Collective excitations and electromagnetic probes of momentum-anisotropic quark-gluon plasma

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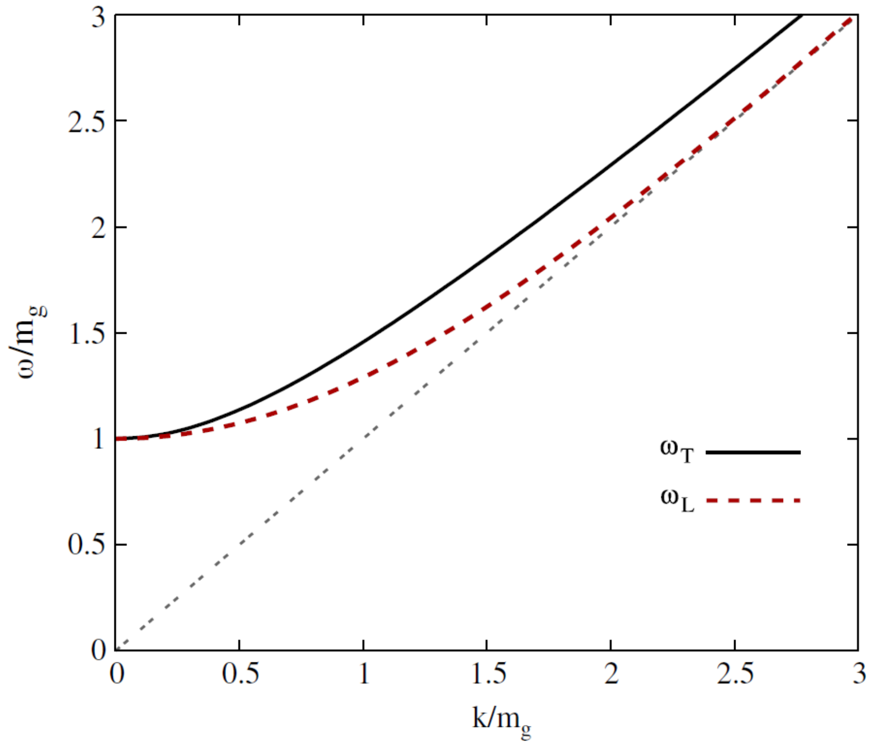
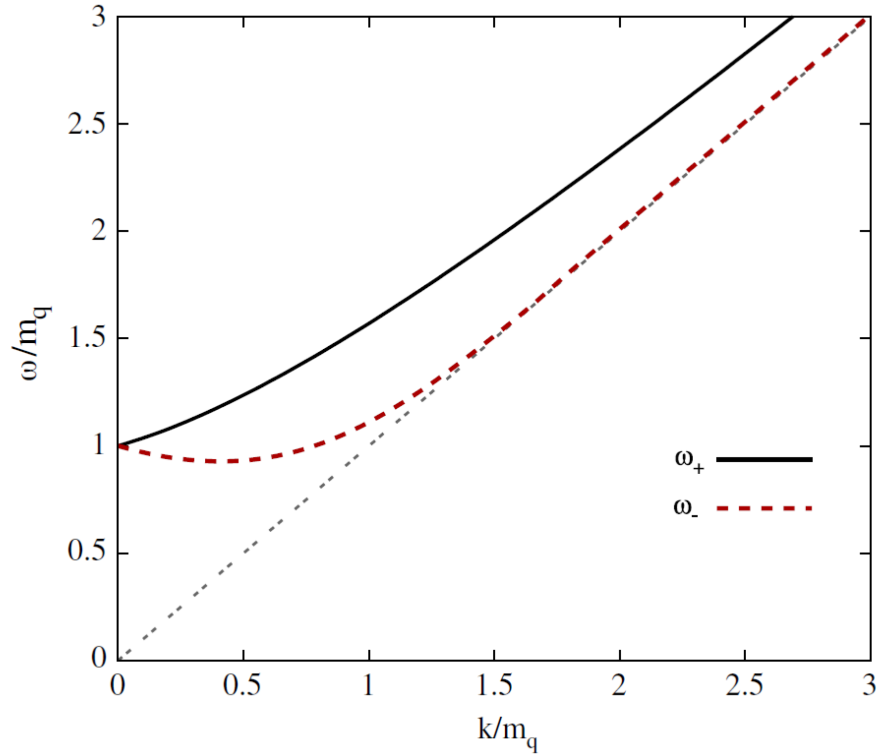
Collective excitations-many-body systems

- A large number of interacting particles → small number of almost-free quasiparticles (~ collective excitations)
- Dressed collective fermions
- (Anti-)Screened collective bosonic interactions
- Complex frequency poles of Green's functions
- Medium-dependent charactristics

Collective excitation- isotropic QGP



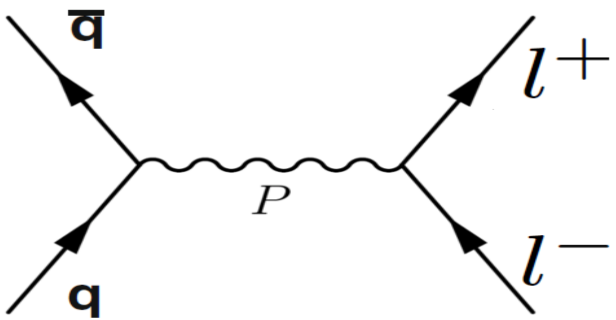


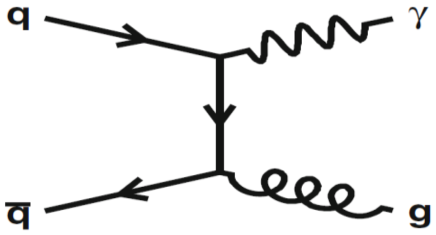
VP Silin - Sov. Phys. JETP, 1960 | VV Klimov - Sov. Phys. JETP, 1982|
HA Weldon - Physical Review D, 1982

