

GHP meeting, April 12, 2019, Denver

Gluon TMDs from Quarkonium Production in proton collisions

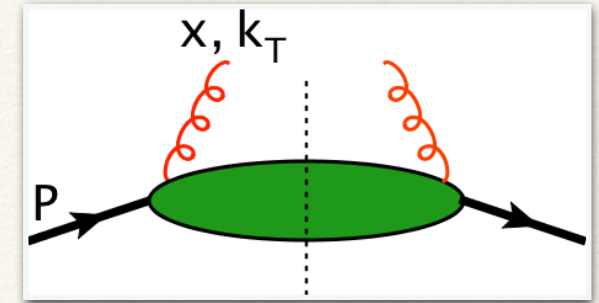
Marc Schlegel
Department of Physics
New Mexico State University

in collaboration with J.-P. Lansberg, C. Pisano, F. Scarpa
based on **PLB 784, 217 (2018)** ; **NPB 920, 192 (2017)**

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of gluons in nucleon**

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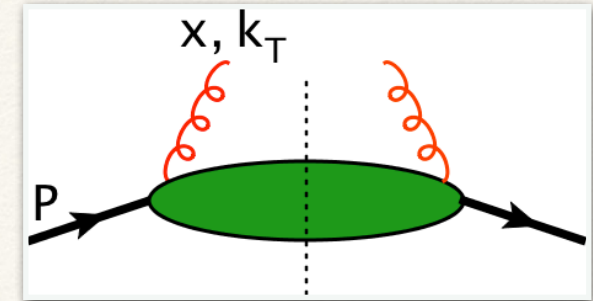
TMD gluonic matrix element



$$\Gamma^{\alpha\beta; [\mathcal{W}, \mathcal{W}']} (x, \mathbf{k}_T) = \int \frac{d\lambda d^2 z_T}{(2\pi)^3} e^{ix\lambda + i\mathbf{k}_T \cdot \mathbf{z}_T} \langle P, S | F^{n\alpha}(0) \mathcal{W} F^{n\beta}(\lambda n + z_T) \mathcal{W}' | P, S \rangle$$

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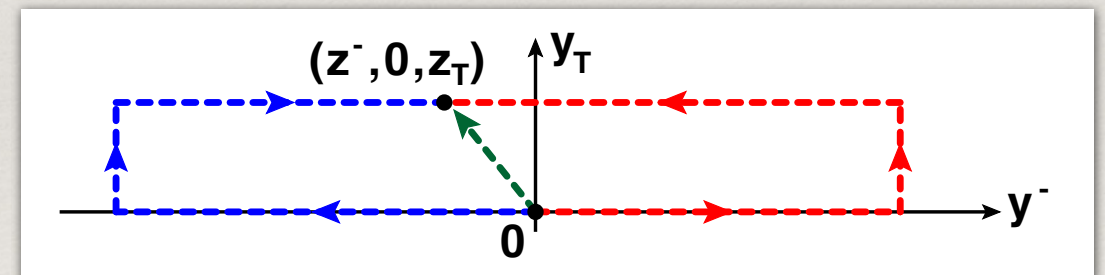
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Wilson line in fundamental representation

Color Gauge invariant definition of TMDs $\rightarrow$ Wilson line

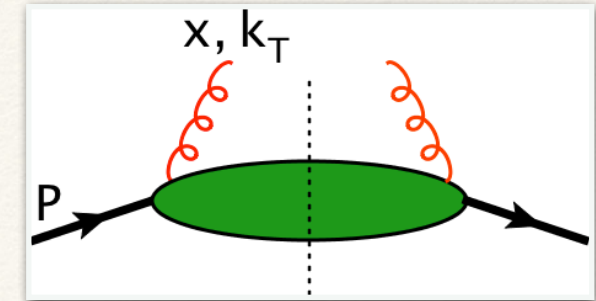
$$\mathcal{W}[a, b] = \mathcal{P} e^{-ig \int_a^b ds \cdot A^a(s) t^a}$$

$\rightarrow$ Wilson line for TMD:
nontrivial, process dependent:



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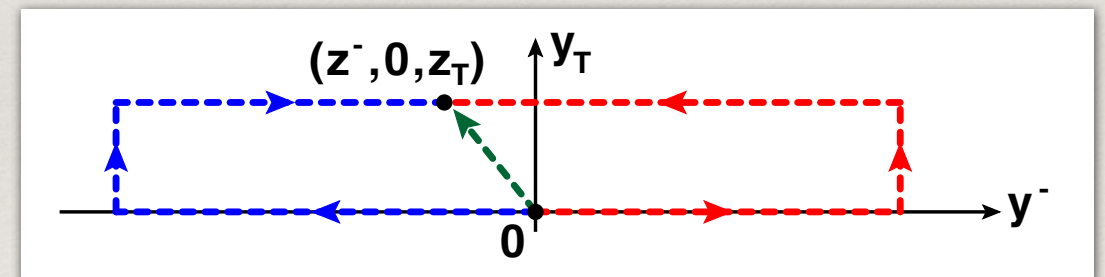
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past-pointing WL $\rightarrow$
Initial State Interactions:
pp $\rightarrow$ color singlet + X (DY-type)

$$\Gamma^{\alpha\beta; [-, -]}$$

future-pointing WLs $\rightarrow$
Final State Interactions:
ep $\rightarrow$ jets + X (SIDIS-type)

$$\Gamma^{\alpha\beta; [+, +]}$$

Proper Definition & Soft function

[Collins; Sun, Xiao, Yuan; Aybat, Rogers; see also: Echevarria, Kasemats, Mulders, Pisano, JHEP 07, 158 (2015)]

