Extraction of Observables from Deeply Virtual ep-scattering Experiments

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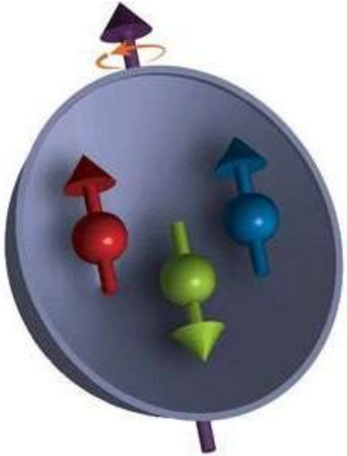
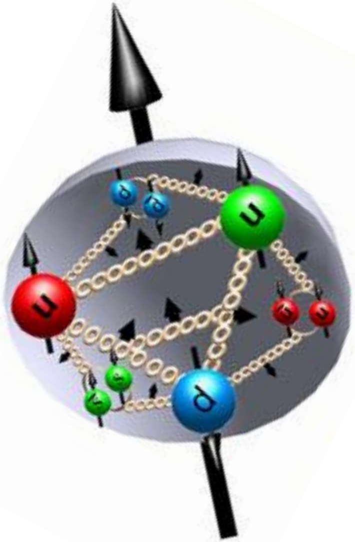


The naive parton model cannot explain the spin of the proton.

Proton Spin Crisis!

$$\frac{\Delta\Sigma}{2} = \frac{1}{2}\sum_q \Delta q + \Delta\bar{q} \neq \frac{1}{2}$$



Gluon contribution while large, still does not account.

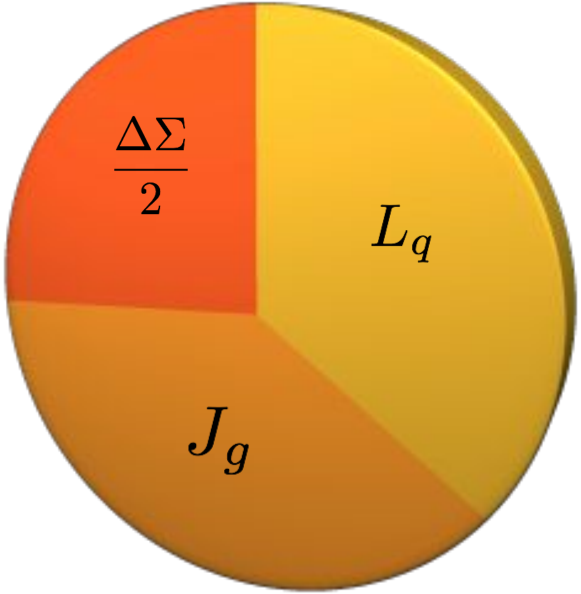
$$\frac{\Delta\Sigma}{2} + \Delta G \neq \frac{1}{2}$$

Partonic Orbital Momentum

$$\frac{\Delta\Sigma}{2} + \Delta G + \mathcal{L}_{q,g} \overset{???}{=} \frac{1}{2}$$

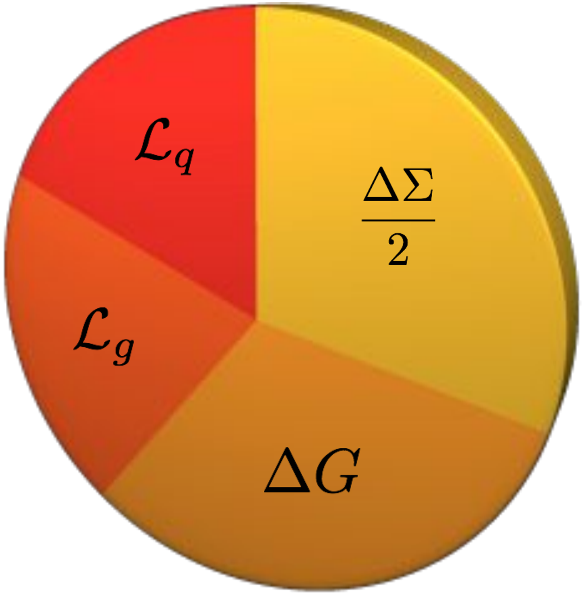
How to define OAM?

Ji



$$\boxed{\frac{1}{2} = \frac{\Delta\Sigma}{2} + L_q + J_g}$$

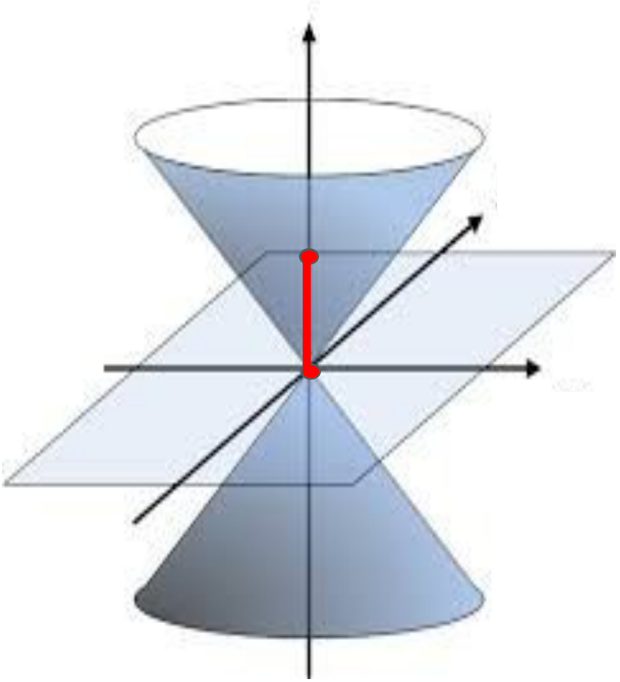
Jaffe Manohar



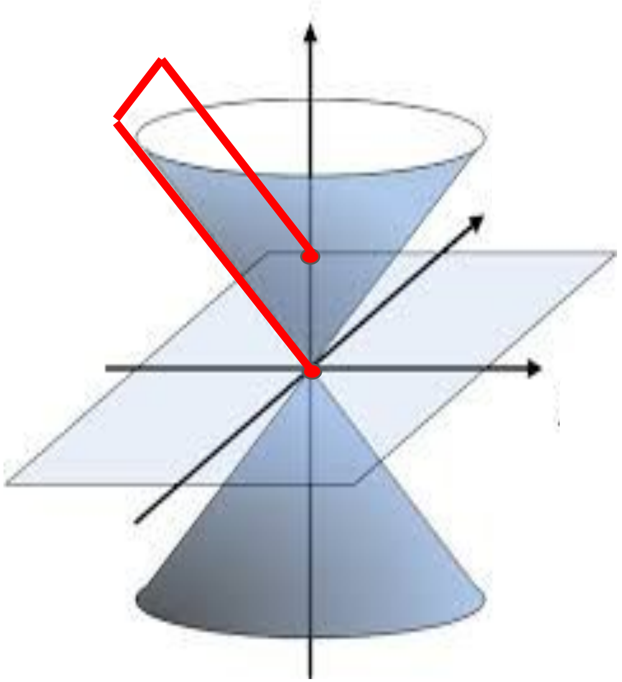
$$\boxed{\frac{1}{2} = \frac{\Delta\Sigma}{2} + \mathcal{L}_q + \Delta G + \mathcal{L}_g}$$

How to define OAM?

Ji



Jaffe Manohar

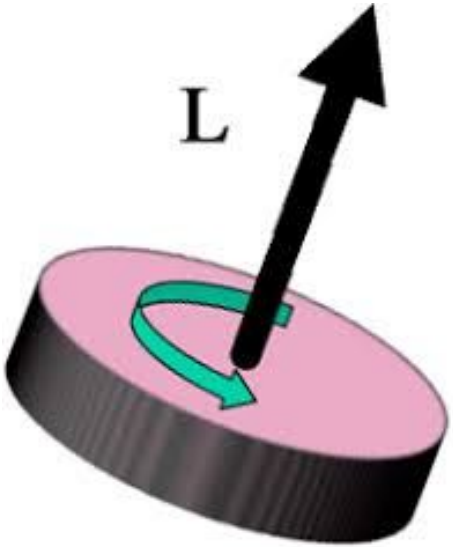


$$L_q = \int d^3r \langle P', \Lambda' | \overline{\psi}\, \gamma^+ \left(\vec{r} \times i\vec{D} \right) \psi | P, \Lambda \rangle$$

$$\mathcal{L}_q = \int d^3r \langle P', \Lambda' | \overline{\psi}\, \gamma^+ \left(\vec{r} \times i\vec{\partial} \right) \psi | P, \Lambda \rangle$$

Liuti, Rajan, et. al. arXiv:1309.7029

How do we describe the orbital angular momentum of the partons?



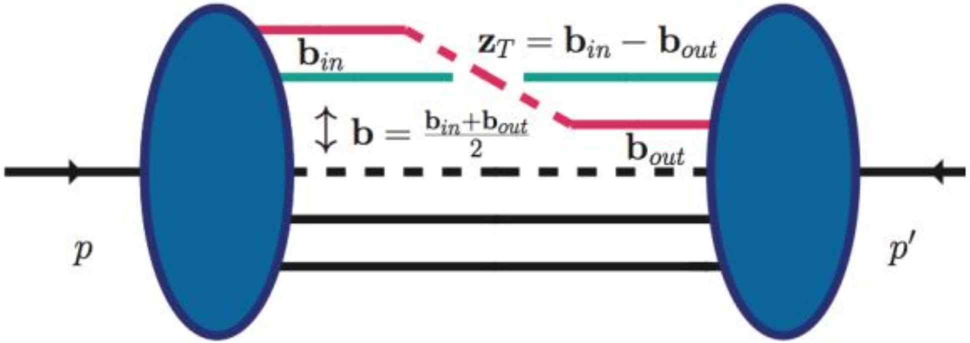
$$\vec{L} = \vec{r} \times \vec{p}$$

Classically

$$L_z^q = -\left(k_T \times b_T\right)_z^q$$

Partonic

b_T	Relative average transverse position from the center of momentum of the system
k_T	Relative average transverse momentum



Lorce, Pasquini arXiv:1106.0139

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Partonic

b_T	Relative average transverse position from the center of momentum of the system
k_T	Relative average transverse momentum

$$l_z^q = \int dx d^2k_T d^2b_T \left(b_T \times k_T\right)_z^q \rho^{[\gamma^+]}(b_T, k_T, x)$$

$$\boxed{l_z^q = -\int dx d^2k_T \frac{k_T^2}{M^2} \boxed{F_{1,4}^q}}$$

Observable?

Lorentz Invariance Relation (LIR) for OAM

No framework yet for GTMD observables

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\underset{\textbf{Twist-2}}{\boxed{F_{1,4} = H + E}} + \underset{\textbf{Twist-3}}{\boxed{\widetilde{E}_{2T}}} + \underset{\textbf{LIR breaking}}{\boxed{\mathcal{A}_{F_{14}}}}$$

Can we disentangle the Twist-3 GPDs from data?

