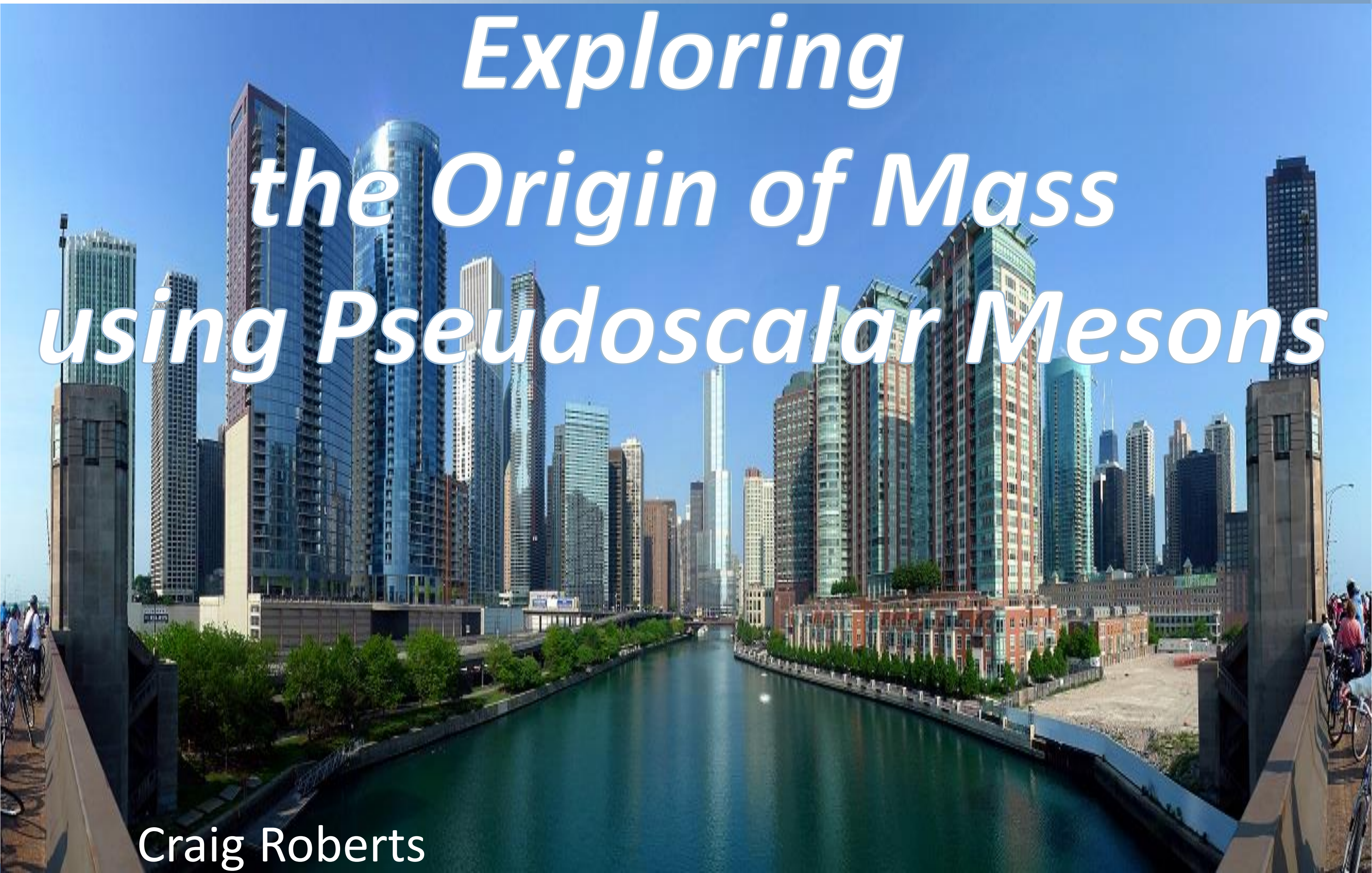
Exploring the Origin of Mass using Pseudoscalar Mesons



Craig Roberts

Strong Interactions in the Standard Model

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i\,(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij})\,\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

- Only apparent scale in chromodynamics is mass of the quark field
- Quark mass is said to be generated by Higgs boson.
- In connection with everyday matter, that mass is 1/250$^{\rm th}$ of the natural (empirical) scale for strong interactions,
  viz. more-than two orders-of-magnitude smaller
- Plainly, the Higgs-generated mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
  - No amount of staring at $L_{\rm QCD}$ can reveal that scale
- Contrast with quantum electrodynamics, *e.g*. spectrum of hydrogen levels measured in units of m_e, which appears in $L_{\rm QED}$

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Classically, in a scale invariant theory
  the *energy-momentum tensor must be traceless:* $T_{\mu\mu} \equiv 0$
- Regularisation and renormalisation of (ultraviolet) divergences in Quantum Chromodynamics introduces a mass-scale
  … *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ
- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, $\mathcal{L}(m=0)$

$$\Rightarrow \partial_\mu \mathcal{D}_\mu = \delta\mathcal{L}/\delta\sigma = \alpha\beta(\alpha)\, d\mathcal{L}/d\alpha = \boxed{\beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0}$$

Trace anomaly

QCD β function

Quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal
  … Indicates only that a mass-scale must exist
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$
$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$
$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0
  … In QCD, Θ_0 measures the strength of gluon self-interactions
  … so, from one perspective,
  m_p is (somehow) completely generated by glue.

On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chral limit

$$\langle \pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- Difficult way to obtain "zero"!
- Easier to imagine that "zero" owes to cancellations between different operator contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.

  It could only arise naturally because

  of some symmetry and/or symmetry-breaking pattern.

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

  "How does the mass of the proton arise?"

  is complete without the additional clause

  "How does the pion remain ***massless****?"*

- Natural visible-matter mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems
  - Expectation value of Θ_0 in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions = m_p

Whence "1" and yet "0" ?

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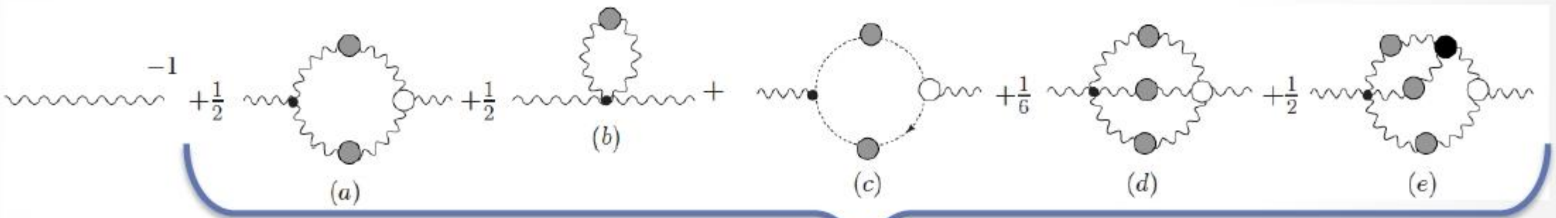
- Natu ... ısly with ... stems
  - Ex ... ıe of th ... = m_p

Elucidate the entire array
of empirical consequences
of the mechanism responsible
so that the theory can be validated

$$\Delta^{-1}_{\mu\nu}(q) =$$



$$\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

Gluon Gap Equation

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons become massive!

- Running gluon mass

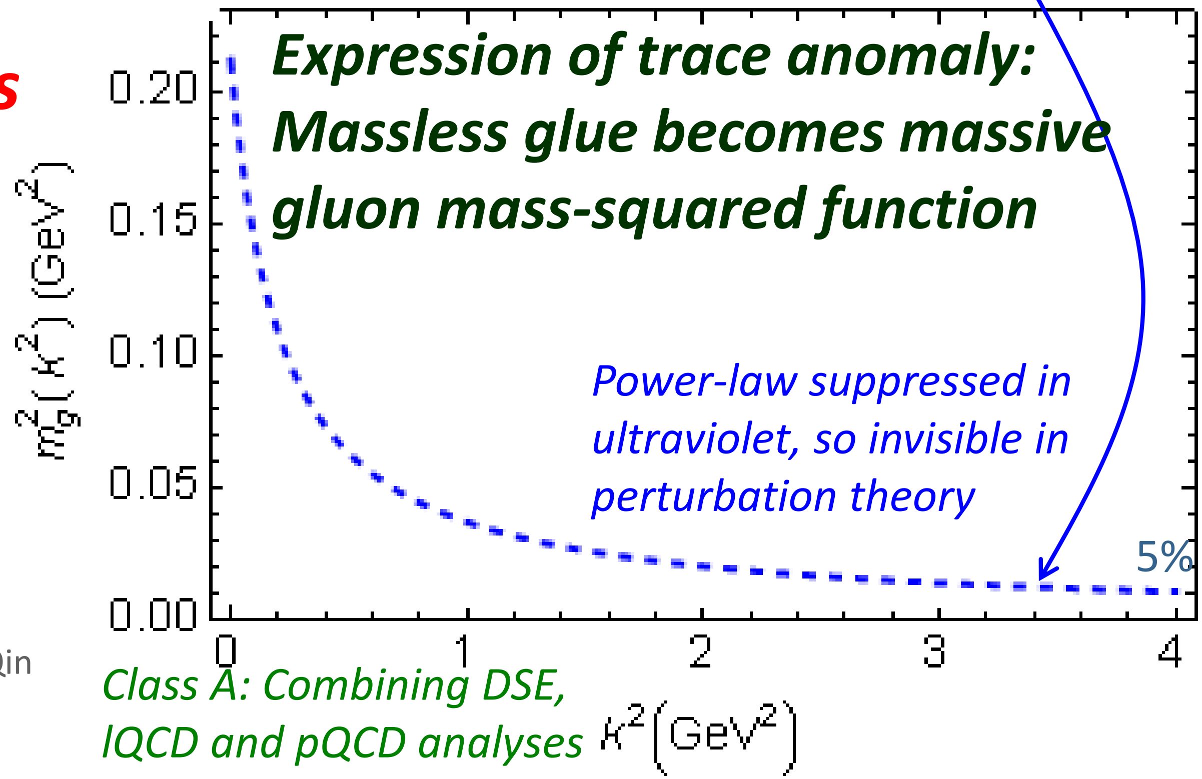
$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_S(0) \approx \pi \qquad m_g^2(0) = (0.46\,\mathrm{GeV})^2$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!

