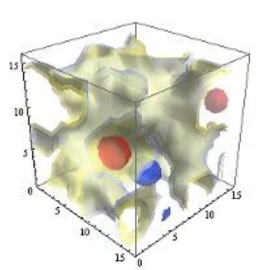
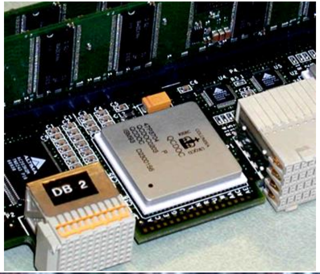
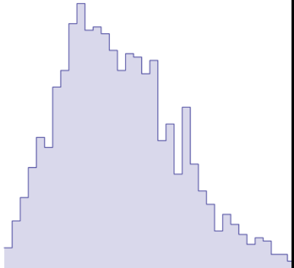
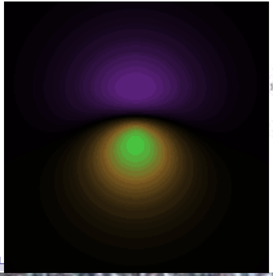
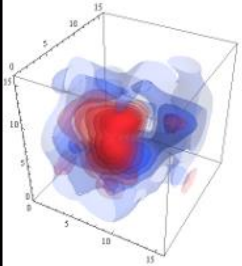
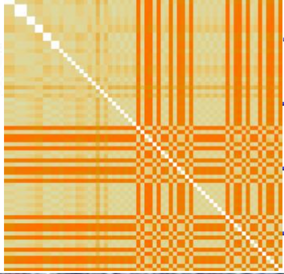
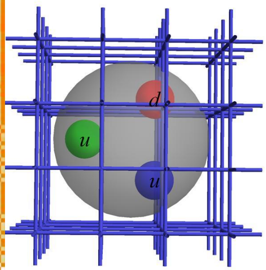
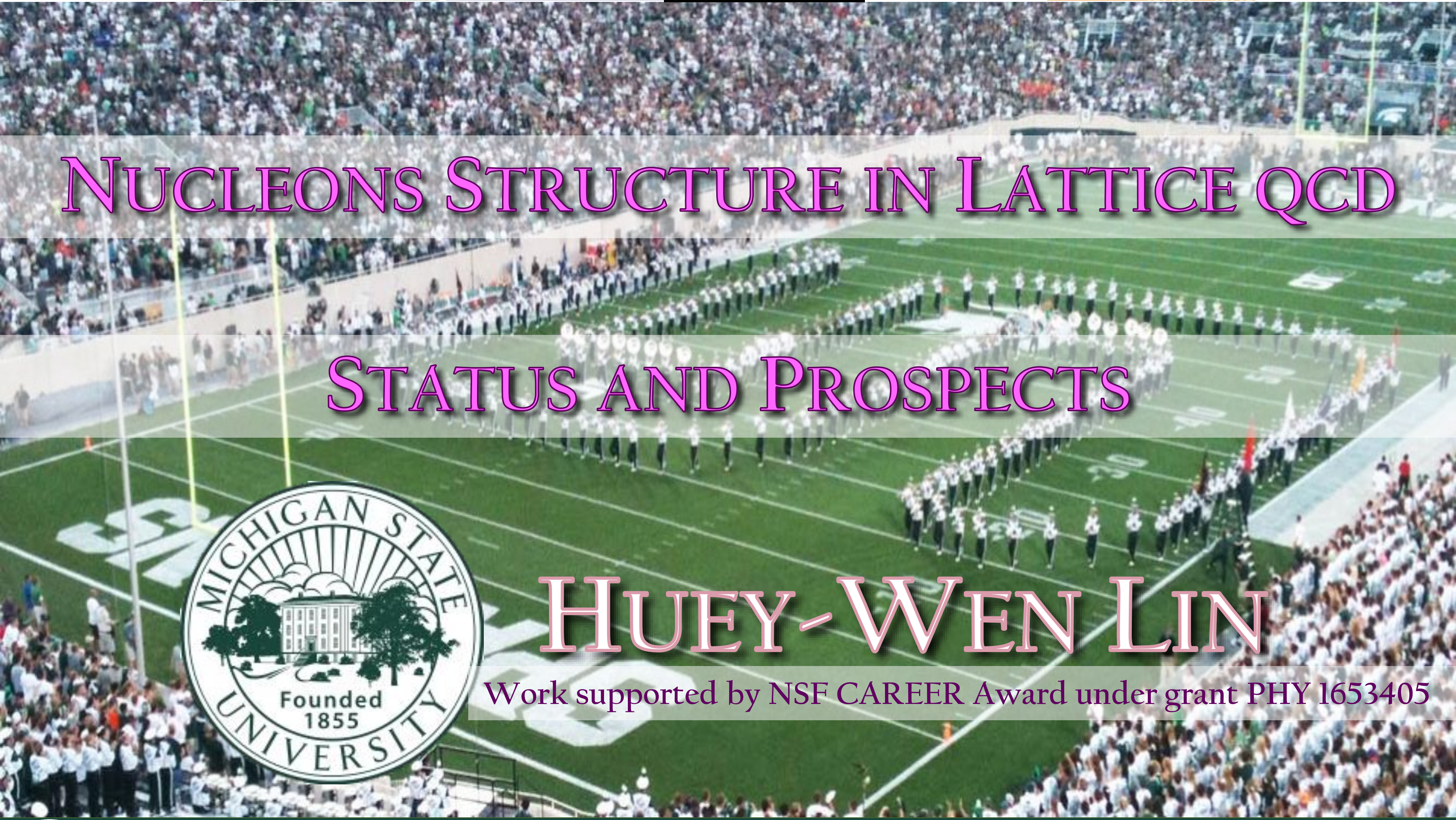





Nucleons Structure in Lattice QCD

Status and Prospects

Huey-Wen Lin

Work supported by NSF CAREER Award under grant PHY 1653405

Outline

§ Consumer's Guide to Lattice Nucleon Calculations

- Nucleon structure with controlled systematics in the physical limit ($m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty$)

§ Origin of Proton Spin

§ Spotlight on Selected Nucleon Structure Results

- As time allows…

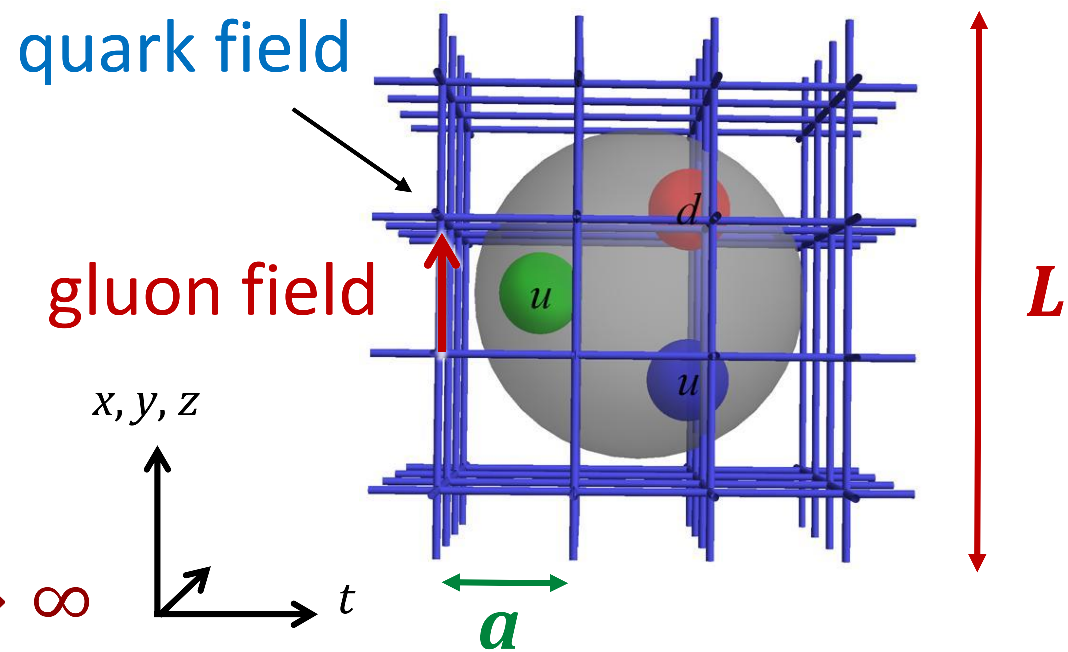
What is Lattice QCD?

- Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{iS(\bar{\psi},\psi,A)}\,O\left(\bar{\psi},\psi,A\right)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

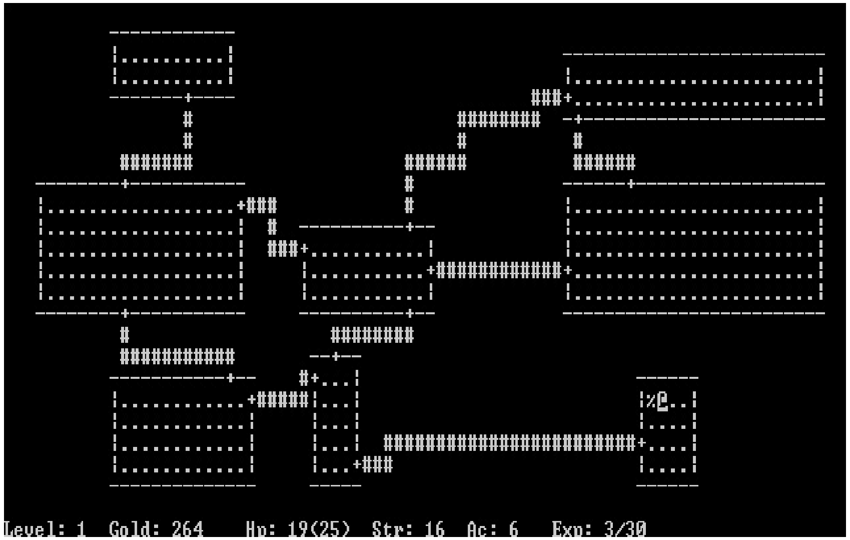
Are We There Yet?

§ Lattice gauge theory was proposed in the 1970s by Wilson

- Why haven't we solved QCD yet?

§ Progress is limited by computational resources

1980s



Today

§ Greatly assisted by advances in algorithms

- Physical pion-mass ensembles are not uncommon!

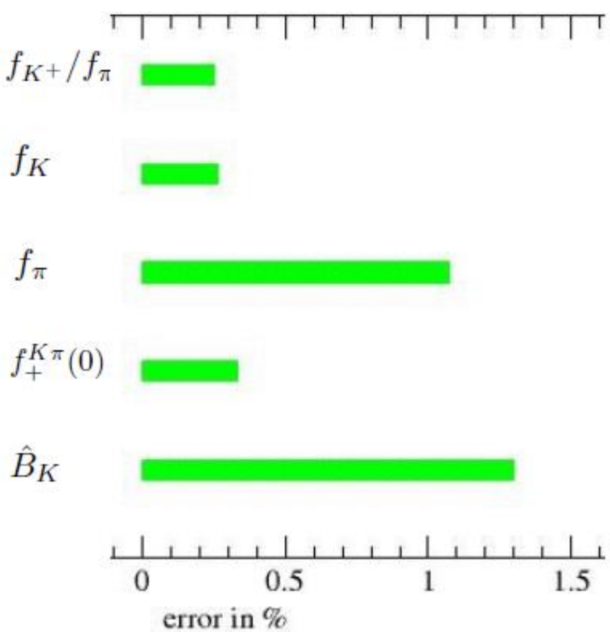
Successful Examples

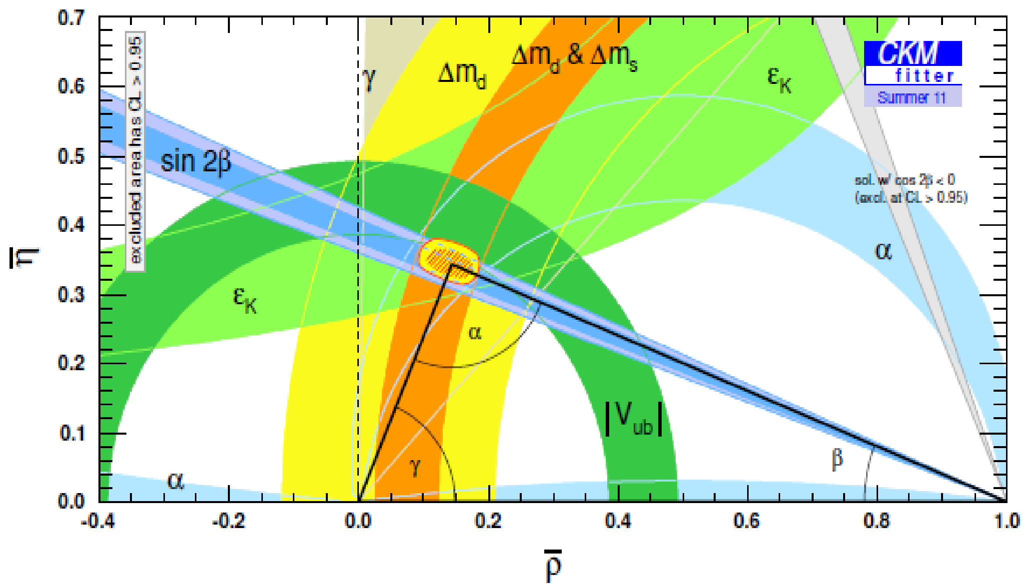
§ Lattice flavor physics provides precise inputs from the SM

A. El-Khadra, Sep. 2015, INT workshop "QCD for New Physics at the Precision Frontier"

- Very precise results in many meson systems

errors (in %) (preliminary) FLAG-3 averages





§ We are beginning to do precision calculations in nucleons

The Trouble with Nucleons

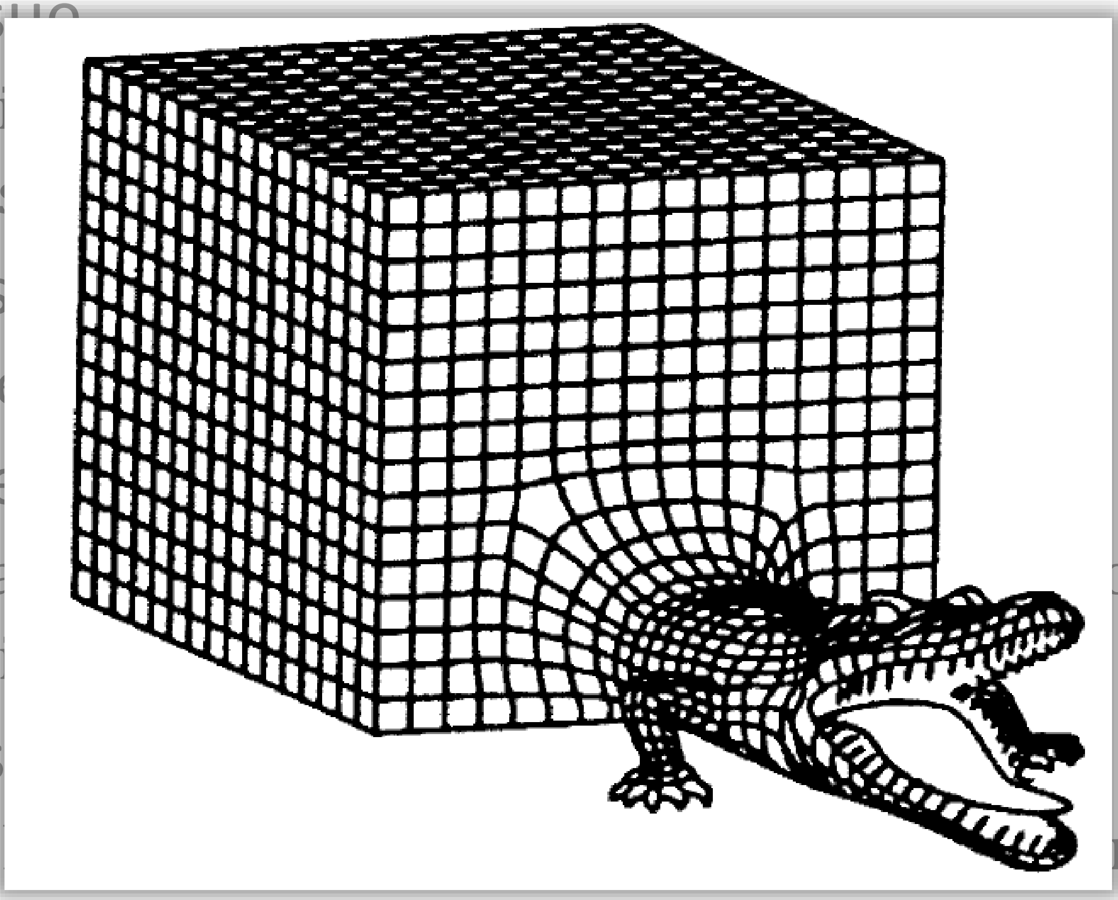
Nucleons are more complicated than mesons because…

- Noise issue
  - Signal diminishes at large t_E relative to noise
  - Get worse when quark mass decreases
- Excited-state contamination
  - Nearby excited state: Roper(1440)
- Hard to extrapolate in pion mass
  - Δ resonance nearby; multiple expansions, poor convergence…
  - Less an issue in the physical pion-mass era
- Requires larger volume and higher statistics
  - Ensembles are not always generated with nucleons in mind
  - **High-statistics**: large measurement and long trajectory

The Trouble with Nucleons

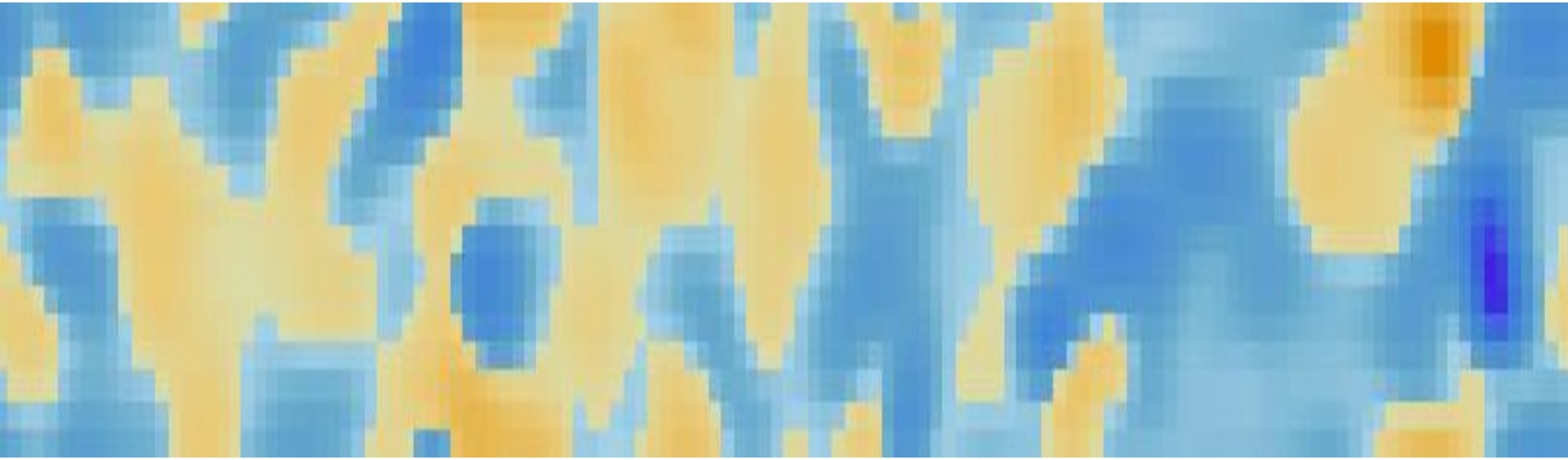
Nucleons are more complicated than mesons because...