Rajan, Englehardt, Liuti **arXiv:1709.05770** Rajan, Courtoy, Englehardt, Liuti **arXiv:1601.06117**

Lorentz Invariance Relation (LIR) for OAM

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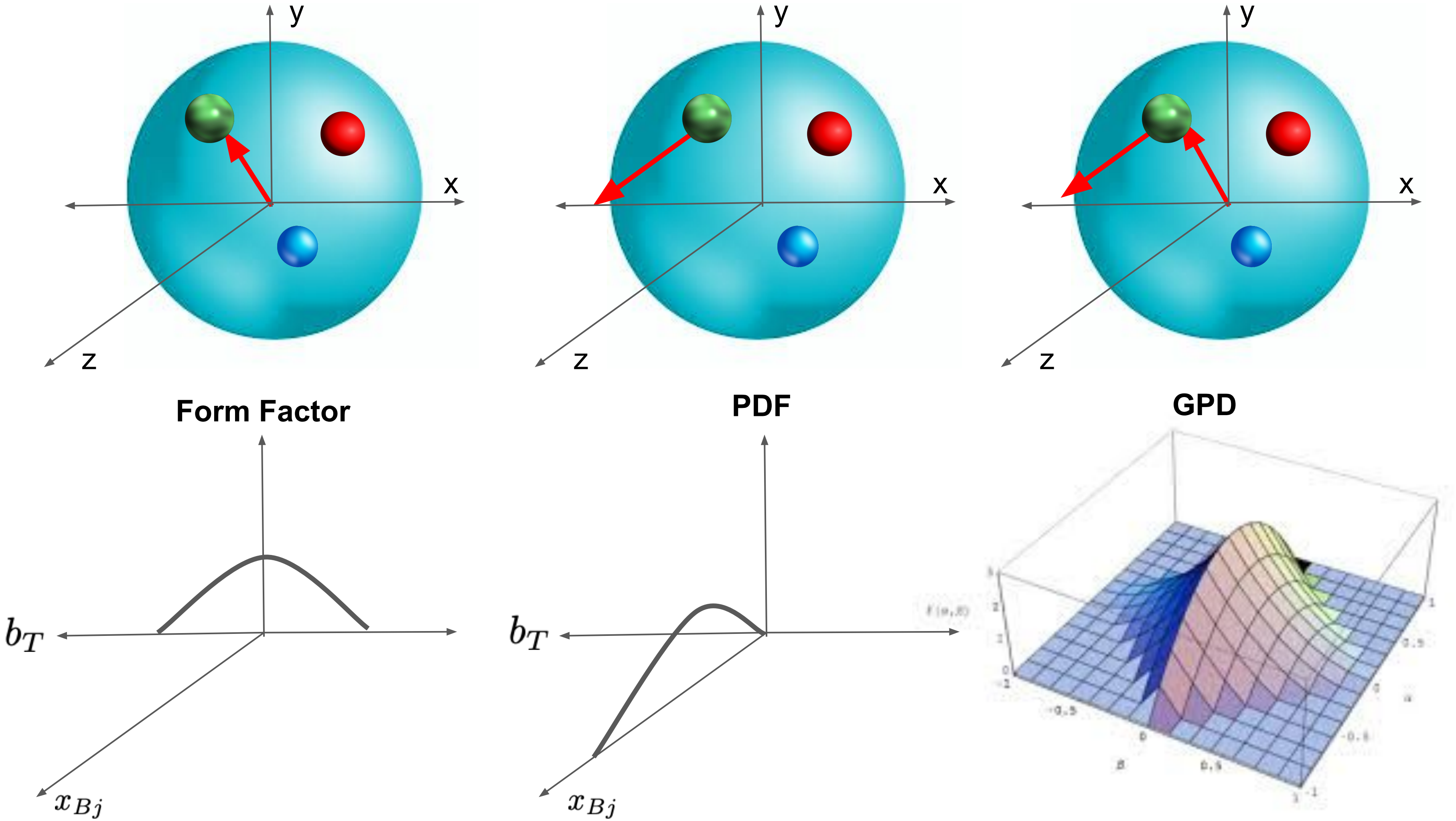
Can we disentangle the Twist-3 GPDs from data?

???

Rajan, Englehardt, Liuti arXiv:1709.05770 Rajan, Courtoy, Englehardt, Liuti arXiv:1601.06117



Form Factor

PDF

GPD

Lorentz Invariance Relation (LIR) for OAM

No framework <u>yet</u> for GTMD observables

???

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\underbrace{F_{1,4} = H + E}_{\textbf{Twist-2}} + \underbrace{\widetilde{E}_{2T}}_{\textbf{Twist-3}} + \underbrace{\mathcal{A}_{F_{14}}}_{\textbf{LIR breaking}}$$

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Wigner Distribution

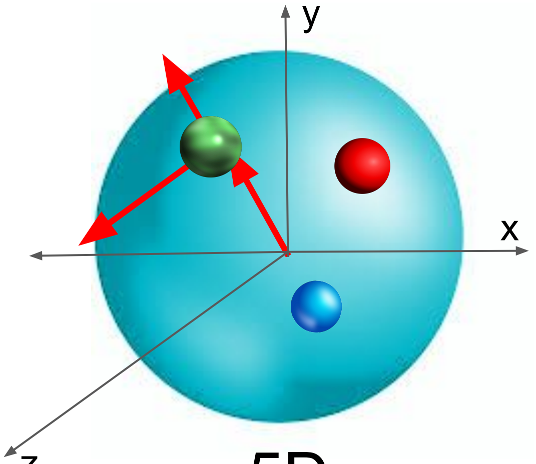
Generalized Transverse Momentum Distribution Functions

Generalized Parton Distribution Functions

Transverse Momentum Distribution Functions

Parton Distribution Functions

$$\begin{tikzcd}
& W(x,\vec{k_T},\vec{b_T}) \arrow[d, "b_T \to \Delta_T"] & \\
& W(x,\vec{k_T},\vec{\Delta_T}) \arrow[dl, "\int d^2k_T"'] \arrow[dr, "\Delta_T \to 0"] & \\
H(x,\vec{\Delta_T}) \arrow[dr, "\Delta_T \to 0"'] & & f_1(x,k_T) \arrow[dl, "\int d^2k_T"] \\
& f(x) &
\end{tikzcd}$$



5D

Lorentz Invariance Relation (LIR) for OAM

No framework yet for GTMD observables

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\underbrace{\boxed{F_{1,4}=H+E}}_{\text{Twist-2}}+\underbrace{\boxed{\widetilde{E}_{2T}}}_{\text{Twist-3}}+\underbrace{\boxed{\mathcal{A}_{F_{14}}}}_{\text{LIR breaking}}$$

Can we disentangle the Twist-3 GPDs from data?

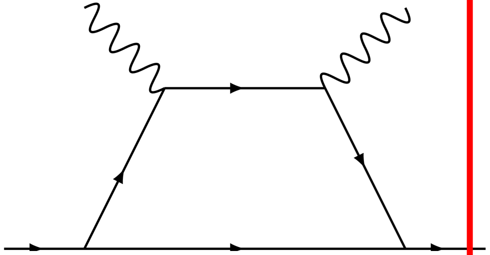
???

Rajan, Englehardt, Liuti arXiv:1709.05770 Rajan, Courtoy, Englehardt, Liuti arXiv:1601.06117

Kinematic Twist

Twist-2

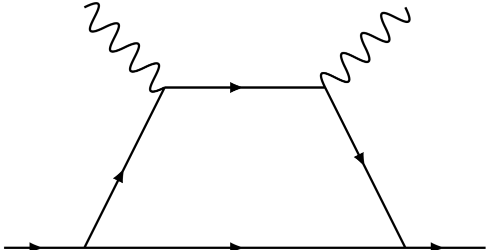
$\gamma^+, \gamma^+\gamma_5, i\sigma^{i+}\gamma_5$



$\mathcal{O}(1)$

Twist-3

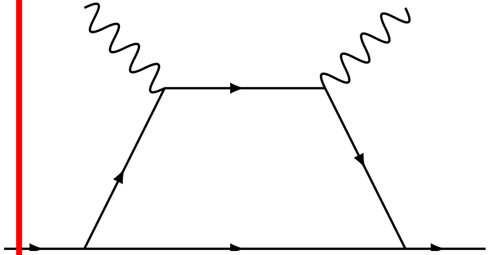
$1, \gamma_5, \gamma^i, \gamma^i\gamma_5, i\sigma^{ij}\gamma_5, i\sigma^{+-}\gamma_5$



$\mathcal{O}\left(\frac{1}{P^+}\right)$

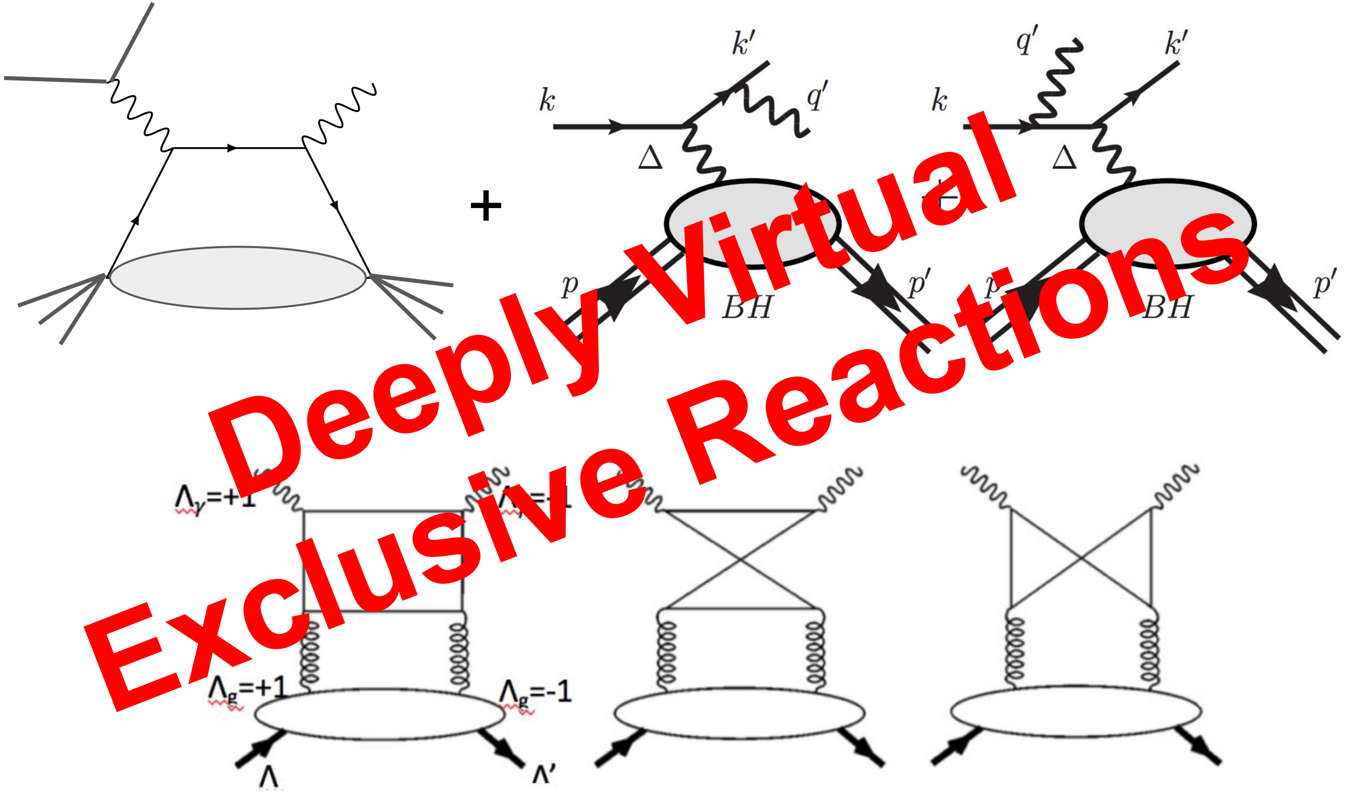
Twist-4

$\gamma^-, \gamma^-\gamma_5, i\sigma^{i-}\gamma_5$



$\mathcal{O}\left(\frac{1}{(P^+)^2}\right)$

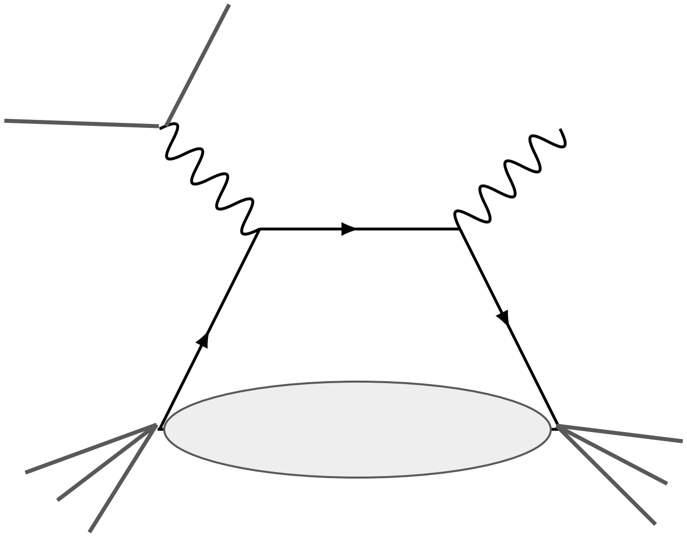
How do we access GPDs?



