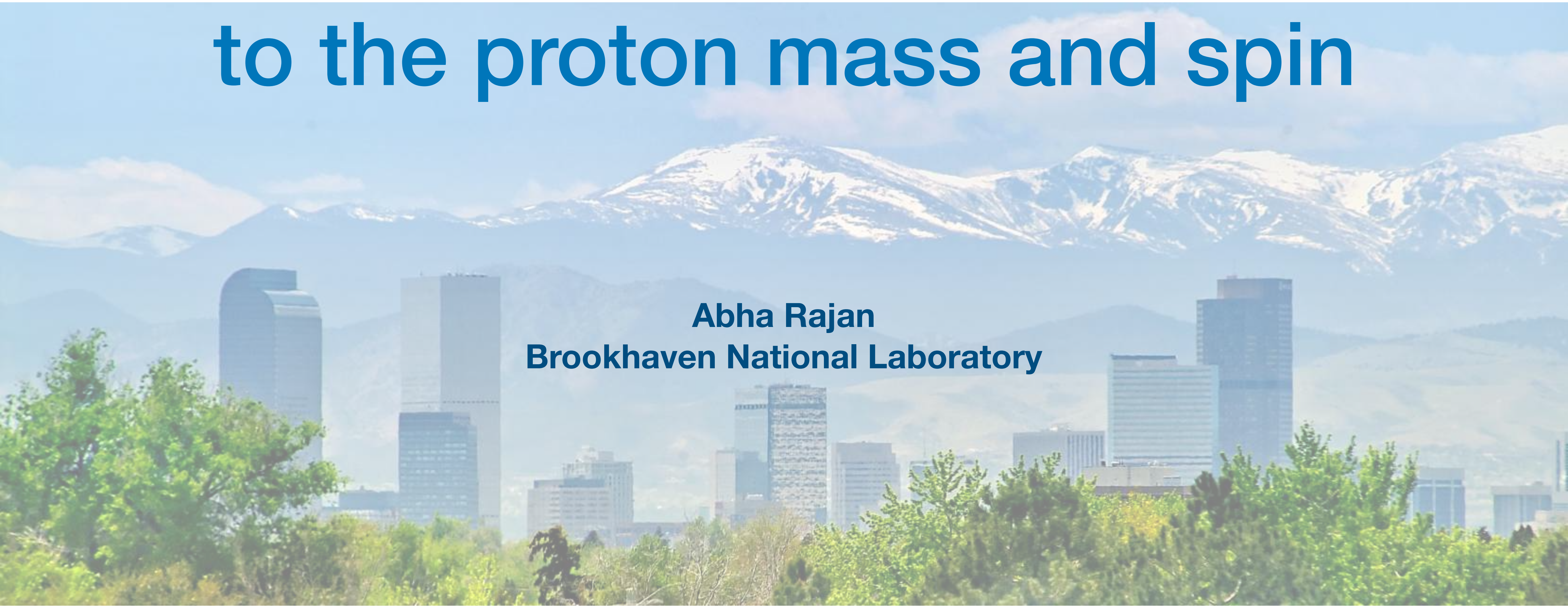
Quark and gluon contributions to the proton mass and spin



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8th Workshop of the APS Topical Group on Hadronic Physics
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Collaborators

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- Michael Engelhardt (New Mexico State University)
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Orbital Angular Momentum

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- Kazuhiro Tanaka (Juntendo University, KEK)
- Di-Lun Yang (Nishina Center, RIKEN)

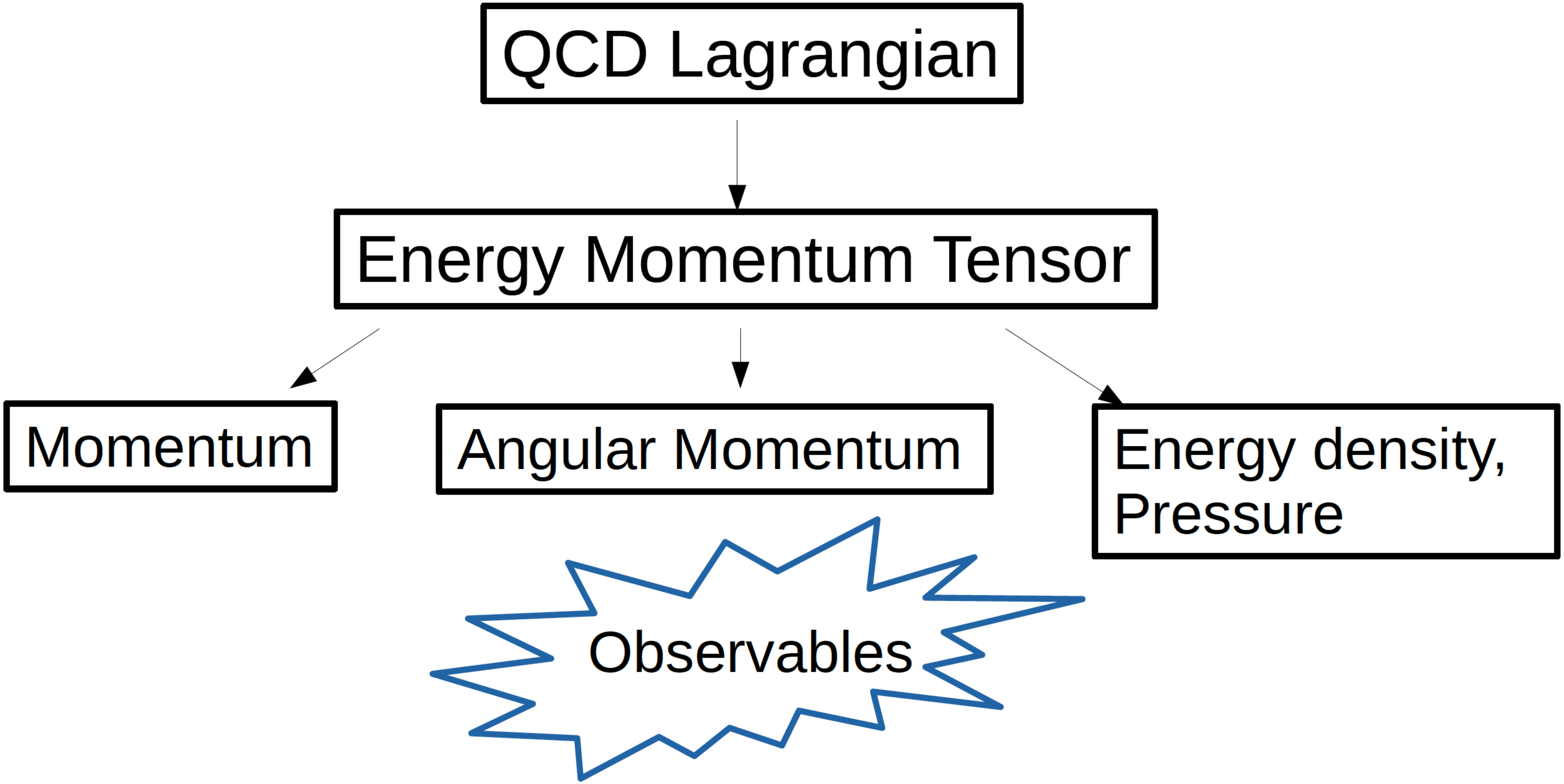
Trace Anomaly and Proton Mass

Thanks!!