$\mu_g \approx \frac{1}{2} m_p$

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$



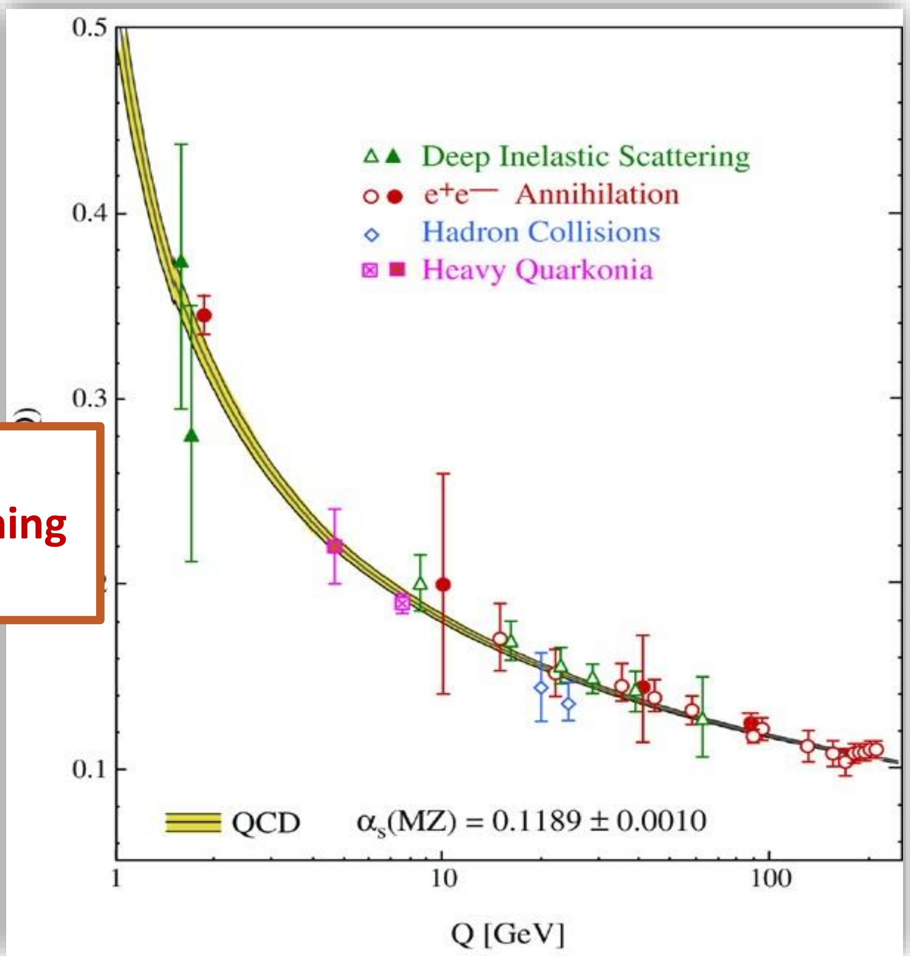
Expression of trace anomaly: Massless glue becomes massive gluon mass-squared function

Power-law suppressed in ultraviolet, so invisible in perturbation theory

Class A: Combining DSE, lQCD and pQCD analyses of QCD's gauge sector

Interaction model for the gap equation, S.-x.Qin *et al.*, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

This is where we live



⇐
What's happening out here?!

QCD's Running Coupling

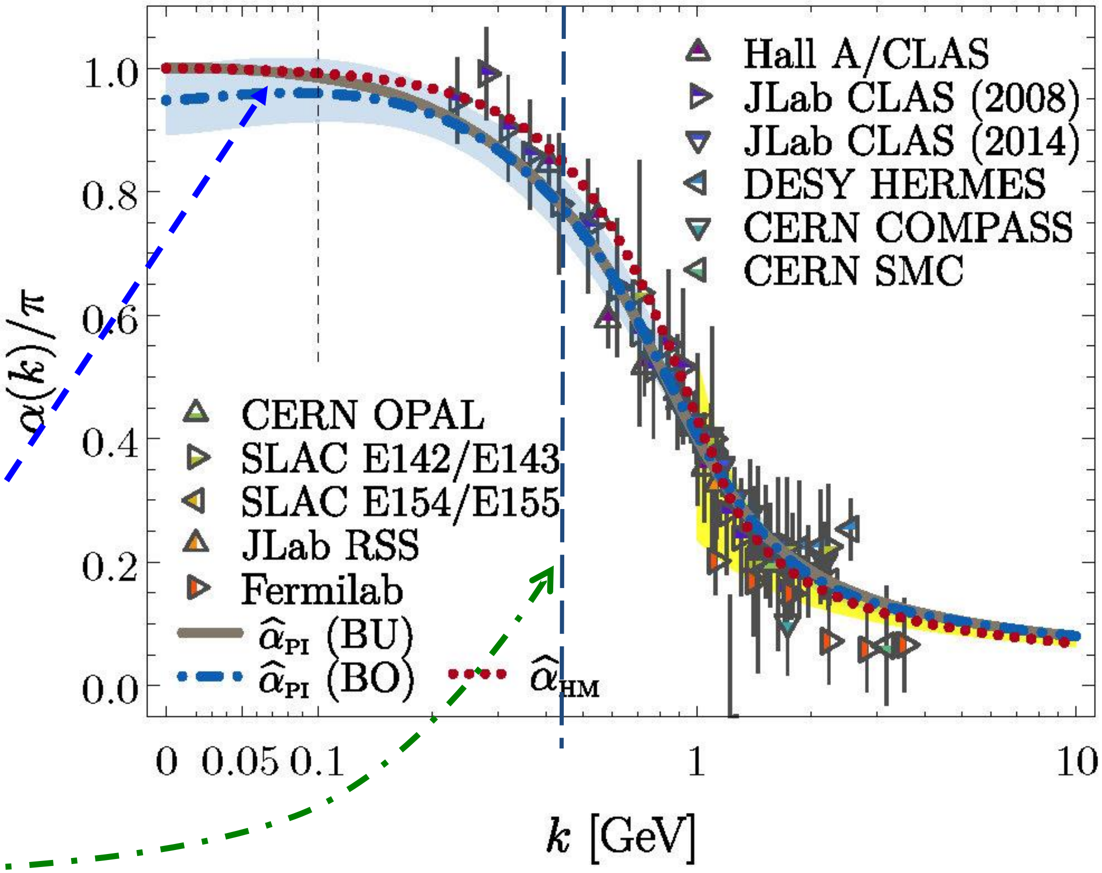
Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7
The QCD Running Coupling,
A. Deur, S. J. Brodsky and G. F. de Teramond,
Prog. Part. Nucl. Phys. **90** (2016) 1-74

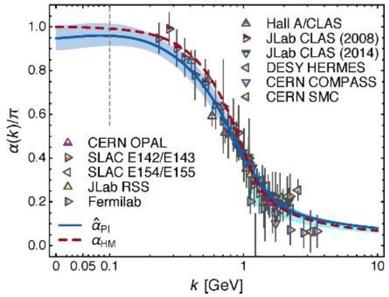
Process-independent effective-charge in QCD

- Modern continuum & lattice methods for analysing gauge sector enable

  "Gell-Mann – Low"

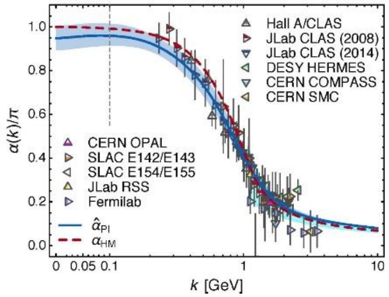
  running charge to be defined in QCD
- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$





QCD Effective Charge

- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge-boson two-point function.
- $\hat{\alpha}_{PI}$ is
  - process-independent
  - appears in every one of QCD's dynamical equations of motion
  - known to unify a vast array of observables
- $\hat{\alpha}_{PI}$ possesses an infrared-stable fixed-point
  - Nonperturbative analysis demonstrating absence of a Landau pole in QCD
- QCD is IR finite, owing to dynamical generation of gluon mass-scale
- Asymptotic freedom ⇒ QCD is well-defined at UV momenta
- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***