- § Noise iss
  - Signal di
  - Get wor
- § Excited-s
  - Nearby
- § Hard to e
  - Δ resona onvergence...
  - Less an
- § Requires
  - Ensemb ns in mind
  - **High-s**tatistics measurement and long trajectory



PROCEED WITH CAUTION

Nucleon Matrix Elements

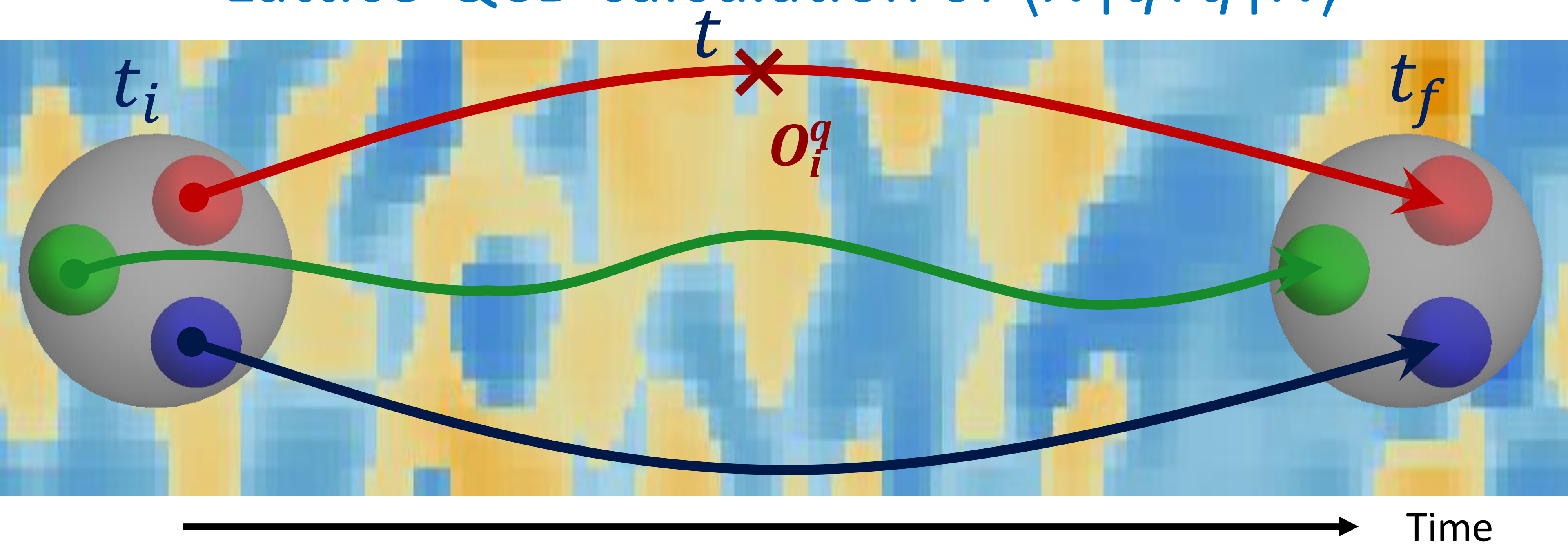


§ Pick a QCD vacuum

- Gauge/fermion actions, flavour (2, 2+1, 2+1+1), m_π, a, L, …

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

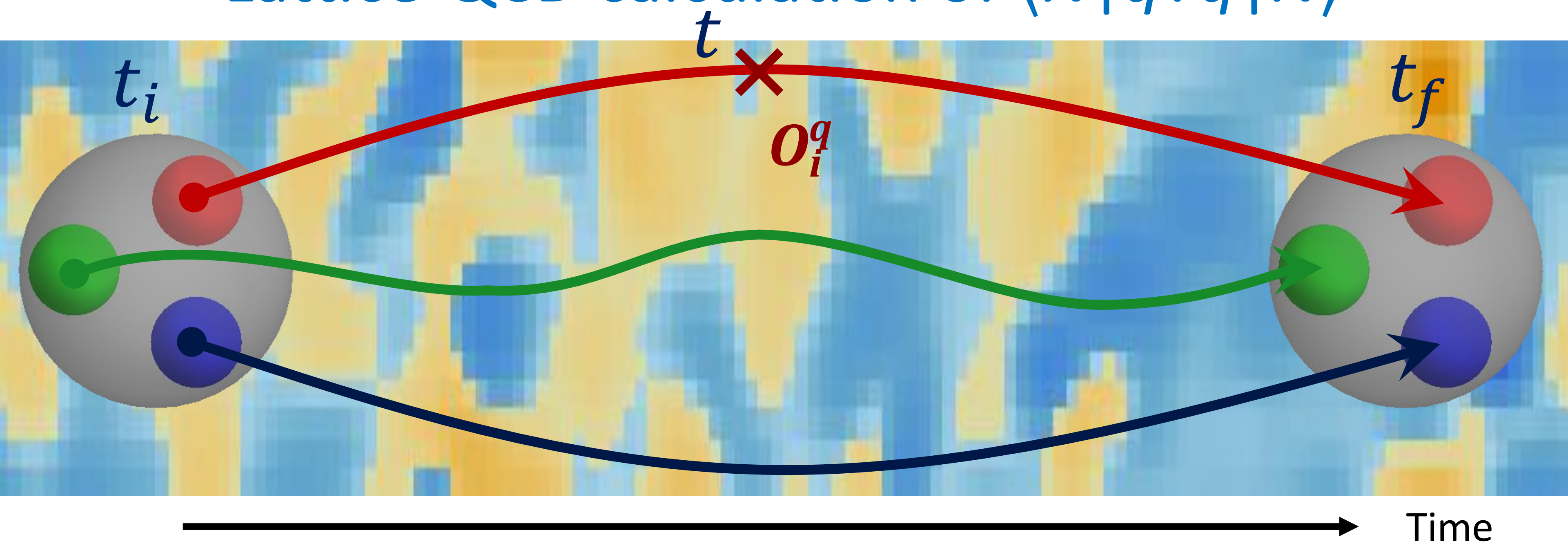


§ Construct correlators (hadronic observables)

- Requires "quark propagator"
  Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



§ Analysis (extract couplings)

$$C^{3\mathrm{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

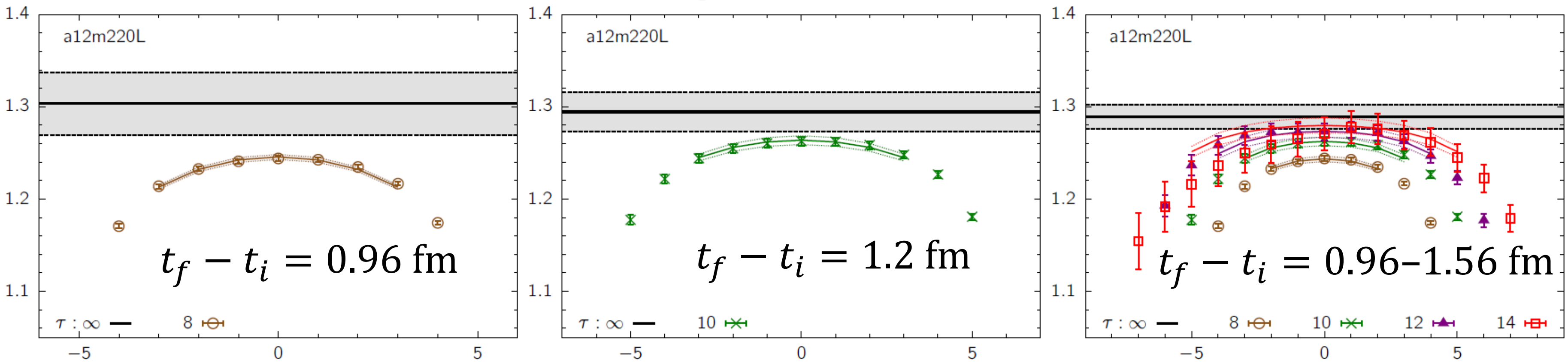
$$C^{2\mathrm{pt}}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)}+\ldots$$

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

§ More careful analysis in removing excited-state systematics

- Move the **excited-state systematic** into the statistical error



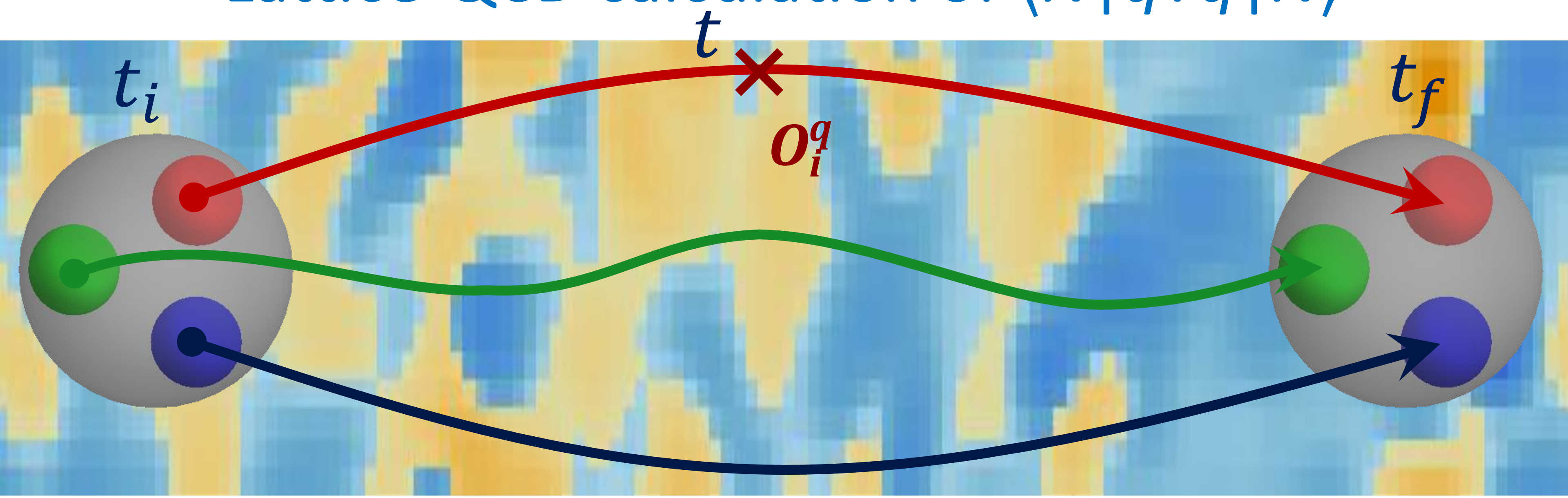
§ Analysis (extract couplings)

$$C^{\rm 3pt}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

$$C^{\rm 2pt}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)} + \ldots$$

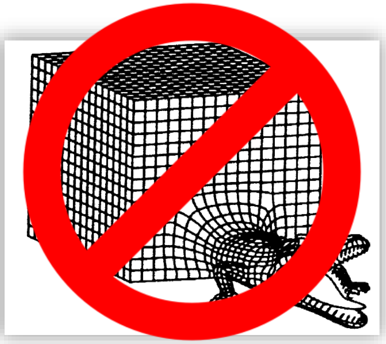
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



§ Systematic uncertainty (nonzero a, finite L, etc.)

- Nonperturbative renormalization
  e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



Precision Nucleon Couplings

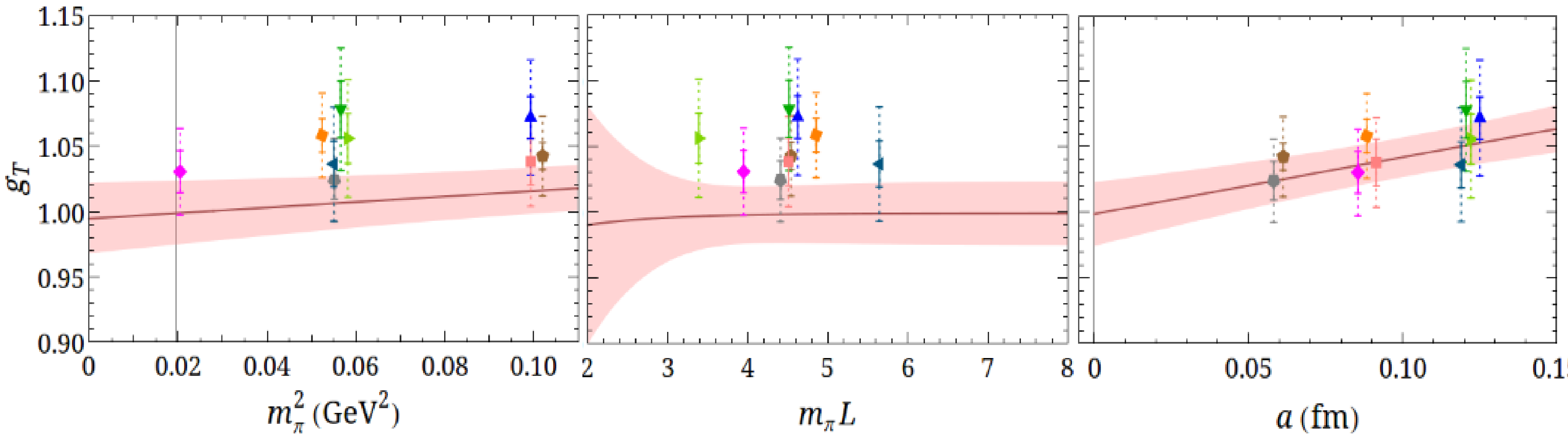
§ g_T: zeroth moment of transversity $\Gamma = \sigma_{\mu\nu}$

$$g_T = \int_{-1}^{1} dx\, \delta q(x)$$

§ A state-of-the art calculation (PNDME)

- Extrapolate to the physical limit

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

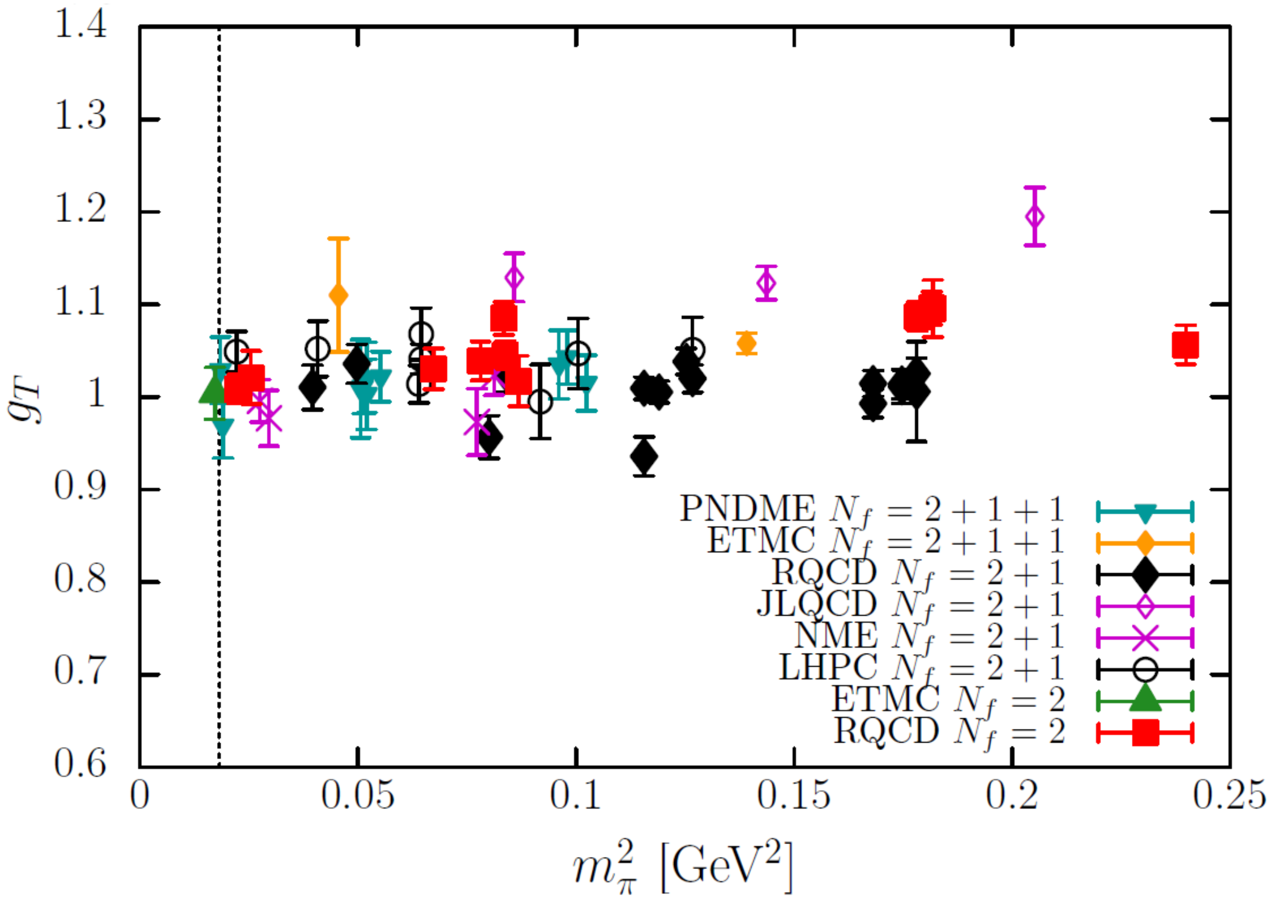


First extrapolation to the physical limit
of a nucleon matrix element!

