8th Workshop of the APS Topical Group on Hadronic Physics

10-12 April 2019 Denver, CO

Minkowski space 4D dynamics for hadrons

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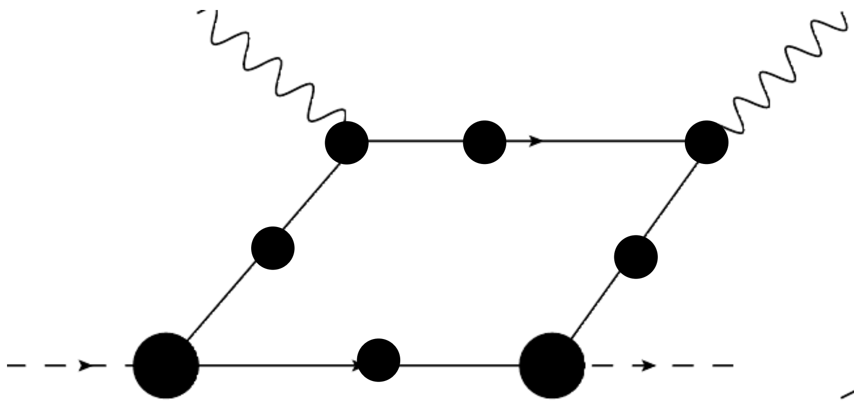
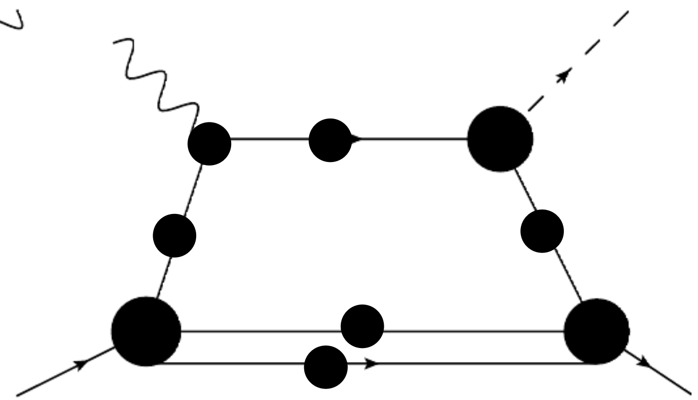
Thanks to:

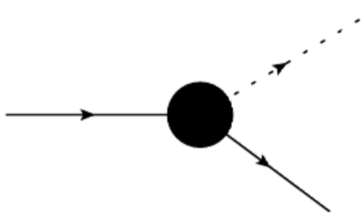
FULBRIGHT BRASIL
CNPq
FAPESP
65 anos CAPES
INCT-FNA
INSTITUTO NACIONAL DE CIÊNCIA E TECNOLOGIA
FÍSICA NUCLEAR E APLICAÇÕES

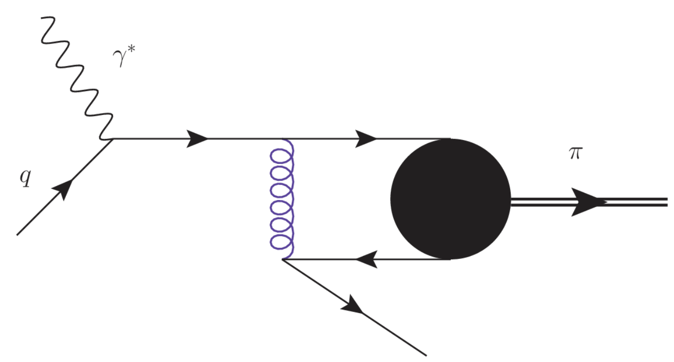
BS amplitude→ beyond the valence

Form-factors, DVCS in ERBL – DGLAP regions



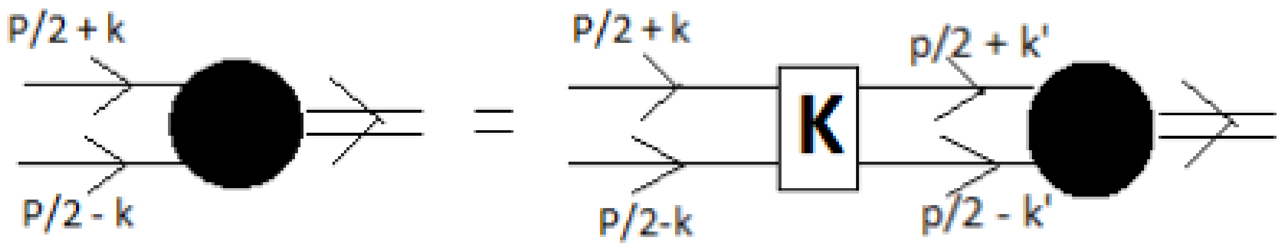






Fragmentation function

BSE for for fermion-antifermion: 0^- state



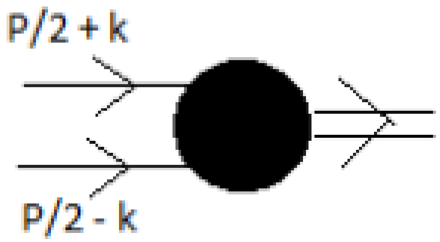
Ladder approximation (L): suppression of XL (non-planar diagram) for N_c=3

[A. Nogueira, CR Ji, Ydrefors, TF, PLB 777 (2018) 2017]

Vector $$i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2-\mu^2+i\epsilon}$$

Vertex Form-Factor $$F(q) = \frac{\mu^2-\Lambda^2}{q^2-\Lambda^2+i\epsilon}$$

BS amplitude fermion-antifermion: 0^-



$$\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu\gamma_5$$

Nakanishi Integral Representation (NIR)

$$\phi_i(k,p) = \int_{-1}^{+1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{(k^2 + p\cdot k\ z' + M^2/4 - m^2 - \gamma' + i\epsilon)^3}$$

Kusaka and Williams, PRD 51 (1995) 7026 (bosons)

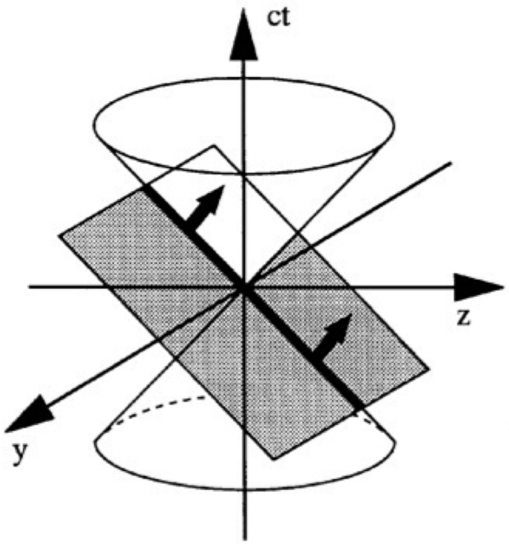
Light-front projection: integration in k^-