Inclusion of Soft Function $\implies$

$$S(z_T) = \frac{\text{Tr}_c}{N_c} \langle 0 | \mathcal{W}_n^\dagger(-z_T/2) \mathcal{W}_{\bar{n}}(-z_T/2) \mathcal{W}_{\bar{n}}^\dagger(z_T/2) \mathcal{W}_n(z_T/2) | 0 \rangle$$

$$\tilde{\Gamma}^{\alpha\beta}(x, \mathbf{b}_T) \rightarrow \frac{\tilde{\Gamma}^{\alpha\beta}(x, \mathbf{b}_T)}{\sqrt{S(\mathbf{b}_T)}}$$

Modification needed in order to have:

- 1) Renormalizable Matrix Element
- 2) Finite matching coefficients at small b_T

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Parameterization

unpolarized & linearly polarized gluons :
helicity flip TMDs $\rightarrow$ azimuthal modulations

$$\Gamma^{\alpha\beta}(x, k_T) = \frac{1}{2x} \left[-g_T^{\alpha\beta} f_1^g(x, k_T^2) + \frac{k_T^\alpha k_T^\beta - \frac{1}{2} k_T^2 g_T^{\alpha\beta}}{M^2} h_1^{\perp g}(x, k_T^2) \right]$$

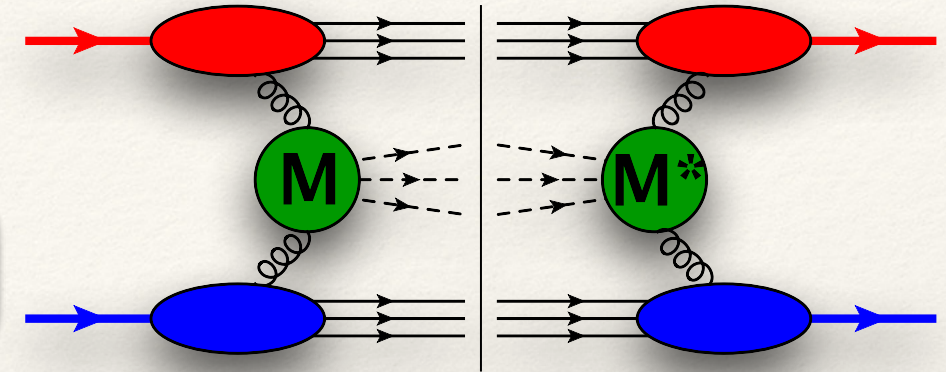
Gluon TMDs from pp - collisions

[J.-P. Lansberg, C. Pisano, M.S., NPB 920, 192 (2017)]

General TMD expression for gluon fusion:

$$\frac{d\sigma}{d^4q...}(q_T \ll Q) \propto \mathcal{C}[\Gamma^{\alpha\alpha'}(x_a, k_{aT}) \Gamma^{\beta\beta'}(x_b, k_{bT})] (\mathcal{M}_{\alpha\beta} (\mathcal{M}_{\alpha'\beta'})^*)$$

$$\mathcal{C}[w \textcolor{blue}{f} \textcolor{red}{g}] = \int d^2k_{aT} \int d^2k_{bT} \delta^{(2)}(k_{aT} + k_{bT} - q_T) w(k_{aT}, k_{bT}) \textcolor{blue}{f}(x_a, k_{aT}) \textcolor{red}{g}(x_b, k_{bT})$$

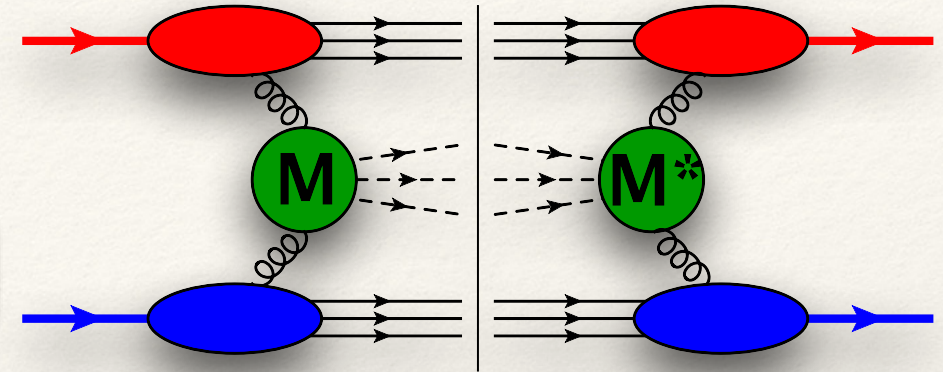


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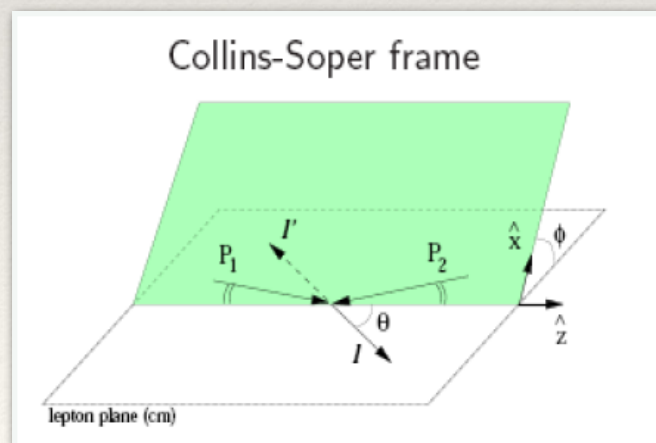
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2-particle (pair) final states in pp - collisions

