Deutsche Forschungsgemeinschaft

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Topics in QCD from DSEs

Richard Williams

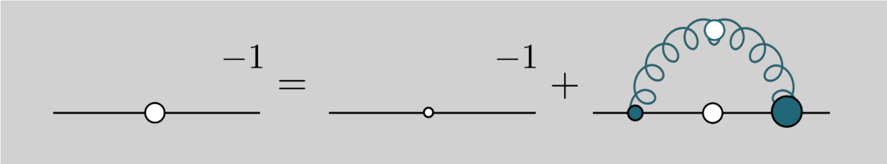
University of Giessen

APS Topical Group on Hadron Physics

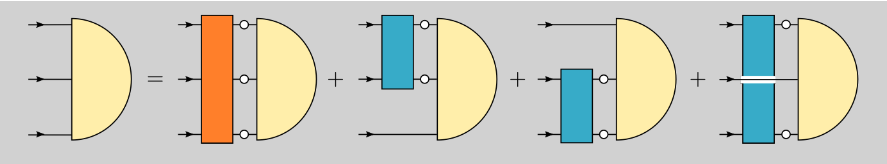
10-12 Apr. 2019, Denver, Colorado

Contents

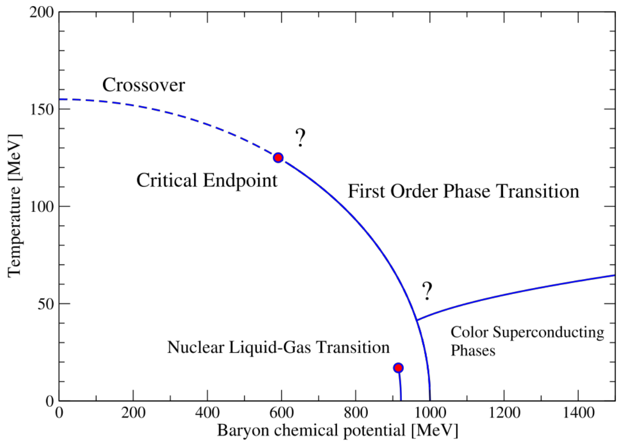
Dynamical Mass Generation



Hadronic Bound-states



Phase transitions in QCD



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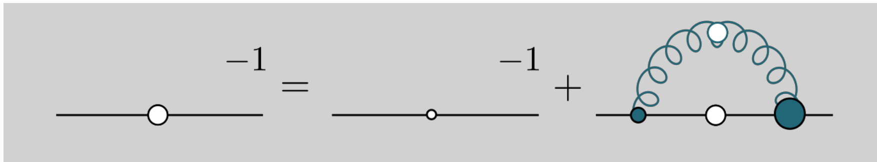

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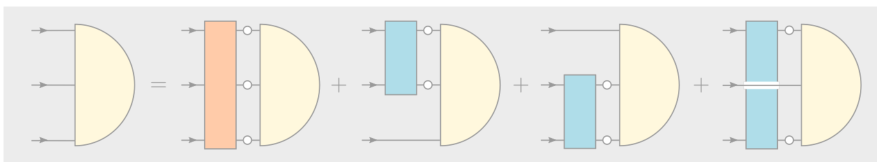

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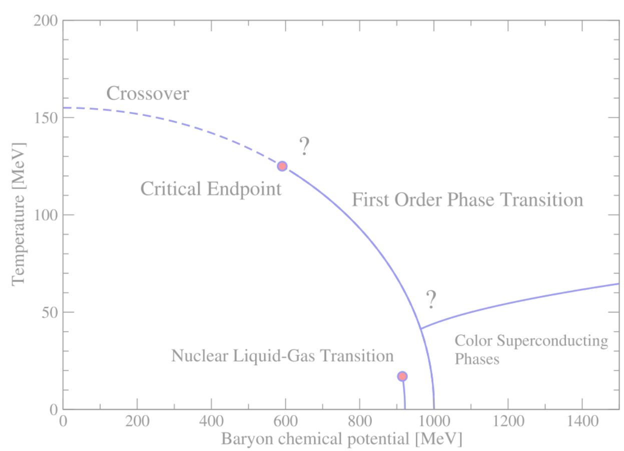

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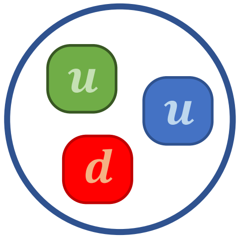


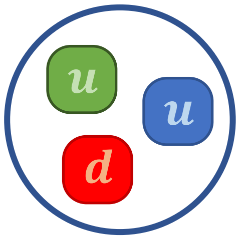
$$m_{\text{proton}} = 938\ MeV$$

quarks	u	d	s	c	b	t
M_{weak}	3	5	80	1200	4500	176000

see Roberts

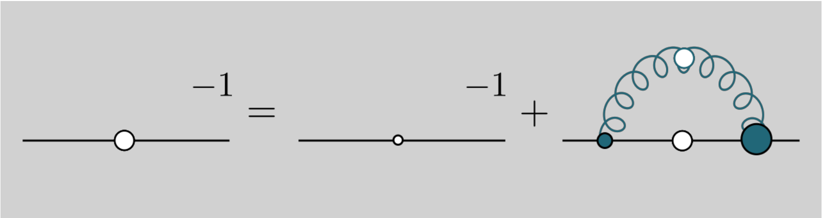
Introduction: Properties of QCD

Dynamical Mass Generation



$m_{\text{proton}} = 938\ MeV$

quarks	u	d	s	c	b	t
M_{weak}	3	5	80	1200	4500	176000
M_{strong}	350	350	350	350	350	350

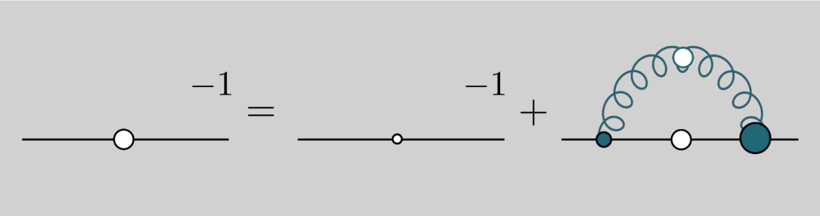


see Roberts

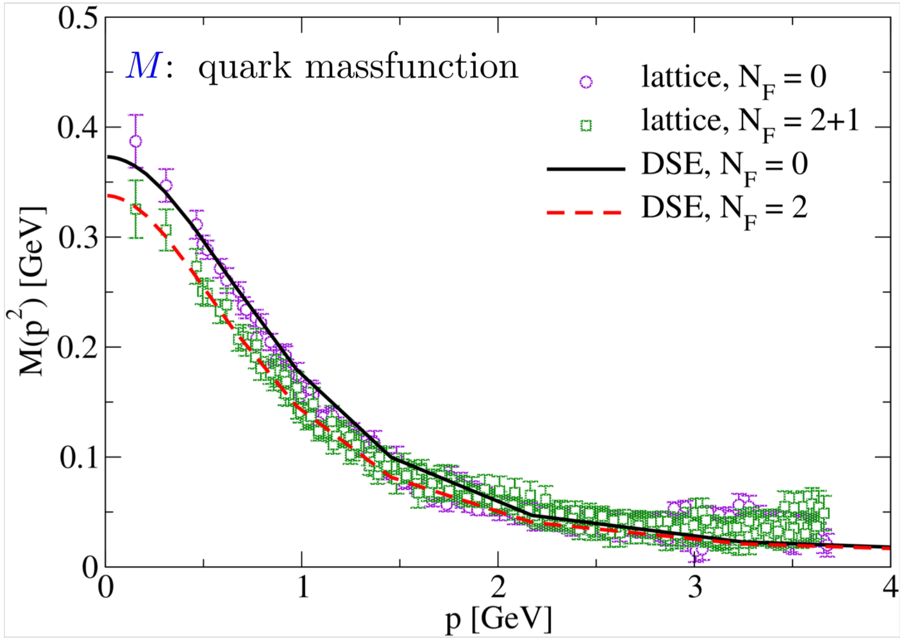
- Work in **Euclidean** space (for Minkowski space, *see Frederico)*
- Work in **Landau** gauge

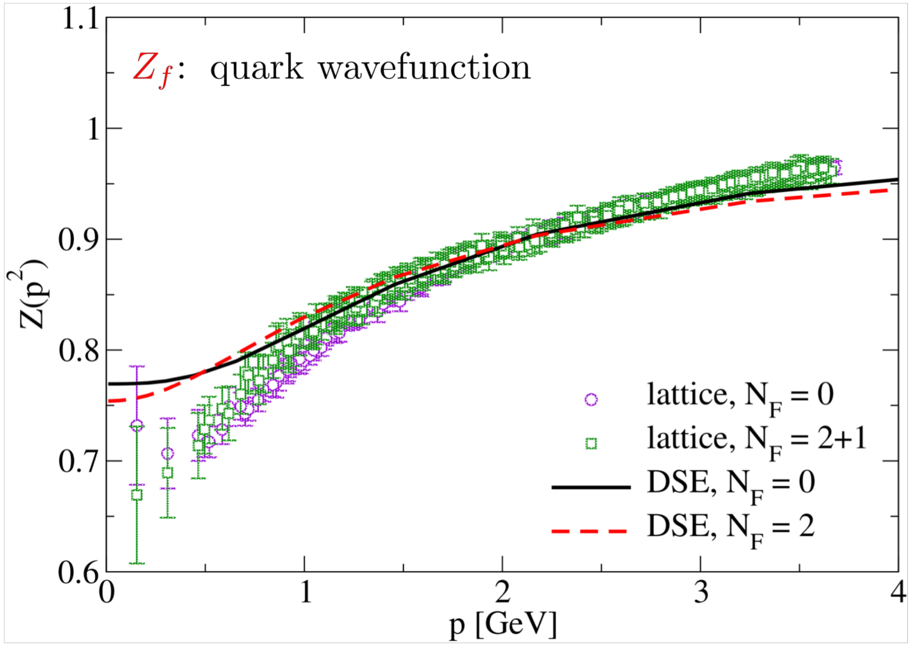
Introduction: Properties of QCD

Dynamical Mass Generation



$$S(p) = Z_f(p^2)\,\frac{-i\,\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$





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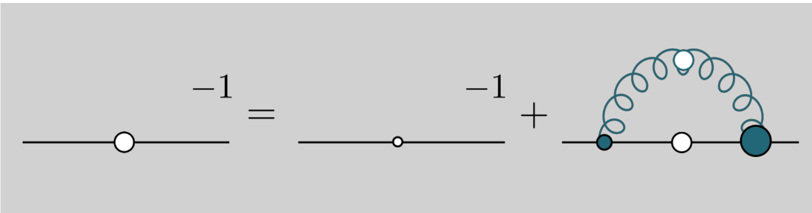

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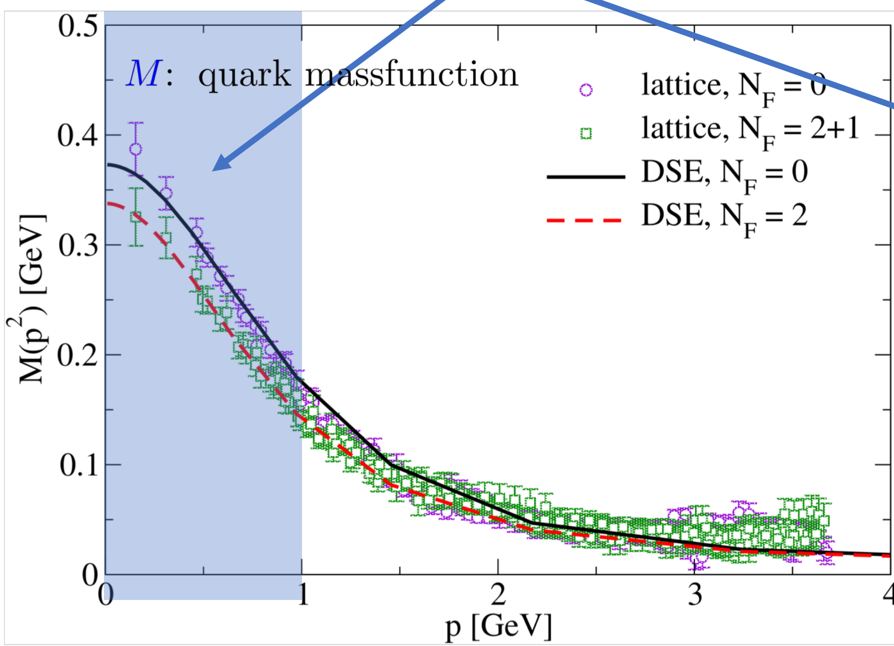
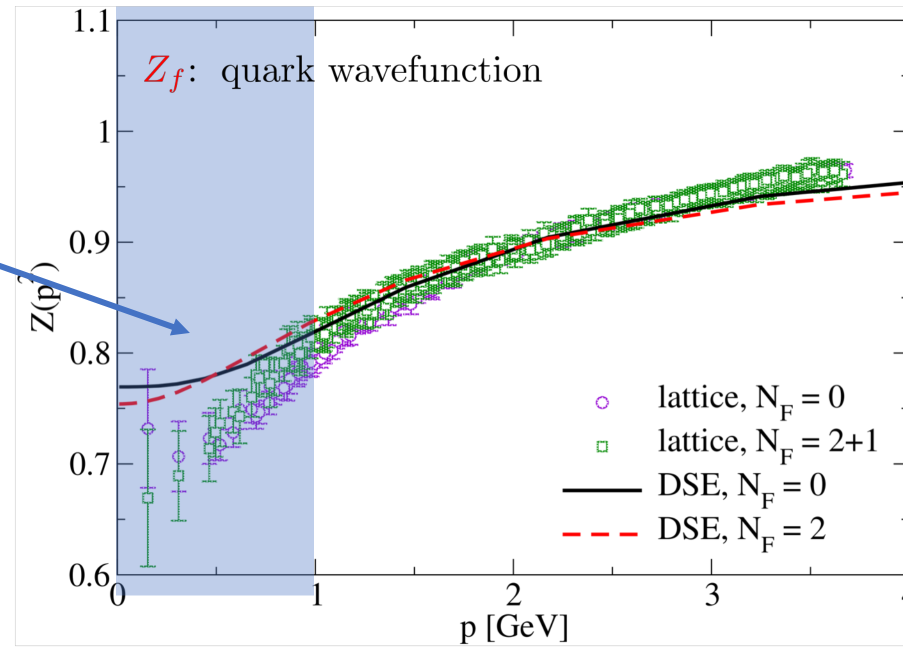
constituent quark: massive/composite