QCD Effective Charge

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- $\hat{\alpha}_{PI}$ is
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- ***QCD is therefore unique amongst known 4D quantum field theories***
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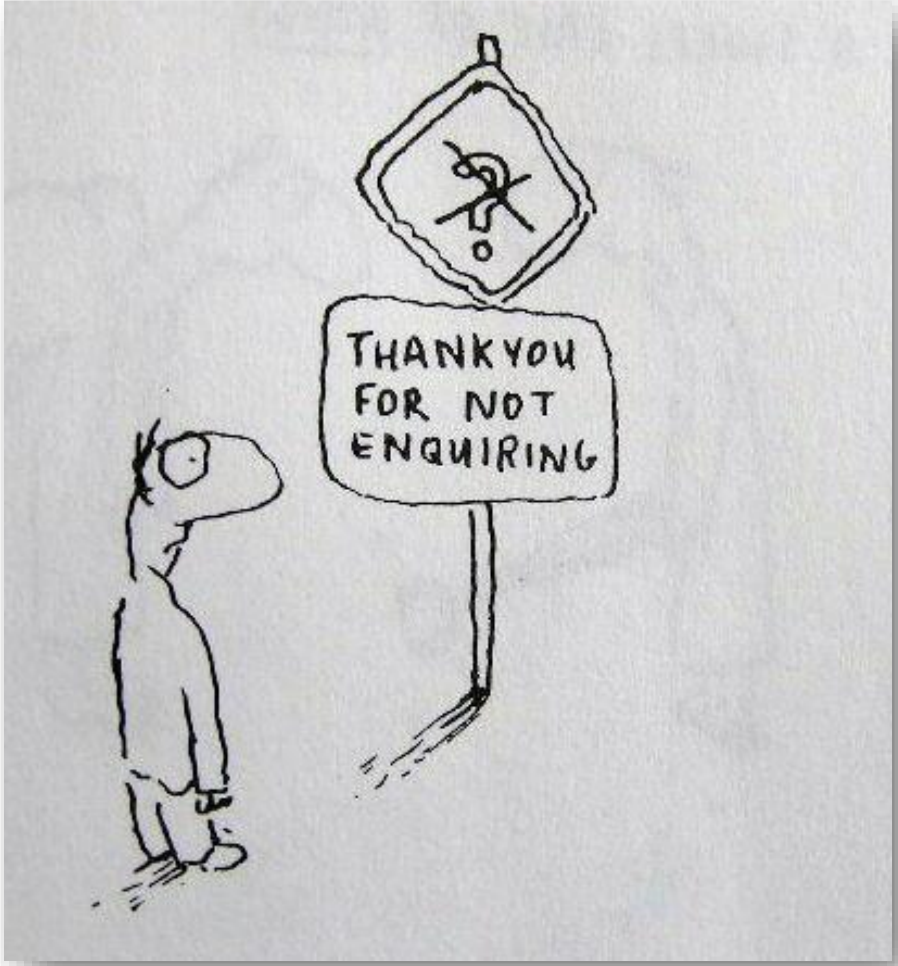
Conceivably, therefore, QCD can serve as a basis for theories that take physics beyond the Standard Model



Enigma of Mass

Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]$$

This means that π necessarily has dressed-quark L=0 & L=1 components in any frame
Twist-3 on light-front

- Dressed-quark propagator $S(p)=\dfrac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

$$f_\pi E_\pi(p^2) = B(p^2)$$

Enigma of mass

This algebraic identity is why QCD's pion is massless in the chiral limit

- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

The answer is algebraic

*Munczek, H. J., Phys. Rev. D **52** (1995) pp. 4736-4740*
*Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B **380** (1996) pp. 7-12*
*Maris, P. , Roberts, C.D. and Tandy, P.C., Phys. Lett. B **420** (1998) pp. 267-273*
*Binosi, Chang, Papavassiliou, Qin, Roberts, Phys. Rev. D **93** (2016) 096010/1-7*

Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations
  - Bethe-Salpeter Kernel:
    - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
    - Interactions of arbitrary but enumerable complexity involving these "basis vectors"
  - Chiral limit:
    - Algebraic proof
      - at any & each finite order in symmetry-preserving construction of kernels for
        - » the gap (quark dressing)
        - » and Bethe-Salpeter (bound-state) equations,
      - there is a precise cancellation between
        - » mass-generating effect of dressing the valence-quarks
        - » and attraction introduced by the scattering events
    - Cancellation guarantees that
      - simple system, which began massless,
      - becomes a complex system, with
        - » a nontrivial bound-state wave function
        - » attached to a pole in the scattering matrix, which remains at $P^2=0$ …
    - Interacting, bound system remains massless!

Munczek, H. J., Phys. Rev. D **52** (1995) pp. 4736-4740
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Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations

Quantum field theory statement:
In the pseudsocalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi E_\pi(p^2) = B(p^2)$$

- Cancellation guarantees that
  - simple system, which began massless,
  - becomes a complex system, with
    - a nontrivial bound-state wave function
    - attached to a pole in the scattering matrix, which remains at $P^2=0$ …
- Interacting, bound system remains massless

Revealing Mass

Consequences … 1

- Mass is dynamically generated in QCD: Scale $\sim \Lambda_{\mathrm{QCD}}$
  - Empirically $\Lambda_{\mathrm{QCD}} \approx 0.2$ GeV … Standard Model can't predict this value.
- Gluon self-interactions make $\Lambda_{\mathrm{QCD}} \approx 0.2$ GeV possible.
  They do not guarantee it.
- *Understanding* of observables (almost always) depends on frame of reference and scale of probe
  - gluons and quarks → dressed quasiparticles:
    - massless in perturbation theory
    - possess mass functions which are large at infrared momenta $\lesssim m_g \approx 2\,\Lambda_{\mathrm{QCD}}$
  - at hadronic scale: wave functions, cross-sections, *etc*. are most readily understood using evolving quasiparticle operators for dressed-g, -q
    - Each contains a (distinct) countable infinity of partons

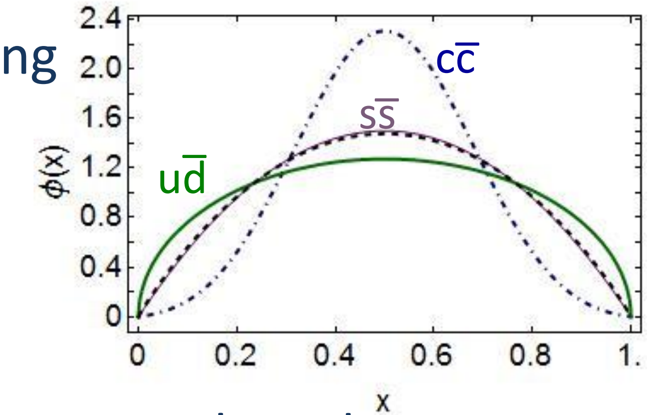
⇒ All bound-states have GeV-scale masses

⇒ Except Nambu-Goldstone modes

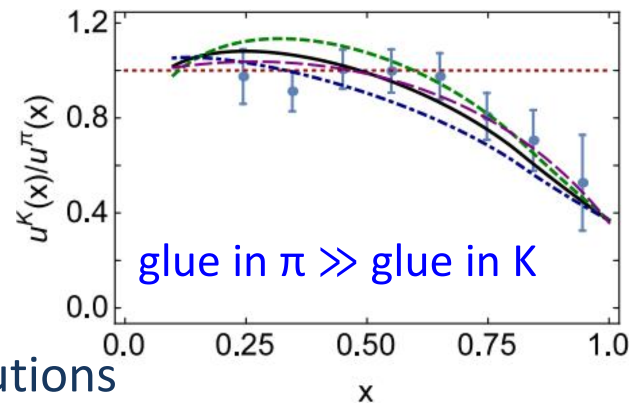
✓ DCSB: whilst constituents are massive, NG-modes are (nearly) massless

Consequences … 2

- QCD's unique Gell-Mann–Low effective coupling
  - ✓ Infrared finite … $\alpha(\sim 0) \approx \pi$
  - ✓ Landau pole of perturbation theory is eliminated by emergence of gluon mass
  - ✓ Cross-sections are free of infrared divergences



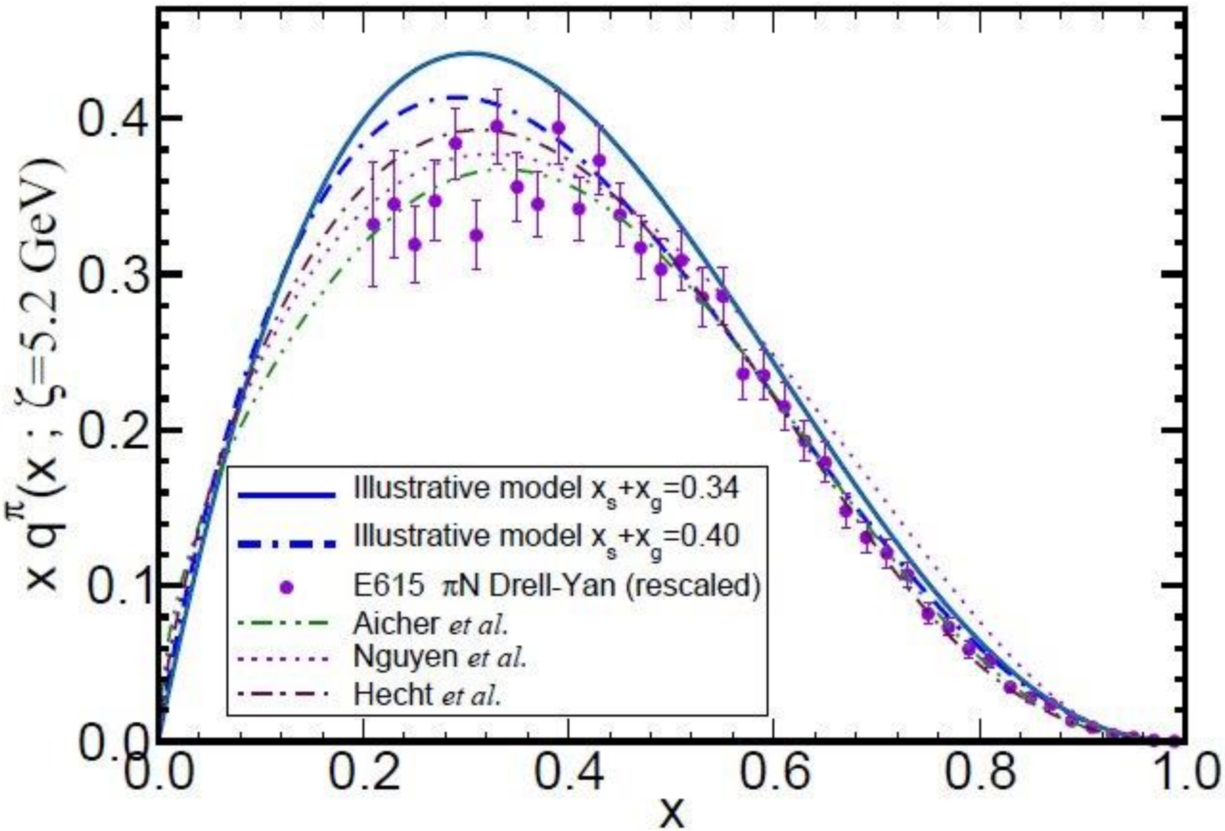
- PDAs of ground-state *S*-wave mesons and baryons are broad, concave functions
  - Numerous empirical consequences ⇒ empirically verifiable
    - Hadron elastic and transition form factors
- Emergent vs Explicit (Higgs) mass generation
  - s-quark defines a boundary:
    - emergent mass generation dominates for $m < m_s$
    - but explicit (Higgs) mass is most important for $m > m_s$
  - s-quark/u-quark comparisons in parton distributions are a sensitive probe of emergent mass and its distribution