Deeply Virtual Exclusive Reactions

2

DVCS



🟦 Twist - 2

🟩 Twist - 3

$$W^{\mu\nu} \propto \gamma^{\mu}\gamma^{+}\gamma^{\nu} = \begin{bmatrix} \gamma^{-} & \gamma^{1} - i\gamma^{2}\gamma_{5} & \gamma^{2} + i\gamma^{1}\gamma_{5} & i\gamma^{-}\gamma_{5} \\ \fcolorbox{green}{white}{$\gamma^{1} + i\gamma^{2}\gamma_{5}$} & \fcolorbox{blue}{white}{γ^{+}} & \fcolorbox{blue}{white}{$i\gamma^{+}\gamma_{5}$} & \fcolorbox{green}{white}{$-\gamma^{1} - i\gamma^{2}\gamma_{5}$} \\ \fcolorbox{green}{white}{$\gamma^{2} - i\gamma^{1}\gamma_{5}$} & \fcolorbox{blue}{white}{$-i\gamma^{+}\gamma_{5}$} & \fcolorbox{blue}{white}{γ^{+}} & \fcolorbox{green}{white}{$-\gamma^{2} + i\gamma^{1}\gamma_{5}$} \\ -i\gamma^{-}\gamma_{5} & -\gamma^{1} + i\gamma^{2}\gamma_{5} & -\gamma^{2} - i\gamma^{1}\gamma_{5} & \gamma^{-} \end{bmatrix}$$

Kinematics

$$x_{Bj} = \frac{Q^2}{2(pq)} \approx \frac{2\xi}{1+\xi}$$

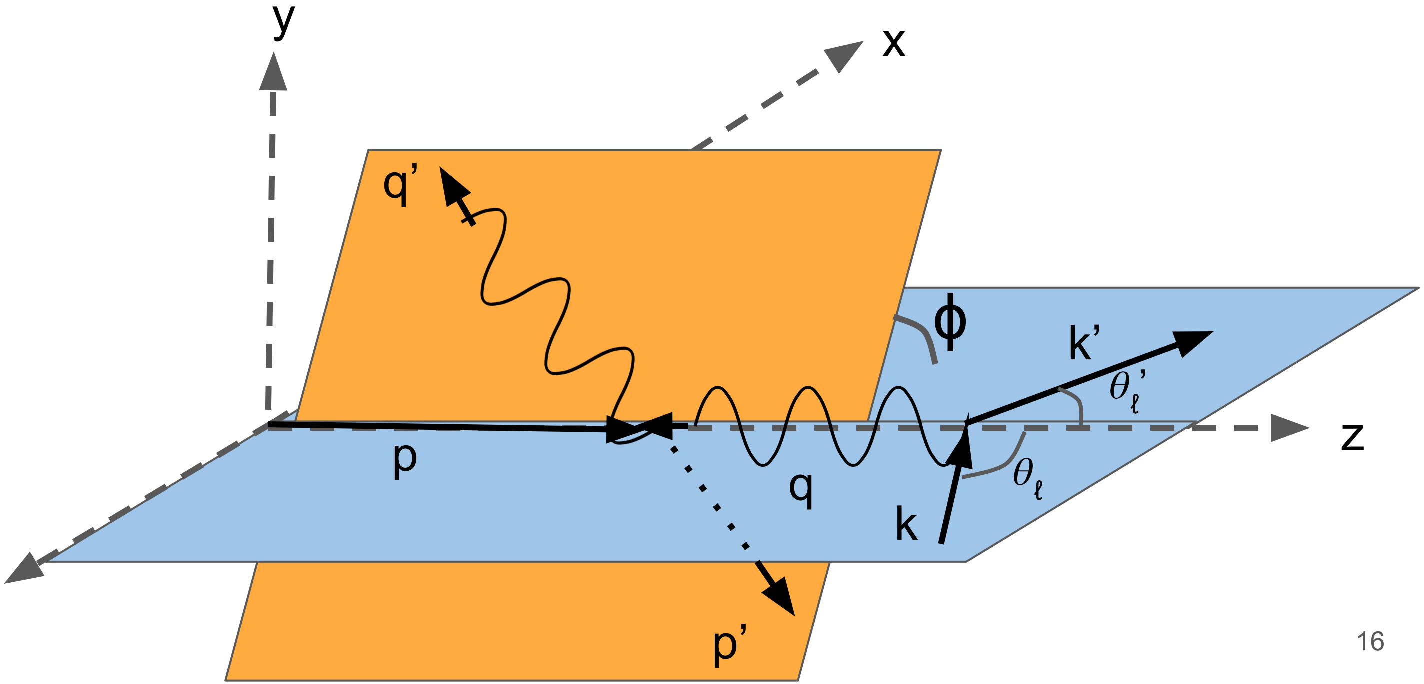
$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \Gamma|T|^2$$

$$Q^2 = -q^2 = -(k-k')^2$$

$$\xi = -\frac{\Delta^+}{2P^+}$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$



B.K., S.L., et. al. arXiv:1903.05742

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T_{DVCS}|^2$$

Unpolarized

$$= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{\Big[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi}$$

$$+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \sin\phi F_{UU}^{\sin\phi}\right]$$

LU polarized

$$+ (2h)F_{LU} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi F_{LU}^{\cos\phi}\Big]$$

UL polarized

$$+ (2\Lambda)\Big[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}$$

LL polarized

$$+ (2h)\sqrt{1-\epsilon^2}F_{LL} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi F_{LL}^{\sin\phi}\Big]$$

UT polarized

$$+ |\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)$$

$$\epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}$$

$$+\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big]$$

LT polarized

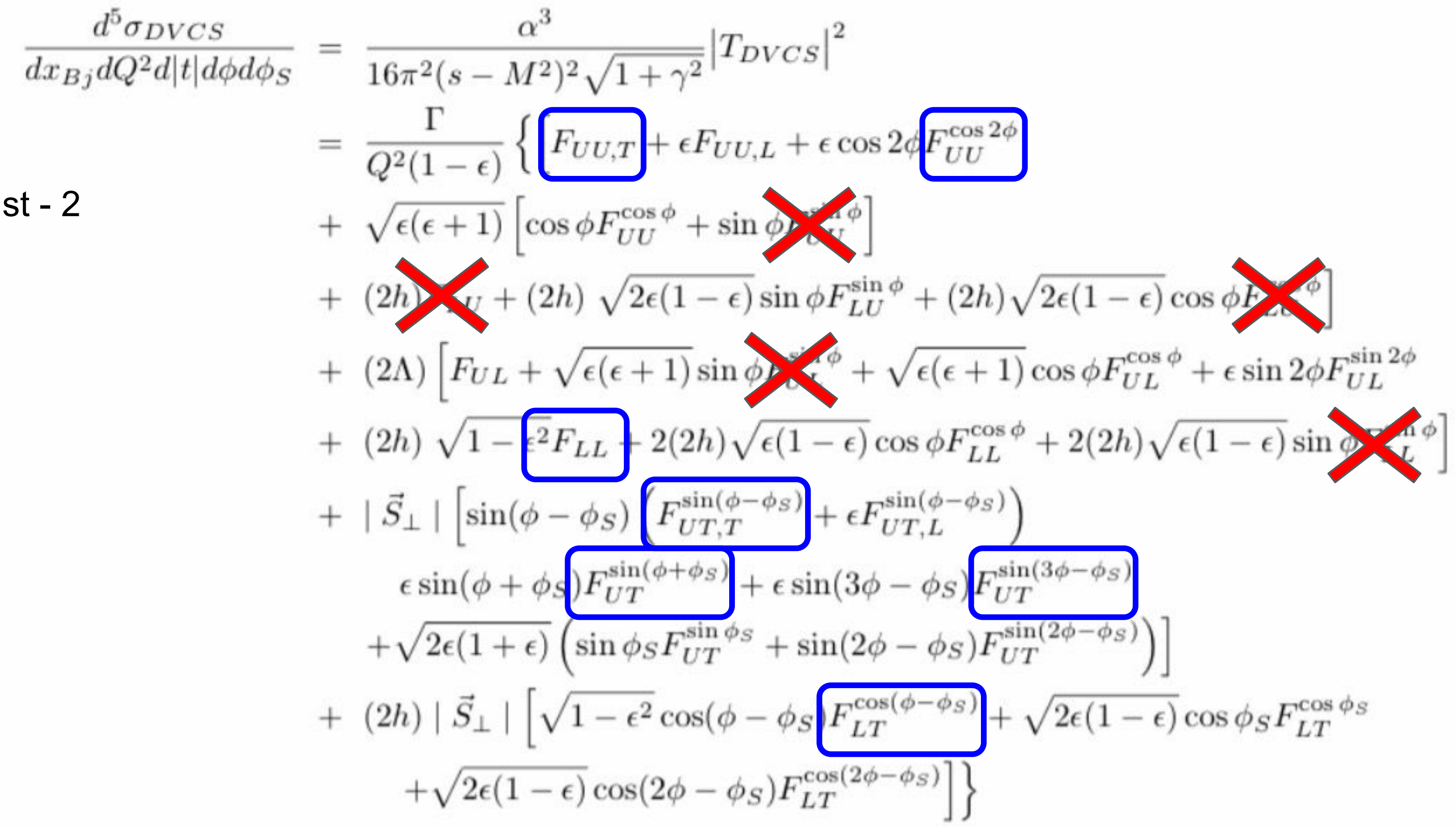
$$+ (2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}$$

B.K., S.L., et. al. arXiv:1903.05742

$$
\begin{aligned}
\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} &= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left|T_{DVCS}\right|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{\Big[F_{UU,T}+\epsilon F_{UU,L}+\epsilon\cos 2\phi F_{UU}^{\cos 2\phi} \\
&+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi}+\sin\phi \xcancel{F_{UU}^{\sin\phi}}\right] \\
&+ (2h)\xcancel{F_{LU}}+(2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi}+(2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi \xcancel{F_{LU}^{\cos\phi}}\Big] \\
&+ (2\Lambda)\Big[F_{UL}+\sqrt{\epsilon(\epsilon+1)}\sin\phi \xcancel{F_{UL}^{\sin\phi}}+\sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi}+\epsilon\sin 2\phi F_{UL}^{\sin 2\phi} \\
&+ (2h)\sqrt{1-\epsilon^2}F_{LL}+2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi}+2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi \xcancel{F_{LL}^{\sin\phi}}\Big] \\
&+ |\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right) \\
&\quad \epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)}+\epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)} \\
&\quad +\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S}+\sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big] \\
&+ (2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)}+\sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
&\quad +\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}
\end{aligned}
$$

B.K., S.L., et. al. [arXiv:1903.05742]