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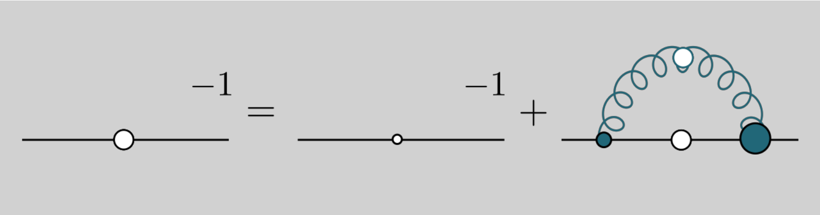

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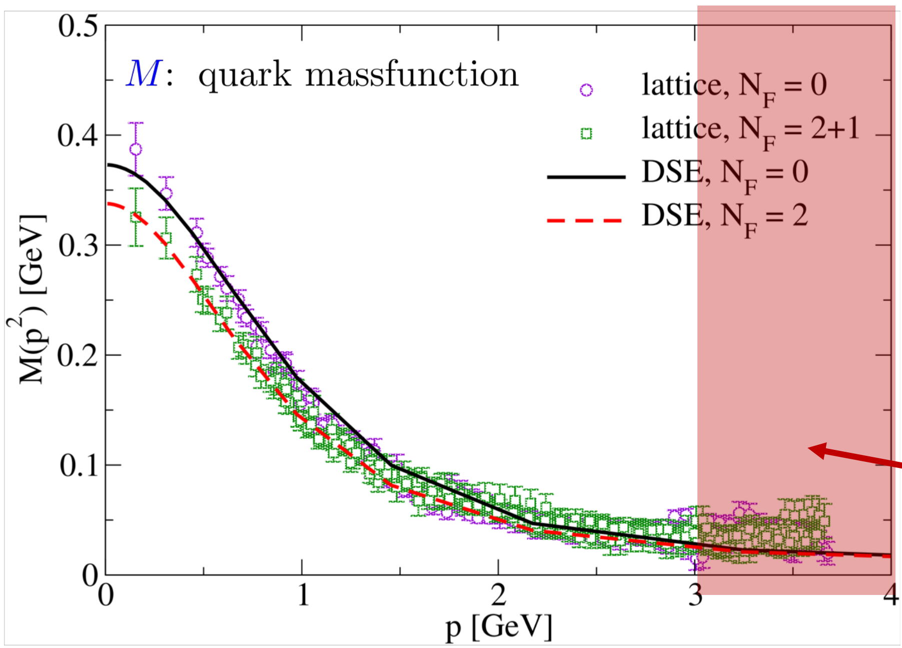
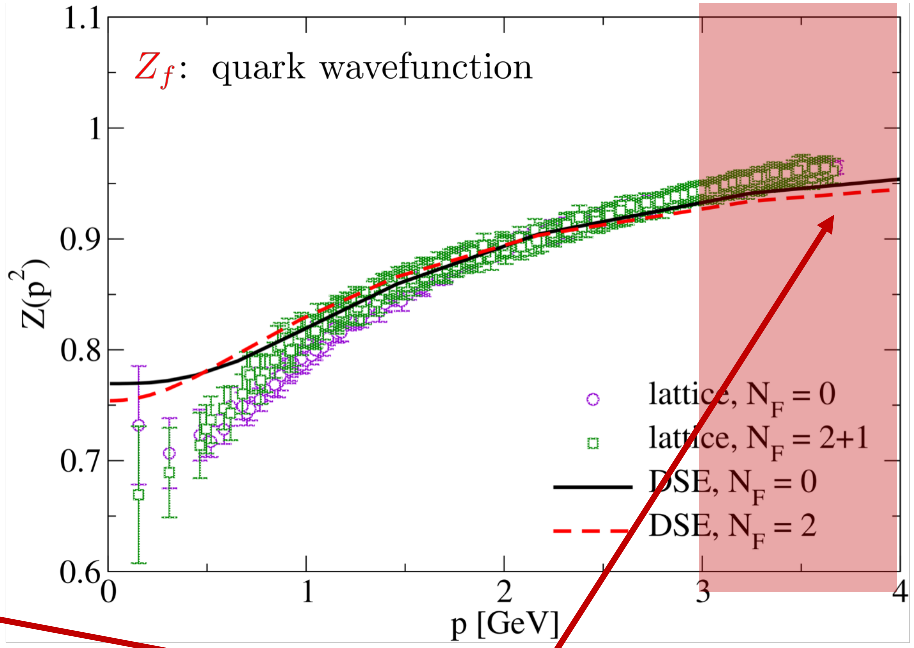


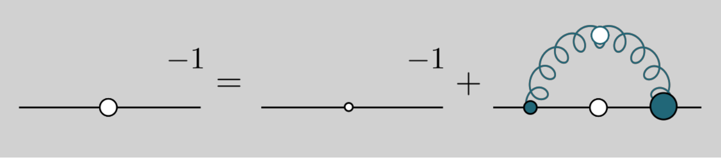


current quark: light/non-composite

Introduction: DSEs and truncation

$$\mathcal{Z}_{\mathrm{QCD}} = \int \mathcal{D}\Phi \exp\left\{ - \int d^4x \left[\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right] \right\}$$

Propagators



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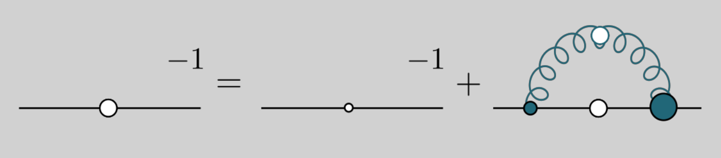

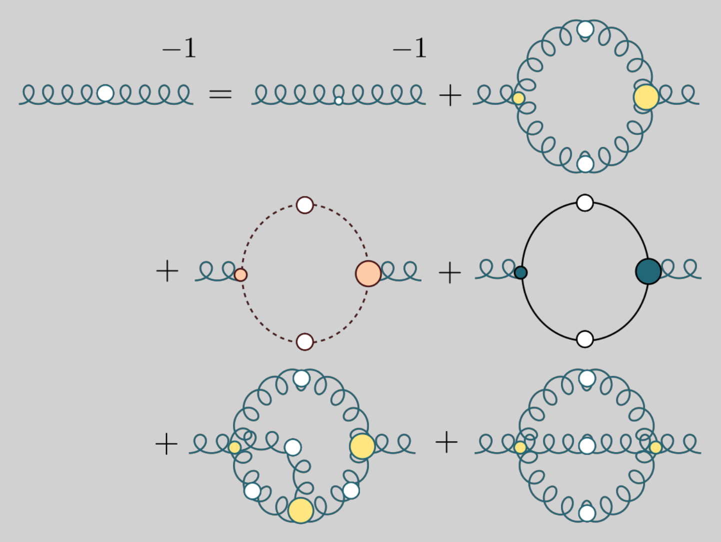

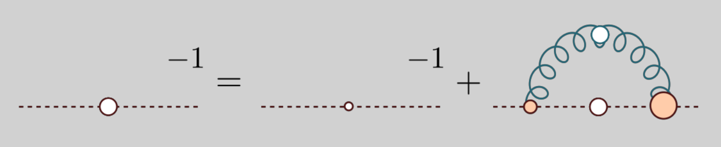

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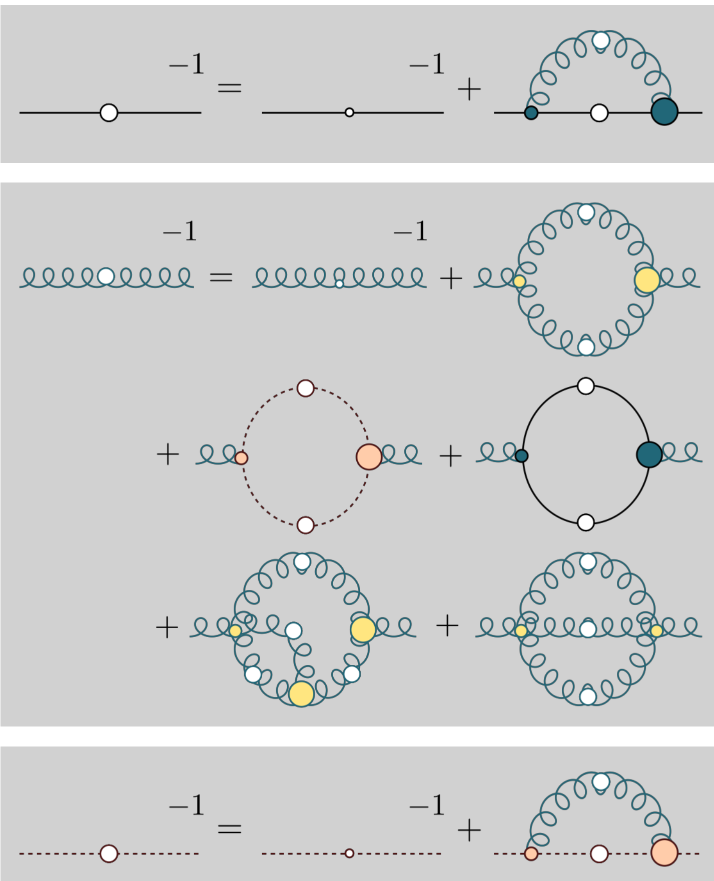
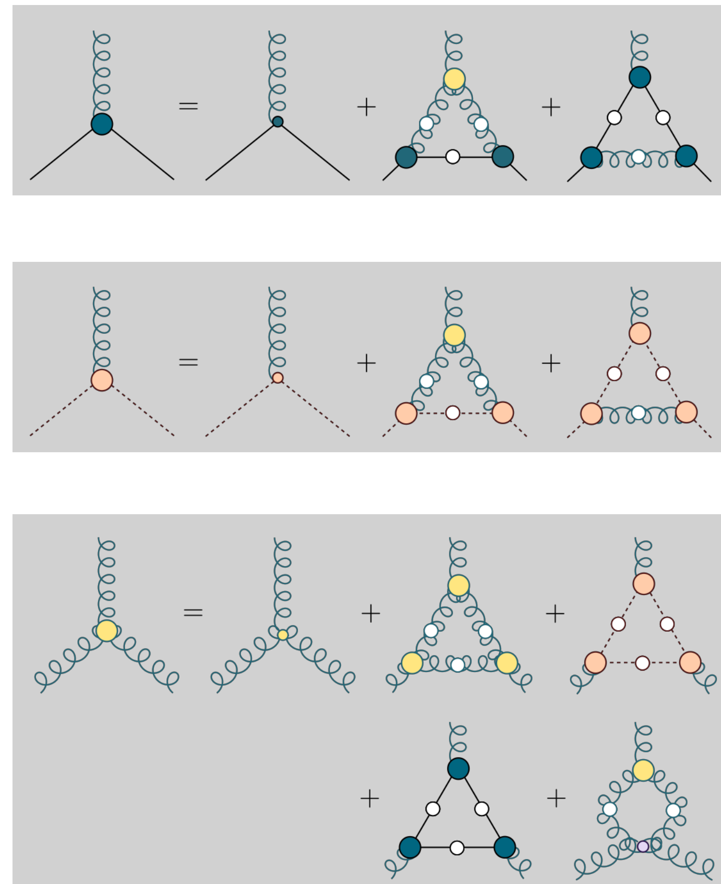