Precision Nucleon Couplings

§ Usually more than one LQCD calculation

- For example, tensor charge
- Lattice results should agree in the continuum limit



Plot by Sara Collins @ Lattice 2016

Precision Nucleon Couplings

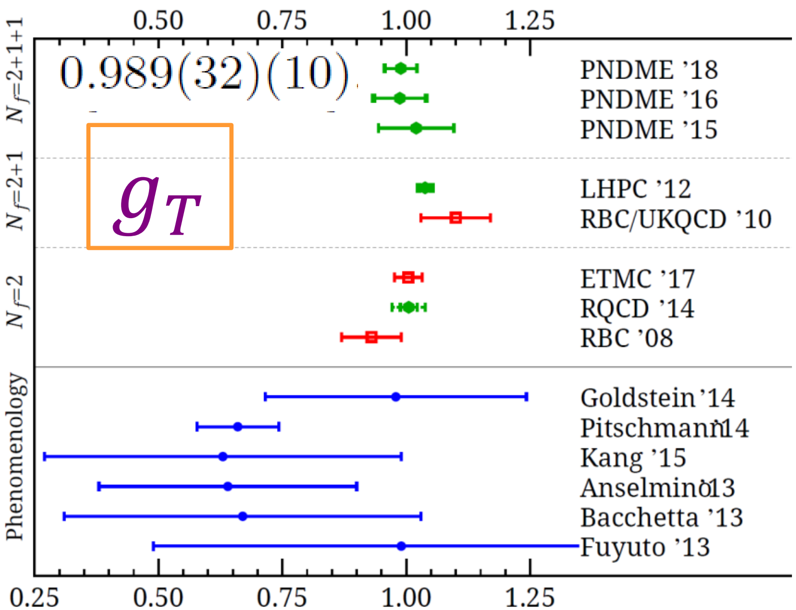
FLAG rating system

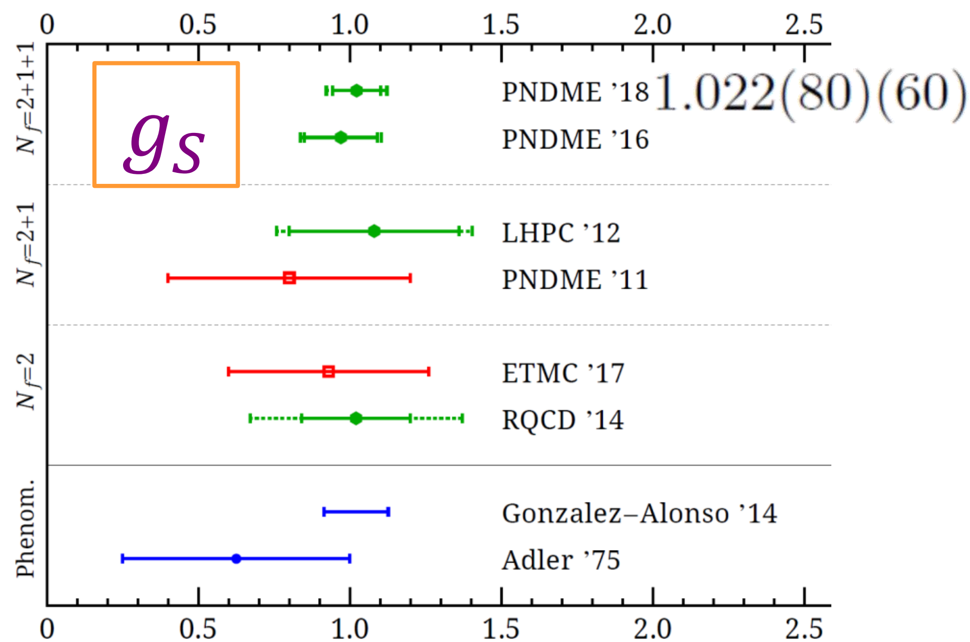
PNDME, 1506.06411; 1606.07049

New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]

PNDME, 1806.09006



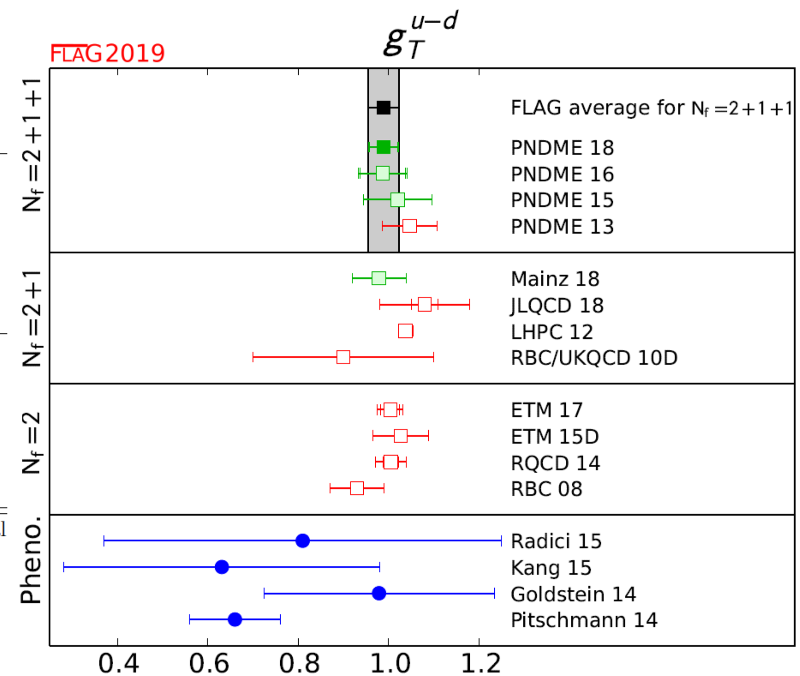


FLAG 2019

§ Finally adopted by FLAG! https://arxiv.org/pdf/1902.08191.pdf

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	g_T^{u-d}
PNDME 18	[84]	2+1+1	A	★‡	★	★	★	★	0.989(32)(10)
PNDME 16	[830]	2+1+1	A	○‡	★	★	★	★	0.987(51)(20)
PNDME 15	[828, 829]	2+1+1	A	○‡	★	★	★	★	1.020(76)
PNDME 13	[827]	2+1+1	A	■‡	■	★	★	★	1.047(61)
Mainz 18	[915]	2+1	C	★	○	★	★	★	0.979(60)
JLQCD 18	[839]	2+1	A	■	○	○	★	★	1.08(3)(3)(9)
LHPC 12	[920]	2+1	A	■‡	★	★	★	★	1.038(11)(12)
RBC/UKQCD 10D	[834]	2+1	A	■	■	○	★	■	0.9(2)
ETM 17	[826]	2	A	■	○	○	★	★	1.004(21)(2)(19)
ETM 15D	[822]	2	A	■	○	○	★	★	1.027(62)
RQCD 14	[819]	2	A	○	★	★	★	■	1.005(17)(29)
RBC 08	[918]	2	A	■	■	■	★	■	0.93(6)

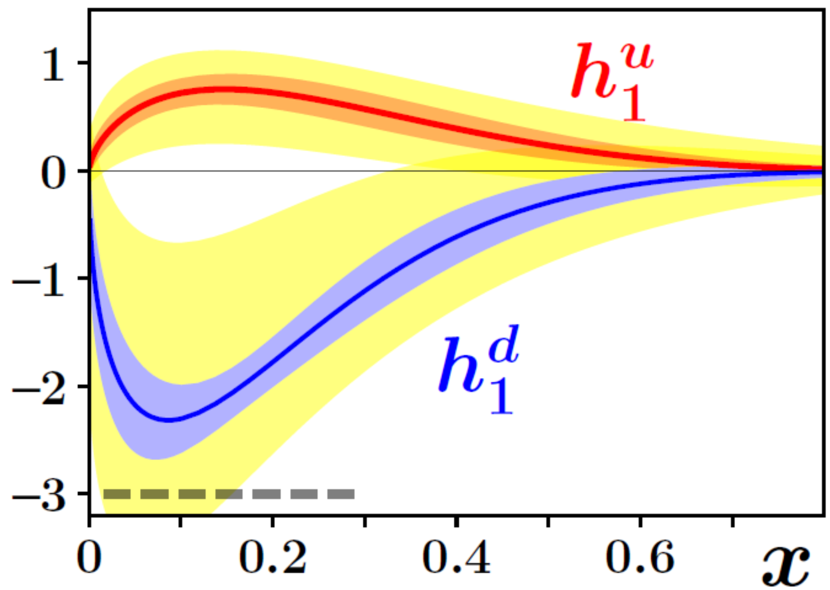
‡ The rating takes into account that the action is not fully O(a) improved by requiring an additional lattice spacing.

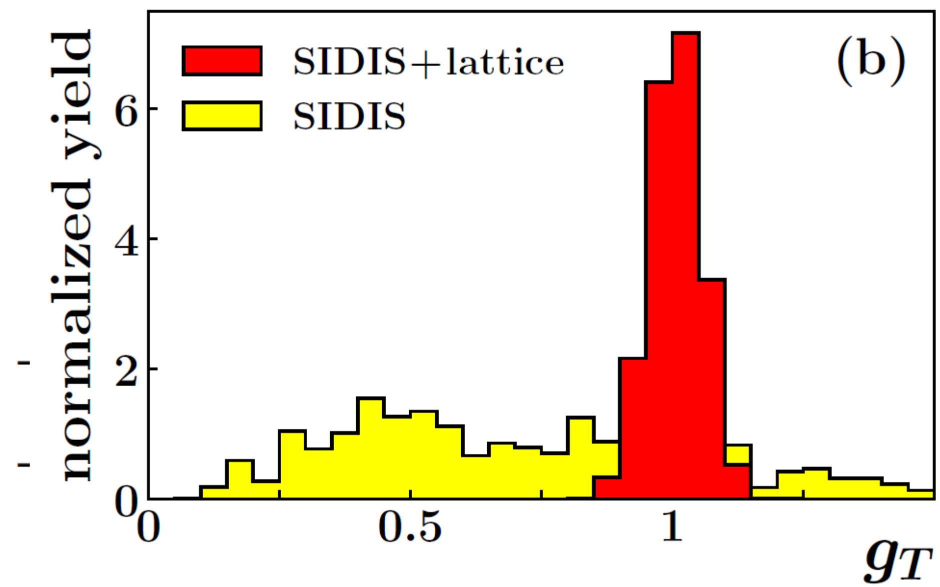


From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets





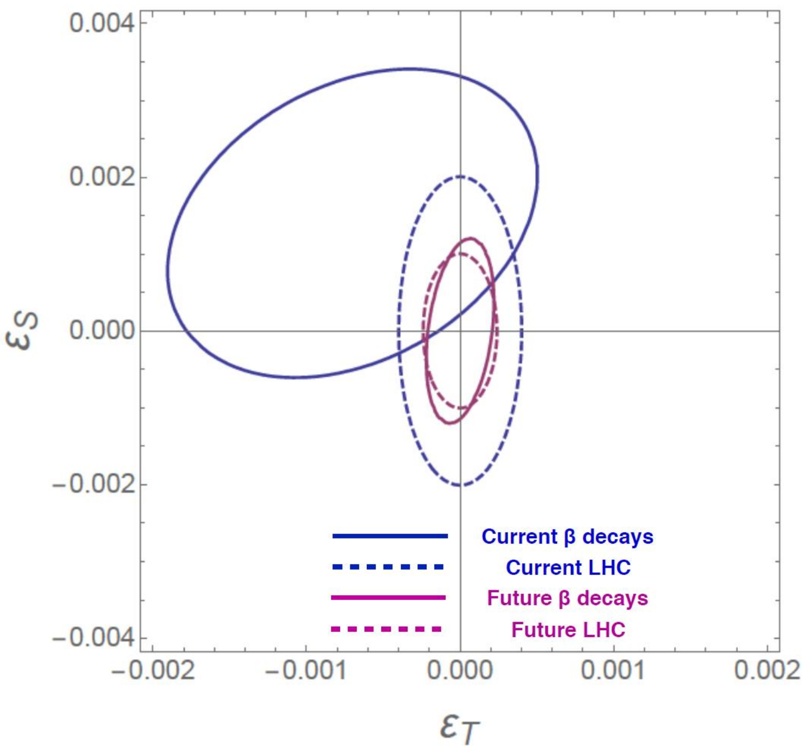
Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

BSM Applications

§ Given precision $g_{S,T}$ and O_{BSM}, predict new-physics scales

Low-Energy Expt → $\boxed{O_{\text{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})}$ ← Precision LQCD input ($m_\pi \to 140$ MeV, $a \to 0$)

