

The impact of η photoproduction on the resonance spectrum

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Messier 87

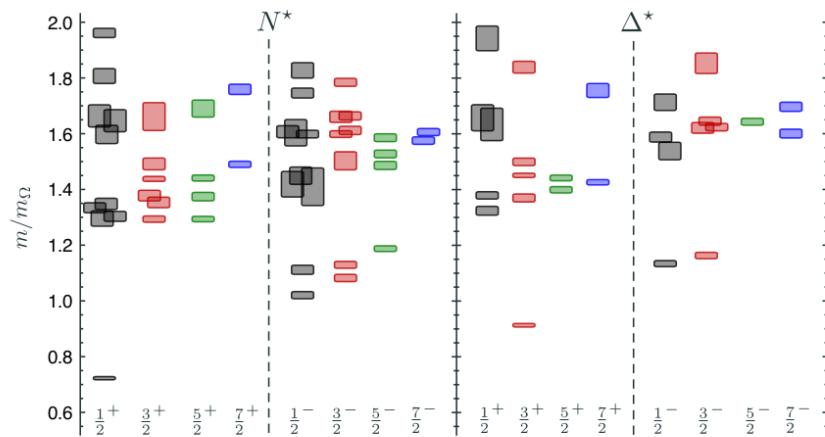
Degrees of freedom: Quarks or hadrons?

The Missing Resonance Problem

- above 1.8 GeV much more states are predicted than observed,

“Missing resonance problem”

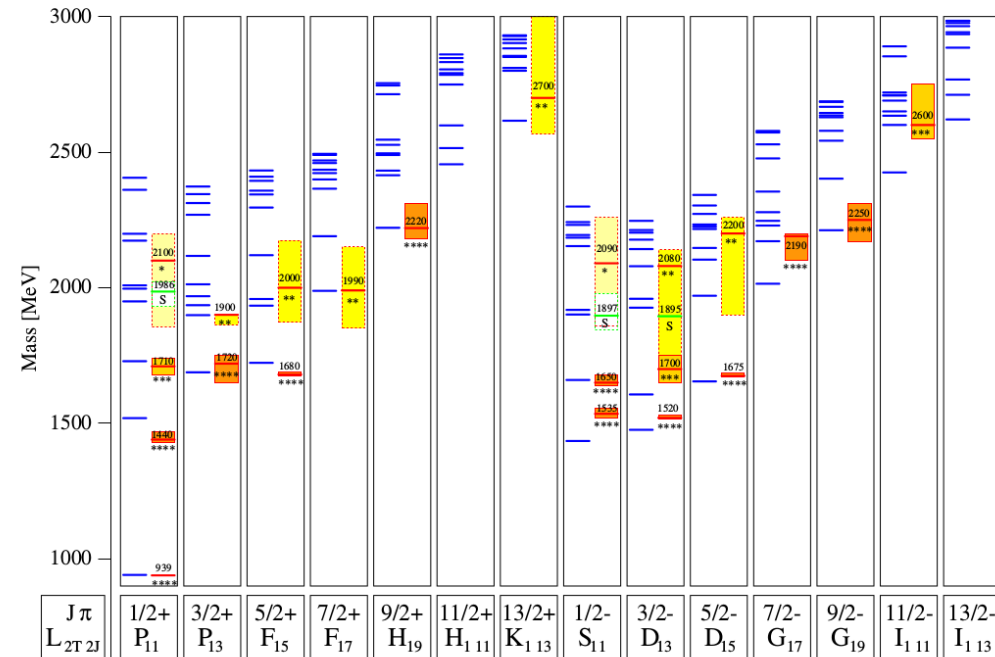
Lattice calculation (single hadron approximation):



[Edwards *et al.*, Phys.Rev. D84 (2011)]

- only 15 established N^* states (PDG 2015)
- $\sim 48\%$ of the states have **** or *** status (PDG 1982: 58% with **** or ***)

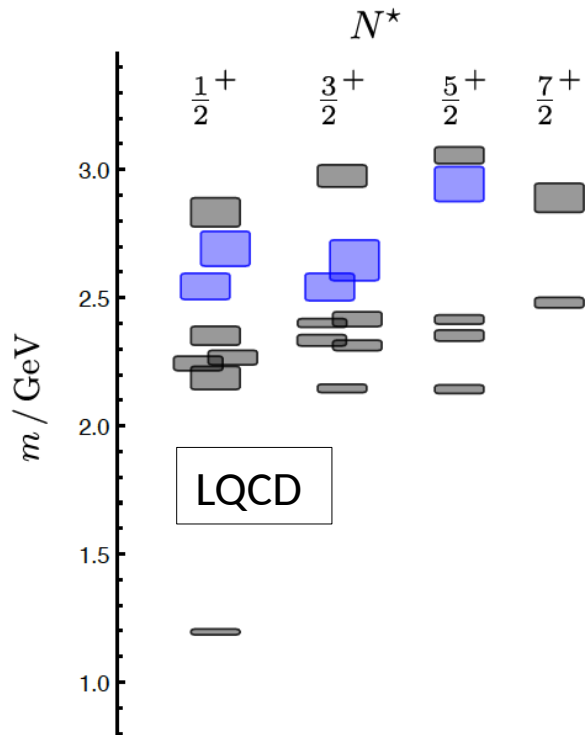
N^* spectrum in a relativistic quark model:



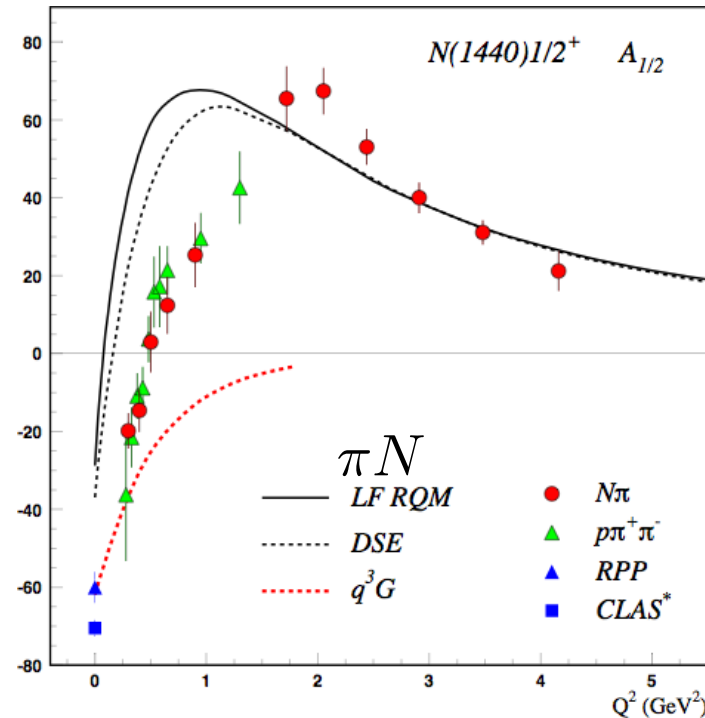
Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Overviews: Crede, Roberts, Rep. Prog. Phys. 76 (2013)
Aznauryan *et al.*, Int. J. Mod. Phys. E 22 (2013)

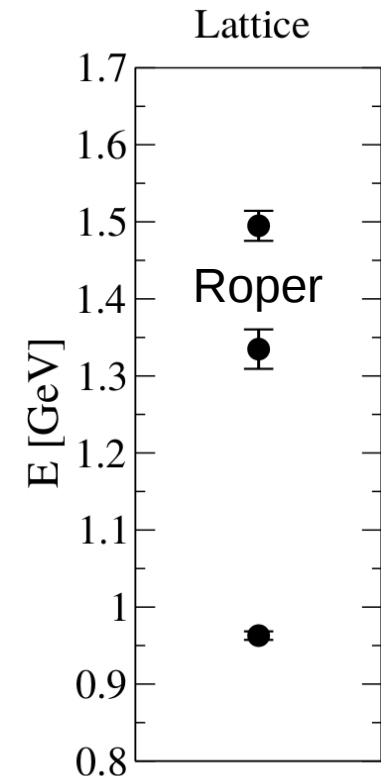
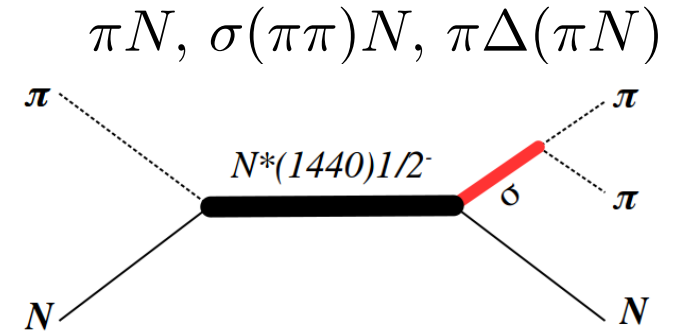
Hybrid Baryons



J.J. Dudek and R.G. Edwards,
PRD85 (2012)



Rel. quark model: Aznauryan (2007)
Dyson-Schwinger: Wilson, Cloet, Chang,
C. D. Roberts (2012)
[source: Int. J. Mod. Phys. (2013)]



Data: [Lang et al., Phys.Rev. D95 (2017), 014510]

Hybrid states: same J^P values as q^3 baryons.
Identification? Measure Q^2 dependence of
electro-couplings (**CLAS 12**)

[parts of slide courtesy of V. Burkert]

Talk by Maxim Mai
tomorrow,
Row H, 5:05pm

- **QCD** at low energies

→ *mass generation & confinement*

- Non-perturbative dynamics

→ rich spectrum of excited states

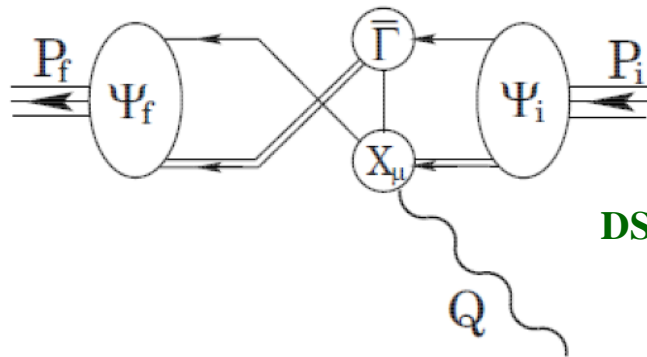
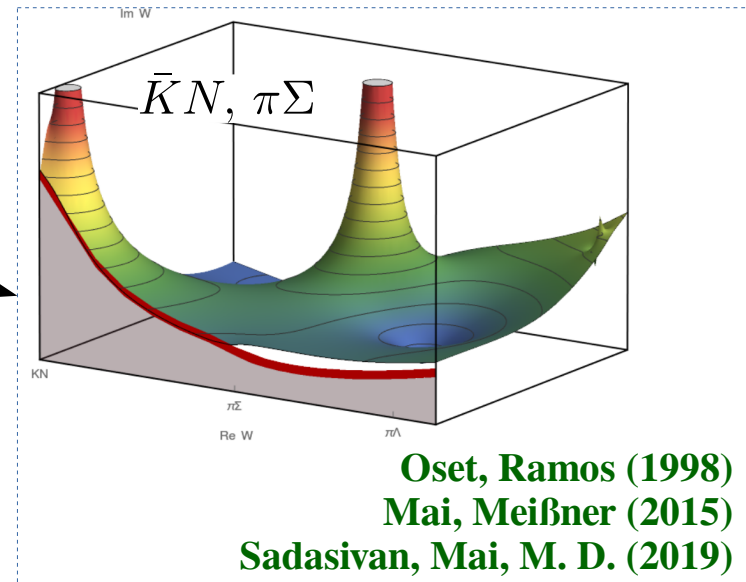
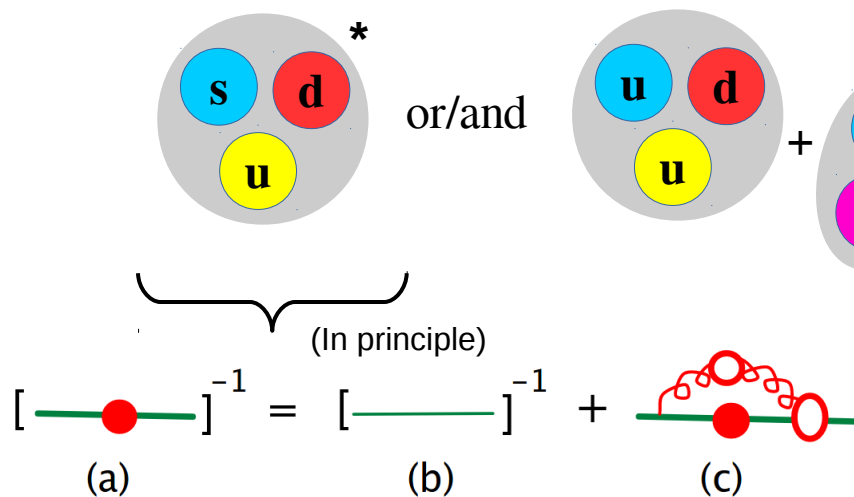
Q1: how many are there?

(missing resonance problem)

Q2: *what are they?*

(2-quark/3-quark, hadron molecules, exotics,...)

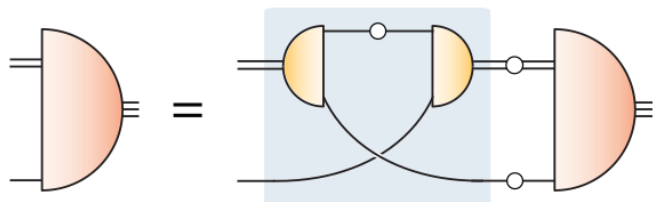
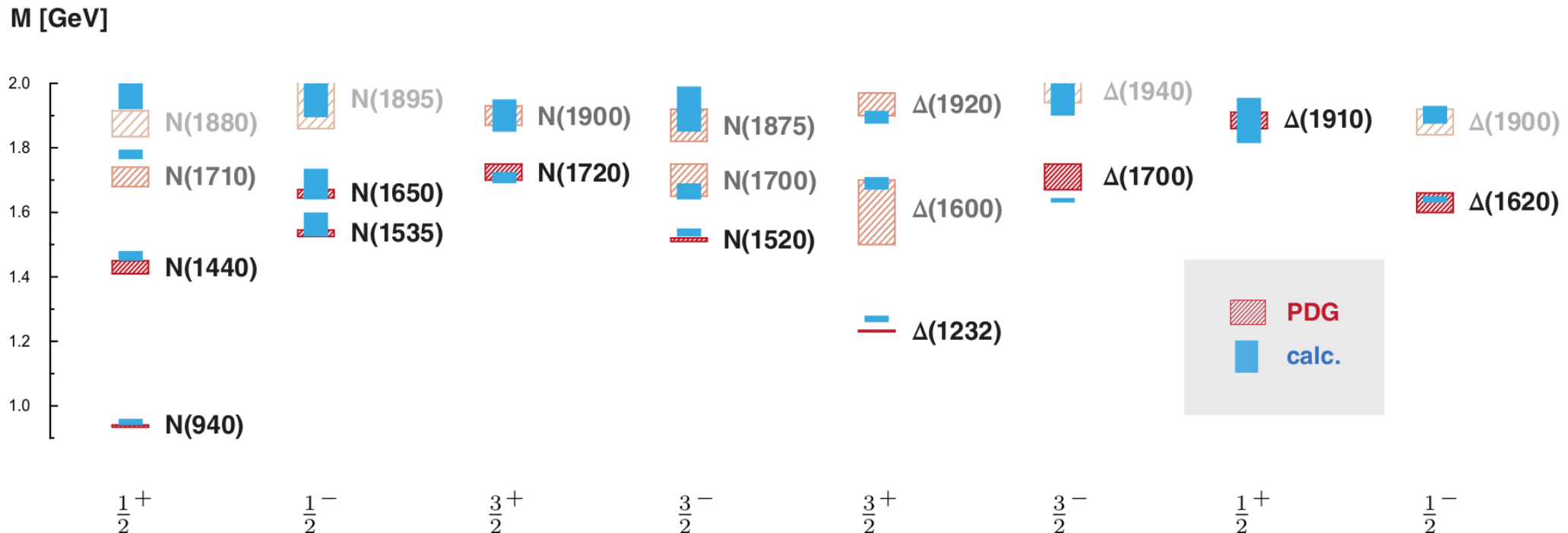
$\Lambda(1405)$