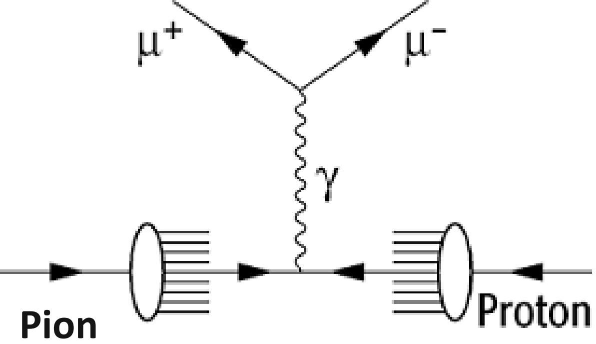
Consequences … 3

- Existence of nonpointlike scalar and axial-vector diquarks in nucleon.
  - Axial-vector correlations are essential
- Empirically verifiable consequences
  - Example … proton's tensor charges:
    - $\delta_T d \neq 0 \Rightarrow$ rules-out scalar-diquark-only nucleon
    - $\delta_T u \approx 4\,|\,\delta_T d\,|$ … understood via highly-correlated proton wave function
- Baryon resonances:
  - Relative strengths of 0+ and 1+ change & new diquarks appear (0- & 1-)
  - Vast array of new predictions whose testing is crucial to validating the emergent mass paradigm
- Hybrid Mesons
  - Just like diquark correlations exist in baryons …
  - $q_g = g+q$ and $\bar{q}_g = g+\bar{q}$ correlations very probably also exist in systems with valence glue.
  - Hybrid mesons may be understood as highly-correlated $q_g\bar{q} \leftrightarrow q\bar{q}_g$ bound-states

Observing *Mass*



π & *K* Valence-quark Distribution Functions



Continuum QCD prediction of π valence-quark distributions

- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:
  $\pi\, p \to \mu^+ \mu^- X$
- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 >> m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a resolving scale Q_0

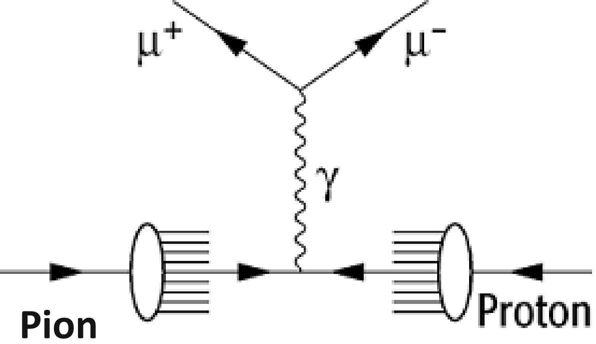
$$u_\pi(x;Q_0) \sim (1-x)^{2\beta}$$

namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

- In QCD, β=1 and hence

QCD: $Q>Q_0 \Rightarrow 2 \to 2+\gamma,\ \gamma > 0$

$$^{QCD}u_\pi(x;Q_0) \sim (1-x)^2$$



Empirical status of the Pion's valence-quark distributions

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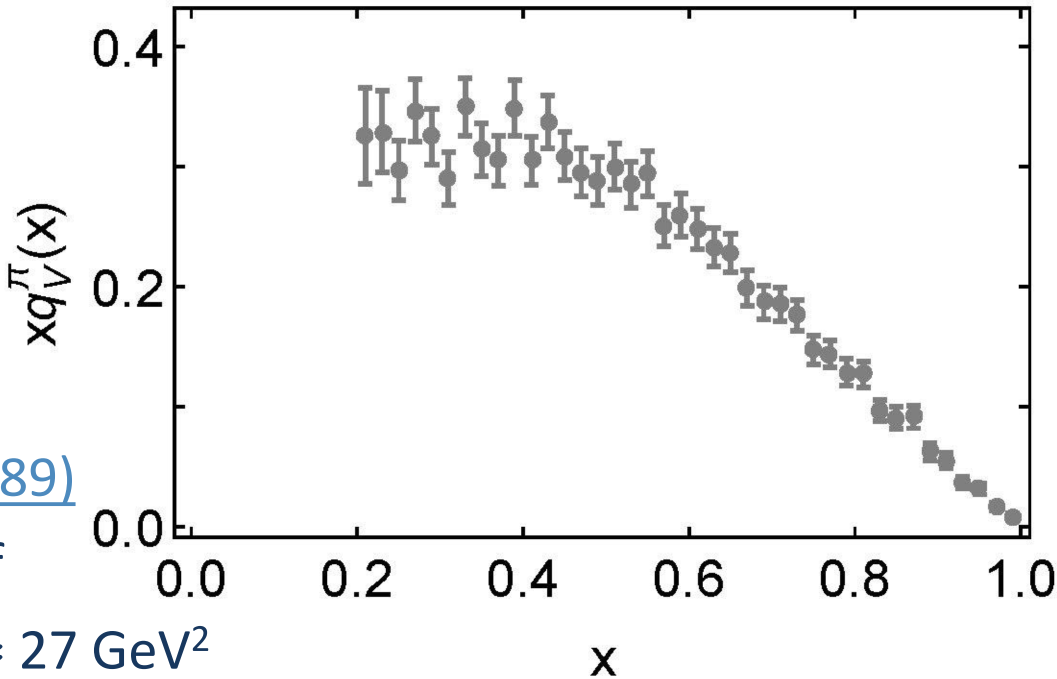
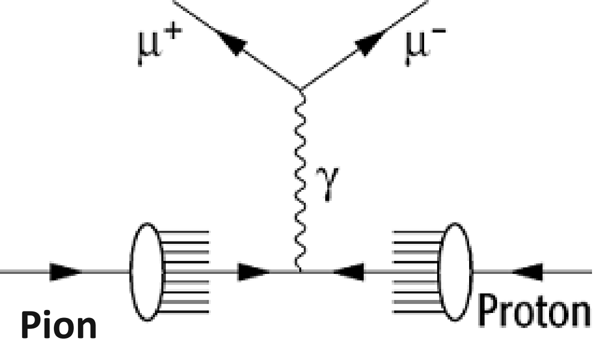
  $\pi\, p \rightarrow \mu^+ \mu^- X$
- Three experiments:
  - CERN (1983 & 1985)
  - FNAL (1989).
- None more recent
- Conway *et al.*
  [Phys. Rev. D **39**, 92 (1989)]
  - Leading-order analysis of the Drell-Yan data: $Q^2 \approx 27\ \text{GeV}^2$
  - ~ 400 citations





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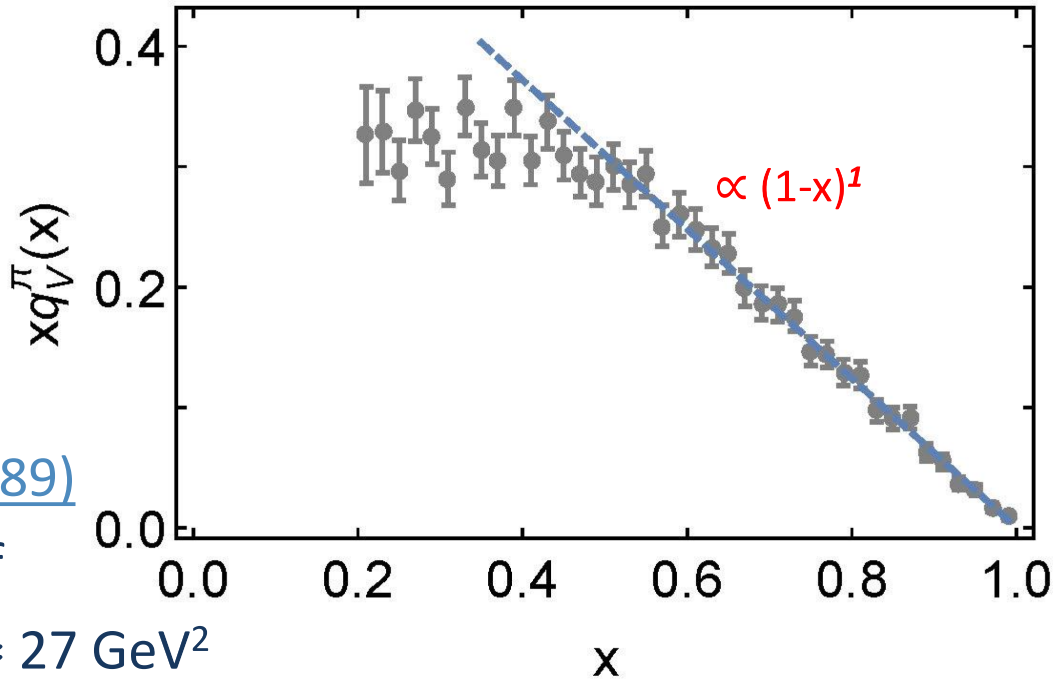
- ***Controversial!***



