1 flavor QCD

What is the nature of the chiral transition in 2+1 flavor QCD for fixed m_s and $m_l \to 0$?

HotQCD, arXiv:1903.04801





$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L) \ ,$$

$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L) \ .$$

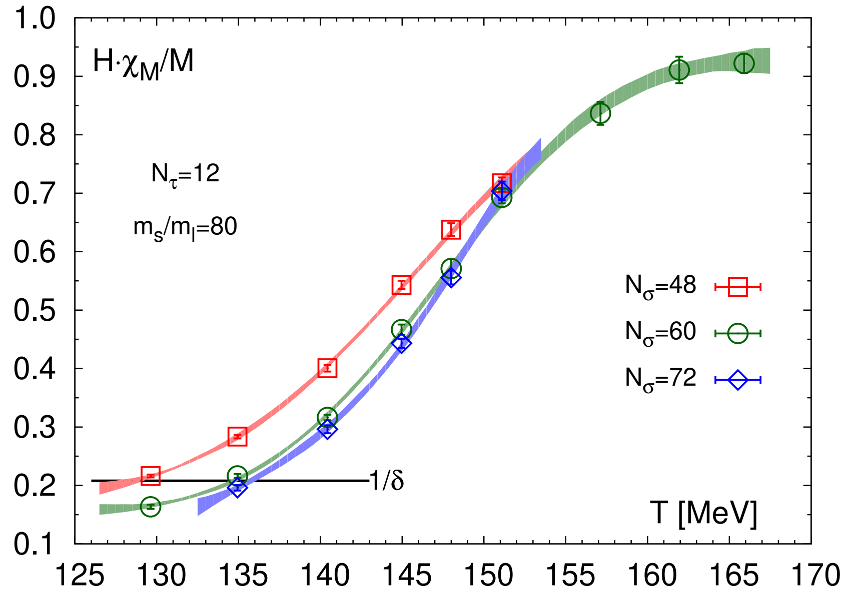
Finite volume:

$$z_L = l_0/(L h^{\nu/\beta\delta})$$

$$T_p(H, L) = T_c^0 \left(1 + \frac{z_p(z_L)}{z_0} H^{1/\beta\delta}\right) \ + \ \text{sub leading}$$

New estimators:

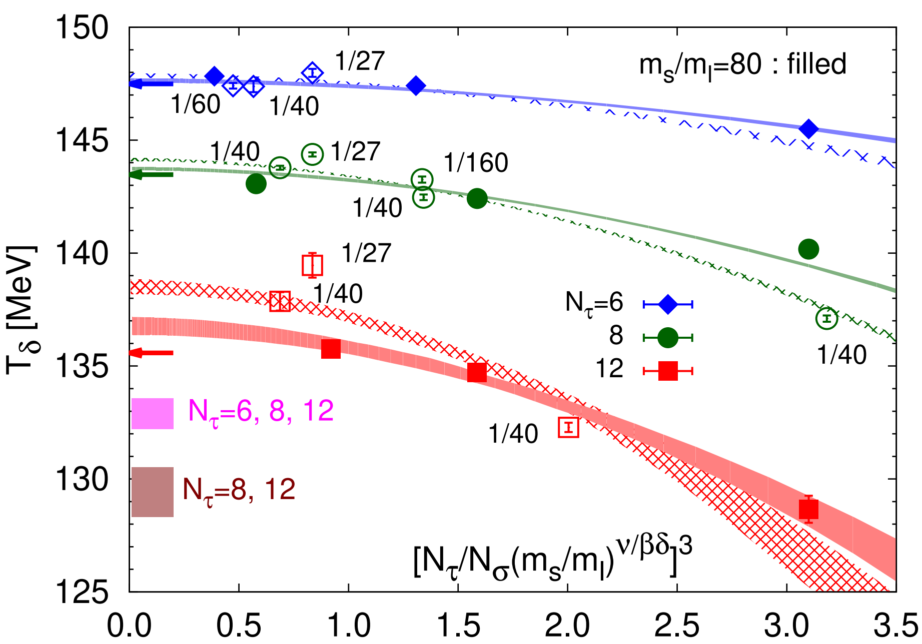
$$T_X(H, L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0}\right) H^{1/\beta\delta}\right)$$