Outline

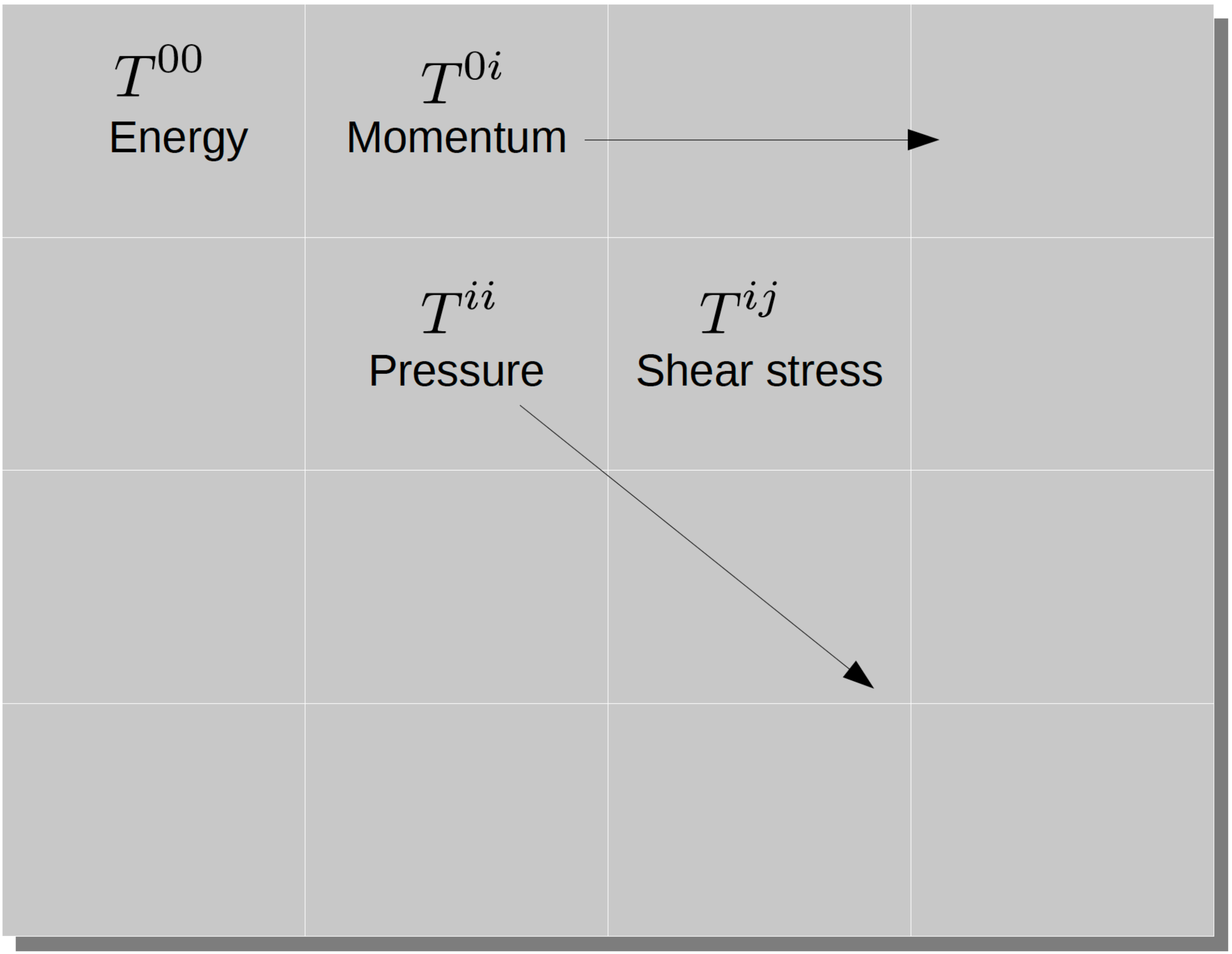
- Partonic Orbital Angular Momentum
- Wandzura Wilczek and genuine twist three contributions to twist 3 GPDs
- Extending to chiral odd sector and twist four
- The mass of the proton and the QCD trace anomaly

QCD Energy Momentum Tensor



Deeply Virtual Compton Scattering, moments of GPDs etc.

QCD Energy Momentum Tensor



GPD based definition of Angular Momentum

$$J^i_{q,g} = \frac{1}{2}\epsilon^{ijk}\int d^3x\left(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k\right)$$

$$\vec{J}_q = \int d^3x \psi^\dagger \left[\vec{\gamma}\gamma_5 + \vec{x}\times i\vec{D}\right]\psi \qquad \vec{J}_g = \int d^3x\left(\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right)$$

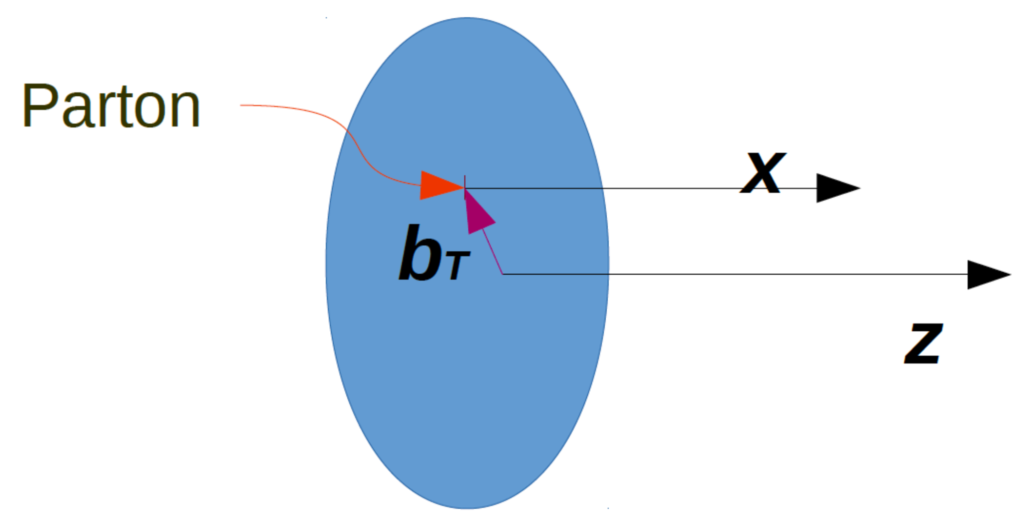
$$J_q = \frac{1}{2}\int dx x(H_q(x,0,0) + E_q(x,0,0))$$

Xiangdong Ji, PRL 78.610,1997

To access OAM, we take the difference between total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

OAM Total Spin



Direct description of OAM

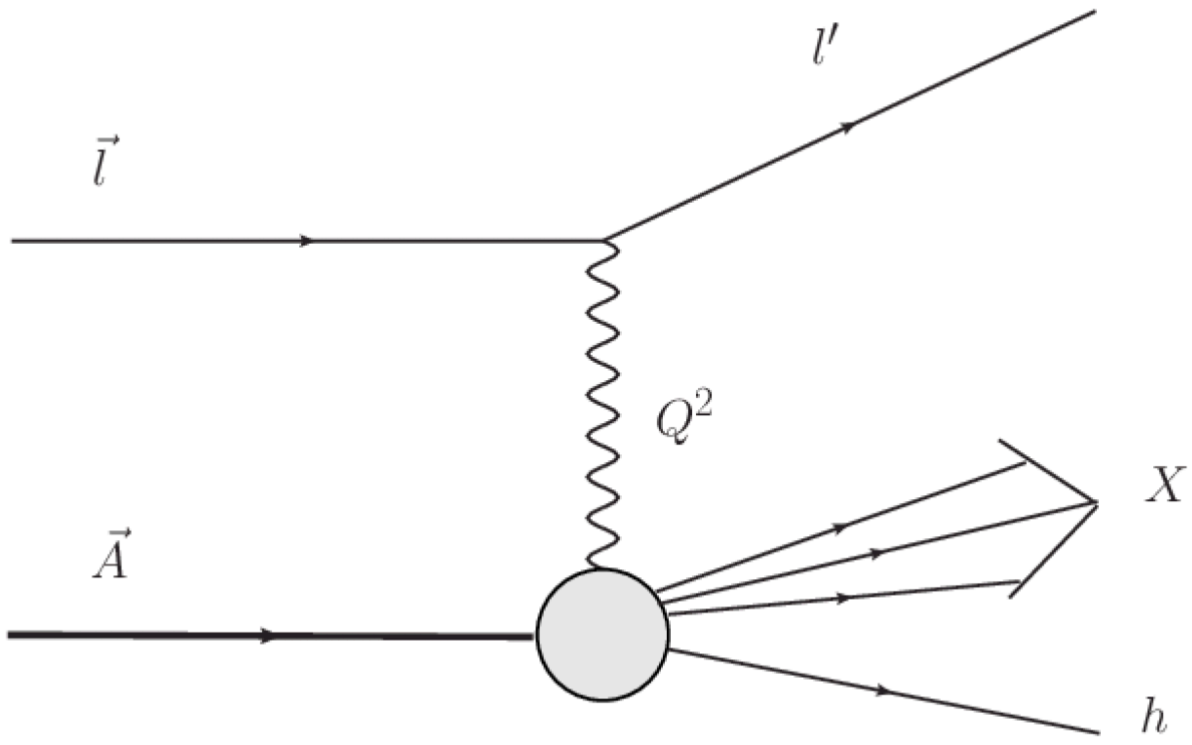
$$\int dx x G_2 = \int dx x(H+E) - \int dx \tilde{H}$$

$$G_2 \equiv \tilde{E}_{2T} + H + E$$

Kiptily and Polyakov, Eur Phys J C 37 (2004)
Hatta and Yoshida, JHEP (1210), 2012

- The moment in x of the GPD G_2 shown to be OAM

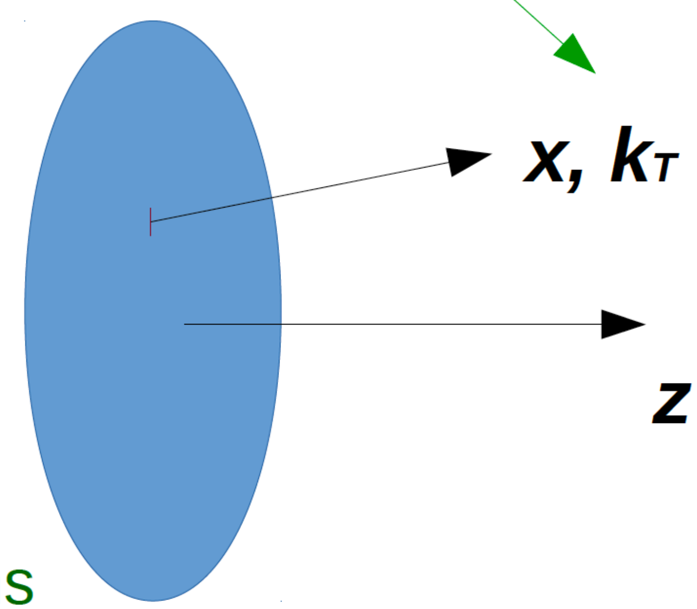
Intrinsic Transverse Momentum



The partonic transverse momentum is correlated with the momentum of the observed hadron