$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$



Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp't

$|B_1 - b|_{\text{BSM}} < 10^{-3}$

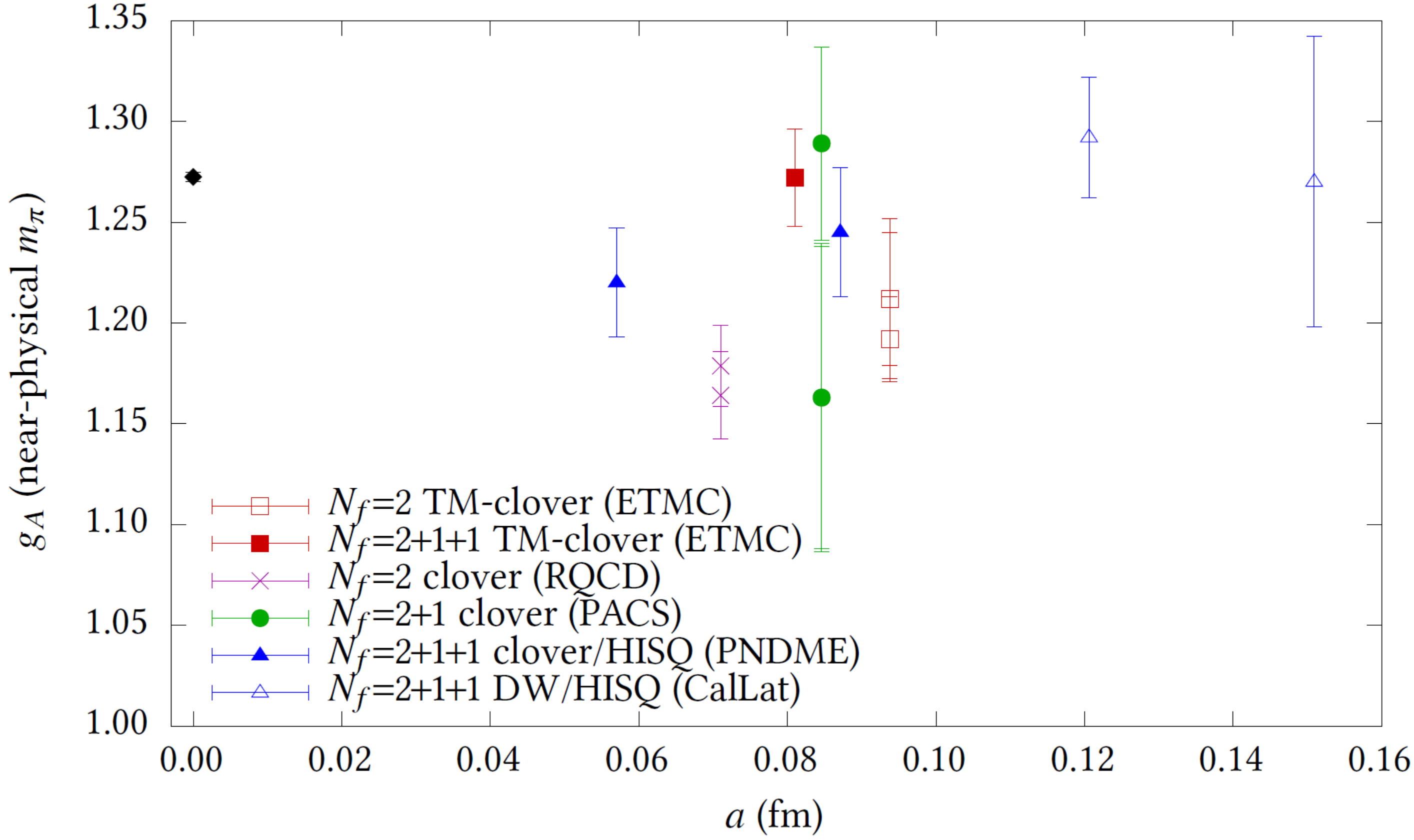
$|b|_{\text{BSM}} < 10^{-3}$

CENPA: $^6\text{He}(b_{\text{GT}})$ at 10^{-3}

PNDME, PRD85 054512 (2012); 1306.5435; 1606.07049; 1806.09006

Nucleon Axial Charge

§ Calculation near physical pion mass

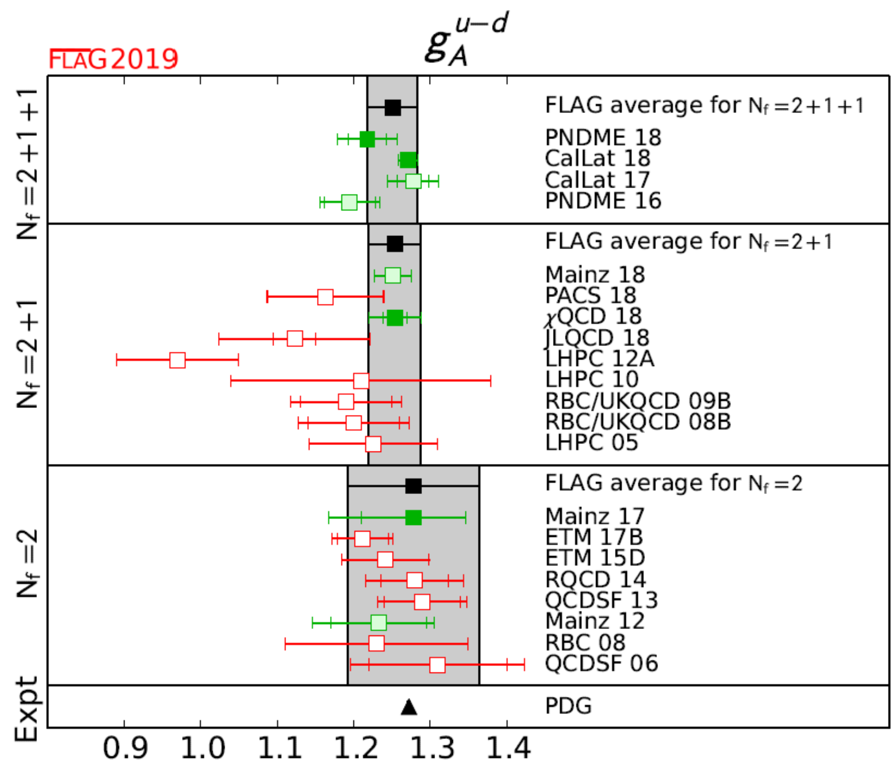


Plot by J. Green @Lattice 2018

Nucleon Axial Charge

§ Summary

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	excited states	g_A^{u-d}
PNDME 18[a]	[84]	2+1+1	A	★‡	★	★	★	★	1.218(25)(30
CalLat 18	[85]	2+1+1	A	○	★	★	★	★	1.271(10)(7)
CalLat 17	[831]	2+1+1	P	○	★	★	★	★	1.278(21)(26
PNDME 16[a]	[830]	2+1+1	A	○‡	★	★	★	★	1.195(33)(20
Mainz 18	[915]	2+1	C	★	○	★	★	★	1.251(24)
PACS 18	[808]	2+1	A	■	■	★	★	■	1.163(75)(14
χQCD 18	[6]	2+1	A	○	★	★	★	★	1.254(16)(30
JLQCD 18	[839]	2+1	A	■	○	○	★	★	1.123(28)(29
LHPC 12A[b]	[916]	2+1	A	■‡	★	★	★	★	0.97(8)
LHPC 10	[846]	2+1	A	■	○	■	★	■	1.21(17)
RBC/UKQCD 09B	[833]	2+1	A	■	■	○	★	■	1.19(6)(4)
RBC/UKQCD 08B	[832]	2+1	A	■	■	○	★	■	1.20(6)(4)
LHPC 05	[917]	2+1	A	■	■	★	★	■	1.226(84)
Mainz 17	[86]	2	A	★	★	★	★	○	1.278(68)(
ETM 17B	[824]	2	A	■	○	○	★	★	1.212(33)(22
ETM 15D	[822]	2	A	■	○	○	★	★	1.242(57)
RQCD 14	[819]	2	A	○	★	★	★	■	1.280(44)(46)
QCDSF 13	[400]	2	A	○	★	■	★	■	1.29(5)(3)
Mainz 12	[818]	2	A	★	○	○	★	○	1.233(63)($^{+0.035}_{-0.060}$)
RBC 08	[918]	2	A	■	■	■	★	■	1.23(12)
QCDSF 06	[817]	2	A	○	■	■	★	■	1.31(9)(7)

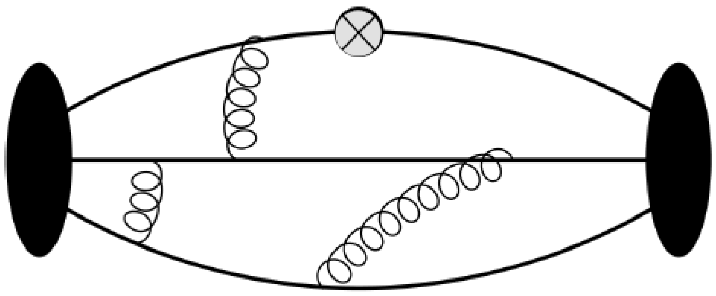


Origin of Proton Spin

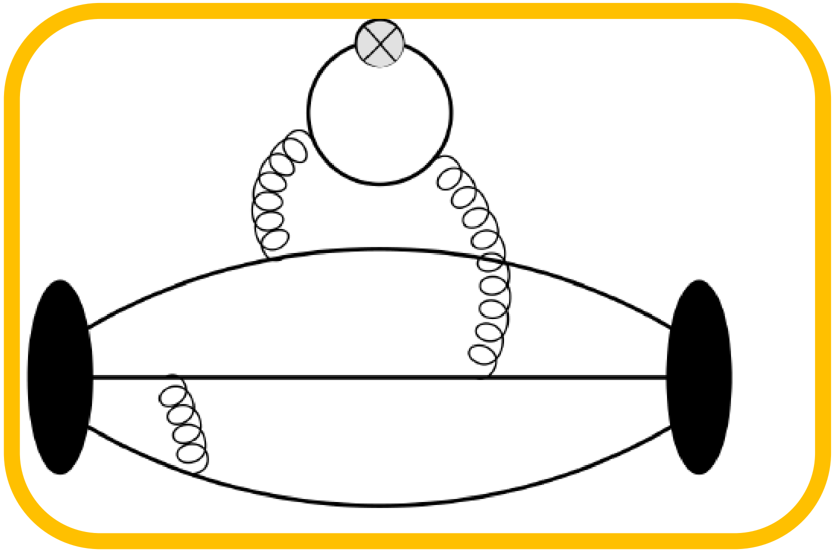
Disconnected Diagrams

§ Disconnected diagram

- Multiple ways to calculate this notorious contribution…
- Truncated solver, hopping-parameter expansion, hierarchical probing, …



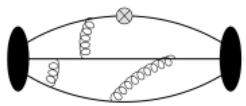
+



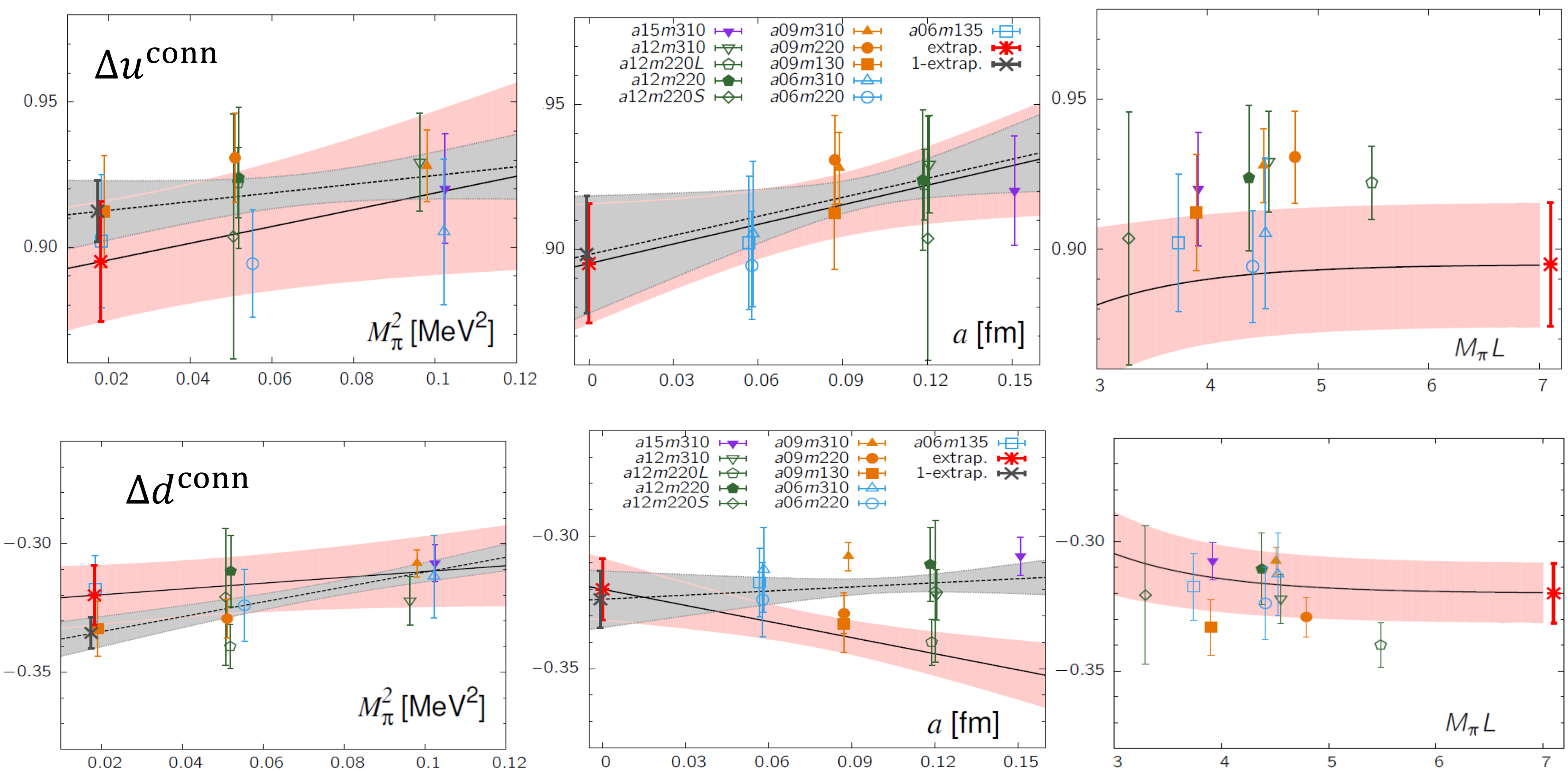
Quark Spin

§ Up and down quark "connected" contribution

PNDME, 1806.09006, 1806.10604

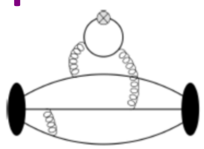


$$\Delta q(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



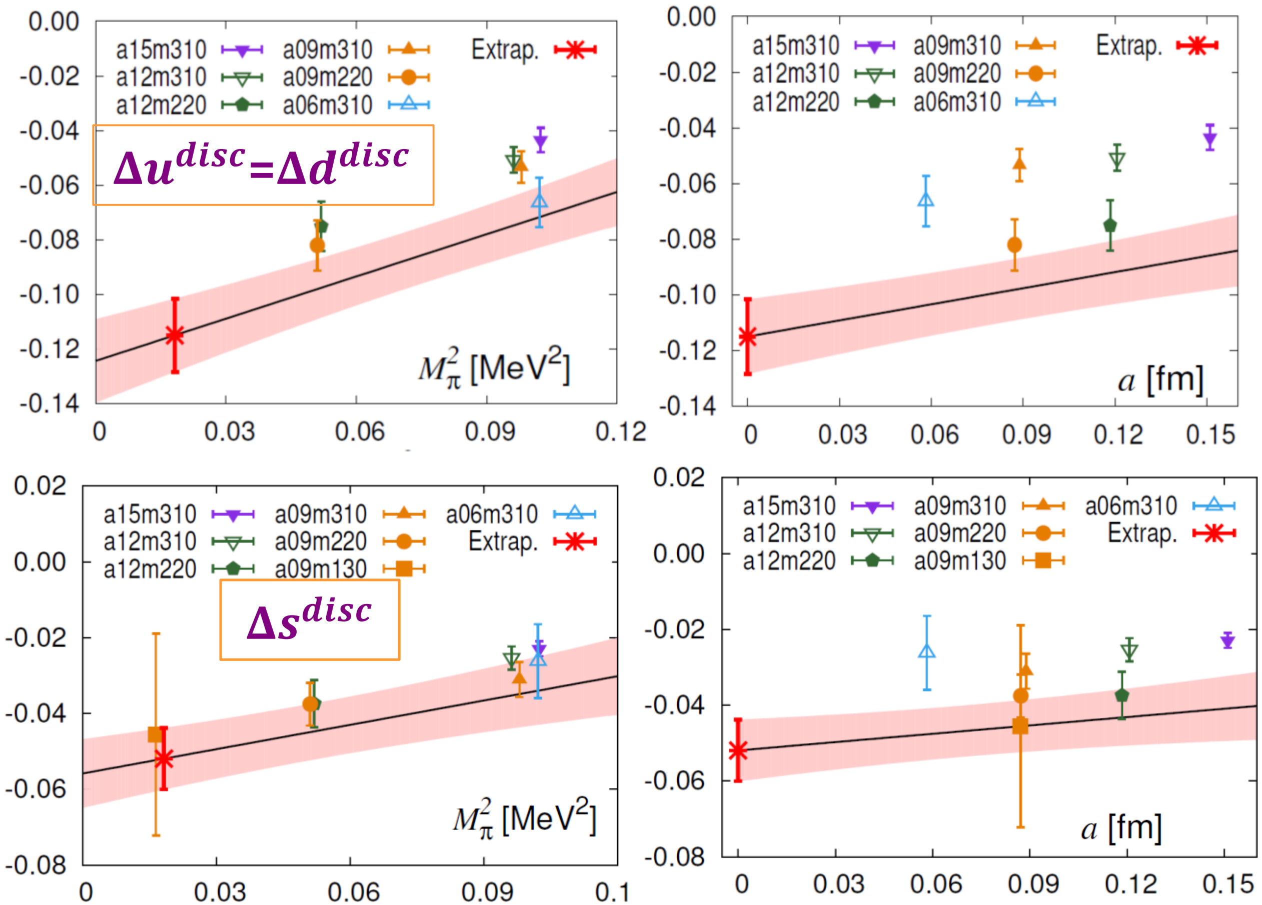
Quark Spin

§ Up and down quark "disconnected" contribution



PNDME, 1806.09006, 1806.10604

$$\Delta q^{\text{disc}} = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



Anticipated pion-mass dependence

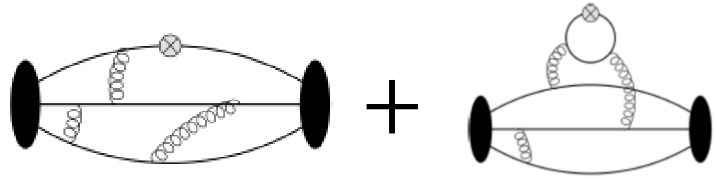
Unexpectedly strong lattice-spacing dependence!

Calculation at $a \approx$ 0.09 fm can have 50% change in Δu^{disc}

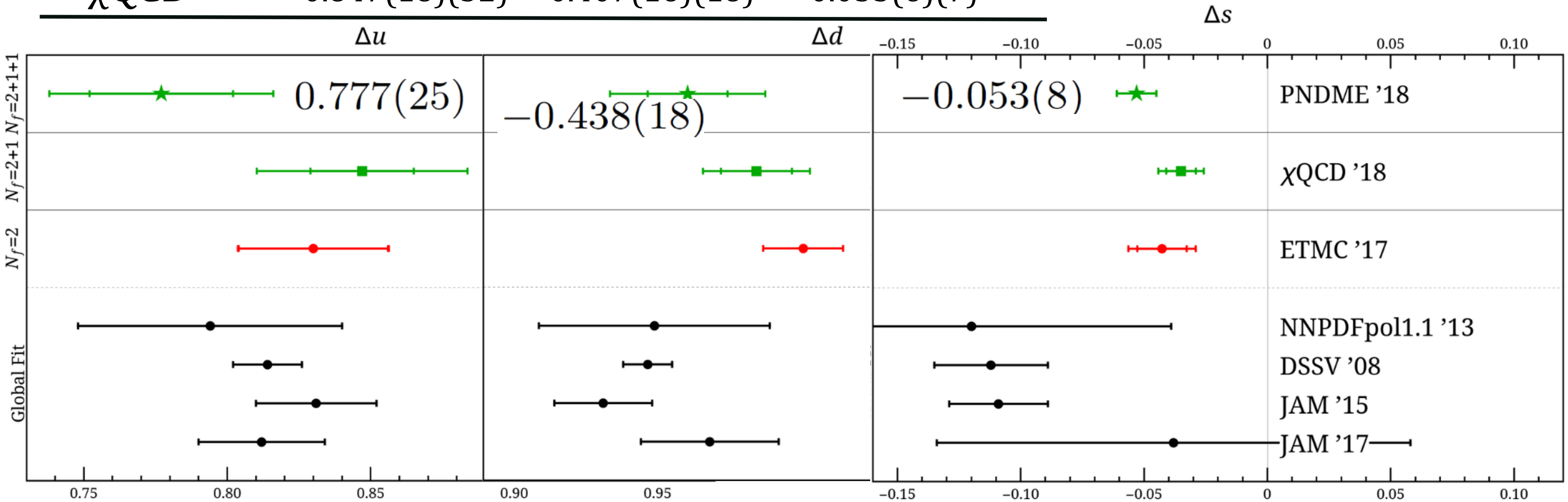
Quark Spin Contribution

§ Total quark contributions

PNDME	$g_A^u \equiv \Delta u$	$g_A^d \equiv \Delta d$	$g_A^s \equiv \Delta s$
Connected	0.895(21)	−0.320(12)	
Disconnected	−0.118(14)	−0.118(14)	−0.053(8)
Sum	0.777(25)	−0.438(18)	−0.053(8)
ETMC	0.830(26)	−0.386(18)	−0.042(10)(2)
χQCD	0.847(18)(32)	-0.407(16)(18)	-0.035(6)(7)



Difference caused by Δq^{disc}



$$\sum_{q=u,d,s}\left(\tfrac{1}{2}\Delta q\right) = 0.143(31)$$

PNDME, 1806.09006, 1806.10604

The work of HL is supported by NSF CAREER Award under grant PHY 1653405

Total Quark Intrinsic Spin

Not "equal": systematics are different?