Vertices



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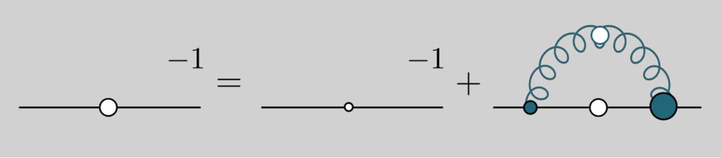

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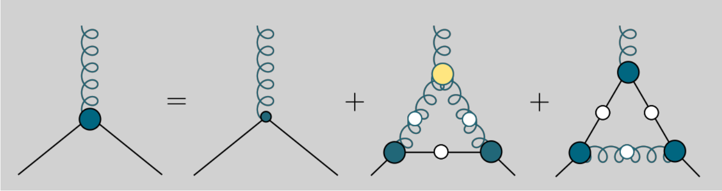
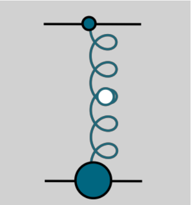


Rainbow-Ladder

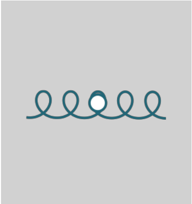
- Tree-level structures for vertex, gluon
- Combination modelled: **effective interaction**
- Connection to Yang-Mills **lost**

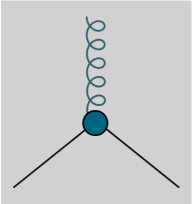


$$\longrightarrow \alpha_{\mathrm{eff}}(k^2)$$

Beyond Rainbow-Ladder

- Gluon **is typically** a gluon
  - Not always so: effective "interactions" still employed and useful
- Vertex can be
  - Modelled – constrained or constructed
  - **Calculated**





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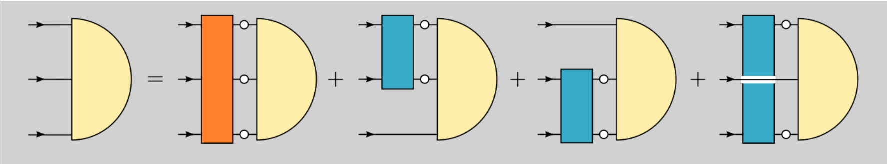

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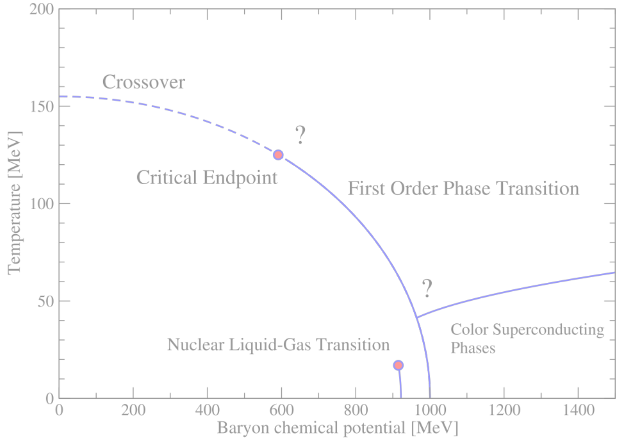
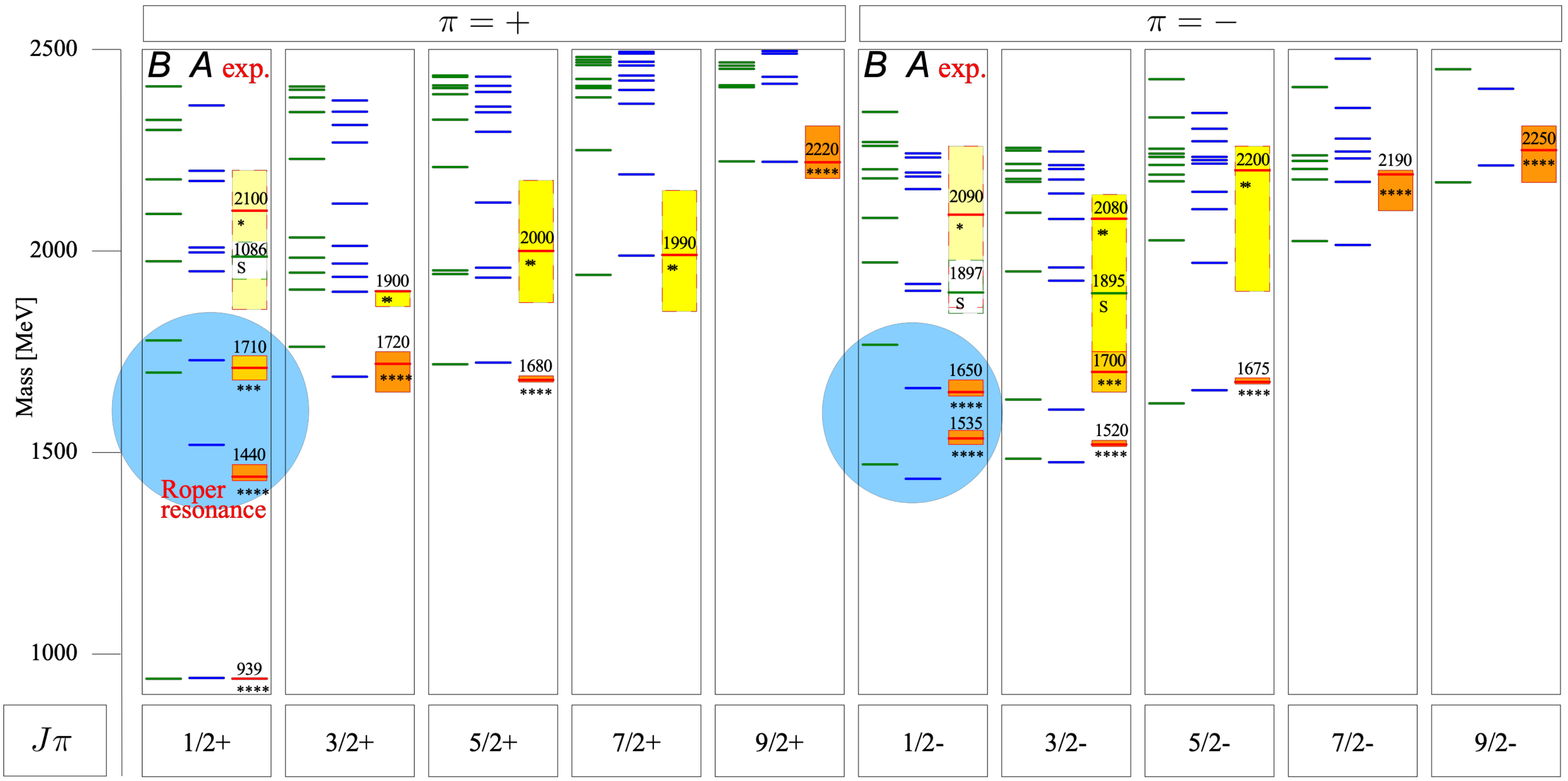


Baryons: motivation

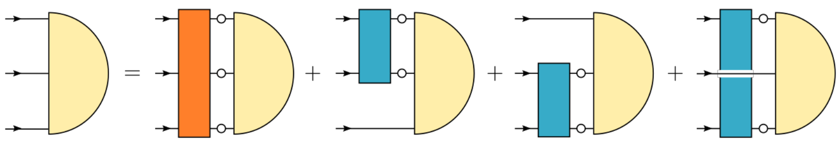


[Loring, Metsch, Petry, EPJA (2001)]

- Missing resonances
- Level ordering of parity partners

Baryons: interaction and amplitude

Faddeev equation



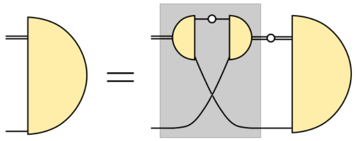
[Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)]

[Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014)]

[Sanchis-Alepuz, RW, PLB 749 (2015)]

- Neglect irreducible three-body forces
  [Sanchis-Alepuz, RW]
- Two-body interactions
  - one-gluon exchange
  - one-pion exchange
  - beyond rainbow-ladder

quark-diquark equation



[Cahill, Roberts, Praschifka, AJP 42 (1989)]

[Oettel, Hellstern, Alkofer, Reinhardt PRC 58 (1998)]

- Two-body bound-state of quark and diquark via quark exchange
  - Diquark has structure

Non-relativistic amplitude

$$P = (-1)^L$$

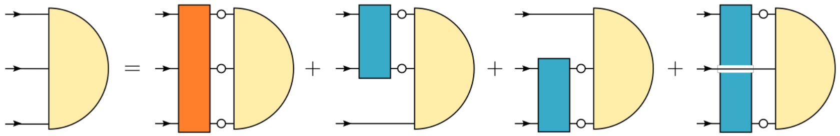
$N \qquad (S, L) = (1/2, 0), (3/2, 2)$

Relativistic amplitude

	s-wave	p-wave	d-wave
S=1/2	8	24	
S=3/2		12	20

Baryons: light spectrum, rainbow-ladder

Faddeev equation



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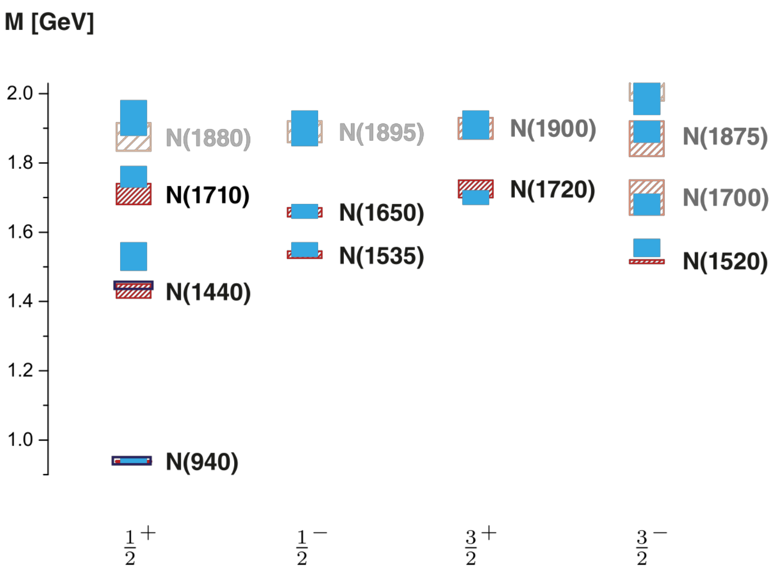
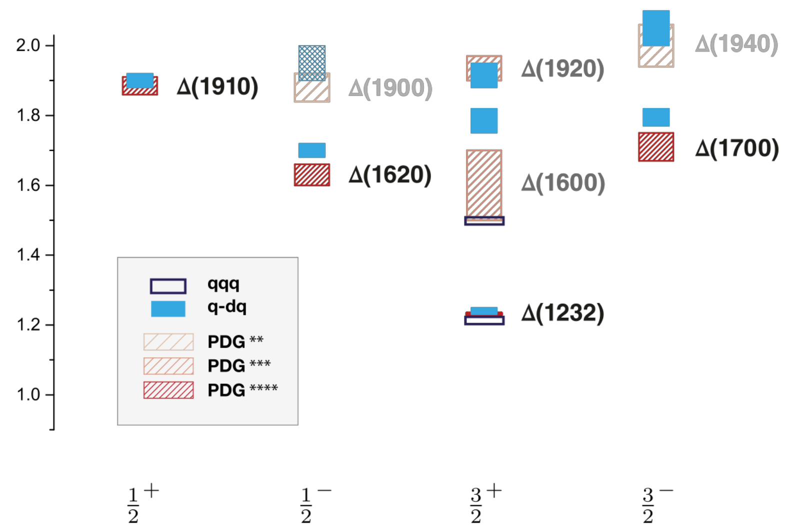
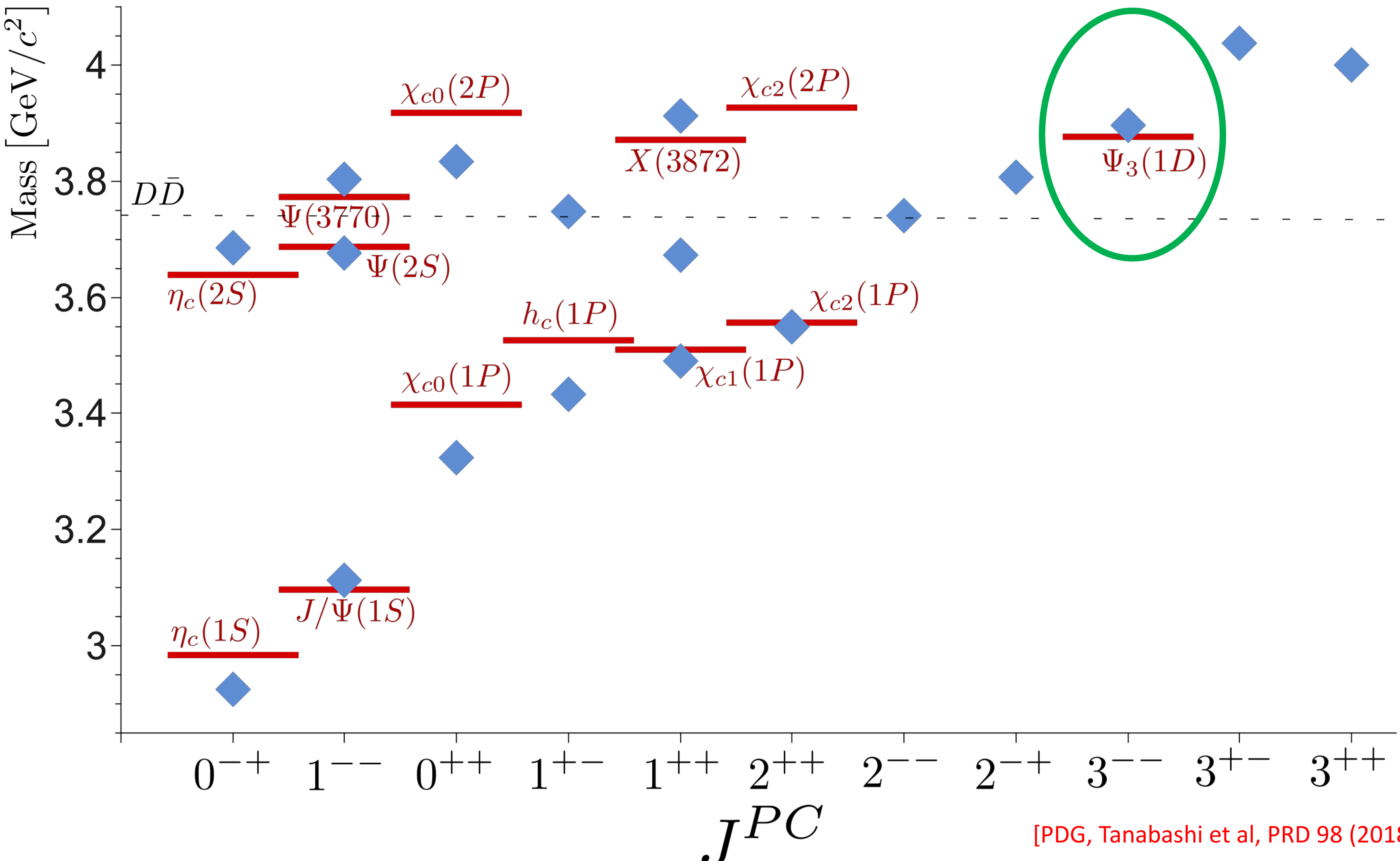
- Correct level ordering
- Number of states matches that of experiment
- Agreement between quark-diquark and three-body