Semi inclusive Deep Inelastic Scattering

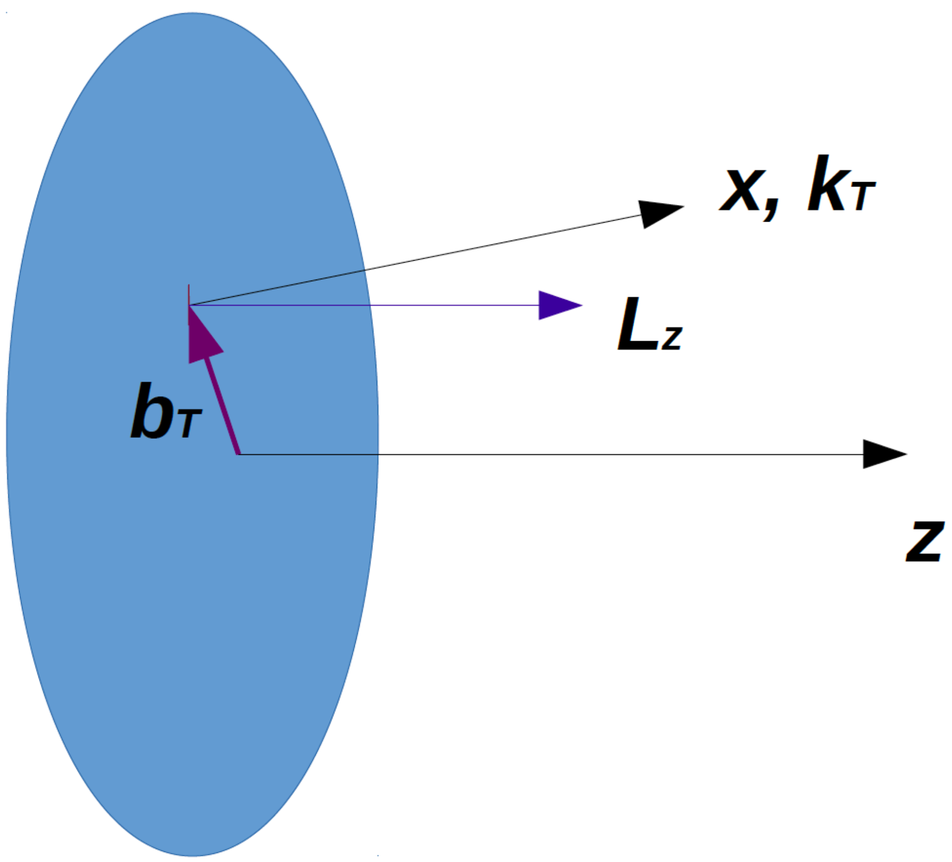
However, the target does not remain intact, no access to the spatial distribution of partons



Transverse Momentum Distributions

Partonic Orbital Angular Momentum II

- Consider measuring both the intrinsic transverse momentum and the spatial distribution of partons
- $L_{q,z} = \mathbf{b}_T \times \mathbf{k}_T$



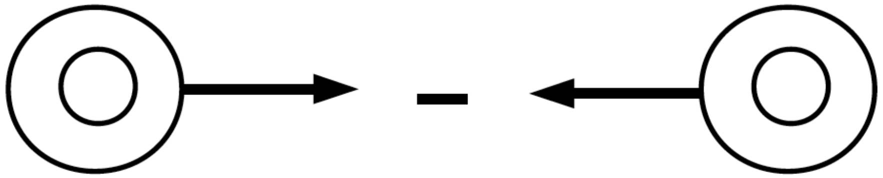
$$W_{\Lambda,\Lambda'}^{[\gamma^+]} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

Generalized Transverse Momentum Distributions (related by Fourier transform to Wigner Distributions)

Meissner Metz and Schlegel, JHEP 0908 (2009)

GTMDs that describe OAM

- How does F_{14} connect to OAM ?



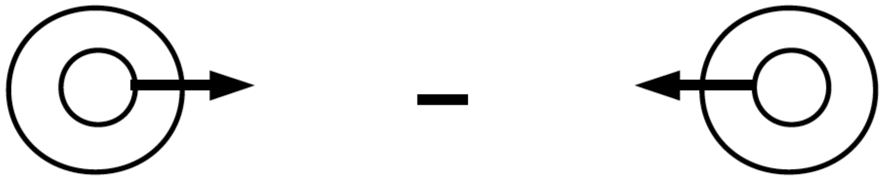
Unpolarized quark in a longitudinally polarized proton

$$\mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{ib.\Delta_T} \left[W^{\gamma^+}_{++} - W^{\gamma^+}_{--} \right]$$

$$L = \int dx \int d^2k_T \int d^2\mathbf{b} (\mathbf{b} \times \mathbf{k}_T) \mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce et al PRD84, (2011)

- Another GTMD relevant to OAM



G_{11} describes a longitudinally polarized quark in an unpolarized proton. Measures spin orbit correlation.

The Two Definitions

- Weighted average of b_T X k_T

GTMD

$$L_z = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx\, x(H_q + E_q)$$

$$\frac{1}{2}\int_{-1}^{1} dx\, \tilde{H}_q$$

The Two Definitions

- Weighted average of b_T X k_T

$$F_{14}^{(1)}$$

GTMD

$$L_z = -\int dx \boxed{\int d^2 k_T \frac{k_T^2}{M^2} F_{14}}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

Is there a connection ?

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

AR, Engelhardt and Liuti PRD 98 (2018)

AR, Courtoy, Engelhardt and Liuti PRD 94 (2016)

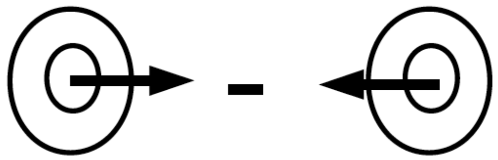
- This is a form of Lorentz Invariant Relation (LIR)
- This is a distribution of OAM in x
- Derived for a straight gauge link

Generalized Lorentz Invariance Relations

Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

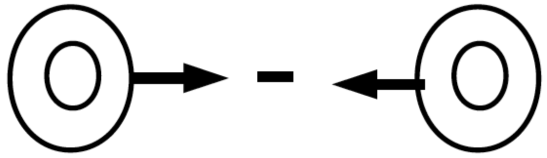


Twist two

Twist three

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



The GTMDs are complex in general.

$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

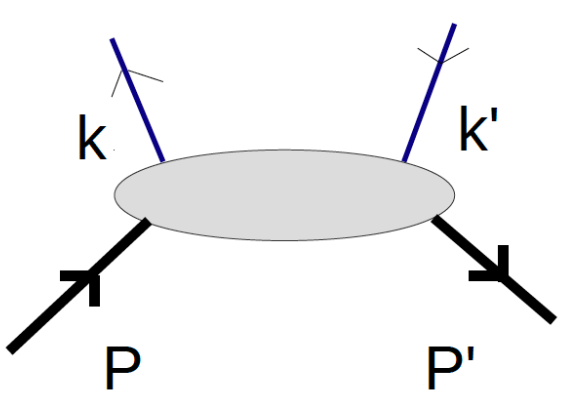
Higher Twist

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

$$\gamma^+, \gamma^+\gamma^5, \sigma^{i+}\gamma^5$$

Leading twist – twist 2

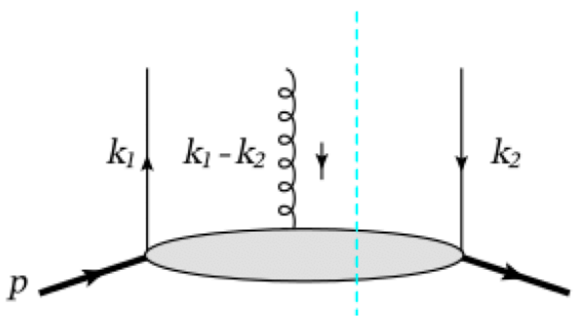
- Involve only good components
- Simple interpretation in terms of parton densities



$$\gamma^i, \gamma^i\gamma^5, \sigma^{ij}\gamma^5, 1, \gamma^5, \sigma^{+-}\gamma^5$$

Higher twist – twist 3

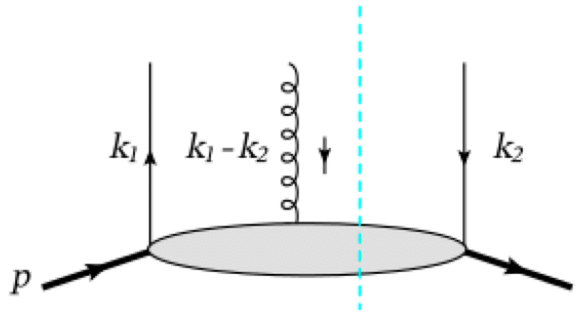
- Involve one good and one bad component
- The bad component represents a quark gluon composite



Collinear Picture : Transverse Quark Current, Higher Twist

$\bar{\psi}(-z/2)\gamma^{+}\psi(z/2) \longrightarrow$ Leading order quark current

$\bar{\psi}(-z/2)\gamma_T^{i}\psi(z/2) \longrightarrow$ Transverse quark current, implicitly involves quark gluon interactions



Probabilistic parton model interpretation works well at leading order, with transverse quark projection operator need to include quark gluon interactions

Both in Collinear Picture

Derivation of Generalized LIRs

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2z_T}{2\pi} e^{ixP^+z^- - k_T.z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