$$\left. \frac{d\sigma^{gg}}{d^4q d\Omega} \right|_{q_T \ll Q} = \hat{F}_1 [f_1^g \otimes f_1^g] + \hat{F}_2 [h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi) \hat{F}_3 [h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi) \hat{F}_4 [h_1^{\perp g} \otimes h_1^{\perp g}]$$



Evaluate cross section in
c.m. frame of the produced pair
Collins - Soper angles θ , ϕ

Gluon TMDs do not appear in Drell-Yan

Double $J/\psi(\Upsilon)$ - production

[Lansberg, Pisano, Scarpa, PLB 784, 217 (2018)]

TMD - formalism: J/ψ pair back-to-back; Color singlet

Here: Sample diagram (of about $O(40)$)

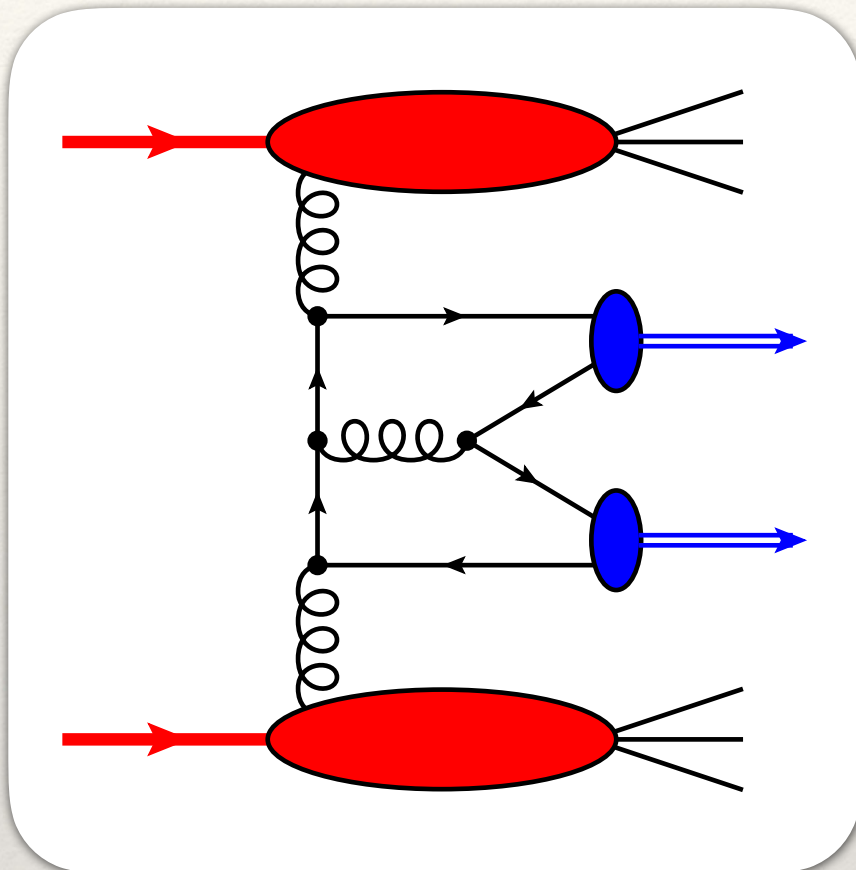
Full analytical LO amplitude

$$g + g \rightarrow J/\psi + J/\psi$$

(J/ψ in color singlet configuration)

Presented in Appendix of
[Qiao, Sun, Sun; 0903.0954] (Kudos!)

Helicity formalism: contract with polarization vectors
→ perturbative factors F_1, F_2, F_3, F_4



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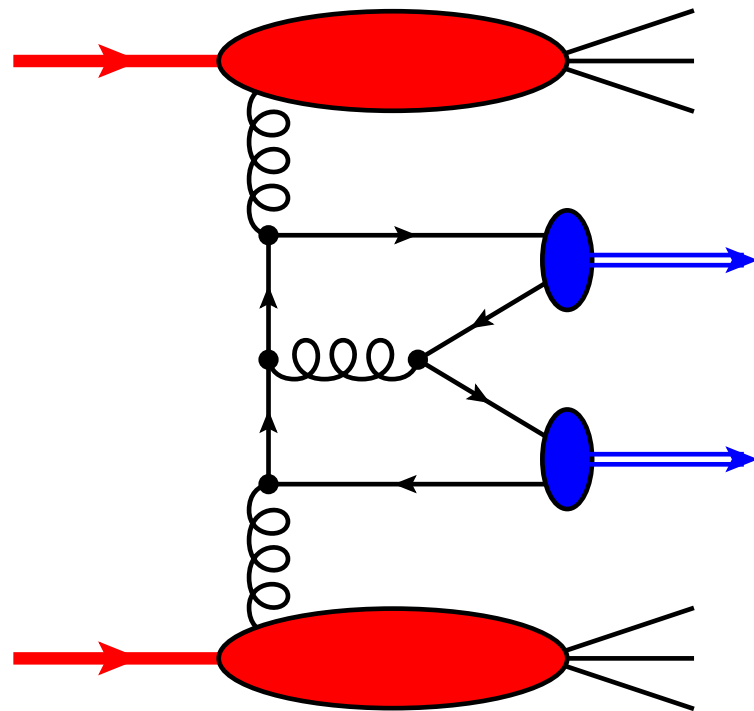
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$$F_i(Q, \cos^2 \theta) \propto \frac{\sum_{k=0}^{N_i} c_k(\alpha) (\cos^2 \theta)^k}{(1 - (1 - \alpha^2) \cos^2 \theta)^4}$$