π valence-quark distributions 20 Years of Evolution

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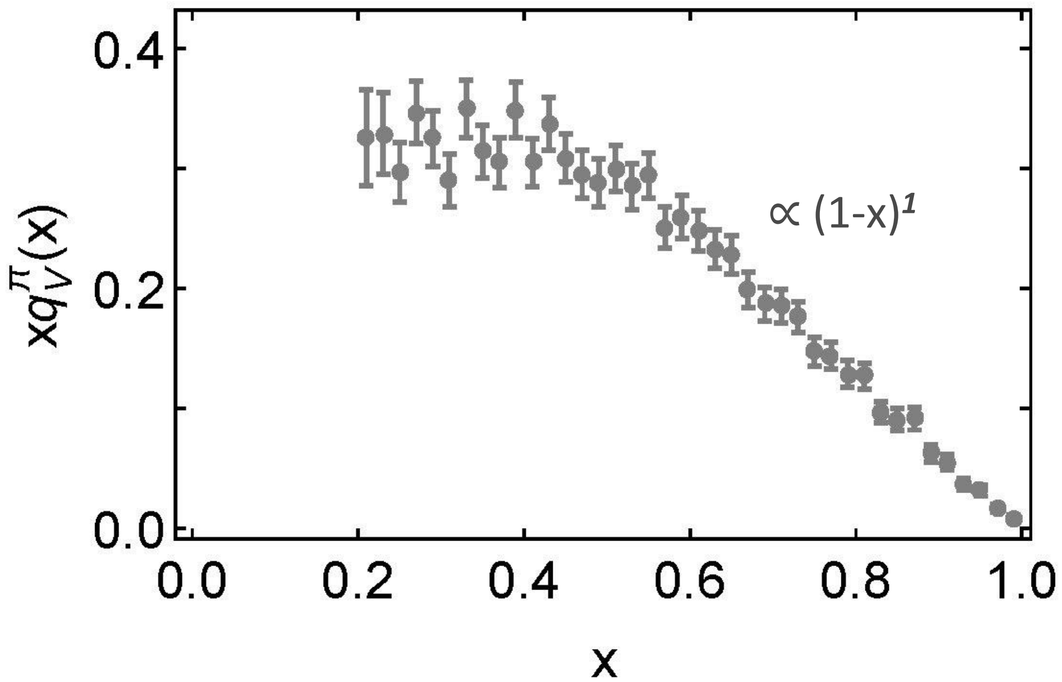

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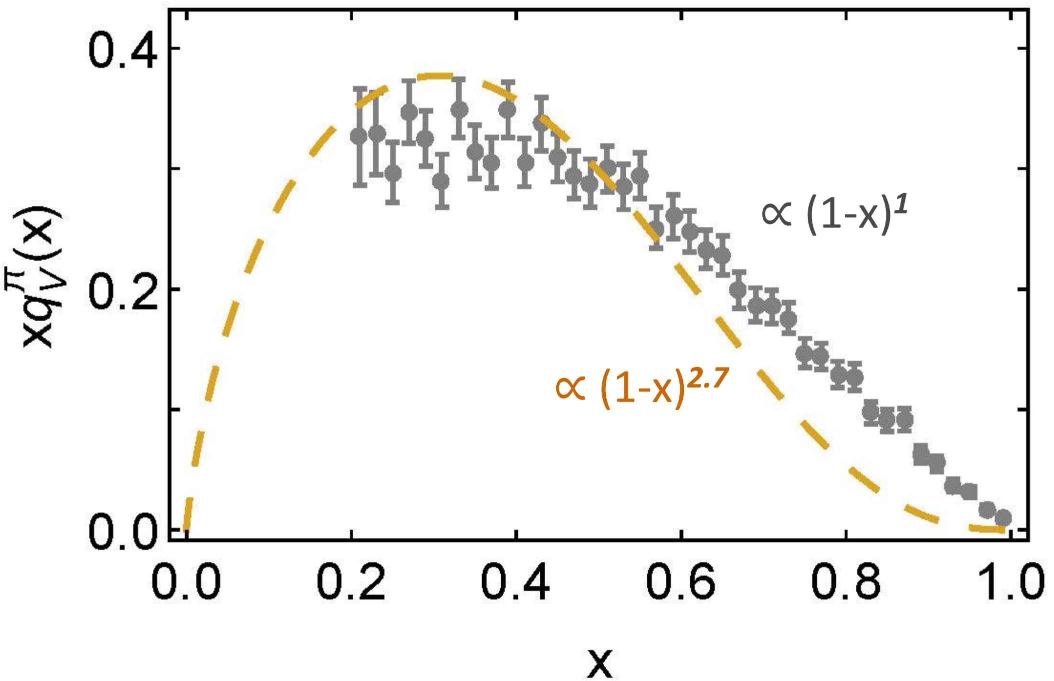

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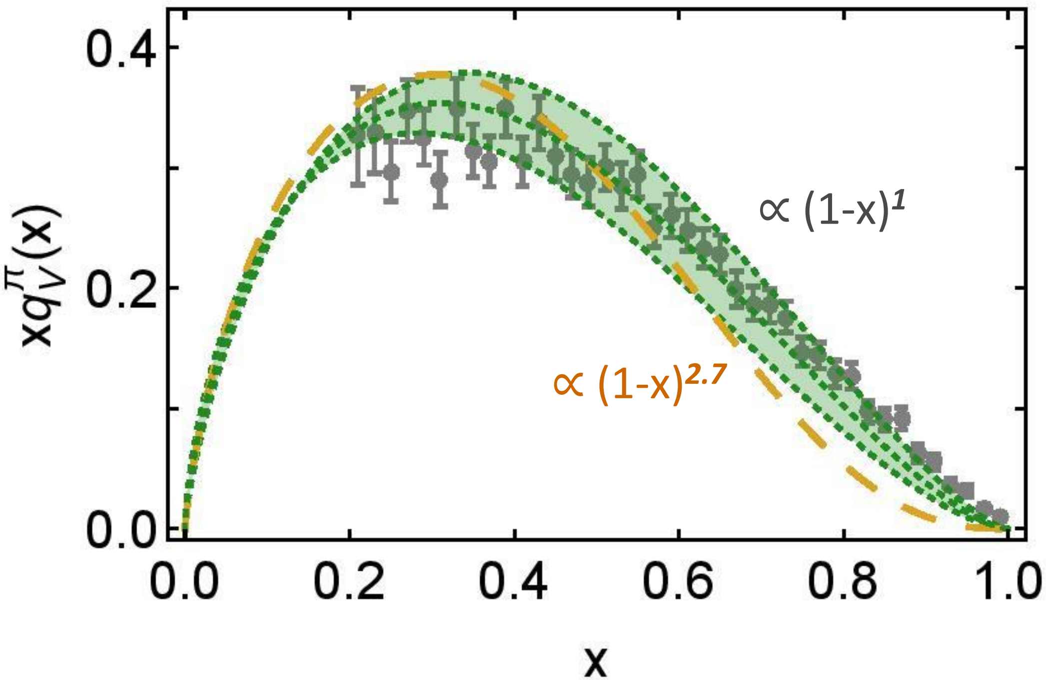

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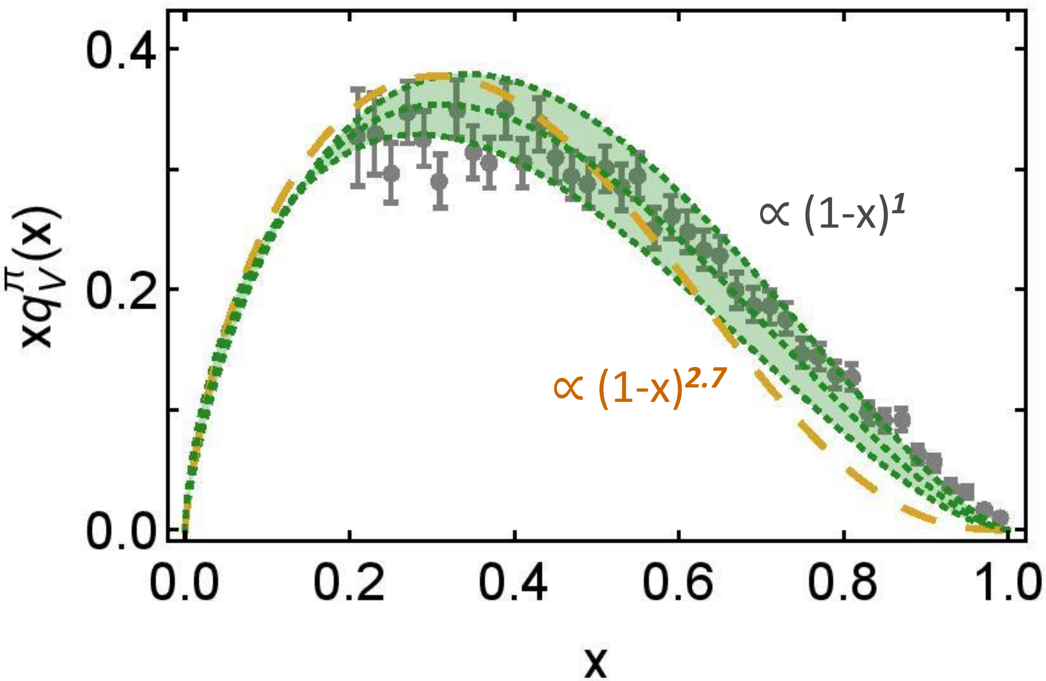