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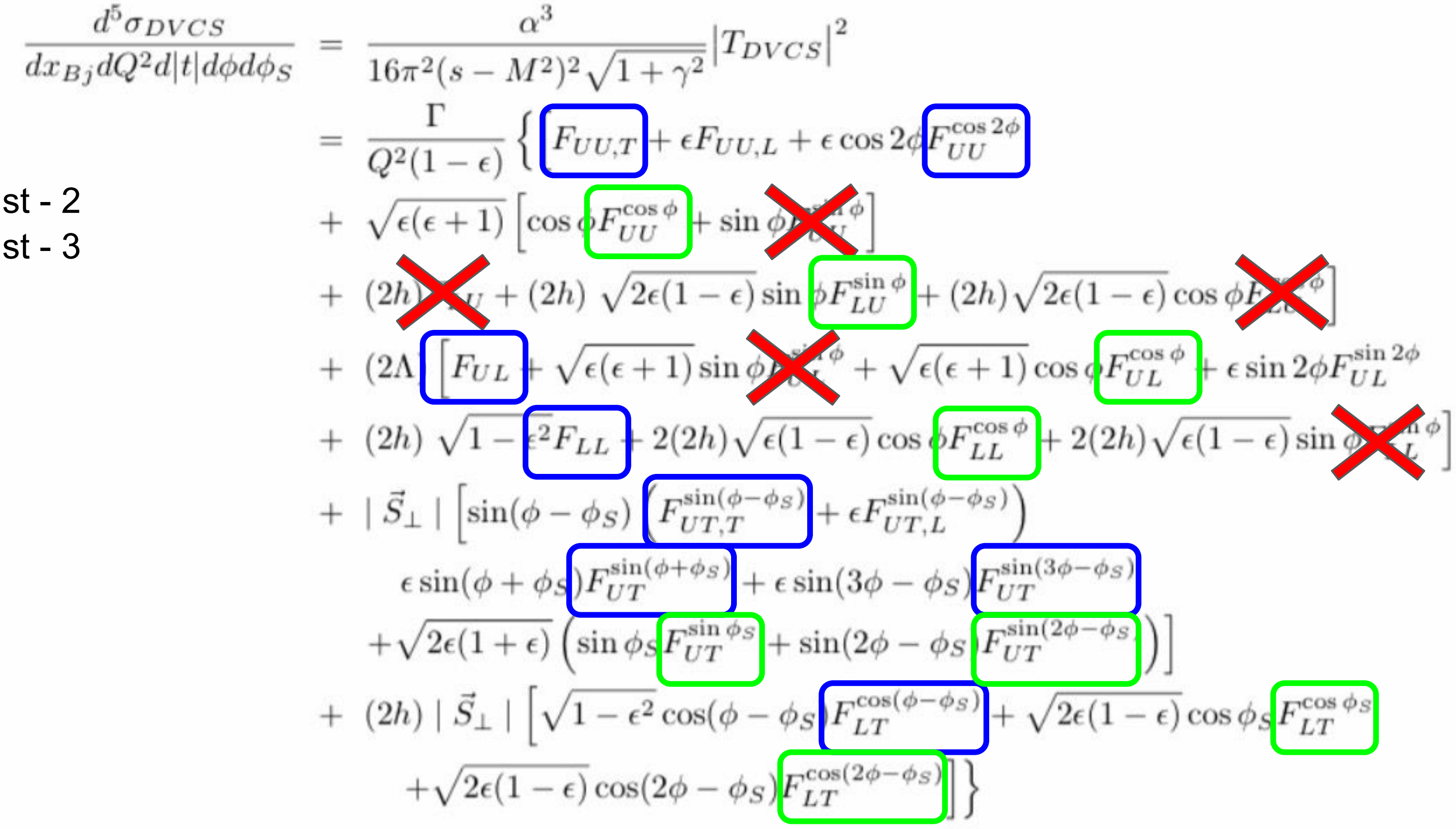
&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{\boxed{F_{UU,T}} + \epsilon F_{UU,L} + \epsilon\cos 2\phi \boxed{F_{UU}^{\cos 2\phi}} \\
&+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi F_{UU}^{\cos\phi} + \cancel{\sin\phi F_{UU}^{\sin\phi}}\right] \\
&+ \cancel{(2h) F_{LU}} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cancel{\cos\phi F_{LU}^{\cos\phi}}] \\
&+ (2\Lambda)\left[F_{UL} + \sqrt{\epsilon(\epsilon+1)}\cancel{\sin\phi F_{UL}^{\sin\phi}} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UL}^{\cos\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}\right. \\
&+ (2h)\sqrt{1-\boxed{\epsilon^2 F_{LL}}} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cancel{\sin\phi F_{LL}^{\sin\phi}}] \\
&+ |\vec{S}_\perp|\left[\sin(\phi-\phi_S)\left(\boxed{F_{UT,T}^{\sin(\phi-\phi_S)}} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)\right. \\
&\quad \epsilon\sin(\phi+\phi_S)\boxed{F_{UT}^{\sin(\phi+\phi_S)}} + \epsilon\sin(3\phi-\phi_S)\boxed{F_{UT}^{\sin(3\phi-\phi_S)}} \\
&\quad \left.+\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\right] \\
&+ (2h)|\vec{S}_\perp|\left[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)\boxed{F_{LT}^{\cos(\phi-\phi_S)}} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}\right. \\
&\quad \left.\left.+\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\right]\right\}
\end{aligned}
$$

— Twist - 2

B.K., S.L., et. al. arXiv:1903.05742



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&= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{\boxed{F_{UU,T}}+\epsilon F_{UU,L}+\epsilon\cos 2\phi \boxed{F_{UU}^{\cos 2\phi}}\\
&+\sqrt{\epsilon(\epsilon+1)}\left[\cos\phi \boxed{F_{UU}^{\cos\phi}}+\sin\phi \cancel{F_{UU}^{\sin\phi}}\right]\\
&+(2h)\cancel{F_{LU}}+(2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi \boxed{F_{LU}^{\sin\phi}}+(2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi \cancel{F_{LU}^{\cos\phi}}]\\
&+(2\Lambda)\Big[\boxed{F_{UL}}+\sqrt{\epsilon(\epsilon+1)}\sin\phi \cancel{F_{UL}^{\sin\phi}}+\sqrt{\epsilon(\epsilon+1)}\cos\phi \boxed{F_{UL}^{\cos\phi}}+\epsilon\sin 2\phi F_{UL}^{\sin 2\phi}\\
&+(2h)\sqrt{1-\epsilon^2}\boxed{F_{LL}}+2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi \boxed{F_{LL}^{\cos\phi}}+2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi \cancel{F_{LL}^{\sin\phi}}\Big]\\
&+|\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\left(\boxed{F_{UT,T}^{\sin(\phi-\phi_S)}}+\epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)\\
&\quad \epsilon\sin(\phi+\phi_S)\boxed{F_{UT}^{\sin(\phi+\phi_S)}}+\epsilon\sin(3\phi-\phi_S)\boxed{F_{UT}^{\sin(3\phi-\phi_S)}}\\
&\quad +\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S \boxed{F_{UT}^{\sin\phi_S}}+\sin(2\phi-\phi_S)\boxed{F_{UT}^{\sin(2\phi-\phi_S)}}\right)\Big]\\
&+(2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)\boxed{F_{LT}^{\cos(\phi-\phi_S)}}+\sqrt{2\epsilon(1-\epsilon)}\cos\phi_S \boxed{F_{LT}^{\cos\phi_S}}\\
&\quad +\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)\boxed{F_{LT}^{\cos(2\phi-\phi_S)}}\Big]\Big\}
\end{aligned}
$$

🟦 Twist - 2

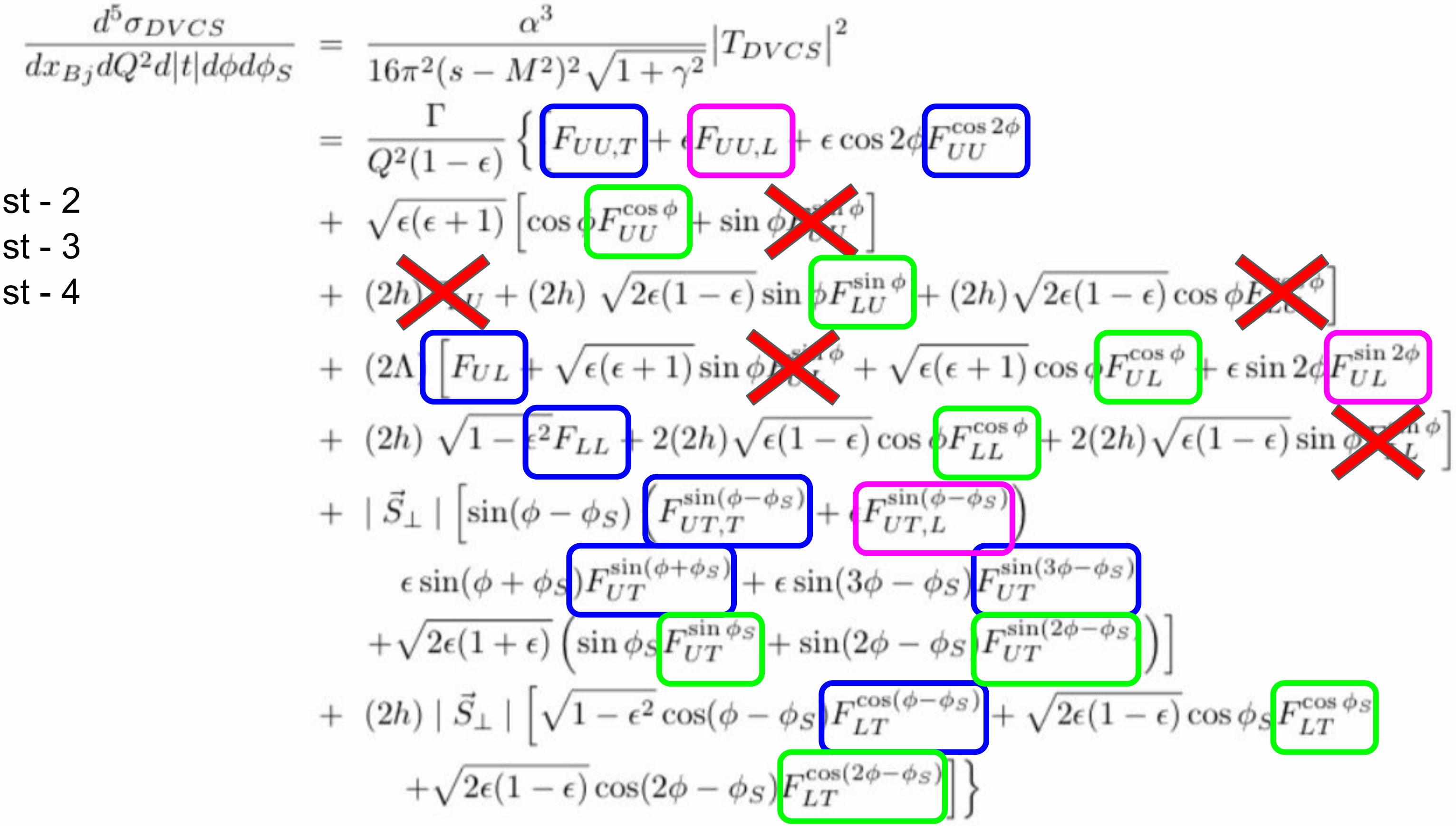
🟩 Twist - 3

B.K., S.L., et. al. arXiv:1903.05742

🟦 Twist - 2

🟩 Twist - 3

🟪 Twist - 4



$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T_{DVCS}|^2$$

$$= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{\boxed{F_{UU,T}} + \boxed{\epsilon F_{UU,L}} + \epsilon\cos 2\phi \boxed{F_{UU}^{\cos 2\phi}}$$