Degree of polynomial $N_i = 6, 4, 5, 6$

Q : invariant mass of Quarkonium pair

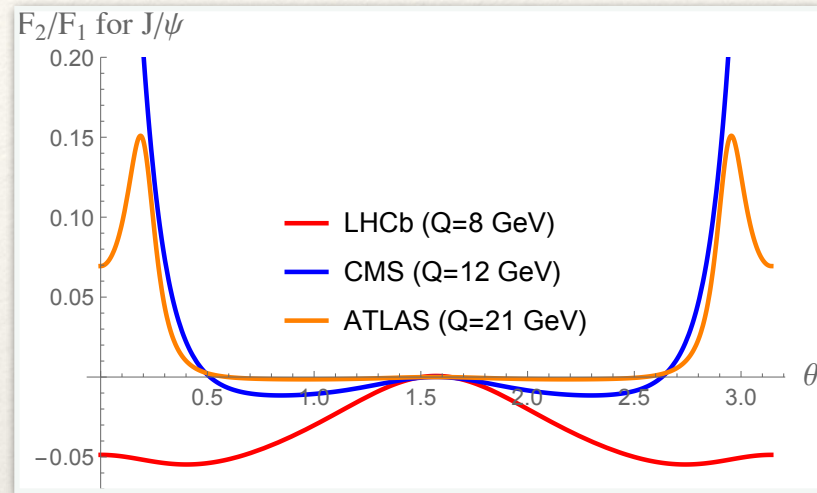
$$\alpha = \frac{2M_{J/\psi}}{Q}$$

$\alpha = 1$ threshold limit

$\alpha = 0$ high-energy limit

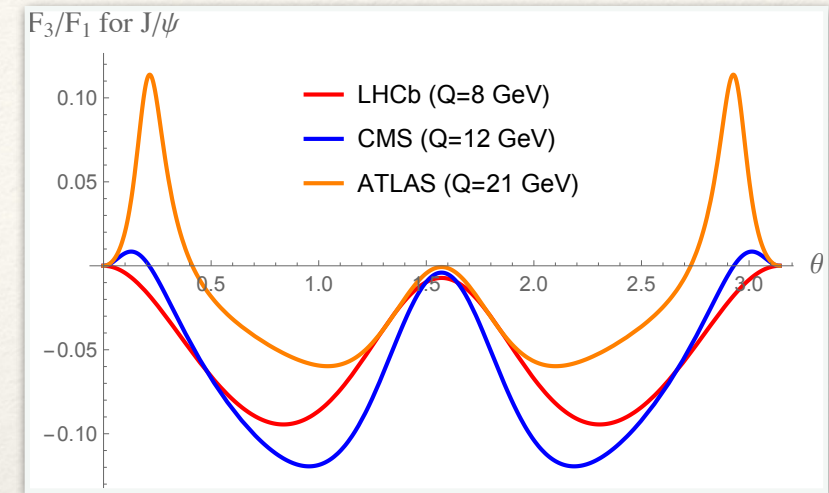
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Ratios:



q_T - spectrum:

Modification from lin. pol. glue mostly negligible

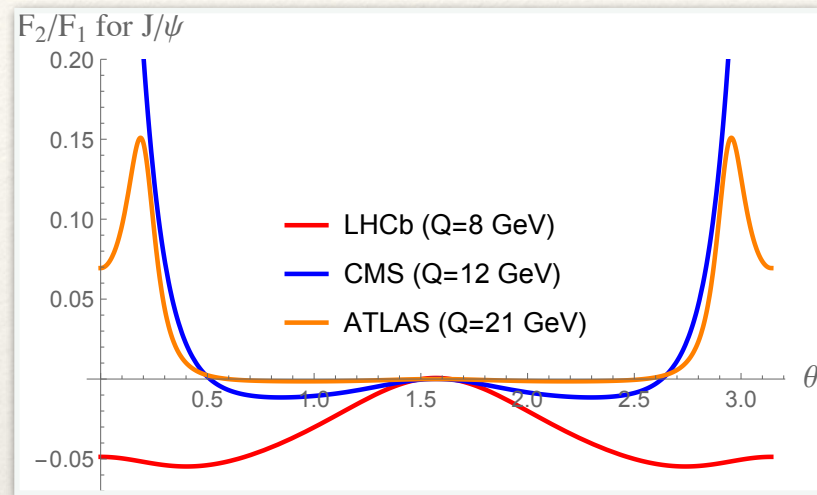


$\cos(2\phi)$:

about -10% at $\theta = \pi/4, 3\pi/4$

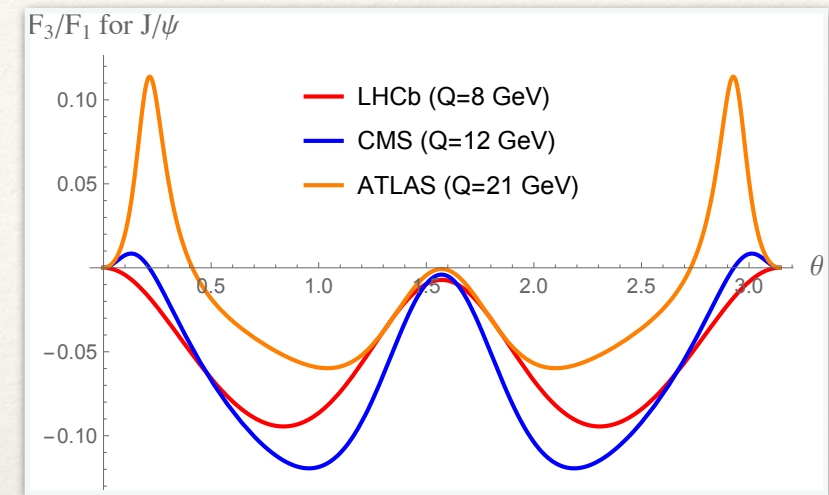
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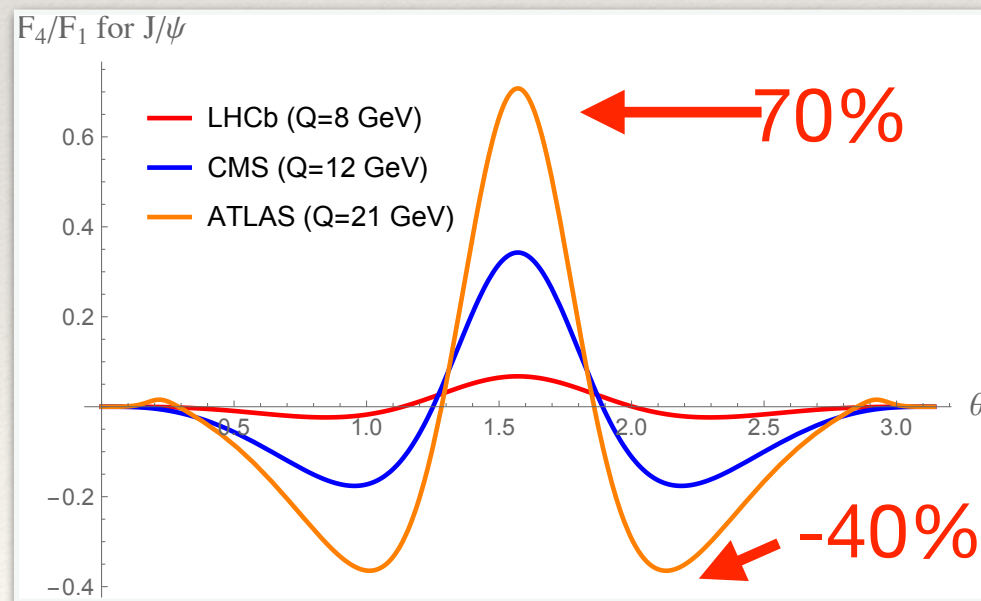
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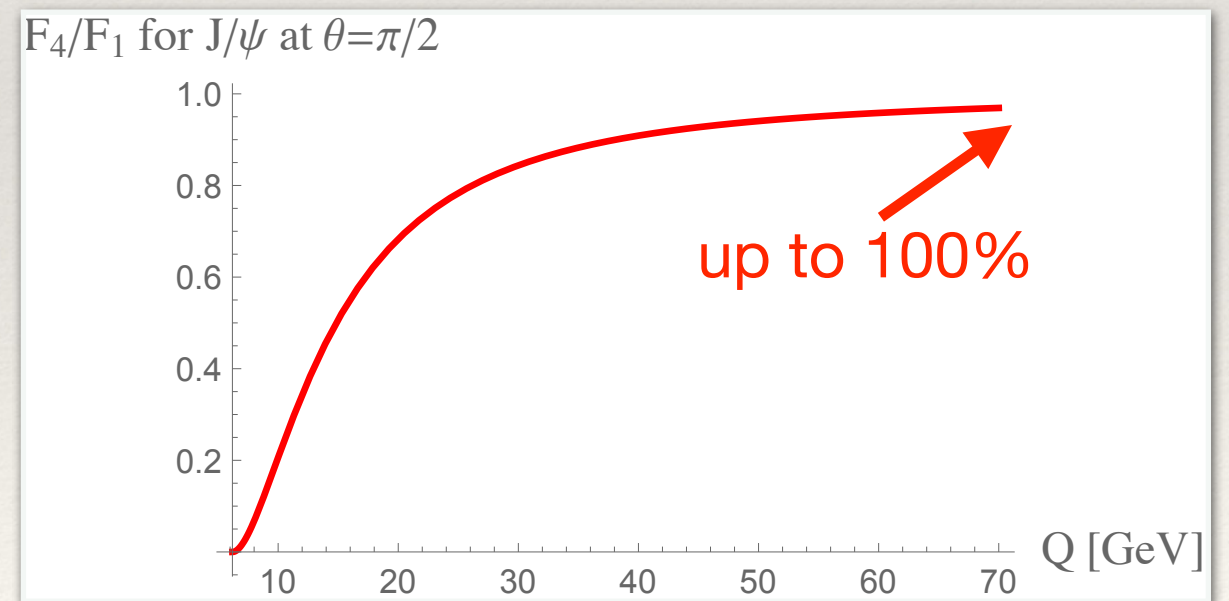
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$\cos(4\phi)$: Large at $\theta = \pi/2$!



