The LPM effect in QCD revisited

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Yacine Mehtar-Tani (BNL)

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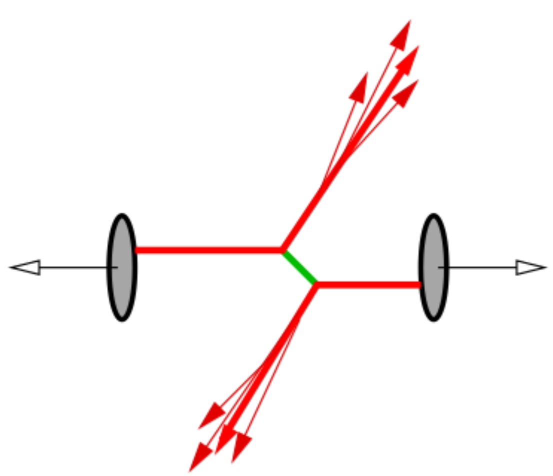
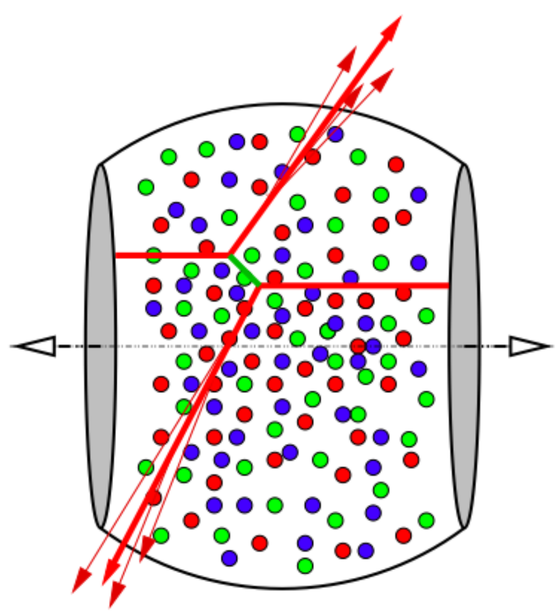
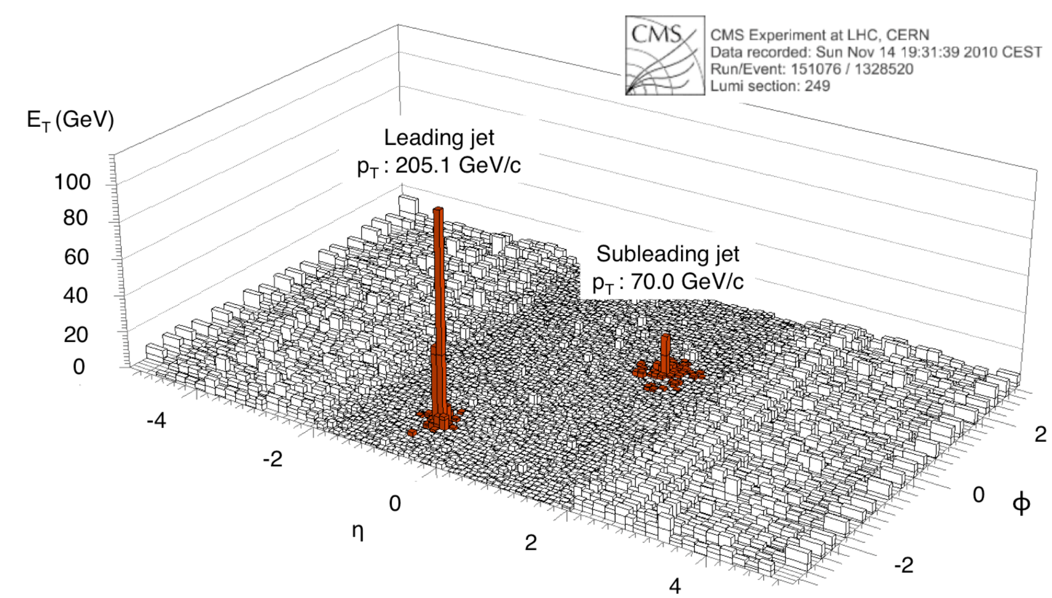
Motivation

- High energy jets traversing a deconfined matter lose energy via medium-induced radiation → Jet quenching







- Jet quenching provides a quantitative measure of the transport properties of QGP in heavy ion collisions (RHIC, LHC)

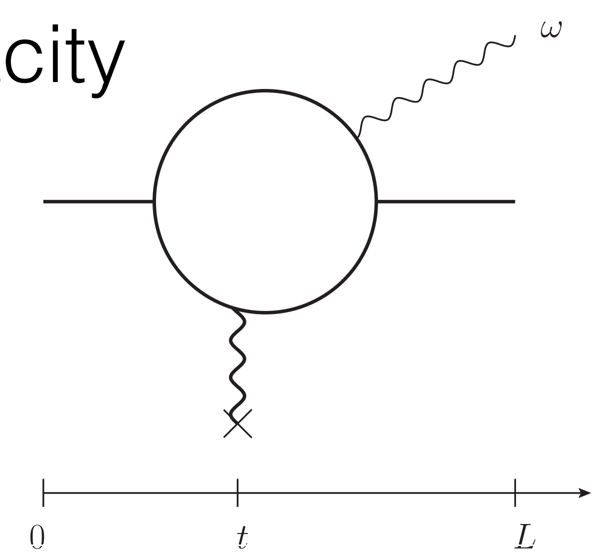
See talk by A. Majumder

Motivation

- Elementary process: in-medium gluon bremsstrahlung
- Two (orthogonal) analytic approximations in the literature (implemented in various MC event generators)

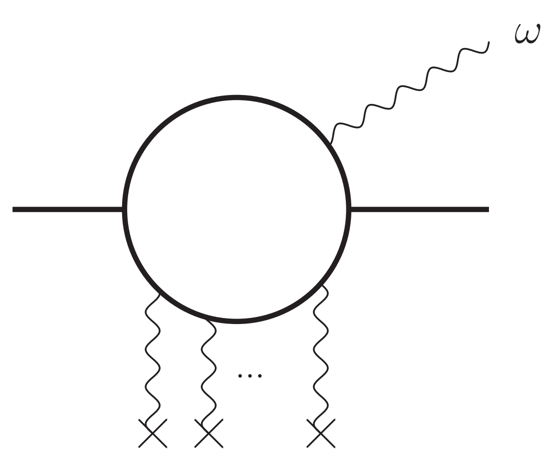
1. Dilute medium: single-hard scattering (Opacity expansion, Higher-Twist)

   Gyulassy-Levai-Vitev (2000) Guo, Wang (2000)



2. Dense medium: multiple-soft scattering

   Baier, Dokshitzer, Mueller, Peigné, Schiff (1996)
   Zakharov (1997)

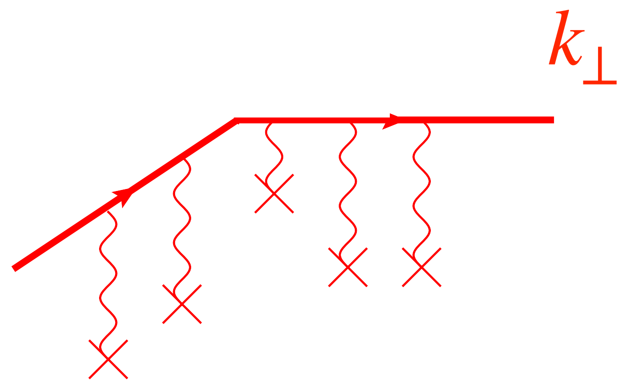


→ This talk: unify these two approaches

Momentum broadening and $\hat{q}$

- Jet constituents traversing the plasma may suffer frequent soft elastic collisions. To leading order the diffusion coefficient that characterized the jet-plasma coupling reads

$$\hat{q} \equiv \frac{\mathrm{d}\langle k_T^2 \rangle_{typ}}{\mathrm{d}t} \sim \alpha_s^2\, C_R\, n\, \ln \frac{Q^2}{m_D^2} \sim \alpha_s^2 T^3$$