Physics Reports 682 (2017) 1–97

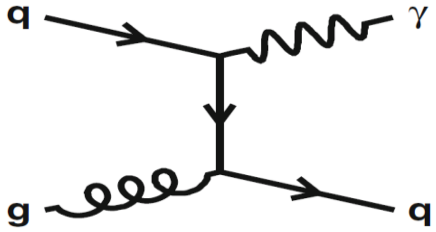
EM probes of QGP

Leading order





Annihilation

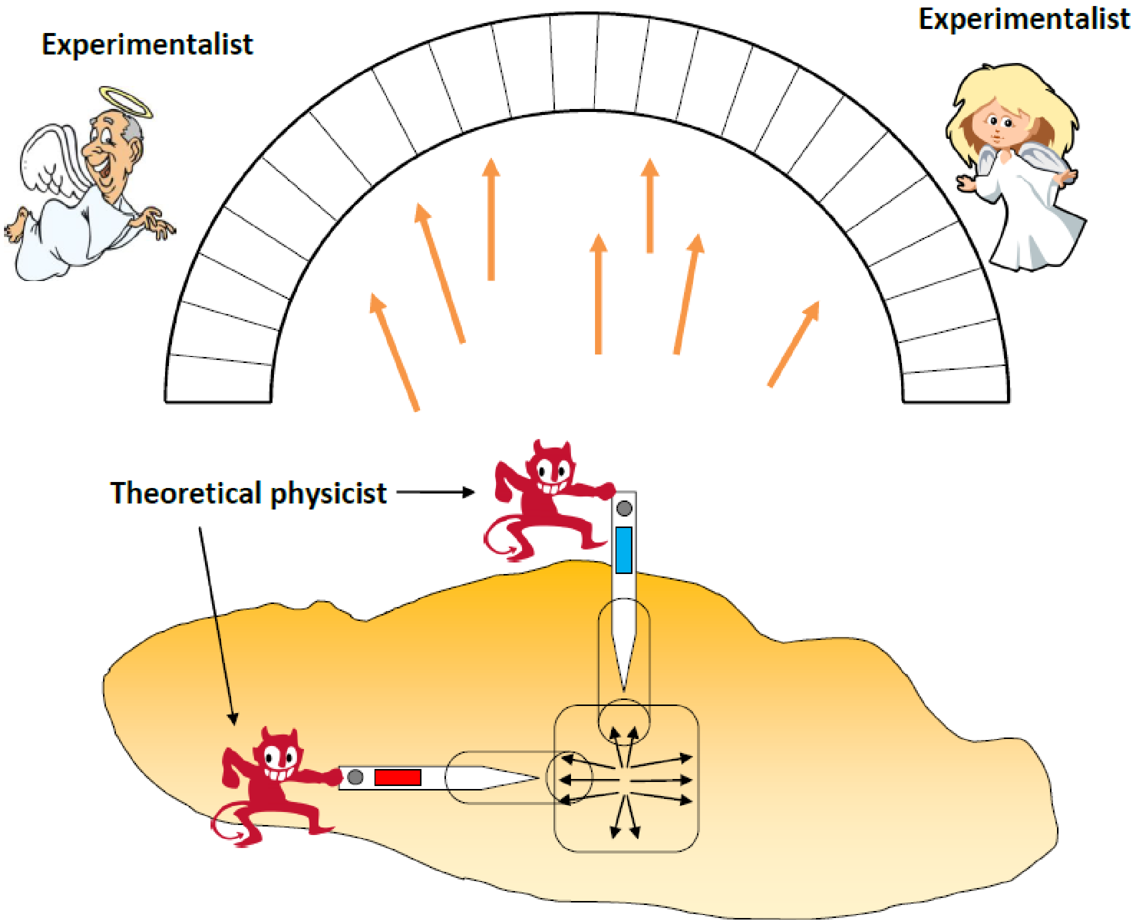


Compton

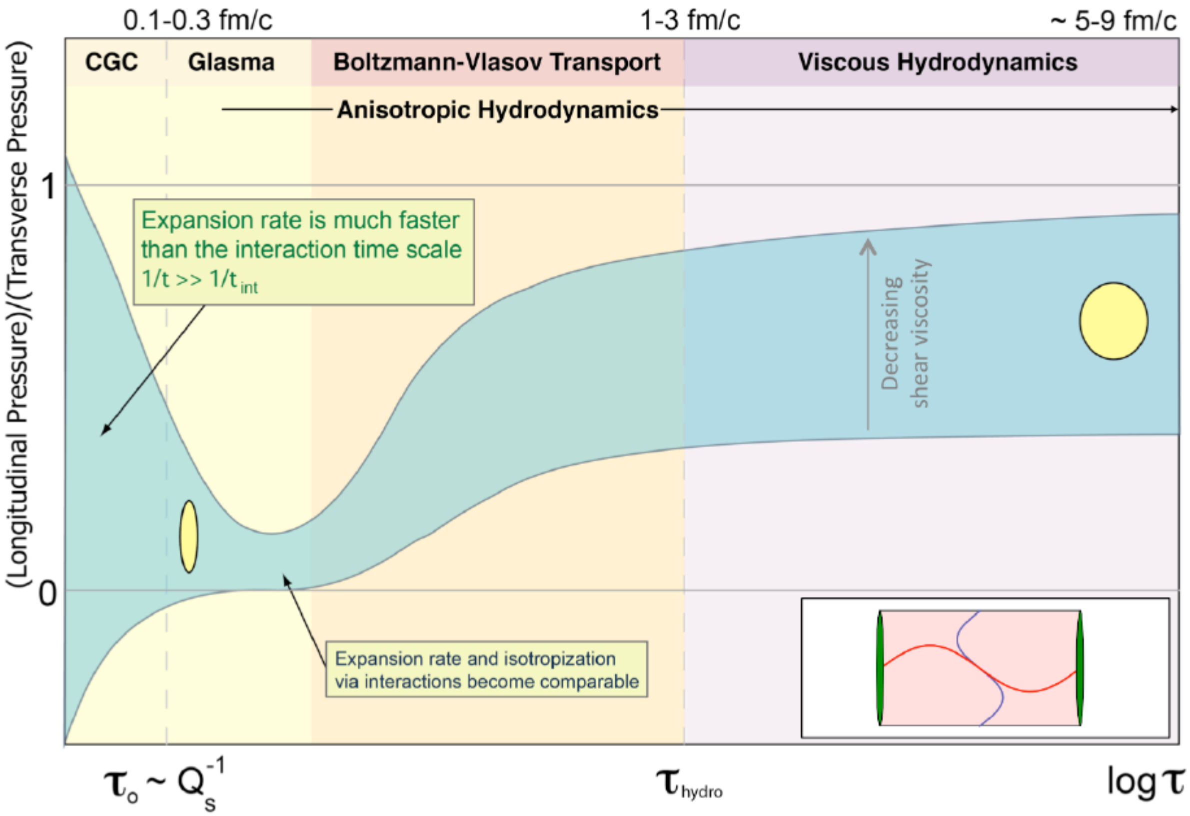
Early stages of QGP evolution

Collective modes

Momentum Anisotropy ≠ Momentum Anisotropy

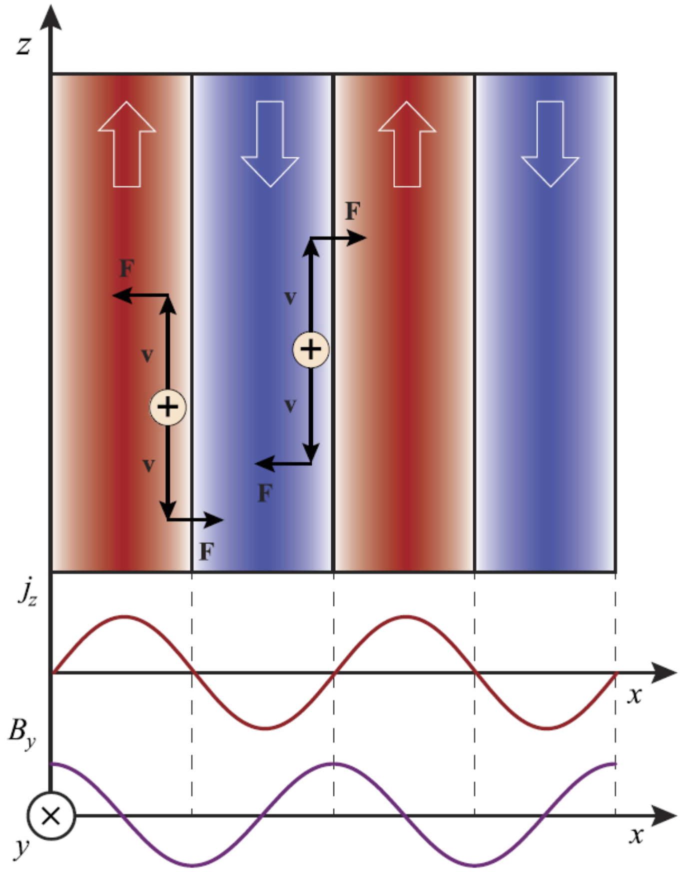


Evolution of momentum anisotropy



M Strickland , Acta Physica Polonica B 45, 2355 (2014)

Weibel plasma instability



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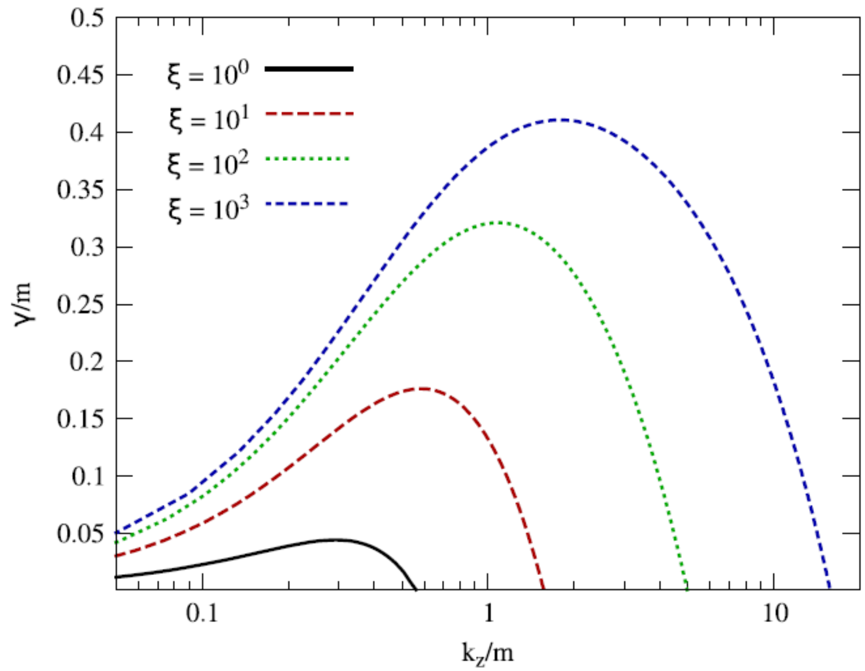


S Mrówczyński - Physics Letters B, 1988 |
P Arnold, J Lenaghan, GD Moore
Journal of High Energy Physics 2003 (08), 002

Hard-Loop Self-energies

Gluon self-energy (LO):

$$\Pi^{ij}(k) = -g^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} v^i \frac{\partial f(\mathbf{p})}{\partial p_l} \left[\delta^{jl} + \frac{k^l v^j}{\omega - \mathbf{k}.\mathbf{v} + i0^+}\right]$$

$$k_\mu \Pi^{\mu\nu} = 0$$

Quark self-energy (LO):

$$\Sigma(K) = \frac{g^2 C_F}{4} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|} \frac{\gamma_0 + \mathbf{v}.\boldsymbol{\gamma}}{\omega - \mathbf{k}.\mathbf{v} + i0^+}$$

$$C_F = (N_c^2 - 1)/(2N_c) \qquad \Sigma = \gamma_\mu \Sigma^\mu$$

Same forms, with:

$$f(\mathbf{p}) = \begin{cases} 2N_c n_g(\mathbf{p}) + N_f[n_q(\mathbf{p}) + n_{\bar{q}}(\mathbf{p})] & \text{for the gluon self-energy integral,} \\ 2n_g(\mathbf{p}) + N_f[n_q(\mathbf{p}) + n_{\bar{q}}(\mathbf{p})] & \text{for the quark self-energy integral,} \end{cases}$$

$$f(\mathbf{p}) = f(-\mathbf{p})$$

Stanisław Mrówczyński and Markus H. Thoma
Phys. Rev. D 62, 2000

Momentum Anisotropic Distributions

- Deformed isotropic distributions $$f_{\{\alpha\}}(\mathbf{p}) = f_{\rm iso}\left(\frac{|\mathbf{p}|}{\Lambda(\{\alpha\})} H(\hat{\mathbf{v}};\{\alpha\})\right)$$ $$\boxed{f(\mathbf{p}) = f(-\mathbf{p})}$$
- Spheroidal $$H_s(\hat{\mathbf{v}};\{\xi,\hat{\mathbf{n}}\}) = \sqrt{1+\xi(\hat{\mathbf{n}}\cdot\hat{\mathbf{v}})^2}$$
- Ellipsoidal $$H_e(\hat{\mathbf{v}};\{\xi_1,\xi_2,\hat{\mathbf{n}}_1,\hat{\mathbf{n}}_2\}) = \sqrt{1+\xi_1(\hat{\mathbf{n}}_1\cdot\hat{\mathbf{v}})^2+\xi_2(\hat{\mathbf{n}}_2\cdot\hat{\mathbf{v}})^2}$$
- Quadratic $$H_\Xi(\hat{\mathbf{v}};\Xi^{\mu\nu}) = \sqrt{v^\mu \Xi_{\mu\nu} v^\nu}$$

$$\Sigma(K) = m_q^2 \int \frac{d\Omega}{4\pi}\frac{1}{[H(\theta,\phi)]^2}\frac{\gamma_0+\hat{\mathbf{v}}\cdot\boldsymbol{\gamma}}{\omega-\mathbf{k}\cdot\hat{\mathbf{v}}+i0^+}$$

$$m_q^2 = \frac{g^2 C_F}{8\pi^2}\int_0^\infty dp\, p\, f_{\rm iso}(p/\Lambda)$$

$$\Pi^{ij}(K) = m_D^2 \int \frac{d\Omega}{4\pi} v^i \frac{W^l}{[H_\Xi(\theta,\phi)]^4}\left[\delta^{jl}+\frac{k^l v^j}{\omega-\mathbf{k}\cdot\hat{\mathbf{v}}+i0^+}\right]$$

$$m_D^2 = -\frac{g^2}{2\pi^2}\int_0^\infty dp\, p^2 \frac{df_{\rm iso}(p/\Lambda)}{dp}$$

Spheroidal parameterization-1

Tensor decomposition: $\Pi^{ij} = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij}$

$$A^{ij} = \delta^{ij} - k^i k^j / k^2,$$

$$B^{ij} = k^i k^j / k^2,$$

$$C^{ij} = \tilde{n}^i \tilde{n}^j / \tilde{n}^2,$$

$$D^{ij} = k^i \tilde{n}^j + k^j \tilde{n}^i$$

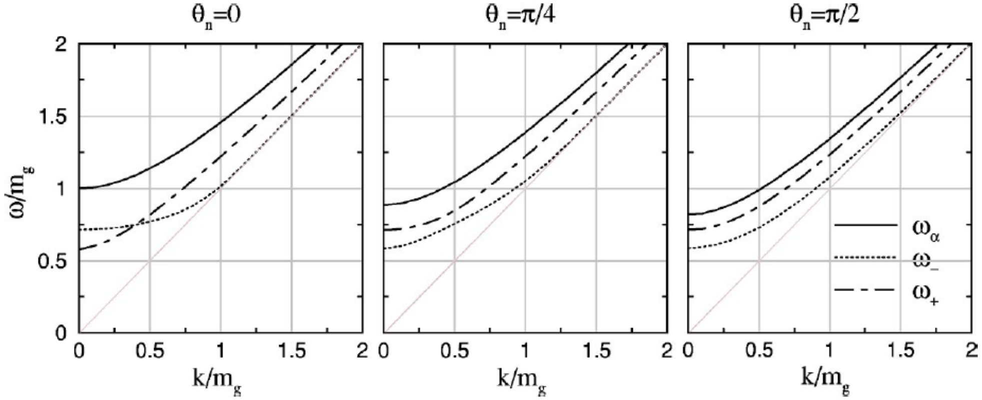
$$\tilde{n}^i = A^{ij} n^j$$

$$k^i \Pi^{ij} k^j = k^2 \beta,$$

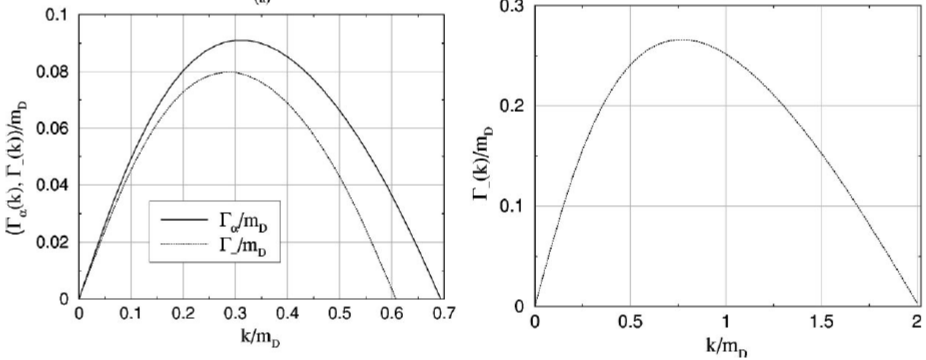
$$\tilde{n}^i \Pi^{ij} k^j = \tilde{n}^2 k^2 \delta,$$

$$\tilde{n}^i \Pi^{ij} \tilde{n}^j = \tilde{n}^2 (\alpha + \gamma),$$

$$\mathrm{Tr}\, \Pi^{ij} = 2\alpha + \beta + \gamma.$$



(a) (b)



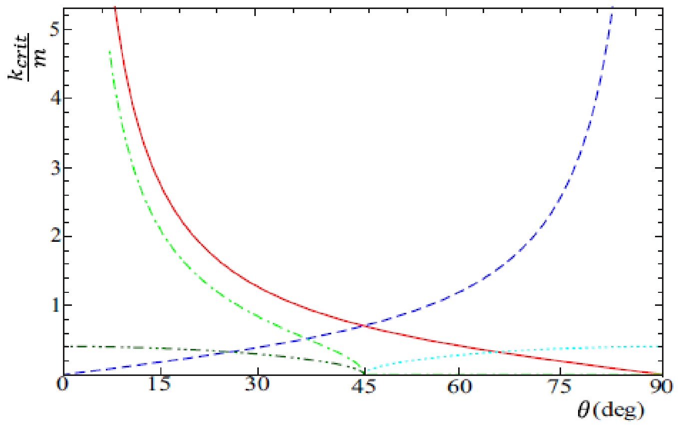
Paul Romatschke and Michael Strickland
Phys. Rev. D 68, 2003