[Eichmann, Fischer, Sanchis-Alepuz, PRD 94 (2016)]

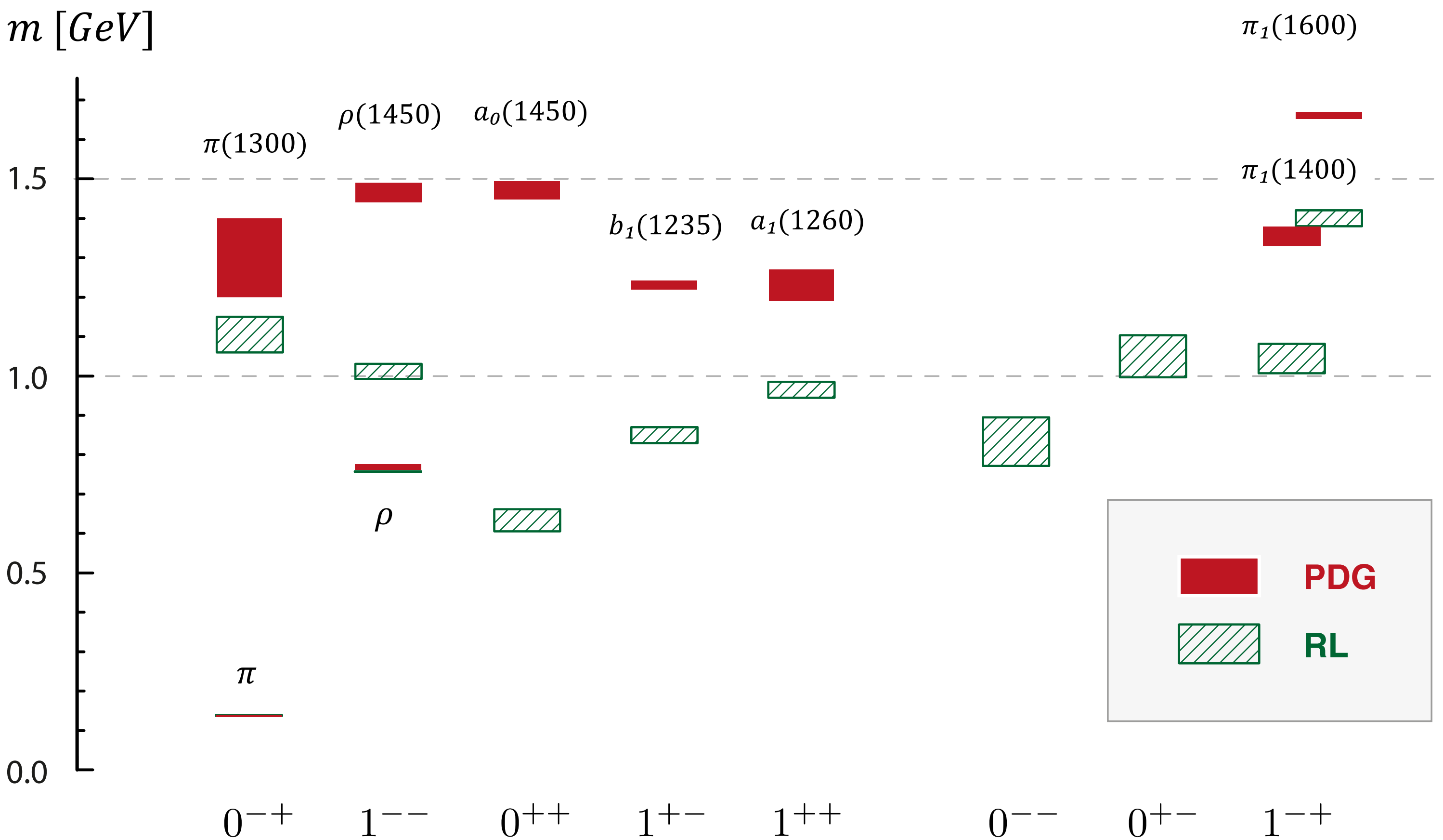
Mesons: charmonium, rainbow-ladder



[PDG, Tanabashi et al, PRD 98 (2018)]
[Fischer, Kubrak, RW, EPJA 51 (2015)]
[Hilger, Gomez-Rocha, Krassnigg, Lucha, EPJA 53 (2017)]
[LHCb Collaboration, Aaij et al, arXiv:1903.12240]

- Same Regge trajectory: $J^{PC} = 1^{--}, 2^{++}, 3^{--}$

Mesons: light, rainbow ladder



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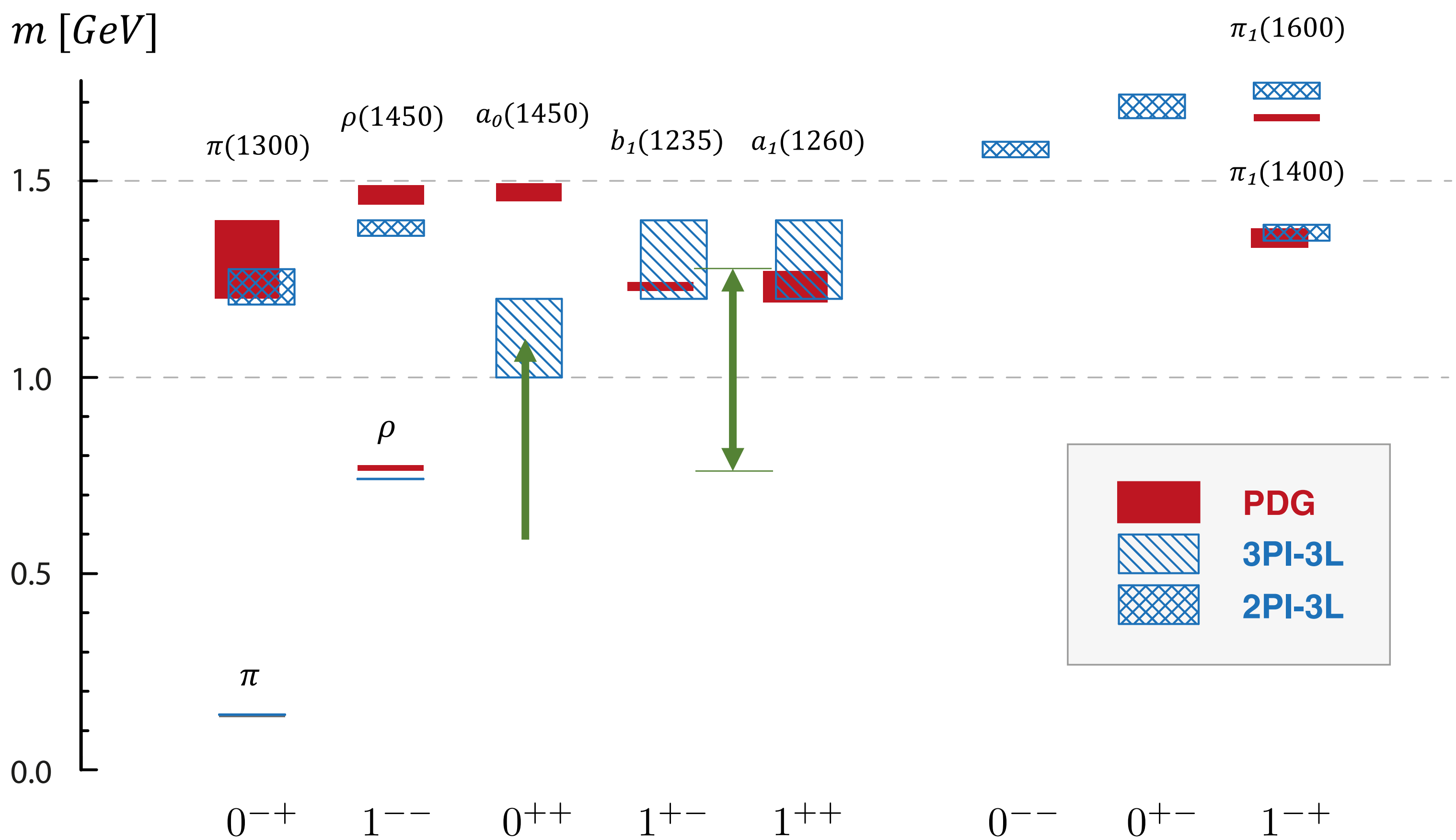


[RW, Fischer, Heupel, PRD93 (2016)]

- Correct $\rho - a_1$ splitting. Degeneracy in axial-vectors
- Lightest $q\bar{q}$ scalar pushed above 1 GeV.