DSE (Wilson, Cloet, Chang, Roberts)

New results for the baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: [GE, Fischer, Sanchis-Alepuz, PRD 94 \(2016\)](#)



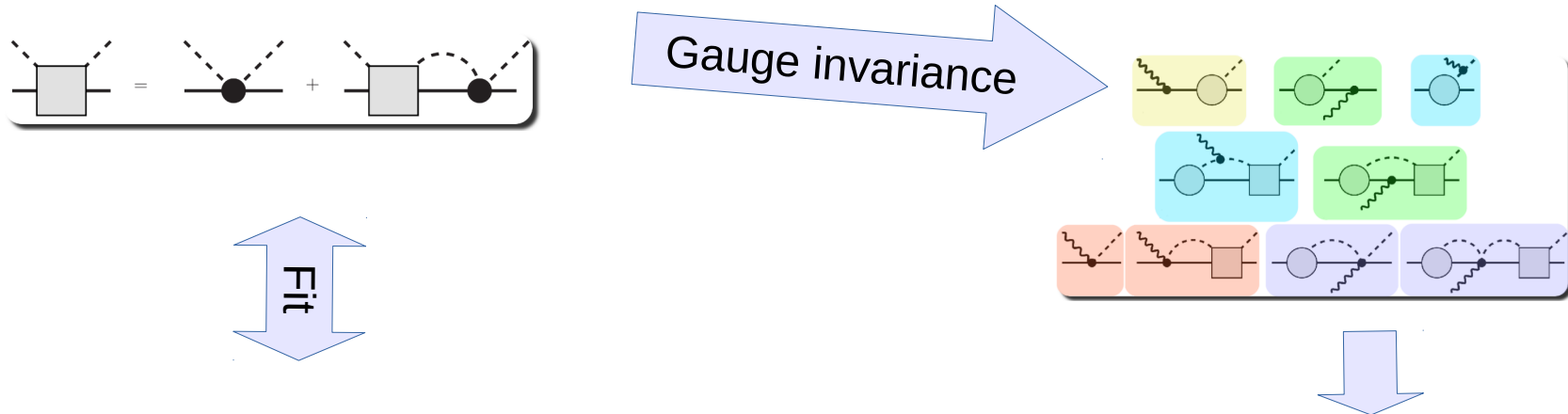
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ - a_1 splitting
- η doesn't change much

[Slide courtesy of G. Eichmann, Few Body 2018]

Using ONLY meson-baryon degrees of freedom (no explicit quark dynamics):

Manifestly gauge invariant approach based on full BSE solution

[Ruic, M. Mai, U.-G. Meissner PLB 704 (2011)]



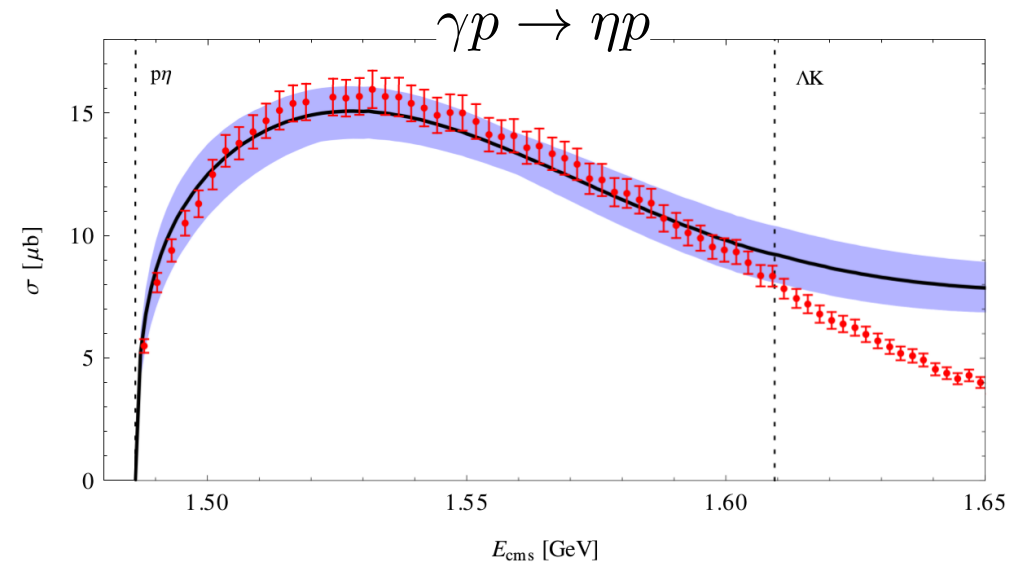
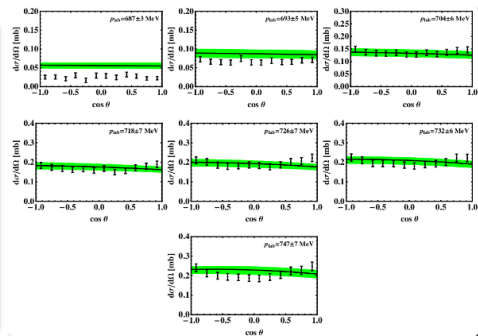
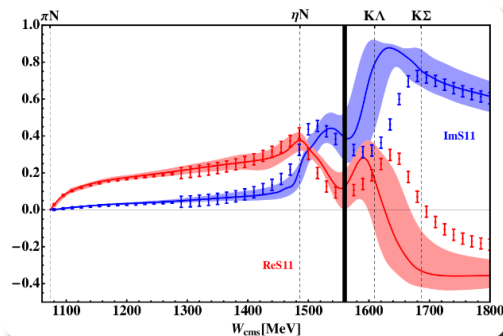
- Exact unitary meson-baryon scattering amplitude T with parameters, fixed to reproduce:

- πN -partial wave S_{11} and S_{31} for $\sqrt{s} < 1560$ MeV

Arndt et al. (2012)

- $\pi^- p \rightarrow \eta n$ differential cross sections

Prakhov et al. (2005)

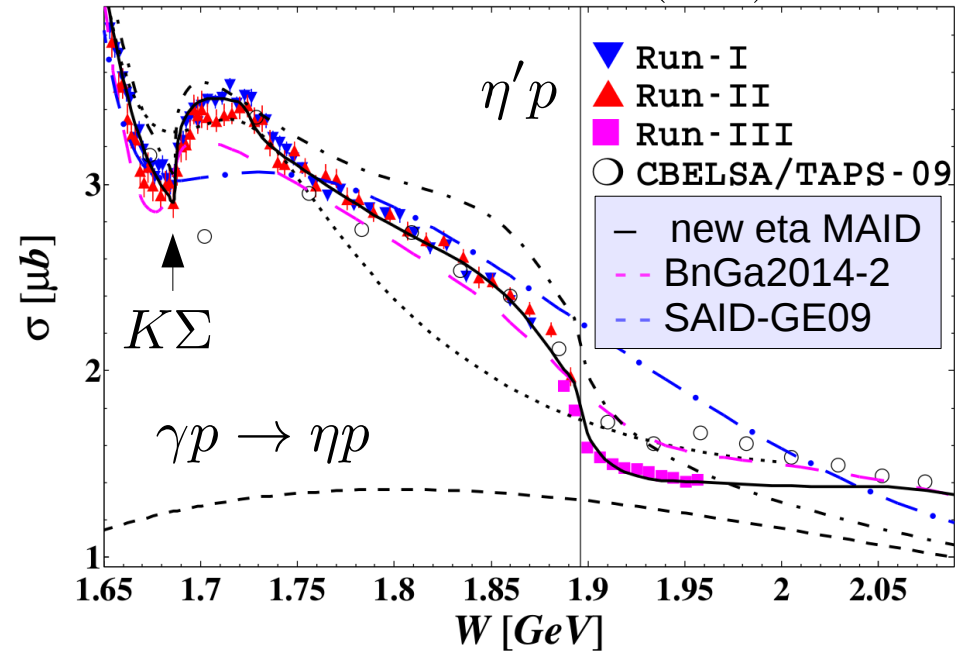


→ Making the “Missing resonance problem” worse ?!

Phenomenology

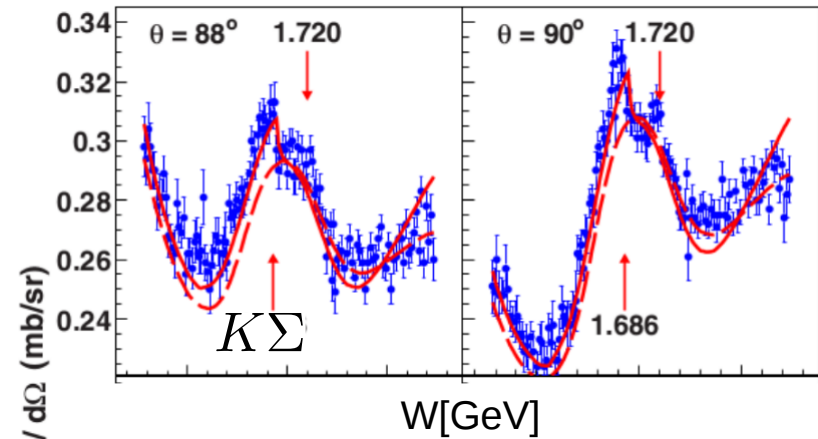
Resonances or not?

A2 MAMI, PRL 118 (2017) $S_{11}(1895)$

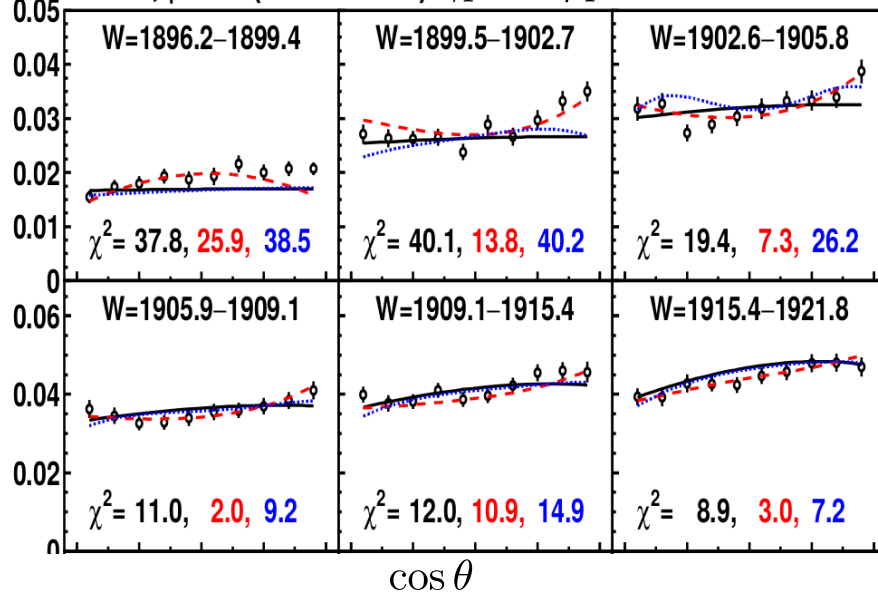


$\pi N \rightarrow \pi N$

EPECUR/SAID PRC 93 (2016)



$d\sigma/d\Omega, \mu\text{b/sr}$ (MAMI 2017) $\gamma p \rightarrow \eta' p$



BnGa

PLB785 (2018):

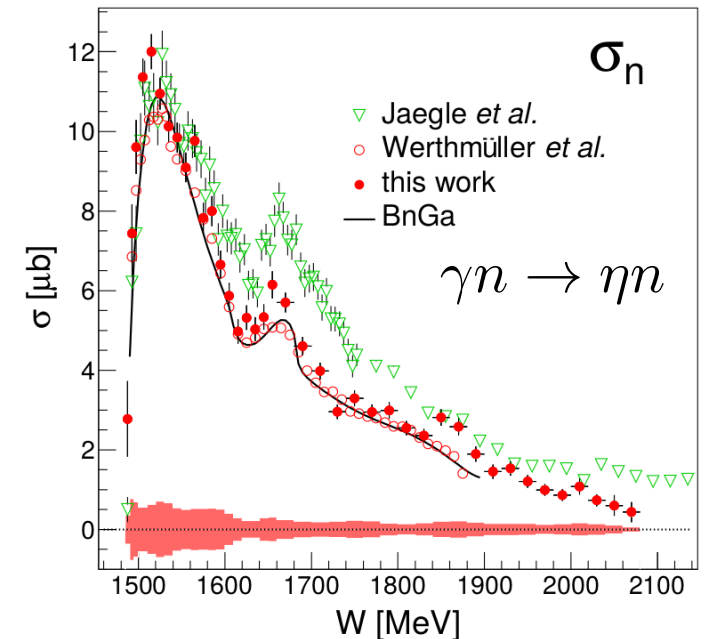
No narrow
resonance

$3/2^-$ narrow
Resonance

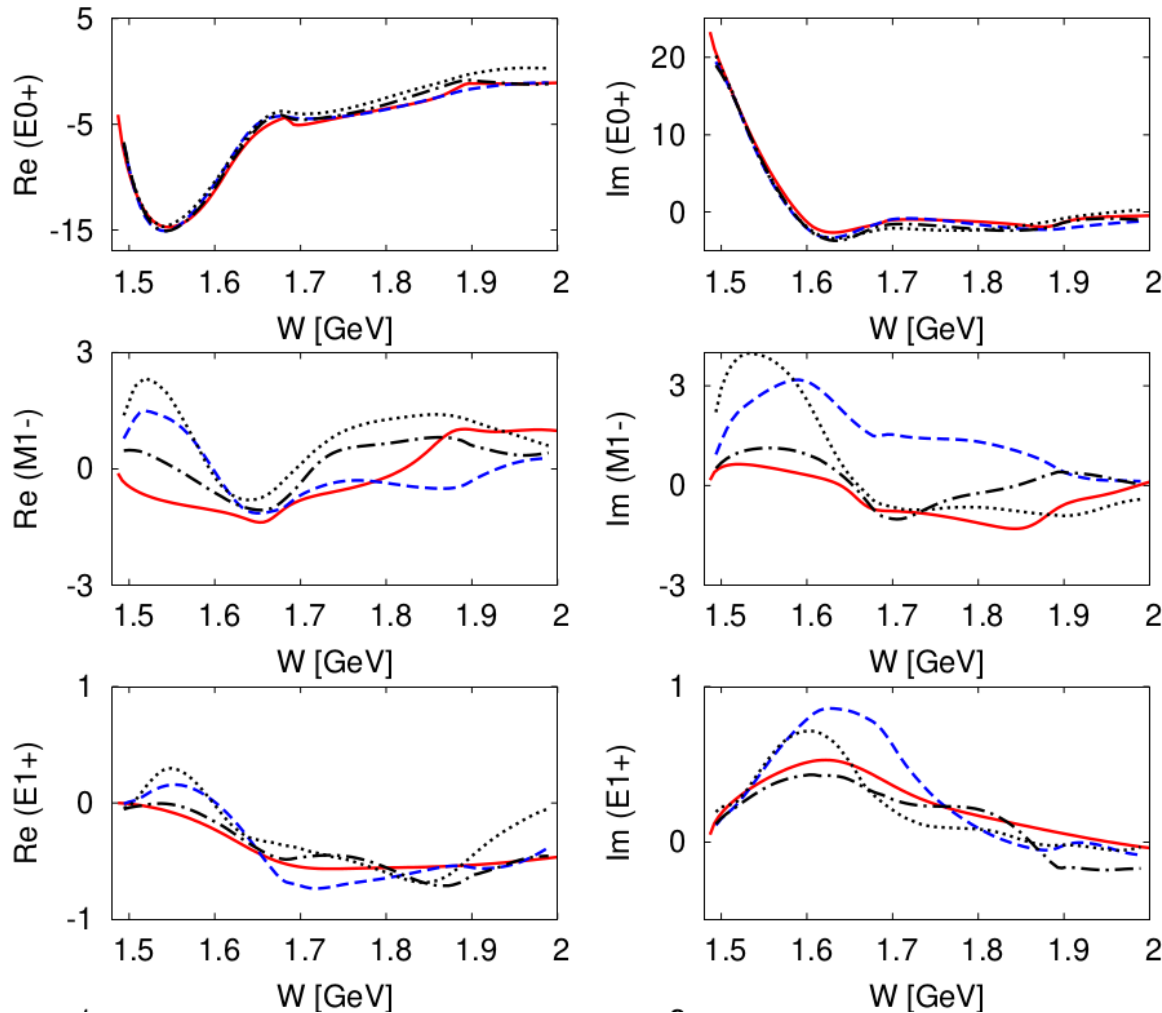
$5/2^-$ narrow
Resonance

Data: A2.Mami
PRL 118 (2017)