soft multiple interactions

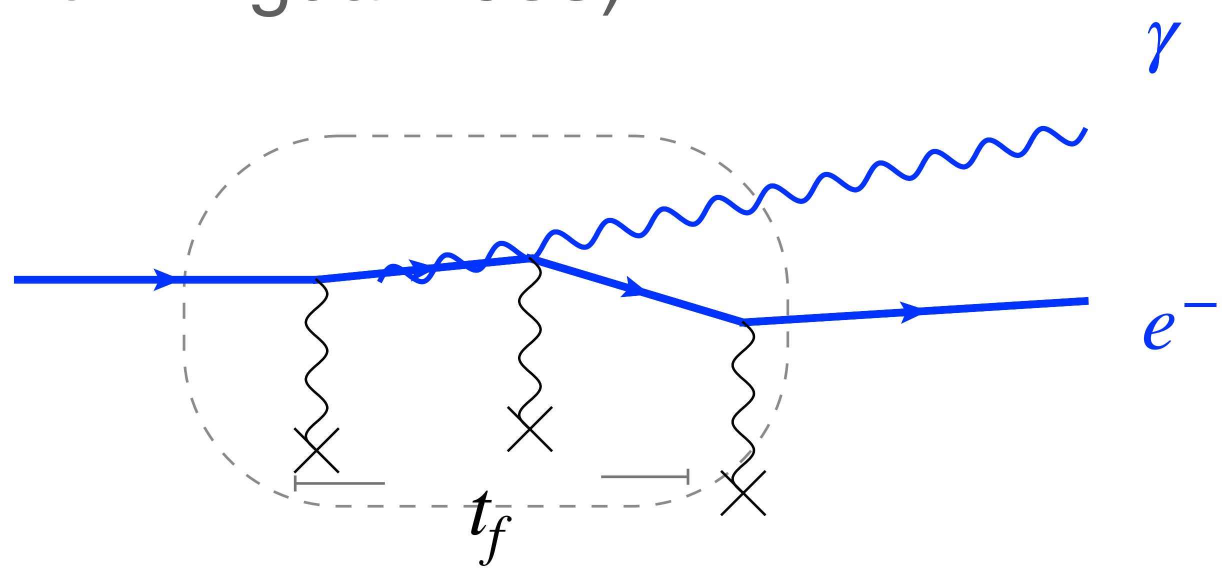
- At weak coupling $\alpha_s \ll 1$ (kinetic description):

$$1/m_D \ll \ell_{mfp} \ll L$$

- Note that $Q^2 \sim \hat{q}L \gg \Lambda_{QCD}^2$ which implies that the jet quenching parameter stays under perturbative control for large medium length L

The LPM effect on the back of the envelop

- The energy spectrum of photons caused by the propagation of a relativistic charge in a medium is suppressed due to coherence effects (Landau-Pomeranchuk-Migdal 1953)



- Coherence length: during the quantum mechanical formation time N_{coh} scattering centers act coherently reducing the radiation spectrum

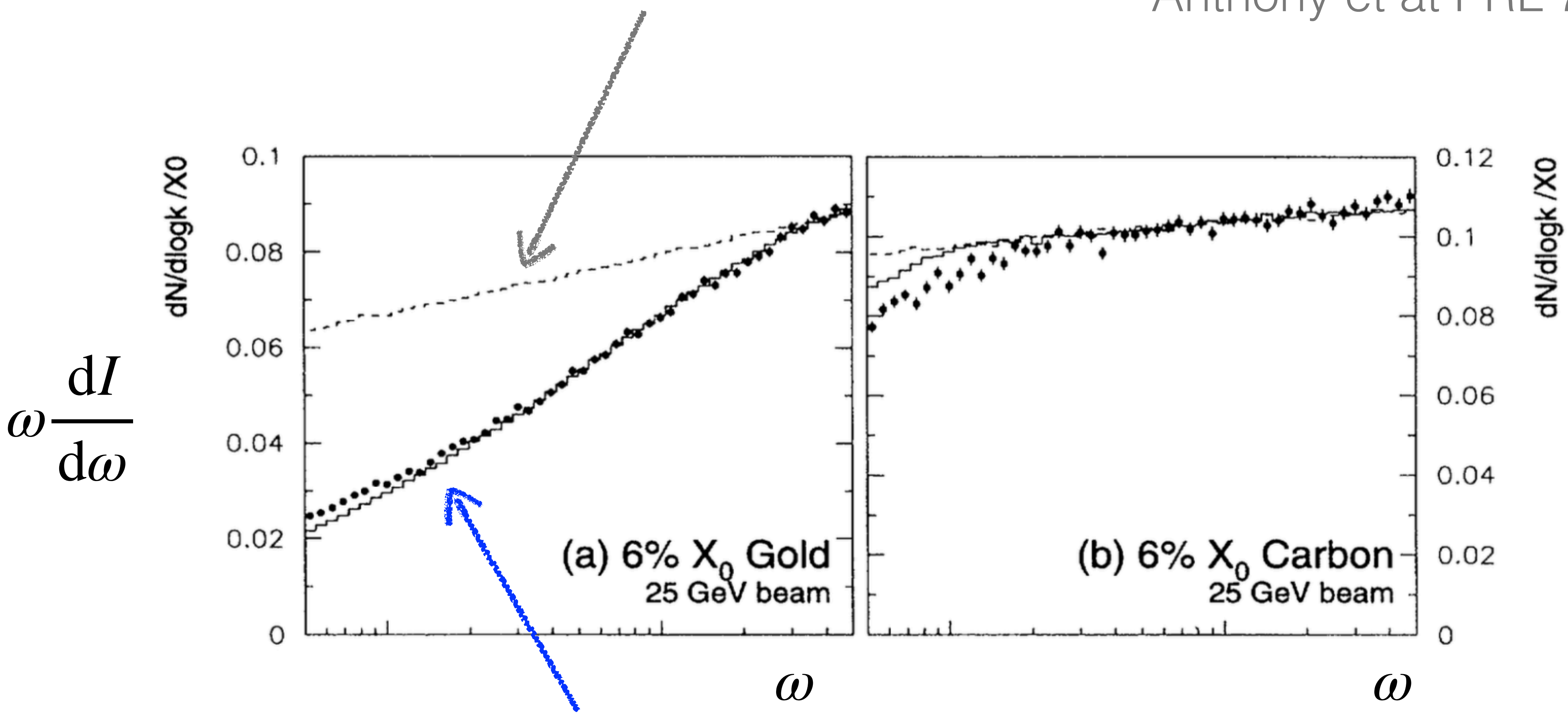
$$\omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_e N_{eff} \sim \alpha_e \frac{N_{scatt}}{N_{coh}} \sim \alpha_e \frac{L}{t_f(\omega)}$$

The LPM effect on the back of the envelop

- The LPM effect was observed at SLAC in 1995

BH (incoherent radiation)

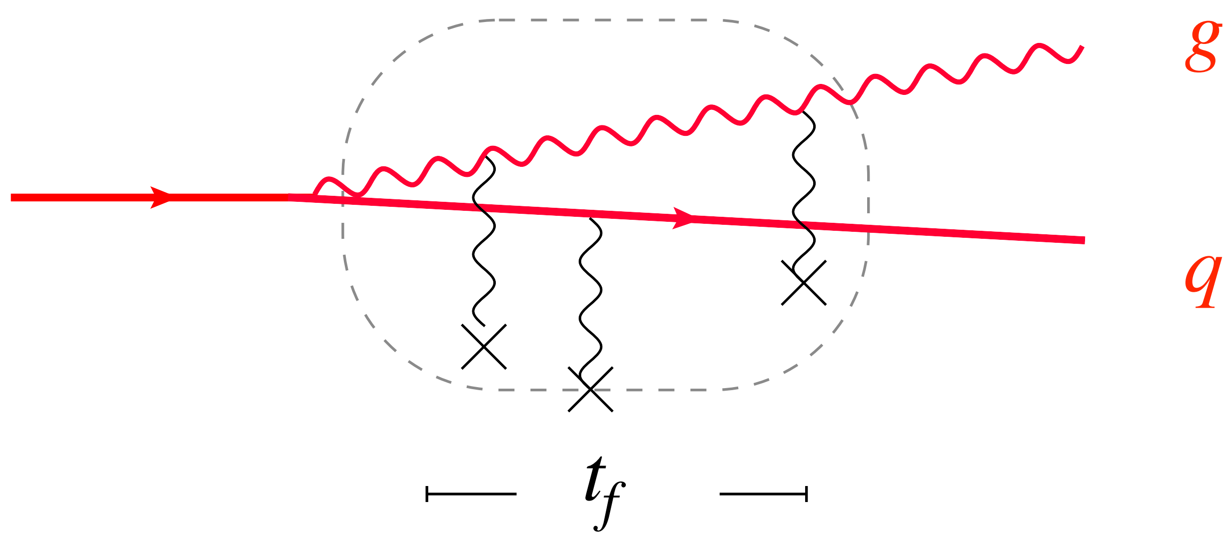
Anthony et at PRL 75 (1995)