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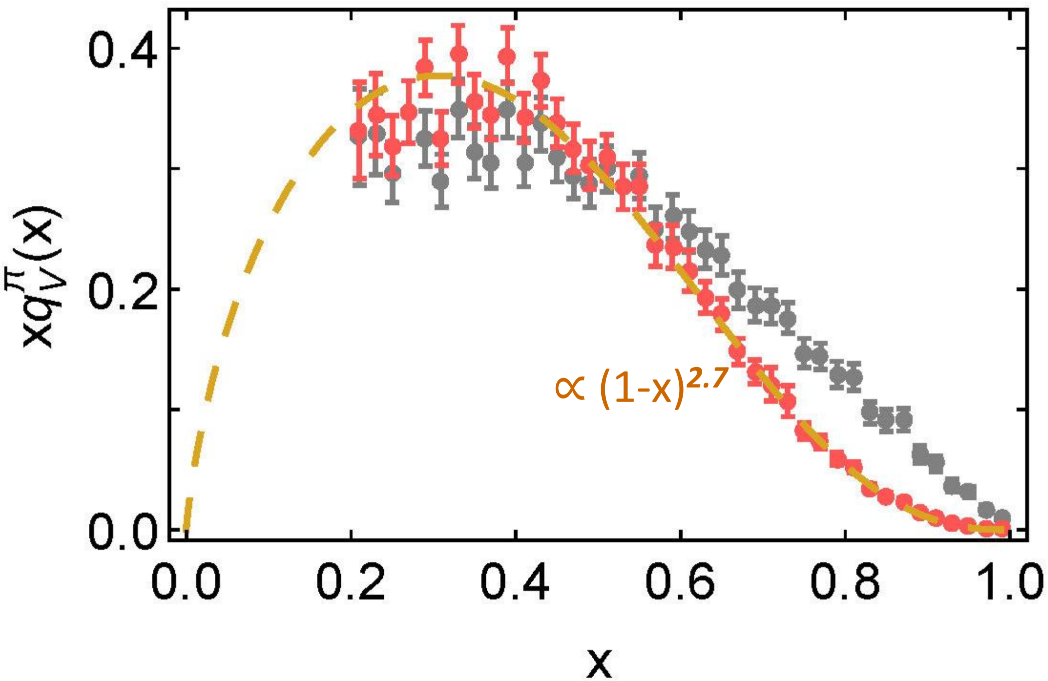

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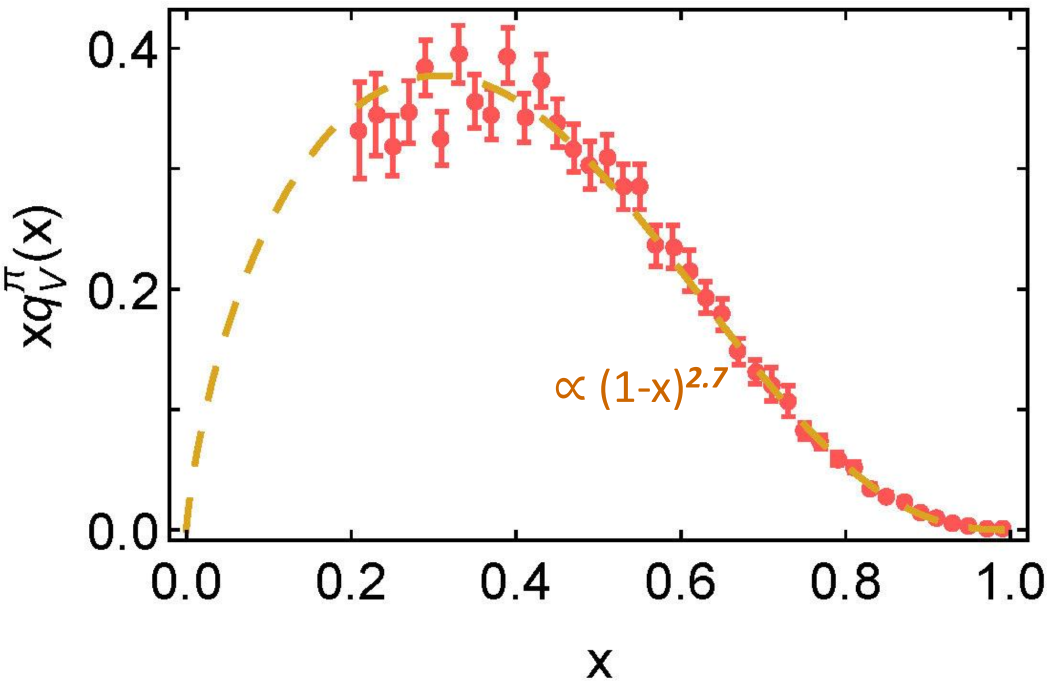
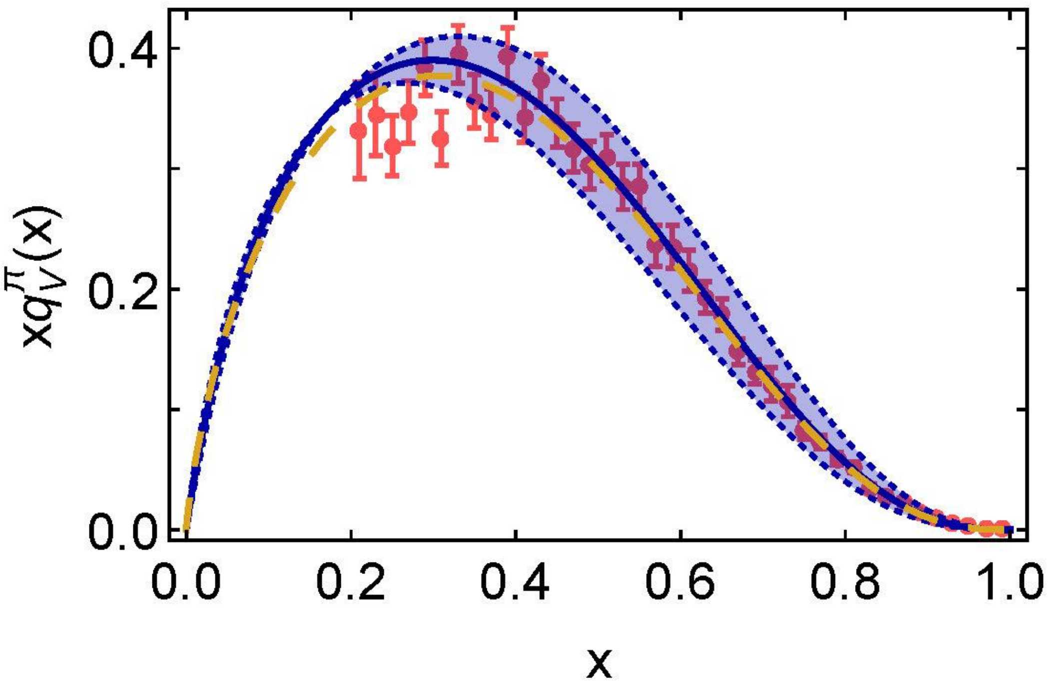


π valence-quark distributions 20 Years of Evolution

- 2010/09 … Reconsideration of data: Aicher *et al.*, Phys. Rev. Lett. **105** (2010) 252003
  - Consistent next-to-leading-order analysis
- 2019/04 … Ding, *et al.*
  - Continuum QCD prediction, using most sophisticated bound-state kernels currently available



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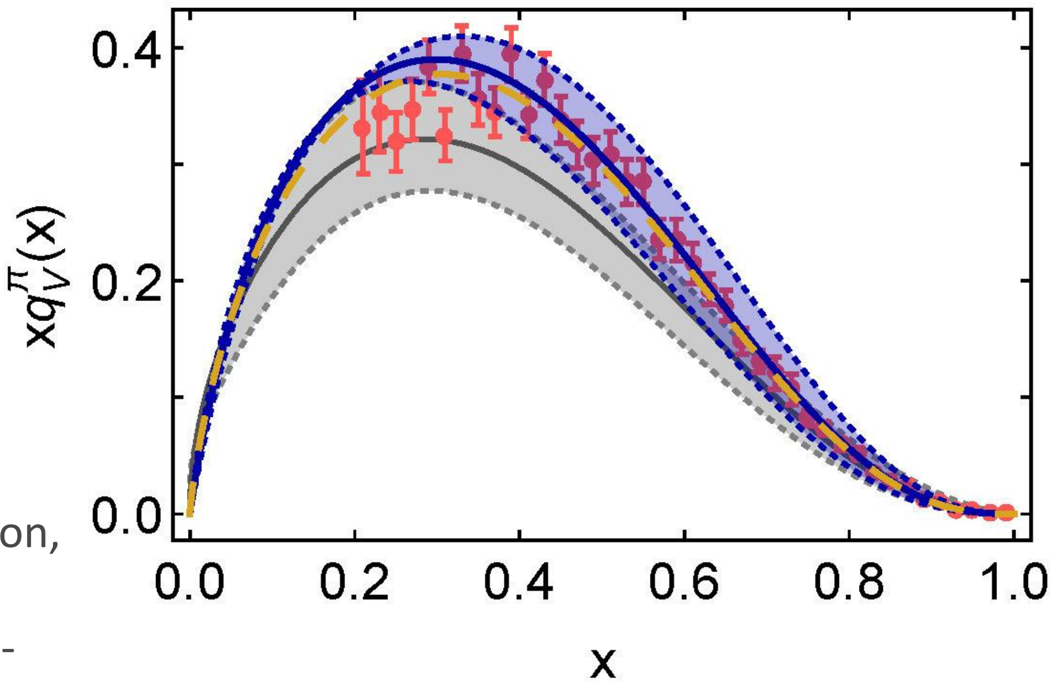
  - $m_\pi^2 = 9\ m_\pi^2$-physical

Large-*x* exponent $^{\zeta = 5.2\text{GeV}}$ and momentum$^{\zeta = 2\text{GeV}}$