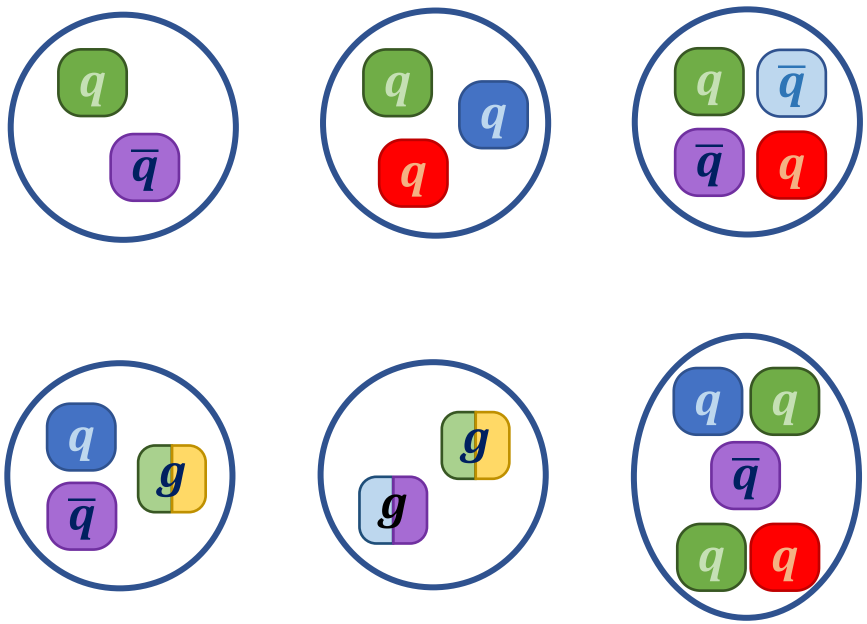
Uses 3PI formalism: *see Carrington*

[Chang, Roberts, PRL 103 (2009)]

[Chang, Roberts, PRC 85 (2012)]

Tetraquarks as light mesons

- Interesting candidates for tetraquarks
- Lightest scalar nonet
- Mass ordering



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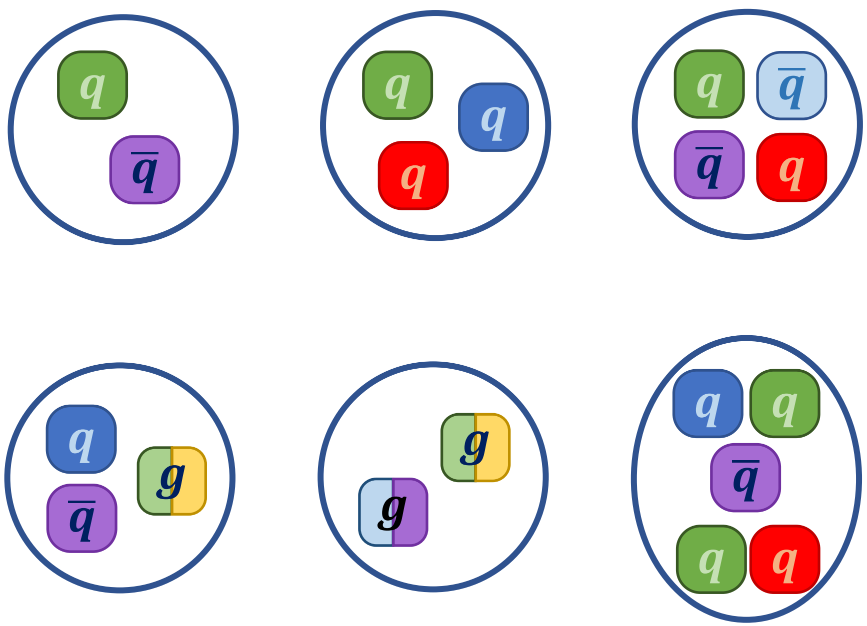

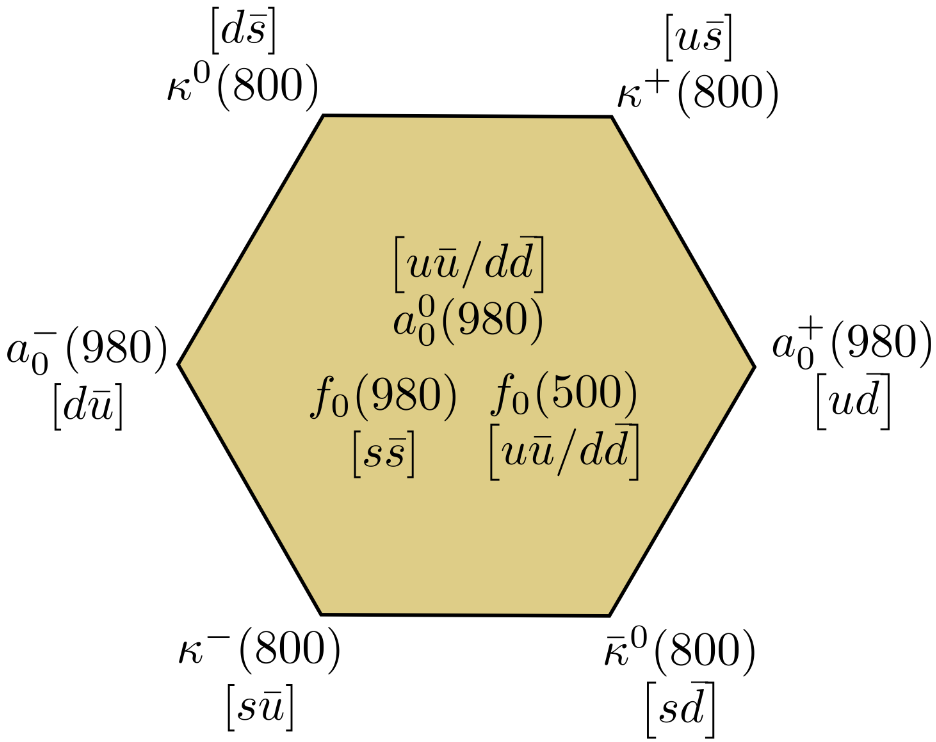

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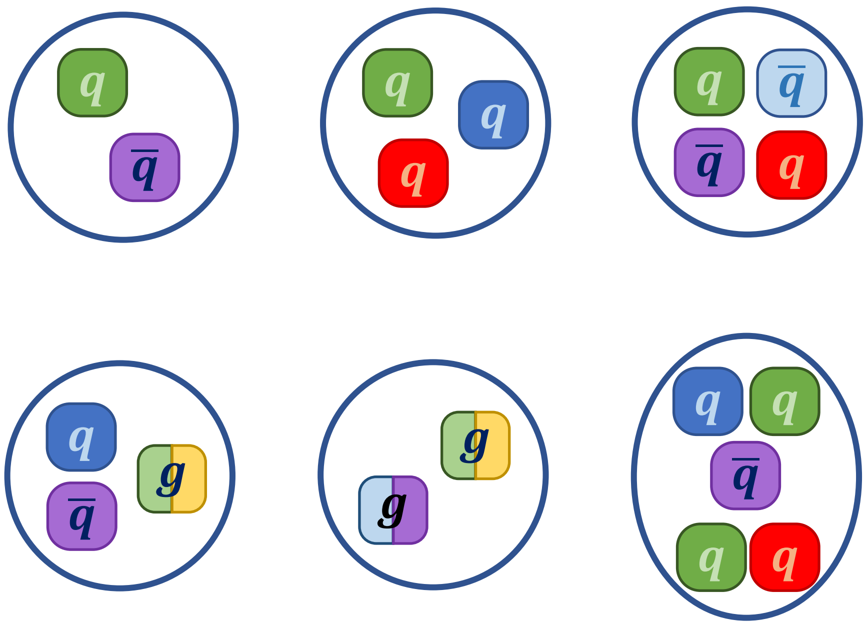
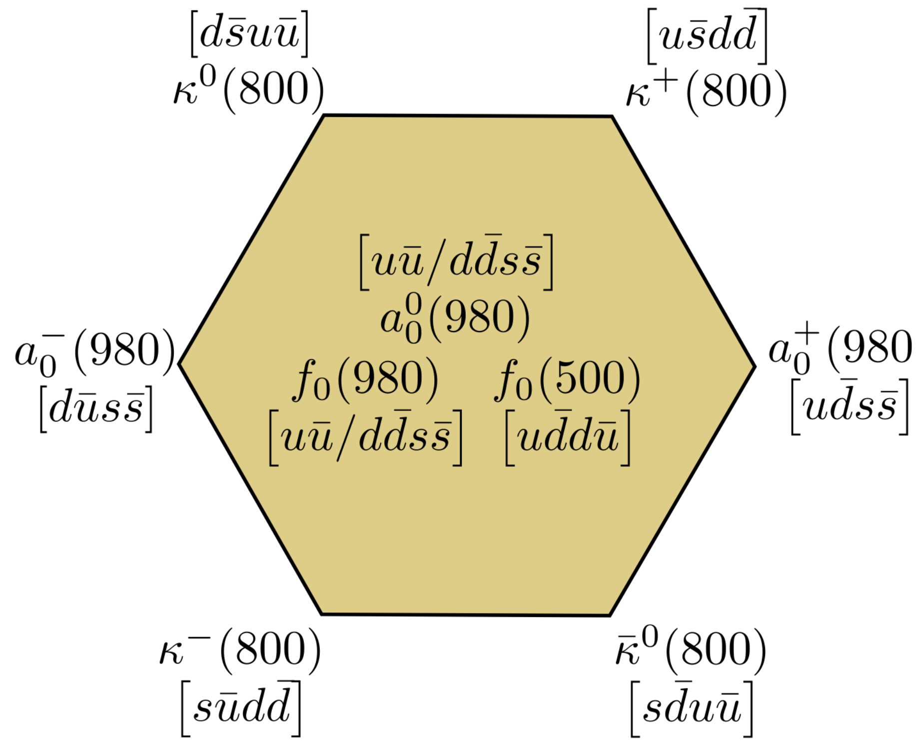
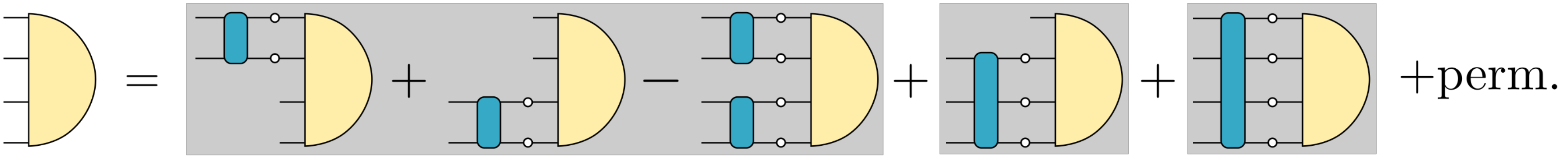




Tetraquarks from BSE

Exact equation



[Kvinikhidze, Kvedelidze, TMP 90 (1992)]
[Heupel, Eichmann, Fischer, PLB 718 (2012)]
[Eichmann, Fischer, Heupel, PLB 753 (2016)]

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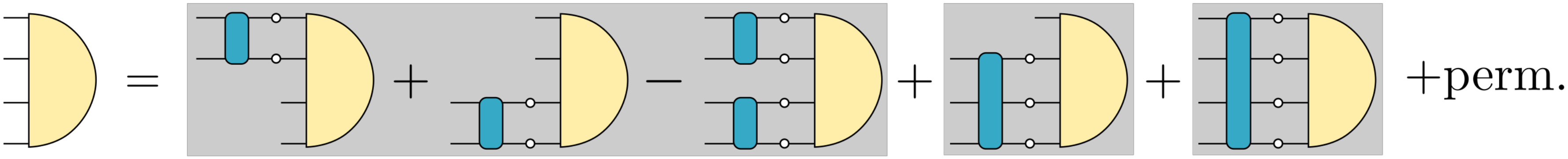


[Kvinikhidze, Kvedelidze, TMP 90 (1992)]
[Heupel, Eichmann, Fischer, PLB 718 (2012)]
[Eichmann, Fischer, Heupel, PLB 753 (2016)]

$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times \text{color} \times \text{flavour}$$

$f_i(s_1,\cdots,s_9)$ → Nine **Lorentz** scalars

$\tau_i(P,p,q,k)$ → 256 **Tensor** structures (scalar)

- **Approximate:**
  - Keep s-waves only, neglect three- and four-body interactions
  - Restricting nine Lorentz scalars enables exploration of *clustering*

Tetraquarks: Internal clustering

[Eichmann, Fischer, Heupel, PLB 753 (2016)]

Tetraquarks: Internal clustering

Without π clustering

Bound-state of four massive quarks

[Eichmann, Fischer, Heupel, PLB 753 (2016)]

Tetraquarks: Internal clustering

With π clustering

Two-pion resonance

Without π clustering

Bound-state of four massive quarks

[Eichmann, Fischer, Heupel, PLB 753 (2016)]