LPM suppression (coherent radiation)

The LPM effect on the back of the envelop

- Same effect in QCD except the gluon interacts with the plasma and suffers “brownian kicks”



$$t_f(\omega) = \frac{\omega}{k_\perp^2}$$

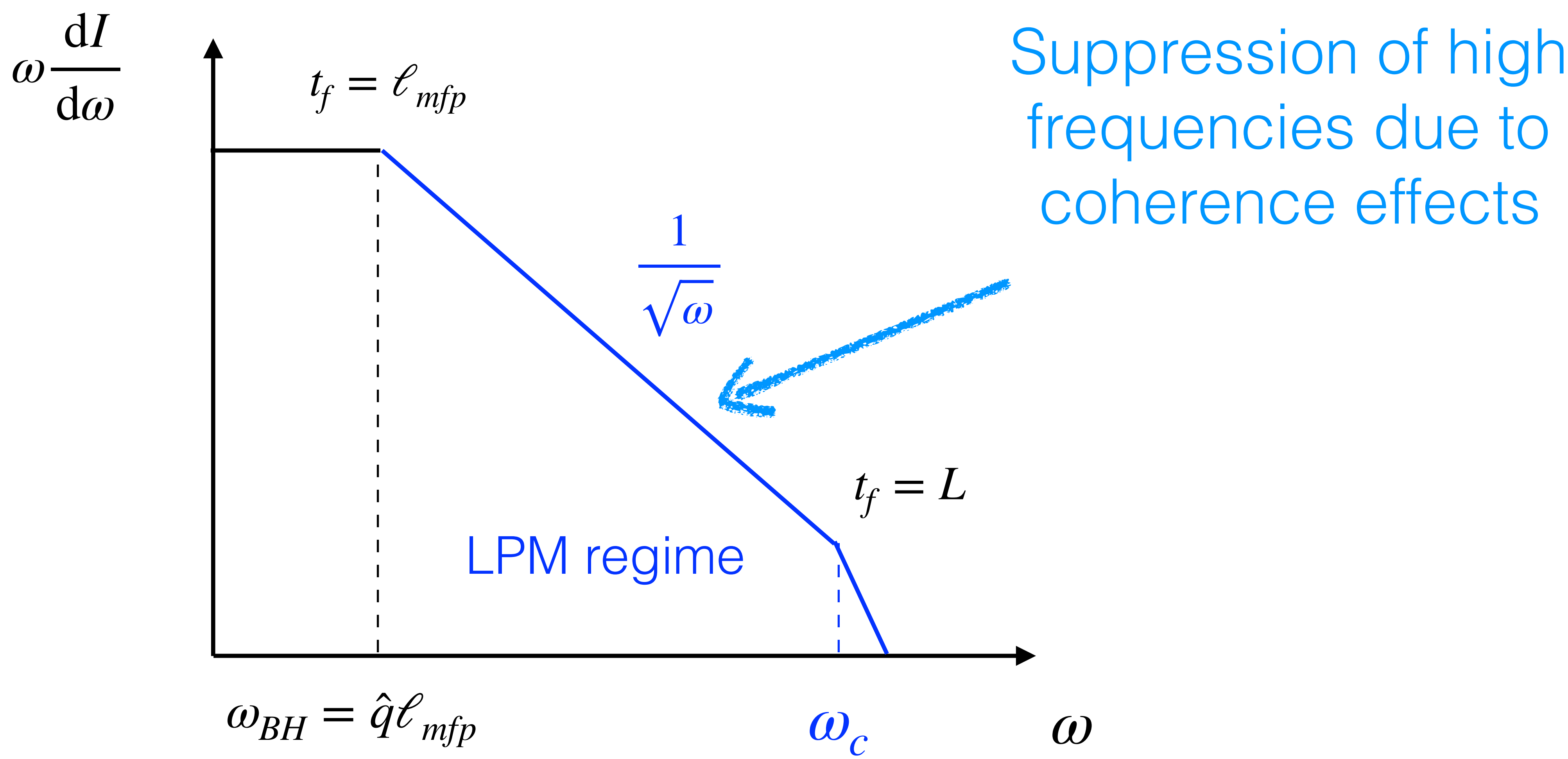
$$k_\perp^2 \sim \hat{q}\, t_f$$

- In QCD the spectrum is suppressed in the UV

$$t_f(\omega) = \sqrt{\frac{\omega}{\hat{q}}} \quad \text{and} \quad \omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_s \sqrt{\frac{\omega}{\hat{q}}}\, L \propto \frac{1}{\sqrt{\omega}}$$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

The LPM effect on the back of the envelop



- Maximum radiation frequency: $\omega_c = \hat{q}L^2$
- Minimum radiation angle (no mass singularity): $\theta_c = \frac{1}{\sqrt{\hat{q}L^3}}$

A model for the medium

- Static scattering centers

  Gyulassy-Wang (1992)
  Gyulassy-Levai-Vitev (2000)

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp} \equiv \frac{g^4 n}{(q_\perp^2 + \mu^2)^2}$$

- Thermal medium (HTL)

  Aurenche-Gelis-Zakaret (2000)

$$\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2 q_\perp} \equiv \frac{g^2 m_D^2 T}{q_\perp^2 (q_\perp^2 + \mu^2)}$$

- Large momentum transfer is given by the 2 to 2 QCD matrix element:

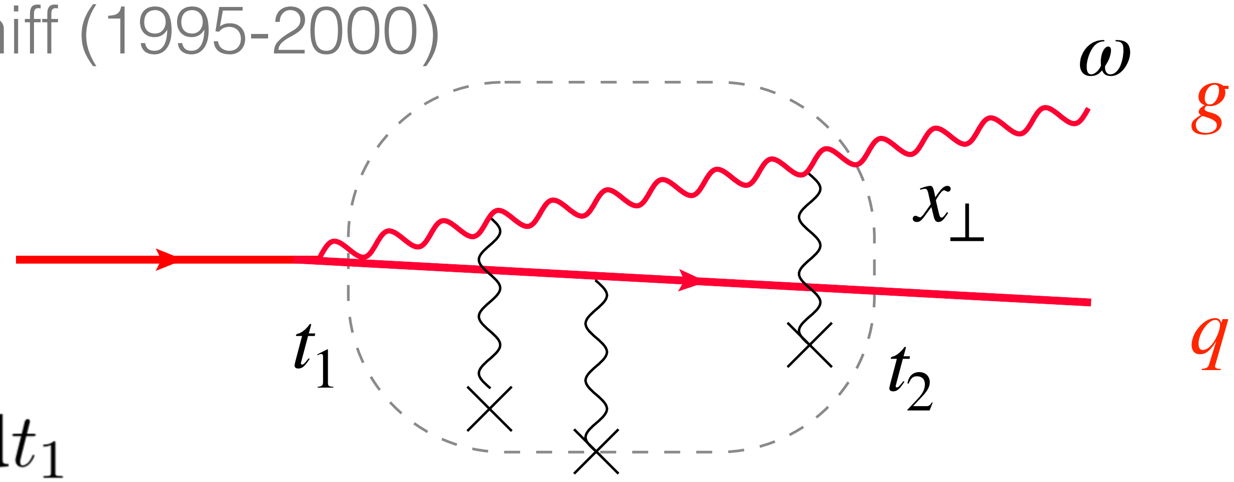
$$1/q_\perp^4 \qquad \text{for} \qquad q_\perp \gg \mu$$