Continuum … 2.3(1) & <2*x*> = 0.52±0.04

Lattice … 2.2(1) & <2x> = 0.42±0.05



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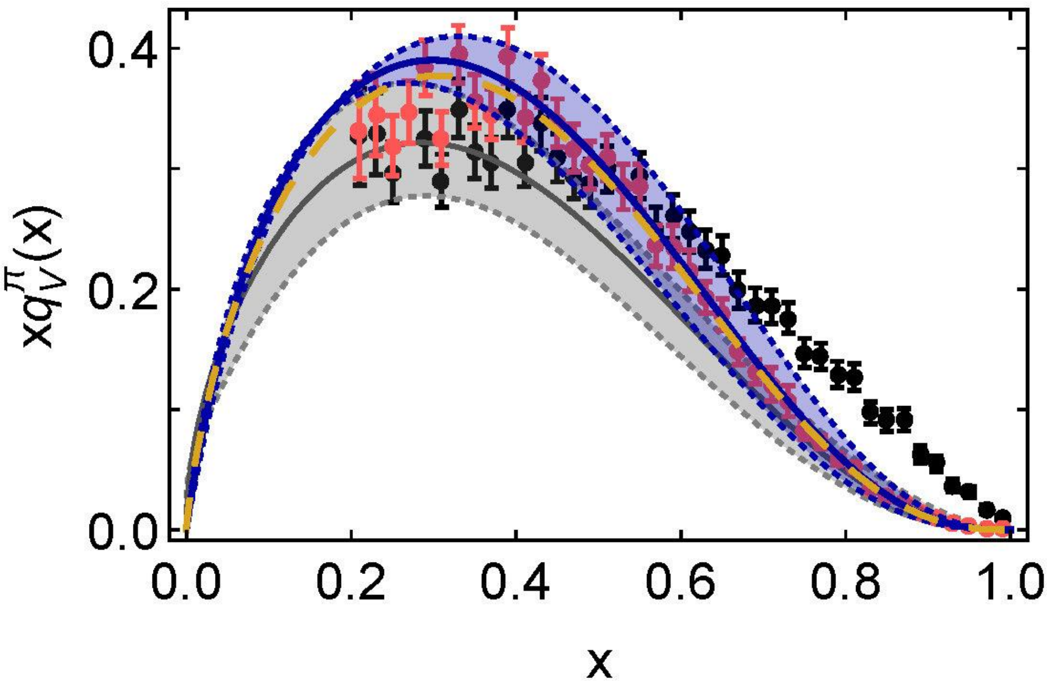


Modellers still ignoring QCD & violating symmetries

Phenomenologists question analysis of Aicher *et al.*

π & K PDFs

- Urgent need for Newer Data
  - Persistent controversy regarding the Bjorken-$x \simeq 1$ behaviour of the pion's valence-quark PDF
  - Single modest-quality measurement of $u^K(x)/u^\pi(x)$ *(1980)* cannot be considered definitive.
- Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question
- Similar technique might also serve for the kaon … experiment approved
- Future:
  - New mesonic Drell-Yan measurements at modern facilities (COMPASS at LHC) could yield valuable information on π and K PDFs
    - Discussed extensively in "Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER)" [http://arxiv.org/abs/arXiv:1808.00848]
  - EIC would be capable of providing access to π and K PDFs through measurements of forward nucleon structure functions.

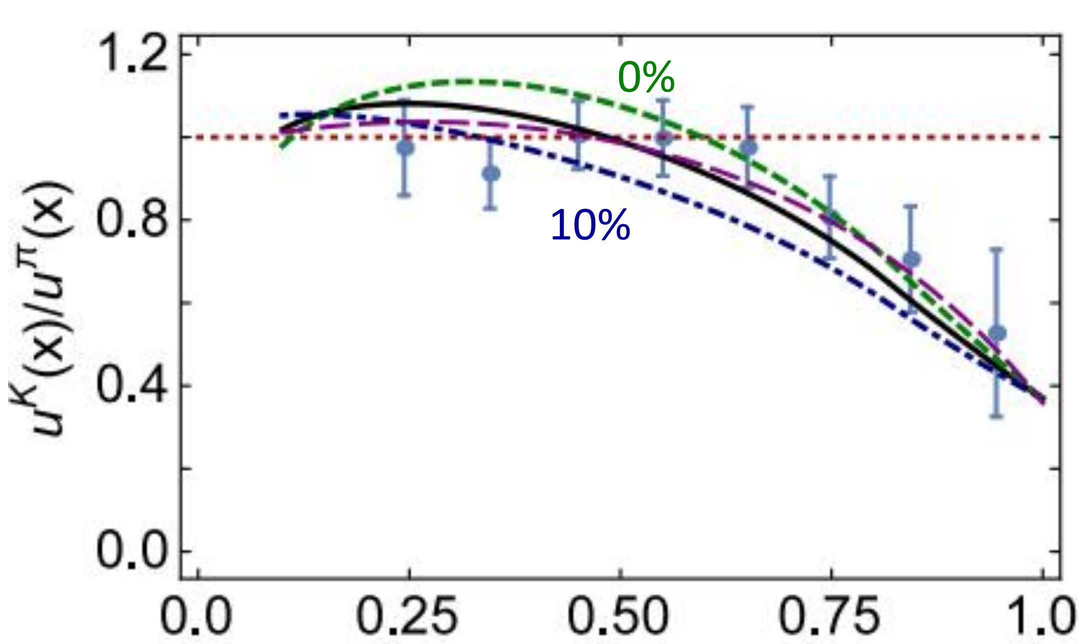
Kaon's gluon content

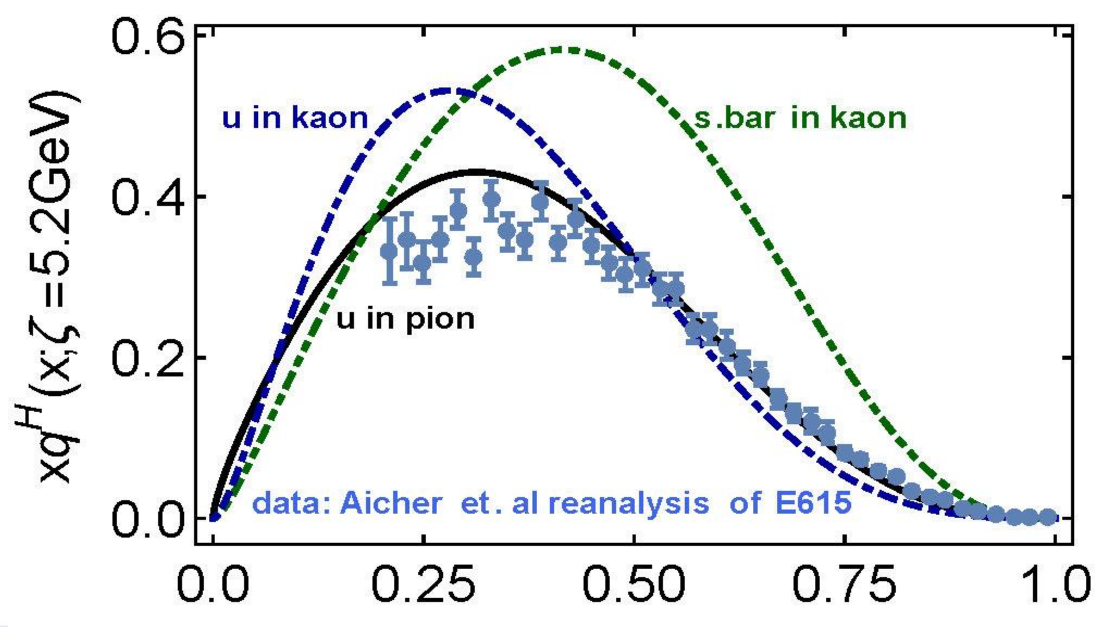
- $\langle x\rangle_g^K(\zeta_H) = 0.05 \pm 0.05$
  $\Rightarrow$ Valence quarks carry 95% of kaon's momentum at ζ_H
- DGLAP-evolved to ζ_2

q	$\langle x\rangle_q^K$	$\langle x^2\rangle_q^K$	$\langle x^3\rangle_q^K$
u	0.28	0.11	0.048
$\bar{s}$	0.36	0.17	0.092

Valence-quarks carry ⅔ of kaon's light-front momentum

Cf. Only ½ for the pion





Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang *et al.*
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & K PDFs