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD85(2012)036009;PRD89(2014) 016010,EPJC75(2015)398

(application to scattering)

Light-front projection: integration over k^-



Fermion-antifermion: Carbonell and Karmanov EPJA 46 (2010) 387

$$\boxed{\psi_{LF}(\gamma,z)=\frac{1}{4}(1-z^2)\int_0^\infty \frac{g(\gamma',z)d\gamma'}{\Big[\gamma'+\gamma+z^2m^2+\kappa^2(1-z^2)\Big]^2}}$$

$$\gamma=k_\perp^2 \qquad z=2x-1 \qquad \kappa^2=m^2-\frac{M^2}{4}$$

LF singularities: de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901

MOCK PION MODEL

W. de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

- **Gluon effective mass ~ 500 MeV – Landau Gauge LQCD**

[Oliveira, Bicudo, JPG 38 (2011) 045003;
Duarte, Oliveira, Silva, Phys. Rev. D 94 (2016) 01450240]

- **Mquark ~ 250 MeV**

[Parappilly, et al, PR D73 (2006) 054504]

- **Λ/m = 2**

Results

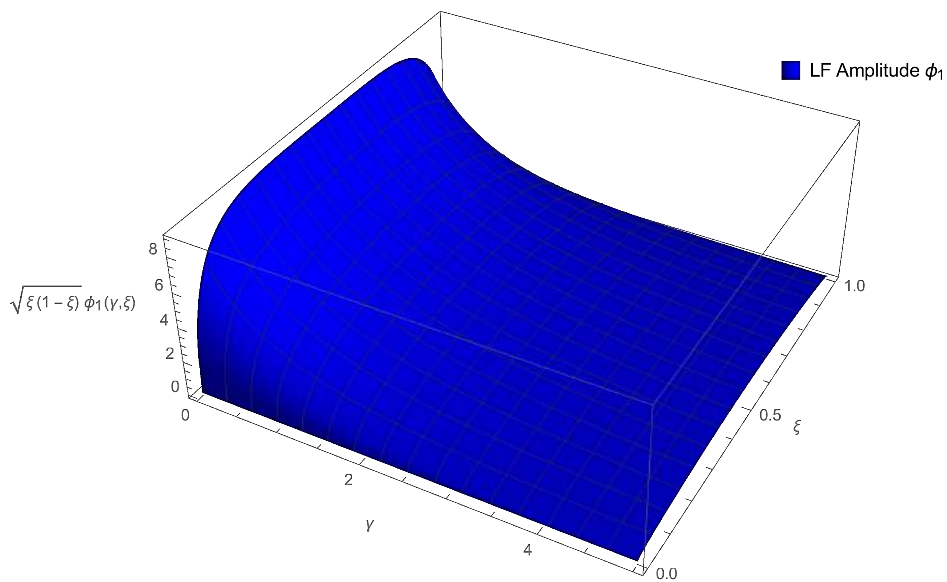
- **4 amplitudes in LF variables**
- **valence probability, decay constant**
- **momentum distributions**
- **Electromagnetic form factor**

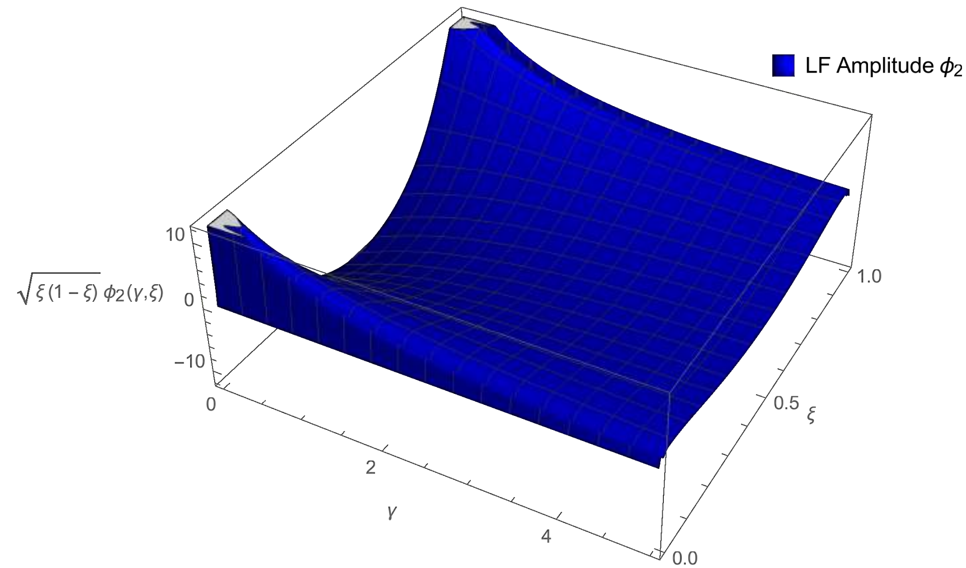
Light-front amplitudes

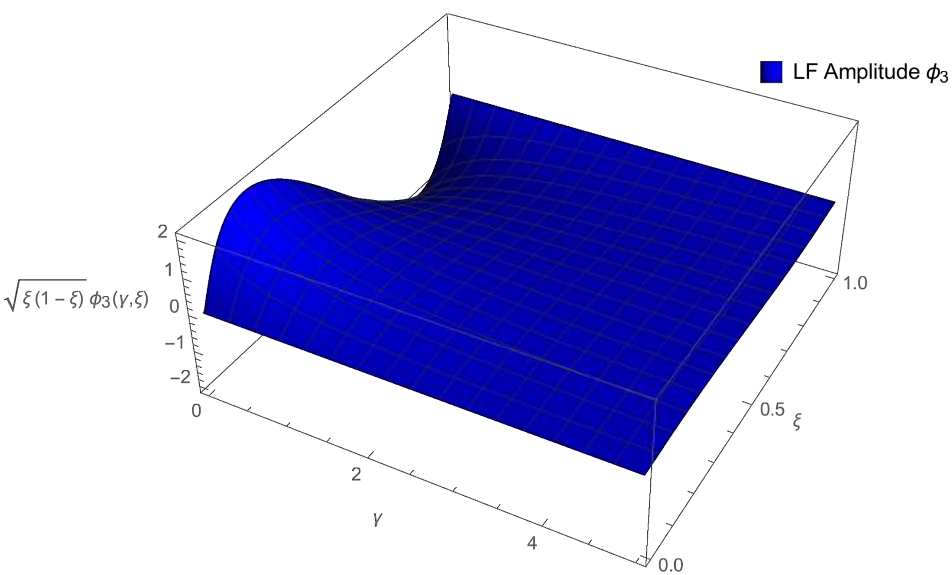
$(B/m = 1.35, \mu/m = 2.0, \Lambda/m = 1.0, \tilde{m}_q$=215 MeV): $f_\pi = 96$ MeV