⇒ Large Coulomb logarithm in the perturbative regime

$$k_\perp^2 \sim L \int^{Q^2} \mathrm{d}q_\perp q_\perp^2 \frac{\mathrm{d}\sigma}{\mathrm{d}q_\perp} \propto nL \ln \frac{Q^2}{\mu^2}$$

Medium-induced gluon spectrum

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000)
Zakharov (1996)



$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{\alpha_s C_R}{\omega^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1 \times \boldsymbol{\partial}_x \cdot \boldsymbol{\partial}_y \left[\mathcal{K}(\boldsymbol{x}, t_2|\boldsymbol{y}, t_1) - \mathcal{K}_0(\boldsymbol{x}, t_2|\boldsymbol{y}, t_1)\right]_{\boldsymbol{x}=\boldsymbol{y}=\boldsymbol{0}}$$

- The Green's function K obeys a Schrödinger equation

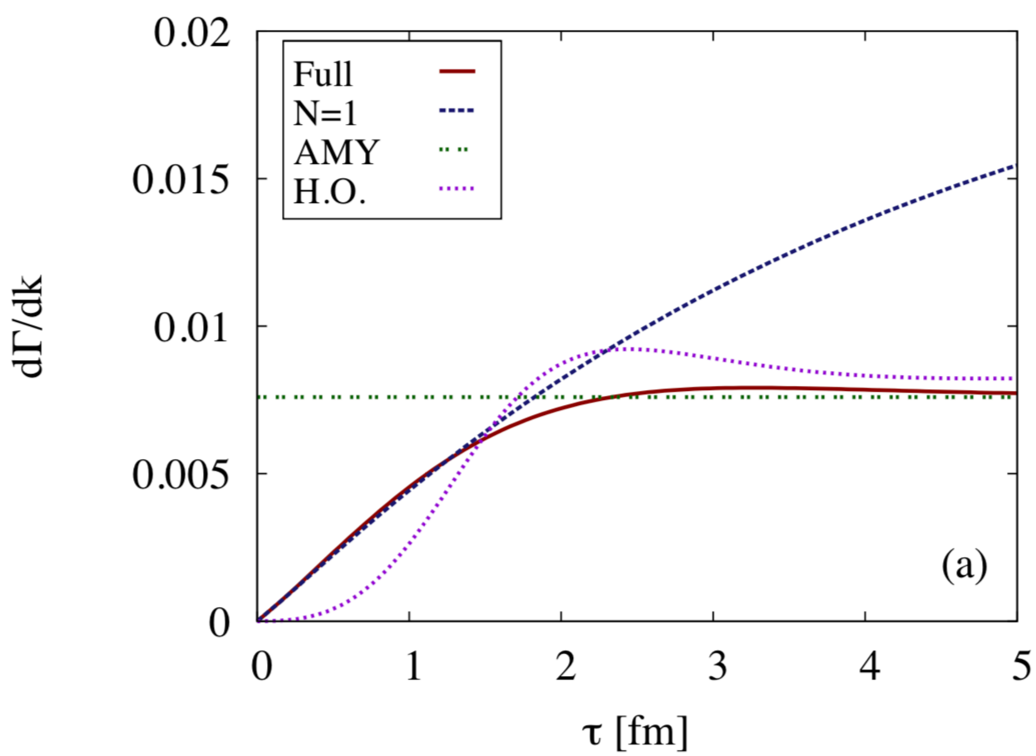
$$\left[i\frac{\partial}{\partial t} + \frac{\boldsymbol{\partial}^2}{2\omega} + i\sigma(\boldsymbol{x})\right] \mathcal{K}(\boldsymbol{x}, t|\boldsymbol{y}, t_1) = i\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t - t_1)$$

- where the imaginary potential is given by

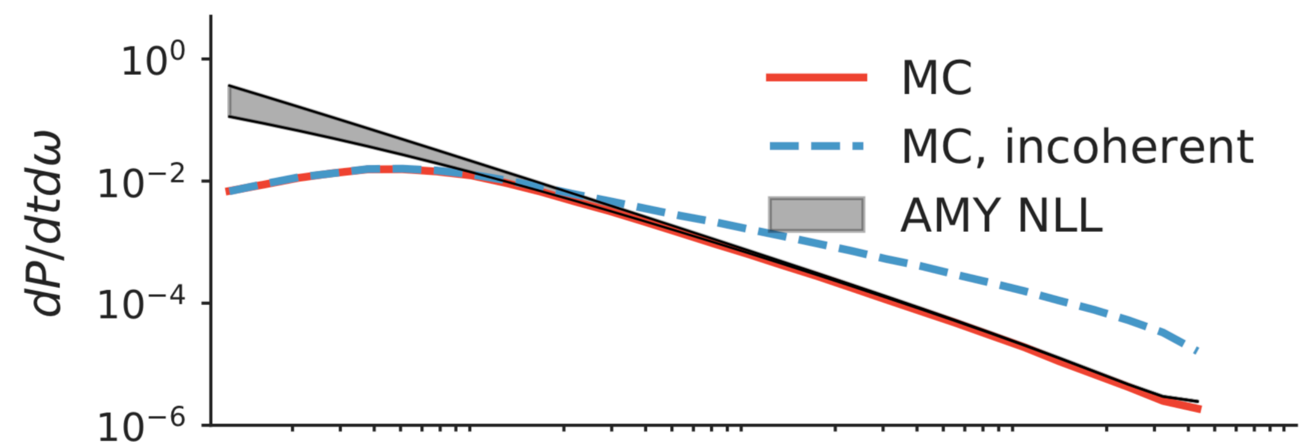
$$\sigma(\boldsymbol{x}, t) = N_c \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2} \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2\boldsymbol{q}} \left(1 - \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right) \quad \sim x_\perp^2 \left(\ln \frac{1}{x_\perp^2 \mu^2} + O(x_\perp^2 \mu^2)\right)$$

Medium-induced gluon spectrum

- Difficult to solve. Numerical solutions



Caron-Huot and Gale (2010)



Ke , Xu, Bass (2018)

- Analytic limits:
  - N=1 opacity (GLV) dilute medium or hard radiation
  - Harmonic Oscillator (HO) (BDMPS) dense medium $\sigma(x_\perp) \sim x_\perp^2$
- This talk: opacity expansion around HO to account for both regimes

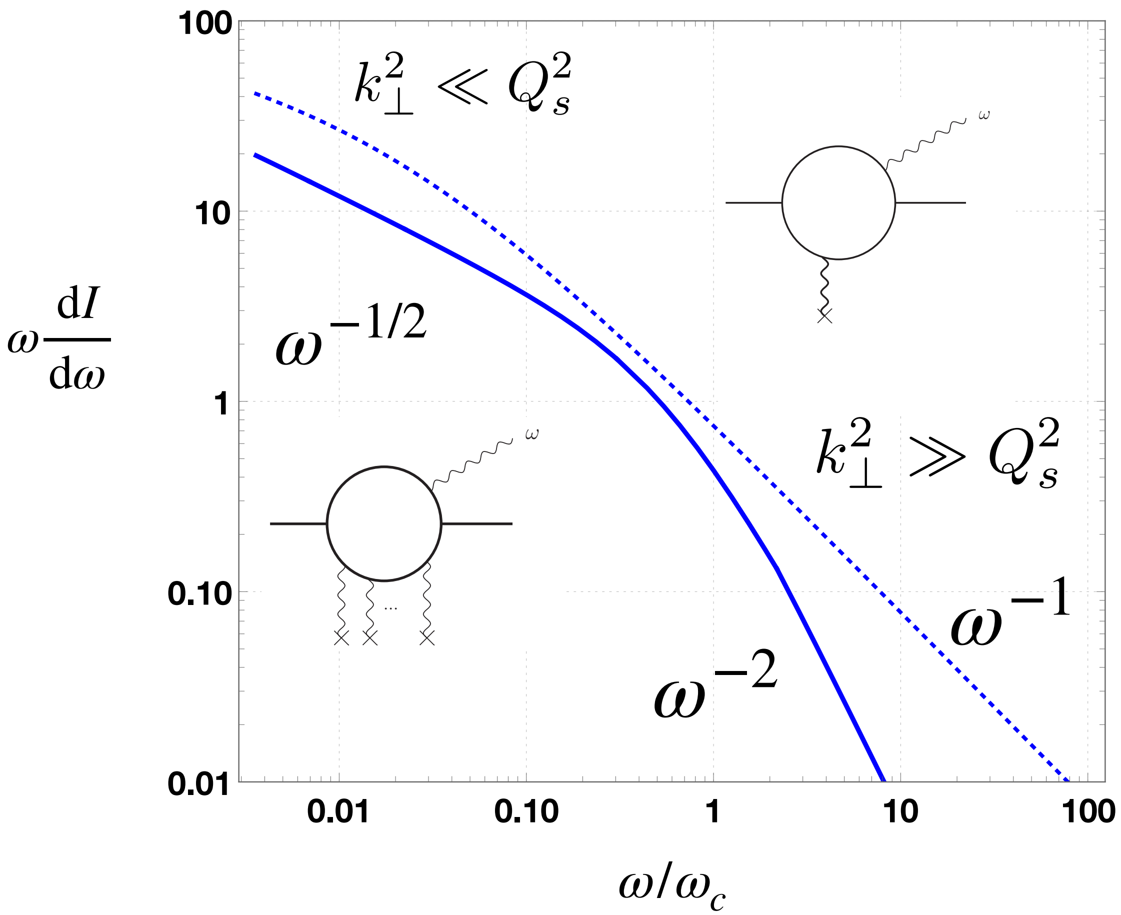
Multiple-soft scattering (BDMPS (1997))