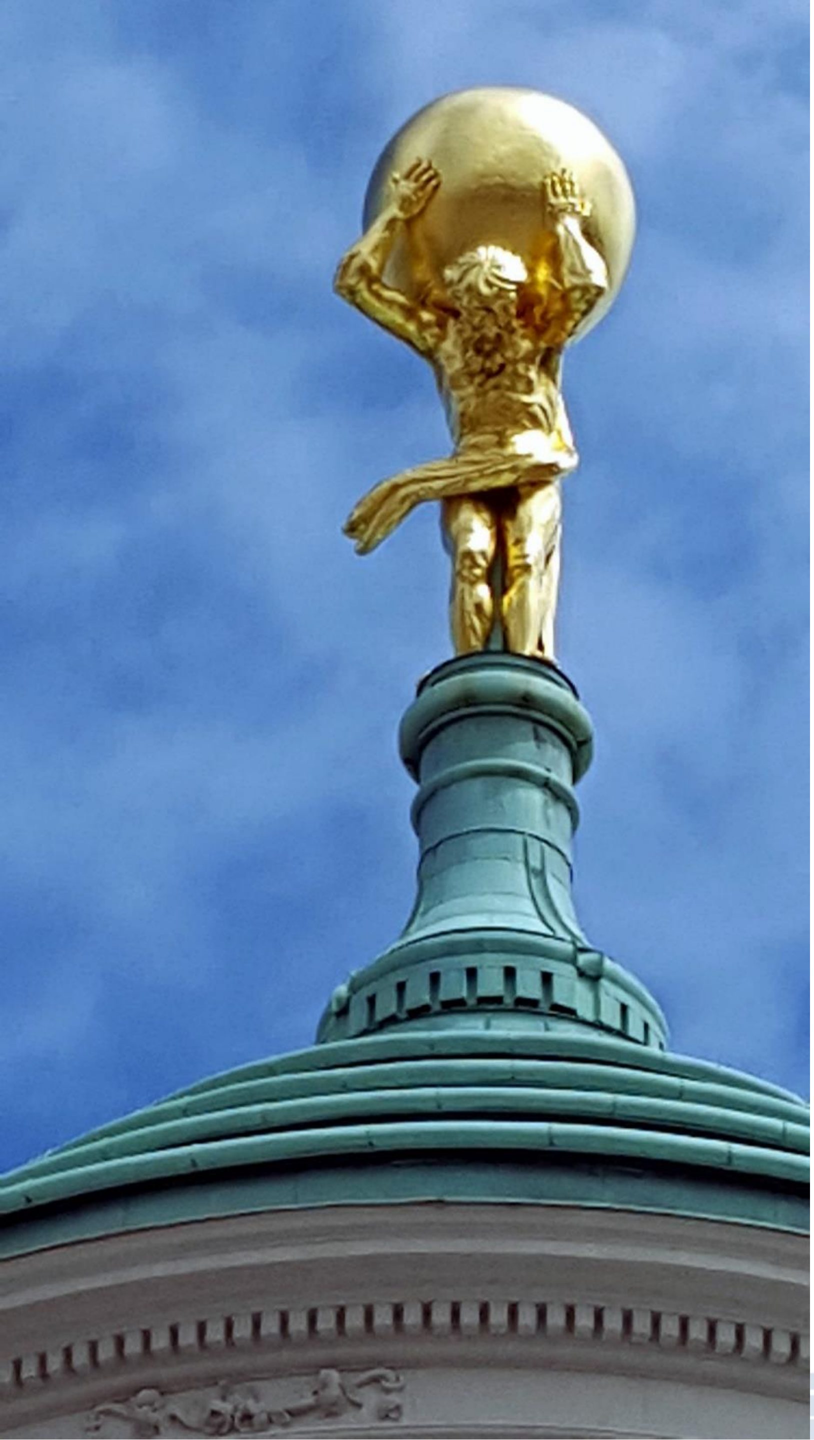
- Marked differences between π & K gluon content
  - ζ_H:
    - Whilst $\frac{1}{3} \sim \frac{1}{5}$ of pion's light-front momentum carried by glue
    - $Only\ \frac{1}{20}$ of the kaon's light-front momentum lies with glue
  - $\zeta_2{}^2 = 4\ \mathrm{GeV}^2$
    - Glue carries $\frac{1}{2}$ of pion's momentum and $\frac{1}{3}$ of kaon's momentum
  - Evident in differences between large-*x* behaviour of valence-quark distributions in these two mesons
- Signal of Nambu-Goldstone boson character of π
  - Nearly complete cancellation between one-particle dressing and binding attraction in this almost-massless pseudoscalar system

$$2\ \mathrm{Mass}_Q + U_g \approx 0$$

π & K PDFs

- Understanding the emergence and character of Nambu-Goldstone modes in the Standard Model is critical
  - Nambu-Goldstone modes are nonpointlike!
  - Intimately connected with origin of mass!
  - Possibly/Probably(?) inseparable from expression of confinement!

- Difference between gluon content of π & K is measurable … using well-designed EIC
- Write a definitive new chapter in future textbooks on the Standard Model

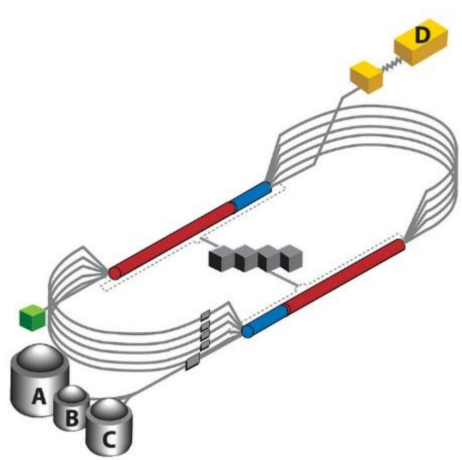


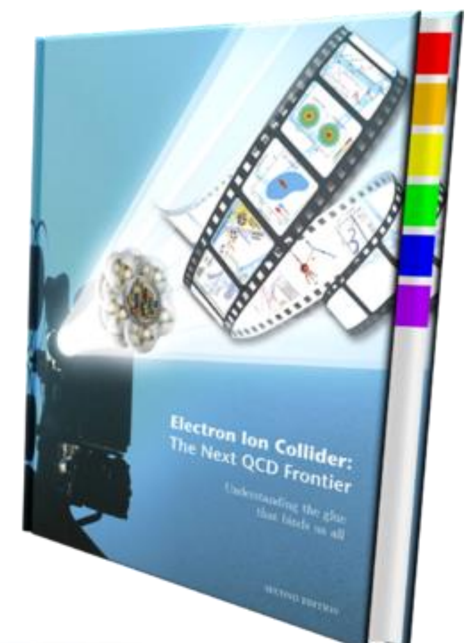
Epilogue

Epilogue

- Challenge: Explain and Understand
  the Origin and Distribution of the Vast Bulk of Visible Mass
- Current Paradigm: Quantum Chromodynamics
- QCD is plausibly a mathematically well-defined quantum field theory,
  The only one we've ever produced
  - Consequently, it is a worthwhile paradigm for developing Beyond-SM theories
- Challenge is to reveal the content of strong-QCD
- **Tough Problem**
- *Progress* and *Insights*

  being delivered by amalgam of
  - Experiment
  - Phenomenology
  - Theory
- Must continue into eras of





Trace Anomaly

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

- Knowing that a trace anomaly exists does not deliver a great deal
  ... Indicates only that a mass-scale must exist
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude ... consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$
$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2 = \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0
  ... In QCD, Θ_0 measures the strength of gluon self-interactions
  ... so, from one perspective, m_p is (somehow) completely generated by glue.

Trace Anomaly

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

- In the chiral limit

$$\langle \pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \Rightarrow \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e.* gluons contribute nothing to the pion mass?
- Difficult way to obtain "zero"!
- Easier to imagine that "zero" owes to cancellations between different operator contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.
  It could only arise naturally because
  of some symmetry and/or symmetry-breaking pattern.

In QCD: Gluons become massive!

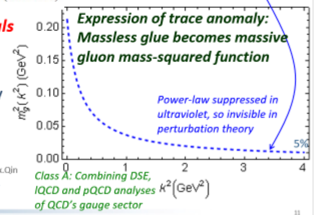
Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

- Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)} \qquad m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2} \qquad \mu_g \approx \tfrac{1}{2} m_p$$

$$\alpha_S(0) \approx \pi \qquad m_g^2(0) = (0.46\,\mathrm{GeV})^2$$

- Gluons are *cannibals* – a particle species whose members become massive by eating each other!

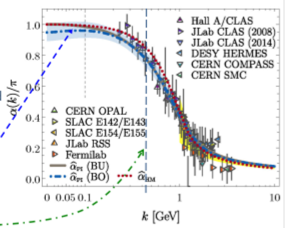


Interaction model for the gap equation, S.-x.Qin et al., arXiv:1108.0603 [nucl-th], Phys. Rev. C 84 (2011) 042202(R) [5 pages]

Process-independent effective-charge in QCD

Process independent strong running coupling, Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero, arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7
The QCD Running Coupling, A. Deur, S. J. Brodsky and G. F. de Teramond, Prog. Part. Nucl. Phys. 90 (2016) 1-74

- Modern continuum & lattice methods for analysing gauge sector enable "Gell-Mann – Low" running charge to be defined in QCD
- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx \tfrac{1}{2} m_p$



Pion's Goldberger -Treiman relation

Maris, Roberts and Tandy, nucl-th/9707003, Phys.Lett. B420 (1998) 267-273

- Pion's Bethe-Salpeter amplitude
  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot PF_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)\right]$$

This means that π necessarily has dressed-quark $L=0$ & $L=1$ components in any frame
Twist-3 on light-front

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

Miracle: two body problem solved, almost completely, once solution of one body problem is known

Pion masslessness

Munczek, H. J., Phys. Rev. D 52 (1995) pp. 4736-4740
Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B 380 (1996) pp. 7-12
Maris, P., Roberts, C.D. and Tandy, P.C., Phys. Lett. B 420 (1998) pp. 267-273
Binosi, Chang, Papavassiliou, Qin, Roberts, Phys. Rev. D 93 (2016) 096010/1-7

- Obtain a coupled set of gap- and Bethe-Salpeter equations

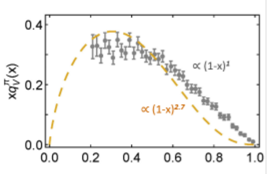
Quantum field theory statement:
In the pseudoscalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi E_\pi(p^2) = B(p^2)$$

- Cancellation guarantees that
  - simple system, which began massless,
  - becomes a complex system, with
    - a nontrivial bound-state wave function
    - attached to a pole in the scattering matrix, which remains at $P^2=0$ ...
- Interacting, bound system remains massless

π valence-quark distributions 20 Years of Evolution

- 1989 ... Conway *et al.* Phys. Rev. D 39, 92 (1989)
  - Leading-order analysis of Drell-Yan data: $Q^2 \approx 27$ GeV^2
- 2000 ... Hecht et al. Phys.Rev. C 63 (2001) 025213
  - QCD-connected model prediction



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