Tetraquarks: Internal clustering

With π clustering

Two-pion resonance

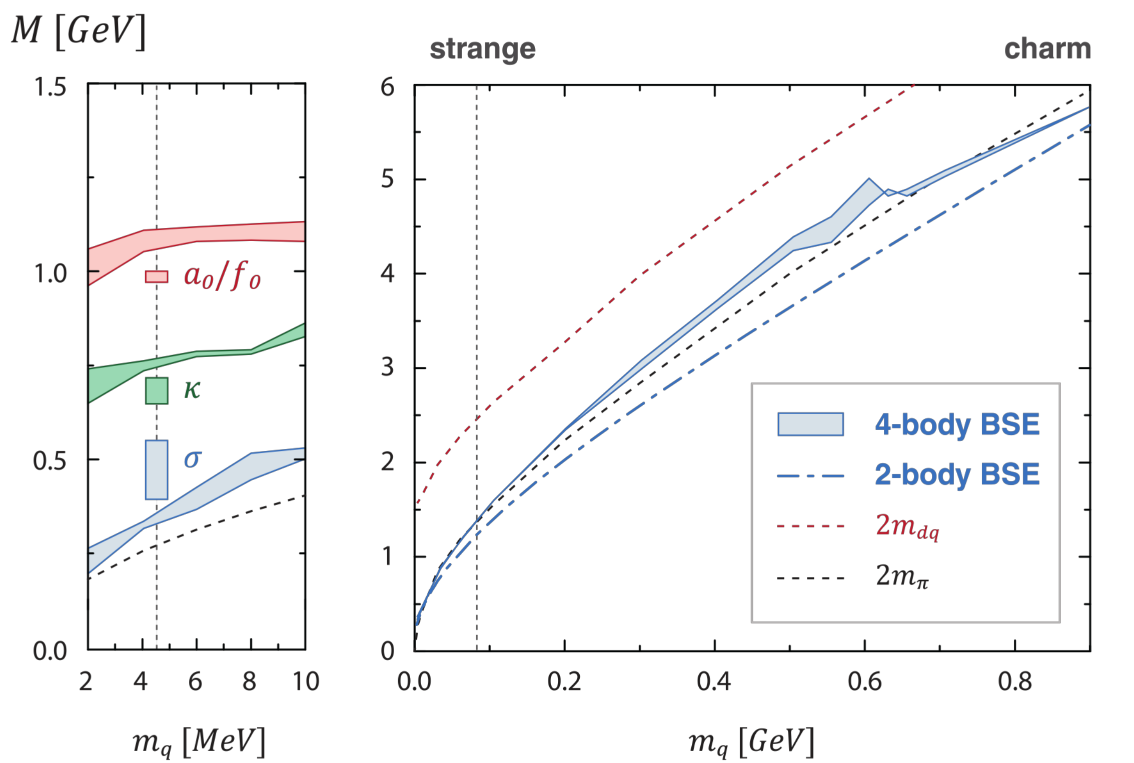
Without π clustering

Bound-state of four massive quarks

- **Identify** with the sigma meson: $f_0(500)$

$$M_\sigma = 348(13)\ \text{MeV}$$
$$M_\kappa = 750(12)\ \text{MeV}$$
$$M_{a_0/f_0} = 1081(28)\ \text{MeV}$$

- Meson cluster not diquark is a **dynamical decision**



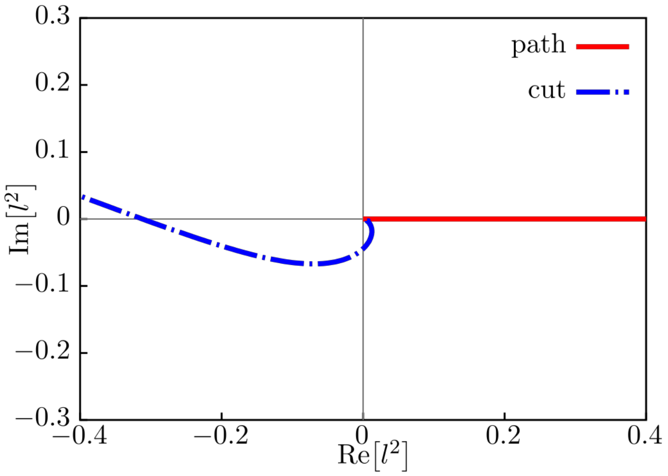
[Eichmann, Fischer, Heupel, PLB 753 (2016)]

Resonances

Intermediate bound-state poles enter integration region and sweep out **cuts** in integration plane

Intermediate bound-state poles enter integration region and sweep out **cuts** in integration plane

Without clustering/below threshold



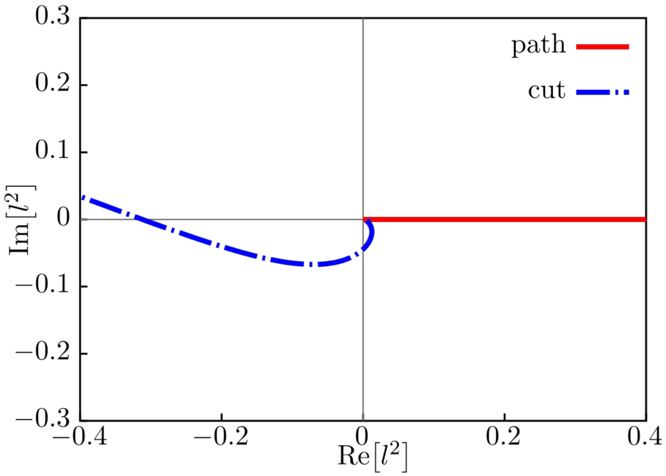
No obstruction

- Integrate along spacelike axis without problem.

Resonances

Intermediate bound-state poles enter integration region and sweep out **cuts** in integration plane

Without clustering/below threshold



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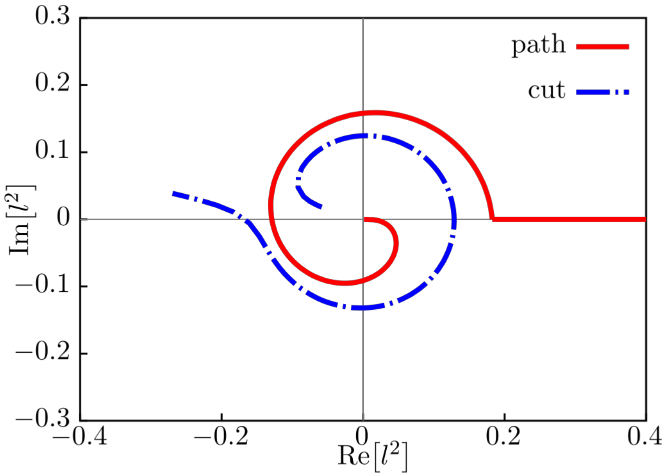
With clustering/above threshold

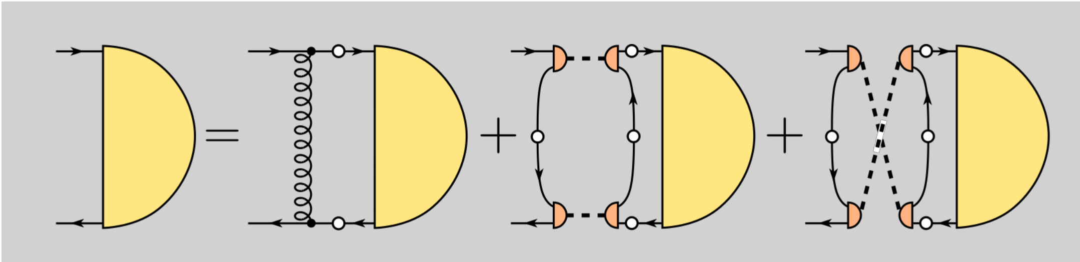


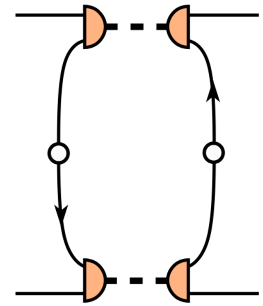
Path deformation required

- Avoid cut(s) during integration.

Resonances: meson example

Dynamical decay of rho meson

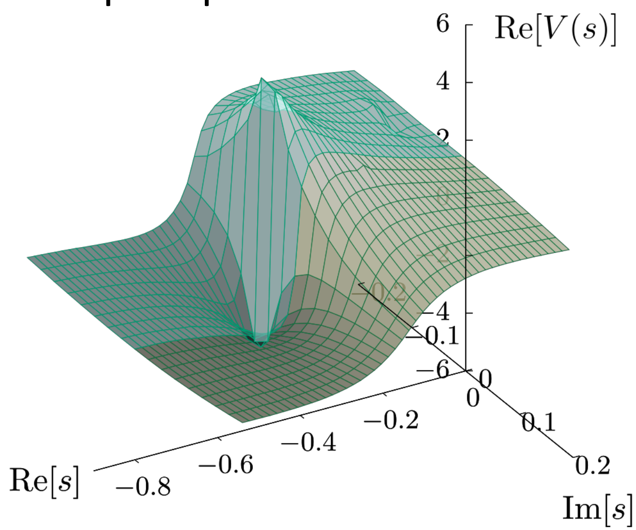


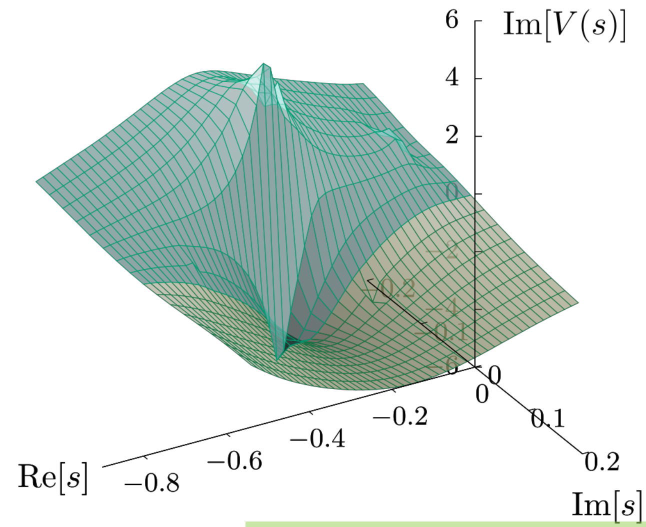


[RW, arXiv:1804.11161]

Intermediate pions

- Require path deformation

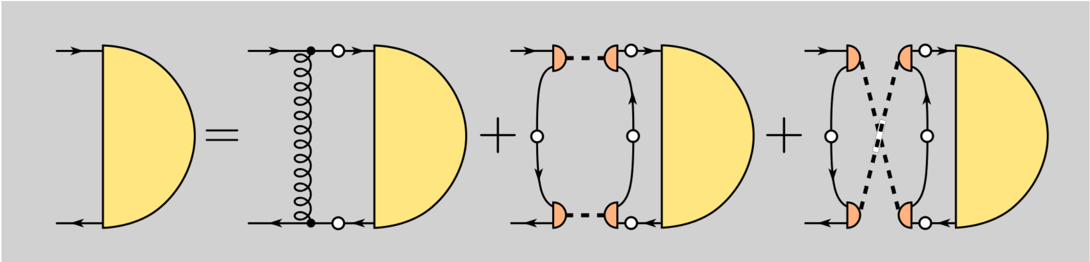


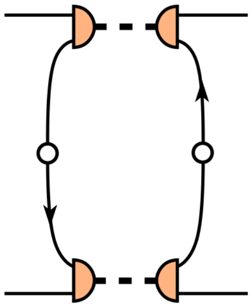


Solution yields both Mass and Width

Resonances: meson example

Dynamical decay of rho meson

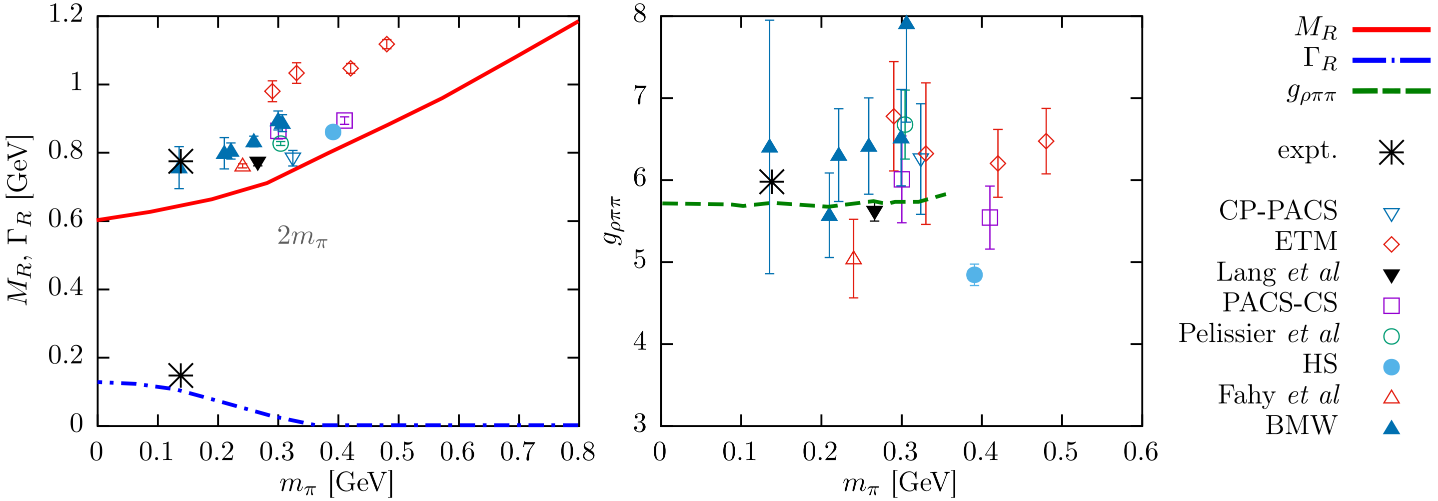




[RW, arXiv:1804.11161]