[CBELSA/TAPS EPJA 53 (2017)]



Current state in η photoproduction: Multipoles from different groups



From: **EtaMAID2018**
[Tiator et al., EPJA54 (2018)]
Analyzes:

$$\gamma p \rightarrow \eta p$$

$$\gamma p \rightarrow \eta' p$$

$$\gamma n \rightarrow \eta n$$

$$\gamma n \rightarrow \eta' n$$

EtaMAID2018

BnGa [PLB 772 (2017)]

JuBo (dotted) [EPJA 54 (2018)]

KSU [1804.06031]

Review: Krusche, Wilkins,
[Prog.Part.Nucl.Phys. 80 (2014)]

Observable	σ	Σ	T	P	E	F	G	H	T_x	T_z	L_x	L_z	O_x	O_z	C_x	C_z
$p\pi^0$	✓	✓	✓		✓	✓	✓	✓								
$n\pi^+$	✓	✓	✓		✓	✓	✓	✓								
$p\eta$	✓	✓	✓		✓	✓	✓	✓								
$p\eta'$	✓	✓	✓		✓	✓	✓	✓								
$K^+\Lambda$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^+\Sigma^0$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$p\omega/\phi$	✓	✓	✓		✓	✓	✓	✓	✓ SDME							
$K^{*+}\Lambda$	✓			✓					SDME							
$K^{0*}\Sigma^+$	✓	✓									✓	✓	SDME			
$p\pi^-$	✓	✓			✓	✓	✓									
$p\rho^-$	✓	✓			✓	✓	✓									
$K^-\Sigma^+$	✓	✓			✓	✓	✓									
$K^0\Lambda$	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
$K^0\Sigma^0$	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
$K^{0*}\Sigma^0$	✓	✓									✓	✓				

Phys.Lett. B771 (2017)
Phys.Lett. B755 (2016)



$\gamma p \rightarrow X$

$\gamma n \rightarrow X$

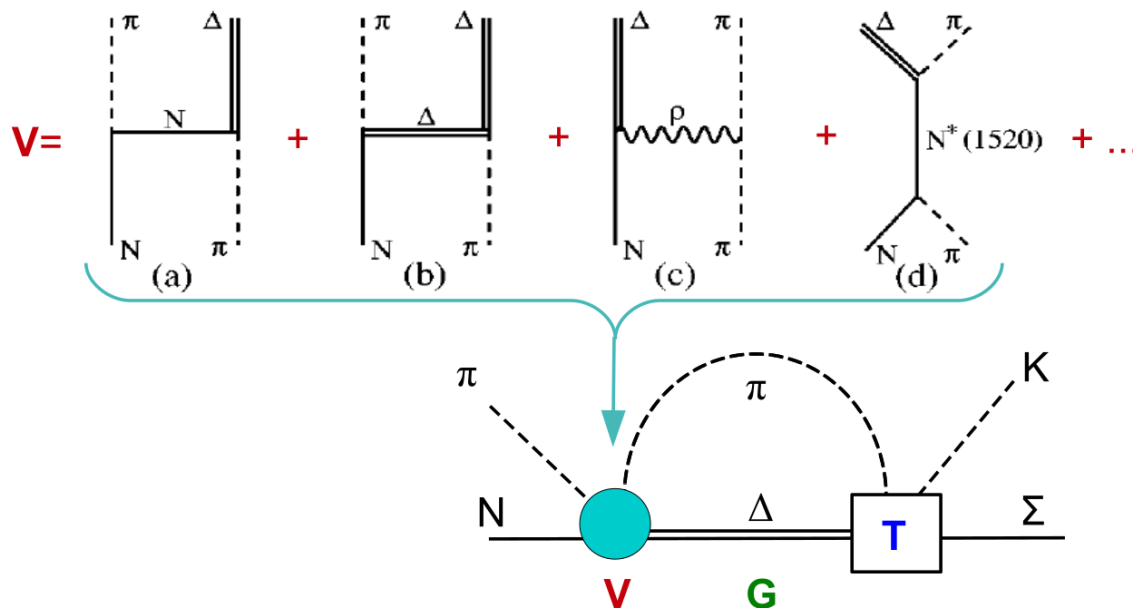
The Julich-Bonn Dynamical Coupled-Channel Approach

e.g. EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): simultaneous analysis of different reactions

The scattering equation in partial-wave basis

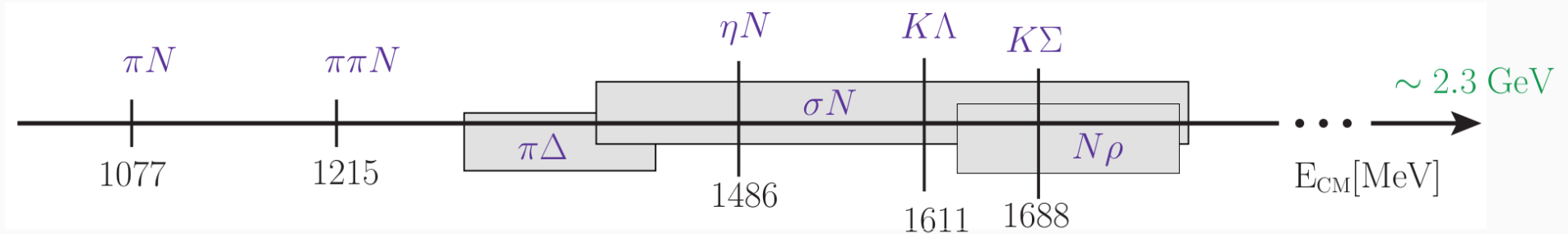
$$\langle L' S' p' | T_{\mu\nu}^{IJ} | L S p \rangle = \langle L' S' p' | V_{\mu\nu}^{IJ} | L S p \rangle + \sum_{\gamma, L'' S''} \int_0^\infty dq \, q^2 \langle L' S' p' | V_{\mu\gamma}^{IJ} | L'' S'' q \rangle \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L'' S'' q | T_{\gamma\nu}^{IJ} | L S p \rangle$$



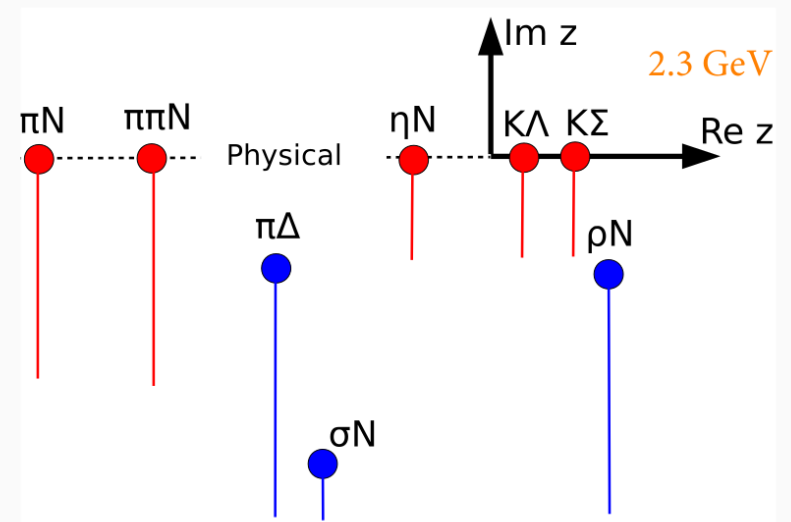
- potentials V constructed from effective \mathcal{L}
- s-channel diagrams: T^P
genuine resonance states
- t - and u -channel: T^{NP}
dynamical generation of poles
partial waves strongly correlated

JuBo: Channels and Analytic Structure

Channels included:



- (2-body) unitarity and analyticity respected
 - 3-body $\pi\pi N$ channel:
 - parameterized effectively as $\pi\Delta$, σN , ρN
 - $\pi N/\pi\pi$ subsystems fit the respective phase shifts
- ↳ branch points move into complex plane



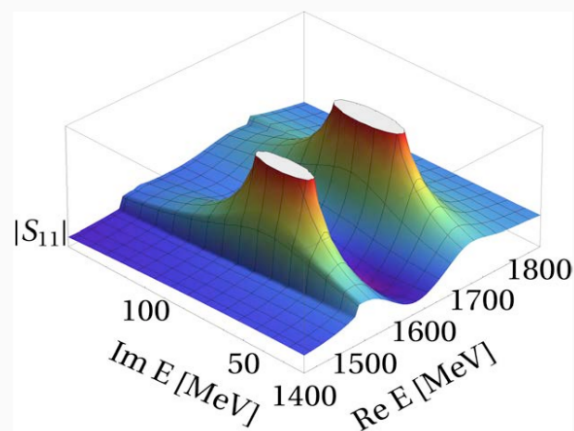
JuBo: Data base

[D. Roenchen, M. D., U.-G. Meißner, EPJ A 54, 110 (2018)]

Reaction	Observables (# data points)	p./channel
$\pi N \rightarrow \pi N$	PWA GW-SAID WI08 (ED solution)	3,760
$\pi^- p \rightarrow \eta n$	$d\sigma/d\Omega$ (676), P (79)	755
$\pi^- p \rightarrow K^0 \Lambda$	$d\sigma/d\Omega$ (814), P (472), β (72)	1,358
$\pi^- p \rightarrow K^0 \Sigma^0$	$d\sigma/d\Omega$ (470), P (120)	590
$\pi^- p \rightarrow K^+ \Sigma^-$	$d\sigma/d\Omega$ (150)	150
$\pi^+ p \rightarrow K^+ \Sigma^+$	$d\sigma/d\Omega$ (1124), P (551) , β (7)	1,682
$\gamma p \rightarrow \pi^0 p$	$d\sigma/d\Omega$ (10743), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140), G (393), H (225), E (467), F (397), $C_{x'_L}$ (74), $C_{z'_L}$ (26)	17,564
$\gamma p \rightarrow \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231), G (86), H (128), E (903)	9,748
$\gamma p \rightarrow \eta p$	$d\sigma/d\Omega$ (5680), Σ (403), P (7), T (144), F (144), E (129)	6,507
$\gamma p \rightarrow K^+ \Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383), $C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
	in total	48,050

Resonance Couplings

Resonance states: Poles in the T -matrix on the 2^{nd} Riemann sheet



- $\text{Re}(E_0)$ = “mass”, $-2\text{Im}(E_0)$ = “width”
- elastic πN residue ($|r_{\pi N}|, \theta_{\pi N \rightarrow \pi N}$), normalized residues for inelastic channels ($\sqrt{\Gamma_{\pi N} \Gamma_{\mu}} / \Gamma_{\text{tot}}, \theta_{\pi N \rightarrow \mu}$)
- photocouplings at the pole: $\tilde{A}_{\text{pole}}^h = A_{\text{pole}}^h e^{i\vartheta^h}$, $h = 1/2, 3/2$

Inclusion of $\gamma p \rightarrow K^+ \Lambda$ in JüBo (“JuBo2017-1”): **3 additional states**

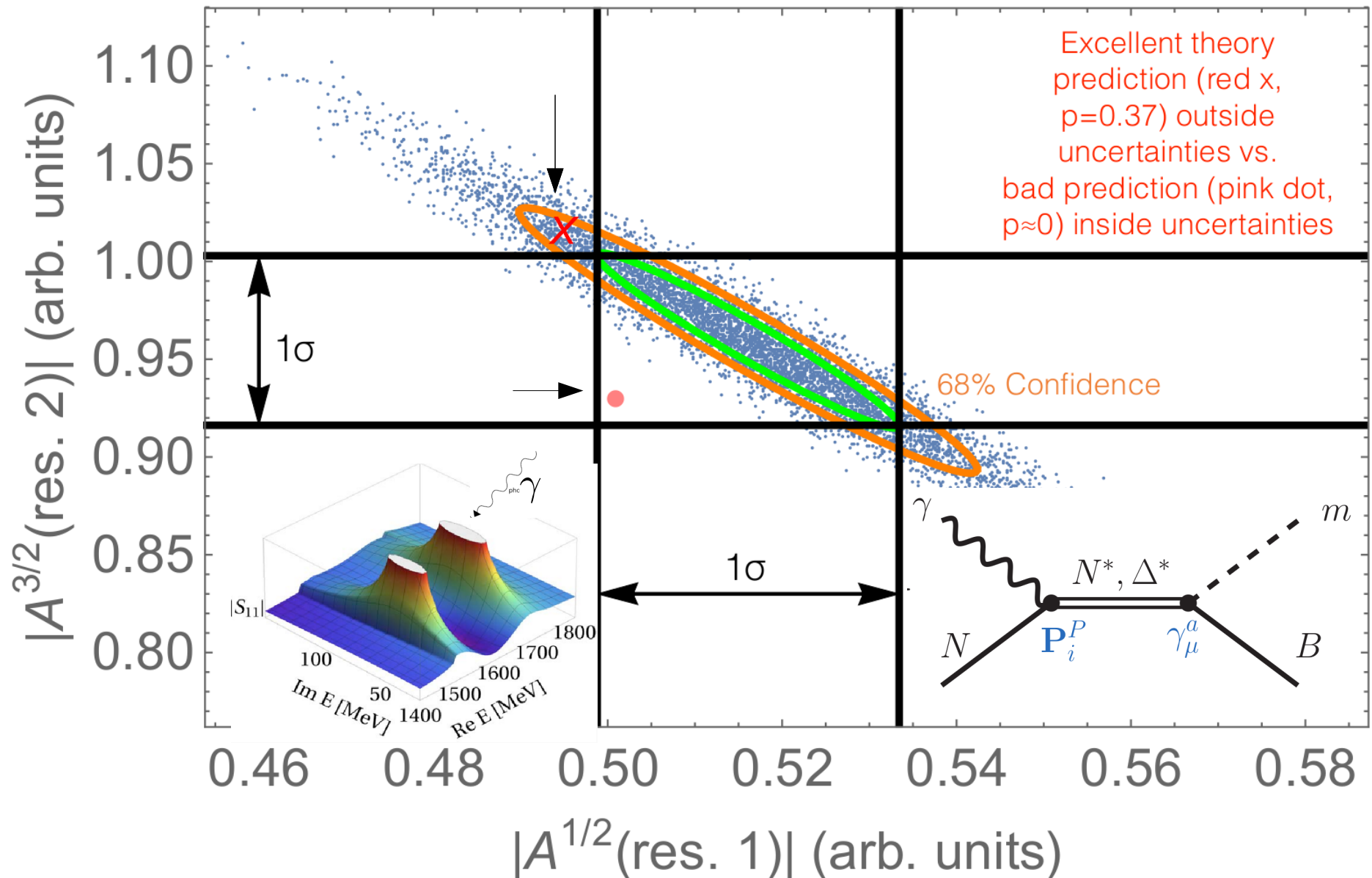
	z_0 [MeV]	$\frac{\Gamma_{\pi N}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{\eta N}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{K\Lambda}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{K\Sigma}}{\Gamma_{\text{tot}}}$
N(1900)3/2 ⁺	1923 − i 108.4	1.5 %	0.78 %	2.99 %	69.5 %
N(2060)5/2 [−]	1924 − i 100.4	0.35 %	0.15 %	13.47 %	27.02 %
$\Delta(2190)$ 1/2 ⁺	2191 − i 103.0	33.12 %			3.78 %

- N(1900)3/2⁺: s-channel resonances, seen in many other analyses of kaon photoproduction (BnGa), 3 stars in PDG
- N(2060)5/2[−]: dynamically generated, 2 stars in PDG, seen e.g. by BnGa
- $\Delta(2190)$ 3/2⁺: dyn. gen., no equivalent PDG state

How to quantify the impact of new measurements?