$$F_4(\theta = \frac{\pi}{2}, Q \gg 2M_{J/\psi}) \rightarrow F_1$$



unique to this process!

q_T - spectrum: Data on J/ψ - pairs available from LHC

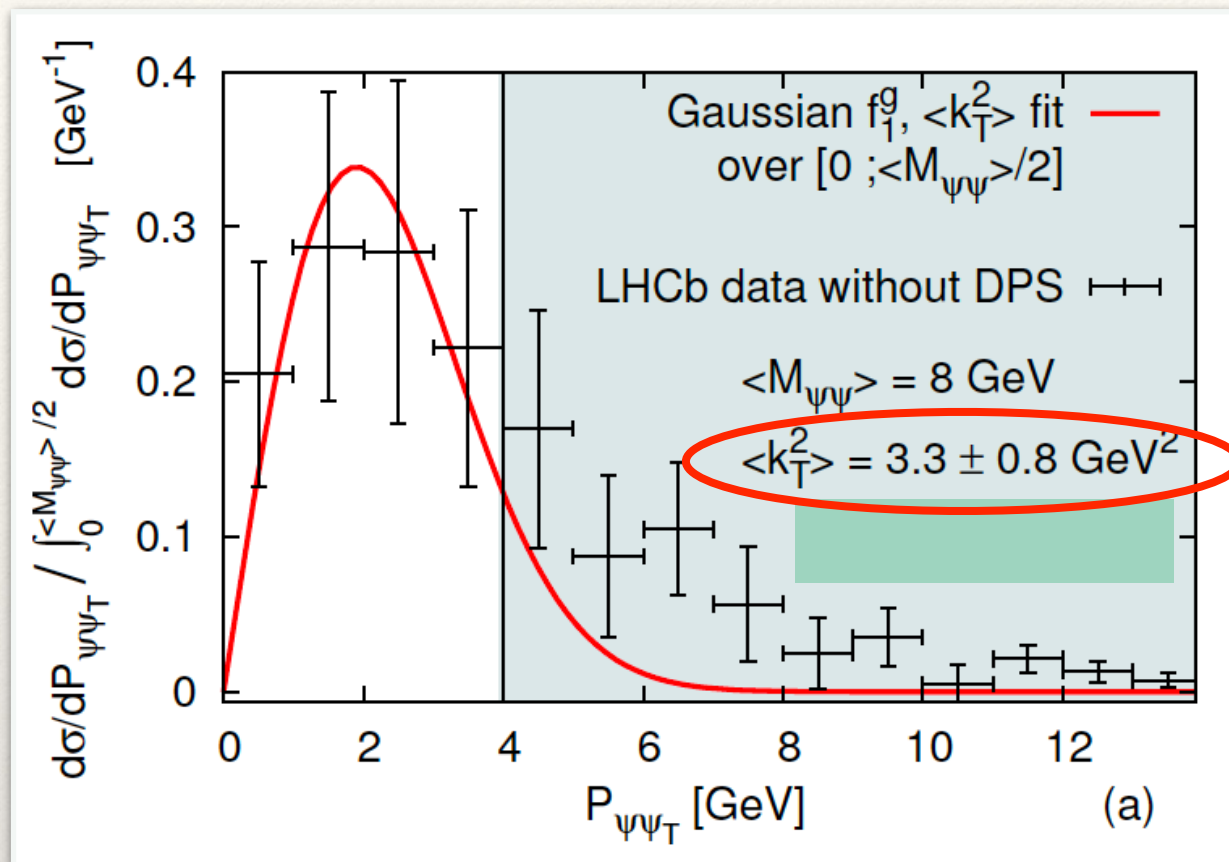
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LHCb data (2017) at 13 TeV slightly above threshold, $Q = 8$ GeV

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Fit observable

$$\frac{\left[\frac{d\sigma}{dq_T} \right]_{\text{bins}}}{\int_0^{Q/2} dq_T \left[\frac{d\sigma}{dq_T} \right]_{\text{bins}}} \propto \frac{\mathcal{C}[f_1^g f_1^g]}{\int_0^{Q/2} dq_T \mathcal{C}[f_1^g f_1^g]}$$

First glimpse on unpol. gluon TMD:

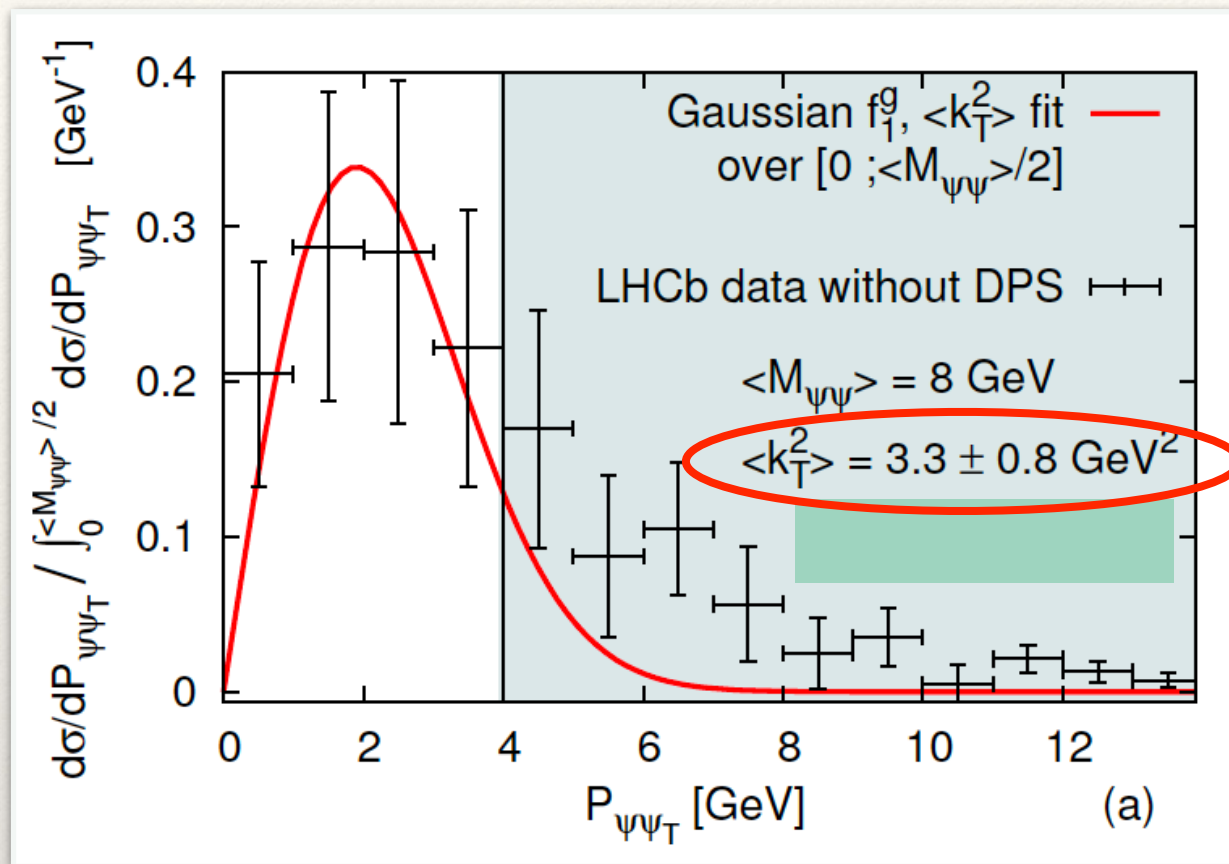
No fits for f_1 so far, Gaussian ansatz

$$f_1^g(x, k_T^2; Q = 8 \text{ GeV}) = \frac{g(x)}{\pi \langle k_T^2 \rangle} \exp(-k_T^2 / \langle k_T^2 \rangle)$$

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- One-parameter fit for $\langle k_T^2 \rangle$, $\chi^2 = 1.08$: *effective*, but not *intrinsic* width
- TMD evolution not included, needed if more data sets at different Q are available
- LHCb data corrected for double-parton scattering
- Expectation: Color Singlet Mode dominant

[Lansberg, Shao, PLB 2015; Ko, Yu, Lee, JHEP 2011; Li, Xu, Liu, Zhang, JHEP 1023]

Azimuthal modulation: no data available → predictions

Suggested observables: weighted cross section ratios

$$\frac{\int d\phi \cos(2\phi) \left[\frac{d\sigma d\phi}{dq_T} \right]_{\text{bins}}}{\int_0^{Q/2} dq_T \left[\frac{d\sigma}{dq_T} \right]_{\text{bins}}} \propto \frac{\mathcal{C}[w_3 h_1^{\perp g} f_1^g] + \mathcal{C}[w_3 f_1^g h_1^{\perp g}]}{\int_0^{Q/2} dq_T \mathcal{C}[f_1^g f_1^g]}$$

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Input for linearly pol. gluon TMD: models

Model 1

$$h_1^{\perp g}(x, k_T^2) = \frac{M^2}{\langle k_T^2 \rangle} \frac{g(x)}{\pi \langle k_T^2 \rangle} \exp \left(1 - \frac{3 k_T^2}{2 \langle k_T^2 \rangle} \right)$$