Intermediate pions

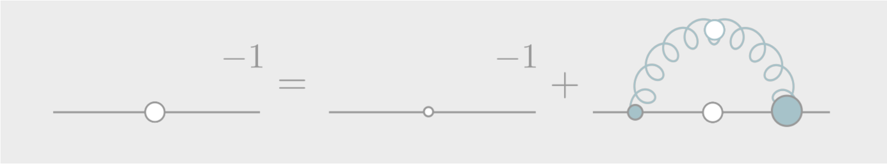
- Require path deformation



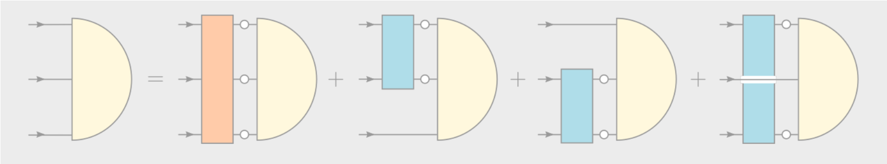
Solution yields both Mass and Width

Contents

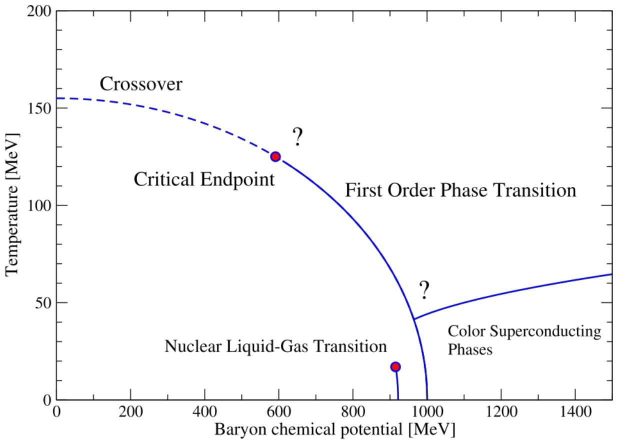
Dynamical Mass Generation



Hadronic Bound-states

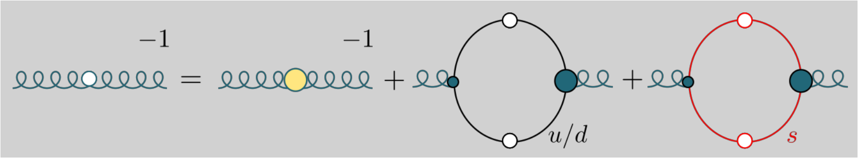


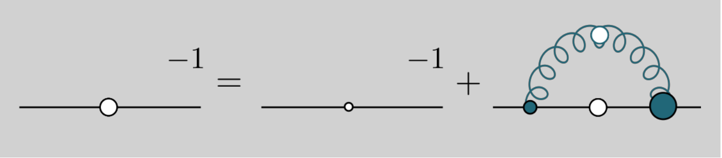
Phase transitions in QCD



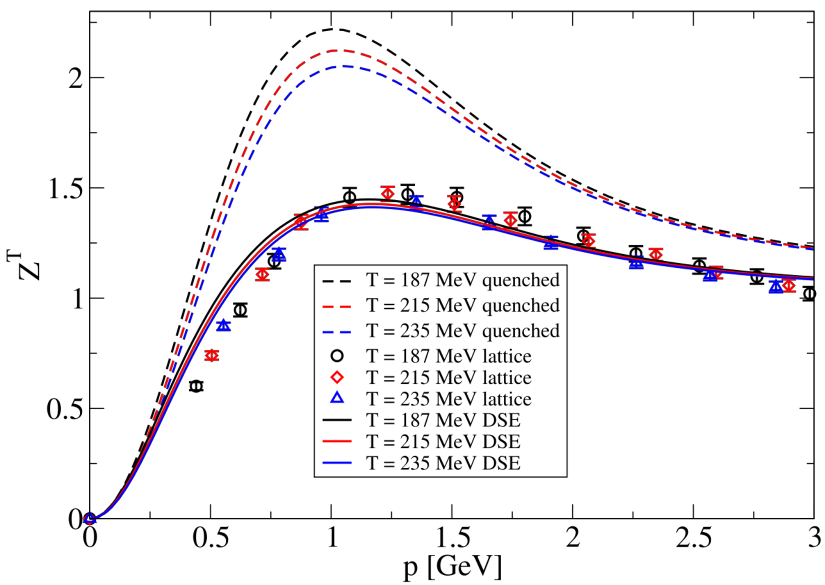
2+1 flavour QCD with DSEs

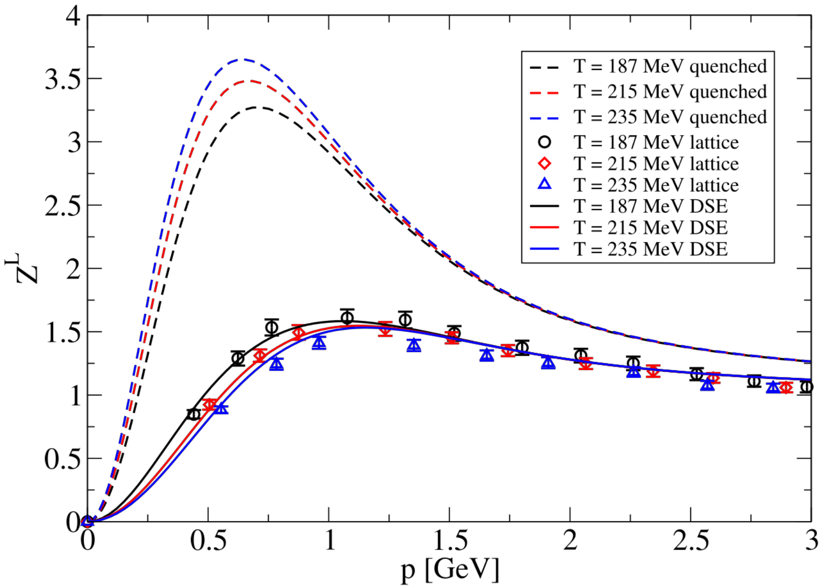
quenched propagator from lattice





Unquenching via DSEs plus STI constrained quark-gluon vertex Ansatz (BRL)





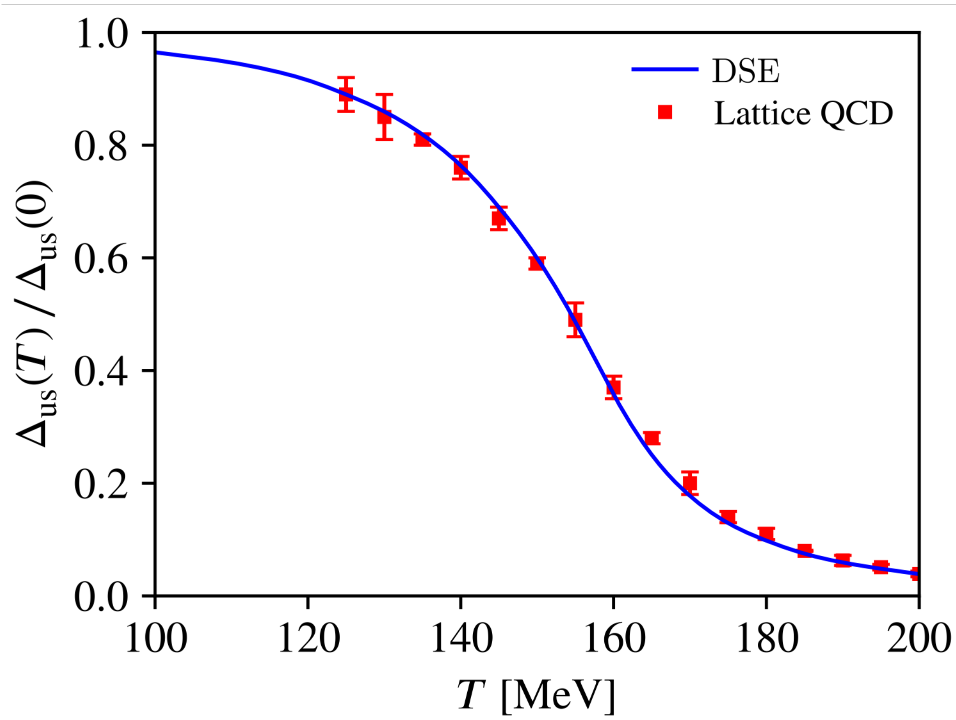
ZL: Quenched vs Unquenched:

- Inverted ordering with respect to temperature

[Aouane et al, PRD 87 (2013)]

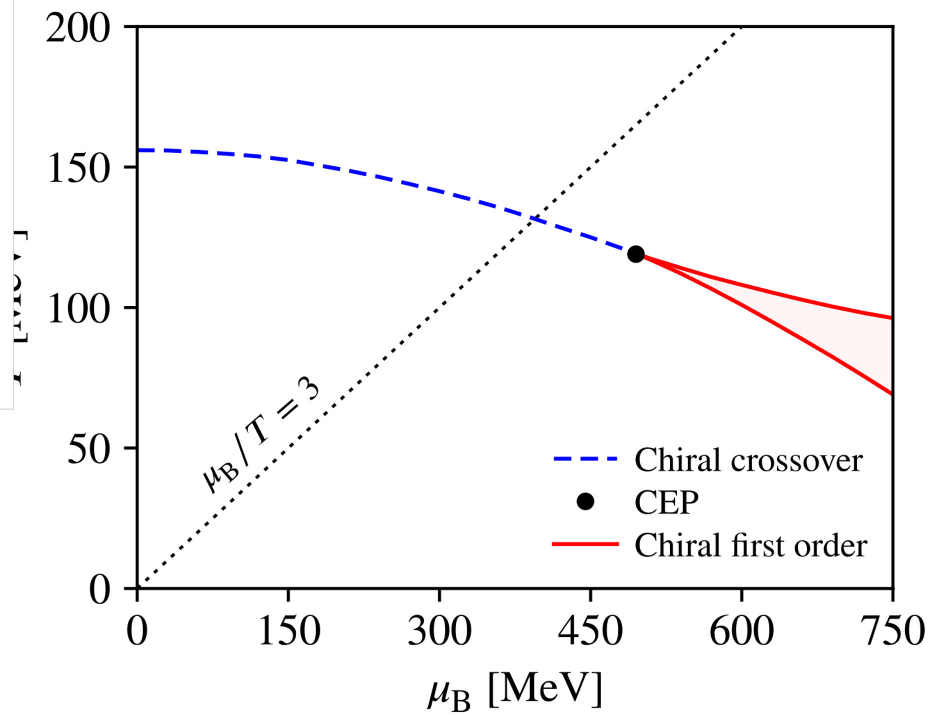
[Fischer, Luecker, PLB 718 (2013)]