Consider correlations of helicity couplings extracted from experiment

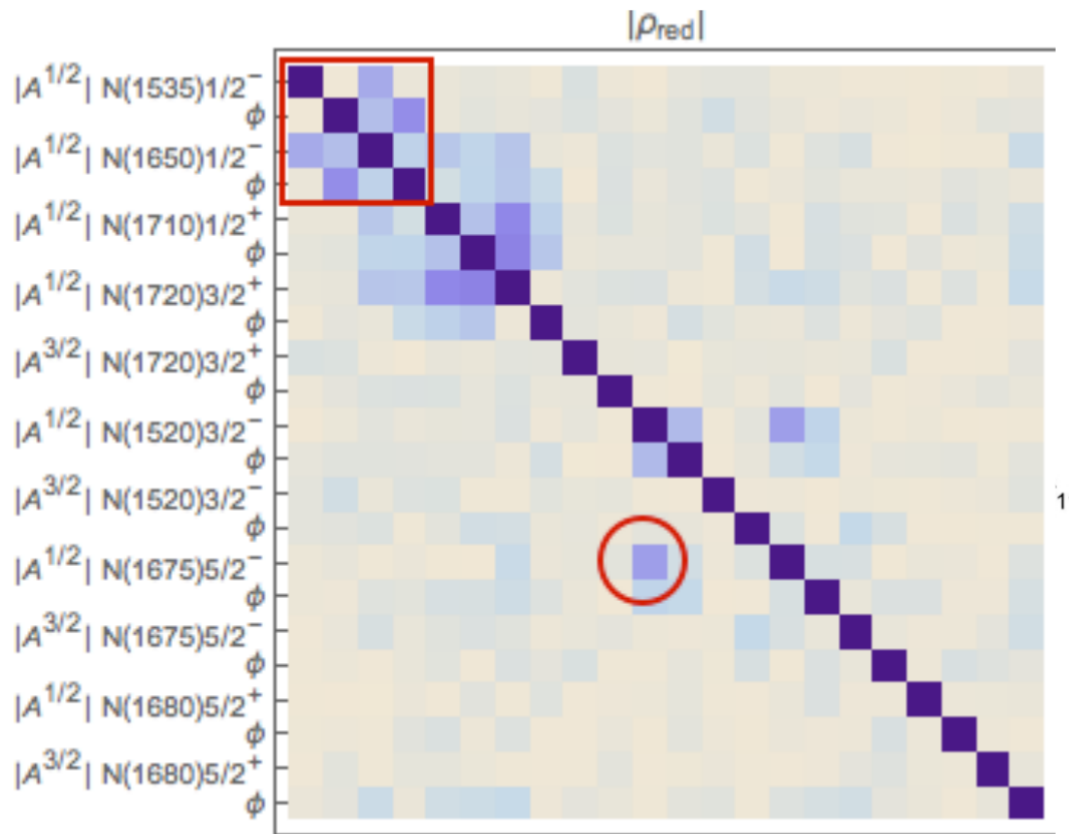
[D. Sadasivan, M.D., M. Mai, in preparation]



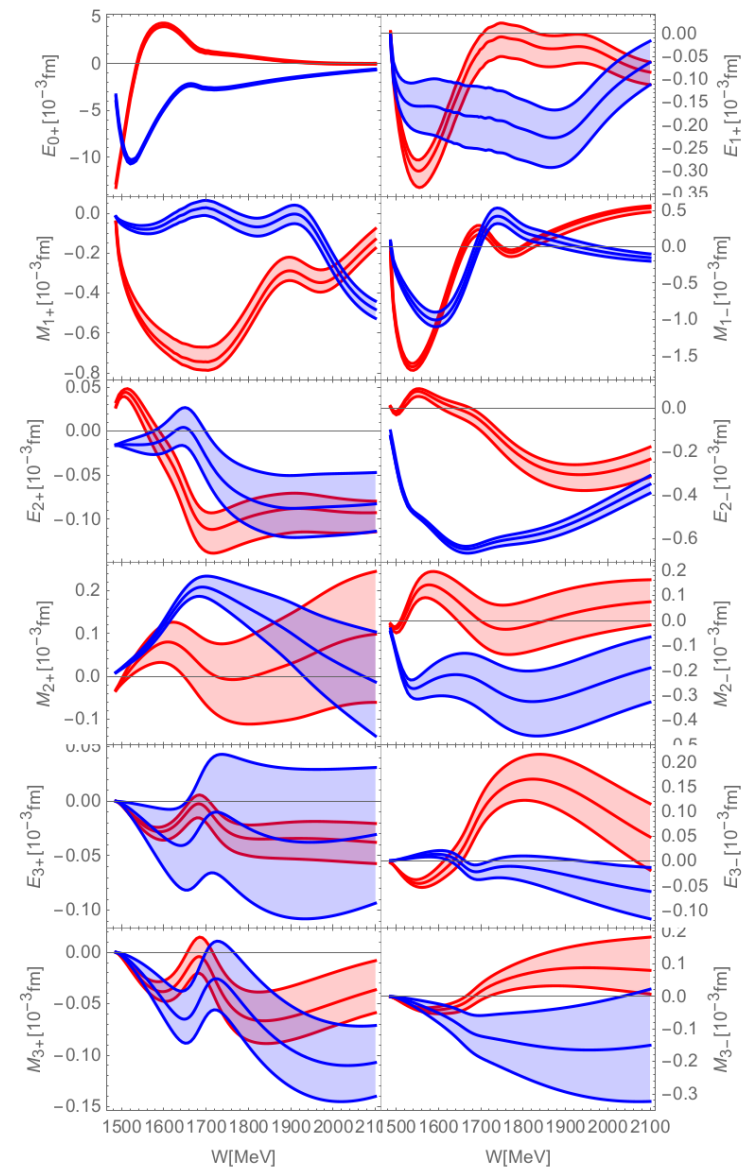
Results from analysis of world data of η photoproduction

[D. Sadasivan, M.D., M. Mai, in preparation]

Here $A = |A|e^{i\phi}$ defined at the resonance pole.



Correlation matrix



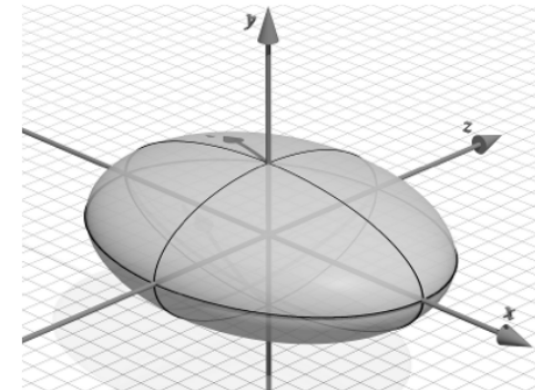
Bulk properties of uncertainties from different data sets

Helicity Coupling	All	No E	No F	No T	No Σ
Number of Data Points	6425	6369	6281	6281	6022
Generalized Variance	<u>0.0494</u>	0.0521	0.1288	0.1239	<u>6.664</u>
$\sqrt{\text{Tr } C}$	10.4965	10.51	12.00	11.423	19.85
Multicollinearity	8.173	8.203	9.280	9.5323	10.371
Condition number	133.61	132.10	173.664	164.1	322.66

C=Covariance Matrix

Generalized Variance
= $\text{Det}[C]$ \sim Volume of
the Error Ellipsoid

Helicity Coupling	No artificial data	Cx	Cz	Cx and Cz
Number of Data Points	6425	6569	6569	6713
Generalized Variance	0.0494	0.03758	0.0362	<u>0.0132</u>
$\sqrt{\text{Tr } C}$	10.4965	10.72	10.487	10.102
Multicollinearity	8.173	7.599	6.770	6.157
Condition number	133.61	112.47	109.69	107.683

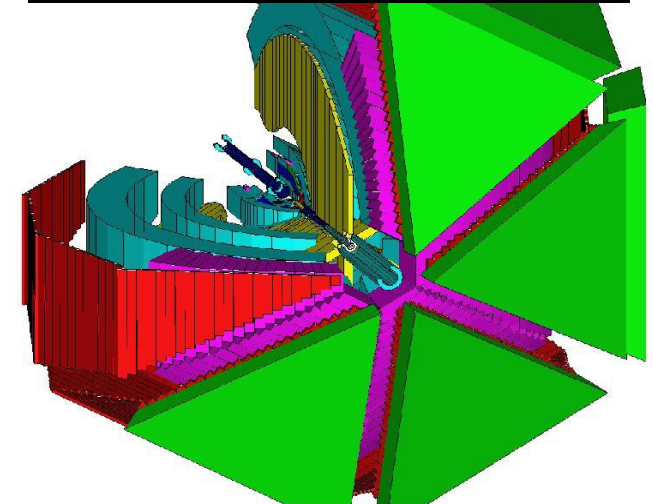
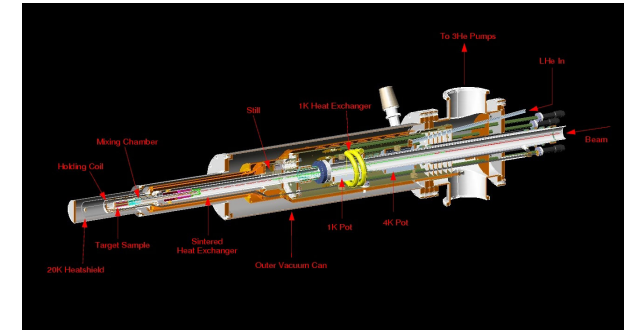
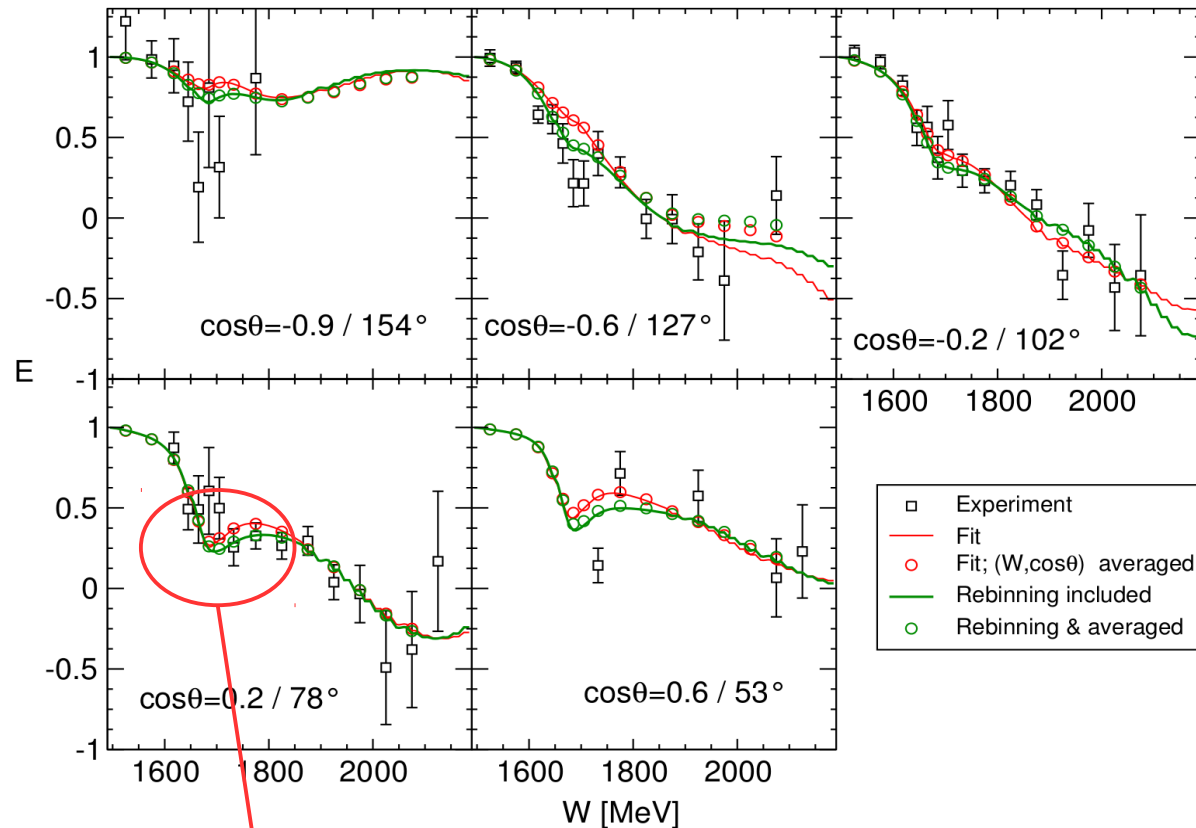


- Allows to trace quantitatively the impact of data sets and observables
- Helpful in design of new measurements
- Correlations allow to assess quality of theory predictions

Resonances and other structures

CLAS/JuBo (M. D., D. Rönchen), Phys.Lett. B755 (2016)

- First-ever measurement of observable E in η photo-production, enabled through the CLAS FROST target



Is this a new narrow baryonic resonance?
→ Conventional explanation in terms of interference effects.