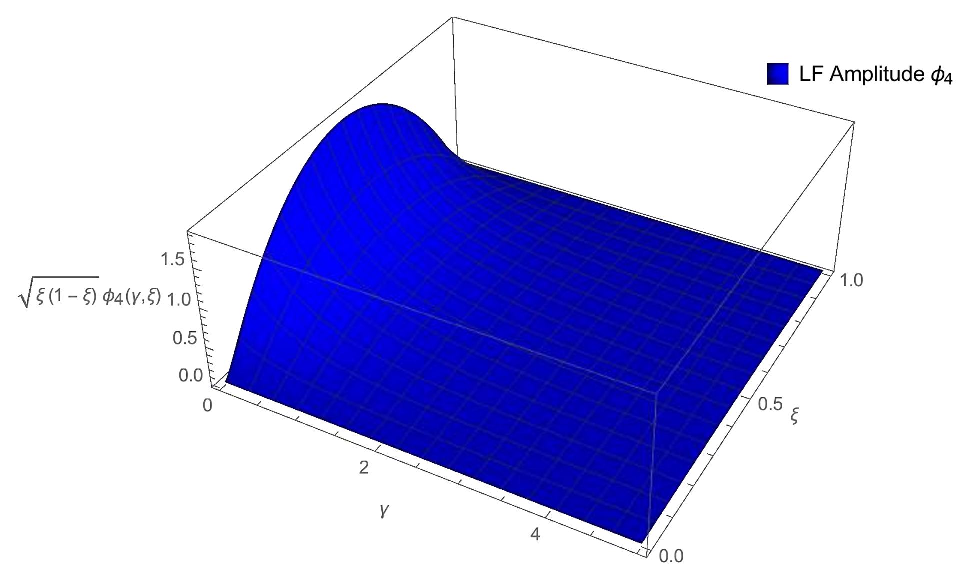
$P_{val} = 0.68$

W. de Paula, et. al, in preparation









Valence distribution functions: longitudinal and transverse

W. de Paula, et. al, in preparation

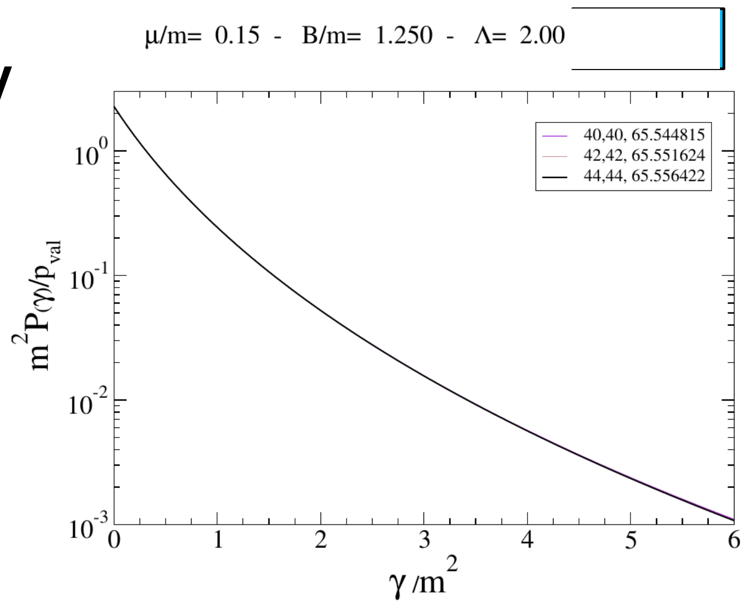
Mquark 187 MeV

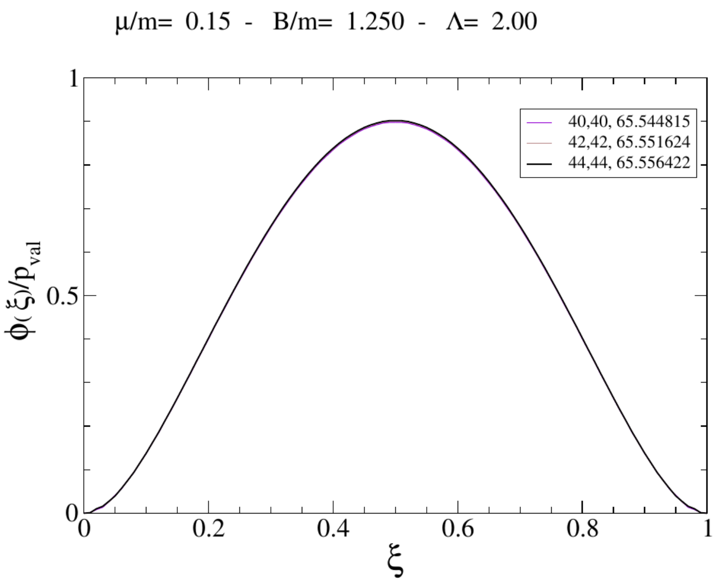
Mgluon 28 MeV

Λ/m =2

Pval=0.64

$$f_\pi = 77\,\mathrm{MeV}$$





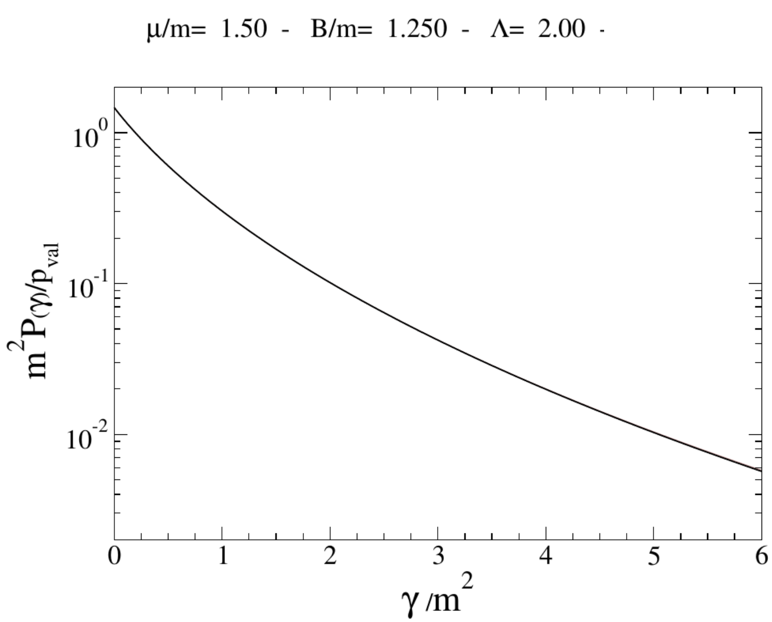
Mquark 187 MeV

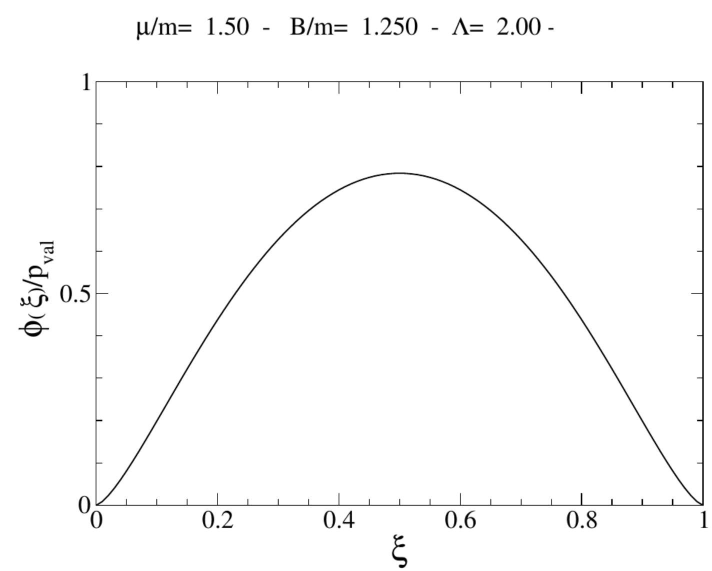
Mgluon 280 MeV

Λ/m =2

Pval=0.78

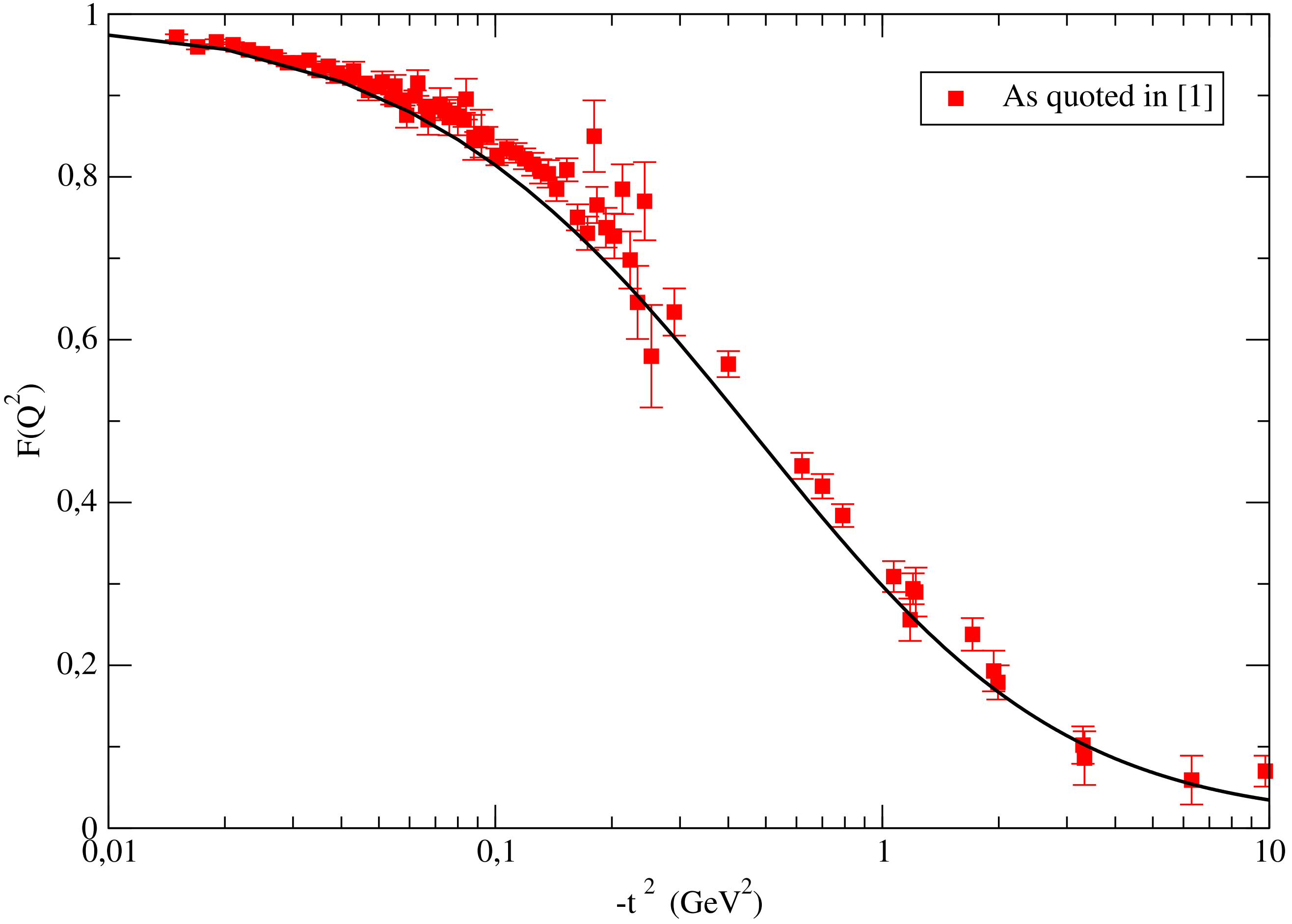
$$f_\pi = 99\,\mathrm{MeV}$$





Preliminary: Pion's electromagnetic form factor

Valence contribution renormalized to the charge



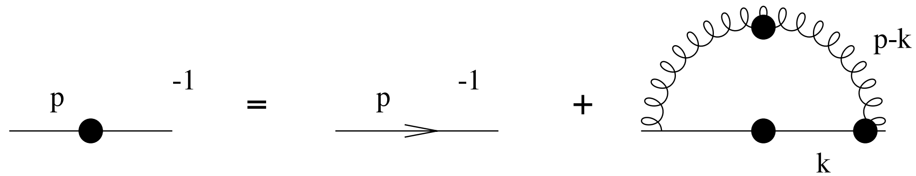
[1] Collection of R. Baldini, et al., Eur. Phys. J. C 11, 709 (1999); Nucl. Phys. A 666 & 667, 3 (2000);

Thanks to Jorge Alvarenga Nogueira

Dyson-Schwinger equation in Rainbow ladder truncation

from Euclidean to Minkowski: Un-Wick rotating

In collaboration with Duarte, de Paula, Maris,Shaoyang Ji, Nogueira, Ydrefors



$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)$$

QED-like, Landau Gauge, bare vertices, massive vector boson, Pauli-Villars regulator