$$\frac{H\chi_M(T_\delta,H,L)}{M(T_\delta,H,L)} = \frac{1}{\delta} ,$$
$$\chi_M(T_{60},H) = 0.6\chi_M^{max} .$$

$$T_X(H,L) = T_c^0\left(1+\left(\frac{z_X(z_L)}{z_0}\right)H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta} , \quad X=\delta,\ 60$$

$$z_{60} \simeq z_\delta \simeq 0$$



Use $O(4)$ fits for m_l and volume dependence

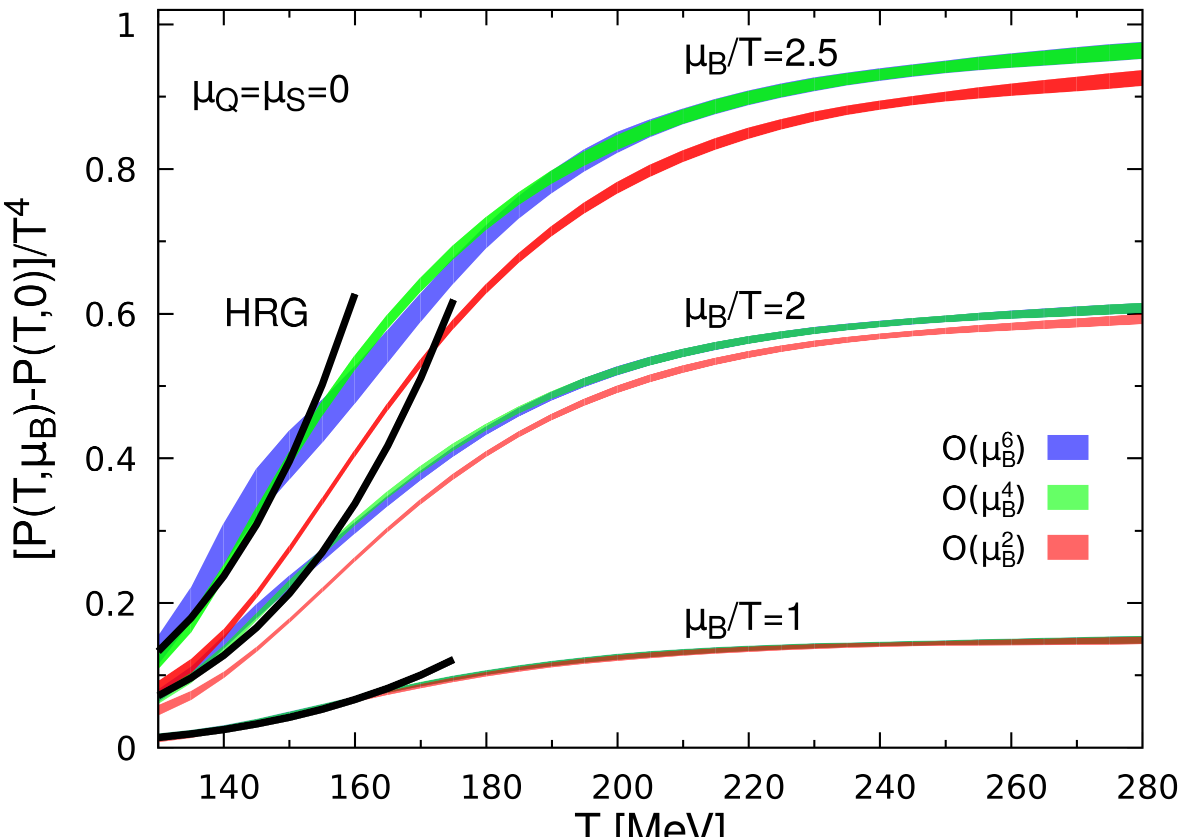
Continuum extrapolations:

$T_c^0 = 132^{+3}_{-6}$ MeV

HotQCD, arXiv:1903.04801

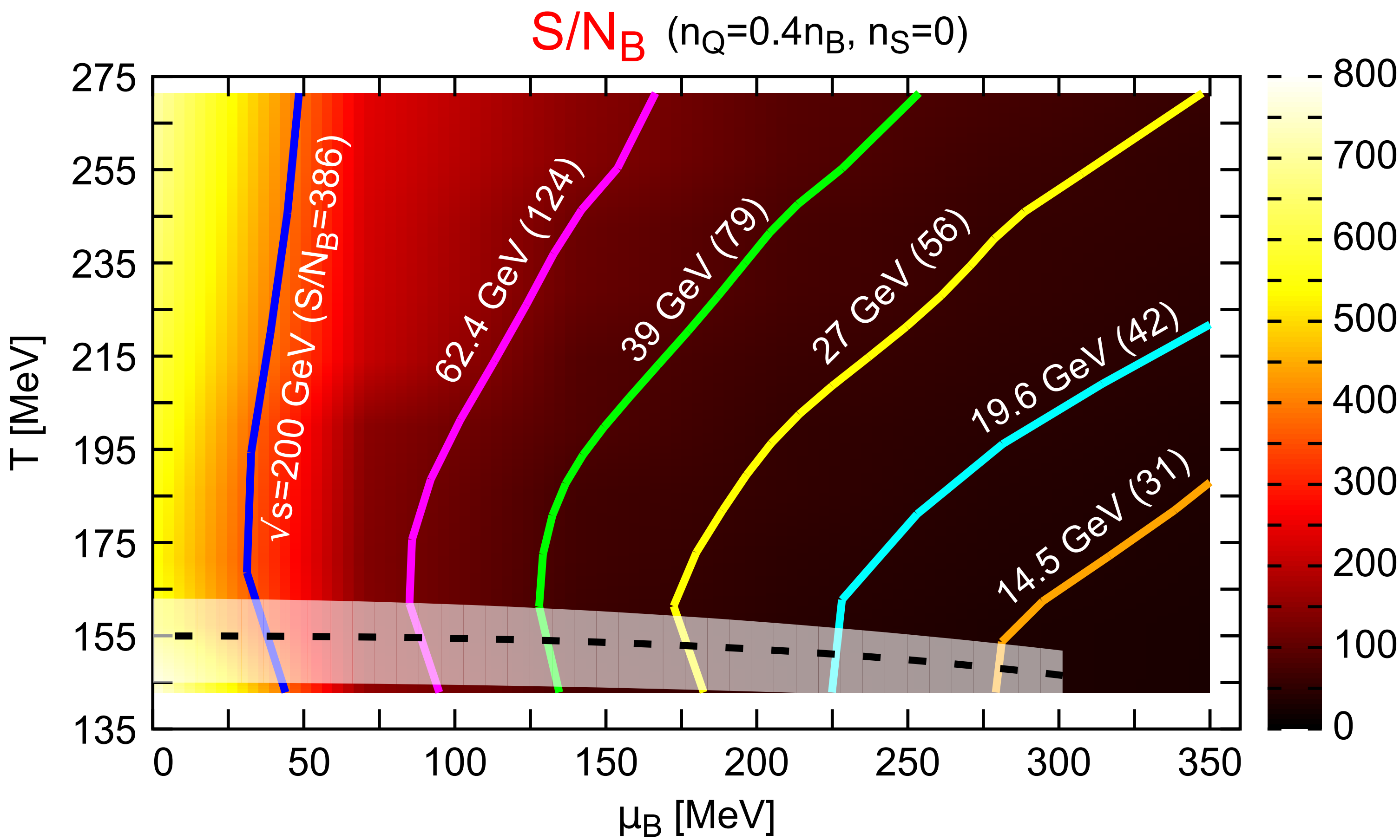
Equation of state at non-zero net baryon density