Resonance selection $(K^- p \rightarrow K \Xi)$

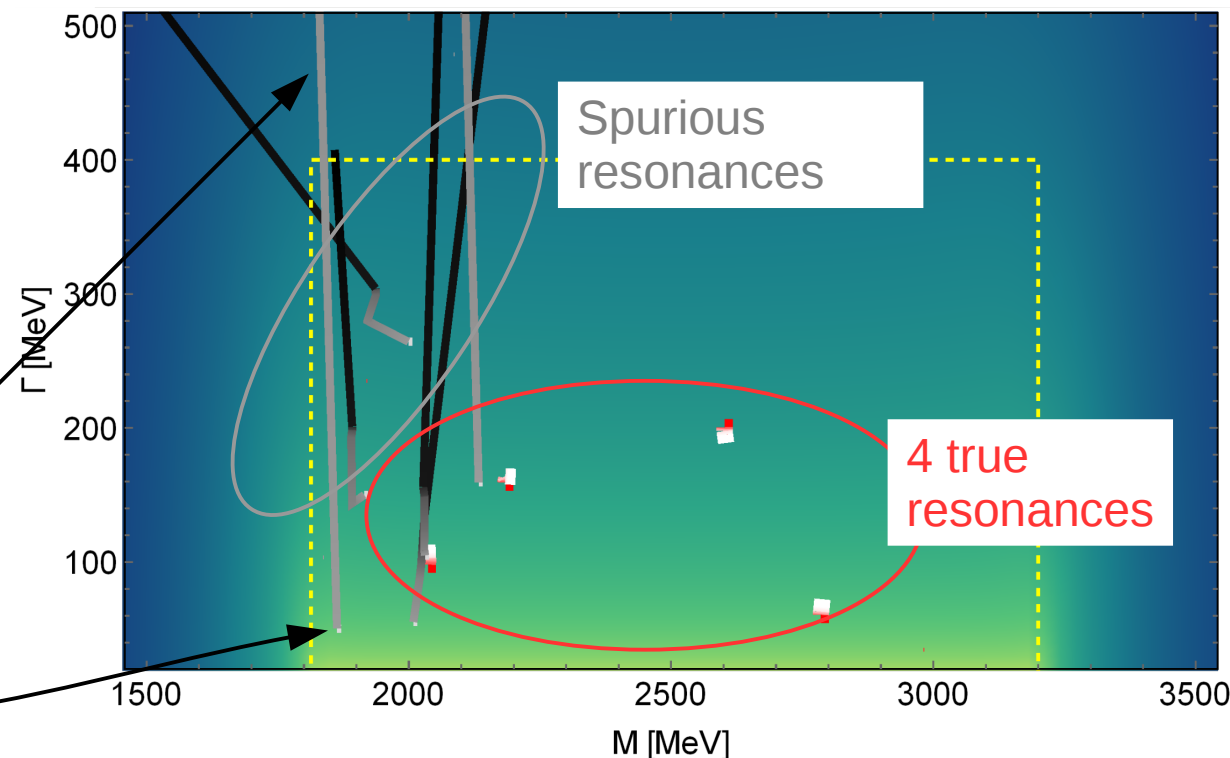
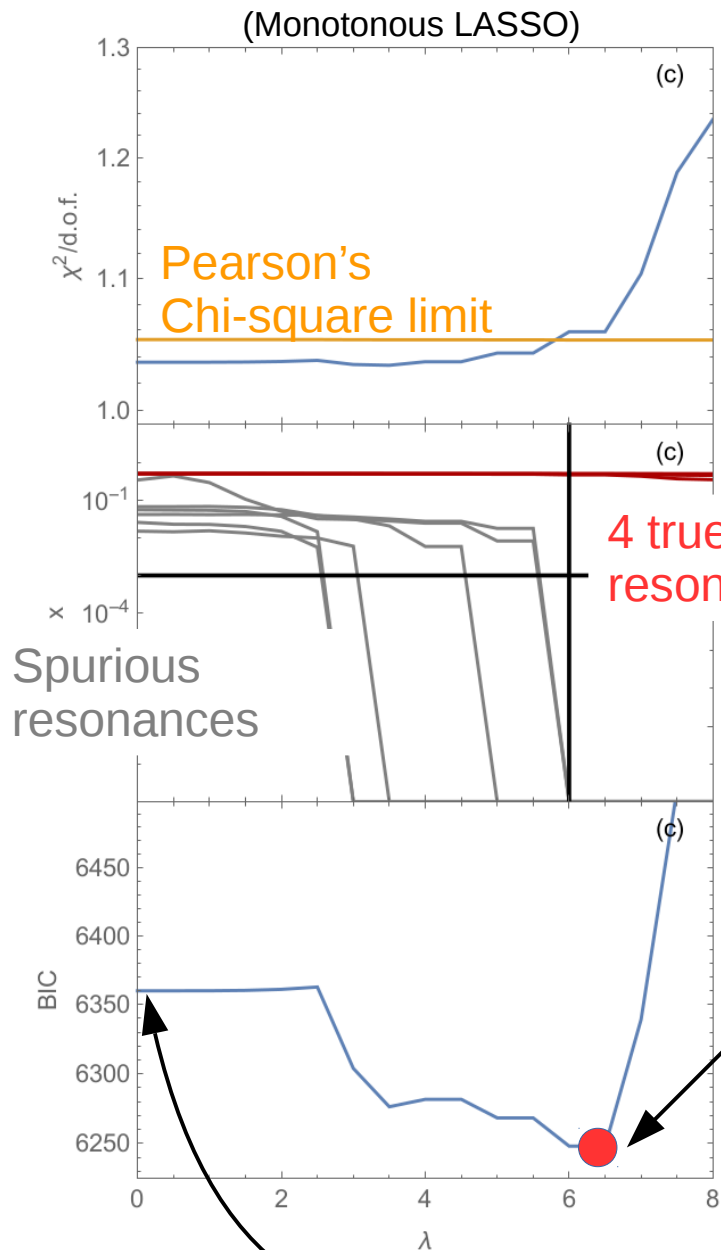
[M.D., J. Landay, H. Haberzettl, M. Mai, K. Nakayama, PRC 2018]

- Ten partial waves; 10 resonances in Ansatz

- Penalty: $\chi^2 = \chi_{\text{stat.}}^2 + P$

$$P(\lambda) = \lambda^5 \sum_{i=0}^9 \int_{m_K+m_\Xi}^{3200} \frac{\partial^2}{\partial W^2} \left(\left| -x_i e^{i\Phi_i} \frac{\Gamma_i}{2(W - M_i + i\frac{\Gamma_i}{2})} \right|^2 \right) dW$$

- LASSO picks the 4 correct ones

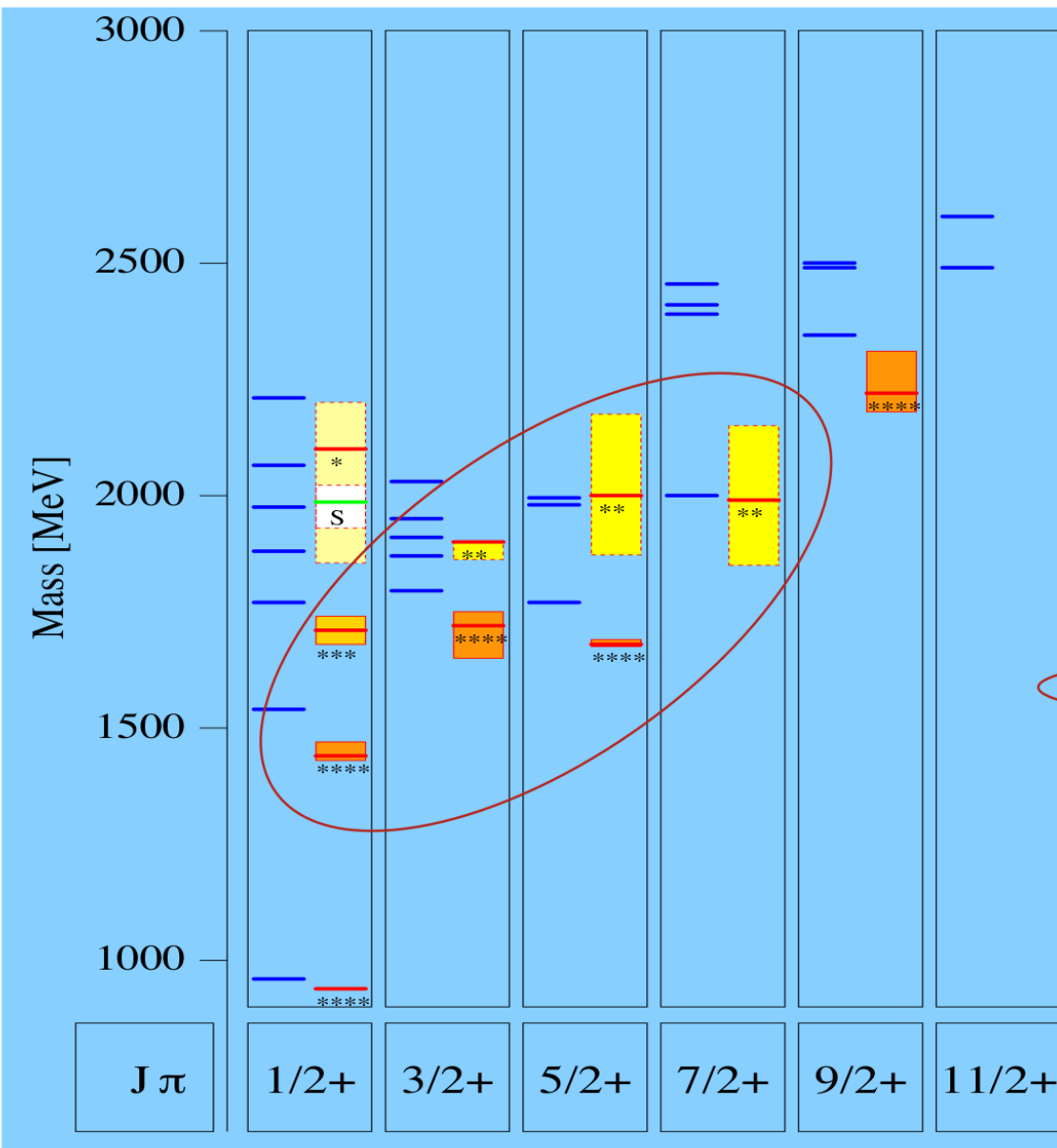


Summary

- Complicated phenomenology of excited baryons through coupled-channel and three-body effects
 - Conceptual progress needed to connect to lattice QCD calculations.
- η photoproduction ideally suitable to study excited baryons
 - Isospin filter, good channel for missing resonances
- Global analyses of pion and photon-induced reactions
 - Jülich-Bonn analysis confirms new states in analysis of photoproduction
- Model selection techniques to extract minimal spectrum of excited baryons

Spare slides

Spectrum of N* resonances



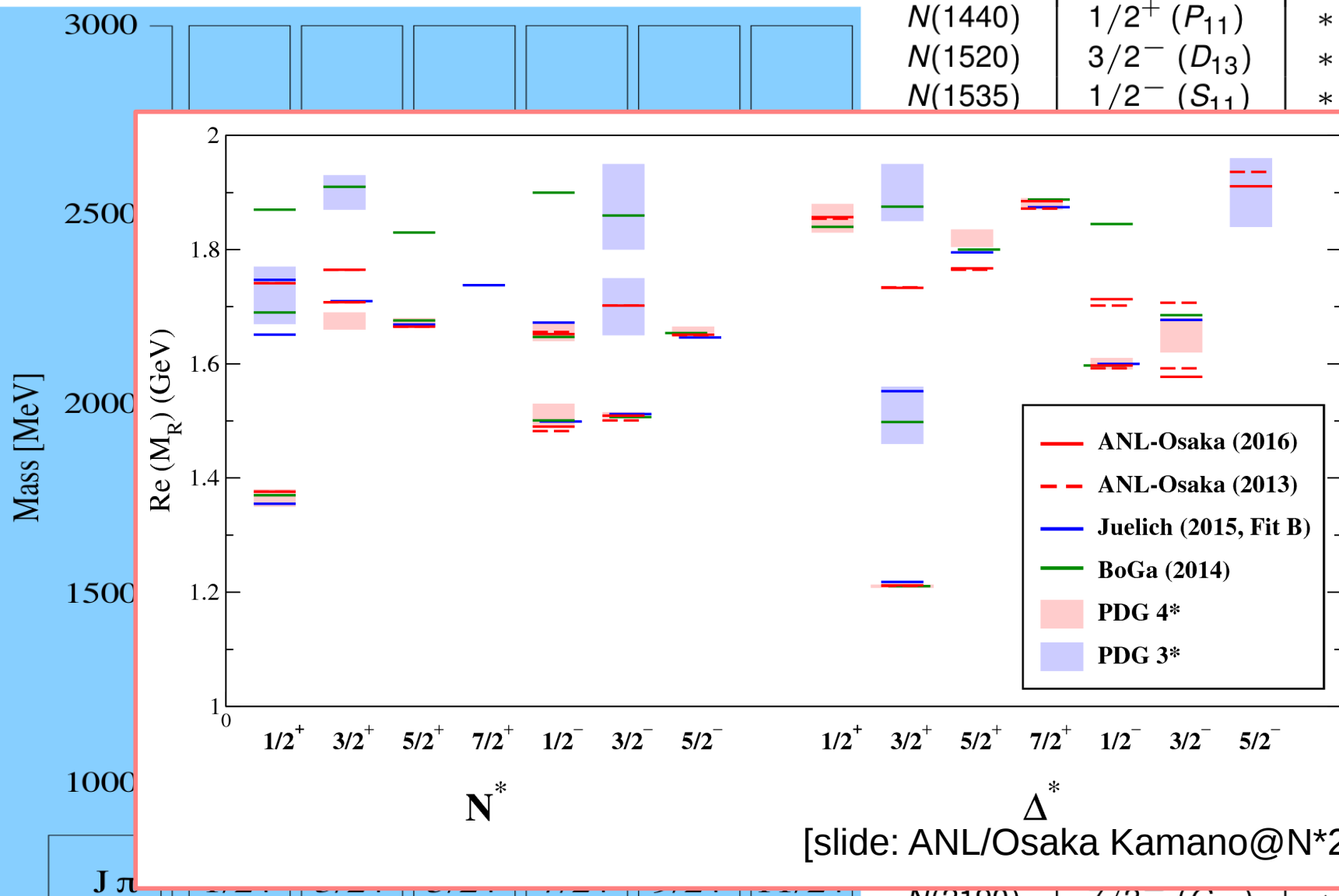
N^*	$J^P (L_{2I,2J})$	2010	2014
$N(1440)$	$1/2^+ (P_{11})$	* * *	* * *
$N(1520)$	$3/2^- (D_{13})$	* * *	* * *
$N(1535)$	$1/2^- (S_{11})$	* * *	* * *
$N(1650)$	$1/2^- (S_{11})$	* * *	* * *
$N(1675)$	$5/2^- (D_{15})$	* * *	* * *
$N(1680)$	$5/2^+ (F_{15})$	* * *	* * *
$N(1685)$			*
$N(1700)$	$3/2^- (D_{13})$	* * *	* * *
$N(1710)$	$1/2^+ (P_{11})$	* * *	* * *
$N(1720)$	$3/2^+ (P_{13})$	* * *	* * *
$N(1860)$	$5/2^+$		**
$N(1875)$	$3/2^-$		* * *
$N(1880)$	$1/2^+$		**
$N(1895)$	$1/2^-$		**
$N(1900)$	$3/2^+ (P_{13})$	**	* * *
$N(1990)$	$7/2^+ (F_{17})$	**	**
$N(2000)$	$5/2^+ (F_{15})$	**	**
$N(2080)$	D_{13}	**	
$N(2090)$	S_{11}	*	
$N(2040)$	$3/2^+$		*
$N(2060)$	$5/2^-$		**
$N(2100)$	$1/2^+ (P_{11})$	*	*
$N(2120)$	$3/2^-$		**
$N(2190)$	$7/2^- (G_{17})$	* * *	* * *
$N(2200)$	D_{15}	**	

- Most new resonances by Bonn-Gatchina group;
- Many from kaon photoproduction

[Slide: V. Crede/Nstar 2017, slight modifications]

[See also: Crede, Roberts, Rep. Prog. Phys. 76 (2013)]

Spectrum of N* resonances



[slide: ANL/Osaka Kamano@N*2017]

N^*	$J^P (L_{2I,2J})$	2010	2014
$N(1440)$	$1/2^+ (P_{11})$	* * *	* * *
$N(1520)$	$3/2^- (D_{13})$	* * *	* * *
$N(1535)$	$1/2^- (S_{11})$	* * *	* * *

$N(2190)$	$1/2^- (G_{17})$	* * *	* * *
$N(2200)$	D_{15}	**	**

- Most new resonances by Bonn-Gatchina group;
- Many from kaon photoproduction

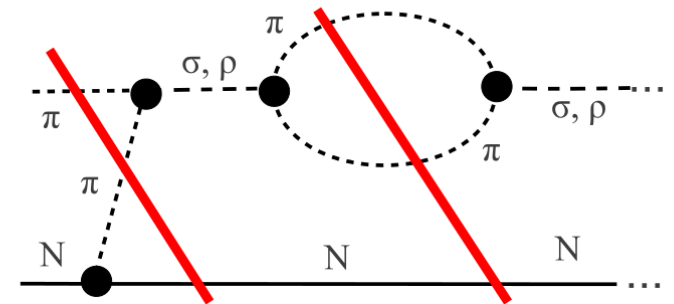
[Slide: V. Crede/Nstar 2017, slight modifications]

[See also: Crede, Roberts, Rep. Prog. Phys. 76 (2013)]

$$S = \mathbb{1} + iT$$

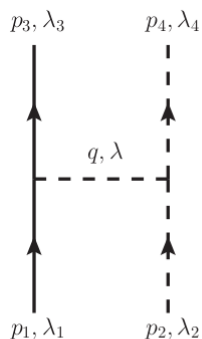
Unitarity: $SS^\dagger = 1 \Leftrightarrow -i(T - T^\dagger) = T T^\dagger$

- 3-body unitarity:
discontinuities from t -channel exchanges
→ Meson exchange from requirements of the S -matrix

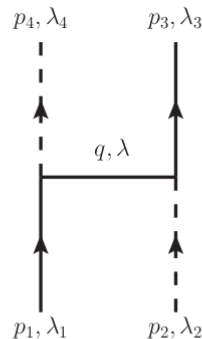


Other cuts

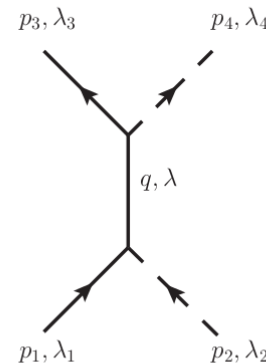
- to approximate left-hand cut → Baryon u -channel exchange
- σ , ρ exchanges from crossing plus analytic continuation.