Model 2: Saturation of positivity bound

$$h_1^{\perp g}(x, k_T^2) = \frac{2M^2}{k_T^2} f_1^g(x, k_T^2)$$

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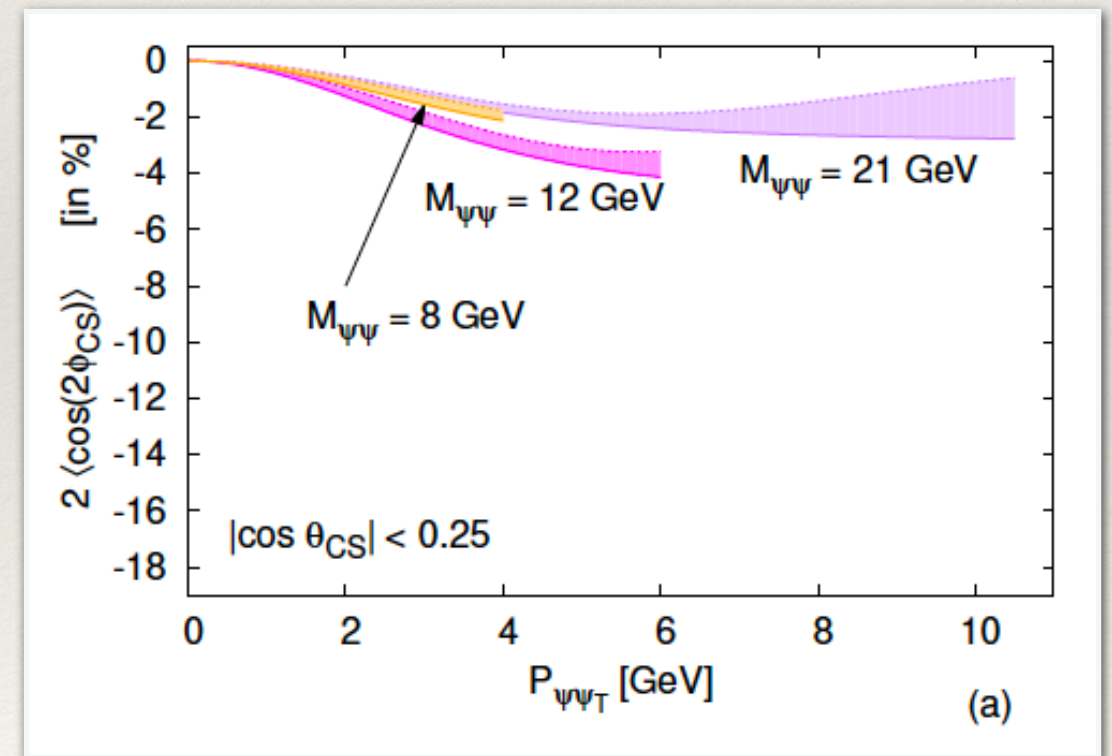
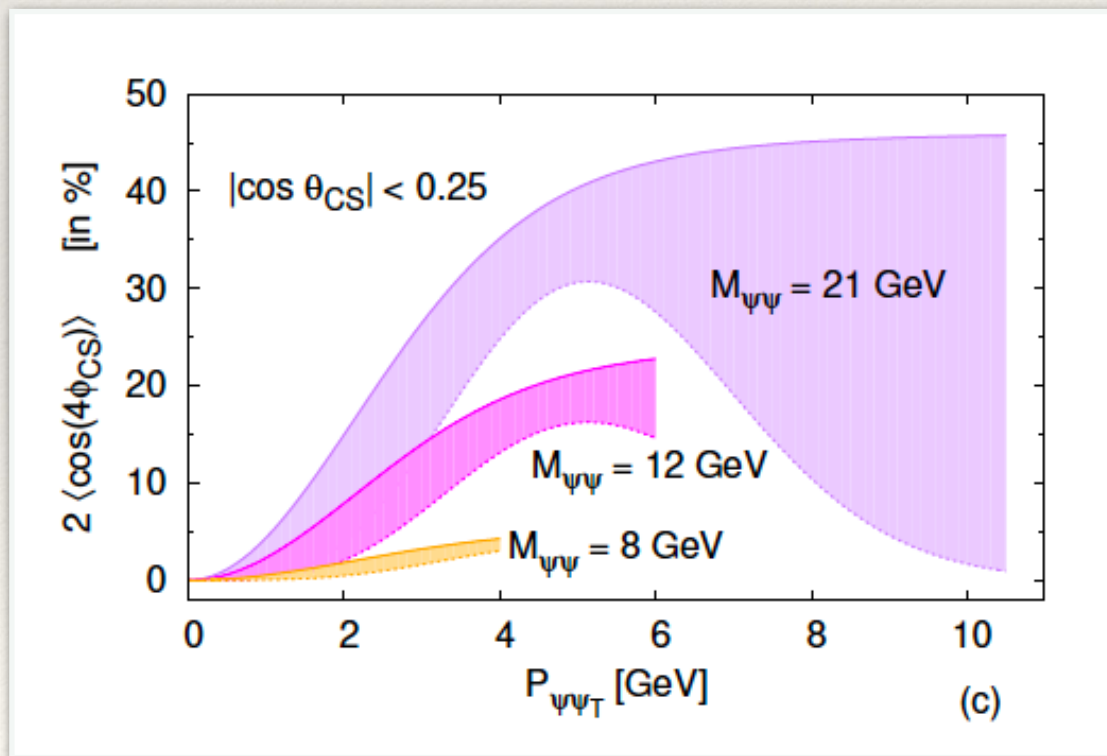
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- Large effects for $\cos(4\phi)$ for Q larger than threshold: 10% - 20%, $\cos(2\phi)$ few percent

Summary & Outlook

- ❖ Gluon TMDs → new aspects on the 3D gluonic structure of the nucleon → linear gluon polarization
- ❖ Several final states in pp - collisions: Leptons, Photons, Heavy Quarkonium states
- ❖ Promising: J/ψ pairs at the LHC, particularly large: $\cos(4\phi)$ azimuthal modulation
- ❖ Data is coming: first extraction of unpolarized gluon TMD
- ❖ Important for EIC: An idea what to expect for gluon TMDs
- ❖ Outlook:
 - Improve fit for unpolarized gluon TMD including ATLAS, CMS data and TMD evolution
 - Extend helicity formalism to spin-dependent observables and gluon TMDs (sPHENIX)