6th order Taylor expansion, HotQCD, PRD 95 (2017) 054504



Truncation errors of the 6th order Taylor expansions are small for $\mu_B/T<2.5$

Iso-entropic Equation of state at non-zero net baryon density

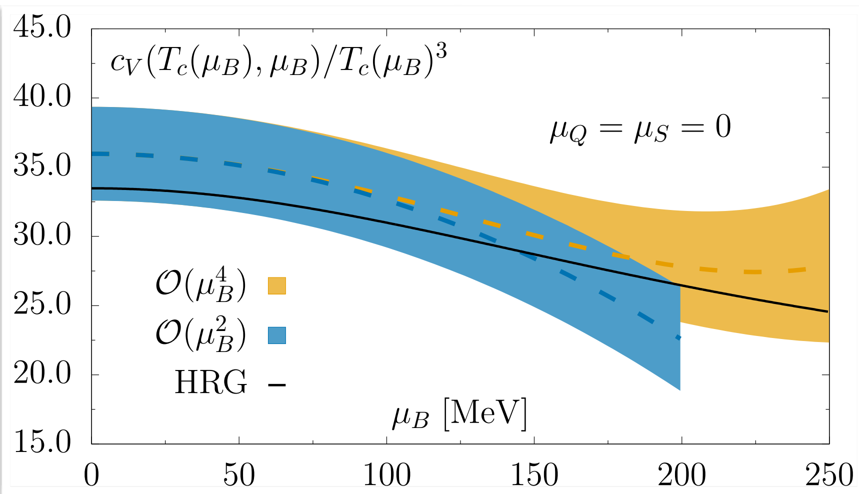


approximate trajectories in heavy-ion collisions follow constant entropy to baryon number ratio

Equation of state along the "phase boundary"

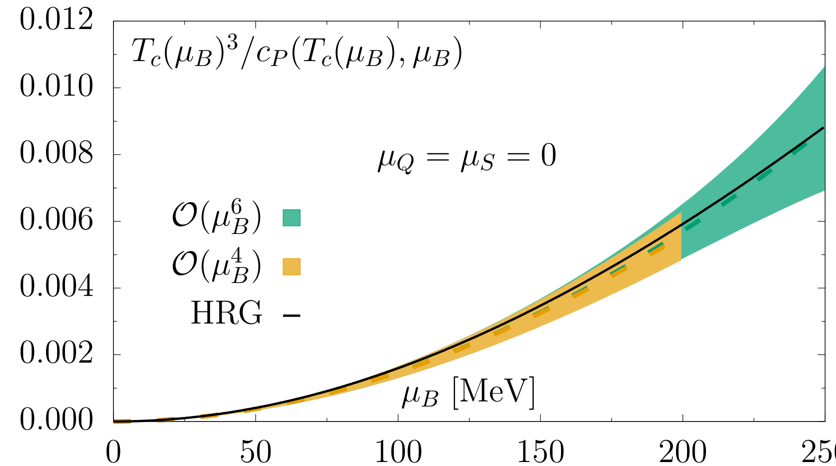
specific heat @ constant volume:

$$c_V = T \left(\frac{\partial s}{\partial T} \right)_{n_B}$$



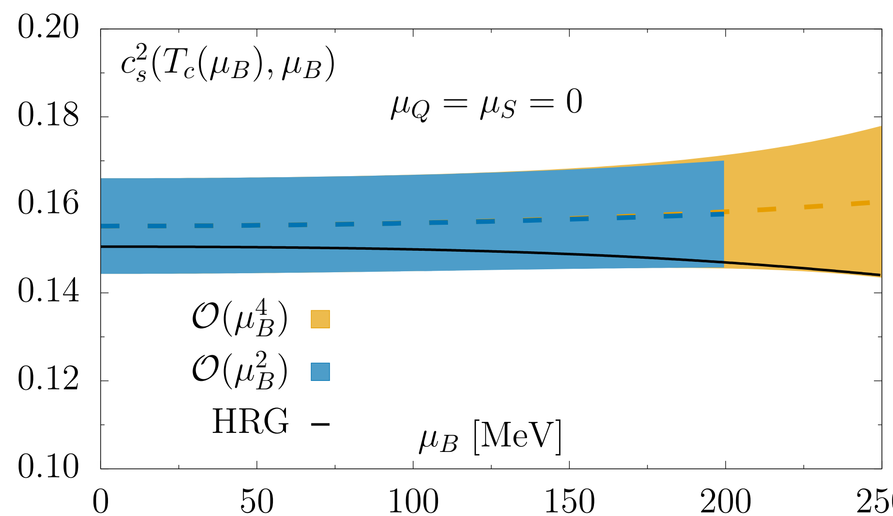
(inverse) specific heat@ constant pressure:

$$c_p = \frac{T}{(s/n_B)} \left(\frac{\partial (s/n_B)}{\partial T} \right)_p$$



isentropic speed of sound:

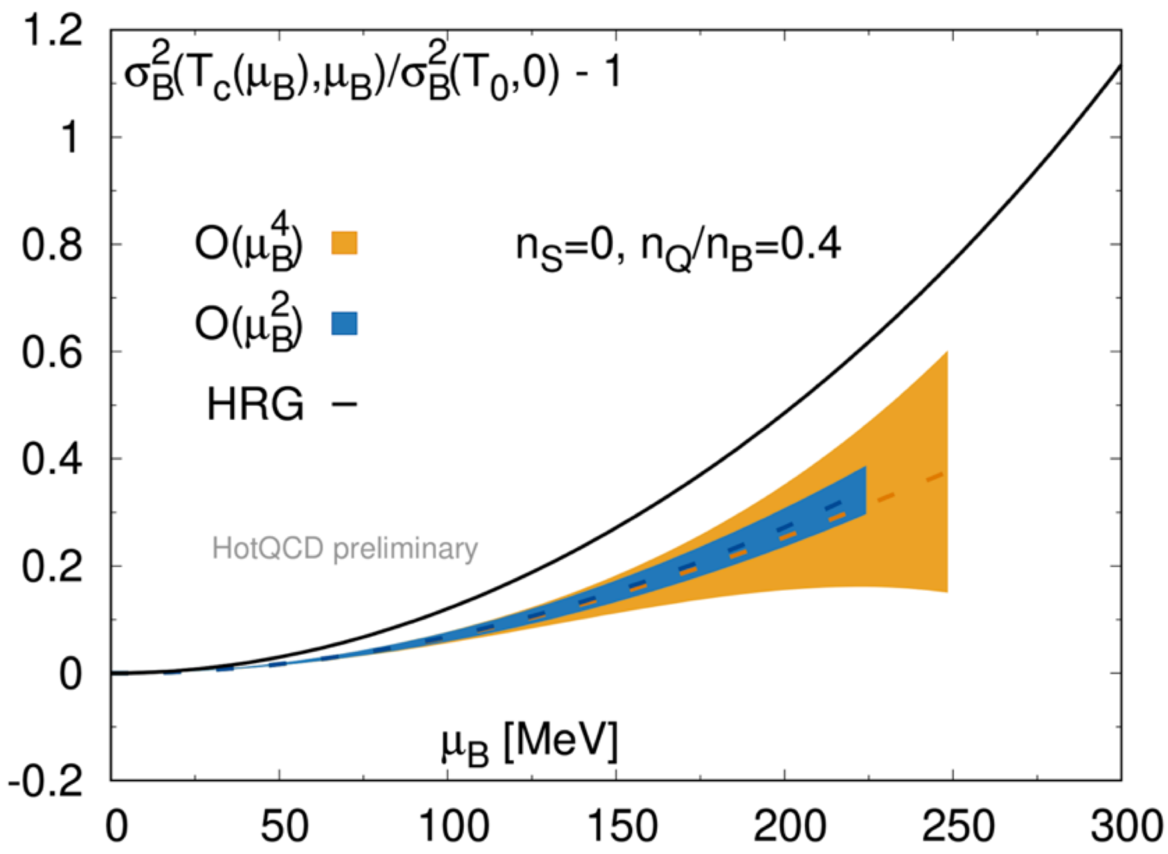
$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon} \right)_{s/n_B}$$



Net baryon number fluctuations along the "phase boundary"

$$\frac{\sigma_B^2}{Vf_K^3} = \frac{1}{Vf_K^3}\frac{\partial \ln Z}{\partial(\mu_B/T)^2} = \sum_{n=1}^{\infty} \chi_{2n}^B \left(\frac{\mu_B}{T}\right)^{2n}$$

Baryon number fluctuations are expected to increase as we approach the critical point