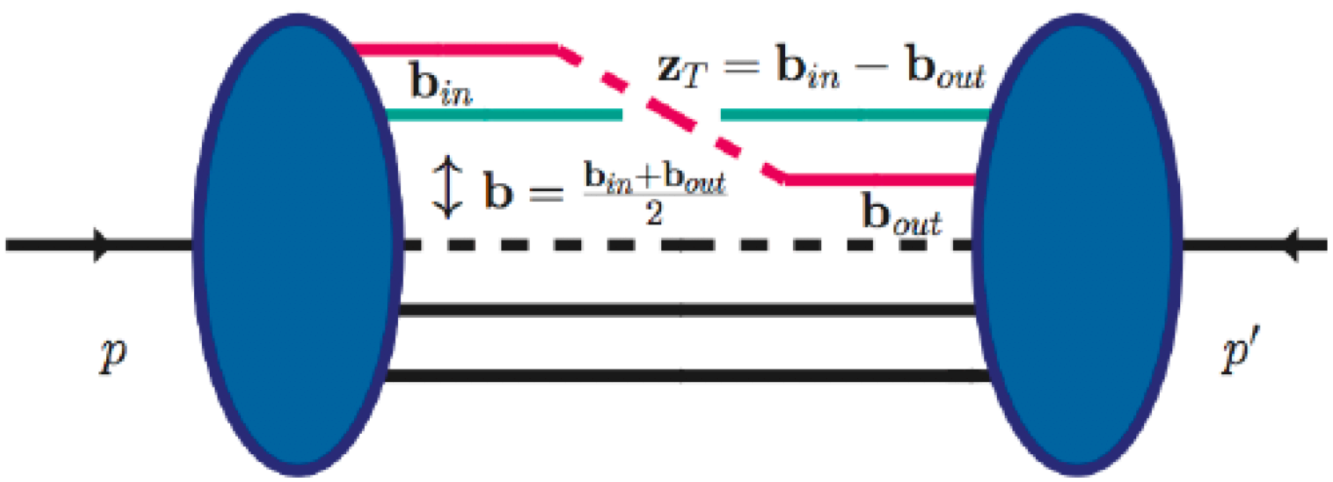
GTMDs

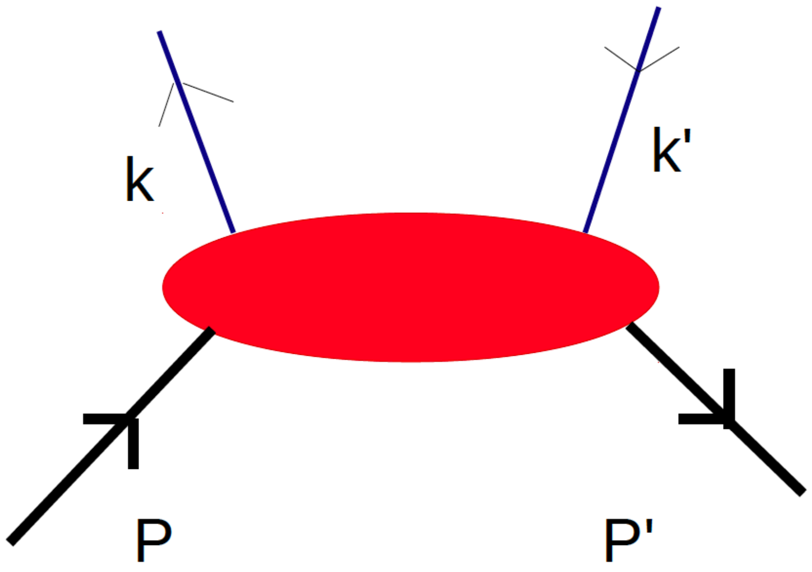
Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

GPDs

Intrinsic Momentum vs Momentum Transfer Δ





$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

Equations of Motion Relations

How do we obtain these ?

$$
\begin{aligned}
(i\not{D} - m)\psi(z_{out}) &= (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

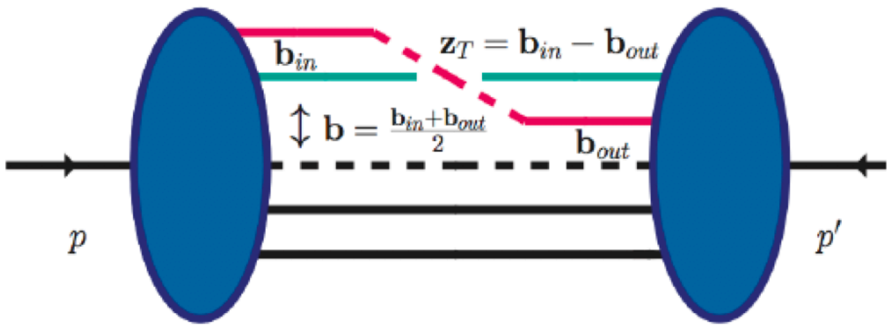
$$
\begin{aligned}
\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5(i\not{D} - m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5(i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$
$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5\mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5\mathcal{U} = 0$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2b_T e^{-ib\cdot\Delta} \int dz^- d^2z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5(i\overrightarrow{\not{D}} - m) \right] \psi | p, \Lambda \rangle = 0$$

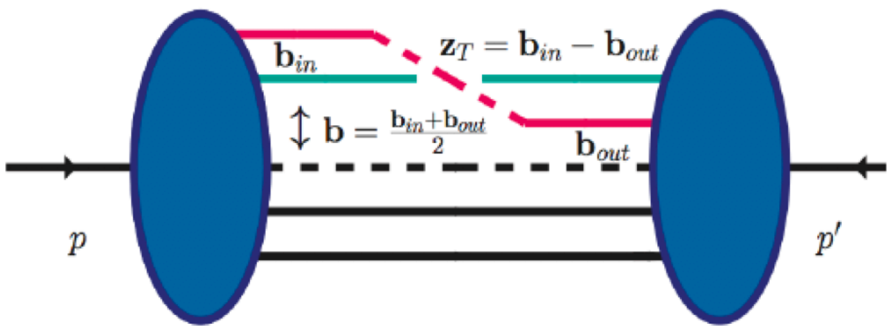
Equations of Motion R

Crucial for understanding qgq contribution to GPDs!!

How do we obtain these ?

$$
\begin{aligned}
\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0
\end{aligned}
$$

$$
b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}
$$



$$
\int db^- d^2 b_T e^{-ib\cdot\Delta} \int dz^- d^2 z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{D} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{D} - m) \right] \psi | p, \Lambda \rangle = 0
$$

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + \boxed{2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14}} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3 **Twist 2** **Genuine Twist 3 (explicit gluon)**

$$\boxed{\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E}$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\partial\!\!\!/} - ig A\!\!\!/)\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial\!\!\!/} + igA\!\!\!/)\Big|_{z/2}\right]\psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0) | p, \Lambda\rangle$$

Wandzura Wilczek Relations

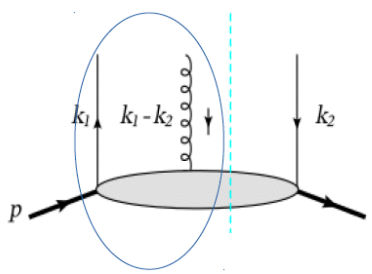
$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

Twist three vector GPD

Twist two

Axial vector GPD contributes to a vector GPD

Genuine Tw 3



AR, Engelhardt and Liuti PRD 98 (2018)

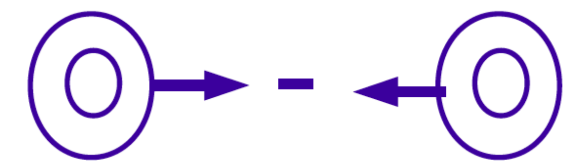
$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

Twist three PDF

Twist two

Genuine Tw 3

Moments of twist three GPDs -Quark gluon structure



$$\int dx \widetilde{E}_{2T} = -\int dx (H+E) \quad \Rightarrow \int dx \left(\widetilde{E}_{2T} + H + E \right) = 0$$

$$\int dx \underline{x} \widetilde{E}_{2T} = -\frac{1}{2}\int dx x (H+E) - \frac{1}{2}\int dx \widetilde{H}$$