Wick-rotated SD equation

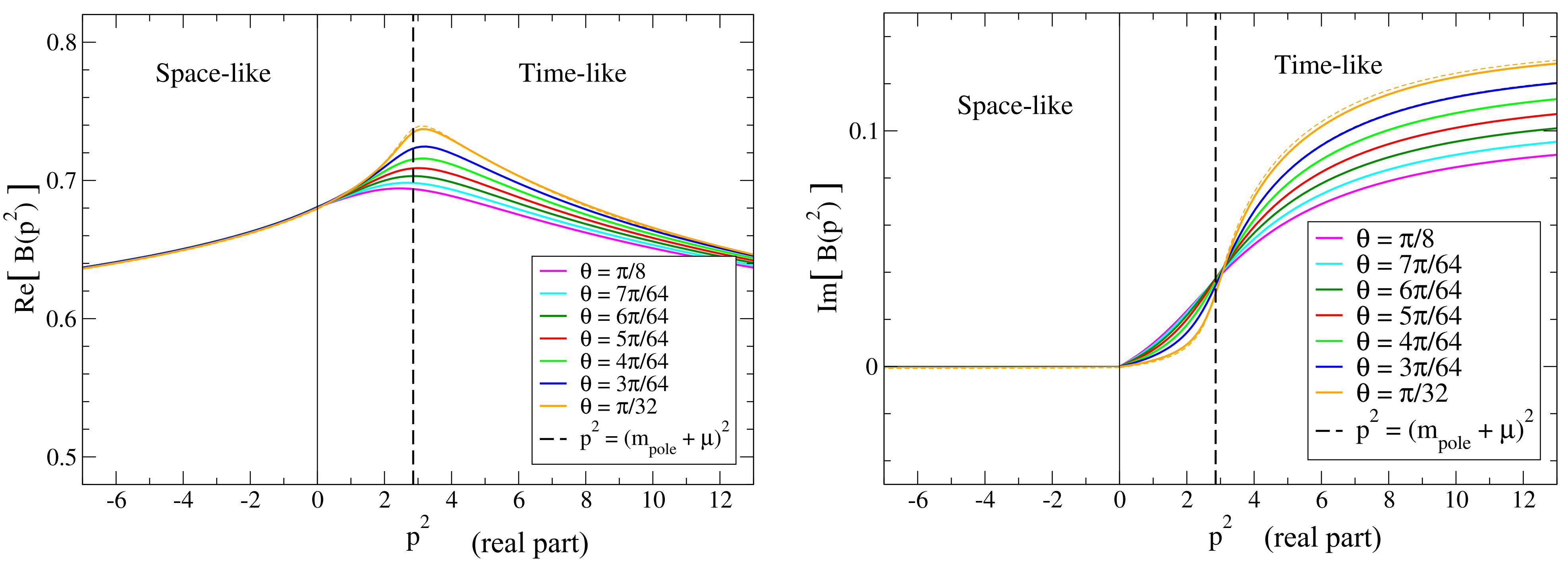
$$A(p_0,\vec{p}^2) \quad = \quad 1 + g^2 \int_{-\infty}^{\infty} dk_0 \int \frac{d^3k}{(2\pi)^4} \frac{A(k_0,\vec{k}^2)\, K^A(p_0,k_0,\vec{p},\vec{k},\vec{p}\cdot\vec{k})}{(k_0^2+\vec{k}^2)\, A^2(k_0,\vec{k}^2) + B^2(k_0,\vec{k}^2)}$$

$$B(p_0,\vec{p}^2) \quad = \quad m_0 + g^2 \int_{-\infty}^{\infty} dk_0 \int \frac{d^3k}{(2\pi)^4} \frac{B(k_0,\vec{k}^2)\, K^B(p_0,k_0,\vec{p},\vec{k},\vec{p}\cdot\vec{k})}{(k_0^2+\vec{k}^2)\, A^2(k_0,\vec{k}^2) + B^2(k_0,\vec{k}^2)}$$

Un-Wick rotation: $k_0 \to k_0 \exp\imath(\theta - \frac{\pi}{2}) \quad p_0 \to p_0 \exp\imath(\theta - \frac{\pi}{2})$

Euclidean $\theta = \frac{\pi}{2}$ **Minkowski** $\theta = 0$

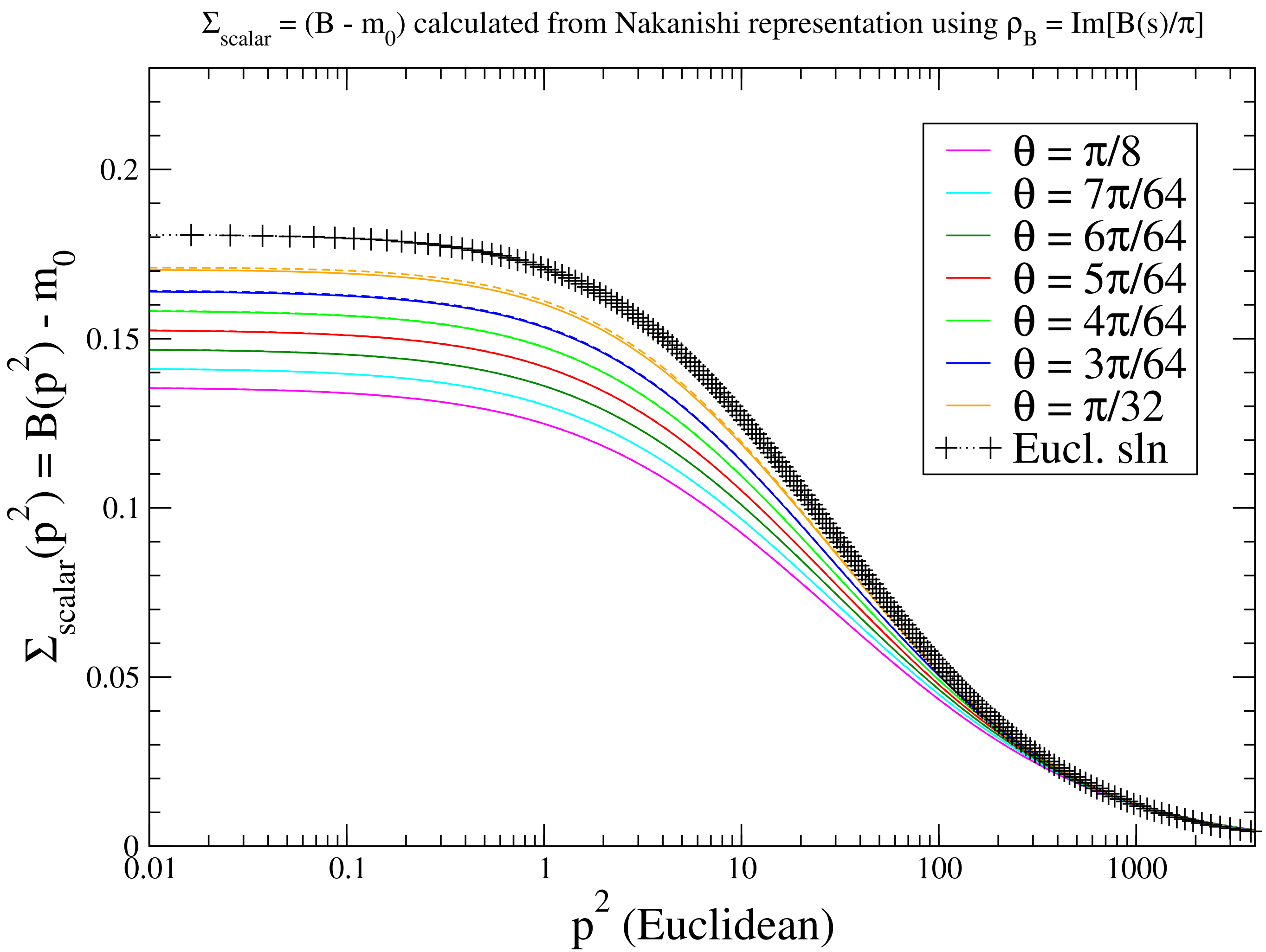
Parameters $m_0 = 0.5 \quad \mu = 1 \quad \alpha = 0.3 \quad \Lambda = 10$