- PNDME: 0.143(31)(36) (2+1+1 flavor clover-on-HISQ)
- ETMC: 0.201(17)(5) (2 flavor twisted mass)
- χQCD: 0.202(13)(19) (2+1 flavor overlap-on-Domain Wall)

	g_A^{u-d}	$a \to 0$	M_π MeV	$M_\pi L$	Z_A
PNDME N_f =2+1+1	1.218(25)(30)	Yes 11 ensembles 0.15 – 0.06 fm	135 220 310	3.3 – 5.5	Assume $Z_A^s = Z_A^{ns}$
ETMC N_f =2	1.212(40)	0.094 fm	130	2.93	Checked $Z_A^s = Z_A^{ns}$
χQCD N_f =2+1	1.254(16)(30)	"No" a variation 0.143 fm 0.11 fm 0.083 fm	 171 337 302	 3.97 4.53 4.06	Checked $Z_A^s = Z_A^{ns}$

In perturbation theory $Z_A^s \neq Z_A^{ns}$ at 2 loops . ETMC & χQCD show a ~1% difference

Slide from Rajan Gupta @ Spin 2018

Spin Decomposition

§ Orbital angular momentum from Ji definition

$$\vec{J}_g = \int d^3x\, [\vec{x} \times (\vec{E} \times \vec{B})]$$

$$\vec{J}_q = \int d^3x\, \psi^\dagger [\vec{\gamma}\gamma_5 + \vec{x} \times (-i\vec{D})]\psi = \frac{1}{2}(A_{20}^q + B_{20}^q)$$

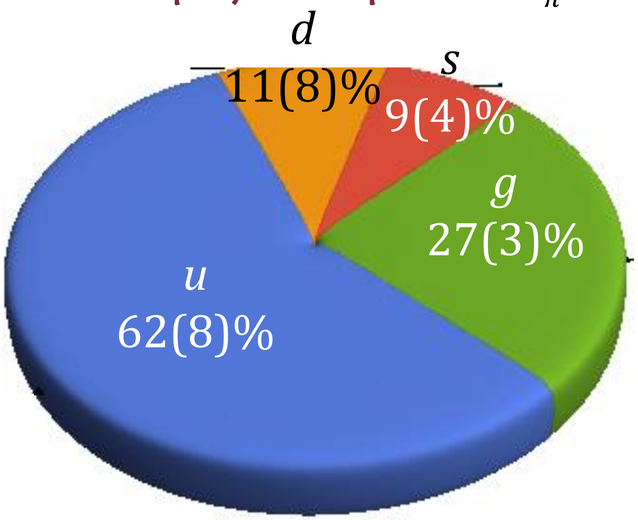
quark spin

quark orbital angular momentum

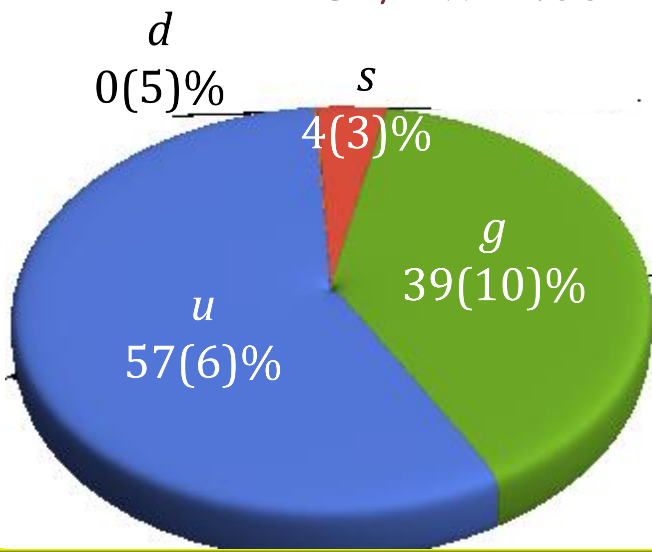
obtained using GPD moment

§ Total quark and gluon contributions

ETMC, 1706.02973
2f TM+clover, physical quark, $M_\pi L < 3$



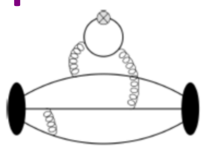
χQCD, 1904.04138
2+1f Ov/DWF 400 MeV



Selected Lattice Nucleon Results

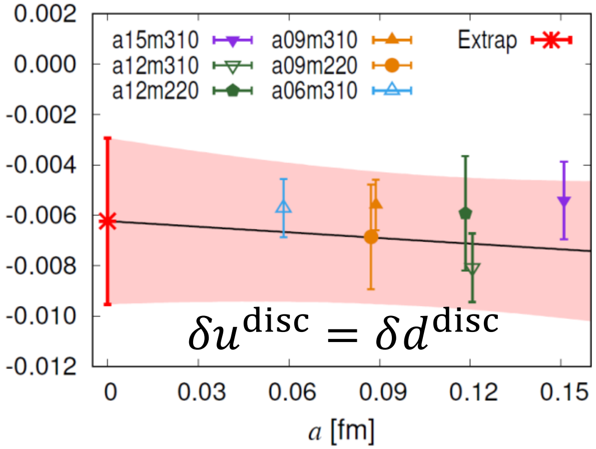
u/d/s Tensor Charges

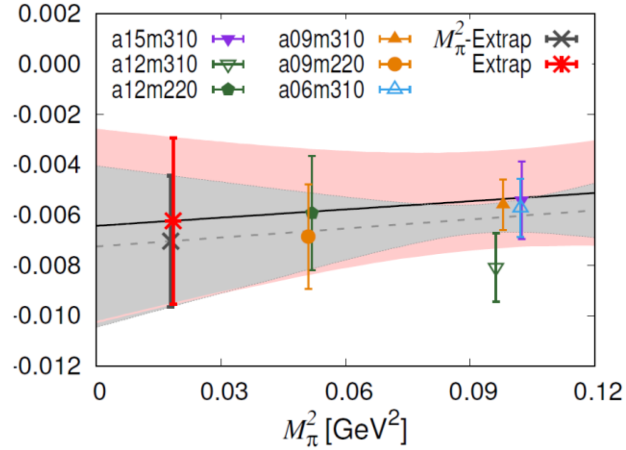
§ Up and down quark “disconnected” contribution



PNDME, 1806.09006, 1808.07597

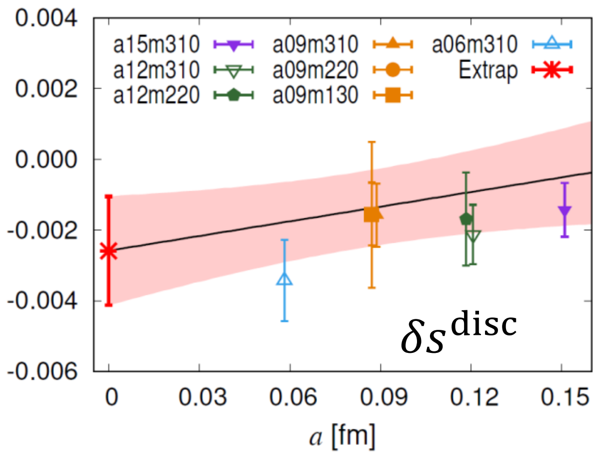
$$\delta q^{\mathrm{disc}} = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

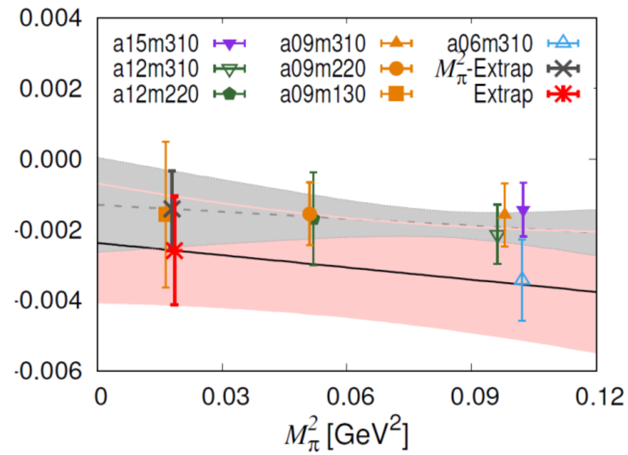




First time in LQCD

Mild dependence on a and pion mass!

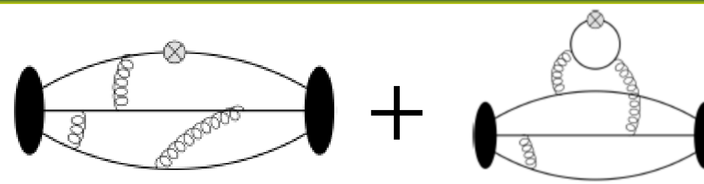




Yong-Chull Jang

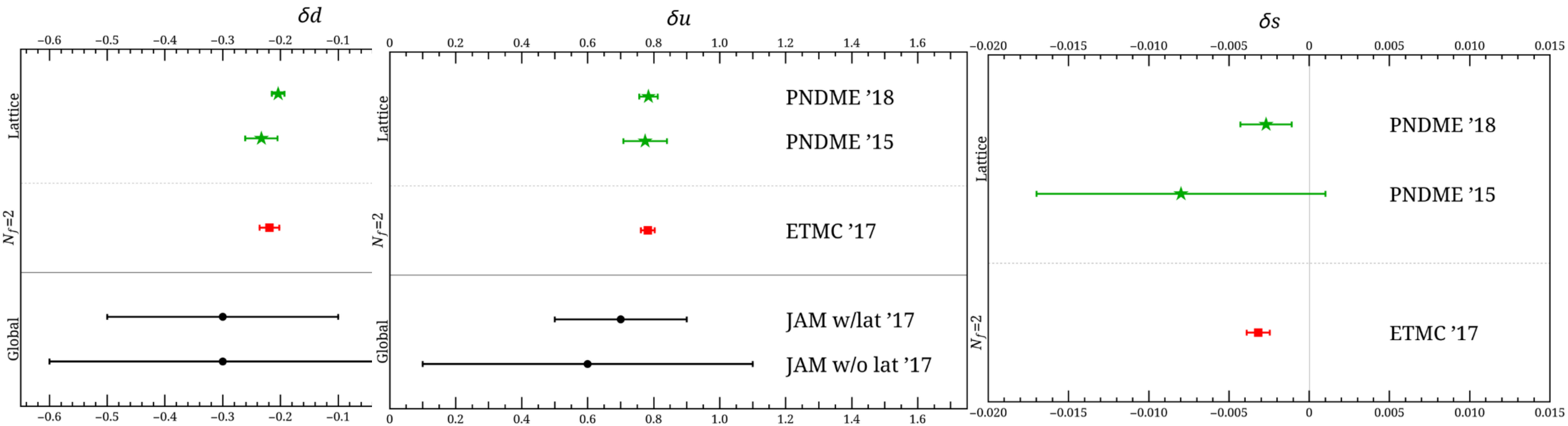
u/d/s Tensor Charges

§ Sum up both contributions



	g_T^u	g_T^d	g_T^s
Connected	0.790(27)	−0.198(10)	
Disconnected	−0.0064(33)	−0.0064(33)	−0.0027(16)
PDNME'18 (Sum)	0.784(28)	−0.204(11)	−0.0027(16)
ETMC'17 [14]	0.782(21)	−0.219(17)	−0.00319(72)
PNDME'15 [5]	0.774(66)	−0.233(28)	0.008(9)

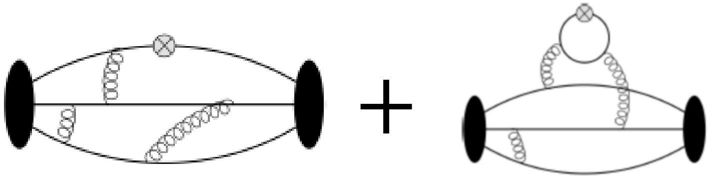
Calculation from one lattice ensemble only
No cont. extrapolation errors



PNDME, 1806.09006, 1808.07597

u/d/s Tensor Charges

§ Sum up both contributions



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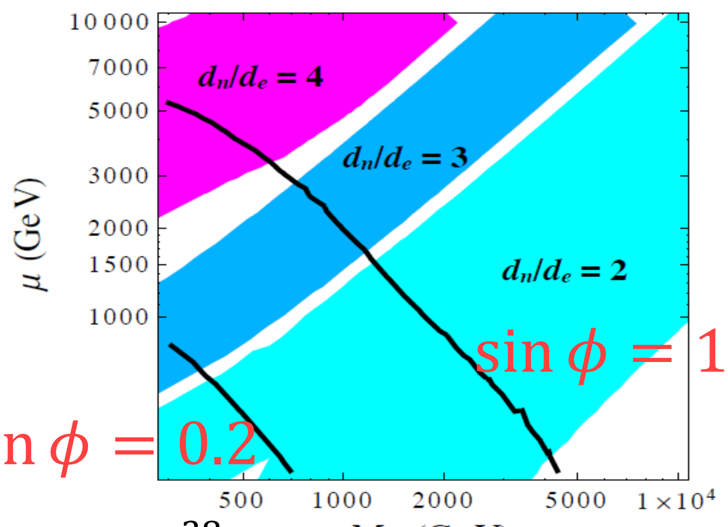
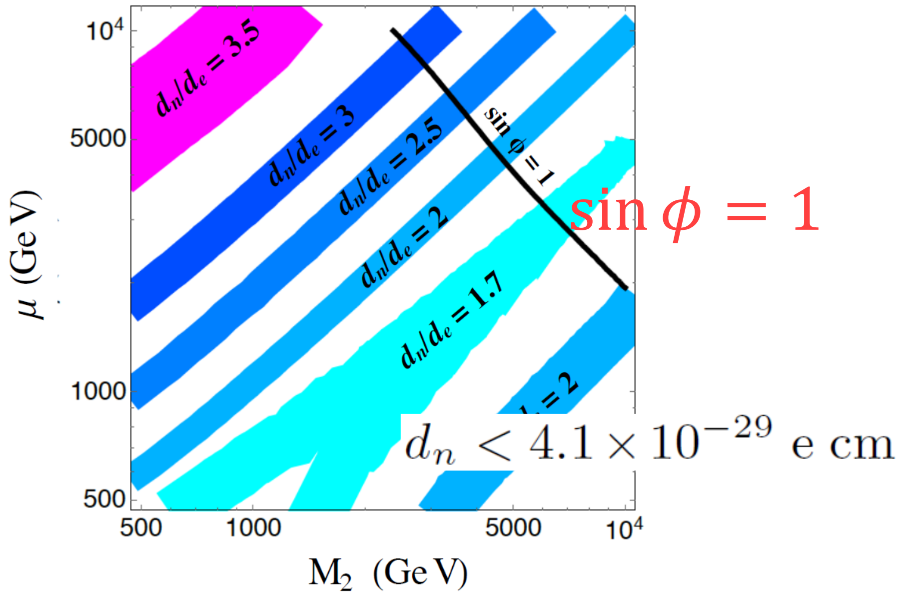
Calculation from one lattice ensemble only
No cont. extrapolation errors

Set limits for nEDM in split-SUSY scenario with gaugino mass unification



$d_n < 4 \times 10^{-28} e\cdot\mathrm{cm}$

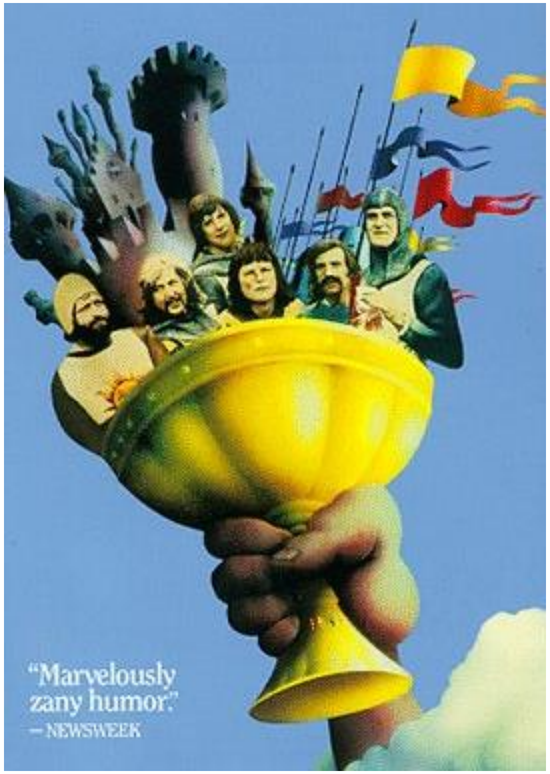
PNDME, 1506.04196; 1506.06411



PNDME, 1806.09006, 1808.07597

Beyond Traditional Moments?