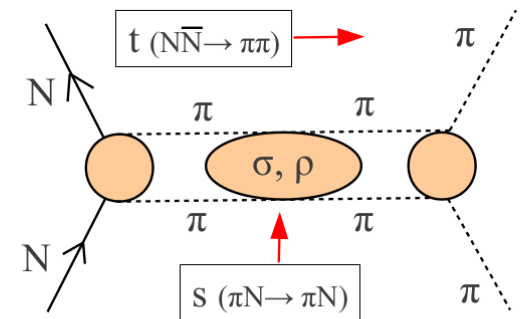
$$\vec{q} = \vec{p}_1 - \vec{p}_3$$



$$\vec{q} = \vec{q}_1 - \vec{p}_4$$

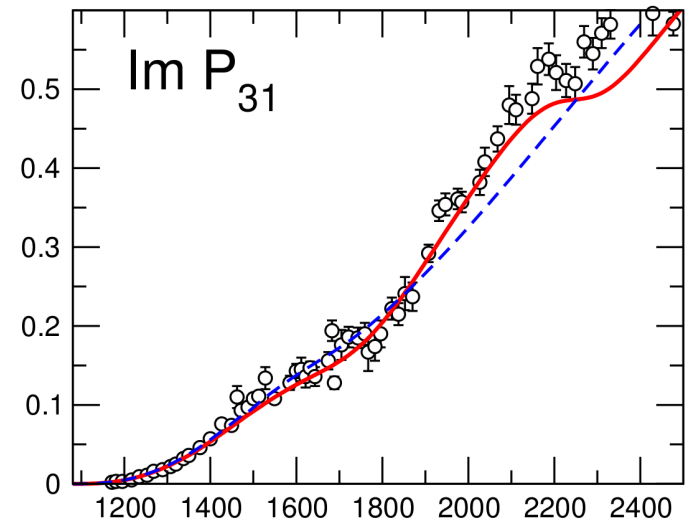
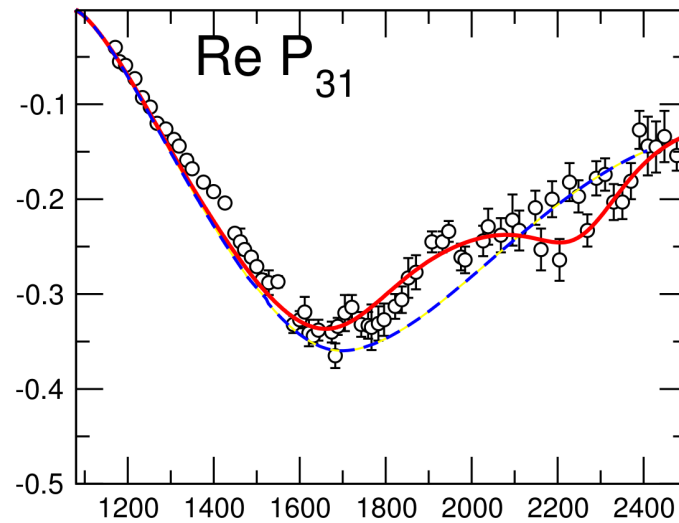


$$\vec{q} = \vec{p}_1 + \vec{p}_2 = 0$$

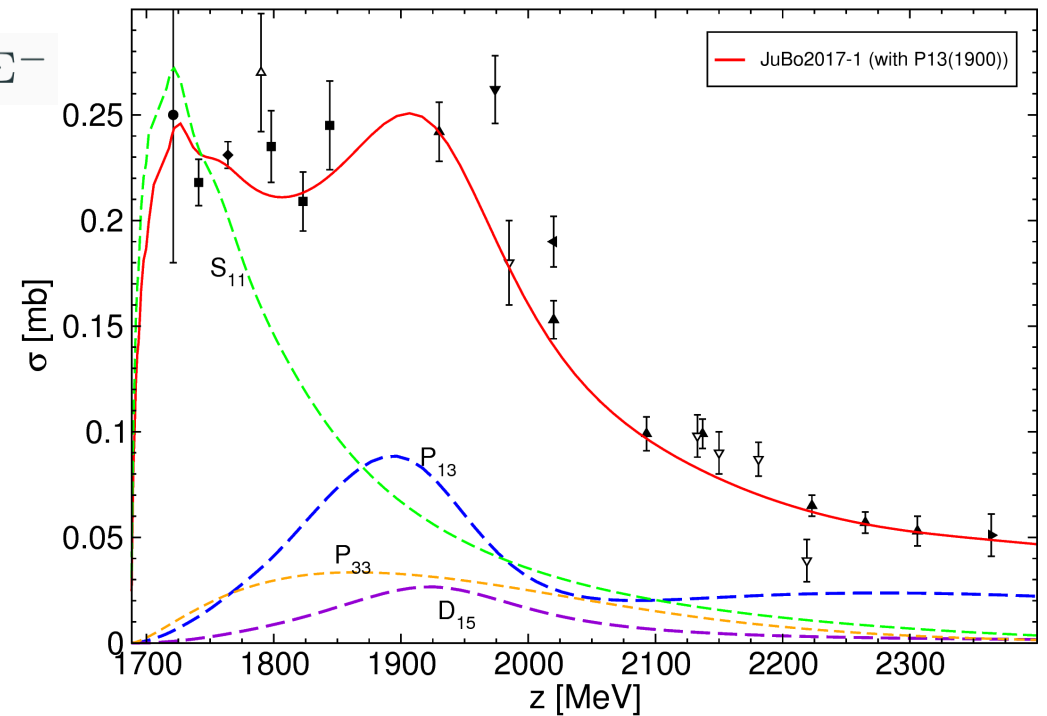


Visible influence of new states

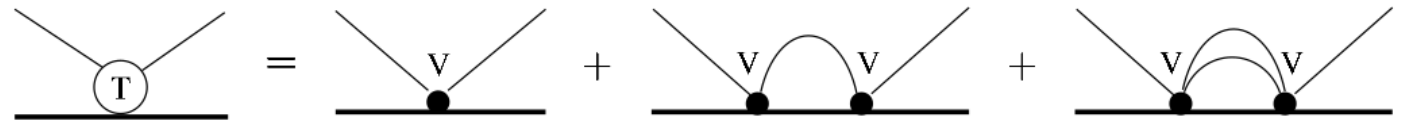
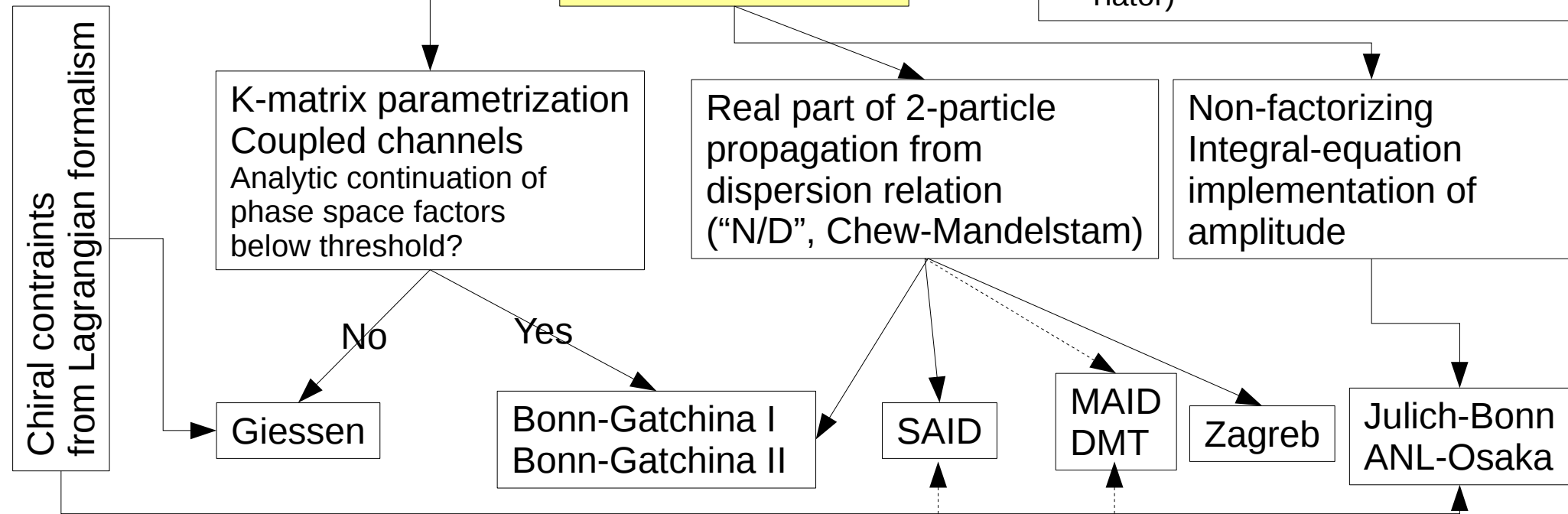
$\Delta(2190)3/2^+$ in πN PW



$N(1900)3/2^+$, $N(2060)5/2^-$ in σ_{tot} in $\pi^- p \rightarrow K^+ \Sigma^-$

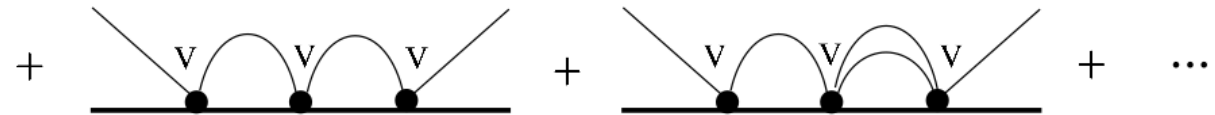
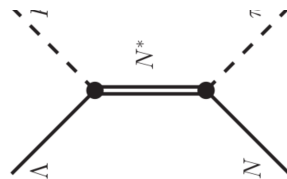


Amplitude parametrization



$$T = V + VGT,$$

Genuine Resonance:

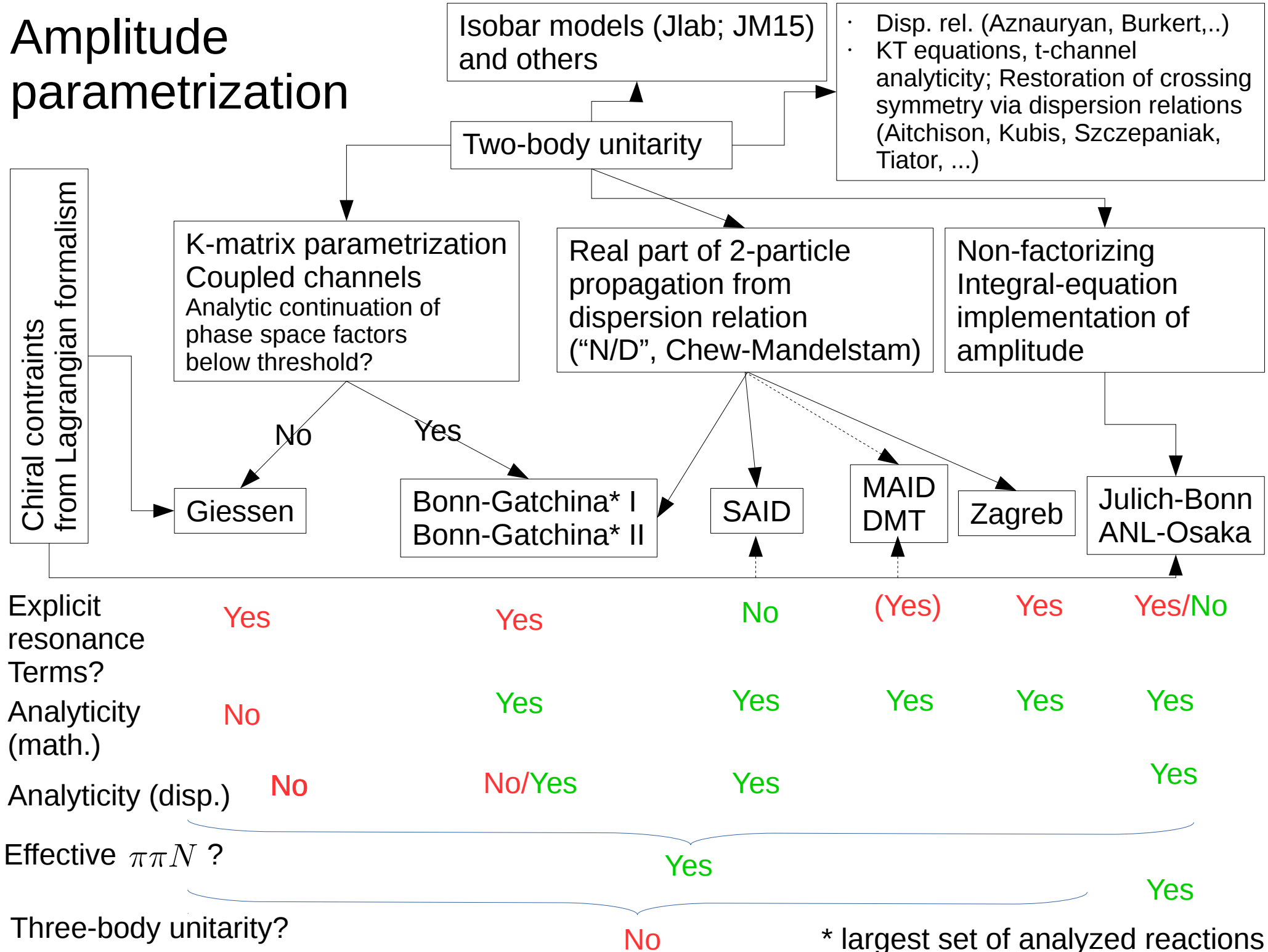


Unitarity loop G:

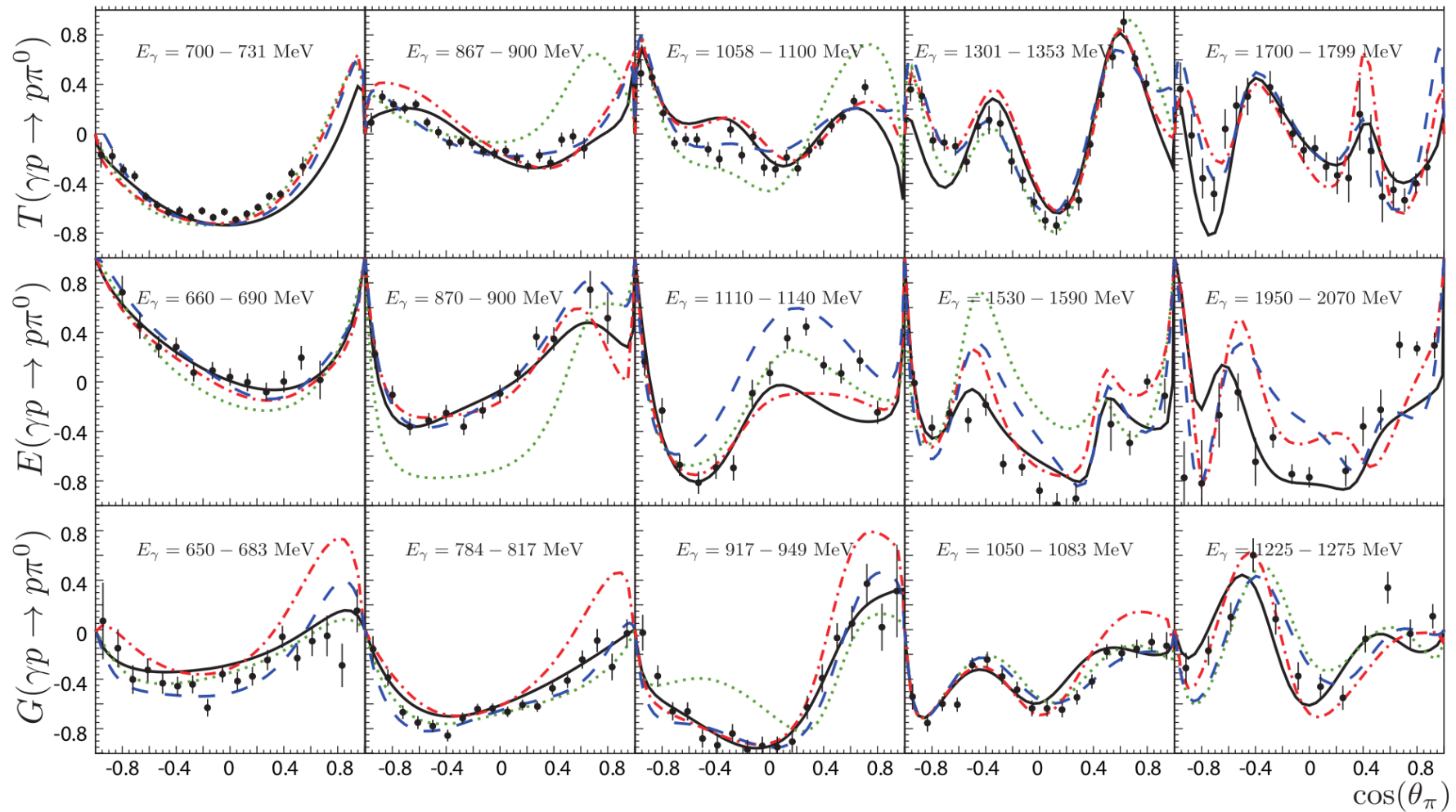
- Re $G \rightarrow 0$: K-matrix
- V point-like: SAID

Integral equation: Julich-Bonn, ANL-Osaka

Amplitude parametrization



Impact of data

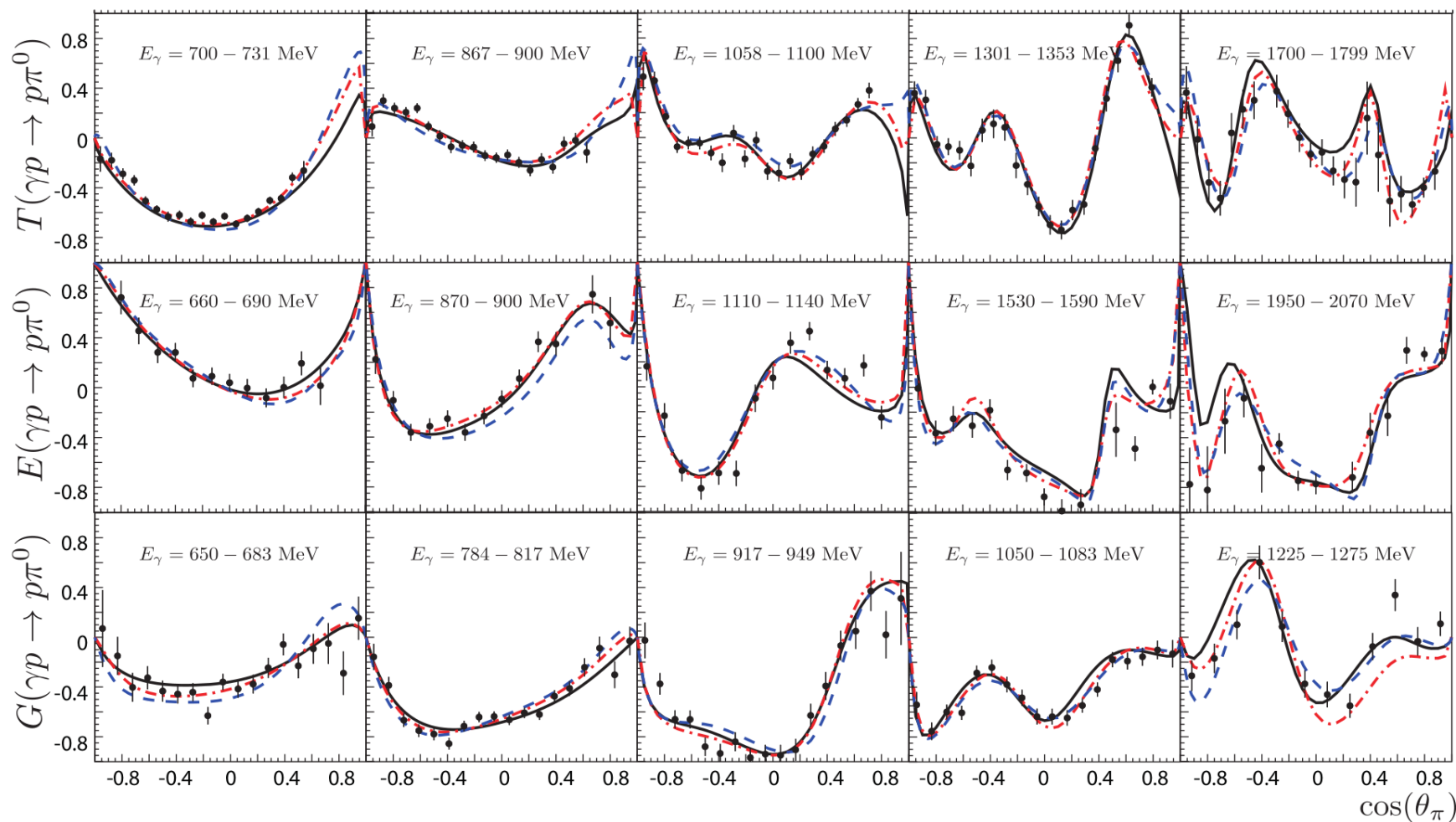


Data: CBELSA/TAPS Collaboration (T : Hartmann et al. PLB 748, 212 (2015) , E : Gottschall et al. PRL 112, 012003 (2014), G : Thiel et al. PRL 109, 102001 (2012), Thiel et al. arXiv:1604.02922)

Predictions: black solid lines: BnGa, red dash-dotted: SAID, blue dashed: JüBo, green dotted: MAID

Impact of new data

EPJA 52, 284 (2016)

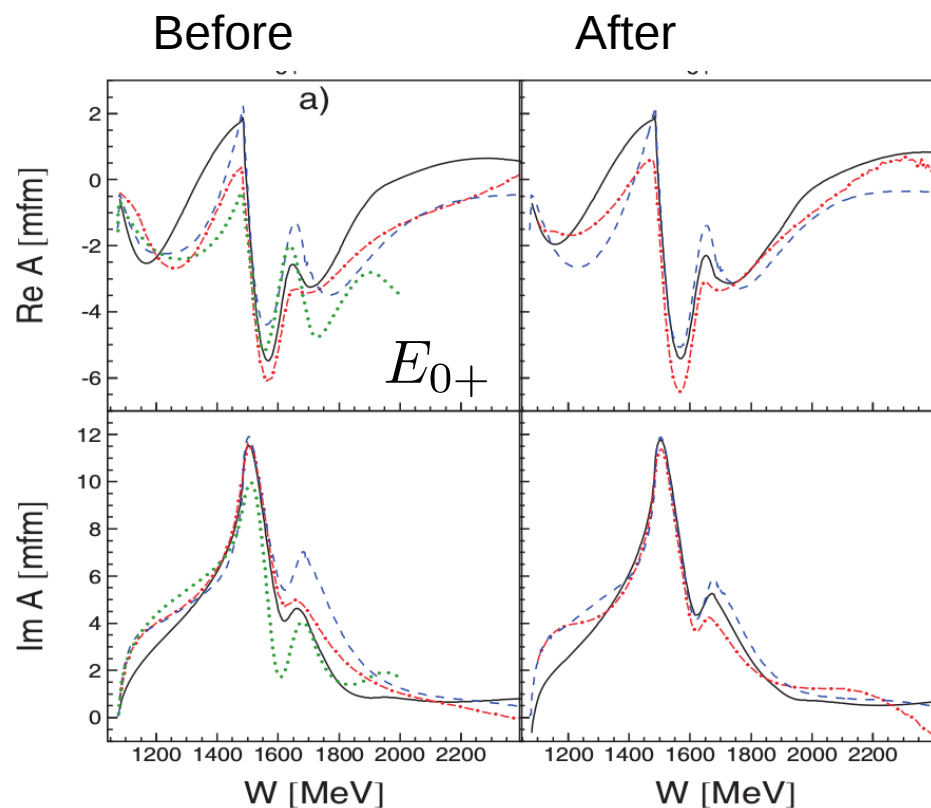


Data: CBELSA/TAPS Collaboration (T : Hartmann et al. PLB 748, 212 (2015) , E : Gottschall et al. PRL 112, 012003 (2014), G : Thiel et al. PRL 109, 102001 (2012), Thiel et al. arXiv:1604.02922)

Fits: black solid lines: BnGa, red dash-dotted: SAID, blue dashed: JüBo

Impact of new data

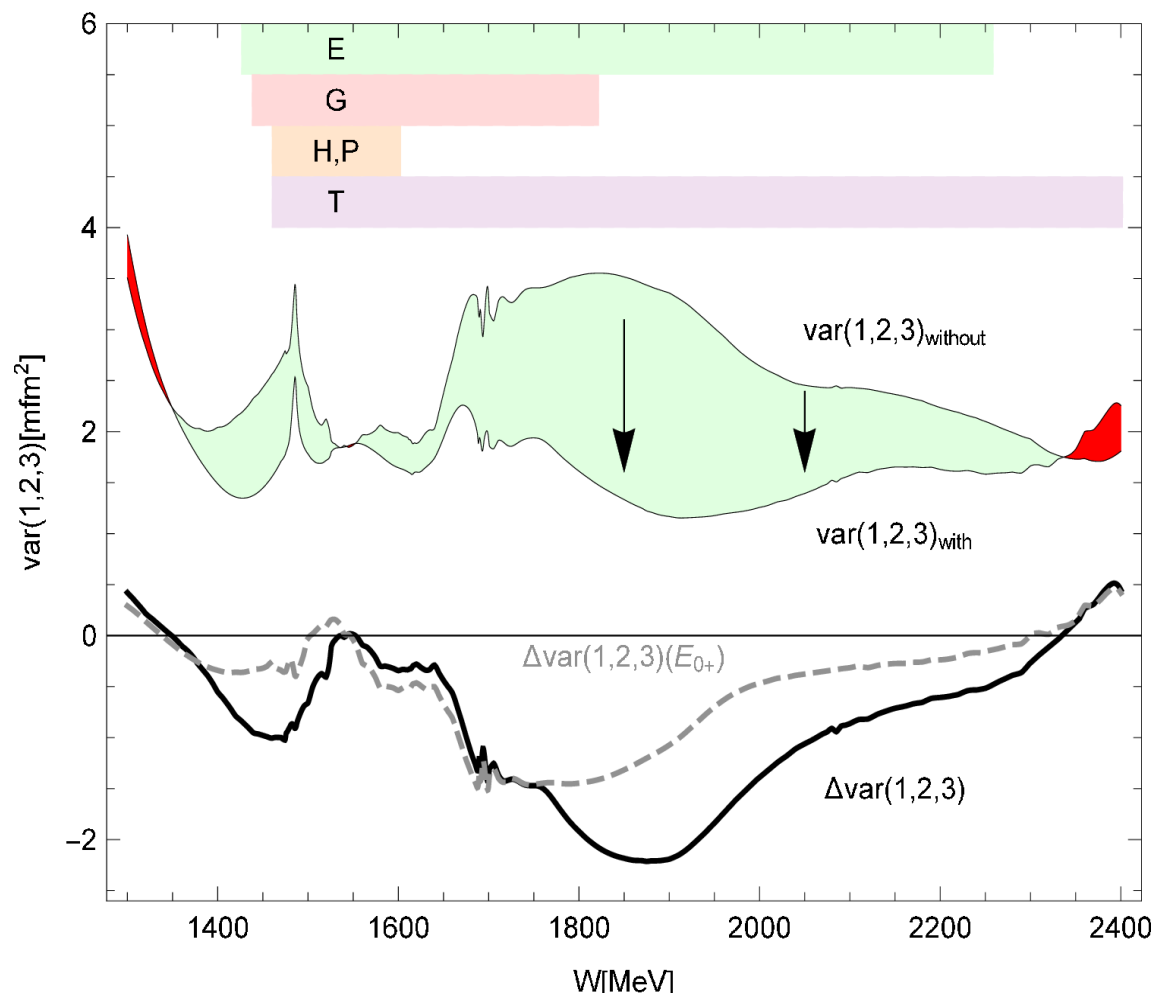
EPJA 52, 284 (2016)



- Multipole solutions approach each other
- Remaining discrepancies

Julich-Bonn, BnGa, SAID

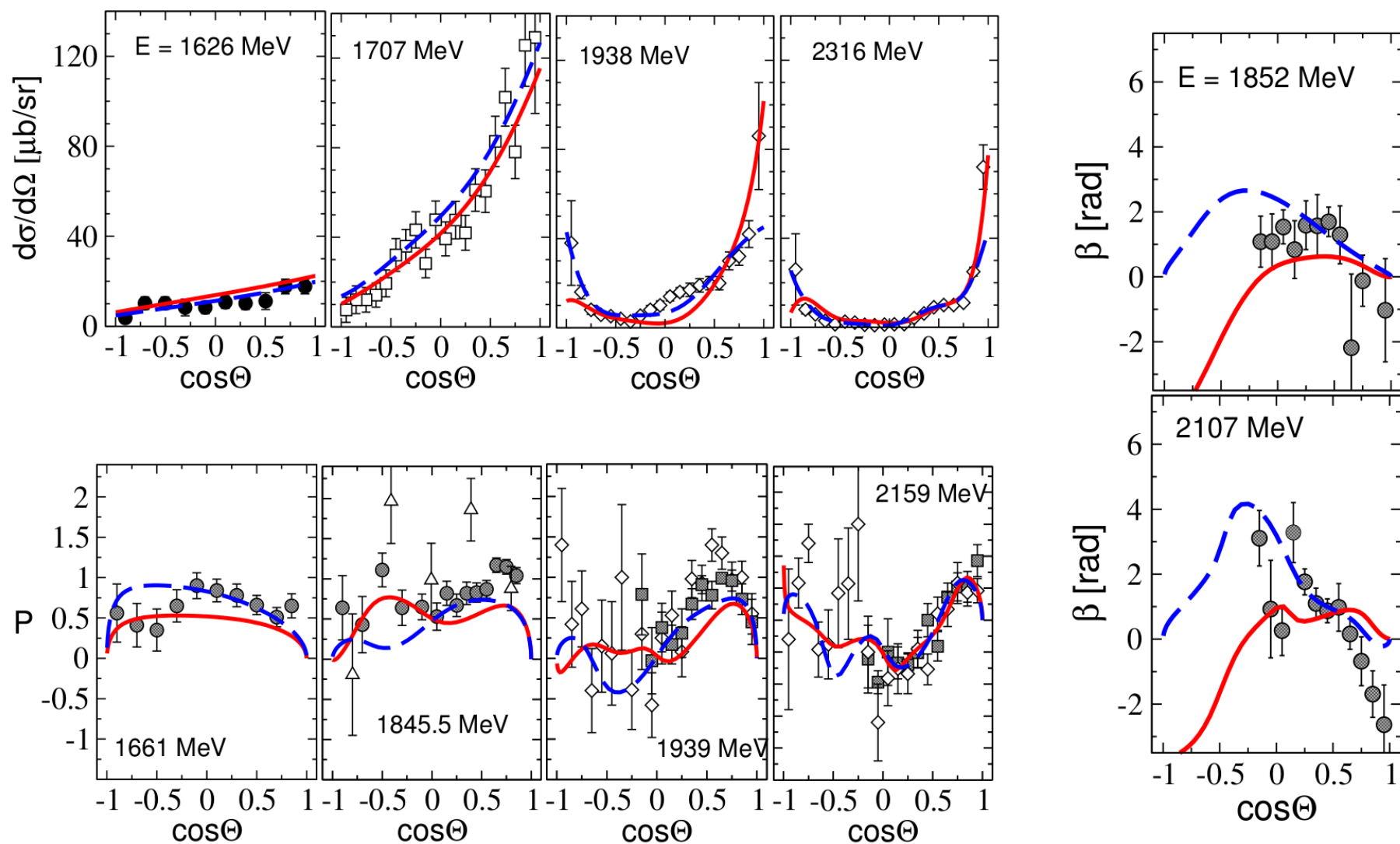
$$\text{var}(1, 2) = \frac{1}{2} \sum_{i=1}^{16} (\mathcal{M}_1(i) - \mathcal{M}_2(i)) (\mathcal{M}_1^*(i) - \mathcal{M}_2^*(i)). \quad (31)$$