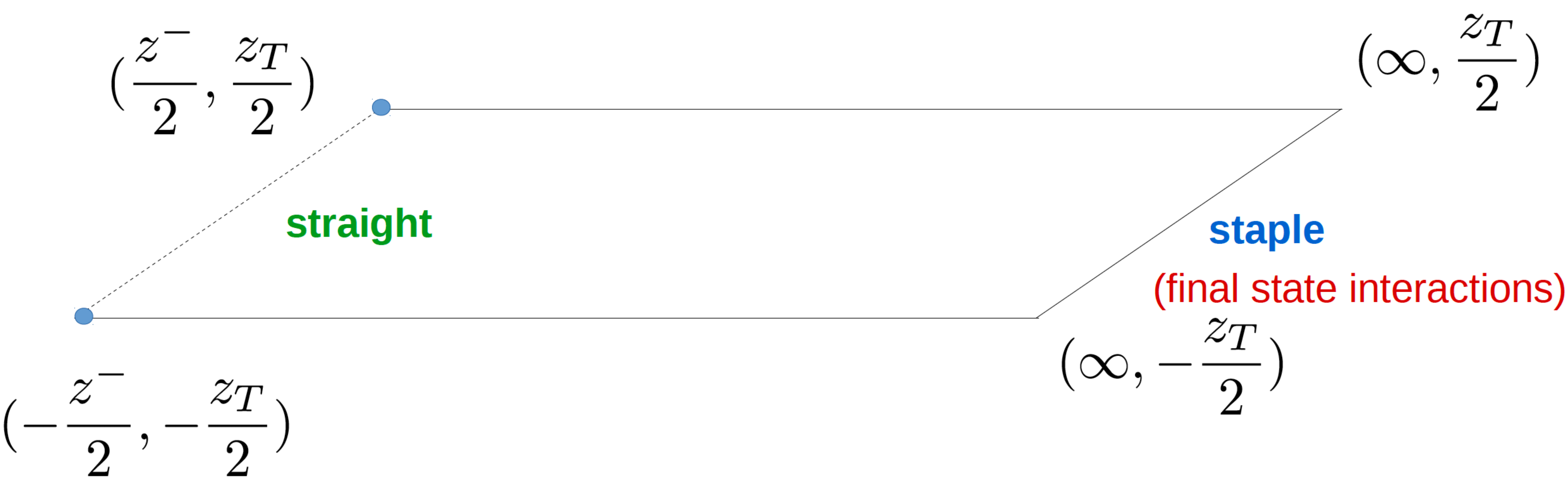
OAM

$$\int dx \underline{x}^2 \widetilde{E}_{2T} = -\frac{1}{3}\int dx x^2 (H+E) - \frac{2}{3}\int dx x \widetilde{H} - \boxed{\frac{2}{3}\int dx x \mathcal{M}_{F_{14}}\Big|_{v=0}}$$

Genuine Twist Three

$$\int dx\, x \int d^2 k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ \gamma^5 F^{+i}(0) \psi(0) | p, \Lambda \rangle$$

Quark gluon quark contributions

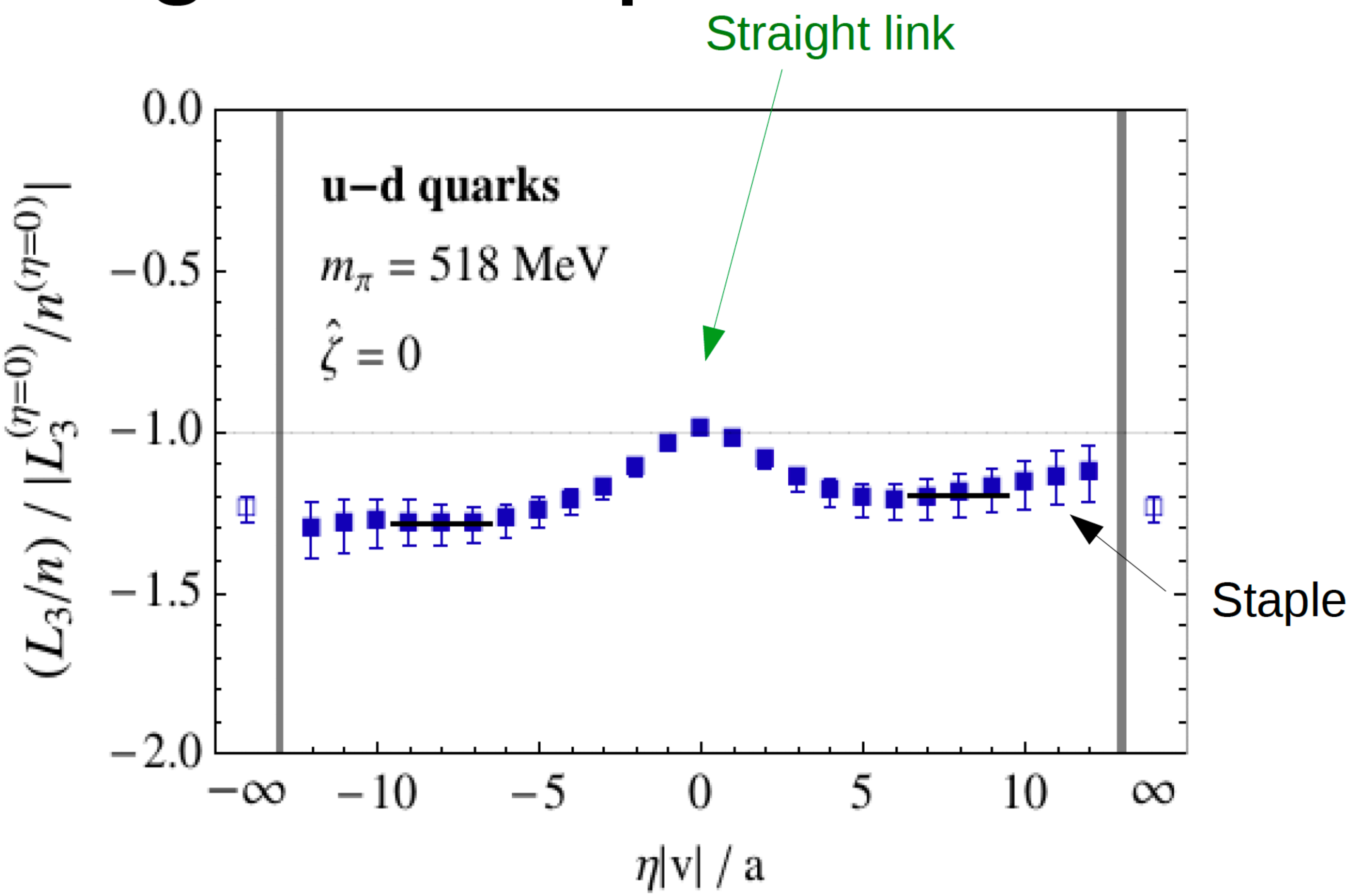


$$\int dx \int d^2 k_T \, \mathcal{M}^{i,S}_{\Lambda' \Lambda} = $$
$$i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds \, \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0) \psi(0) | p, \Lambda \rangle$$

$$\int dx \int d^2 k_T \, \mathcal{M}^{i,A}_{\Lambda' \Lambda} = $$
$$-g v^- \frac{1}{2P^+} \int_0^1 ds \, \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ \gamma^5 U(0, sv) F^{+i}(sv) U(sv, 0) \psi(0) | p, \Lambda \rangle$$

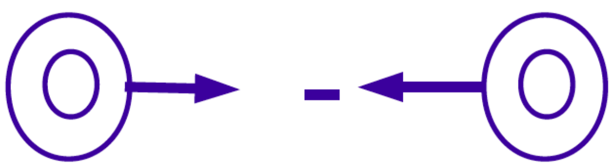
Calculating the torque from Lattice

F_{14}



Michael Engelhardt

Phys. Rev. D95 (2017)



Longitudinally polarized proton

$$\mathcal{L}_{JM} - \mathcal{L}_{Ji} = \mathcal{T}$$

Torque!

Extending to the Chiral Odd Sector

$$\boxed{\frac{dh_{1L}^{\perp(1)}}{dx} = h_1 - h_L}$$

$\downarrow$ Off forward

LIR

$$\boxed{\frac{dH_{17}^{(1)}}{dx} = H_T - \frac{\Delta_T^2}{4M^2} E_T - \tilde{H}_2'}$$

$$\boxed{-x\tilde{H}_2' - \int d^2k_T \frac{(k_T \times \Delta_T)^2}{M^2 \Delta_T^2} H_{17} + \frac{m}{M}\tilde{H} - \frac{1}{2M}\int d^2k_T \left(\mathcal{M}_{++}^{\gamma^+\gamma^5 A} - \mathcal{M}_{--}^{\gamma^+\gamma^5 A}\right) = 0}$$

EoM

$$\Gamma = \gamma^+, \gamma^+\gamma^5$$

for connecting chiral odd GPDs and GTMDs

$$0 = \int \frac{dz_{in}^- \, d^2 z_{in,T}}{(2\pi)^3} \int \frac{dz_{out}^- \, d^2 z_{out,T}}{(2\pi)^3} e^{ik(z_{out}-z_{in})+i\Delta(z_{out}+z_{in})/2}$$

$$\cdot \langle p', \Lambda' \mid \bar{\psi}(z_{in}) \left[(i\overleftarrow{\not{D}} + m)\Gamma\mathcal{U} \pm \Gamma\mathcal{U}(i\not{D} - m) \right] \psi(z_{out}) \mid p, \Lambda \rangle \Big|_{z_{in}^+ = z_{out}^+ = 0}$$

LIRs between Twist 3 and Twist 4 distributions

$$W^{[\gamma_T^j]} = \frac{M}{P^+}\left[\left(\frac{k_T^j}{M}f^\perp + \frac{i\Lambda\epsilon^{ij}k_T^i}{M}f_L^\perp\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2}f_T^\perp + (\Lambda\delta_{j1} + i\delta_{j2})f_T'\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} -\frac{M}{P^+}(\Lambda\delta_{j1} + i\delta_{j2})H_{2T}\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma_T^j\gamma^5]} = \frac{M}{P^+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M}g^\perp + \Lambda\frac{k_T^j}{M}g_L^\perp\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2})g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2}g_T^\perp\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P^+}g_T\delta_{-\Lambda\Lambda'}$$