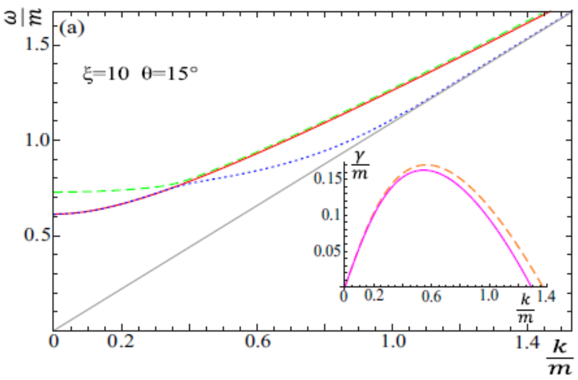
Spheroidal parameterization-2

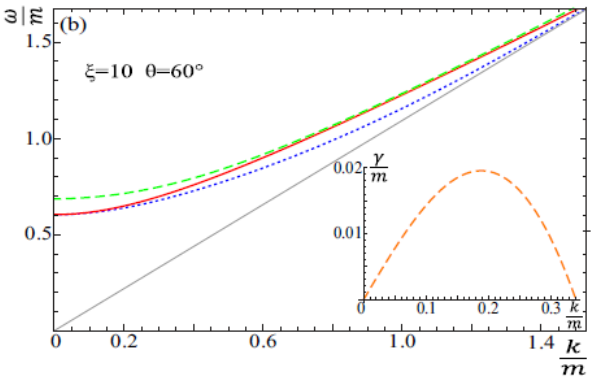
$$\int \frac{d^3p}{(2\pi)^3} \frac{f_\xi(\mathbf{p})}{|\mathbf{p}|} = \int \frac{d^3p}{(2\pi)^3} \frac{f_{\rm iso}(|\mathbf{p}|)}{|\mathbf{p}|}$$

$$f_\sigma(\mathbf{p}) \equiv C_\sigma\, f_{\rm iso}\left(\sqrt{(\sigma+1)\mathbf{p}^2 - \sigma(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

+ Nyquist analysis for analytic results: weak/extreme anisotropy







Margaret E. Carrington, Katarzyna Deja, and Stanisław Mrówczyński
Phys. Rev. C 90, 2014

Spheroidal parameterization-3

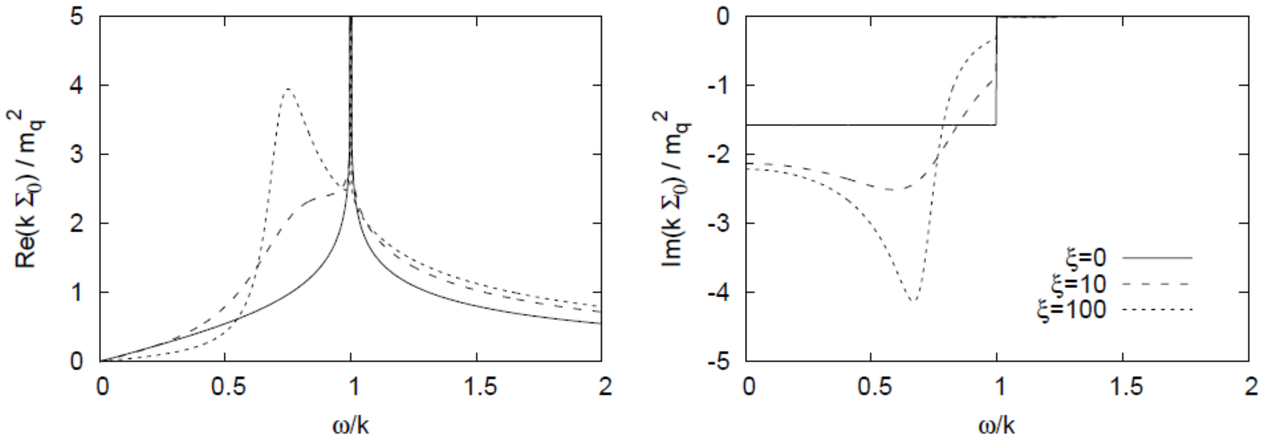
$$\Sigma_0(w,k,\theta_n,\xi) = \frac{1}{2}m_q^2\sqrt{1+\xi}\int_{-1}^{1} dx \frac{R(w-k\cos\theta_n x, k\sin\theta_n\sqrt{1-x^2})}{1+\xi x^2},$$

$$\Sigma_x(w,k,\theta_n,\xi) = \frac{1}{2}m_q^2\sqrt{1+\xi}\int_{-1}^{1} dx \frac{\sqrt{1-x^2}S(w-k\cos\theta_n x, k\sin\theta_n\sqrt{1-x^2})}{1+\xi x^2},$$

$$\Sigma_z(w,k,\theta_n,\xi) = \frac{1}{2}m_q^2\sqrt{1+\xi}\int_{-1}^{1} dx \frac{xR(w-k\cos\theta_n x, k\sin\theta_n\sqrt{1-x^2})}{1+\xi x^2},$$

$$R(a,b) = \left(\sqrt{a+b+i\epsilon}\sqrt{a-b+i\epsilon}\right)^{-1},$$

$$S(a,b) = \frac{1}{b}\left[aR(a,b)-1\right].$$



No unstable fermionic modes

Björn Schenke and Michael Strickland Phys. Rev. D 74, 2006

Ellipsoidal parameterization-1.1

$$\Sigma^i(K) = \frac{m_q^2}{4\pi} \int_0^{2\pi} d\phi \int_{-1}^{1} dx \; \frac{1}{A(1+s\cos^2\phi)} \; \frac{v^i}{a - b\cos\phi - c\sin\phi}$$

$$\Sigma^i = \frac{m_q^2}{4\pi k} \int_{-1}^{1} dx \sum_{j=1}^{8} \lambda_j^i \, D_j \qquad (i = 0, 1, 2, 3)\,,$$

$$D_1 = \int_0^{2\pi} d\phi \, \frac{1}{a - b\cos\phi - c\sin\phi}$$

$$D_2 = \int_0^{2\pi} d\phi \, \frac{\cos\phi}{a - b\cos\phi - c\sin\phi}$$

…

$$D_7 = \int_0^{2\pi} d\phi \, \frac{\sin\phi}{1 + s\cos^2\phi}$$

$$D_8 = \int_0^{2\pi} d\phi \, \frac{\cos\phi\sin\phi}{1 + s\cos^2\phi}$$

$$a = \frac{\omega}{k} - x\cos\theta_k \,,$$

$$b = \sin\theta_k \cos\phi_k \sqrt{1-x^2} \,,$$

$$c = \sin\theta_k \sin\phi_k \sqrt{1-x^2} \,,$$

$$A = 1 + \xi_1 x^2 \,,$$

$$s = \frac{\xi_2(1-x^2)}{A} \,,$$

$$v = (1, \; \sqrt{1-x^2}\cos\phi, \; \sqrt{1-x^2}\sin\phi, \; x)$$

PHYSICAL REVIEW D **94**, 125001 (2016)

Quark self-energy in an ellipsoidally anisotropic quark-gluon plasma

Babak S. Kasmaei, Mohammad Nopoush, and Michael Strickland

Department of Physics, Kent State University, Kent, Ohio 44242, USA

(Received 27 August 2016; published 1 December 2016)

Ellipsoidal parameterization-1.2

$$\Sigma_0 = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_0\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_0\sqrt{\frac{1}{1+s}}\right]$$

$$\Sigma_x = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_x\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_x\sqrt{\frac{1}{1+s}}\right]$$

$$\Sigma_y = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_y\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_y\sqrt{\frac{1}{1+s}}\right]$$

$$\Sigma_z = \frac{m_q^2}{2k}\int_{-1}^{1} dx\, \frac{1}{A\rho}\left[\alpha_z\sqrt{\frac{(a+b)^2}{a^2-(b^2+c^2)}} + s\beta_z\sqrt{\frac{1}{1+s}}\right]$$

$$\rho \equiv (b^2+c^2)^2 + s(2a^2b^2 - 2a^2c^2 + 2b^2c^2 + 2c^4) + s^2(a^2-c^2)^2$$

$$\alpha_0 \equiv \frac{1}{a+b}\left[(b^2+c^2)^2 + s(a^2b^2 - a^2c^2 + b^2c^2 + c^4)\right] ,$$

$$\beta_0 \equiv a(b^2-c^2) + sa(a^2-c^2) ,$$

$$\alpha_x \equiv \frac{ab}{a+b}\left[(b^2+c^2) + s(a^2-c^2)\right]\sqrt{1-x^2} ,$$

$$\beta_x \equiv -b\left[(b^2+c^2) + s(a^2+c^2)\right]\sqrt{1-x^2} ,$$

$$\alpha_y \equiv \frac{ac}{a+b}\left[(b^2+c^2) + s(-a^2+2b^2+c^2)\right]\sqrt{1-x^2} ,$$

$$\beta_y \equiv -c\left[(b^2+c^2) + s(-a^2+b^2+2c^2) + s^2(c^2-a^2)\right]\sqrt{1-x^2} ,$$

$$\alpha_z \equiv x\alpha_0 ,$$

$$\beta_z \equiv x\beta_0 .$$

PHYSICAL REVIEW D **94**, 125001 (2016)

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More efficient and general method-1

For polar integration variable, define $x = \cos\theta = \hat{\mathbf{k}} \cdot \hat{\mathbf{v}}$

Phase velocity $z \equiv \omega/k$

PHYSICAL REVIEW D **97,** 054022 (2018)

Parton self-energies for general momentum-space anisotropy

Babak S. Kasmaei and Michael Strickland
Department of Physics, Kent State University, Kent, Ohio 44242 USA

$$\Sigma(K) = m_q^2 \int \frac{d\Omega}{4\pi} \frac{1}{[H(\theta,\phi)]^2} \frac{\gamma_0 + \hat{\mathbf{v}}\cdot\boldsymbol{\gamma}}{\omega - \mathbf{k}\cdot\hat{\mathbf{v}} + i0^+}$$

$$\mathrm{Im}[\Sigma(z)] = -m_q^2 \Theta(1-z^2) \int_0^{2\pi} \frac{d\phi}{4k} \frac{\gamma_0 + \hat{\mathbf{v}}\cdot\boldsymbol{\gamma}}{[H(z,\phi)]^2}$$

$$\Rightarrow \quad \mathrm{Re}[\Sigma(z)] = -\int_{-1}^{1} \frac{dx}{\pi} \frac{\mathrm{Im}[\Sigma(x)]}{z - x + i0^+}$$

$$\mathrm{Im}[\Pi^{ij}(z)] = -m_D^2 \Theta(1-z^2) \int_0^{2\pi} \frac{d\phi}{4k} \frac{v^i k^l v^j}{[H_\Xi(z,\phi)]^4} W^l$$

$$\mathrm{Re}[\Pi^{ij}(z)] = \Pi_0^{ij} + \Pi_k^{ij}(z) = m_D^2 \int \frac{d\Omega}{4\pi} v^i \frac{W^l(\theta,\phi)}{[H_\Xi(\theta,\phi)]^4} \delta^{jl} - \int_{-1}^{1} \frac{dx\,\mathrm{Im}[\Pi^{ij}(x)]}{\pi\; z - x + i0^+}$$

More efficient and general method-2

PHYSICAL REVIEW D **97,** 054022 (2018)

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$$\mathrm{Re}[\Sigma(z)] = -\int_{-1}^{1}\frac{dx}{\pi}\frac{\mathrm{Im}[\Sigma(x)]}{z-x+i0^{+}}$$

$$\mathrm{Im}[\Sigma^{\mu}(x)] \approx \sum_{n=0}^{n_{\max}} c_n^{\mu} r_n(x),$$

$$I_n(z) = \int_{-1}^{1} dx\,\frac{r_n(x)}{z-x+i0^{+}}$$

$\Rightarrow$

$$\mathrm{Re}[\Sigma^{\mu}(z)] \approx -\frac{1}{\pi}\sum_{n=0}^{n_{\max}} c_n^{\mu}\mathrm{Re}[I_n(z)].$$