- Longstanding obstacle!
- Holy grail of structure calculations
- Applies to many structure quantities:
  - Generalized parton distributions (GPD)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes
  - Wigner distribution



Beyond Traditional Moments?

§ Reaching for higher moments

- Fictitious heavy quarks (Detmold and Lin, hep-lat/0507007)
- Smeared lattice ops (Davoudi et al. 1204.4146)

§ Direct calculation of *x* dependence

- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Inversion method/OPE without OPE (QCDSF, hep-lat/9809171, …1703.01153)
- Euclidean correlation functions (RQCD, 1709.04325)
- Lattice cross-section method (Y.-Q. Ma and J. Qiu, 2014, 2017)
  Colin Egerer (Wed. 14:25)
- Large-momentum effective theory (LaMET) and variations
  - Original LaMET ("quasi-PDF") This talk, Nikhil Karthik (Wed. 15:15)
  - Pseudo-PDF method: differs in FT (A. Radyushkin, 2017)
  - Smeared quasi-PDF (C. Monahan and K. Orginos, 2017)

Ultimate QCD Machine

Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

How are these quark and gluon distributions correlated with overall nucleon properties, such as spin direction?

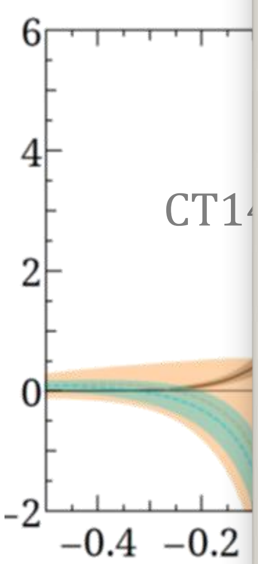
What is the role of the orbital motion of sea quarks and gluons in building the nucleon spin?

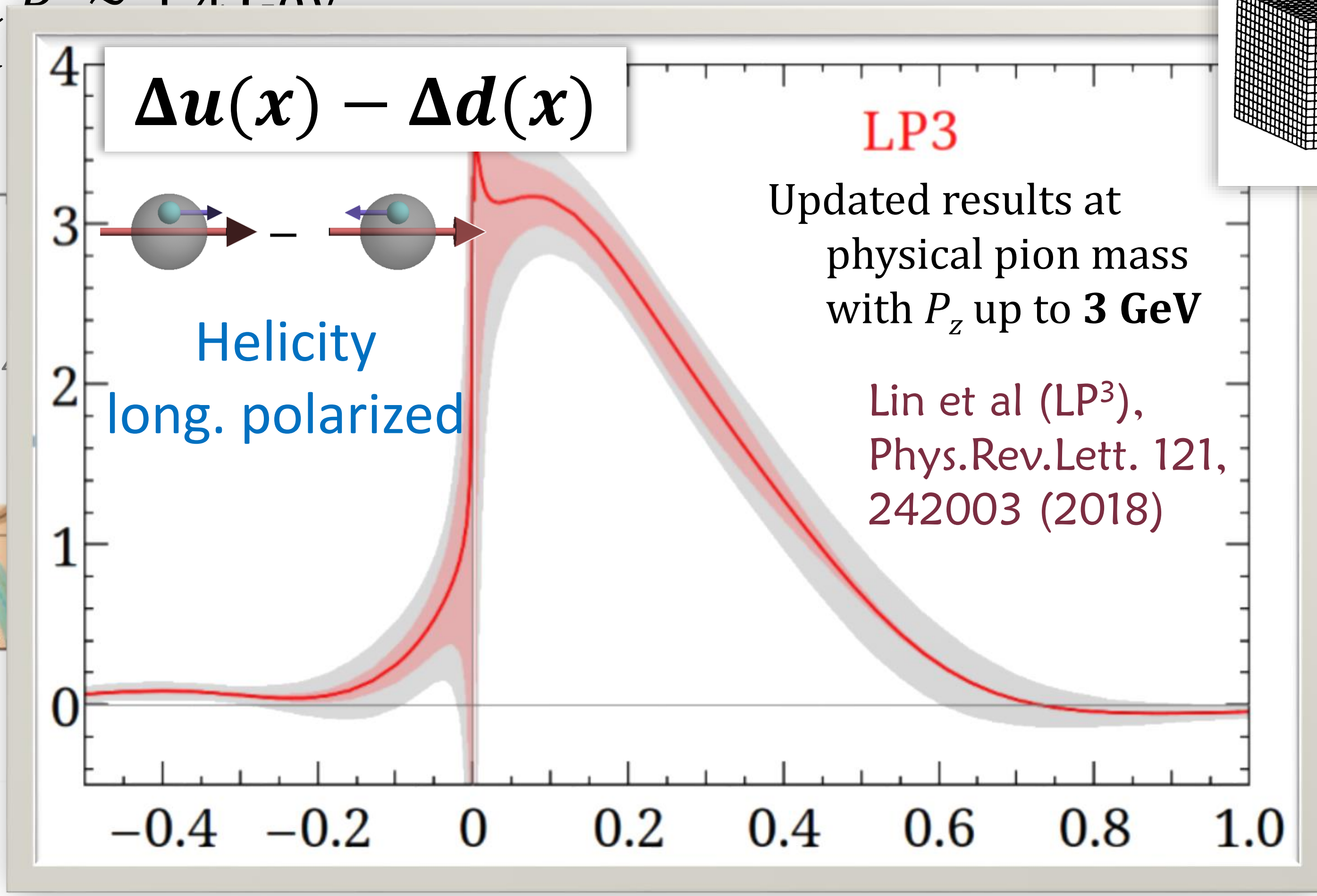
EIC White Paper, 1212.1701

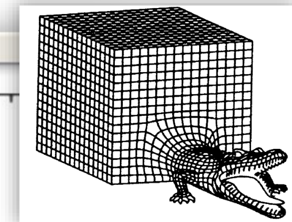
Polarized PDFs

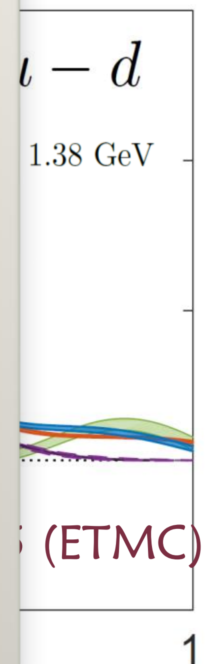
§ Exciting! Two collaborations' results at **physical pion mass**

- Boost $P_z \sim 1.4$ GeV
- Study





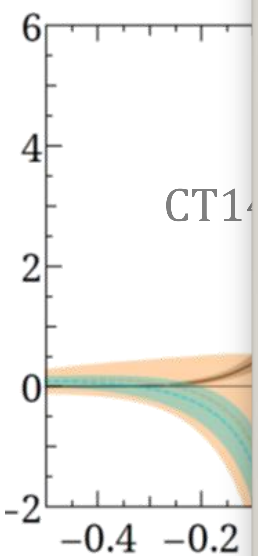


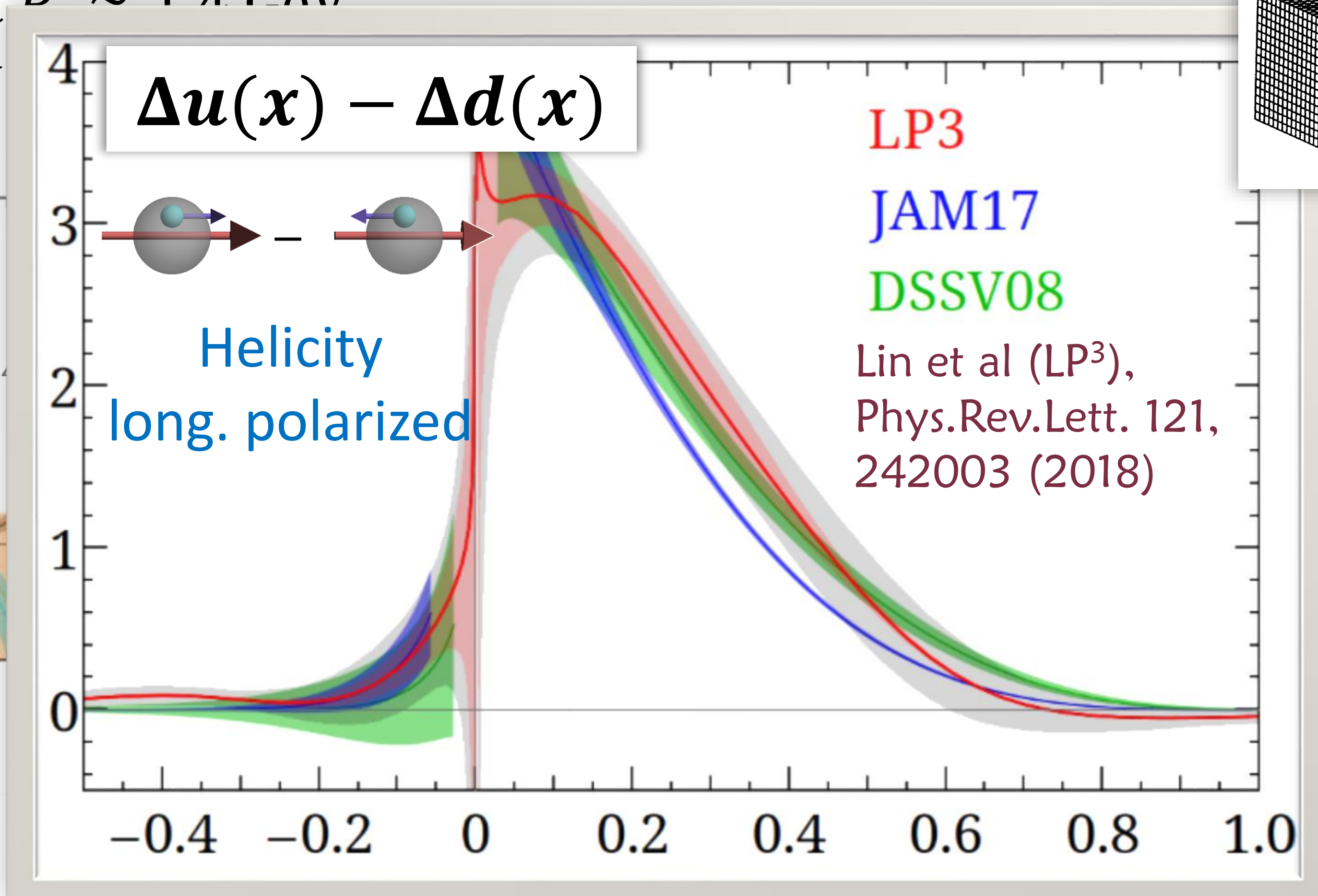


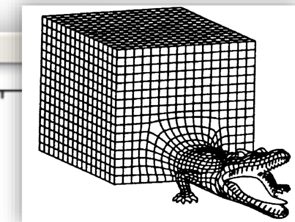
Polarized PDFs

§ Exciting! Two collaborations' results at **physical pion mass**

- Boost P ~ 1.4 GeV
- Study





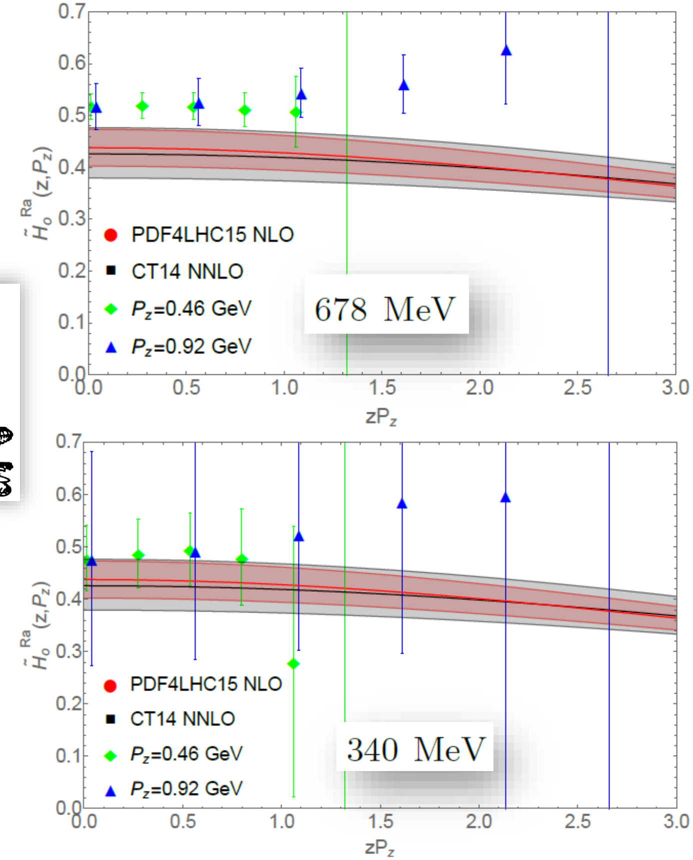


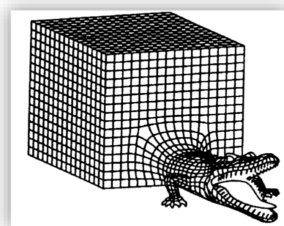
Gluon PDF

§ Pioneering first glimpse into gluon PDF using LaMET

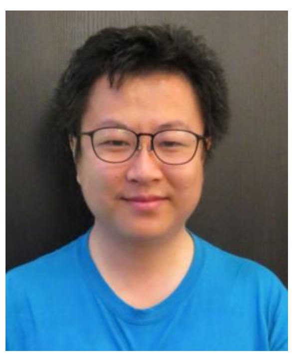
- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Study strange/light-quark
- Promising results using coordinate-space comparison,
  but signal does not go far in z
- Hard numerical problem to be solved

Fan. et al, Phys.Rev.Lett. 121, 242001 (2018)





Zhouyou Fan



Yi-Bo Yang

Gluon Helicity

§ Jaffe & Manohar, 1990 $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q^z + \mathcal{L}_g^z$

§ Can be calculated through large-momentum frame

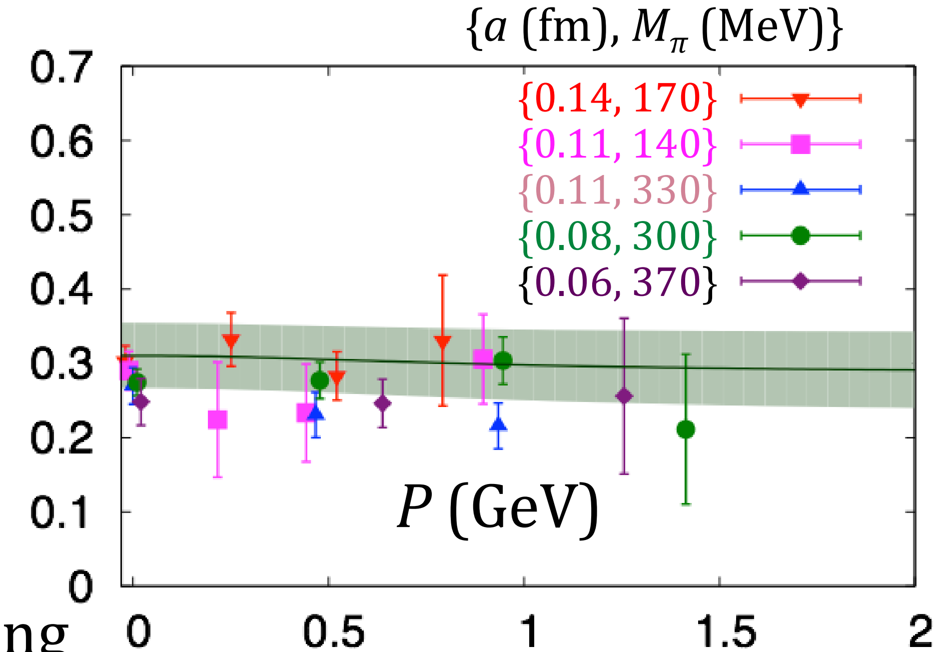
X. Ji et al., PRL. 111 (2013) 112002; 110 (2013) 262002; PRD 89, 085030 (2014)

$$S_G(P)\, S_z = \frac{\left\langle PS \middle| \int d^3x\, \left(\vec{E} \times \vec{A}_{\text{phys}}\right)_z \middle| PS \right\rangle}{2E_P}$$

§ First results by **χQCD**

$$\Delta G\left(\mu^2 = 10\ \text{GeV}^2\right)$$
$$\approx S_G\left(\infty, \mu^2 = 10\ \text{GeV}^2\right)$$
$$= 0.287(55)(16)$$

Yang et al, Phys. Rev. Lett. 118 (2017) 102001



➢ Future improvement on matching

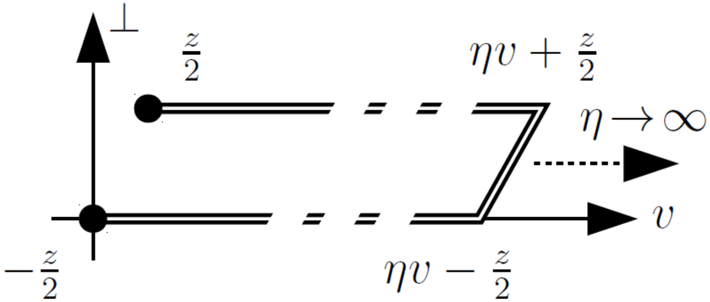
§ Current limit

➢ DSSV14 $\int_{0.05}^{1} dx\, \Delta G\left(10^2\ \text{GeV}, x\right) \approx [0.14, 0.24]$