Compare with lattice



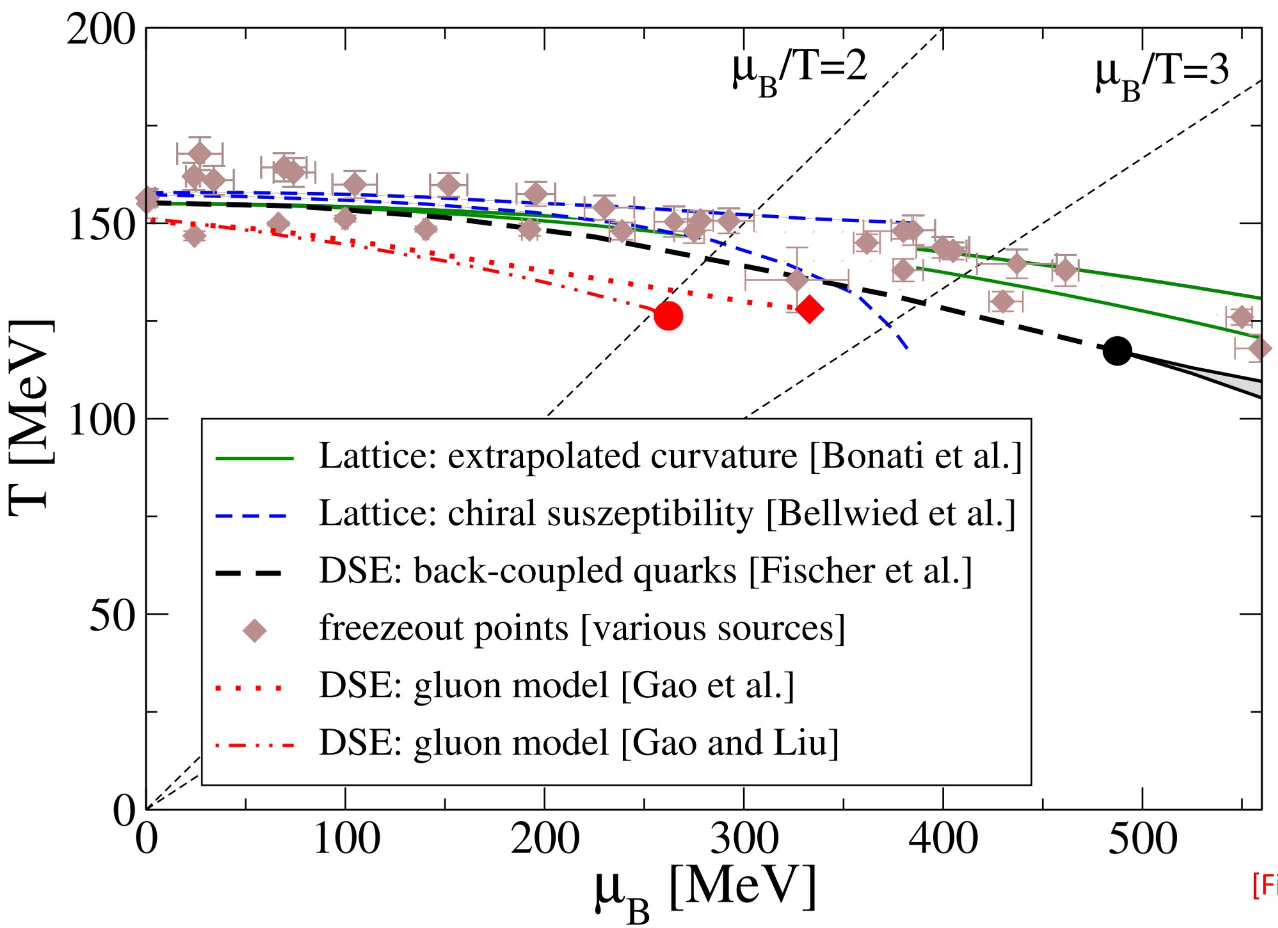
[Fischer, Luecker, PLB 718 (2013)]
[Fischer, Luecker, Welzbacher, PRD 90 (2014)]
[Borsanyi et al, JHEP 1009 (2010)]

- Chiral crossover
- 2nd order phase transition at CEP
- 1st order spinodals
  - Physically preferred phase boundary is that which minimizes the pressure



- T_c fitted in model parameters
- Slope non-trivial

QCD phase diagram



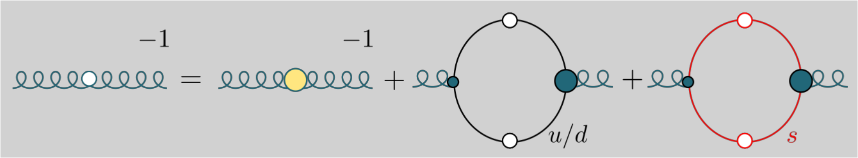
[Fischer, Luecker, PLB 718 (2013)]
[Fischer, Luecker, Welzbacher, PRD 90 (2014)]
[Borsanyi et al, JHEP 1009 (2010)]

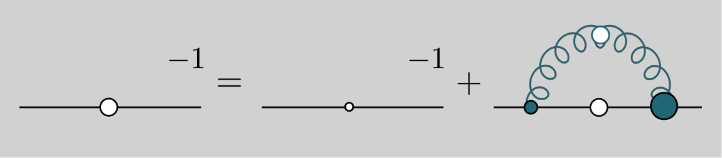
- **Lattice/QCD**: no CEP below $\mu_B/T \simeq 2 \text{ to } 3$
- **CEP**: $(T, \mu_B) = (117, 488)$ MeV

Input from the Yang-Mills sector is crucial

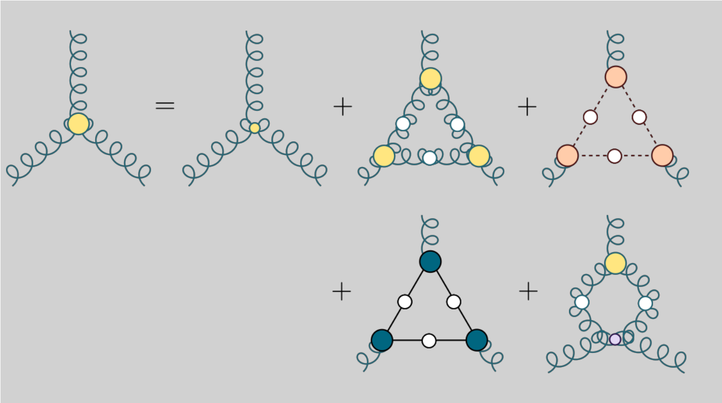
2+1 flavour QCD with DSEs

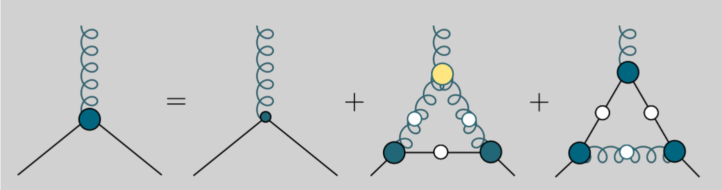
quenched propagator from lattice

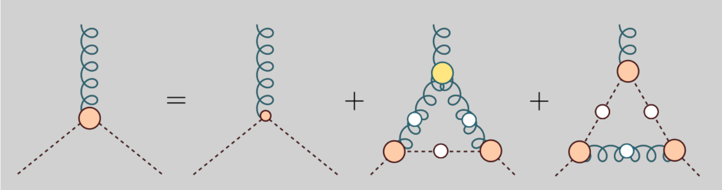




Unquenching via 3PI



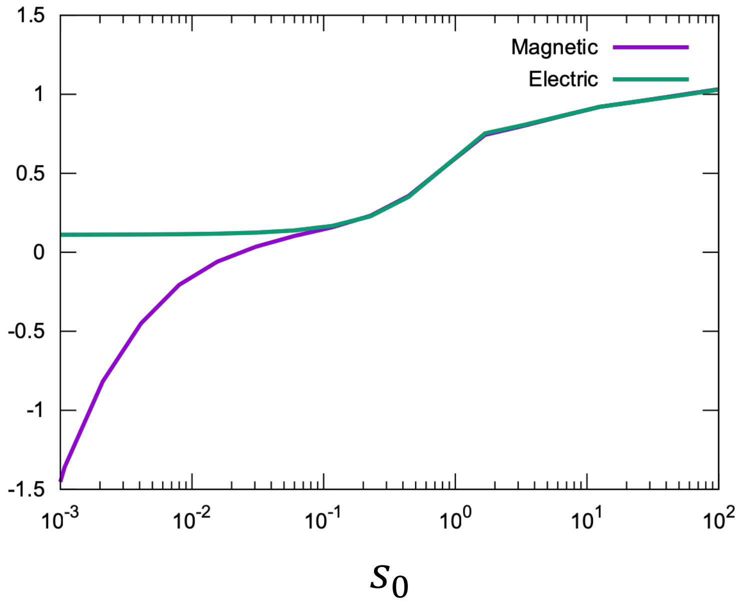


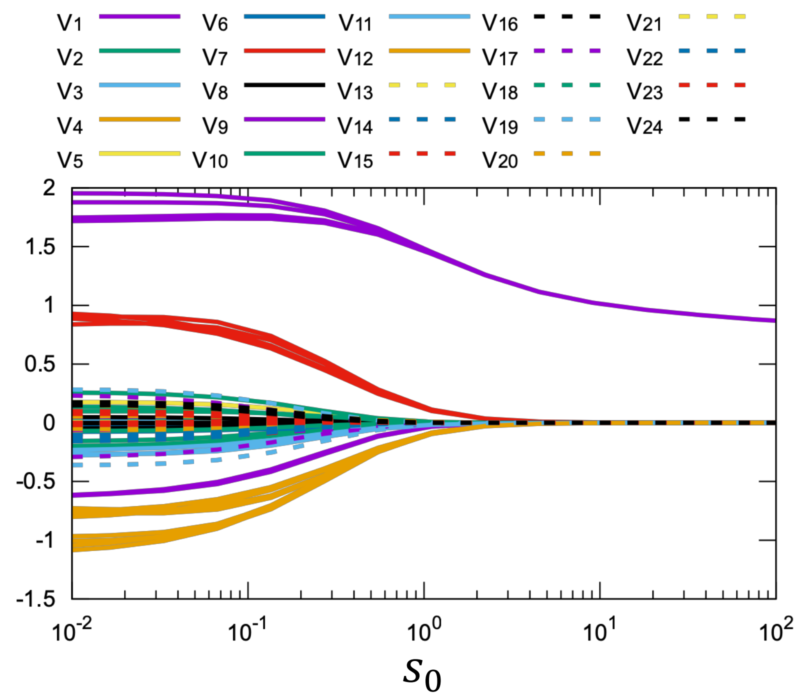


Depends explicitly on: T, μ, m

Roadmap beyond models

Vertices have been calculated (here, T=100 MeV)





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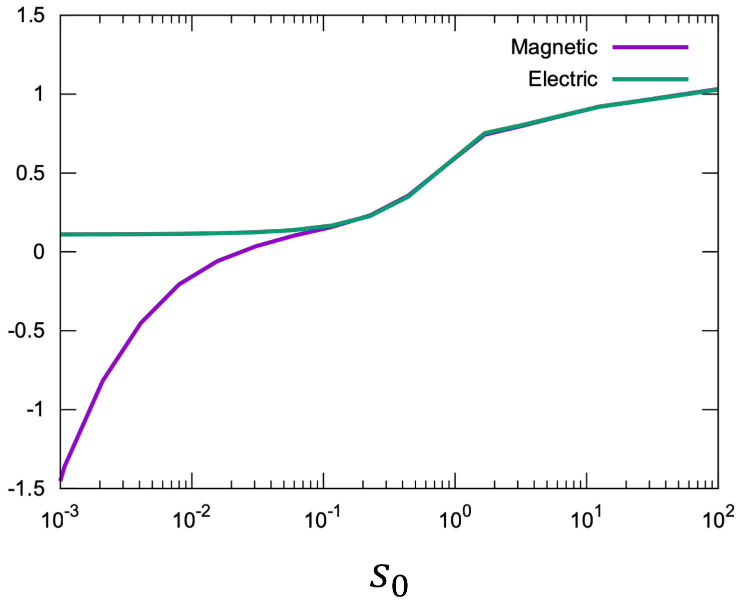

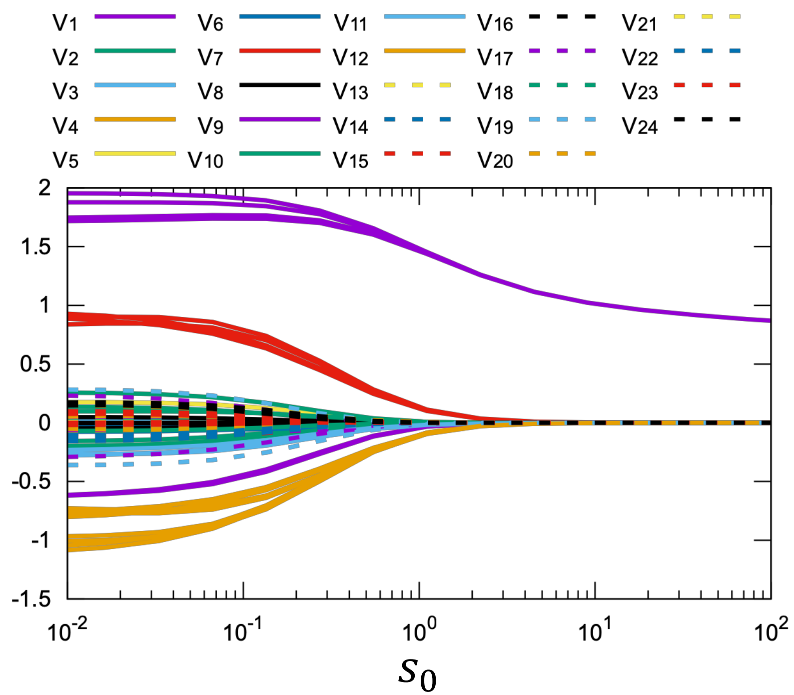

Very much **work in progress**

- Ready to unify DSEs/BSEs with a single truncation (3PI) in both vacuum and in medium

Summary

Main Goals

- one framework
  - Mesons, Baryons, Tetraquarks, Hybrids, Glueballs
  - Electromagnetic form factors, Anomalous magnetic moments
- DCSB, confinement

Main Challenges

- Control of systematic errors:
  - Intrinsic and by comparison to other methods

Main Results

- nPI is a powerful tool *(see Carrington)*
- Not high precision, but quantitative and -- where not -- qualitative.
- Competitive contributions in many wide areas *(see Roberts)*

Review

Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer 1606.9602 PPNP. 91 (2017) 1-100

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Thank you

Giessen

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