If we choose $r_n(x) = x^n$ $\Rightarrow$

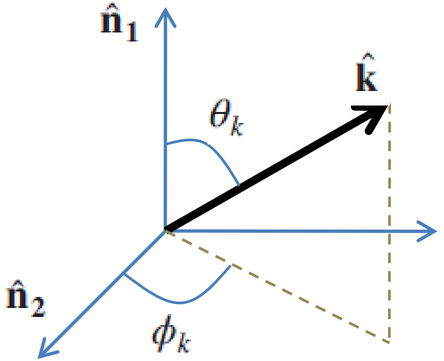
$$I_n(z) = \int_{-1}^{1} dx\frac{x^n}{z-x+i0^{+}} = \frac{{}_2F_1(1,1+n,2+n,-1/z)+(-1)^n F_1(1,1+n,2+n,1/z)}{(1+n)z}$$

Ellipsoidal parameterization-2.1

PHYSICAL REVIEW D **97**, 054022 (2018)

$$\Sigma(z;\{\xi_1,\xi_2,\hat{\mathbf{n}}_1,\hat{\mathbf{n}}_2\}) = \frac{m_q^2}{4\pi k}\int_{-1}^{1}dx\int_0^{2\pi}d\phi\frac{1}{1+\xi_1(\hat{\mathbf{n}}_1\cdot\hat{\mathbf{v}})^2+\xi_2(\hat{\mathbf{n}}_2\cdot\hat{\mathbf{v}})^2}\frac{\gamma_0+\hat{\mathbf{v}}\cdot\boldsymbol{\gamma}}{z-x+i0^+}$$

$$\Pi^{ij}(\omega,\mathbf{k}) = m_D^2\int\frac{d\Omega}{4\pi}v^i\frac{v^l+\xi_1(\mathbf{n}_1\cdot\mathbf{v})n^l+\xi_2(\mathbf{n}_2\cdot\mathbf{v})n^l}{(1+\xi_1(\mathbf{n}_1\cdot\mathbf{v})^2+\xi_2(\mathbf{n}_2\cdot\mathbf{v})^2)^2}\left[\delta^{jl}+\frac{k^l v^j}{\omega-\mathbf{k}\cdot\mathbf{v}+i0^+}\right]$$



$$\hat{\mathbf{k}} = (0,0,1),$$
$$\hat{\mathbf{n}}_1 = (0,-\sin\theta_k,\cos\theta_k),$$
$$\hat{\mathbf{n}}_2 = (\sin\phi_k,\cos\theta_k\cos\phi_k,\sin\theta_k\cos\phi_k),$$
$$\hat{\mathbf{v}} = (\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta).$$

$$\xi_1(\hat{\mathbf{n}}_1\cdot\hat{\mathbf{v}})^2+\xi_2(\hat{\mathbf{n}}_2\cdot\hat{\mathbf{v}})^2 = \xi_1\left(\cos\theta_k\ x-\sin\theta_k\ \sin\phi\ \sqrt{1-x^2}\right)^2$$
$$+\ \xi_2\left(\sin\theta_k\cos\phi_k\ x+\cos\theta_k\cos\phi_k\ \sqrt{1-x^2}\sin\phi+\sin\phi_k\ \sqrt{1-x^2}\cos\phi\right)^2$$

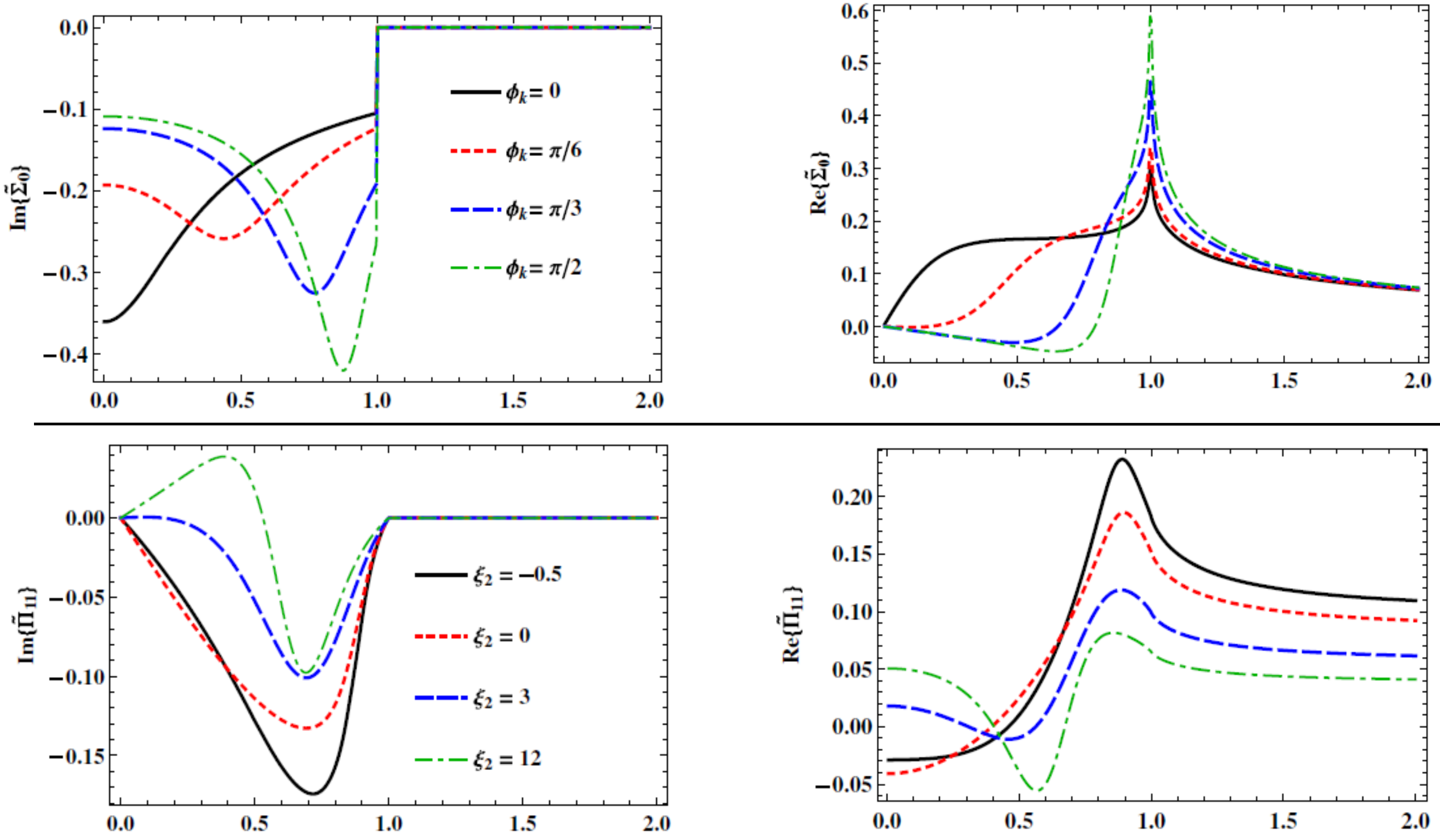
Ellipsoidal parameterization-2.2

Self-energy components (examples)

Quark

Horizontal axes are ω/k

Gluon



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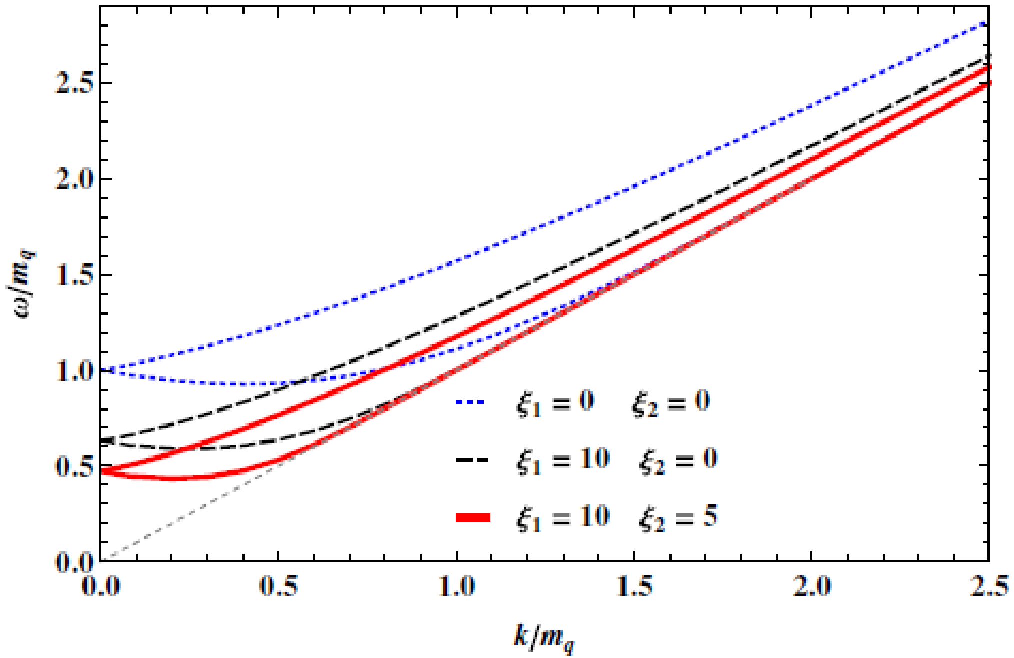
Ellipsoidal parameterization-2.3

Quark modes-dispersion relations

Solutions of $[\omega - \Sigma^0(\omega, \mathbf{k})]^2 - [\mathbf{k} - \mathbf{\Sigma}(\omega, \mathbf{k})]^2 = 0.$



No unstable quark modes

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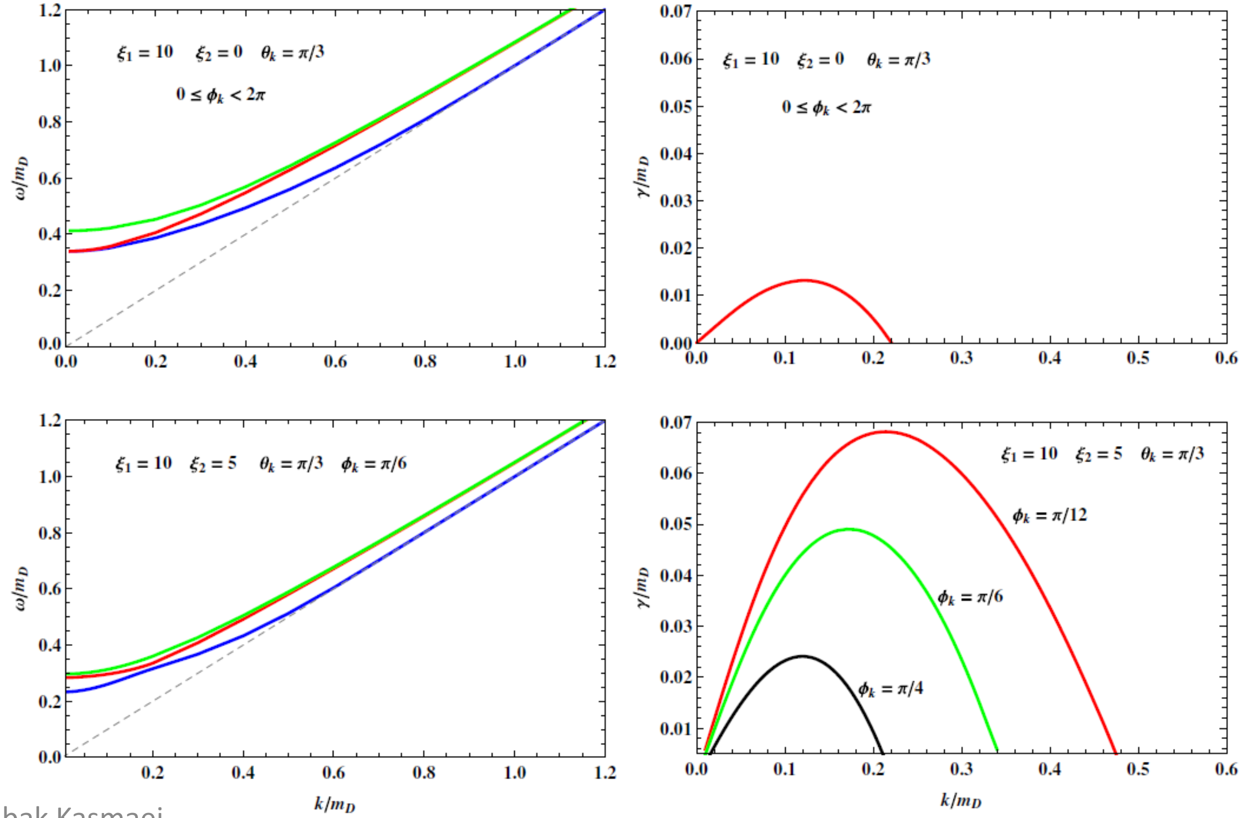
Ellipsoidal parameterization-2.4