$$+ \sqrt{\epsilon(\epsilon+1)}\left[\cos\phi \boxed{F_{UU}^{\cos\phi}} + \sin\phi \cancel{F_{UU}^{\sin\phi}}\right]$$

$$+ (2h)\cancel{F_{LU}} + (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi \boxed{F_{LU}^{\sin\phi}} + (2h)\sqrt{2\epsilon(1-\epsilon)}\cos\phi \cancel{F_{LU}^{\cos\phi}}]$$

$$+ (2\Lambda)\left[\boxed{F_{UL}} + \sqrt{\epsilon(\epsilon+1)}\sin\phi \cancel{F_{UL}^{\sin\phi}} + \sqrt{\epsilon(\epsilon+1)}\cos\phi \boxed{F_{UL}^{\cos\phi}} + \epsilon\sin 2\phi \boxed{F_{UL}^{\sin 2\phi}}\right.$$

$$+ (2h)\sqrt{1-\boxed{\epsilon^2 F_{LL}}} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\cos\phi \boxed{F_{LL}^{\cos\phi}} + 2(2h)\sqrt{\epsilon(1-\epsilon)}\sin\phi \cancel{F_{LL}^{\sin\phi}}\Big]$$

$$+ |\vec{S}_\perp|\Big[\sin(\phi-\phi_S)\Big(\boxed{F_{UT,T}^{\sin(\phi-\phi_S)}} + \boxed{\epsilon F_{UT,L}^{\sin(\phi-\phi_S)}}\Big)$$

$$\epsilon\sin(\phi+\phi_S)\boxed{F_{UT}^{\sin(\phi+\phi_S)}} + \epsilon\sin(3\phi-\phi_S)\boxed{F_{UT}^{\sin(3\phi-\phi_S)}}$$

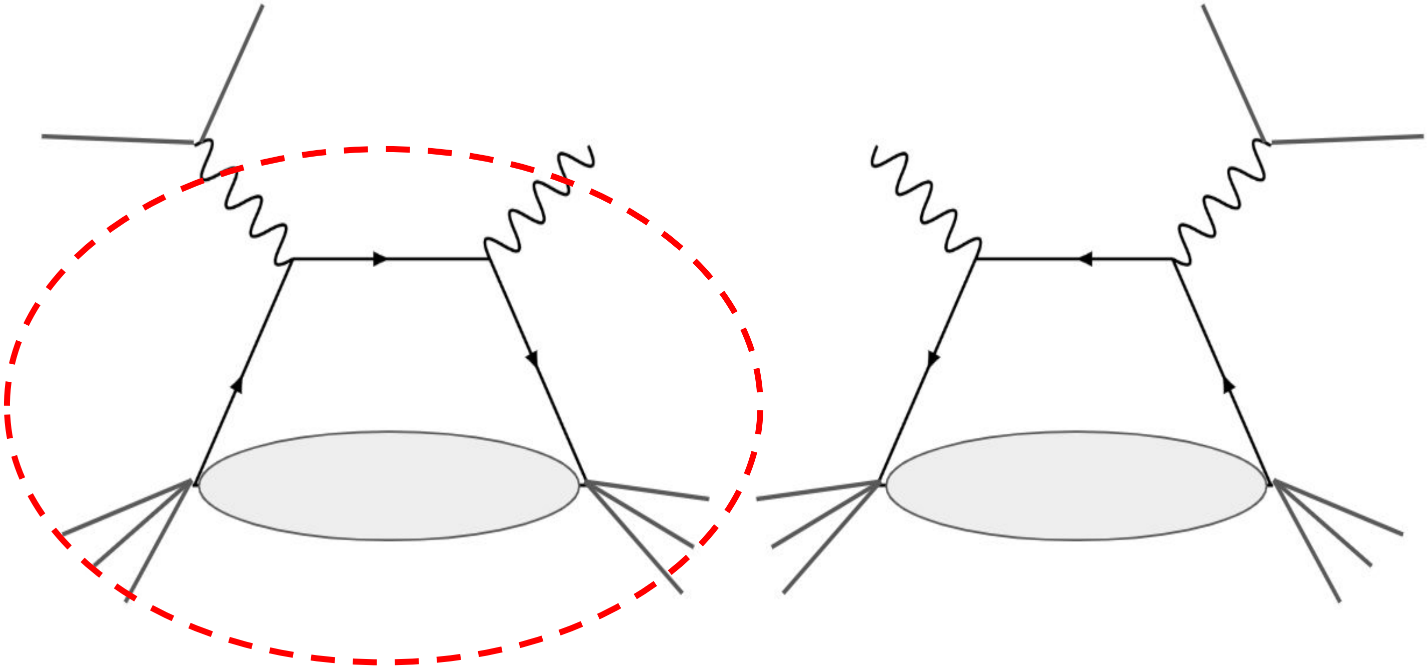
$$+\sqrt{2\epsilon(1+\epsilon)}\Big(\sin\phi_S \boxed{F_{UT}^{\sin\phi_S}} + \sin(2\phi-\phi_S)\boxed{F_{UT}^{\sin(2\phi-\phi_S)}}\Big)\Big]$$

$$+ (2h)|\vec{S}_\perp|\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)\boxed{F_{LT}^{\cos(\phi-\phi_S)}} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S \boxed{F_{LT}^{\cos\phi_S}}$$

$$+\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)\boxed{F_{LT}^{\cos(2\phi-\phi_S)}}\Big]\Big\}$$

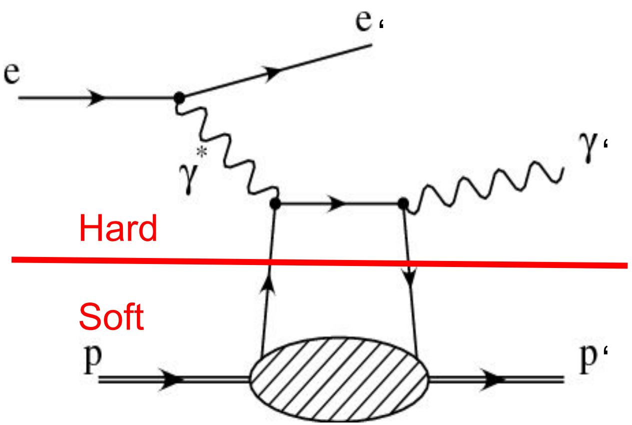
Where do we start?

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} =$$



Photon/Proton Helicity Amplitudes

Goldstein, Gonzalez, Liuti arXiv:1012.3776v2



$$f^{\Lambda^*_\gamma \Lambda'_\gamma}_{\Lambda\Lambda'} = \sum_{\lambda,\lambda'} g^{\Lambda^*_\gamma \Lambda'_\gamma}_{\lambda\lambda'}(x,\xi,t;Q^2) \otimes A_{\Lambda'\lambda',\Lambda\lambda}(x,\xi,t)$$

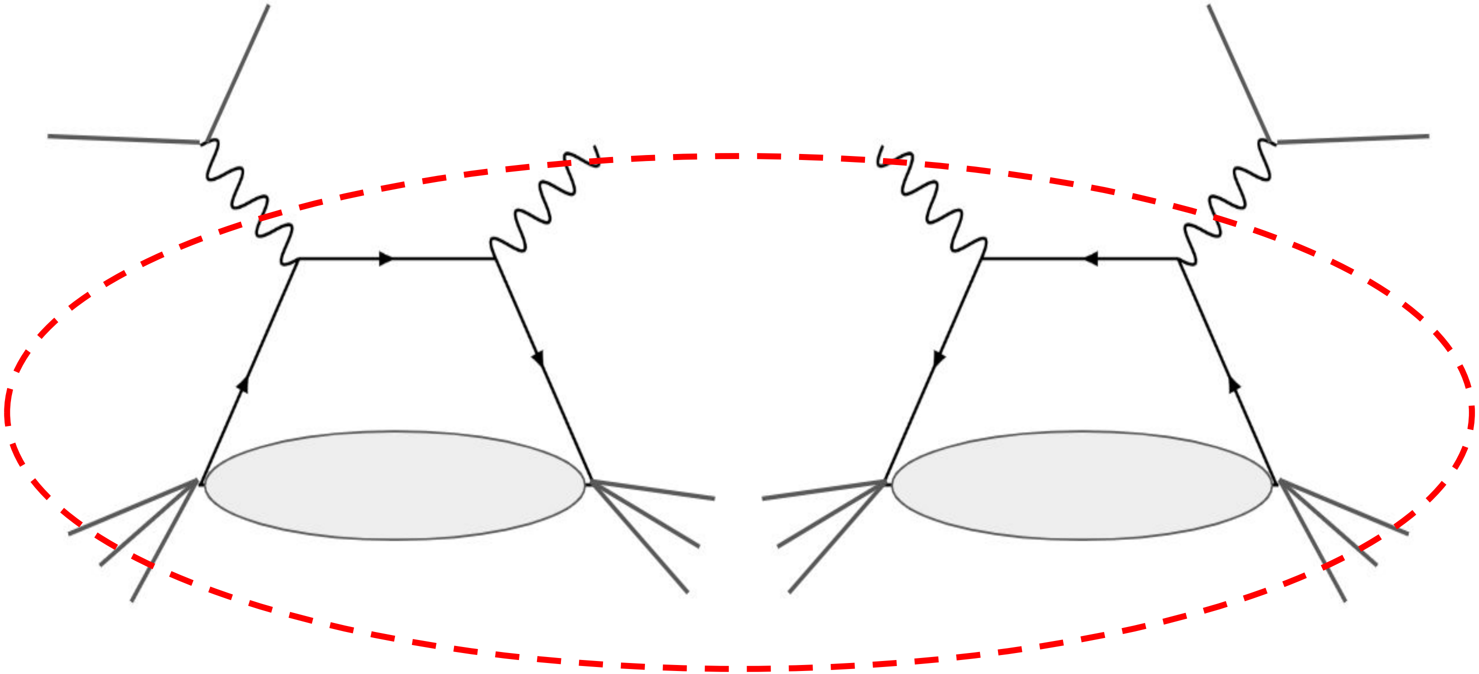
$$g^{\Lambda^*_\gamma \Lambda'_\gamma}_{\lambda\lambda'}(x,\xi,t;Q^2) = \overline{u}(k',\lambda')\gamma^\mu\gamma^+\gamma^\nu \left[\frac{\epsilon^*_\mu(q-\Delta,\Lambda'_\gamma)\epsilon_\nu(q,\Lambda_\gamma *)}{(k+q)^2+i\epsilon} + \frac{\epsilon^*_\nu(q-\Delta,\Lambda'_\gamma)\epsilon_\mu(q,\Lambda_\gamma *)}{(k'-q)^2+i\epsilon} \right] u(k,\lambda) q^-$$

$\mu,\nu \in \{1,2\}$

$\mu \in \{1,2\}, \nu \in \{0,3\}$

Then ...