Fit to world data on $\pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$ ($\sim 10^5$ exp. points)

[Rönchen, M.D. *et al.*, EPJA 49 (2013)]

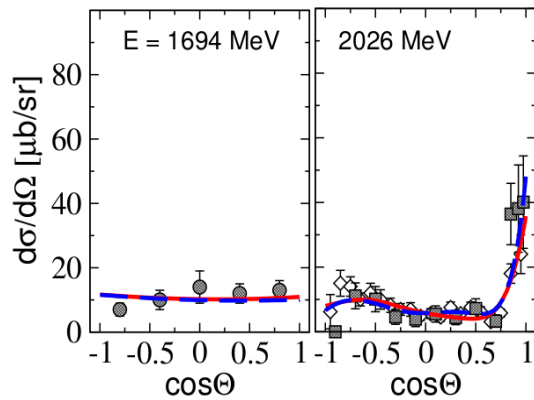
Selected results for $\pi^- p \rightarrow K^0 \Lambda$ [almost complete experiment]



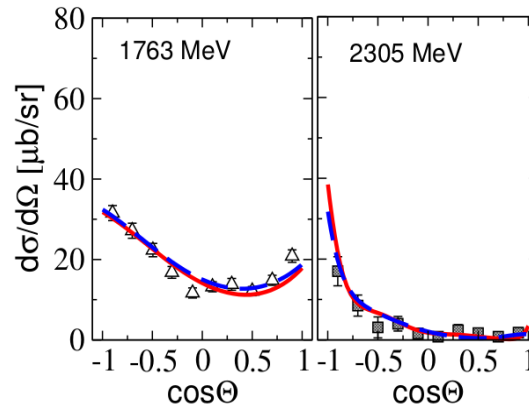
Re-measuring hadron-induced reactions

Fits: D. Rönchen, M.D., et al., EPJ A**49** (2013)

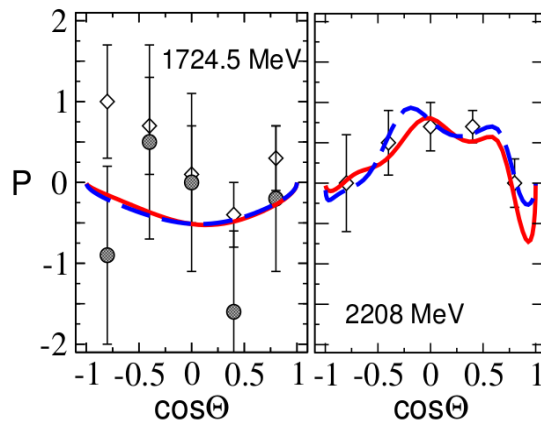
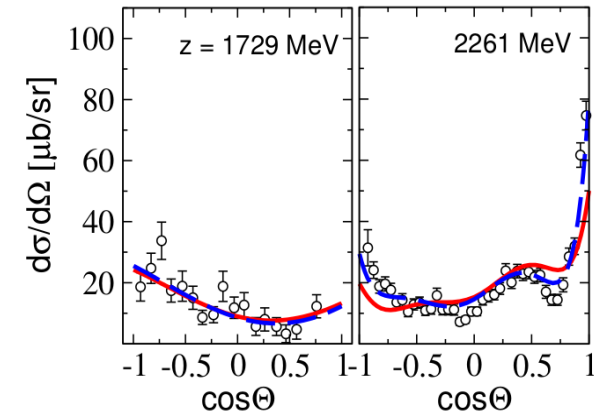
$$\pi^- p \rightarrow K^0 \Sigma^0$$



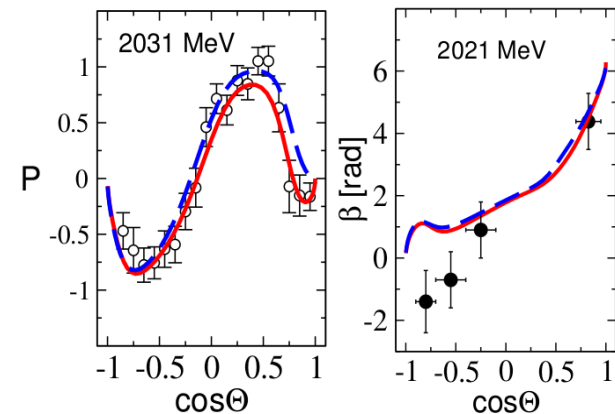
$$\pi^- p \rightarrow K^+ \Sigma^-$$



$$\pi^+ p \rightarrow K^+ \Sigma^+$$



No polarization data!



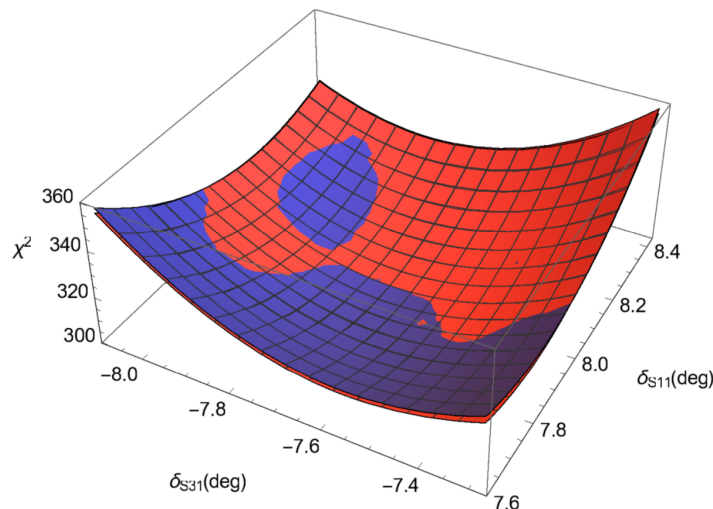
→ *Physics Opportunities with meson beams,*

Briscoe, M.D., Haberzettl, Manley, Naruki, Strakovsky, Swanson, EPJ A**51** (2015)

Toward Data-driven Analyses

[M.D., Revier, Rönchen, Workman, arXiv:1603.07265, PRC 2016]

- Multi-channel analyses to detect faint resonance signals
- All groups use GW/SAID partial waves for $\pi N \rightarrow \pi N$
 - The chi-square obtained in fits to single-energy solutions is not related to chi-square of a fit to data → **Statistical interpretation of resonance signals difficult.**
- Provide online covariance matrices etc. to allow other groups to perform *correlated chi-square* fits.



Slight adaptation of their code allows other groups to obtain a χ^2 (almost) as if they fitted to $\pi N \rightarrow \pi N$ directly.

$$\chi^2(\mathbf{A}) = \chi^2(\hat{\mathbf{A}}) + (\mathbf{A} - \hat{\mathbf{A}})^T \hat{\Sigma}^{-1} (\mathbf{A} - \hat{\mathbf{A}}) + \mathcal{O}(\mathbf{A} - \hat{\mathbf{A}})^3$$

Covariance matrices etc. can be downloaded on the SAID and JPAC web pages.

Amplitude reconstruction from complete experiments and truncated partial-wave expansions

[Workman, Tiator, Wunderlich, M.D.,
H. Haberzettl, PRC (2017)]

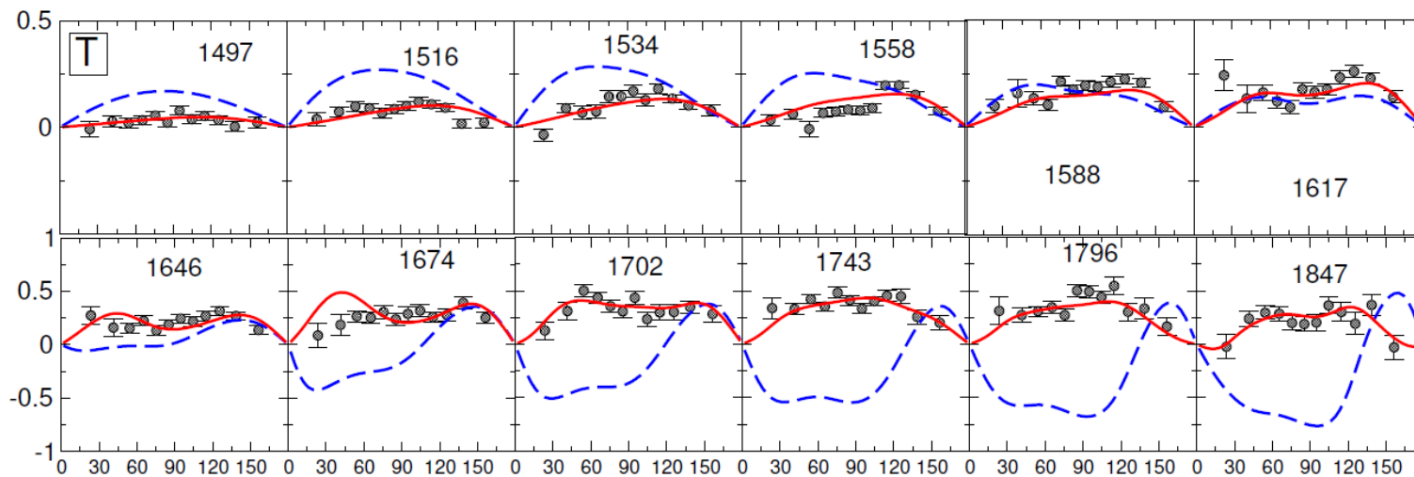
How do complete experiment and truncated partial wave complete experiment compare.
Depending on which partial-wave content is admitted in the amplitude?

Set	Included Partial Waves	CEA	TPWA	Complete Sets for TPWA
1	$L = 0$ (E_{0+})	1(1)	1(1)1	$I[1]$
2	$J = 1/2$ (E_{0+}, M_{1-})	4(4)	4(4)1 4(3)2	$I[1], \check{P}[1], \check{C}_x[1], \check{C}_z[1]$ $I[2], \check{P}[1], \check{C}_x[1]$
3	$L = 0, 1$ (E_{0+}, M_{1-}, E_{1+})	6(6)	6(6)1 6(4)2 6(3)3	$I[1], \check{\Sigma}[1], \check{T}[1], \check{P}[1], \check{F}[1], \check{G}[1]$ $I[2], \check{\Sigma}[1], \check{T}[2], \check{P}[1]$ $I[3], \check{\Sigma}[1], \check{T}[2]$
4	$L = 0, 1$ ($E_{0+}, M_{1-}, E_{1+}, M_{1+}$) full set of 4 S, P wave multipoles	†	8(5)2 8(4)3	TPWA at 1 angle not possible $I[2], \check{\Sigma}[1], \check{T}[2], \check{P}[2], \check{F}[1]$ $I[3], \check{\Sigma}[1], \check{F}[2], \check{H}[2]$
5	$L = 0, 1, 2$ ($E_{0+}, M_{1-}, E_{1+}, E_{2-}$)	8(8)	8(8)1 8(4)2 8(3)3	$I[1], \check{\Sigma}[1], \check{T}[1], \check{P}[1], \check{F}[1], \check{G}[1], \check{C}_x[1], \check{O}_x[1]$ $I[2], \check{\Sigma}[2], \check{T}[2], \check{P}[2]$ $I[3], \check{\Sigma}[2], \check{T}[3]$
6	$J \leq 3/2$ ($E_{0+}, M_{1-}, E_{1+}, M_{1+}, E_{2-}, M_{2-}$)	†	12(5)3 12(4)4	TPWA at 1 or 2 angles not possible $I[3], \check{\Sigma}[2], \check{T}[3], \check{P}[2], \check{F}[2]$ $I[4], \check{\Sigma}[2], \check{F}[3], \check{H}[3]$
7	$L = 0, 1, 2$ (E_{0+}, \dots, M_{2+}) full set of 8 S, P, D wave multipoles	†	16(6)3 16(5)4 16(4)5	TPWA at 1 or 2 angles not possible $I[3], \check{\Sigma}[3], \check{T}[3], \check{P}[3], \check{F}[3], \check{G}[1]$ $I[4], \check{\Sigma}[3], \check{T}[3], \check{P}[3], \check{F}[3]$ $I[5], \check{\Sigma}[3], \check{F}[4], \check{H}[4]$

Order:
of different measurements,
of different observables
of different angles

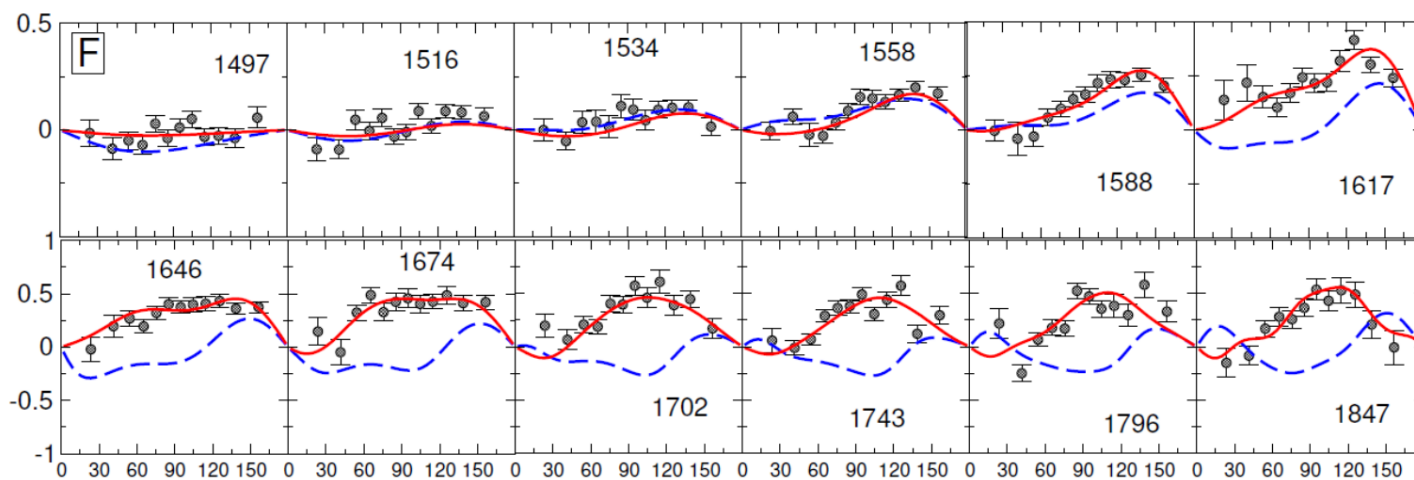
Four are enough!

Data: Akondi *et al.* (A2 at MAMI) PRL 113, 102001 (2014)



--- prediction
— fit

Beam	Target	Recoil
0	$+y$	0
0	$-y$	0



Beam	Target	Recoil
+1	$+x$	0
-1	$+x$	0