- Strong LPM suppression due to multiple soft scattering

$$n = 0.1\, GeV^{-3}$$
$$\mu = 0.3\, GeV$$
$$L = 3\, fm$$
$$\omega_c = nL^2 \simeq 22.5 GeV$$

$$\omega \frac{dI_{\mathrm{HO}}}{d\omega} = 2\bar{\alpha} \ln \left| \cos\left(\frac{1-i}{2} \sqrt{\frac{\omega_c}{\omega}} \right) \right|$$

$$\simeq 2\bar{\alpha} \begin{cases} \sqrt{\frac{\omega_c}{2\omega}} & \text{for} \quad \omega \ll \omega_c \\ \frac{1}{12}\left(\frac{\omega_c}{\omega}\right)^2 & \text{for} \quad \omega \gg \omega_c \end{cases}$$



Validity of approximations:

- Single hard scattering $> \omega_c$
- Multiple-soft scattering $< \omega_c$

$$Q_s^2 \equiv \hat{q} L \sim 5 - 10\, GeV^2$$

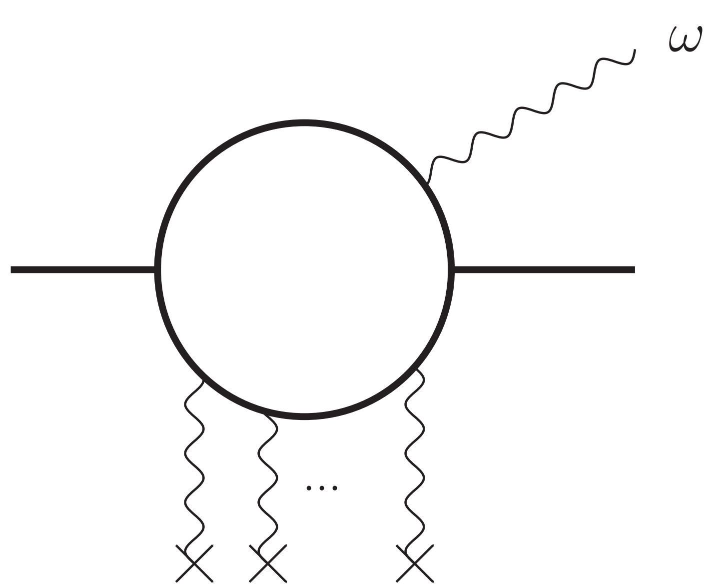
Medium-induced gluon spectrum

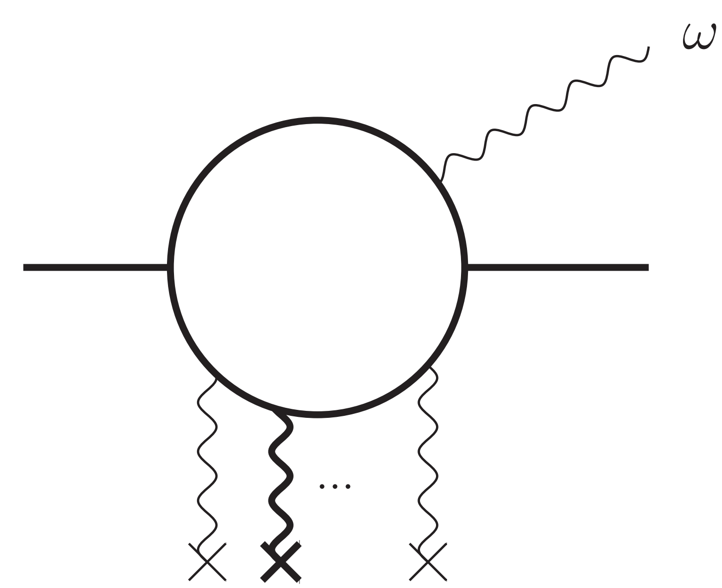
Typical transverse momentum :

$$Q^2 \sim \langle x_\perp^2 \rangle^{-1} \simeq \sqrt{\omega \hat{q}} \sim \sqrt{\omega n \ln(Q^2/\mu^2)}$$

We extract a constant large log from the dipole cross-section and treat the remainder as a perturbation

$$\begin{aligned} \sigma(t,\boldsymbol{x}) &= n(t)\,\boldsymbol{x}^2 \left(\ln \frac{Q^2}{m_D^2} + \ln \frac{1}{\boldsymbol{x}^2 Q^2} \right) \\ &\equiv \sigma_{\rm HO}(t,\boldsymbol{x}) + \sigma_{\rm pert}(t,\boldsymbol{x}), \end{aligned}$$





Molière (1948)

Medium-induced gluon spectrum

Correction to the Harmonic oscillator:

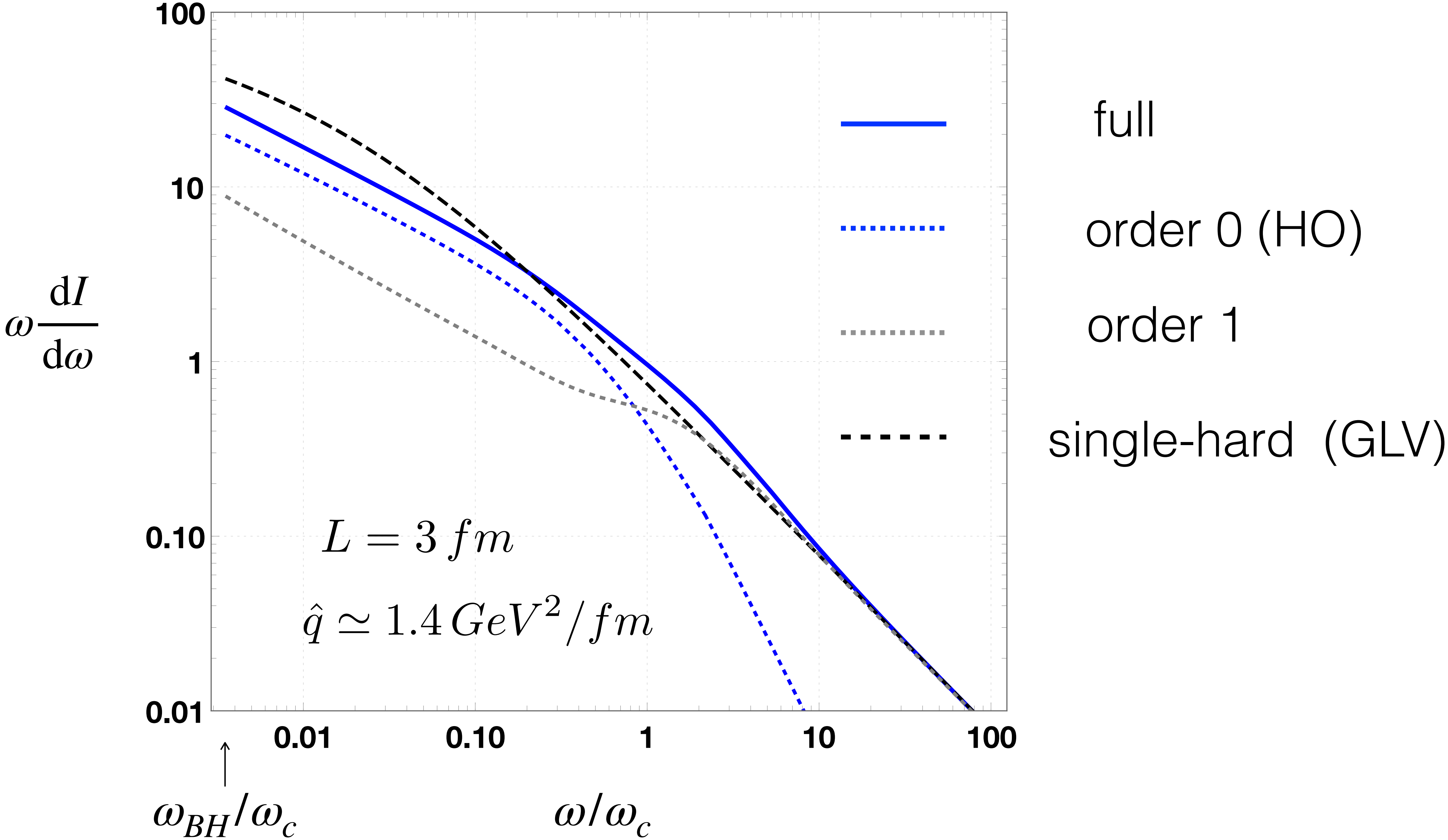
$$\omega \frac{\mathrm{d}I^{(1)}}{\mathrm{d}\omega} = \frac{\alpha_s C_R\, n}{2\pi}\,\mathrm{Re}\int_0^L \mathrm{d}s\,\frac{1}{k^2(s)}\left[\ln\frac{k^2(s)}{Q^2}+\gamma\right]$$