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} =$$



Jakob, Wick Annals Phys 7, 404 (1959)

Photon/Proton Structure Functions and Phase

Definition of a Helicity Amplitude →

$$f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}(\theta, \phi) = e^{-i(\Lambda_{\gamma^*}-\Lambda-\Lambda'_{\gamma}+\Lambda')\phi}\, \tilde{f}_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}(\theta),$$

$$\boxed{e^{i(\Lambda^{(1)}_{\gamma^*}-\Lambda^{(2)}_{\gamma^*})\,\phi}\, F_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}}} = e^{i(\Lambda^{(1)}_{\gamma^*}-\Lambda^{(2)}_{\gamma^*})\,\phi} \sum_{\Lambda_{\gamma'}} \left(\tilde{f}_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda'_{\gamma}}\right)^* \tilde{f}_{\Lambda\Lambda'}^{\Lambda^{(2)}_{\gamma^*}\Lambda'_{\gamma}} = \sum_{\Lambda_{\gamma'}} \left(f_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda'_{\gamma}}\right)^* f_{\Lambda\Lambda'}^{\Lambda^{(2)}_{\gamma^*}\Lambda'_{\gamma}}$$

Enter the observables in your cross section.

$$\boxed{e^{i(\Lambda^{(1)}_{\gamma^*}-\Lambda^{(2)}_{\gamma^*}-2\Lambda)\,\phi}\, F_{T,\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}}} =$$

$$\sum_{\Lambda_{\gamma'}} \left(f_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda'_{\gamma}}\right)^* f_{-\Lambda\Lambda'}^{\Lambda^{(2)}_{\gamma^*}\Lambda'_{\gamma}} = e^{i(\Lambda^{(1)}_{\gamma^*}-\Lambda^{(2)}_{\gamma^*}-2\Lambda)\,\phi} \sum_{\Lambda_{\gamma'}} \left(\tilde{f}_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda'_{\gamma}}\right)^* \tilde{f}_{-\Lambda\Lambda'}^{\Lambda^{(2)}_{\gamma^*}\Lambda'_{\gamma}}$$

B.K., S.L., et. al. arXiv:1903.05742

Jakob, Wick Annals Phys 7, 404 (1959)

Photon/Proton Structure Functions and Phase

Definition of a Helicity Amplitude

$$f^{\Lambda_{\gamma*}\Lambda'_{\gamma}}_{\Lambda\Lambda'}(\theta,\phi) = e^{-i(\Lambda_{\gamma*}-\Lambda-\Lambda'_{\gamma}+\Lambda')\phi}\,\widetilde{f}^{\Lambda_{\gamma*}\Lambda'_{\gamma}}_{\Lambda\Lambda'}(\theta),$$

$$e^{i(\Lambda^{(1)}_{\gamma*}-\Lambda^{(2)}_{\gamma*})\,\phi}\,F^{\Lambda^{(1)}_{\gamma*}\Lambda^{(2)}_{\gamma*}}_{\Lambda\Lambda'} = e^{i(\Lambda^{(1)}_{\gamma*}-\Lambda^{(2)}_{\gamma*})\,\phi}\sum \quad \Lambda^{(2)}\Lambda'$$

Gives access to higher twist terms

El se

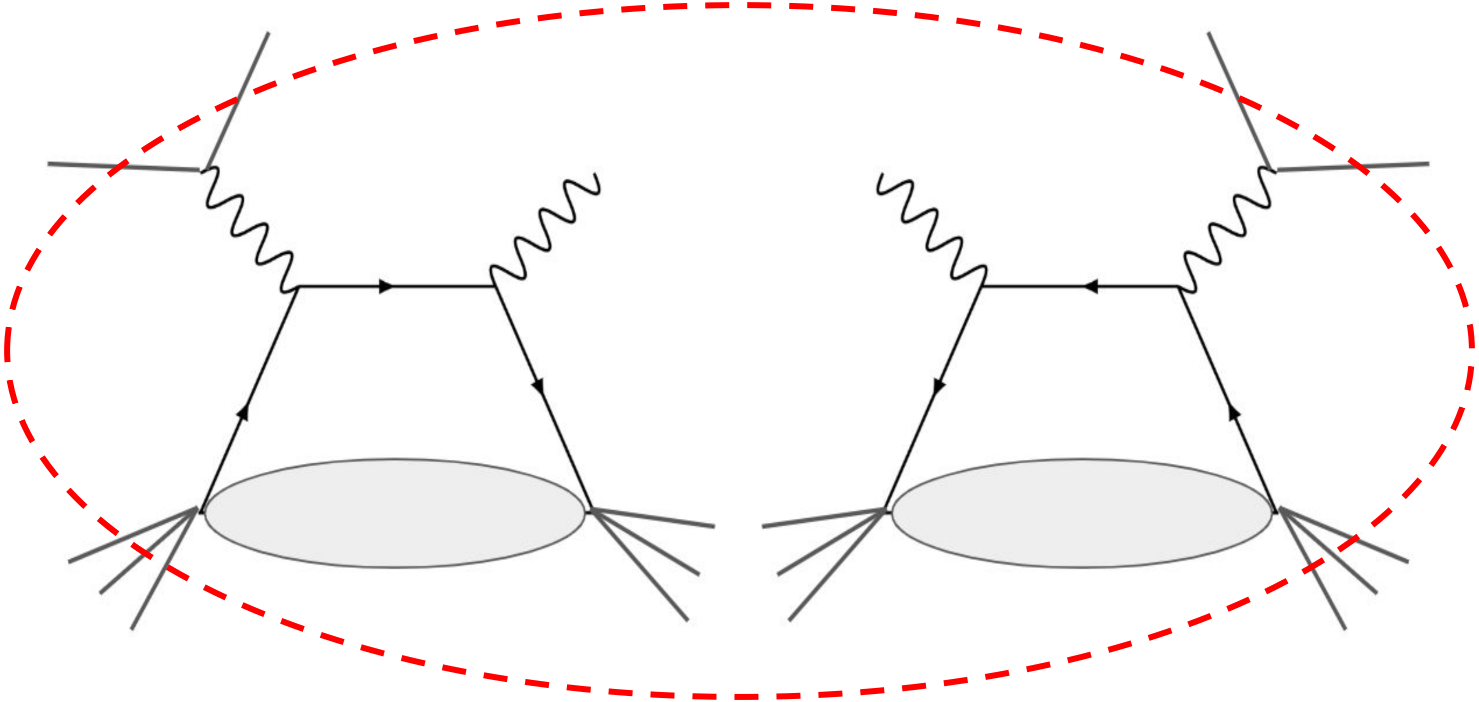
$$e^{i(\Lambda^{(1)}_{\gamma*}-\Lambda^{(2)}_{\gamma*}-2\Lambda)\,\phi}\,F^{\Lambda^{(1)}_{\gamma*}\Lambda^{(2)}_{\gamma*}}_{T,\Lambda\Lambda'} =$$

$$\sum_{\Lambda_{\gamma'}}\left(f^{\Lambda^{(1)}_{\gamma*}\Lambda'_{\gamma}}_{\Lambda\Lambda'}\right)^{*} f^{\Lambda^{(2)}_{\gamma*}\Lambda'_{\gamma}}_{-\Lambda\Lambda'} = e^{i(\Lambda^{(1)}_{\gamma*}-\Lambda^{(2)}_{\gamma*}-2\Lambda)\,\phi}\sum_{\Lambda_{\gamma'}}\left(\widetilde{f}^{\Lambda^{(1)}_{\gamma*}\Lambda'_{\gamma}}_{\Lambda\Lambda'}\right)^{*}\widetilde{f}^{\Lambda^{(2)}_{\gamma*}\Lambda'_{\gamma}}_{-\Lambda\Lambda'}$$

B.K., S.L., et. al. **arXiv:1903.05742**

Finally!

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} =$$



Twist-2 Observables

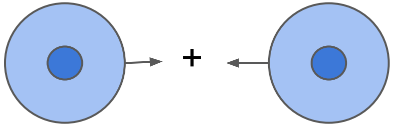
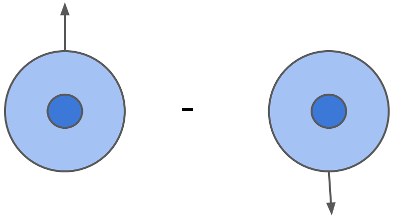
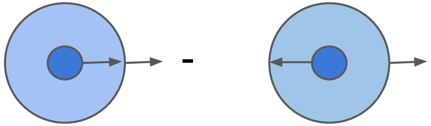
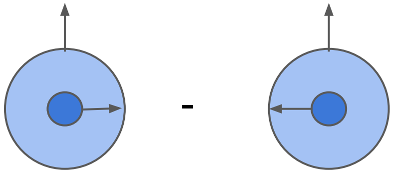
$$F_{UU,T} = 4\left[(1-\xi^2)\left(\mid \mathcal{H} \mid^2 + \mid \tilde{\mathcal{H}} \mid^2\right) + \frac{t_o - t}{2M^2}\left(\mid \mathcal{E} \mid^2 + \xi^2 \mid \tilde{\mathcal{E}} \mid^2\right) - \frac{2\xi^2}{1-\xi^2}\Re\mathrm{e}\left(\mathcal{H}\mathcal{E} + \widetilde{\mathcal{H}}\widetilde{\mathcal{E}}\right)\right]$$

$$F_{LL} = 2\left[2(1-\xi^2)\ \mid \mathcal{H}\,\widetilde{\mathcal{H}} \mid + 4\xi\,\frac{t_o - t}{2M^2}\ \mid \mathcal{E}\widetilde{\mathcal{E}} \mid + \frac{2\xi^2}{1-\xi^2}\Re\mathrm{e}\left(\mathcal{H}\widetilde{\mathcal{E}} + \widetilde{\mathcal{H}}\mathcal{E}\right)\right]$$

$$F_{UT,T}^{\sin(\phi-\phi_S)} = -\frac{\sqrt{t_0-t}}{2M}\left[\Re e\left(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right)\Im m\mathcal{E} - \xi\,\Re e\left(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\right)\Im m\widetilde{\mathcal{E}}\right.$$

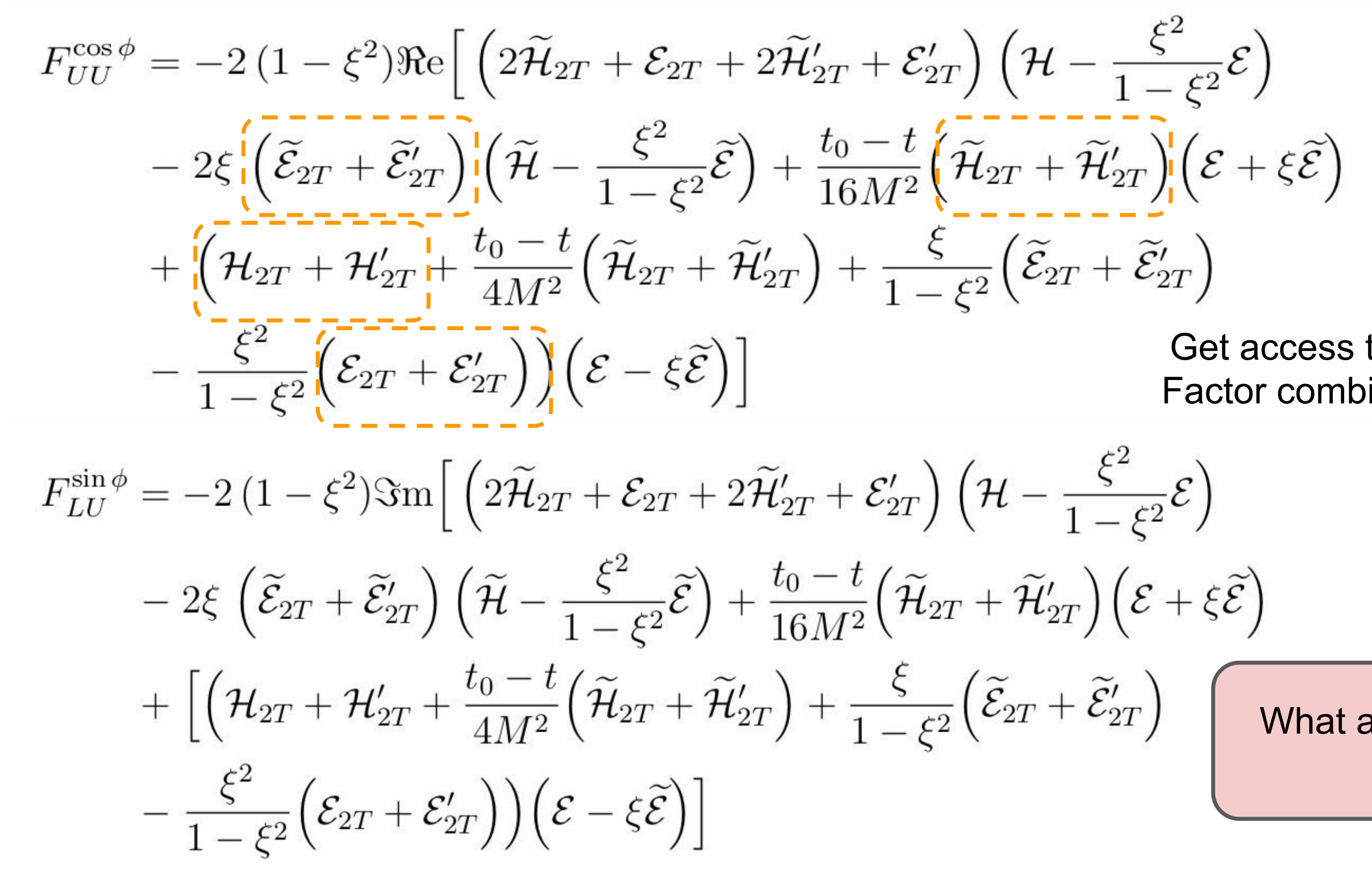
$$\left. - \Im m\left(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\right)\Re e\mathcal{E} + \xi\,\Im m\left(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\right)\Re e\widetilde{\mathcal{E}}\right]$$