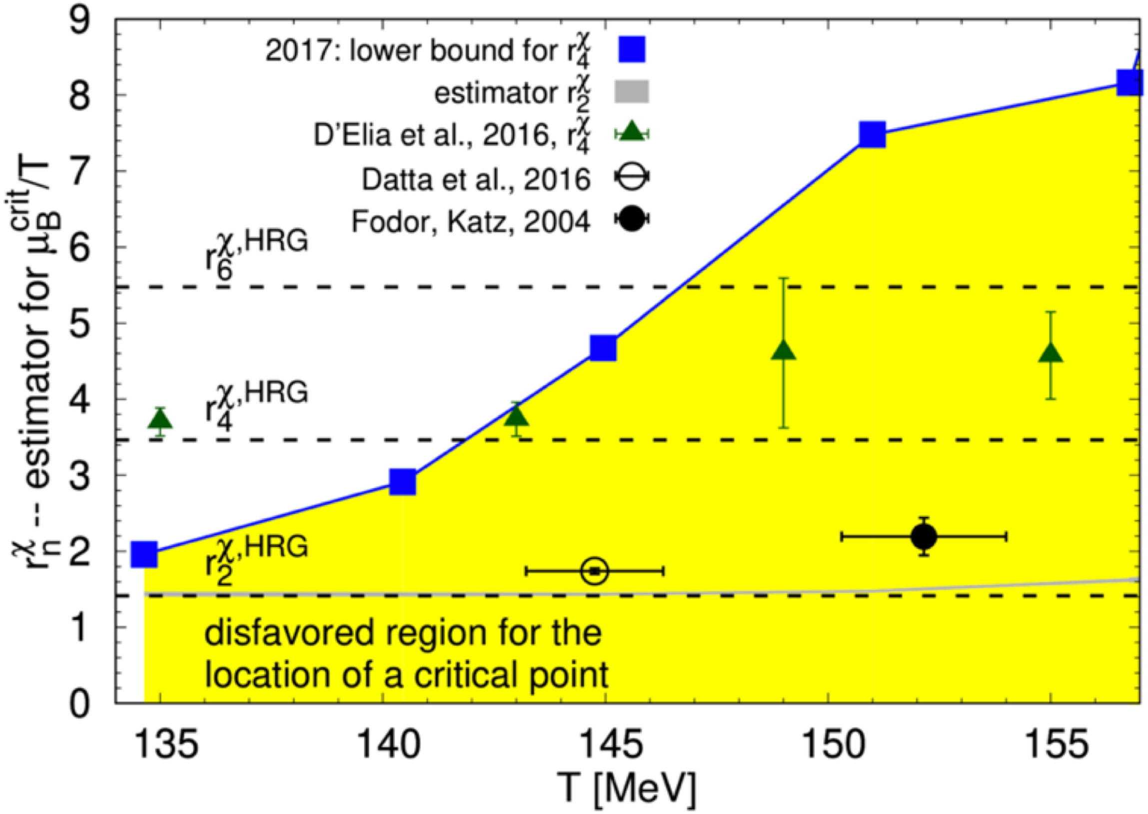


increase remains less than (ideal) hadron gas resonance gas model (HRG)

Radius of convergence of Taylor series and critical point

$$\frac{P(T,\mu_B)-P(T,0)}{T^4}=\sum_{n=1}^{\infty}\frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_2^B(T,\mu_B)=\sum_{n=1}^{\infty}\frac{\chi_{2n+2}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$

$T<T_c$: $\chi_n^B>0 \Rightarrow$ convergence of the Taylor expansion
is limited by a similarity on the real axis $\mu_B=\mu_B^c$ (critical point).



Estimator for radius of convergence:

$$r_n^\chi=\left|\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|^2$$

Critical point is disfavored for $\mu_B<300$ MeV

Summary

- The chiral transition temperature at the physical quark mass can be precisely determined: $T_c = 156 \pm 1.5$ MeV and can be related to chiral phase transition temperature For $m_{u,d} = 0$: $T_c = 132^{+3}_{-6}$ MeV
- Equation of state at non-zero baryon density can be obtained from the Taylor expansion and no indication of limited convergence radius for $\mu_B < 300$ MeV
- The dependence of T_c on μ_B is very small and is consistent with freeze-out curve in HI, the width of the chiral susceptibility and the peak height does not change with μ_B
- The μ_B dependence of EoS and baryon number fluctuations follows the HRG expectation