$$k^2(s) = i\frac{\omega\Omega}{2}\left(\cot(\Omega s) - \tan(\Omega(L-s))\right) \qquad \Omega \equiv \frac{1-i}{2}\sqrt{\frac{\hat{q}}{\omega}}$$

→ Contains the large frequency limit of N=1 opacity (GLV spectrum)

Numerical results

Medium-induced gluon spectrum for $\omega_c = nL^2 = 22.5\,GeV$



Numerical results II

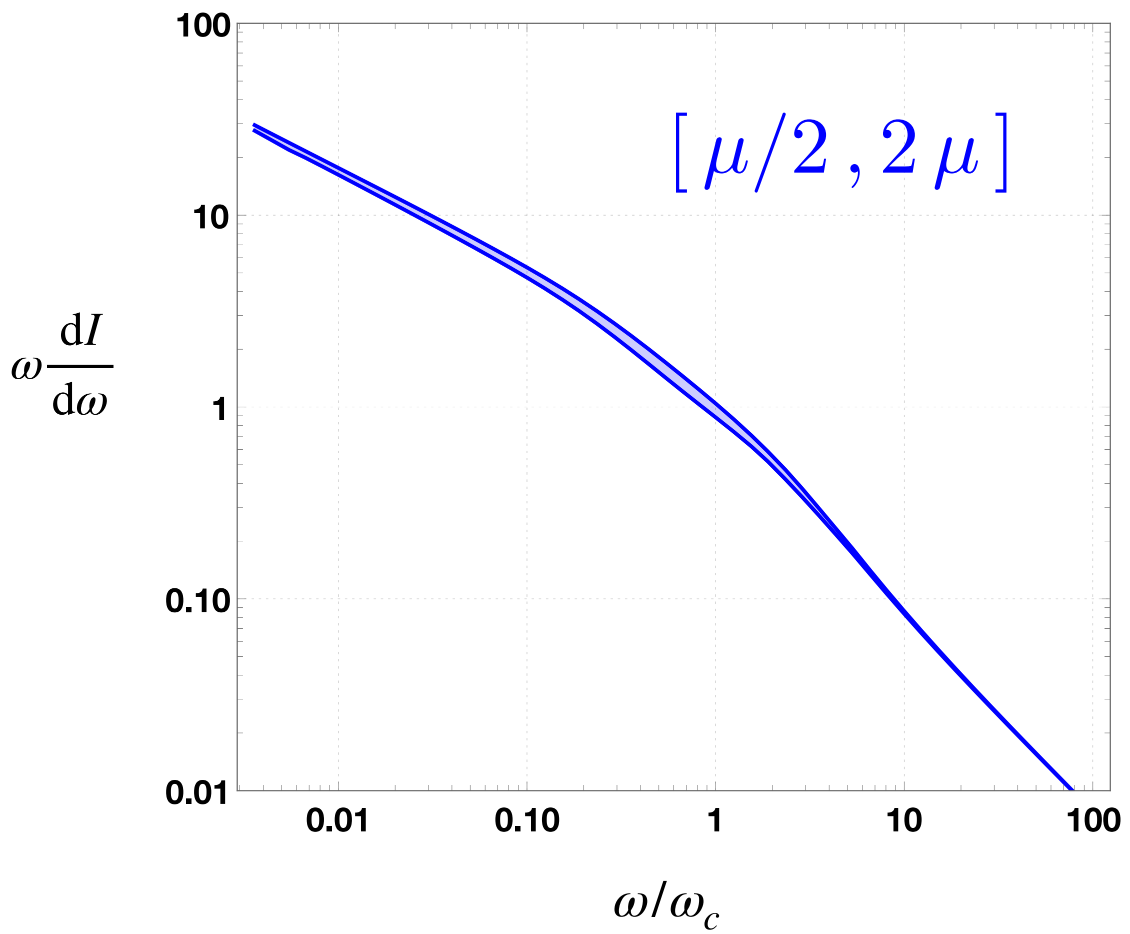
	N=1 (GLV)	full
$\Delta E(\omega < 100\,GeV)$	83 GeV	88 GeV
$N(\omega > 10^{-2}\omega_c)$	40	29

- The mean energy loss is dominated by single hard scattering
- Multiplicity is dominated by multiple soft scattering

IR sensitivity

- Typical transverse momentum scale:

$$Q^2 \sim n\, L\, \ln(nL/\mu^2) \sim 4\, GeV^2 \gg \mu^2 \sim (0.3)^2\, GeV^2$$



$$n = 0.1\, GeV^{-3}$$

$$L = 3\, fm$$

$$\hat{q} \simeq 1.4\, GeV^2/fm$$

$$\omega_c = nL^2 = 22.5\, GeV$$

→ Weak dependence on IR scale for large/dense media:
controlled perturbative expansion

Summary and outlook

- We have developed a new systematic method to perform analytic calculation of the medium-induced gluon spectrum beyond multiple-soft scattering approximation by expanding around the harmonic oscillator

- We have calculated the first two orders that encompass multiple-soft and single hard scattering regimes