Orbital Angular Momentum

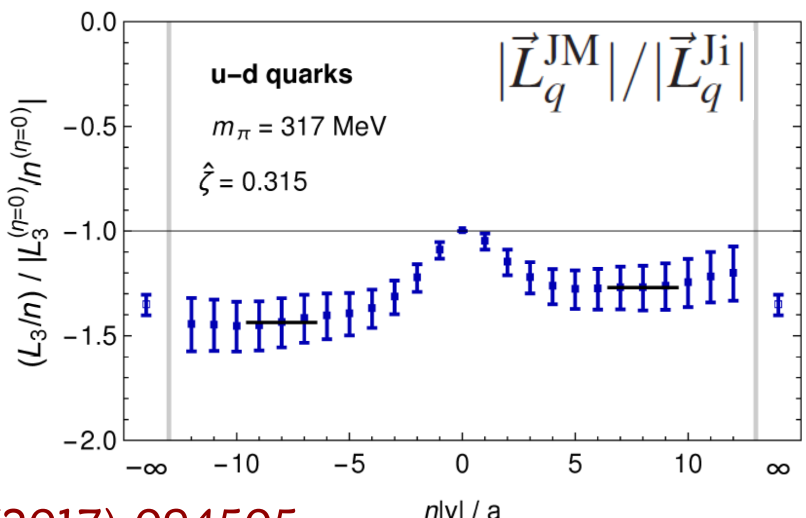
§ Two definitions: Ji vs Jaffe & Manohar

$$\vec{L}_q^{\,\mathrm{Ji}} = \int d^3x\, q^\dagger \left[\vec{x} \times i\vec{D}\right] q \qquad \vec{L}_q^{\,\mathrm{JM}} = \int d^3x\, q^\dagger \left[\vec{x} \times i\vec{\nabla}\right] q\,,$$



§ First result carried out by M. Engelhardt

2+1f clover at 518-MeV pion mass



- $\eta = 0$ gives Ji's OAM
- Staple $\eta \to \infty$ gives Jaffe-Manohar OAM
- Difference is accumulated torque from final-state interaction

Phys. Rev. D95 (2017) 094505

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary

§ Exciting era using LQCD to study nucleon structure

- Well-studied systematics → precision structures
- More nucleon matrix elements with physical pion masses
- Address neglected disconnected contributions
  obtaining flavor-dependent quantities

§ Overcoming longstanding limitations of moment method

- Bjorken-*x* dependence of parton distribution functions are widely studied with LaMET and its variants
- More study of systematics planned in the near future

§ Stay tuned for many more exciting results from LQCD

Titan @ORNL
IC@LANL

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405

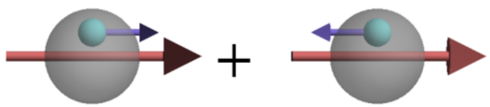
Backup Slides

LaMET Calculation on PDFs

PDFs on the Lattice

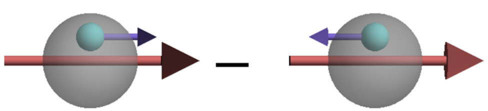
§ Lattice calculations rely on operator product expansion, only provide moments

most well known



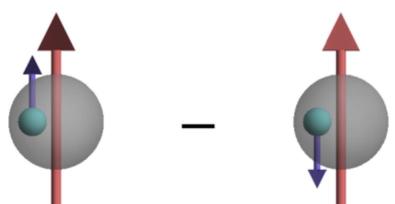
spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

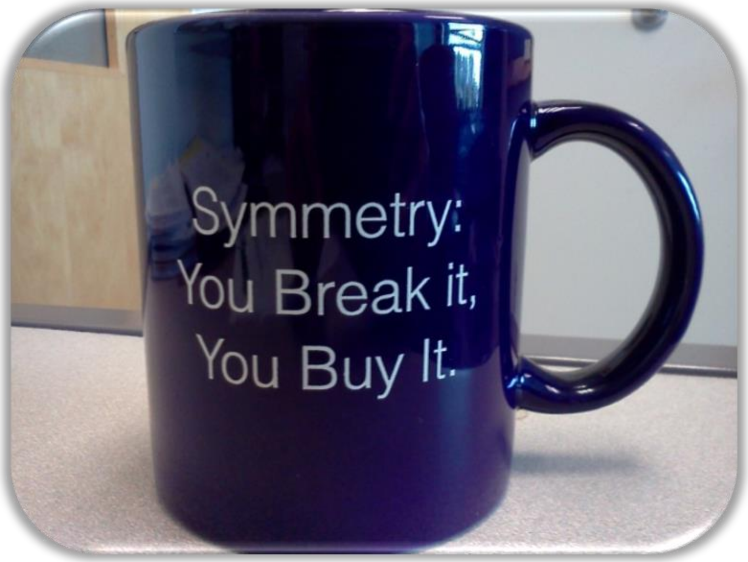
very poorly known

§ True distribution can only be recovered with all moments

PDFs on the Lattice

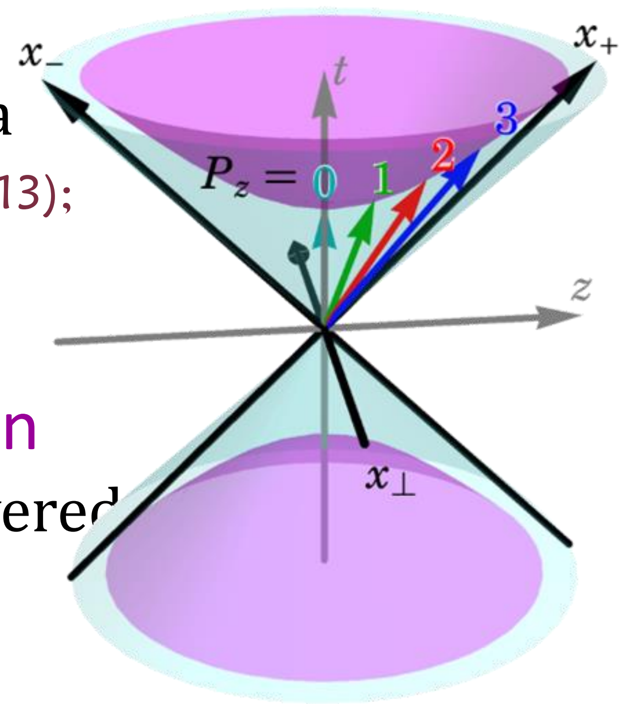
- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea

Z. Davoudi, Tuesday
Lattice



PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea
- New Strategy: Xiangdong Ji, PRL 111, 039103 (2013); J. Chen, Monday Lattce+Helicity
- Adopt lightcone description for PDFs
- Calculate finite-boost quark distribution
  - In $P_z \to \infty$ limit, parton distribution recovered
  - For finite P_z, corrections are applied through effective theory
- Demonstration: Feasible with today's resources!



Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meiβner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanoudes and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

Slide credit: Yong Zhao (LP^3), CIPANP 2018 Plenary talk

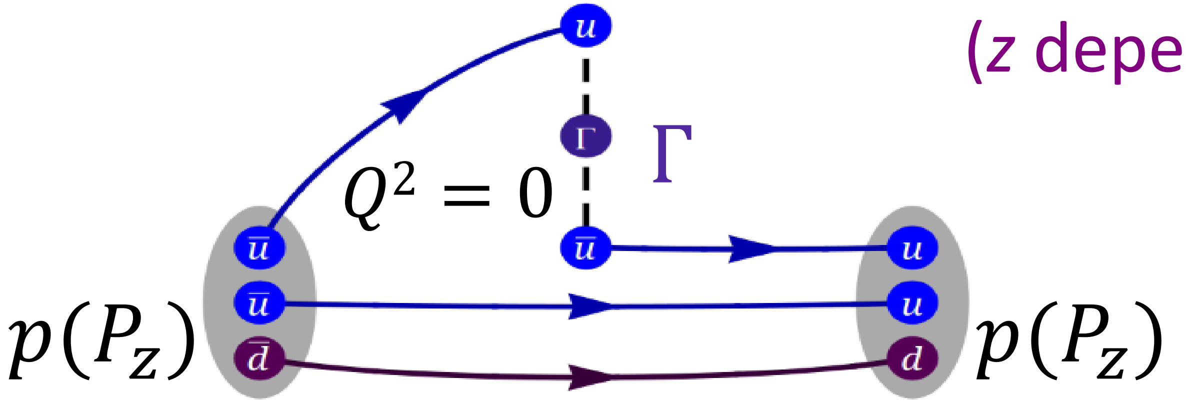
A New Direction

Steps for LaMET

See more details in backup slides

1) Calculate nucleon matrix elements on the lattice

(*z* dependence)



2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \bar{\psi}(z)\, \Gamma \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

3) Recover true distribution (take $P_z \to \infty$ limit)

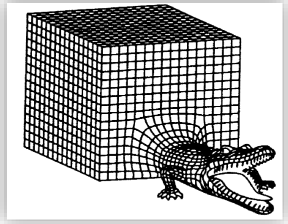
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

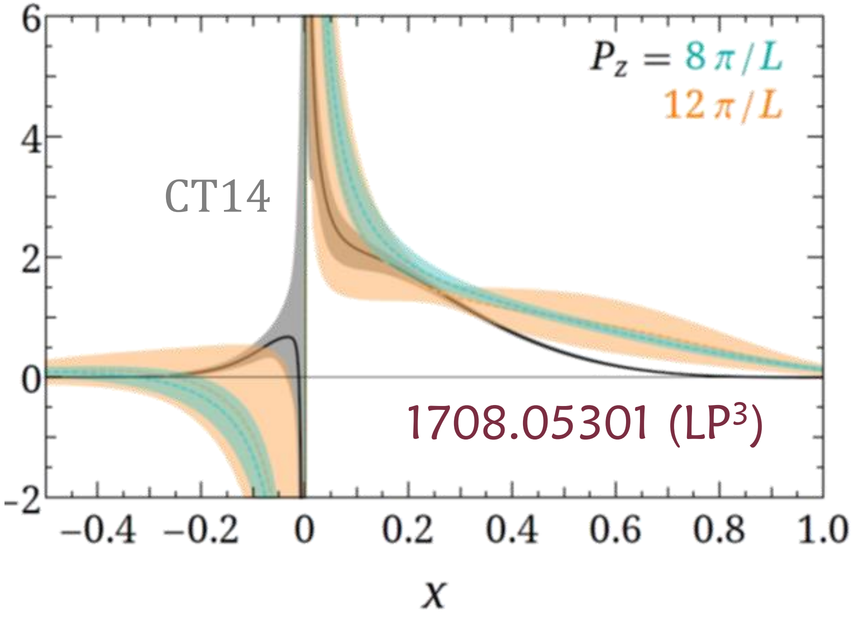
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

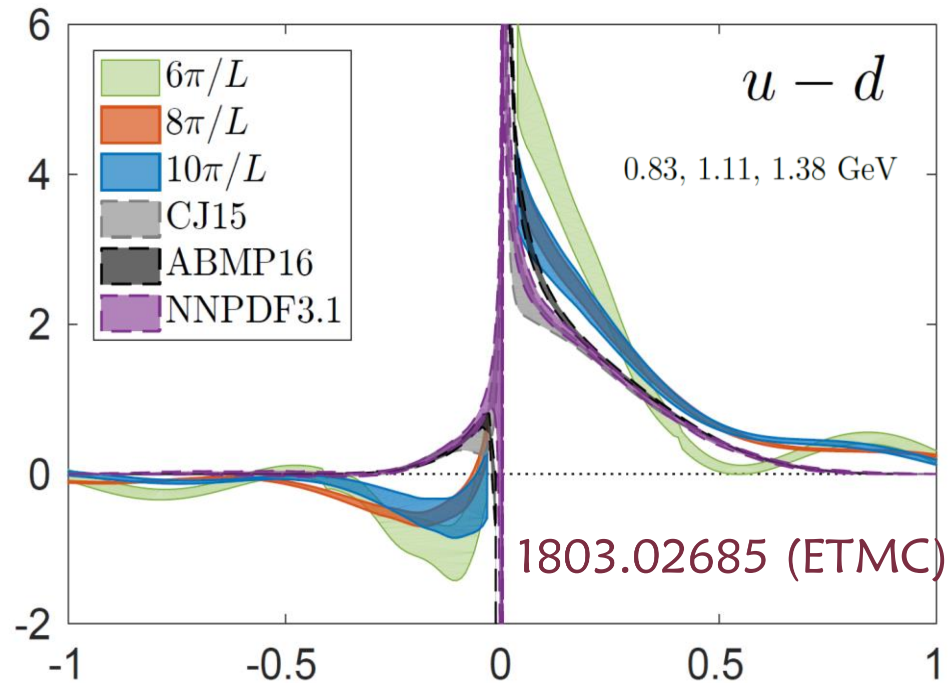
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





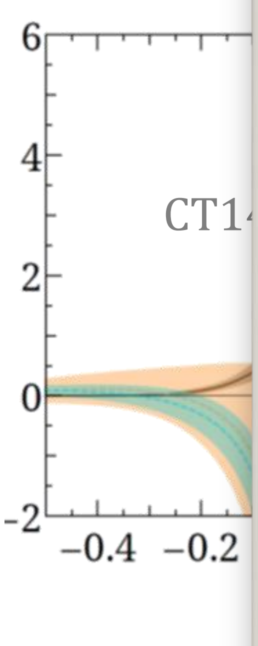


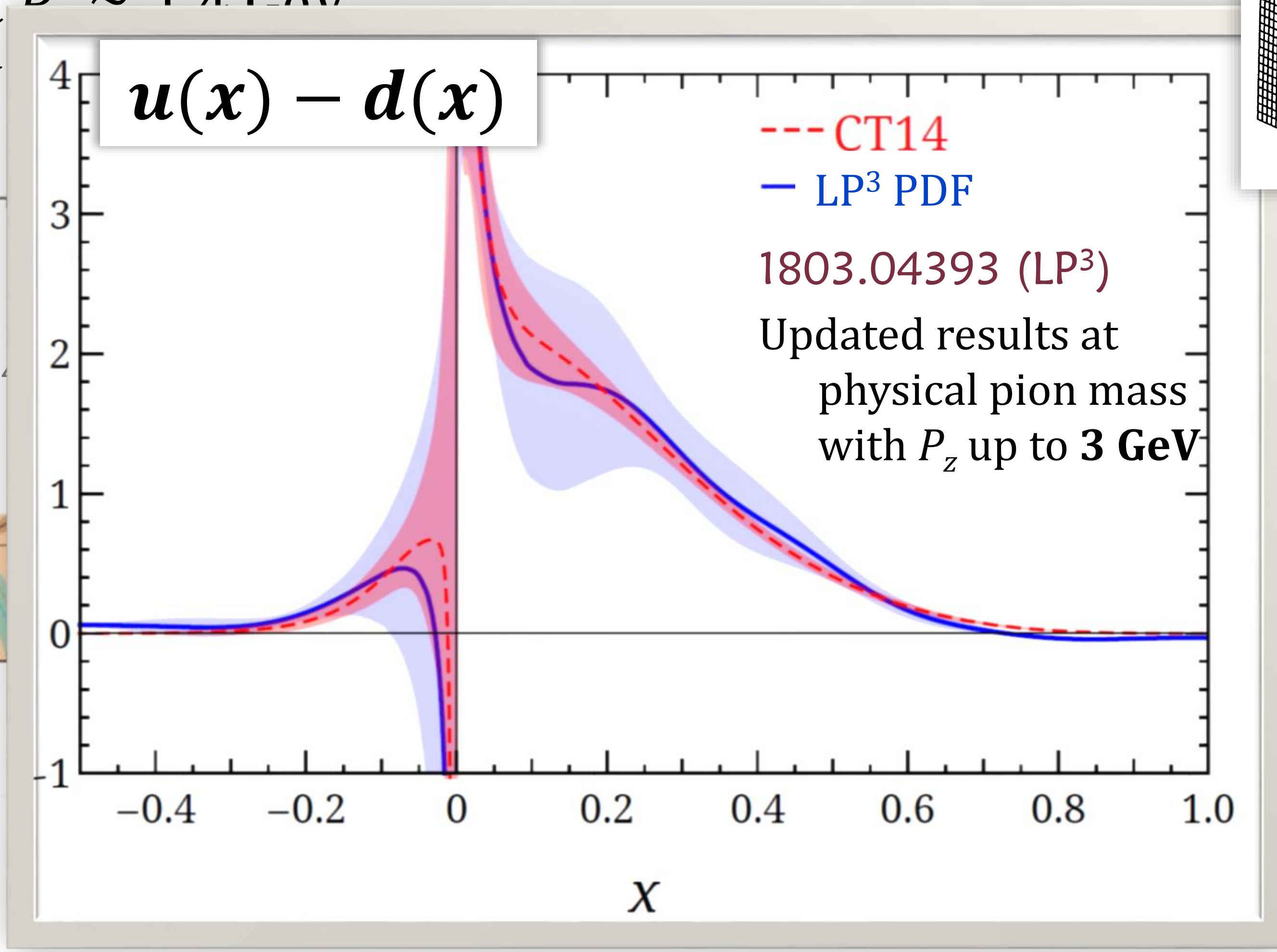
Not using any parametrization form like $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$