LIRs between Twist 3 and Twist 4 distributions

$$
\begin{aligned}
W^{[\gamma_T^j]} &= \frac{M}{P^+}\left[\left(\frac{k_T^j}{M}f^\perp + \frac{i\Lambda\epsilon^{ij}k_T^i}{M}f_L^\perp\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2}f_T^\perp + (\Lambda\delta_{j1} + i\delta_{j2})f_T'\right)\delta_{-\Lambda\Lambda'}\right] \\
&\xrightarrow{\int d^2k_T} -\frac{M}{P^+}(\Lambda\delta_{j1} + i\delta_{j2})H_{2T}\delta_{-\Lambda\Lambda'} \\
W^{[\gamma_T^j\gamma^5]} &= \frac{M}{P^+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M}g^\perp + \Lambda\frac{k_T^j}{M}g_L^\perp\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2})g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2}g_T^\perp\right)\delta_{-\Lambda\Lambda'}\right] \\
&\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P^+}g_T\delta_{-\Lambda\Lambda'}
\end{aligned}
$$

$$
\begin{aligned}
W^{[\gamma^-]} &= \frac{M^2}{(P^+)^2}\left[f_3\delta_{\Lambda\Lambda'} + \frac{\Lambda k_1 + ik_2}{M}f_{3T}^\perp\delta_{-\Lambda\Lambda'}\right] \\
&\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}f_3\delta_{\Lambda\Lambda'} \\
W^{[\gamma^-\gamma^5]} &= \frac{M^2}{(P^+)^2}\left[\Lambda g_{3L}\delta_{\Lambda\Lambda'} + \frac{k_1 + i\Lambda k_2}{M}g_{3T}\delta_{-\Lambda\Lambda'}\right] \\
&\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}\Lambda g_{3L}\delta_{\Lambda\Lambda'}
\end{aligned}
$$

LIRs between Twist 3 and Twist 4 distributions

$$W^{[\gamma_T^j]} = \frac{M}{P+}\left[\left(\frac{k_T^j}{M} f^{\perp} + \frac{i\Lambda\epsilon^{ij}k_T^i}{M} f_L^{\perp}\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2} f_T^{\perp} + (\Lambda\delta_{j1} + i\delta_{j2}) f_T'\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} -\frac{M}{P+}(\Lambda\delta_{j1} + i\delta_{j2}) H_{2T}\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma_T^j\gamma^5]} = \frac{M}{P+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M} g^{\perp} + \Lambda\frac{k_T^j}{M} g_L^{\perp}\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2}) g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2} g_T^{\perp}\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P+} g_T\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma^-]} = \frac{M^2}{(P+)^2}\left[\quad\right]$$

Measured in experiment F_L !

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P+)^2} f_3\delta_{\Lambda\Lambda'}$$

$$W^{[\gamma^-\gamma^5]} = \frac{M^2}{(P+)^2}\left[\Lambda g_{3L}\delta_{\Lambda\Lambda'} + \frac{k_1 + i\Lambda k_2}{M} g_{3T}\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P+)^2}\Lambda g_{3L}\delta_{\Lambda\Lambda'}$$

LIRs between Twist 3 and Twist 4 distributions

$$W^{[\gamma_T^j]} = \frac{M}{P^+}\left[\left(\frac{k_T^j}{M}f^\perp + \frac{i\Lambda\epsilon^{ij}k_T^i}{M}f_L^\perp\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2}f_T^\perp + (\Lambda\delta_{j1} + i\delta_{j2})f_T'\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} -\frac{M}{P^+}(\Lambda\delta_{j1} + i\delta_{j2})H_{2T}\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma_T^j\gamma^5]} = \frac{M}{P^+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M}g^\perp + \Lambda\frac{k_T^j}{M}g_L^\perp\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2})g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2}g_T^\perp\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P^+}g_T\delta_{-\Lambda\Lambda'}$$

$$\boxed{2f_3 - f_1 = \frac{df^{\perp(1)}}{dx}}$$

$$\boxed{g_T + g_{3L} - \frac{1}{2}g_{1L} = \frac{1}{2}\frac{dg_L^{\perp(1)}}{dx}}$$

$$W^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\Big[\quad\Big]$$

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}f_3\delta_{\Lambda\Lambda'}$$

Measured in experiment F_L !

$$W^{[\gamma^-\gamma^5]} = \frac{M^2}{(P^+)^2}\left[\Lambda g_{3L}\delta_{\Lambda\Lambda'} + \frac{k_1 + i\Lambda k_2}{M}g_{3T}\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}\Lambda g_{3L}\delta_{\Lambda\Lambda'}$$

Understanding the mass decomposition of the proton

Mass decomposition of the proton

$$T^{\mu\nu} = \boxed{T^{\mu\nu}_{q,kin} + T^{\mu\nu}_{g,kin}} + T^{\mu\nu}_{m} + T^{\mu\nu}_{a}$$

Traceless

X Ji (1995)

$$M = \frac{\langle P| \int d^3\mathbf{x} T^{00}(0,\mathbf{x})|P\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle$$

Rest frame

$$\langle \bar{T}^{00}\rangle = 3/4M$$ ← Traceless

$$\langle \hat{T}^{00}\rangle = 1/4M$$ ← Trace part

Energy Momentum Tensor Parameterization

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F_{\alpha}^{\nu}$$

- The full energy momentum tensor is a conserved quantity and is scale independent.
- The separate contributions from the quarks and gluons on the other hand are not and do depend on the renormalization scale.

$$\langle P' | (T^{\mu\nu})_R | P \rangle =$$
$$\bar{u}(P') \left[A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + C_{q,g} \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g} M \eta^{\mu\nu} \right] u(P)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

$\big\downarrow$ Trace

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

Trace Anamoly

* $\langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M \longrightarrow M$ **Total**

* $\langle P'|(T^{\mu\nu}_{q,g})_R|P\rangle =$

$$\bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}M\eta^{\mu\nu}\right]u(P)$$

$\Big\downarrow$ Trace

Quark and gluon components separated

$$\langle P'|(T^R_{q,g})^\alpha_\alpha|P\rangle = 2M^2\left(A^R_{q,g}(\mu) + 4\bar{C}^R_{q,g}(\mu)\right)$$

* $$T^{\mu\nu} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^\nu_\alpha$$

$\Big\downarrow$ Trace

$$T^\mu_\mu = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

By studying the Gravitational form factors A and $\bar{C}$ we will know the quark and gluon contributions to the trace anomaly separately.

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}$$

$$\Big\downarrow \text{Trace}$$

$$T^{\mu}_{\mu} = (1+\gamma_m) m \bar{\psi}\psi + \frac{\beta}{2g} F^2$$

Quark and gluon contributions to the trace anomaly

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha}F^{\nu}_{\alpha}$$

Trace

$$T^{\mu}_{\mu} = (1+\gamma_m) m\bar{\psi}\psi + \frac{\beta}{2g}F^2$$

$$(T^{\alpha}_{g\alpha})_R = A^R_g(\mu) + 4\bar{C}^R_g(\mu)$$

$$= \frac{1}{2M^2}\langle P|\frac{\alpha_s}{2\pi}\left(-\frac{11C_A}{6}\underline{(F^2)_R} + \frac{14C_F}{3}\underline{(m\bar{\psi}\psi)_R)}\right)|P\rangle$$

gluons

$$(T^{\alpha}_{q\alpha})_R = A^R_q(\mu) + 4\bar{C}^R_q(\mu)$$

$$= \frac{1}{2M^2}\langle P|(m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}\underline{(F^2)_R} + \frac{4C_F}{3}\underline{(m\bar{\psi}\psi)_R)}\right)|P\rangle$$

quarks

Y Hatta, AR, K Tanaka arxiv:1810.05116

$$\begin{aligned}\langle P|(T_{qR})^\mu_\mu|P\rangle &= a_q\langle P|(m\bar\psi\psi)_R|P\rangle + b_q\langle P|(F^2)_R|P\rangle \\ \langle P|(T_{gR})^\mu_\mu|P\rangle &= a_g\langle P|(m\bar\psi\psi)_R|P\rangle + b_g\langle P|(F^2)_R|P\rangle\end{aligned}$$

$$\begin{aligned}\langle P|(T_{qR})^\mu_\mu|P\rangle &= 2M^2(A_q + 4\bar C_q) \\ \langle P|(T_{gR})^\mu_\mu|P\rangle &= 2M^2(A_g + 4\bar C_g)\end{aligned}$$

Parameterized in terms of quark and gluon gravitational form factors

$$\begin{aligned}\langle P|(T_{qR})^\mu_\mu|P\rangle &= a_q\langle P|(m\bar\psi\psi)_R|P\rangle + b_q\langle P|(F^2)_R|P\rangle\\ \langle P|(T_{gR})^\mu_\mu|P\rangle &= a_g\langle P|(m\bar\psi\psi)_R|P\rangle + b_g\langle P|(F^2)_R|P\rangle\end{aligned}$$