Gluon modes-dispersion relations



Solutions of

$$\det[(\mathbf{k}^2 - \omega^2)\delta^{ij} - k^i k^j - \Pi^{ij}] = 0$$

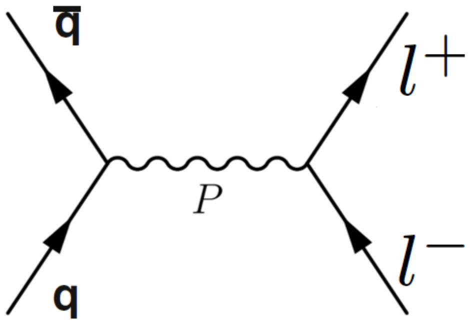
Direction dependence of unstable gluon modes

PHYSICAL REVIEW D **97**, 054022 (2018)

Dilepton emission rate-shperoidal

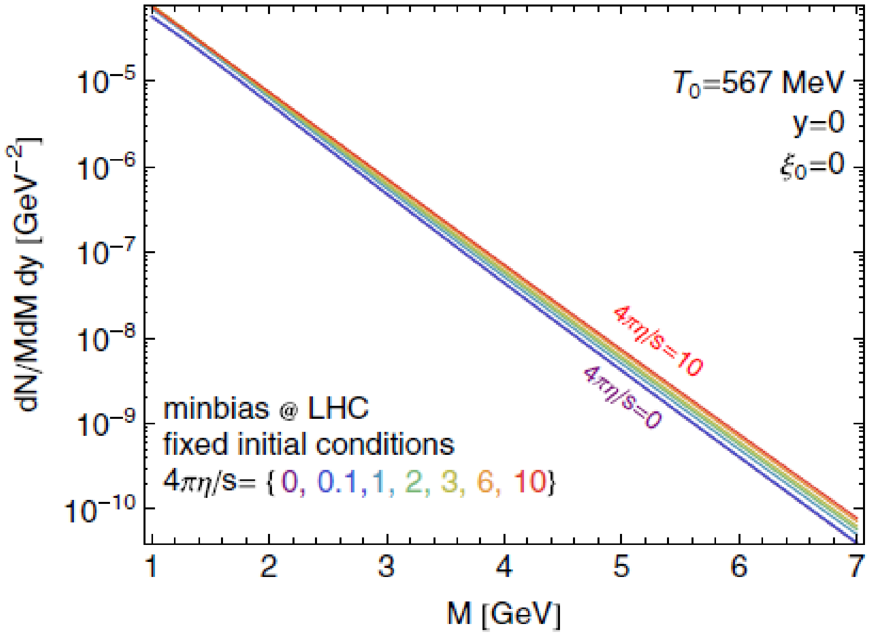
$$\frac{dR^{l^+l^-}}{d^4P} = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} f_q(\mathbf{p}_1) f_{\bar{q}}(\mathbf{p}_2) v_{q\bar{q}} \sigma_{q\bar{q}}^{l^+l^-} \delta^{(4)}(P - p_1 - p_2)$$

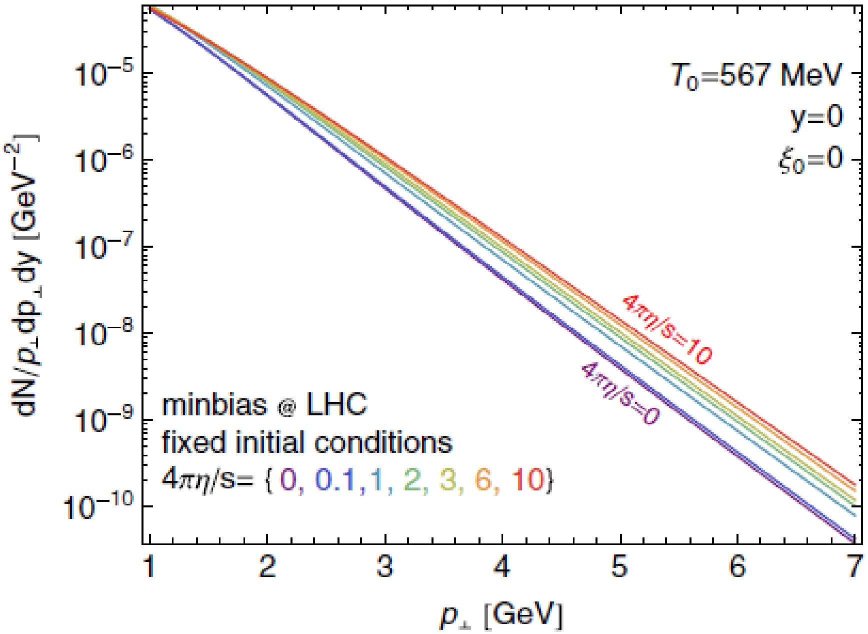
$$\sigma_{q\bar{q}}^{l^+l^-} = \frac{4\pi}{3} \frac{\alpha^2}{M^2} \left(1 + \frac{2m_l^2}{M^2}\right) \left(1 - \frac{4m_l^2}{M^2}\right)^{1/2}$$



Mauricio Martinez and Michael Strickland, Phys. Rev. C 78, 2008.

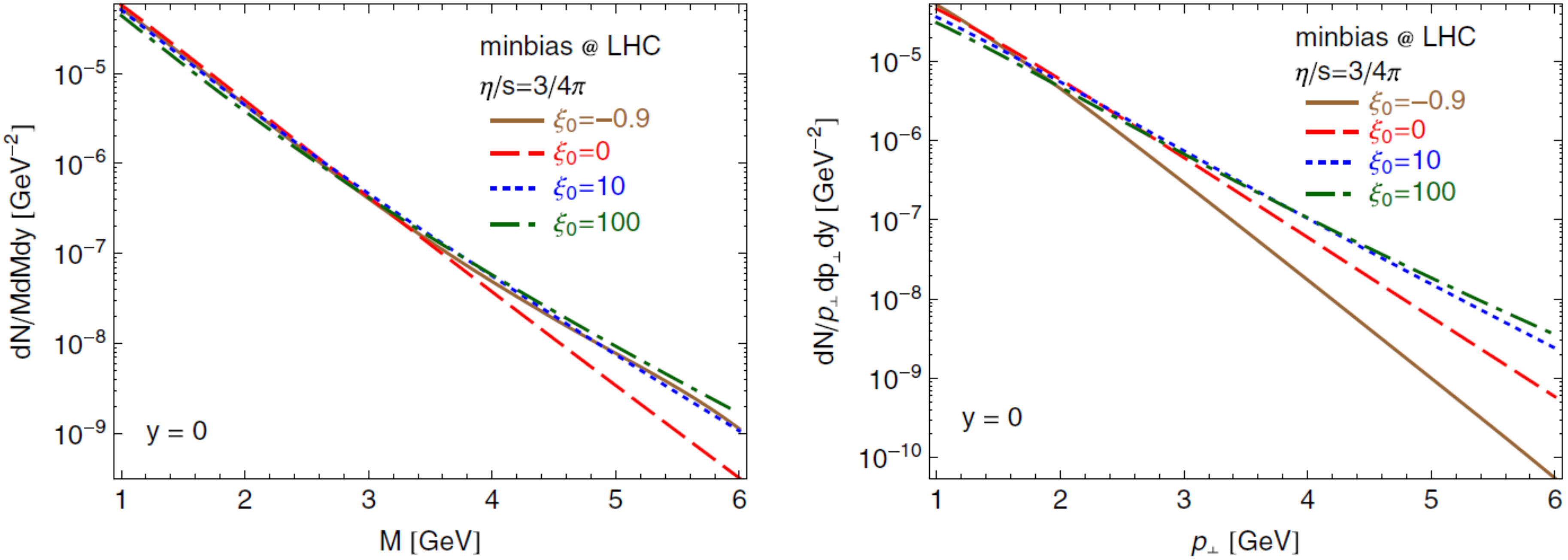
Dilepton yield: spheroidal anisotropy-1





Radoslaw Ryblewski and Michael Strickland
Phys. Rev. D 92, 2015.

Dilepton yield: spheroidal anisotropy-1



Radoslaw Ryblewski and Michael Strickland
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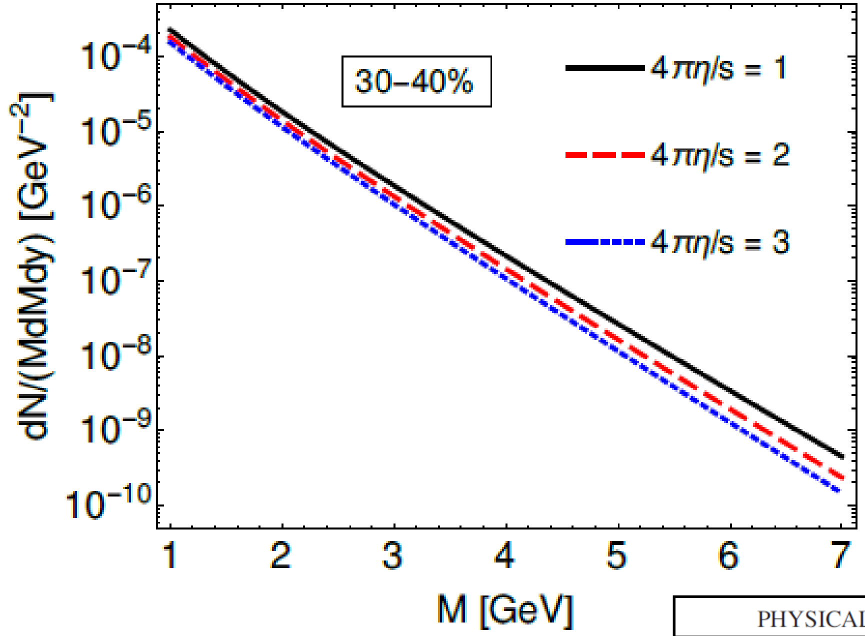
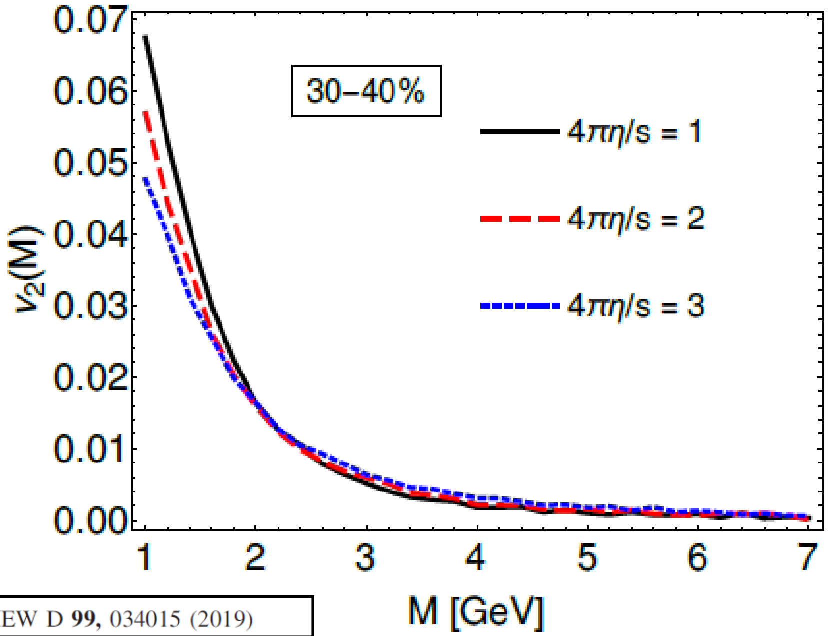
Dilepton emission: ellipsoidal anisotropy-1