B.K., S.L., et. al. arXiv:1903.05742

GPD	Phase	Helicity Composition
H	1	
$\Delta_T E$	$e^{i\phi}$	
$\widetilde{H}$	1	
$\xi \Delta_T \widetilde{E}$	$e^{i\phi}$	

B.K., S.L., et. al. arXiv:1903.05742

Twist-3 Observables

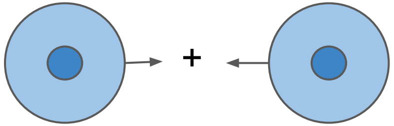
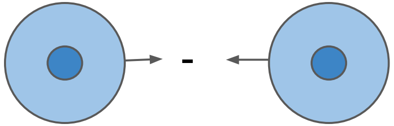
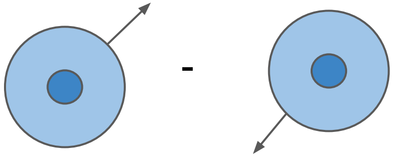
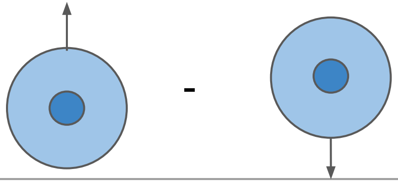


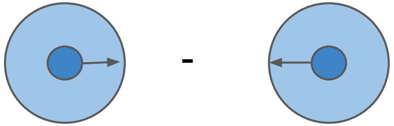
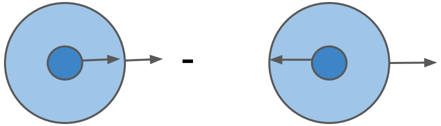
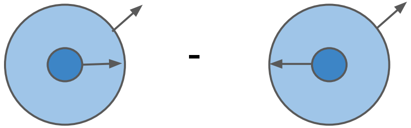
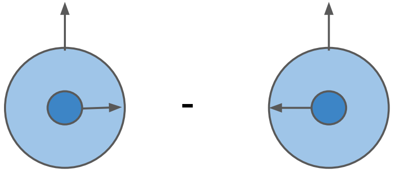
$$
\begin{aligned}
F_{UU}^{\cos\phi} = {}& -2\,(1-\xi^2)\Re\mathrm{e}\Big[\Big(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} + 2\widetilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}\Big)\Big(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\Big)\\
& - 2\xi\boxed{\Big(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\Big)}\Big(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\Big) + \frac{t_0-t}{16M^2}\boxed{\Big(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\Big)}\Big(\mathcal{E} + \xi\widetilde{\mathcal{E}}\Big)\\
& + \boxed{\Big(\mathcal{H}_{2T} + \mathcal{H}'_{2T}} + \frac{t_0-t}{4M^2}\Big(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\Big) + \frac{\xi}{1-\xi^2}\Big(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\Big)\\
& - \frac{\xi^2}{1-\xi^2}\boxed{\Big(\mathcal{E}_{2T} + \mathcal{E}'_{2T}\Big)\Big)}\Big(\mathcal{E} - \xi\widetilde{\mathcal{E}}\Big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
F_{LU}^{\sin\phi} = {}& -2\,(1-\xi^2)\Im\mathrm{m}\Big[\Big(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T} + 2\widetilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}\Big)\Big(\mathcal{H} - \frac{\xi^2}{1-\xi^2}\mathcal{E}\Big)\\
& - 2\xi\,\Big(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\Big)\Big(\widetilde{\mathcal{H}} - \frac{\xi^2}{1-\xi^2}\widetilde{\mathcal{E}}\Big) + \frac{t_0-t}{16M^2}\Big(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\Big)\Big(\mathcal{E} + \xi\widetilde{\mathcal{E}}\Big)\\
& + \Big[\Big(\mathcal{H}_{2T} + \mathcal{H}'_{2T} + \frac{t_0-t}{4M^2}\Big(\widetilde{\mathcal{H}}_{2T} + \widetilde{\mathcal{H}}'_{2T}\Big) + \frac{\xi}{1-\xi^2}\Big(\widetilde{\mathcal{E}}_{2T} + \widetilde{\mathcal{E}}'_{2T}\Big)\\
& - \frac{\xi^2}{1-\xi^2}\Big(\mathcal{E}_{2T} + \mathcal{E}'_{2T}\Big)\Big)\Big(\mathcal{E} - \xi\widetilde{\mathcal{E}}\Big)\Big]
\end{aligned}
$$

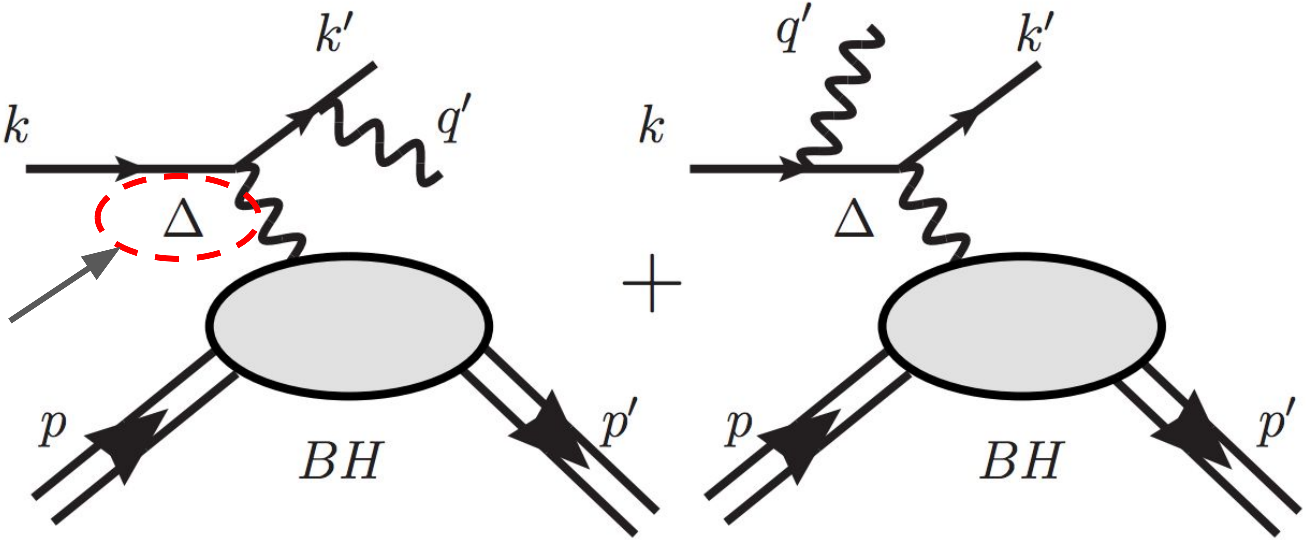
Get access to 8 Compton Form Factor combinations from DVCS alone.

What are these twist 3 GPDs?

GPD		Phase	Helicity Composition
H	$2\widetilde{H}_{2T} + E_{2T}$	$e^{i\phi}$	 +
OAM	$\widetilde{E}_{2T} - \xi E_{2T}$	$e^{i\phi}$	 -
T OAM??	$\frac{\Delta_T^2}{MP^+} 2\widetilde{H}_{2T}$	$e^{2i\phi}$	 -
E	$H_{2T} + \frac{\Delta_T^2}{4M^2}\widetilde{H}_{2T}$	1	 -

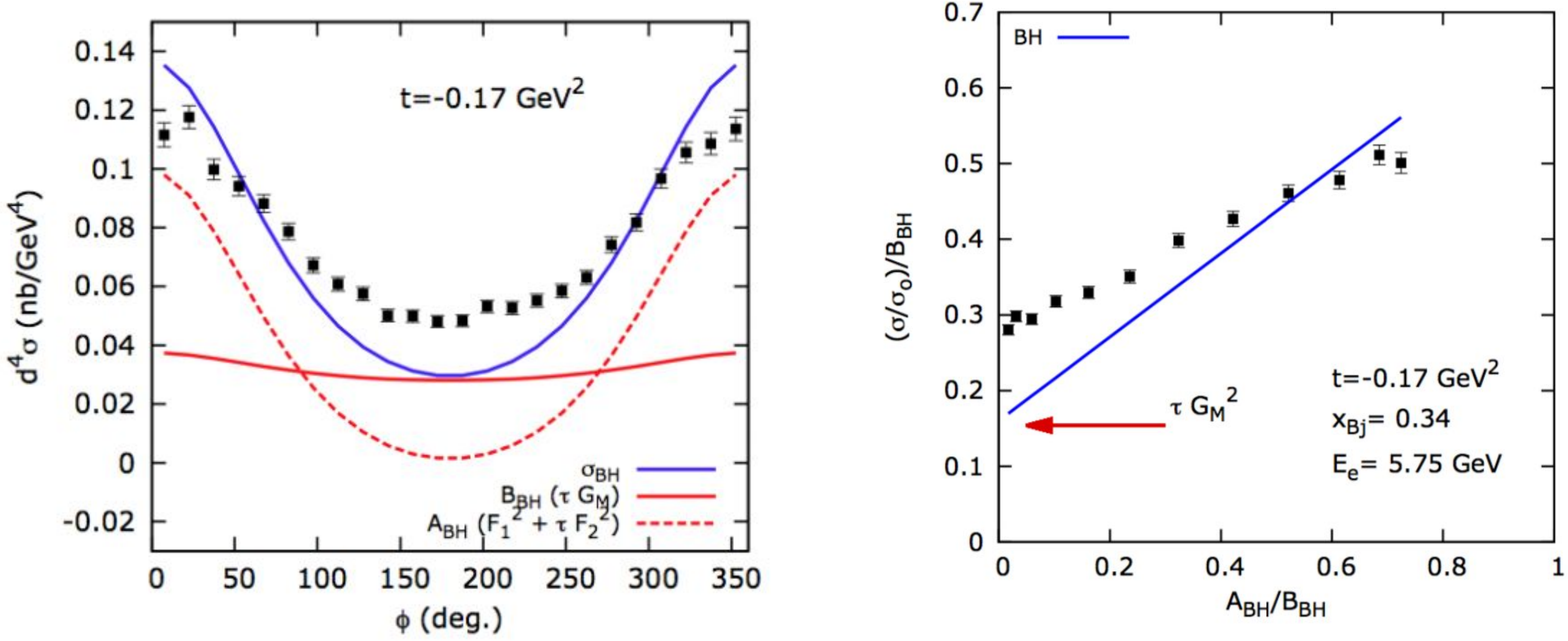
GPD		Phase	Helicity Composition
$\widetilde{H}$	$\frac{\Delta_T}{P^+}\left(\widetilde{E}'_{2T} - \xi E'_{2T}\right)$	$e^{i\phi}$	
Spin Orbit	$\frac{\Delta_T}{P^+}\left(E'_{2T} - \xi \widetilde{E}'_{2T} + 2\widetilde{H}'_{2T}\right)$	$e^{i\phi}$	
T Spin Orbit??	$\frac{\Delta_T^2}{MP^+}\widetilde{H}'_{2T}$	$e^{2i\phi}$	
$\widetilde{E}$	$H'_{2T} + \frac{\Delta_T^2}{4M^2}\widetilde{H}'_{2T}$	1	