Parameterized in terms of quark and gluon gravitational form factors

$$\begin{aligned}\langle P|(T_{qR})^\mu_\mu|P\rangle &= 2M^2(A_q + 4\bar C_q)\\ \langle P|(T_{gR})^\mu_\mu|P\rangle &= 2M^2(A_g + 4\bar C_g)\end{aligned}$$

$$\begin{aligned}\langle P|(m\bar\psi\psi)_R|P\rangle &= \frac{b_g\langle P|(T_{qR})^\mu_\mu|P\rangle - b_q\langle P|(T_{gR})^\mu_\mu|P\rangle}{a_q b_g - a_g b_q}\\ \langle P|(F^2)_R|P\rangle &= \frac{a_g\langle P|(T_{qR})^\mu_\mu|P\rangle - a_q\langle P|(T_{gR})^\mu_\mu|P\rangle}{a_g b_q - a_q b_g}\end{aligned}$$

$$\begin{aligned}\langle P|(T_{qR})^\mu_\mu|P\rangle &= a_q\langle P|(m\bar\psi\psi)_R|P\rangle + b_q\langle P|(F^2)_R|P\rangle \\ \langle P|(T_{gR})^\mu_\mu|P\rangle &= a_g\langle P|(m\bar\psi\psi)_R|P\rangle + b_g\langle P|(F^2)_R|P\rangle\end{aligned}$$

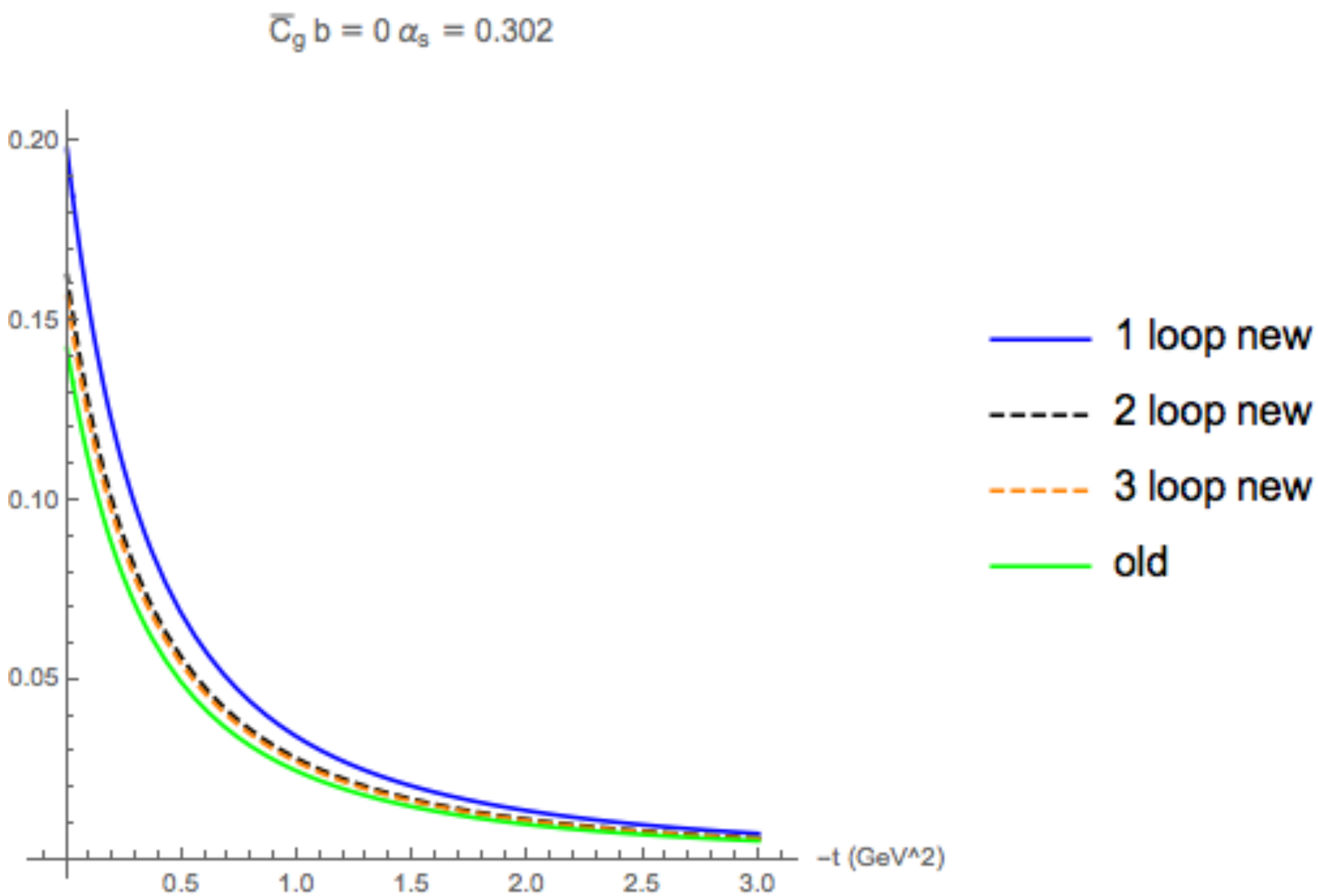
$$\begin{aligned}\langle P|(T_{qR})^\mu_\mu|P\rangle &= 2M^2(A_q + 4\bar C_q) \\ \langle P|(T_{gR})^\mu_\mu|P\rangle &= 2M^2(A_g + 4\bar C_g)\end{aligned}$$

Parameterized in terms of quark and gluon gravitational form factors

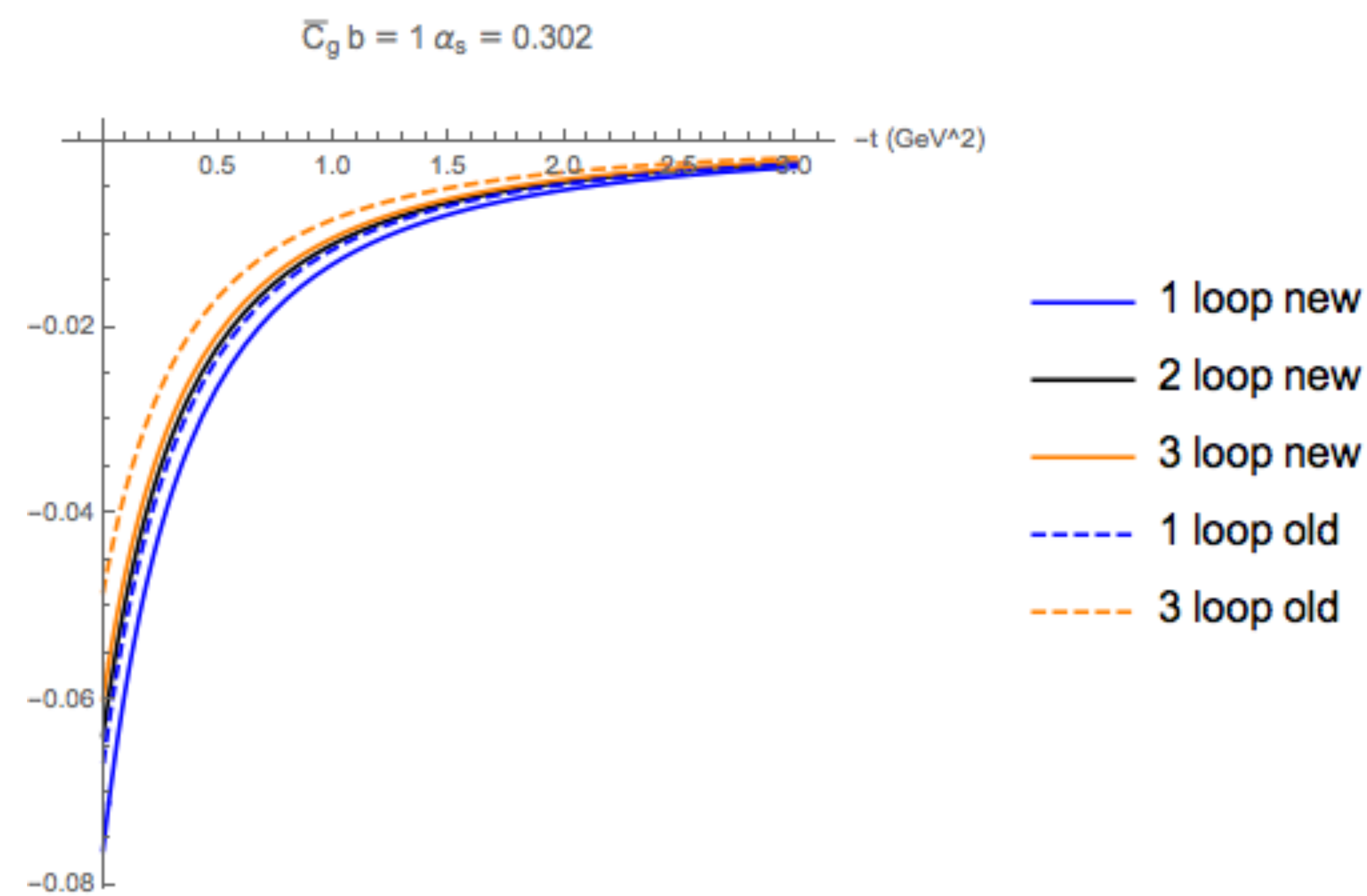
$$\begin{aligned}\langle P|(m\bar\psi\psi)_R|P\rangle &= \frac{b_g\langle P|(T_{qR})^\mu_\mu|P\rangle - b_q\langle P|(T_{gR})^\mu_\mu|P\rangle}{a_q b_g - a_g b_q} \\ \langle P|(F^2)_R|P\rangle &= \frac{a_g\langle P|(T_{qR})^\mu_\mu|P\rangle - a_q\langle P|(T_{gR})^\mu_\mu|P\rangle}{a_g b_q - a_q b_g}\end{aligned}$$

$$\langle P|\frac{\beta}{2g}(F^2)_R|P\rangle = 2M^2(1-b)$$

$$4\bar C_g = \frac{-(a_g b_q - a_q b_g)(1-b) + a_g}{a_q - a_g} - A_g$$



$\overline{C}_g$ b = 0 α_s = 0.302
0.20
0.15
0.10
0.05
0.5
1.0
1.5
2.0
2.5
3.0
−t (GeV^2)
1 loop new
2 loop new
3 loop new
old