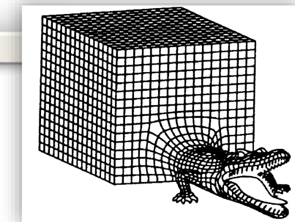
Physical Pion Mass Results

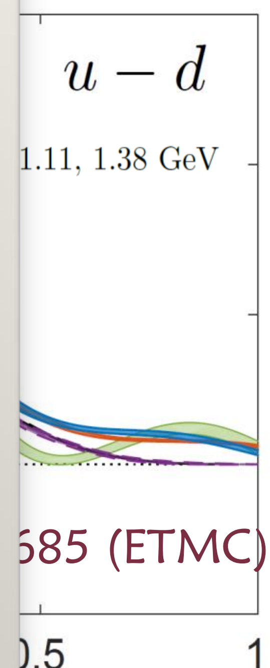
§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \sim 1.4$ GeV
- Study





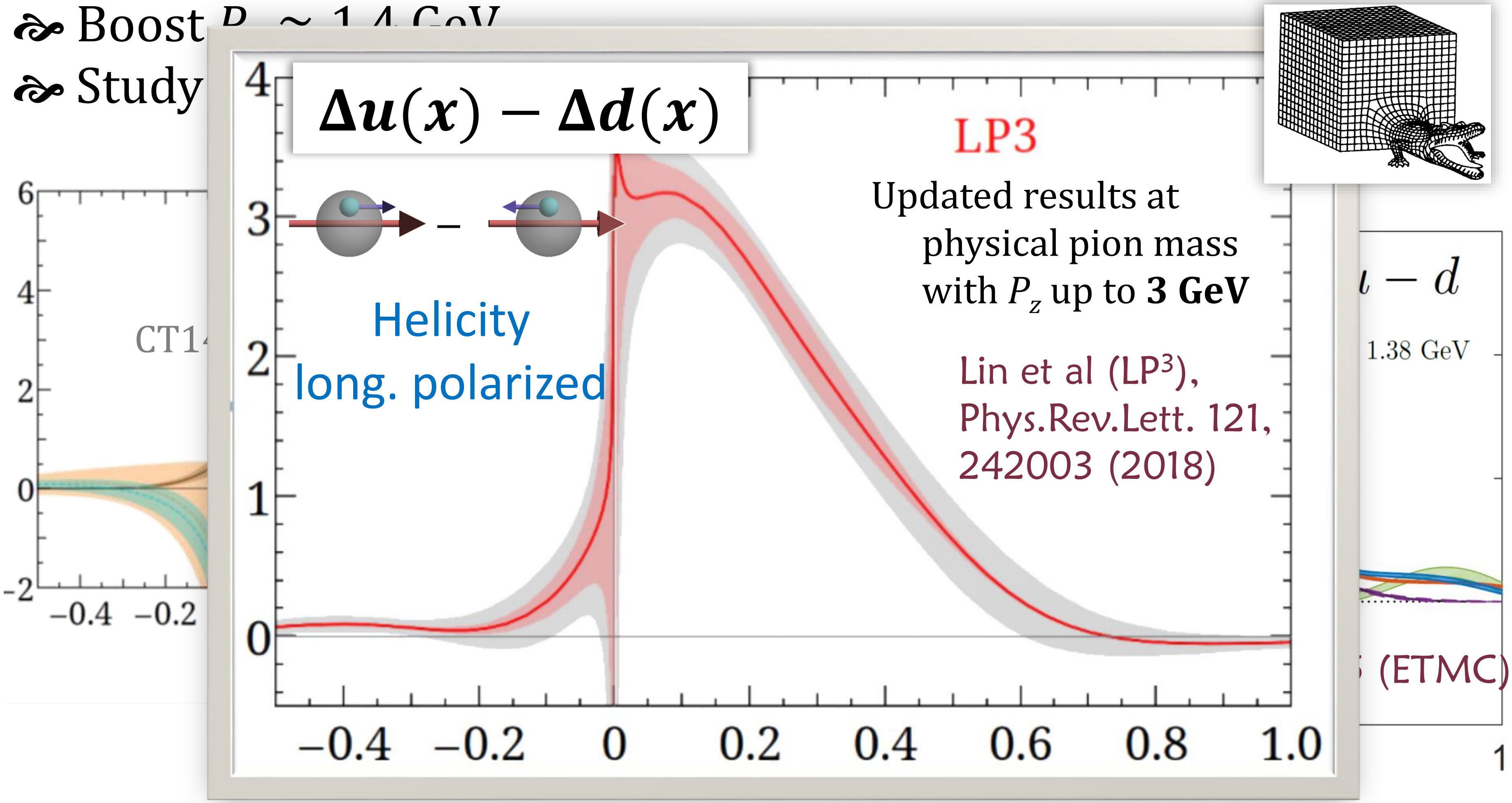




Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \sim 1.4$ GeV
- Study



Physical Pion Mass Results

§ Exciting! Two collaborations’ results at physical pion mass

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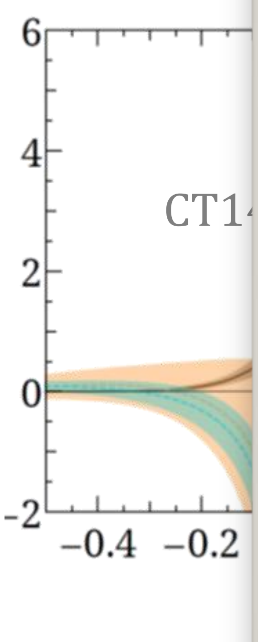

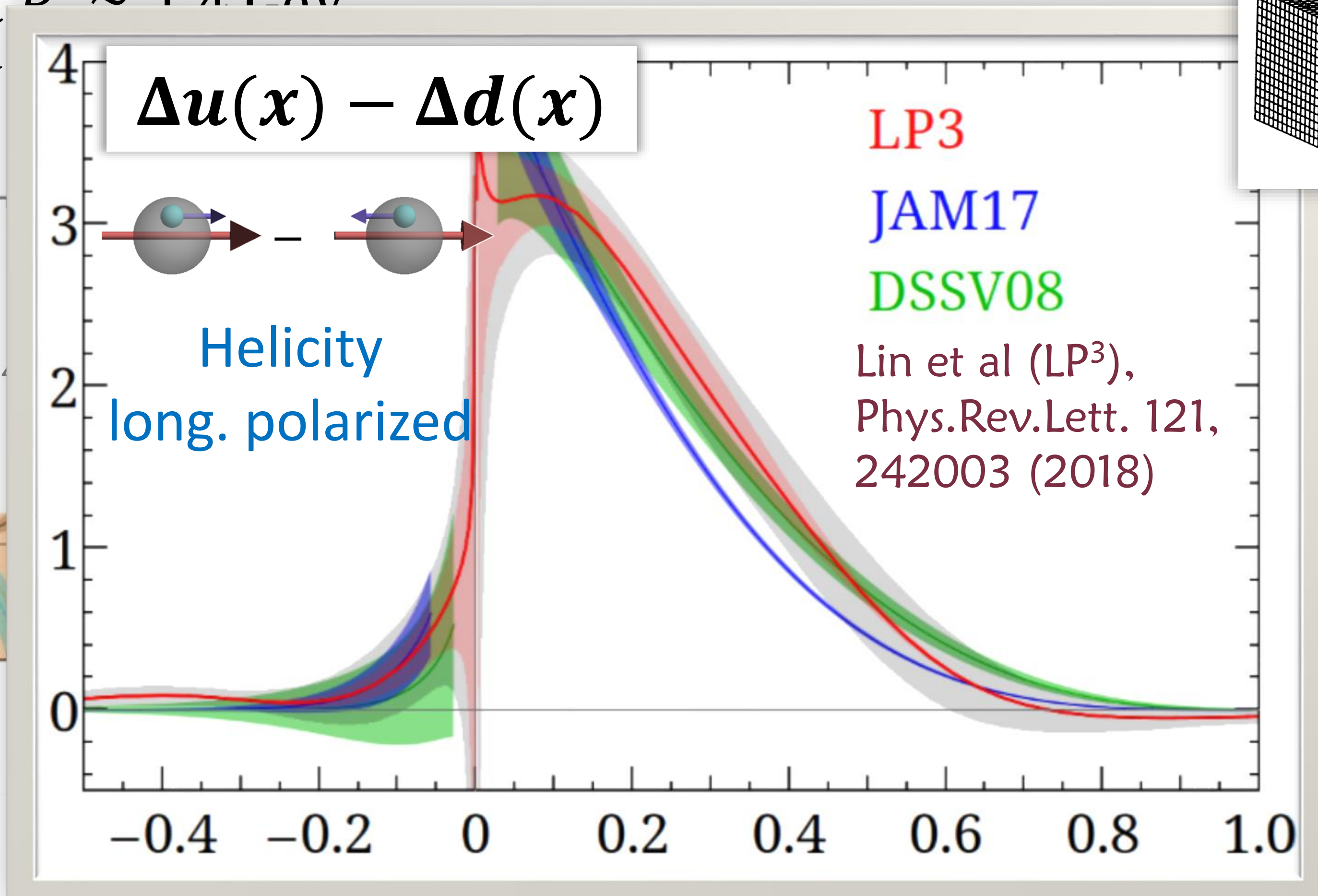

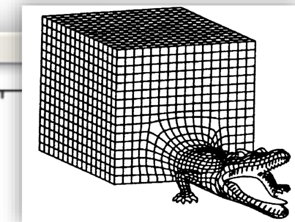

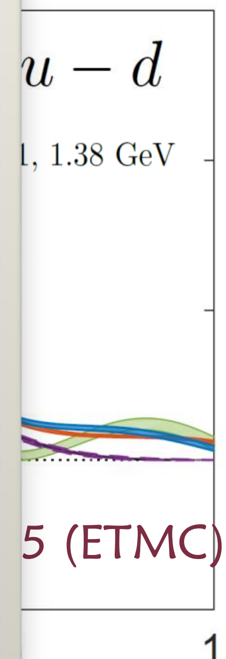

Physical Pion Mass Results

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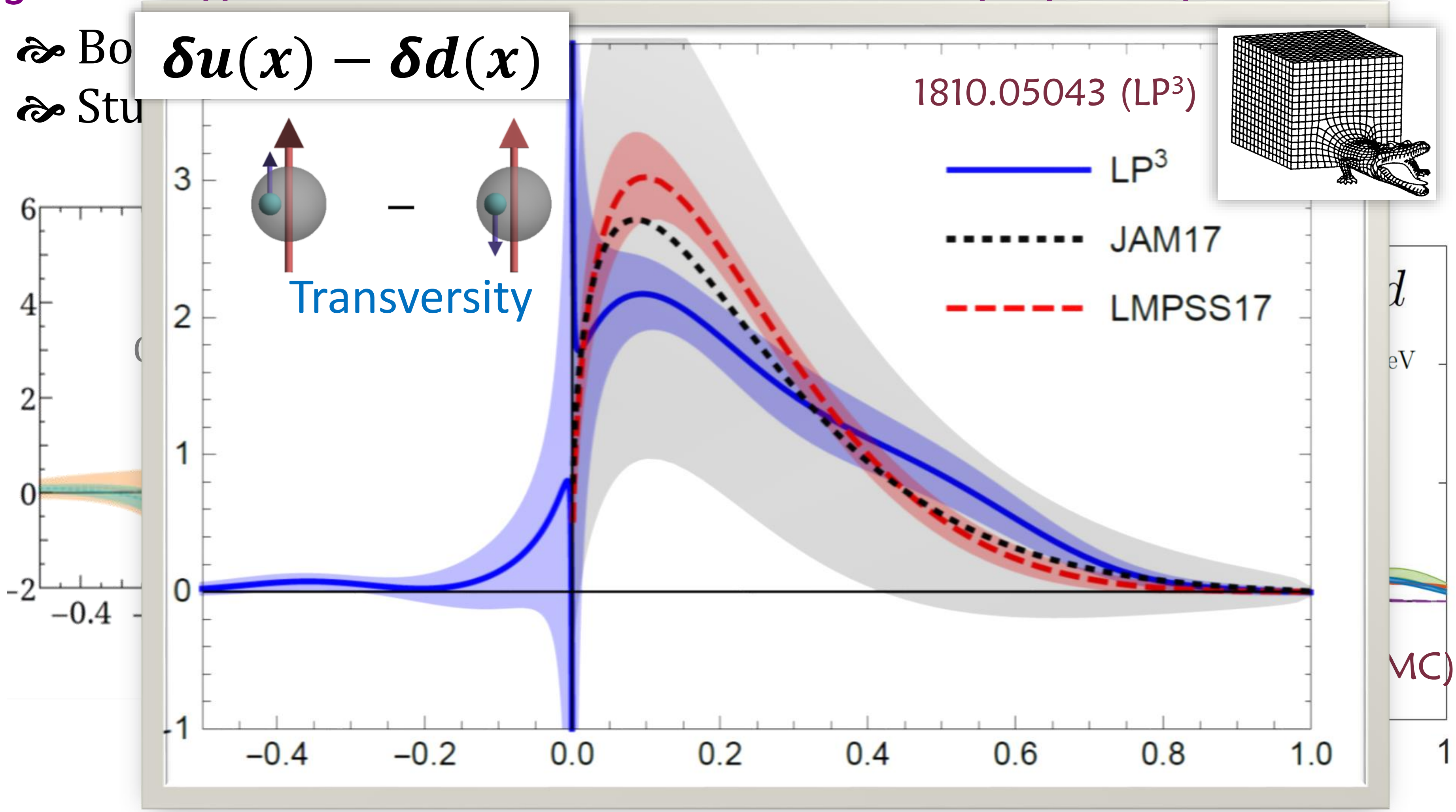
- Bo
- Stu



Physical Pion Mass Results

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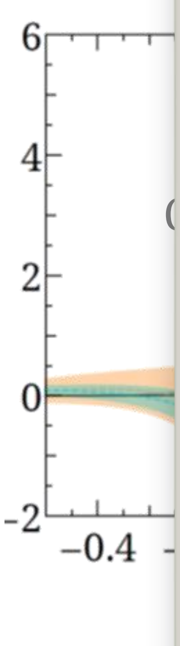

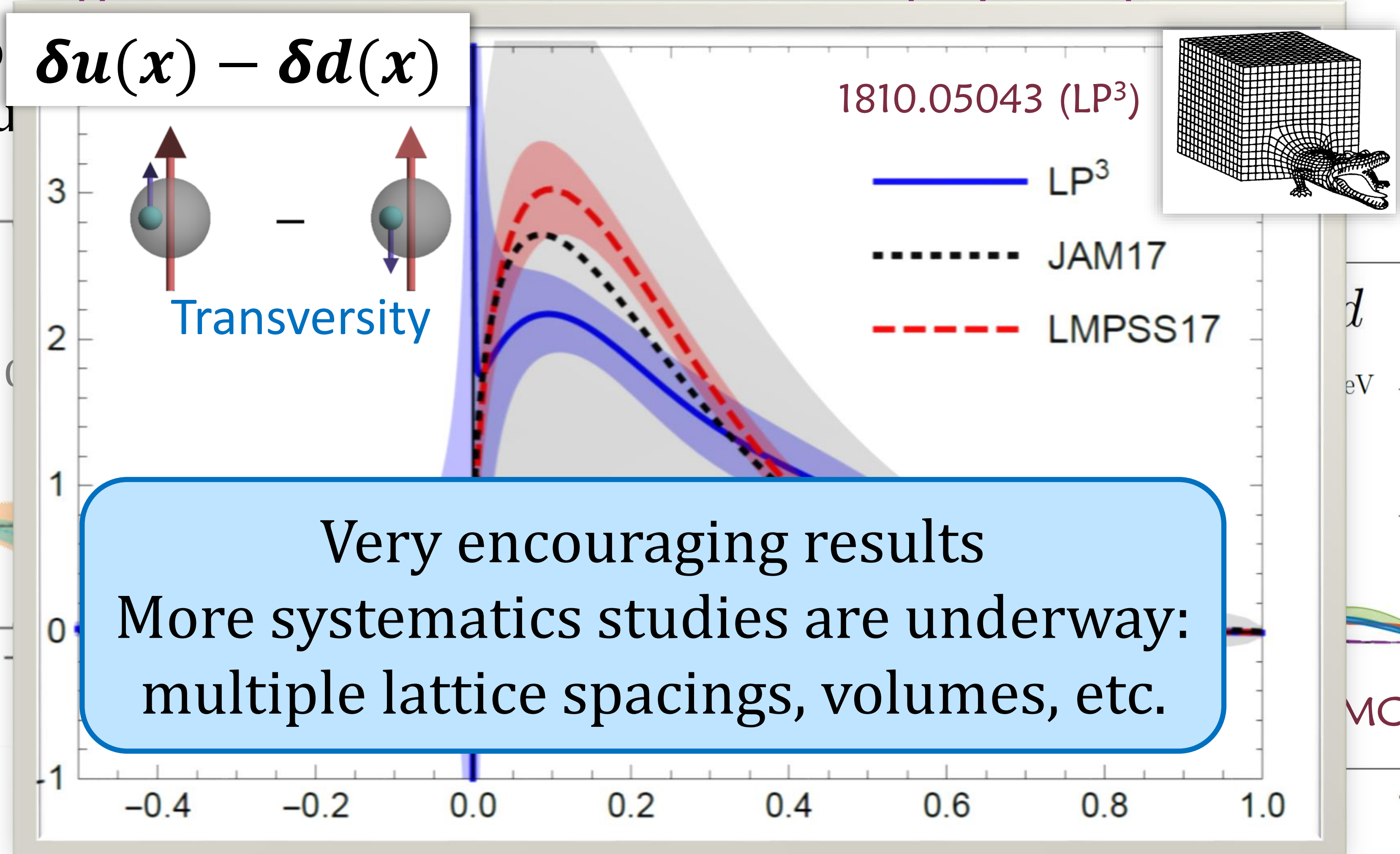

Very encouraging results
More systematics studies are underway:
multiple lattice spacings, volumes, etc.