Bethe-Heitler



Different hard scale due to the radiation.

$$\frac{d^5\sigma^{BH}_{unpol}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\Gamma}{t^2}\left[A_{BH}(F_1^2+\tau F_2^2)+B_{BH}\tau G_M^2\right]$$



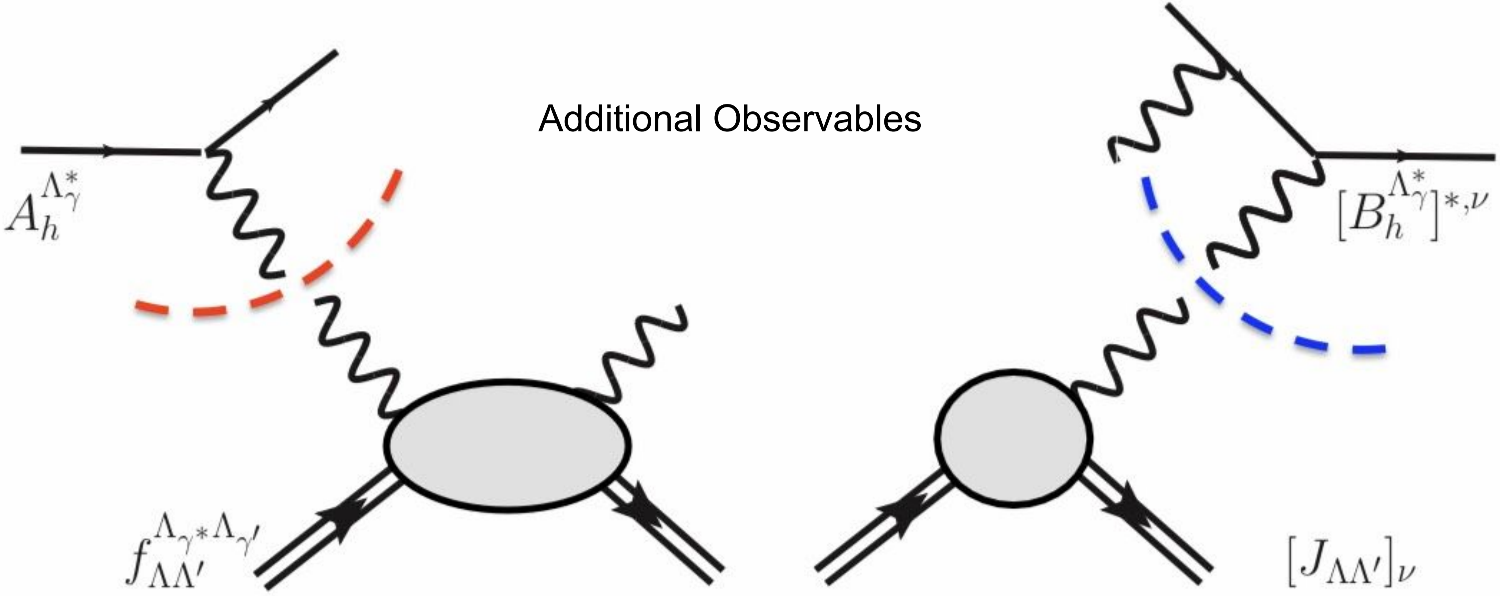
B.K., S.L., et. al. **arXiv:1903.05742**

DVCS/BH Interference

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I} \; .$$

$$\mathcal{I} = T^*_{BH} T_{DVCS} + T^*_{DVCS} T_{BH} .$$

Additional Observables



$$F_{UU}^{\mathcal{I}\,\text{tw2}} = A_{UU}^{\mathcal{I}} \Re e(F_1\mathcal{H} + \tau F_2\mathcal{E}) + B_{UU}^{\mathcal{I}} G_M \Re e(\mathcal{H} + \mathcal{E}) + C_{UU}^{\mathcal{I}} G_M \Re e\widetilde{\mathcal{H}}$$

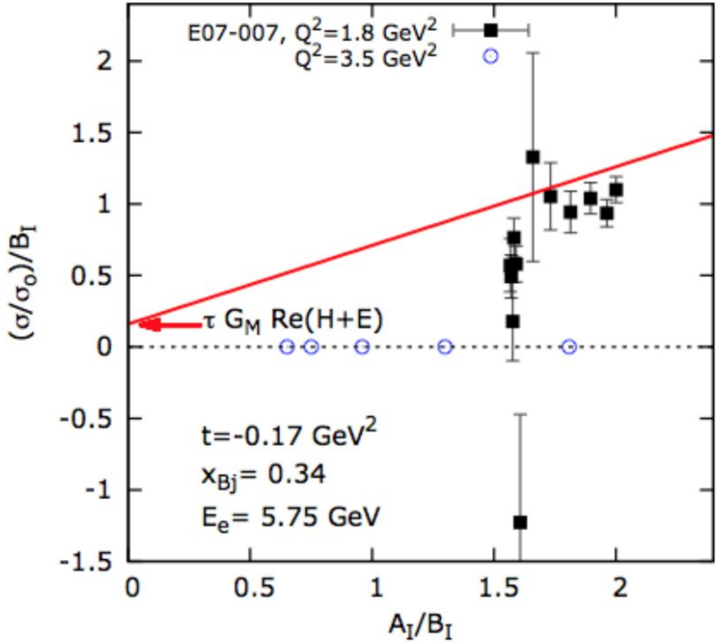
$$\begin{aligned}
F_{UU}^{\mathcal{I}\,\text{tw3}} = \frac{K}{\sqrt{Q^2}} \Re e \Bigg\{ & A_{UU}^{(3)\mathcal{I}} \Big[F_1(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T}) + F_2(\mathcal{H}_{2T} + \tau\widetilde{\mathcal{H}}_{2T}) \Big] \\
& + B_{UU}^{(3)\mathcal{I}} G_M \, \widetilde{E}_{2T} + C_{UU}^{(3)\mathcal{I}} G_M \Big[2\xi H_{2T} - \tau(\widetilde{E}_{2T} - \xi E_{2T}) \Big] \Bigg\} \\
+ \frac{K}{\sqrt{Q^2}} \Im m \Bigg\{ & \widetilde{A}_{UU}^{(3)\mathcal{I}} \Big[F_1(2\widetilde{H}'_{2T} + E'_{2T}) + F_2(H'_{2T} + \tau\widetilde{H}'_{2T}) \Big] \\
& + \widetilde{B}_{UU}^{(3)\mathcal{I}} G_M \widetilde{E}'_{2T} + \widetilde{C}_{UU}^{(3)\mathcal{I}} G_M \Big[2\xi H'_{2T} - \tau(\widetilde{E}'_{2T} - \xi E'_{2T}) \Big] \Bigg\}
\end{aligned}$$

B.K., S.L., et. al. **arXiv:1903.05742**

Rosenbluth-like Separation

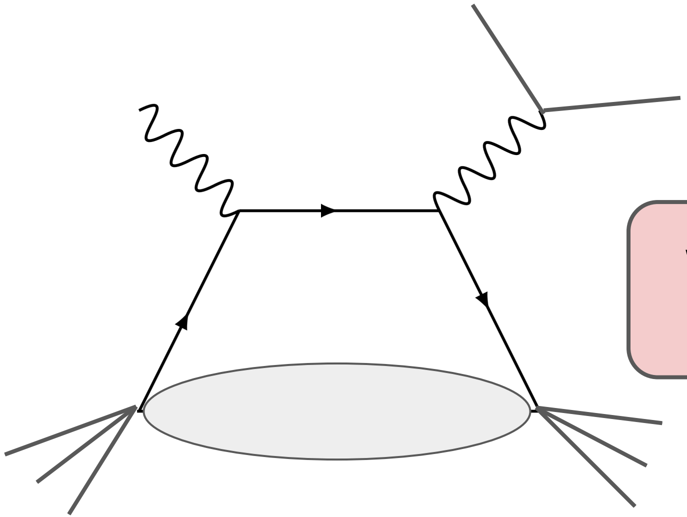
Angular Momentum

$$F_{UU}^{\mathcal{I}\,\mathrm{tw2}} = A_{UU}^{\mathcal{I}} \Re e(F_1\mathcal{H}+\tau F_2\mathcal{E}) + B_{UU}^{\mathcal{I}} G_M \Re e(\mathcal{H}+\mathcal{E}) + C_{UU}^{\mathcal{I}} G_M \Re e\widetilde{\mathcal{H}}$$



S. Liuti arXiv:1903.08760

Coming soon … TCS



🟦 Twist - 2
🟩 Twist - 3

We are developing the formalism for TCS!

Do we gain access to the other 8 Compton Form Factor combinations?

$$W^{\mu\nu} \propto \gamma^{\mu}\gamma^{+}\gamma^{\nu} = \begin{bmatrix} \gamma^{-} & \boxed{\gamma^{1} - i\gamma^{2}\gamma_5} & \boxed{\gamma^{2} + i\gamma^{1}\gamma_5} & i\gamma^{-}\gamma_5 \\ \gamma^{1} + i\gamma^{2}\gamma_5 & \boxed{\gamma^{+}} & \boxed{i\gamma^{+}\gamma_5} & -\gamma^{1} - i\gamma^{2}\gamma_5 \\ \gamma^{2} - i\gamma^{1}\gamma_5 & \boxed{-i\gamma^{+}\gamma_5} & \boxed{\gamma^{+}} & -\gamma^{2} + i\gamma^{1}\gamma_5 \\ -i\gamma^{-}\gamma_5 & \boxed{-\gamma^{1} + i\gamma^{2}\gamma_5} & \boxed{-\gamma^{2} - i\gamma^{1}\gamma_5} & \gamma^{-} \end{bmatrix}$$

Future: DVCS from Spin-1 Deuteron

Angular momentum sum rule for a spin-½ nucleon.

$$J_q = \frac{1}{2}\int dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right]$$

Using the Spin-1 Energy Momentum Tensor, a similar formula has been developed for the Spin-1 deuteron

$$J_q = \frac{1}{2}\int dx\, \boxed{x H_2^q(x,0,0)}$$

Can we identify the observable for the Spin-1 Angular Momentum from DVCS off a spin-1 Deuteron formalism?

S. Taneja, K. Kathuria, S Liuti, and G.R. Goldstein **arXiv:1101.0581**

Summary and future work

- We can isolate the phi-dependence from the kinematic phase contributions.
- Can we isolate Twist-3 GPDs of the vector and axial vector sector through observables of DVCS combined with TCS?
- We are developing the formalism for TCS to investigate possible additional Compton Form Factors
- We are developing the formalism for DVCS off Spin-1 deuteron to identify observables for Spin-1 Angular Momentum.
- OAM in a spin-1 system?