PHYSICAL REVIEW D **99**, 034015 (2019)

Dilepton production and elliptic flow from an anisotropic quark-gluon plasma

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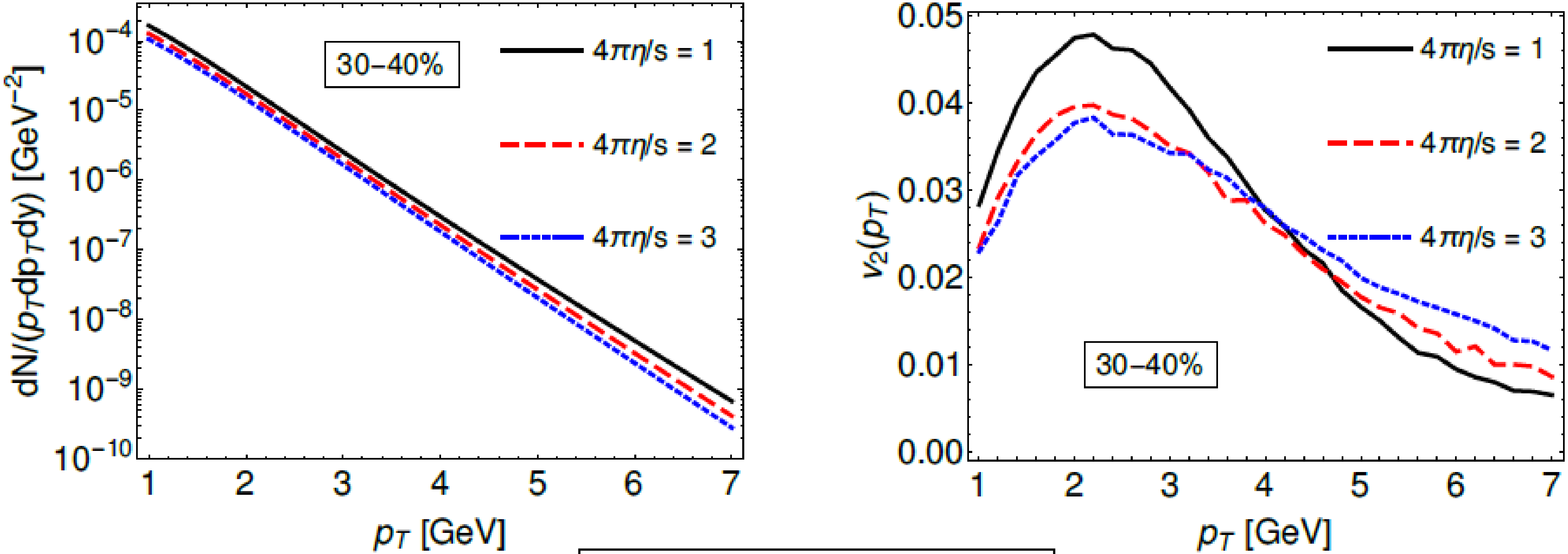
Department of Physics, Kent State University, Kent, Ohio 44242, USA

Dilepton emission: ellipsoidal anisotropy-2



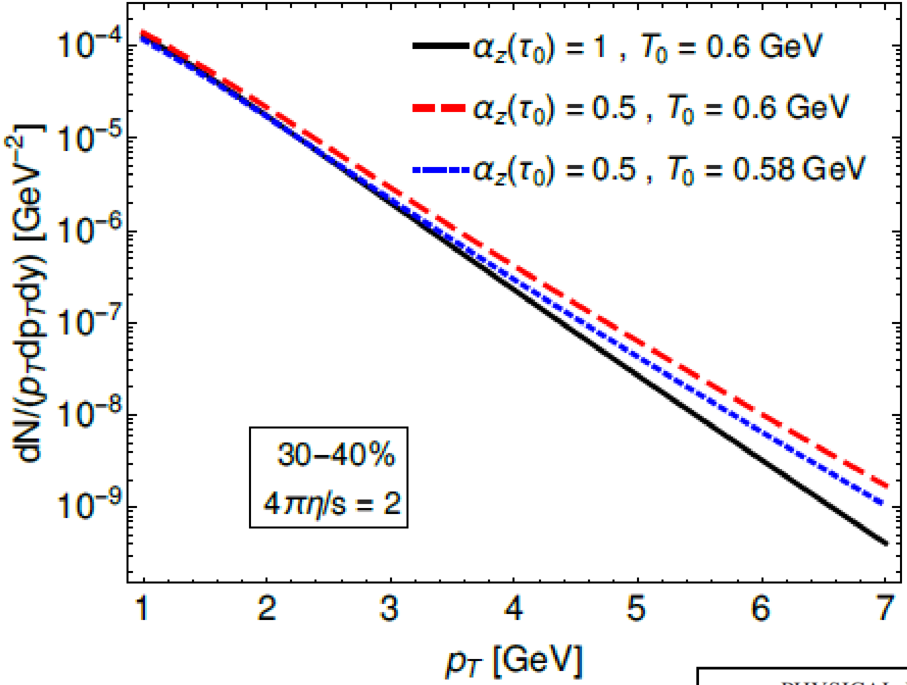
PHYSICAL REVIEW D **99,** 034015 (2019)

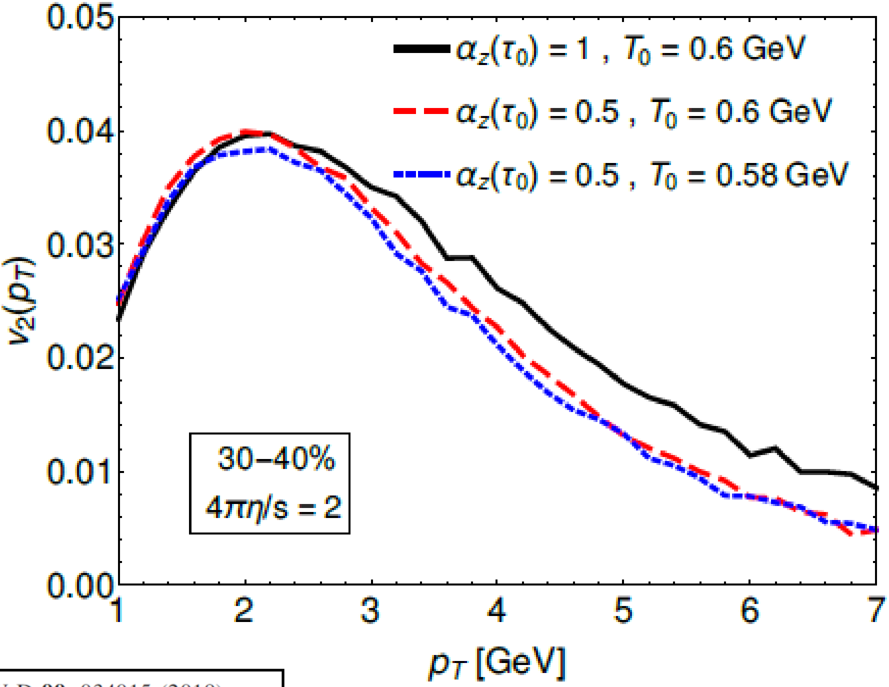
Dilepton production and elliptic flow from an anisotropic quark-gluon plasma

Babak S. Kasmaei and Michael Strickland

Department of Physics, Kent State University, Kent, Ohio 44242, USA

Dilepton emission: ellipsoidal anisotropy-3





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Real photon emission rate

$$E\frac{dR_{\rm hard}}{d^3q} = E\left(\frac{dR_{\rm ann}}{d^3q} + \frac{dR_{\rm com}}{d^3q}\right)$$

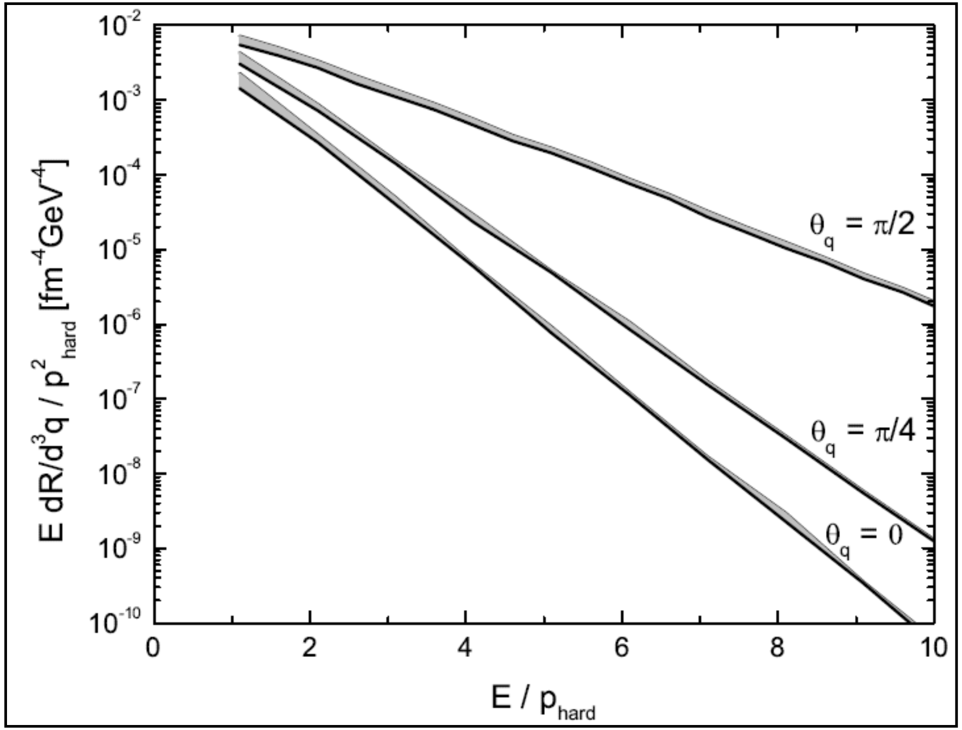
$$E\frac{dR_{\rm soft}}{d^3q} = \frac{i}{2(2\pi)^3}\Pi_{12}{}^{\mu}_{\mu}(Q) \qquad i\Pi_{12}{}^{\mu}_{\mu}(Q) = -e^2e_q^2N_c\frac{8f_q(\mathbf{q})}{q}\int_{\mathbf{p}} Q_\nu\tilde{\Lambda}^\nu(\mathbf{p})$$

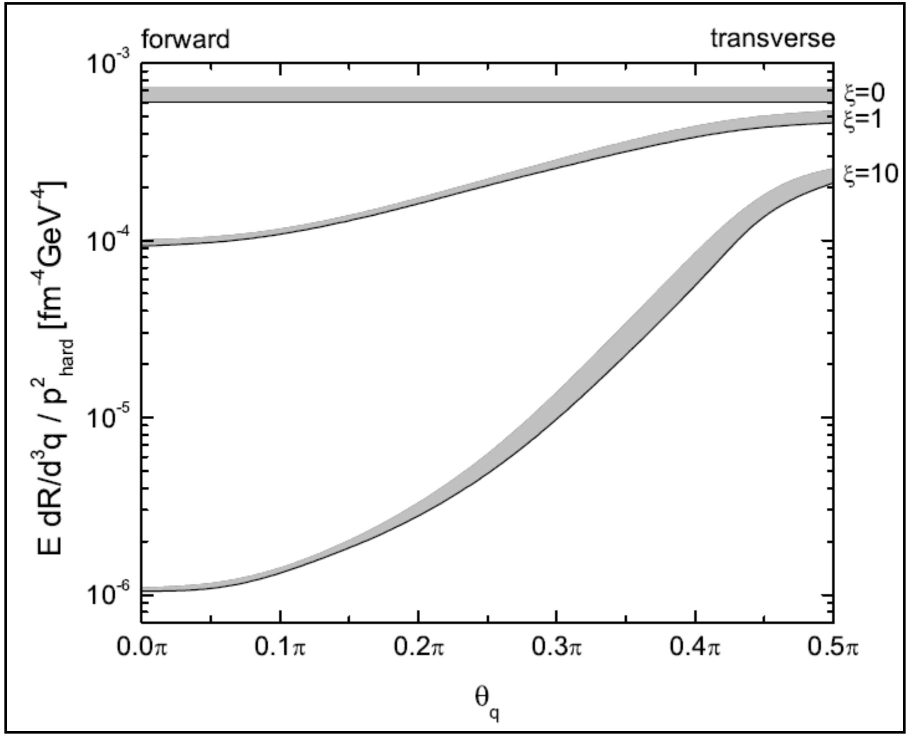
$$\tilde{\Lambda}^\nu(\mathbf{p}) = [\Lambda^{\nu\alpha}{}_{\alpha}(P) - \Lambda_{\alpha}{}^{\nu\alpha}(P) + \Lambda_{\alpha}{}^{\alpha\nu}(P)]_{p_0=p(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})}$$

$$\Lambda_{\alpha\beta\gamma}(P) = \frac{P_\alpha - \Sigma_\alpha(P)}{(P-\Sigma(P))^2}\,\mathrm{Im}\,\Sigma_\beta(P)\frac{P_\gamma - \Sigma^*_\gamma(P)}{(P-\Sigma^*(P))^2}$$

Björn Schenke and Michael Strickland
Phys. Rev. D 76, 2007.