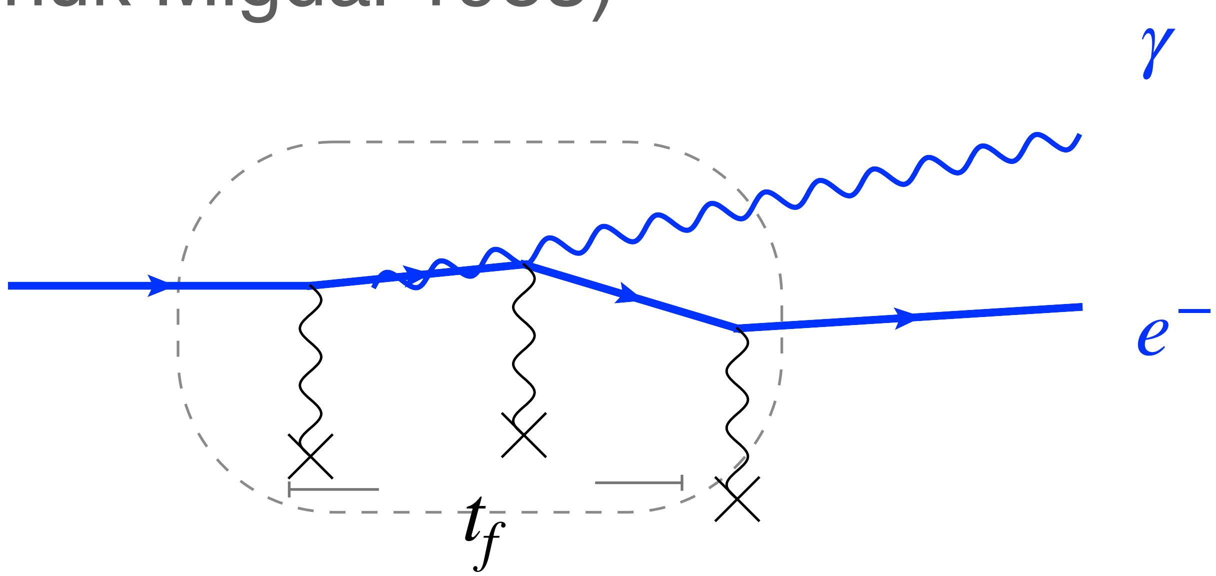
- Under perturbative control for large media

- Outlook: generalize to finite gluon energy and transverse momentum dependence. MC implementation

Back up

The LPM effect on the back of the envelop

- The energy spectrum of photons caused by the propagation of a relativistic charge in a medium is suppressed due to coherence effects (Landau-Pomeranchuk Migdal 1953)



- The Bethe-Heitler spectrum assumes incoherent scatterings center

$$\omega \frac{\mathrm{d}I^{BH}}{\mathrm{d}\omega} \sim \alpha_e N_{scatt} \sim \alpha_e \frac{L}{\ell_{mfp}}$$

The LPM effect on the back of the envelop

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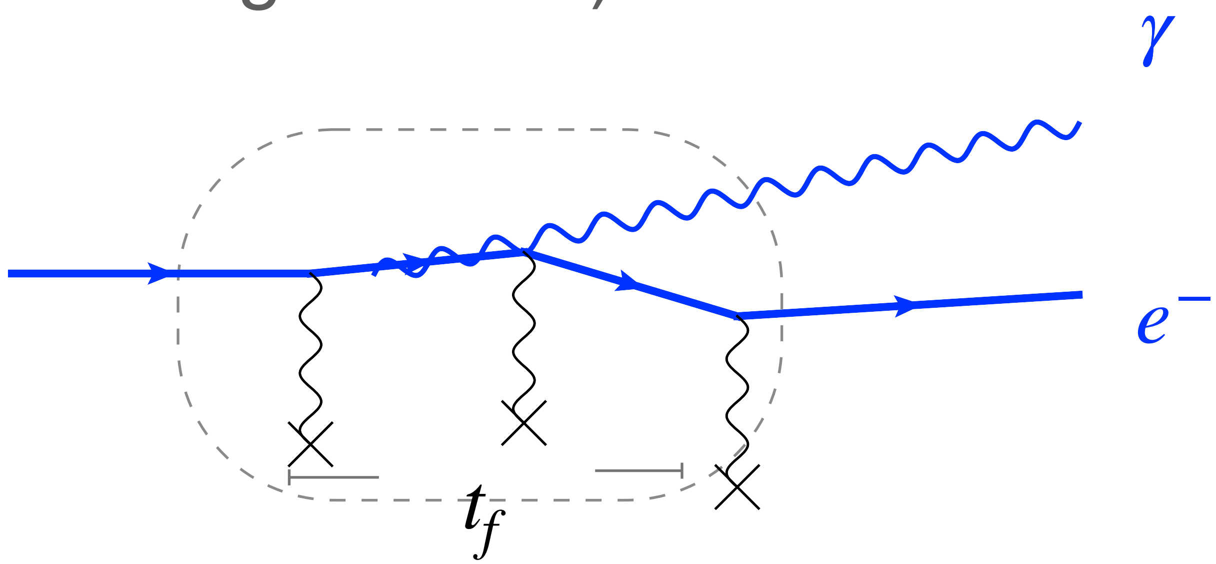


- Coherence length: during the quantum mechanical formation time N_{coh} scattering centers act coherently reducing the radiation spectrum

$$\omega \frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_e N_{eff} \sim \alpha_e \frac{N_{scatt}}{N_{coh}} \sim \alpha_e \frac{L}{t_f(\omega)}$$

The LPM effect on the back of the envelop

- From the uncertainty principle we have:

$$t_f(\omega) = \frac{\omega}{k_\perp^2} = \frac{1}{\omega\theta_\gamma^2}$$

- During that time the electron suffers momentum broadening

$$k_\perp^2 \sim \hat{q}\, t_f \qquad\qquad \theta_\gamma^2 \sim \theta_e^2 = \frac{k_\perp^2}{E_e^2}$$

- Solving for t_f one finds

$$t_f(\omega) = \frac{E_e}{\sqrt{\omega\hat{q}}} \qquad \text{and} \qquad \omega\frac{\mathrm{d}I^{LPM}}{\mathrm{d}\omega} \sim \alpha_e\frac{\sqrt{\omega\hat{q}}}{E_e}L \propto \sqrt{\omega}$$

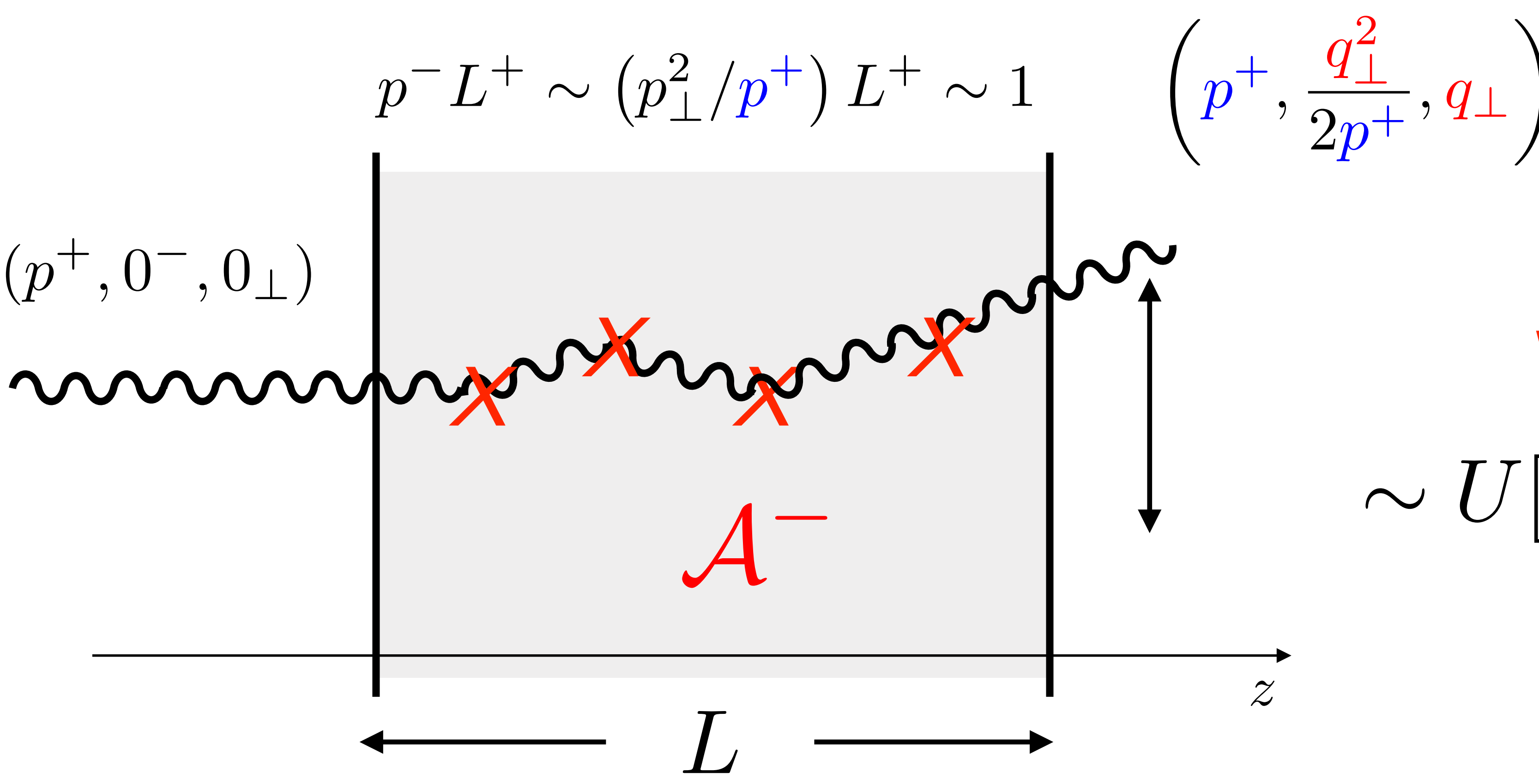
LPM suppression in the IR

General formalism

- Working assumption: neglect power corrections of the small momentum transfer $q^+ \ll p^+$

$$\text{eikonal vertex} \sim \delta(q^+)\, p^\mu \;\Leftrightarrow\; \mathcal{A}^-(x^+, x_\perp)$$