Spectral Representation (Nakanishi Integral representation)

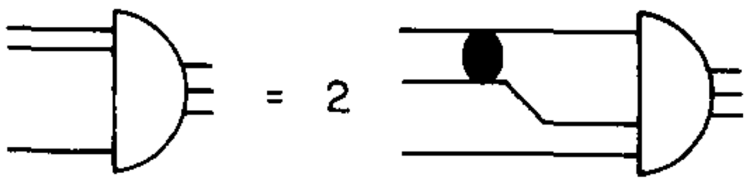
$$\Sigma_{\rm scalar}(p^2) = B(p^2) - m_0 = \int_0^\infty \frac{\rho_B(s)}{p^2 - s + \imath\epsilon}$$

$$\rho_B(s) = -{\rm Im}[B(s)/\pi]$$



Relativistic Three-body Bound states with contact interaction

TF, PLB 282 (1992) 409



$$v(q,p) = 2iF(M_{12}) \int \frac{d^4k}{(2\pi)^4} \frac{i}{[k^2-m^2+i\epsilon]} \frac{i}{[(p-q-k)^2-m^2+i\epsilon]} v(k,p).$$

$$F(M_{12}) = \begin{cases} \dfrac{8\pi^2}{\dfrac{1}{2y'_{M_{12}}}\log\dfrac{1+y'_{M_{12}}}{1-y'_{M_{12}}} - \dfrac{\pi}{2am}}, & \text{if } M_{12}^2 < 0 \\[2ex] \dfrac{8\pi^2}{\dfrac{\arctan y_{M_{12}}}{y_{M_{12}}} - \dfrac{\pi}{2am}}, & \text{if } 0 \le M_{12}^2 < 4m^2 \end{cases}$$

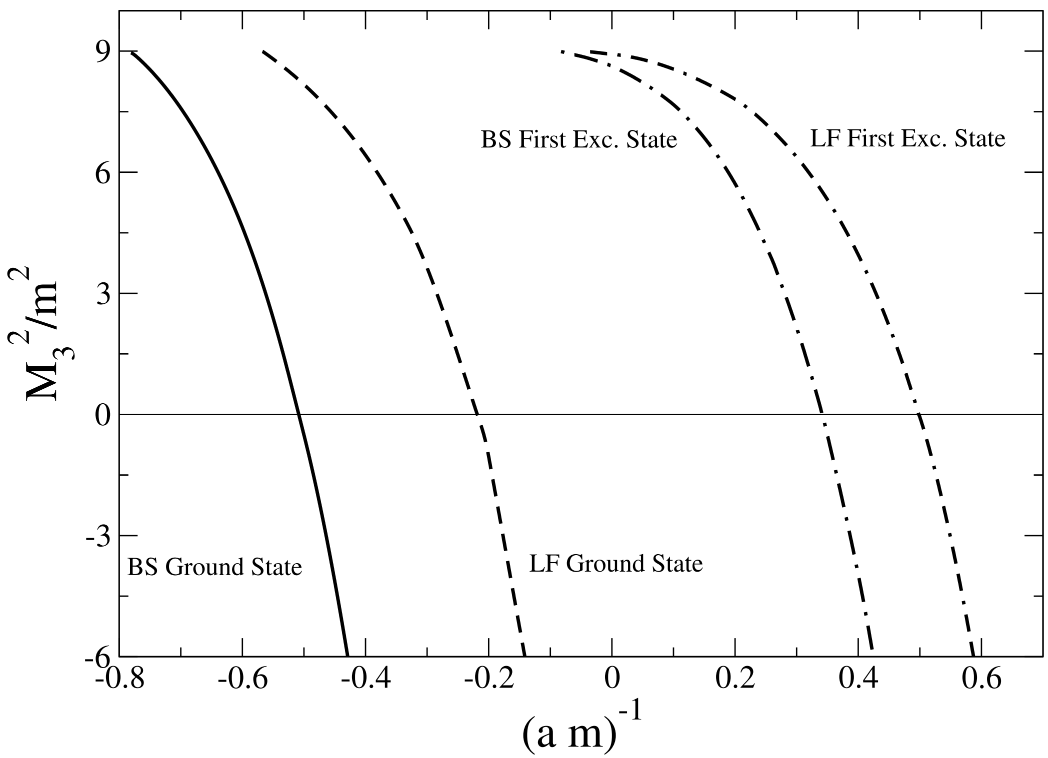
The LF projection in the NREL limit reduces to the SKTM equation!

Euclidean space solution

Ydrefors, Alvarenga Nogueira, TF, Karmanov PLB 770 (2017)131

Wick rotation after the transformation $k = k' + \frac{1}{3}p, \quad q = q' + \frac{1}{3}p.$

Ydrefors, Alvarenga Nogueira, TF, Karmanov PLB 770 (2017)131



Faddeev-BSE in Eucl. space
vs.
Truncation in the LF valence sector

V.A. Karmanov, P. Maris, Few-Body Syst. 46 (2009) 95. ***LF Missing induced three-body forces***

E. Ydrefors et al. / Physics Letters B 770 (2017) 131–137

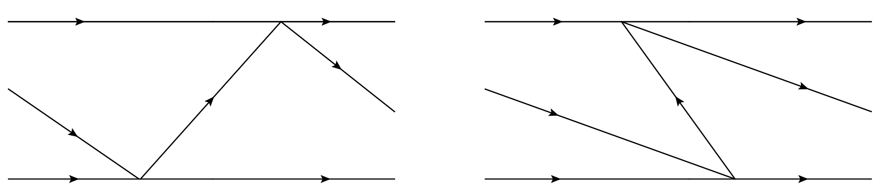


Fig. 2. The three-body LF graphs obtained by time-ordering of the Feynman graph shown in right panel of Fig. 1.