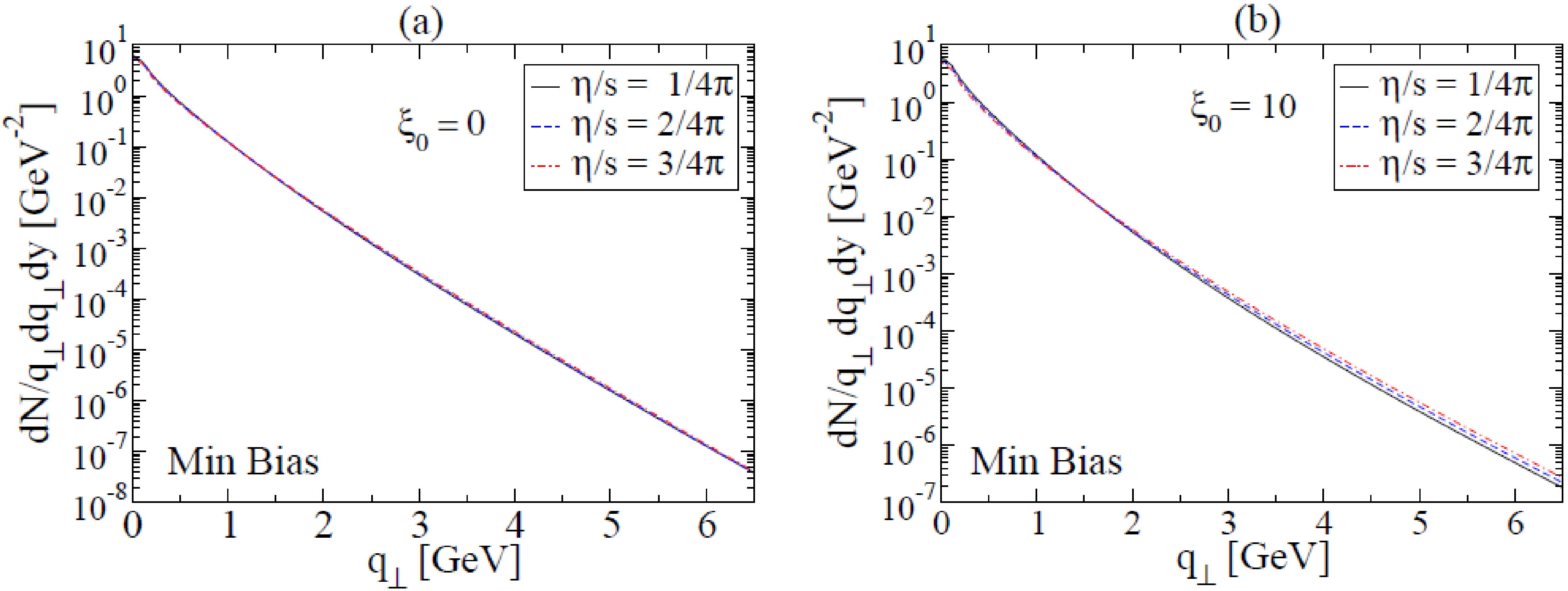
Real photon emission: spheroidal anisotropy-1





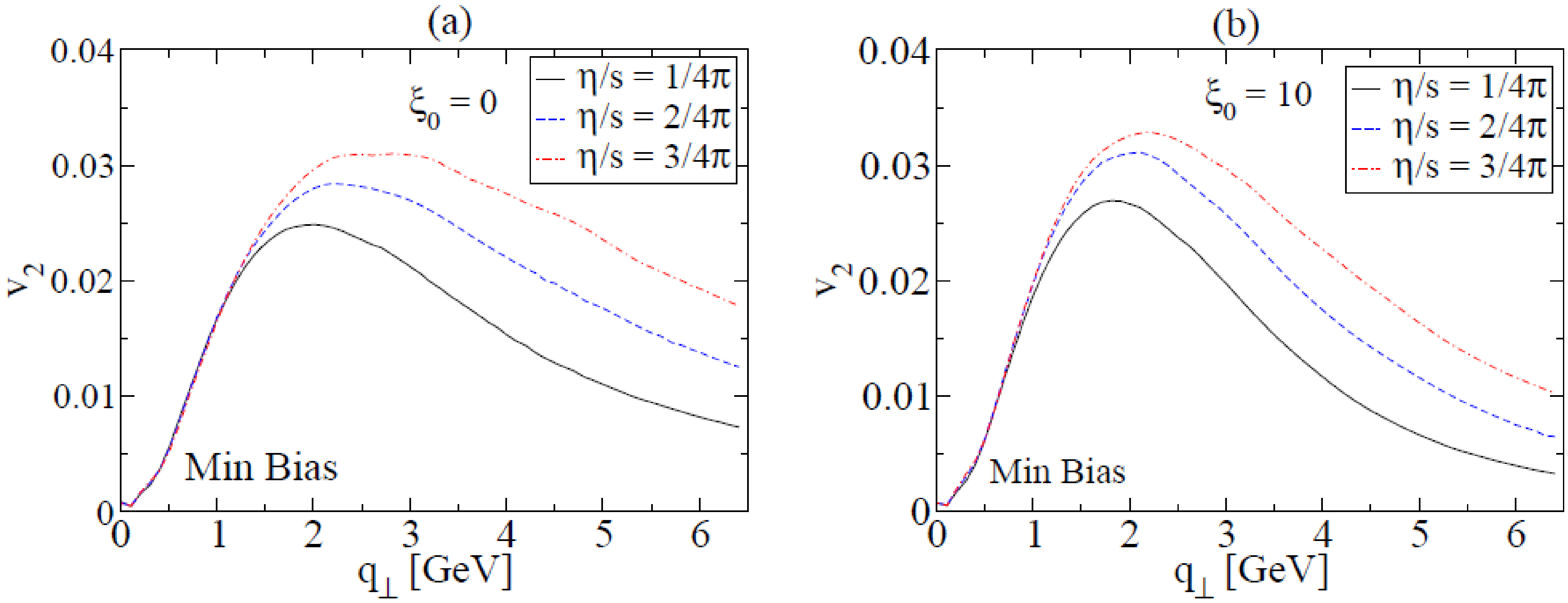
Björn Schenke and Michael Strickland
Phys. Rev. D 76, 2007.

Real photon emission: spheroidal anisotropy-2.1



L Bhattacharya, R Ryblewski, and M Strickland, Phys. Rev. D 93, 2016

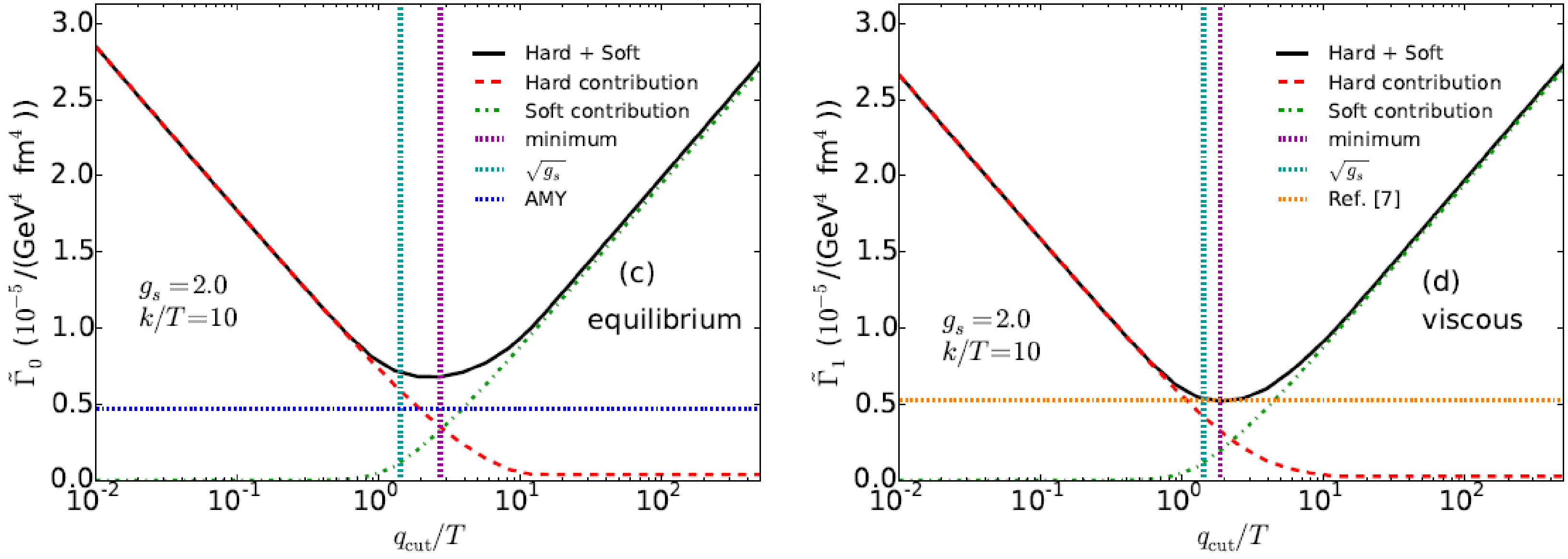
Real photon emission: spheroidal anisotropy-2.2



L Bhattacharya, R Ryblewski, and M Strickland, Phys. Rev. D 93, 2016

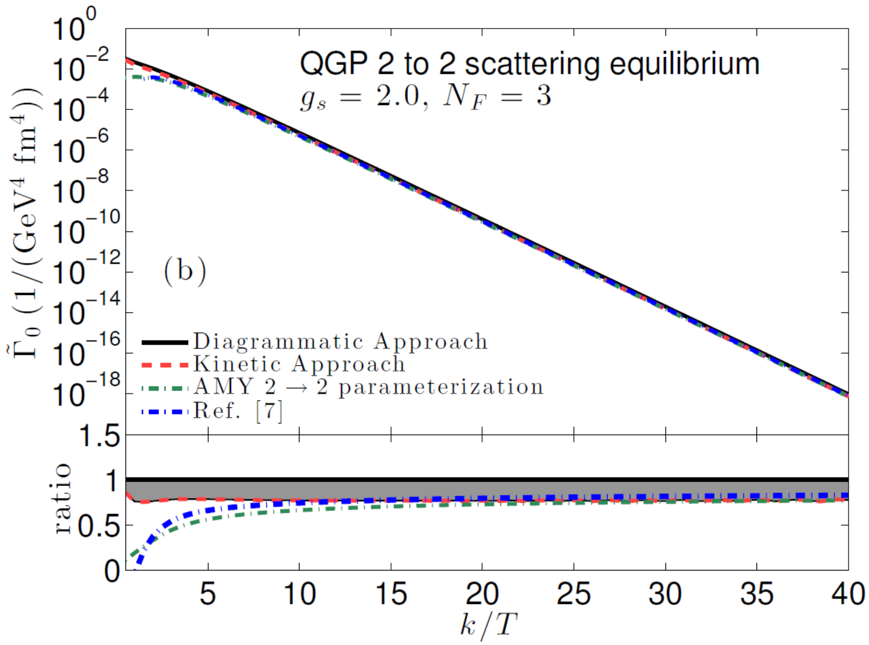
Real photon emission: weak anisotropy (viscous)-1