- Large medium: allow the gluon to explore the transverse plan between two scatterings

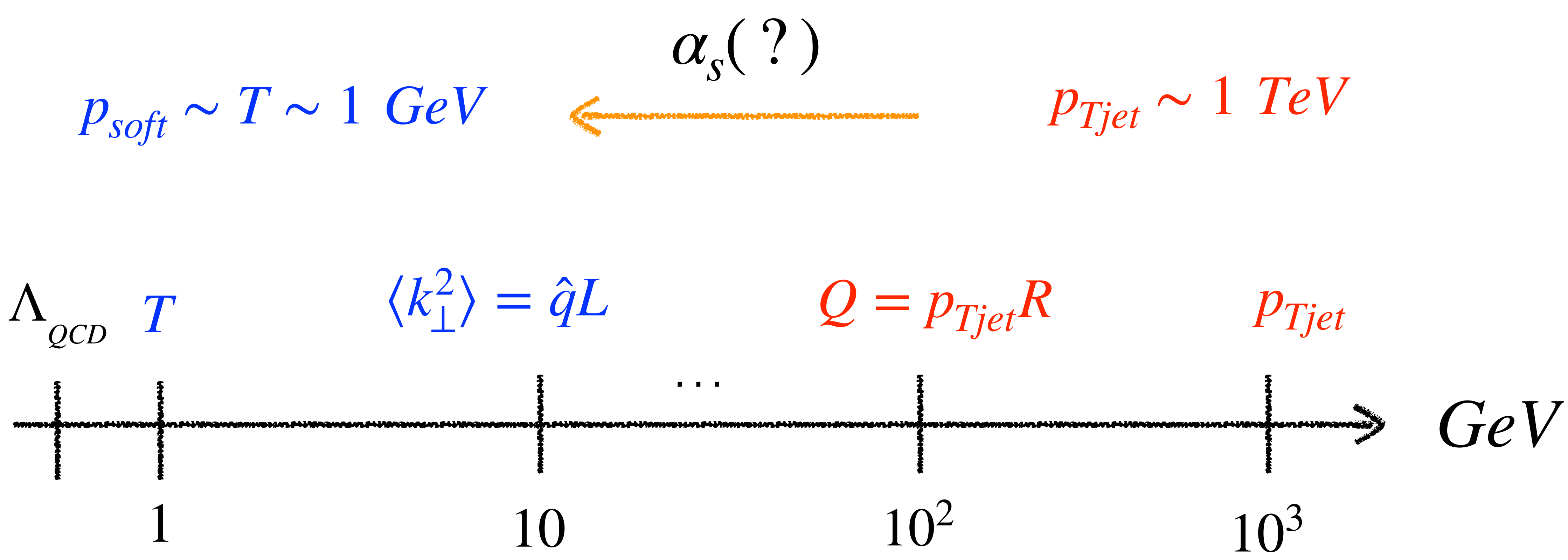


Wilson line

$$\sim U[\mathcal{A}^-(x(t), t)]$$

Motivation

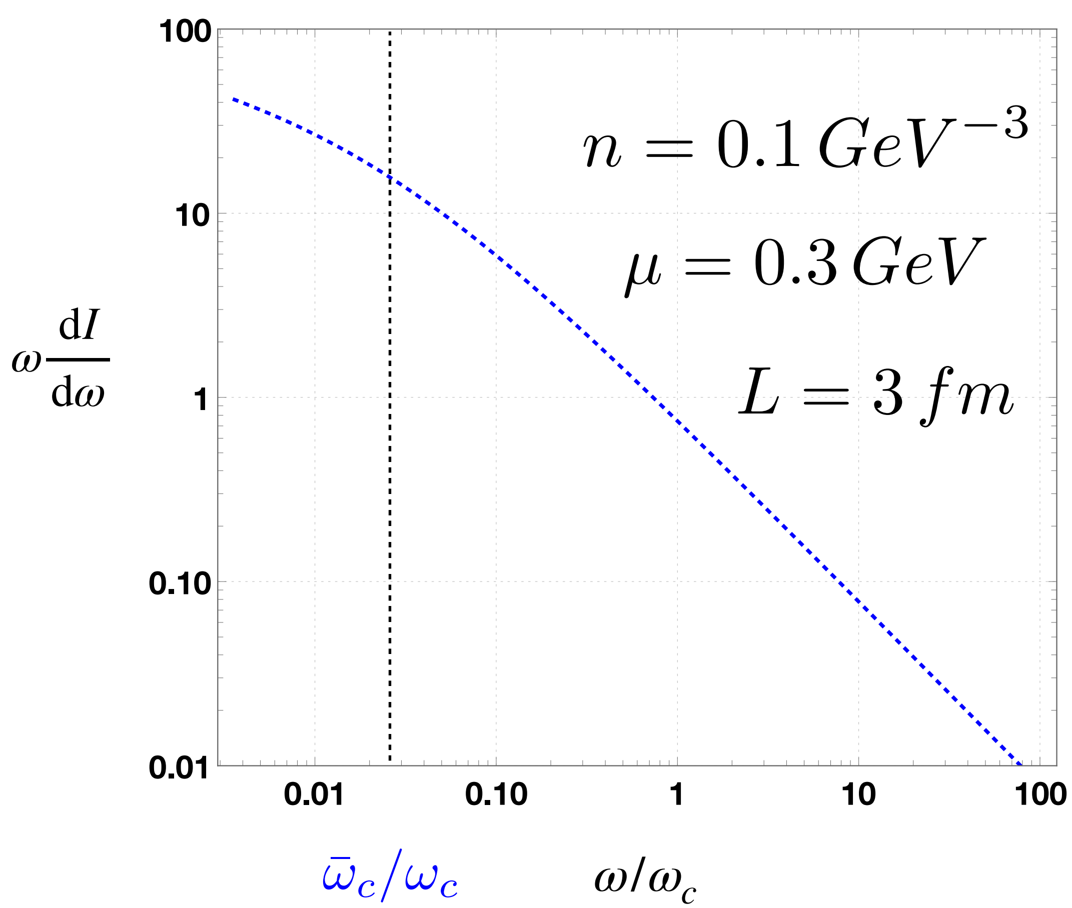
- **Physics question:** How is the jet coupled to the quark gluon plasma? Is perturbation theory applicable?



N=1 Opacity (Gyulassy-Levai-Vitev (2000))

- Assuming a dilute medium and expand to leader order in $\sigma(x_\perp)$

$$\omega \frac{\mathrm{d}I_{\mathrm{GLV}}}{\mathrm{d}\omega} \simeq 2\bar{\alpha} n\, L \begin{cases} \ln \frac{\bar{\omega}_c}{\omega} & \text{for} \quad \omega \ll \bar{\omega}_c \\ \frac{\pi}{4}\left(\frac{\bar{\omega}_c}{\omega}\right) & \text{for} \quad \omega \gg \bar{\omega}_c \end{cases}$$



$$\bar{\omega}_c = \frac{1}{2}\mu^2 L \simeq 0.7\, GeV$$

$$\omega_c = nL^2 \simeq 22.5 GeV$$