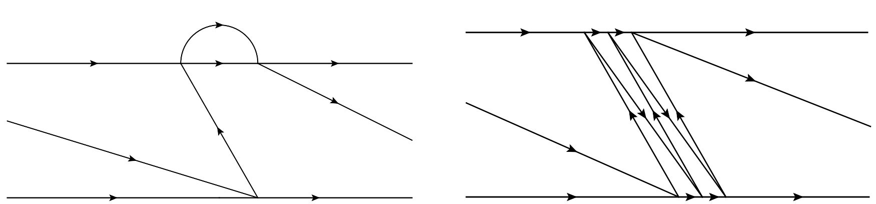
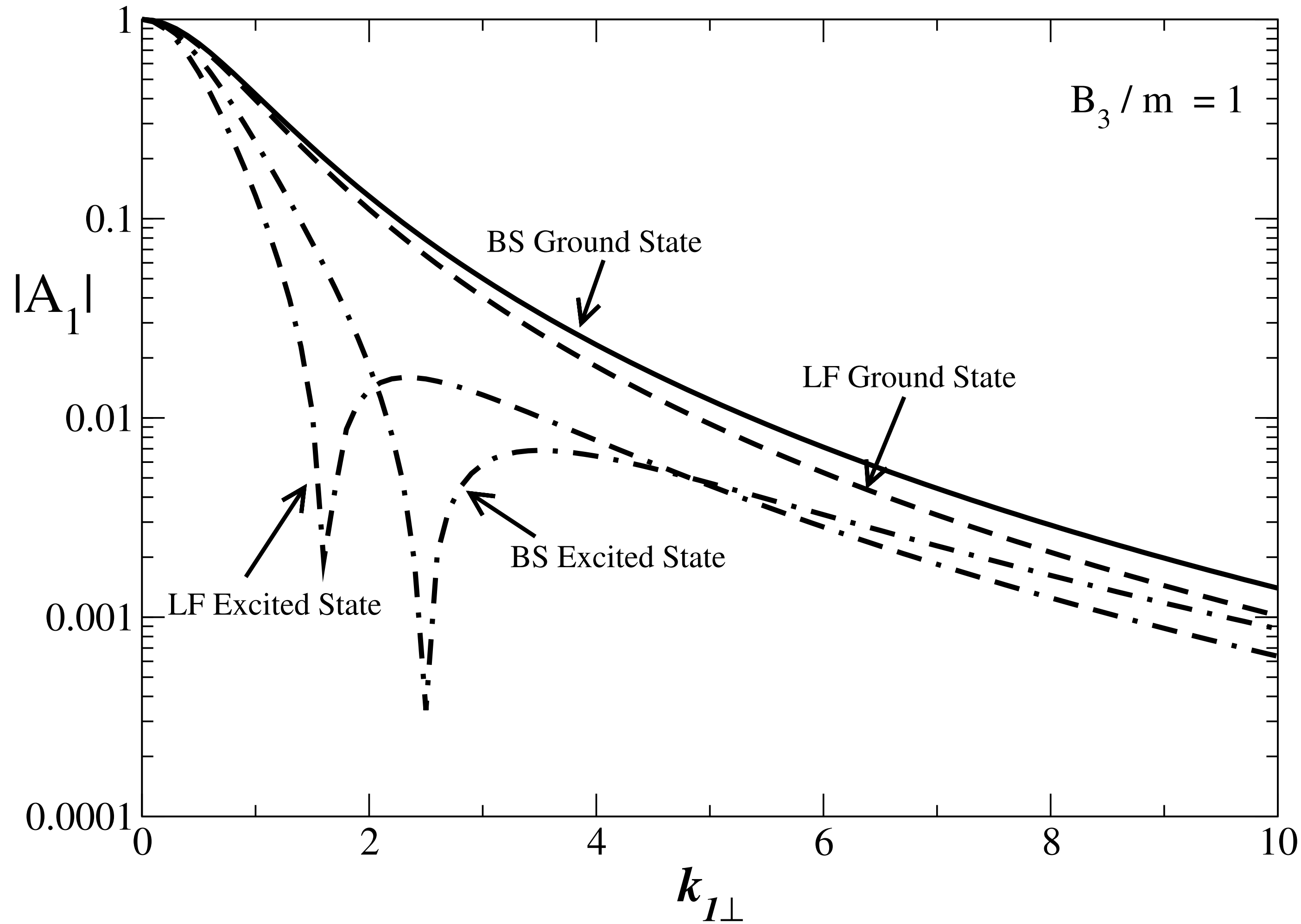


Fig. 3. Examples of many-body intermediate state contributions to the LF three-body forces.

Transverse amplitude

$$A_1(k_{1\perp}) = \int dk_{10}^E \, dk_{1z} \, dk_{20}^E \, dk_{2z} \, d^2k_{2\perp} \, \Phi_E^{i=1}(k_1^\mu, k_2^\mu)$$



Direct solution in Minkowski space

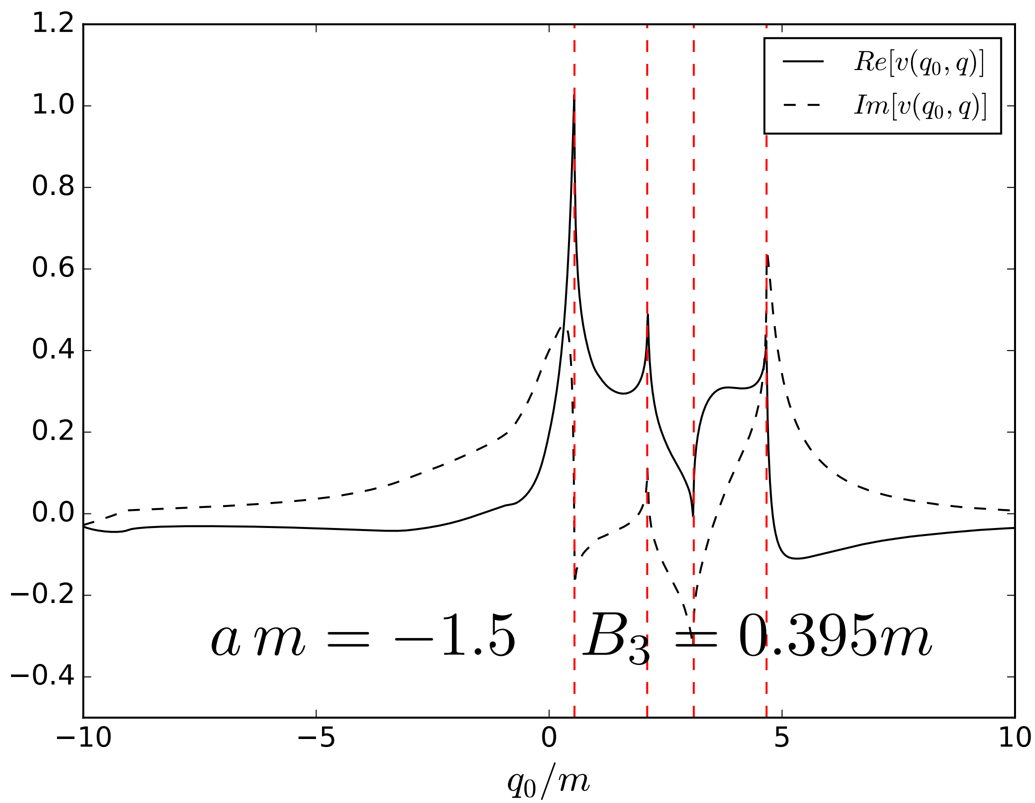
Ydrefors, Alvarenga Nogueira, Karmanov and TF, PL**B791 (2019) 276**