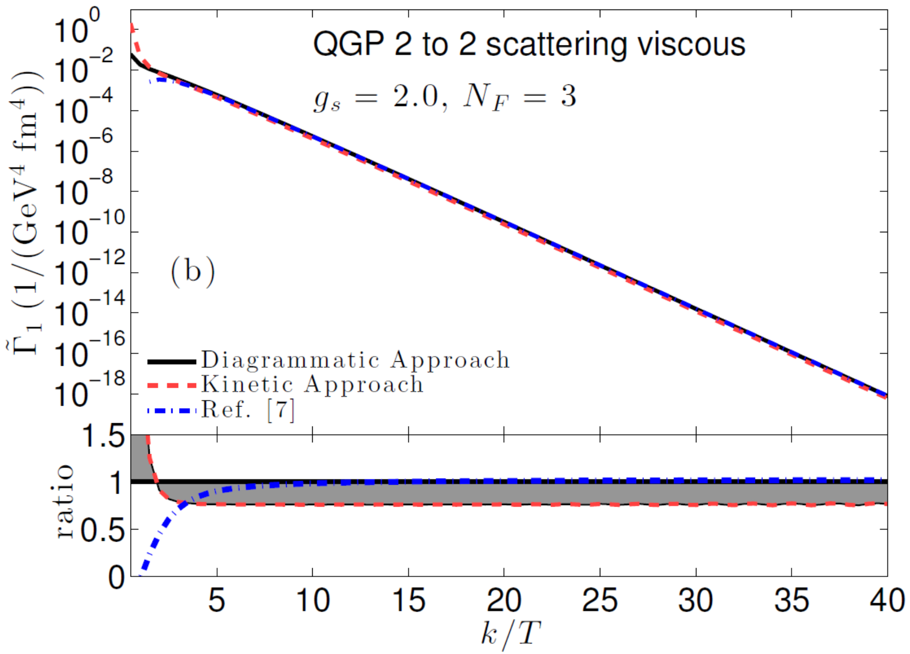
$$f(K) \equiv f_0(k) + \delta f(k) = f_0(k)\left[1 + \left(1 \pm f_0(k)\right)\frac{\pi^{\mu\nu}\hat{k}_\mu \hat{k}_\nu}{2(e+\mathcal{P})}\chi(k/T)\right]$$



C Shen, JF Paquet, U Heinz, C Gale - Phys. Rev. C 91, 2015.

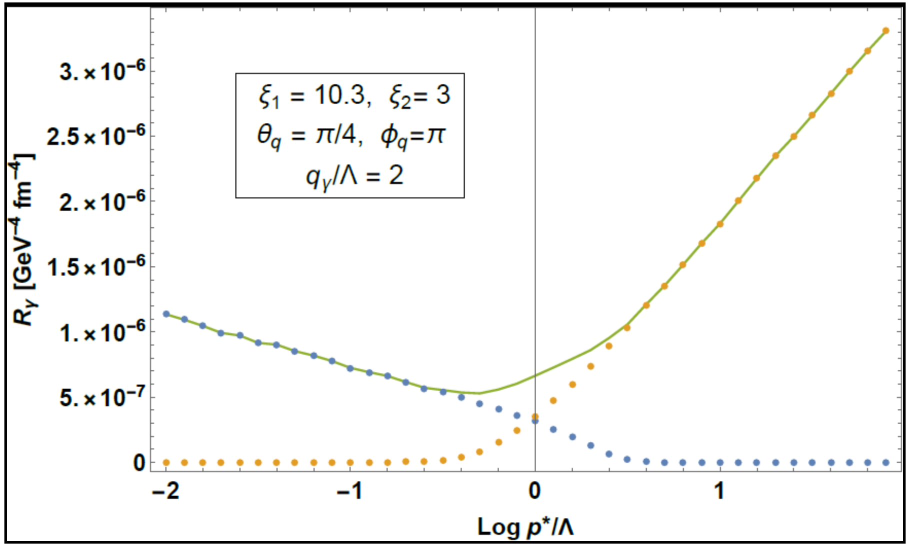
Real photon emission: weak anisotropy (viscous)-2

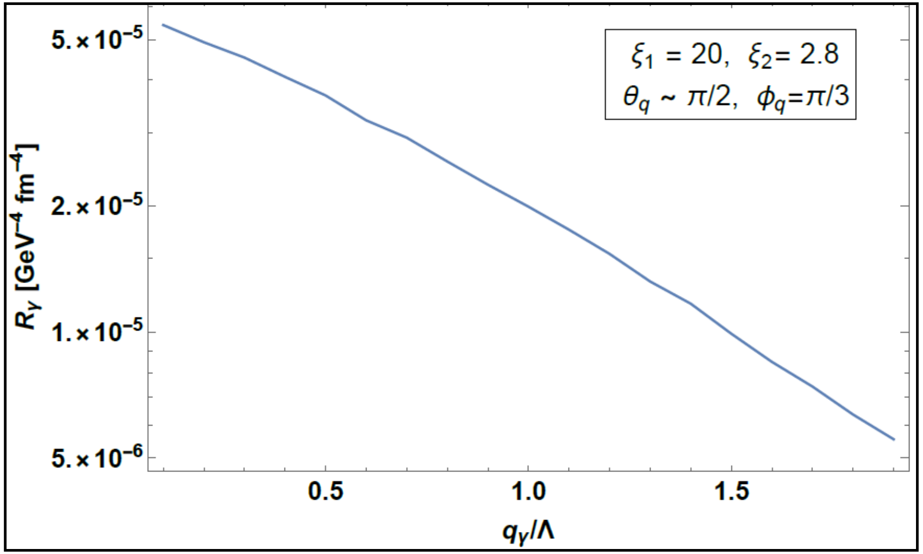




C Shen, JF Paquet, U Heinz, C Gale - Phys. Rev. C 91, 2015

Real photon emission: ellipsoidal anisotropy- preliminary





Outlook

- Real photon yield and flow
- Including hadronic EM production
- Initial anisotropy in transverse plane
- HF and energy loss in ellipsoidally anisotropic case

Extra Slide- Basis functions

$$I_n(z) = \int_{-1}^{1} dx \frac{x^n}{z-x+i0^+}$$

$$= \frac{{}_2F_1(1,1+n,2+n,-1/z)+(-1)^n F_1(1,1+n,2+n,1/z)}{(1+n)z}$$

Recursive relation:

$$I_{n+1}(z) = zI_n(z) - \frac{1+(-1)^n}{n+1}; \quad \text{for } n > 0$$

$${}_2F_1(1,1+n,2+n,x) = -\frac{n+1}{x^{n+1}}\left[\log(1-x) + \sum_{m=1}^{n} \frac{x^m}{m}\right]$$

For large z:

$$I_n(z) \approx \frac{1}{(n+1)z}\left[(-1)^n\left(1-\frac{n+1}{(n+2)z}\right)+\left(1+\frac{n+1}{(n+2)z}\right)\right]$$

Extra Slide- analytic integrations

Imaginary parts can be written in the form of $$\int_0^{2\pi} d\phi \frac{\sum_m C_m \sin^m \phi + D_m \cos^m \phi}{\sum_n A_n \sin^n \phi + B_n \cos^n \phi}.$$

This can be converted into $$\sum_{l,m,n} G_{l,m,n} \int_0^{2\pi} d\phi \frac{e^{im\phi}}{(\sin\phi - a_l)^n}$$

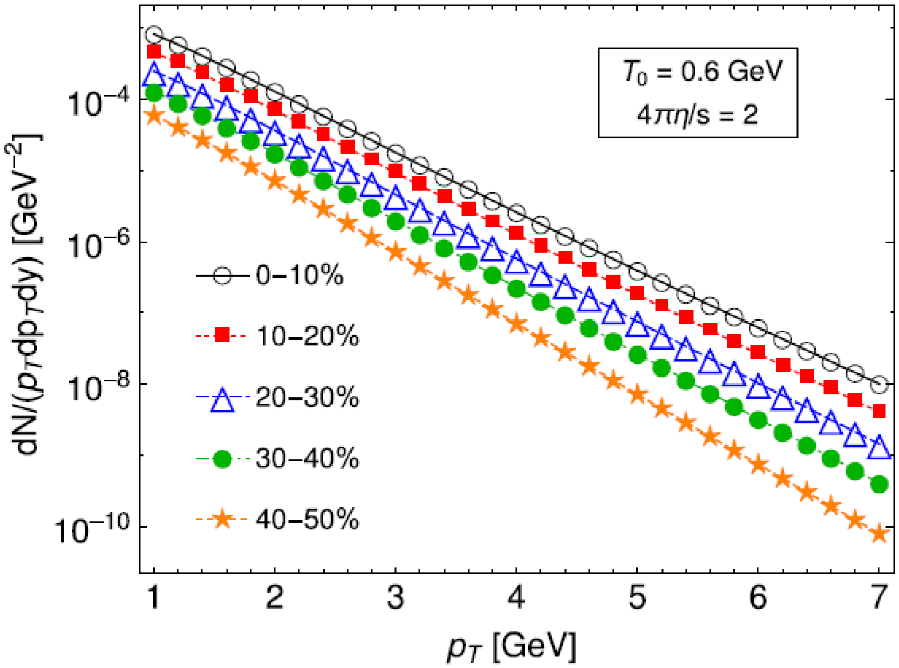
The integrals $R_{l,m,n} = \int_0^{2\pi} d\phi \frac{e^{im\phi}}{(\sin\phi - a_l)^n}$ can be calculated analytically.

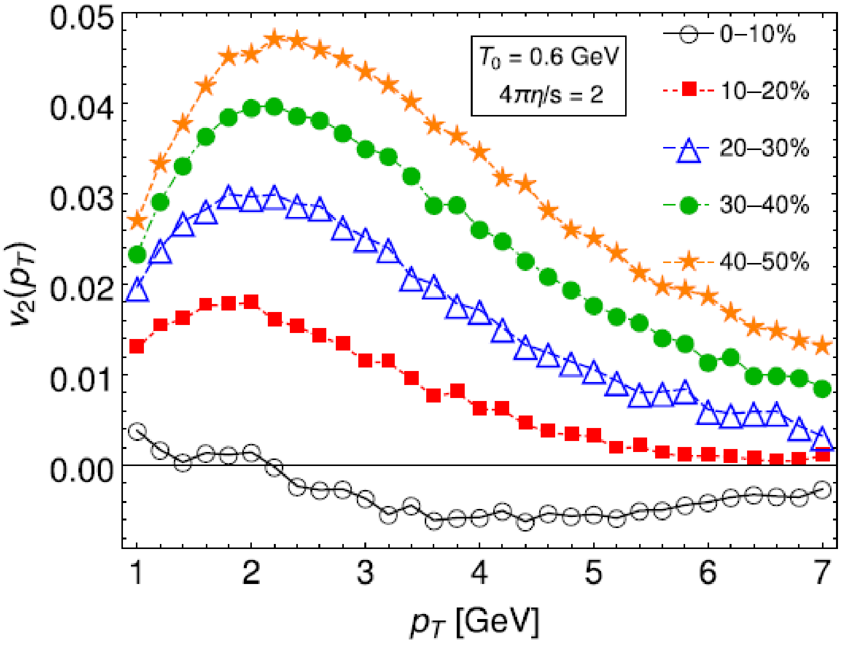
Example (spheroidal): $$\mathrm{Im}\{\boldsymbol{\Sigma}^0(z)\} = -\frac{m_q^2}{4k} \frac{\Theta(1-z^2)}{2\xi\sqrt{-\frac{1}{\xi}\sin\theta_k\sqrt{1-z^2}}}[S(a_+) - S(a_-)]$$

where $$S(a) = -\frac{2\pi\sqrt{\frac{a^2}{a^2-1}}}{a},$$

$$a_\pm = \frac{z\cot\theta_k}{\sqrt{1-z^2}} \pm \csc\theta_k \sqrt{-\frac{1}{\xi(1-z^2)}}$$

Extra Slide- dilepton





Extra Slide- soft hadrons