$\overline{C}_g$ b = 1 α_s = 0.302
−t (GeV^2)
0.5
1.0
1.5
2.0
2.5
3.0
−0.02
−0.04
−0.06
−0.08
1 loop new
2 loop new
3 loop new
1 loop old
3 loop old

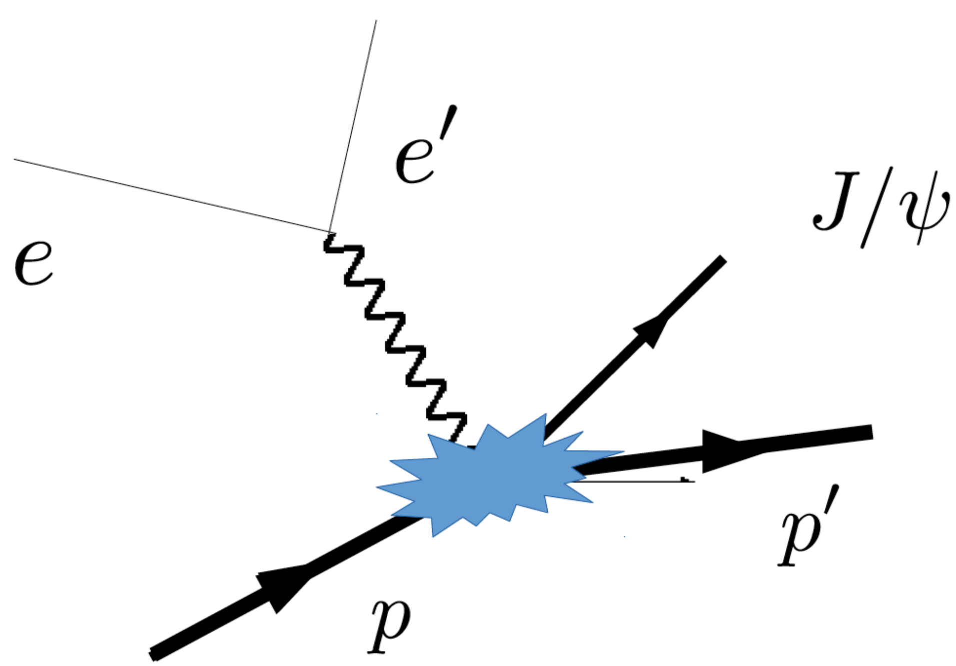
Experimental measurements

The production of a heavy quarkonium near threshold in electron-proton scattering is connected to the origin of the proton mass via the QCD trace anomaly.

D.E Kharzeev (1995)

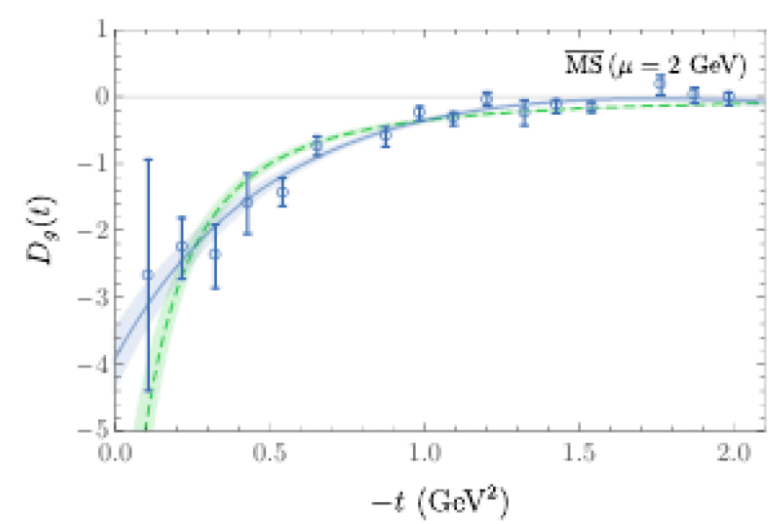
$$ep \to e'\gamma^* p \to e'p'J/\psi$$

Y Hatta, DL Yang PRD98 (2018)



In an actual experiment $p' - p \equiv t \neq 0$

Calculate cross-section using input from latest lattice QCD calculations of gluon gravitational form factors.



Detmold and Shanahan arxiv:1810:04626

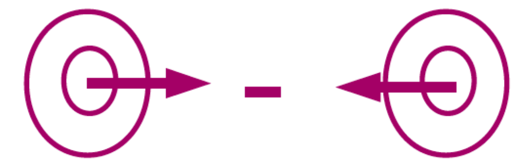
Summary

- Showed a way of deriving Wandzura Wilczek relations. Allows us to write out the quark gluon quark contribution to twist three GPDs precisely. Study the x dependence.

- Gluons play a key role in describing the properties of the nucleon.

Thanks!

Moments of twist three GPDs -Quark gluon structure



$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} \right) = -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H} \right) = 0$$

$$\int dx \underline{x} \left(E'_{2T} + 2\widetilde{H}'_{2T} \right) = -\frac{1}{2} \int dx x \widetilde{H} - \frac{1}{2} \int dx H + \boxed{\frac{m}{2M} \int dx (E_T + 2\widetilde{H}_T)}$$

mass term

$$\int dx \, \underline{x^2} \left(E'_{2T} + 2\widetilde{H}'_{2T} \right) = -\frac{1}{3} \int dx x^2 \widetilde{H} - \frac{2}{3} \int dx x H + \boxed{\frac{2m}{3M} \int dx x (E_T + 2\widetilde{H}_T)} - \boxed{\frac{2}{3} \int dx x \mathcal{M}_{G_{11}} \Big|_{v=0}}$$

Genuine Twist Three d_2

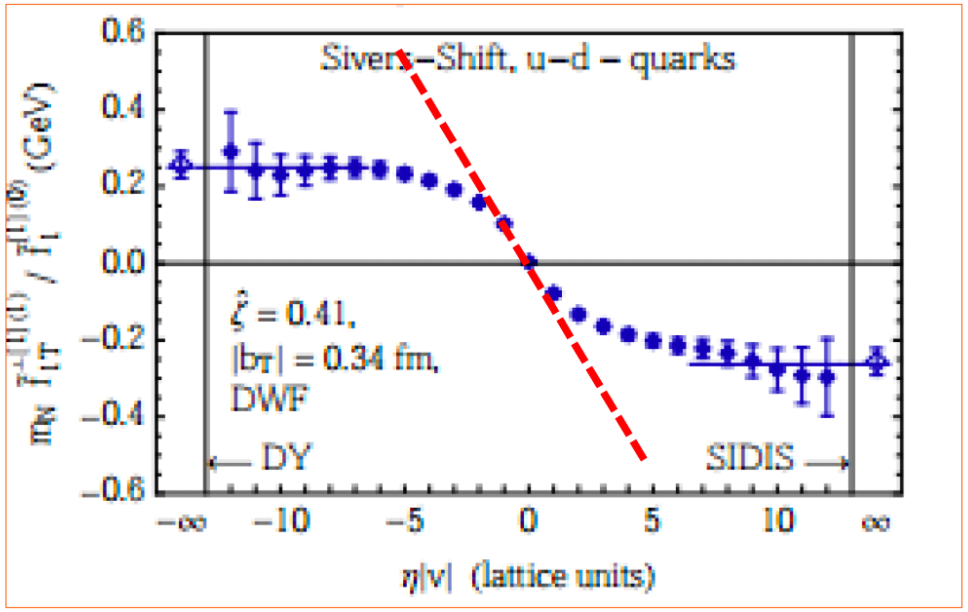
$$\int dx \, x \int d^2 k_T \, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2} \epsilon^{ij} \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ F^{+j}(0) \psi(0) | p, \Lambda \rangle$$

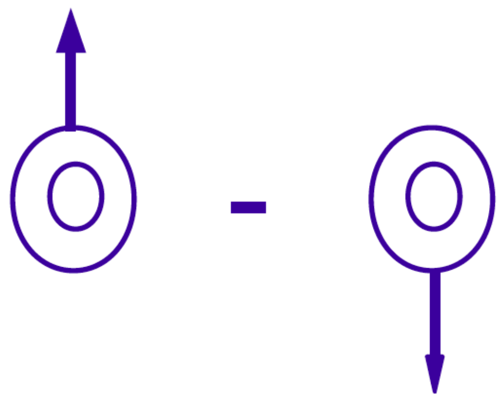
Calculating the force from Lattice data – Sivers function

$$\frac{d}{dv^-}\int dx F_{12}^{(1)}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{F_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,A} - \mathcal{M}_{--}^{i,A}\right) = \mathcal{M}_{G_{12}}^{n=3}$$

$$\frac{d}{dv^-}\int dx G_{12}^{(1)}\Big|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{G_{12}}\Big|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,S} + \mathcal{M}_{--}^{i,S}\right) = \mathcal{M}_{F_{12}}^{n=3}$$

The derivative with respect to the gauge link direction gives the force!





Transversely polarized proton