Generalization of the technique used in the two-boson problem in ladder approximation

Carbonell and Karmanov PRD90(2014) 056002

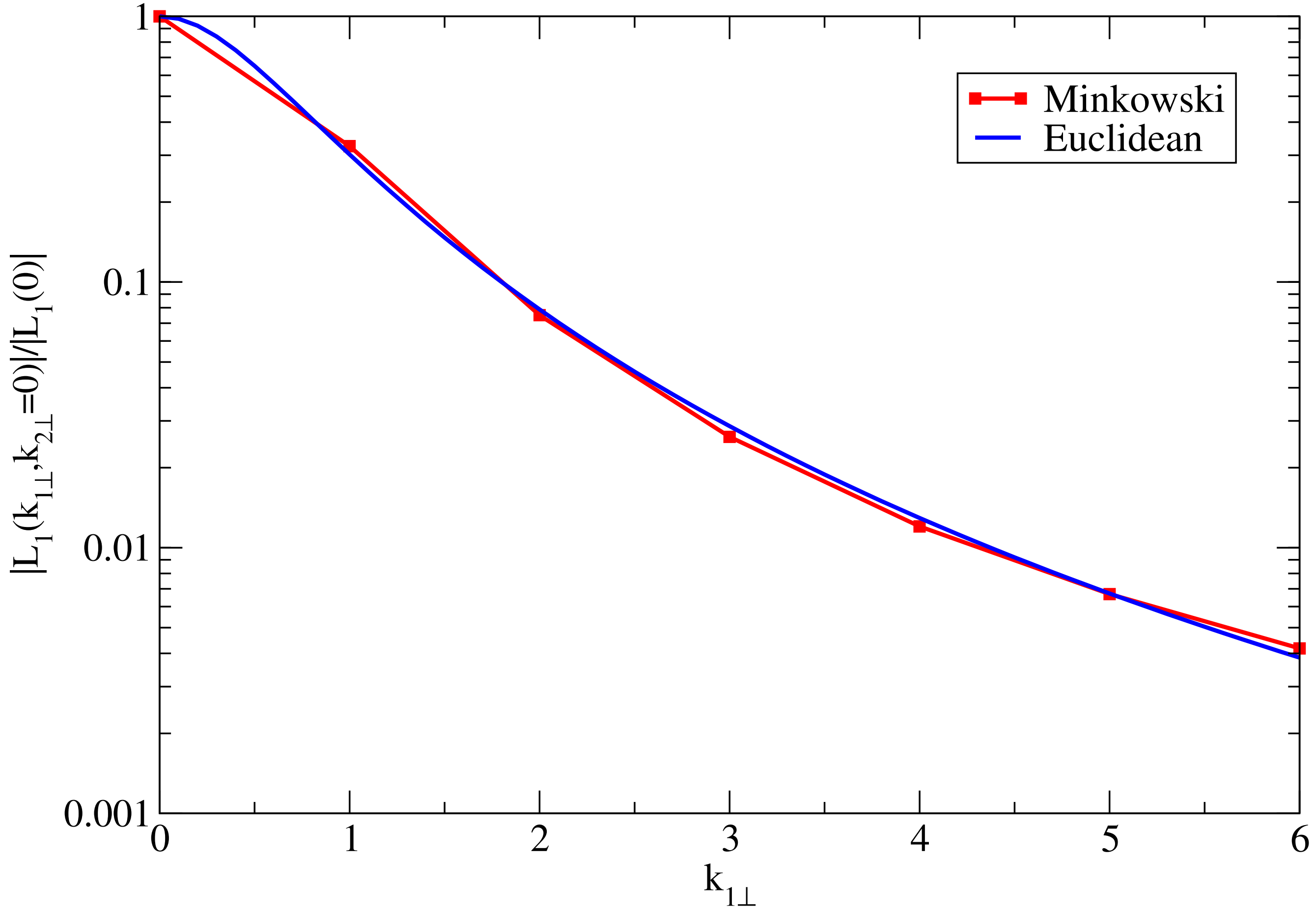
$$
\begin{aligned}
v(q_0,q_v) \;=\; & \frac{\mathcal{F}(M_{12})}{(2\pi)^4}\int_0^\infty k_v^2 dk_v \left\{ \frac{2\pi i}{2\varepsilon_k}\left[\Pi(q_0,q_v;\varepsilon_k,k_v)v(\varepsilon_k,k_v)+\Pi(q_0,q_v;-\varepsilon_k,k_v)v(-\varepsilon_k,k_v)\right]\right.\\
& - 2\int_{-\infty}^0 dk_0 \left[\frac{\Pi(q_0,q_v;k_0,k_v)v(k_0,k_v)-\Pi(q_0,q_v;-\varepsilon_k,k_v)v(-\varepsilon_k,k_v)}{k_0^2-\varepsilon_k^2}\right]\\
& \left. - 2\int_0^\infty dk_0 \left[\frac{\Pi(q_0,q_v;k_0,k_v)v(k_0,k_v)-\Pi(q_0,q_v;\varepsilon_k,k_v)v(\varepsilon_k,k_v)}{k_0^2-\varepsilon_k^2}\right]\right\}, \qquad (9)
\end{aligned}
$$

$v(q_0, q_v = 0.5m)$



Comparison of transverse amplitudes for Euclidean and Minkowski space calculations

$$\int_{-\infty}^{\infty} dk_{10} \int_{-\infty}^{\infty} dk_{1z} \int_{-\infty}^{\infty} dk_{20} \int_{-\infty}^{\infty} dk_{2z}\; i\Phi_M(k_{10}, k_{1z}, k_{20}, k_{2z}; \vec{k}_{1\perp}, \vec{k}_{2\perp}).$$



Conclusions

- **Integral Representation to solve Dyson-Schwinger in diferente gauges;**
- **Un-Wick rotation: BSE and SD - promissing tool allied to Integral Representations;**
- **NIR for bosonic and fermionic BSE: control of the LF singularities-fermions;**
- **Euclidean and Minkowski BSE for 3-bosons;**

Perspectives

- **Self-energies, quark-gluon vertex, ingredients from LQCD**
  (Oliveira, Paula, TF, de Melo, EPJC79(2019)116)….
- **Confinement – How to include with Integral Representation?**
- **Beyond the pion, kaon, D, B, rho…, and the nucleon**
- **Form-Factors (preliminar), PDFs, TMDs, Fragmentation Functions...**