Frame dependence of transition form factors and Dalitz decays in light-front dynamics

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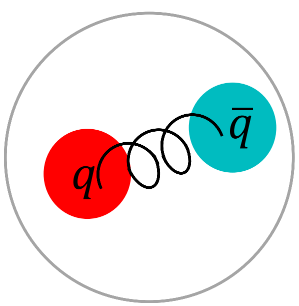
Heavy quarkonia

- The effective Hamiltonian for $|q\bar{q}\rangle$[1]

$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$



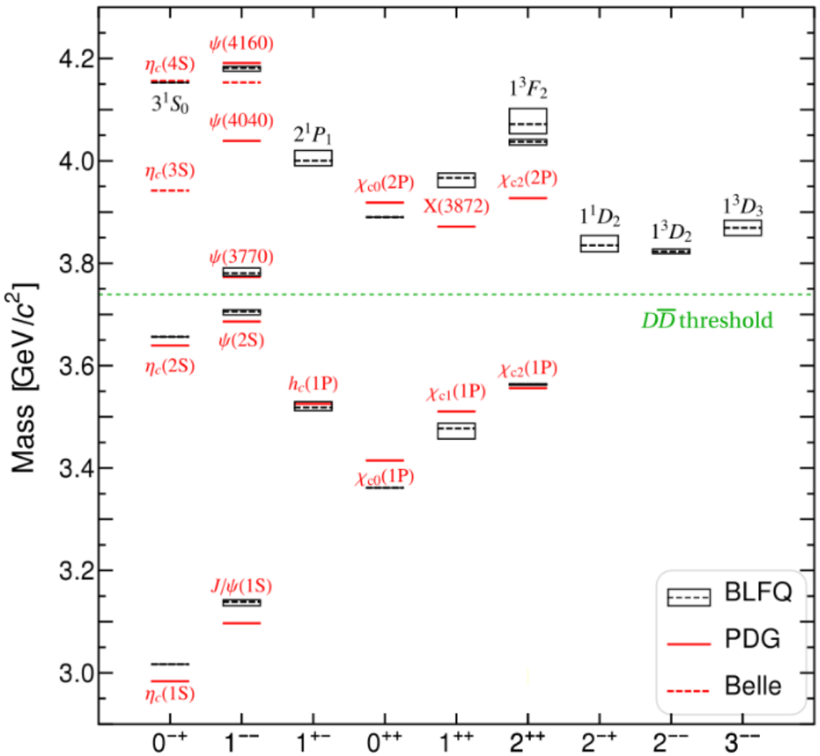
- Confinement
  Transverse (QCD holography)[2]
  Longitudinal (completes the transverse confinement, and produces desirable distribution amplitudes)
- One-gluon exchange

$$x = p_q^+/P^+$$
$$\vec{k}_\perp = \vec{p}_{q\perp} - x\,\vec{P}_\perp$$

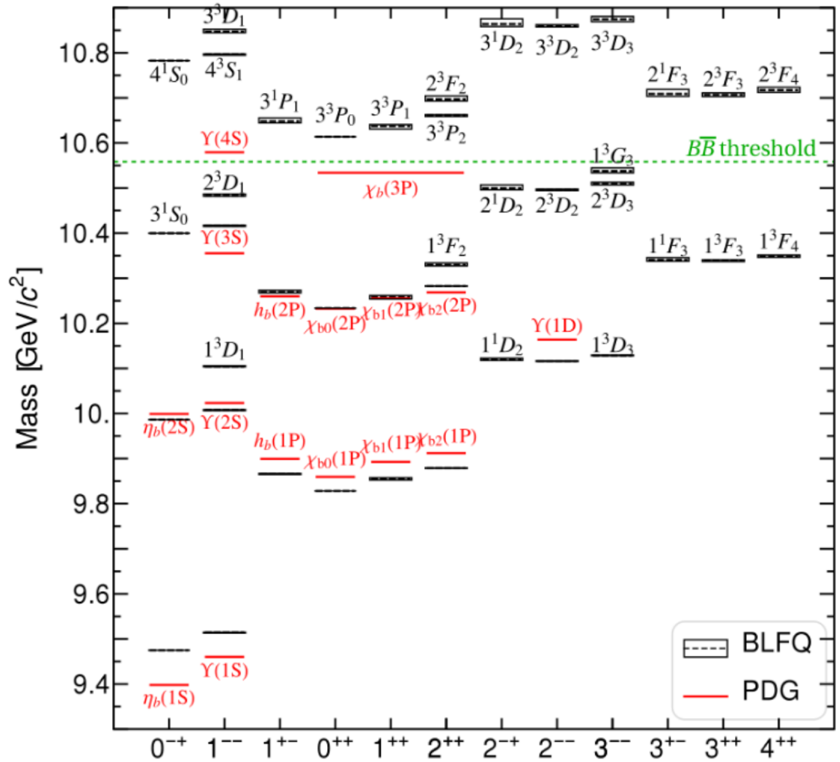
$$V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$$

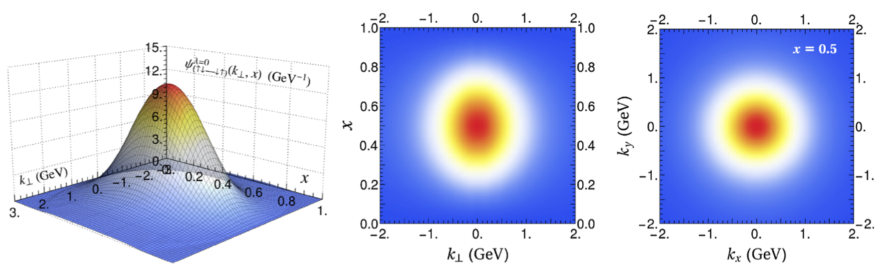
[1] Y. Li, P. Maris, and J. P. Vary, Phys. Rev. D96, 016022 (2017).
[2] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, Phys. Rept. 584, 1 (2015).

Charmonia[1]



Bottomonia[1]



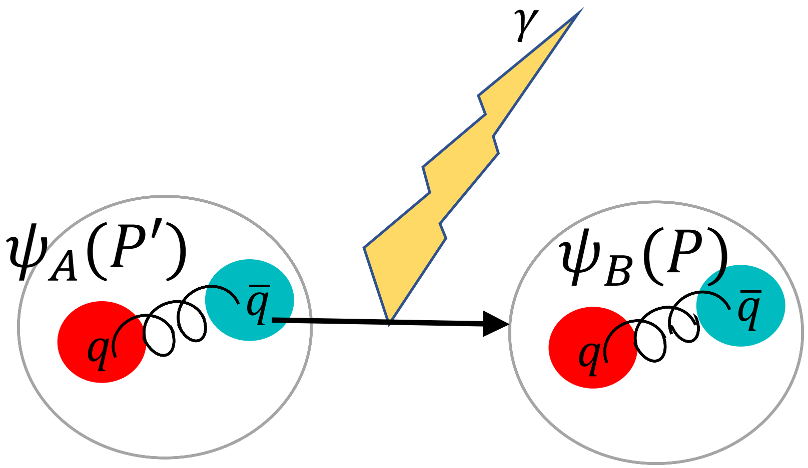


	κ(GeV)	m_q(GeV)	r.m.s. (MeV)
$c\bar{c}$	0.966	1.603	31
$b\bar{b}$	1.389	4.902	38

[1] Y. Li, P. Maris, and J. P. Vary, Phys. Rev. D96, 016022 (2017).

The radiative transition

- $\psi_A \to \psi_B\, \gamma$, sheds light to the internal structure of quark-antiquark bound states.

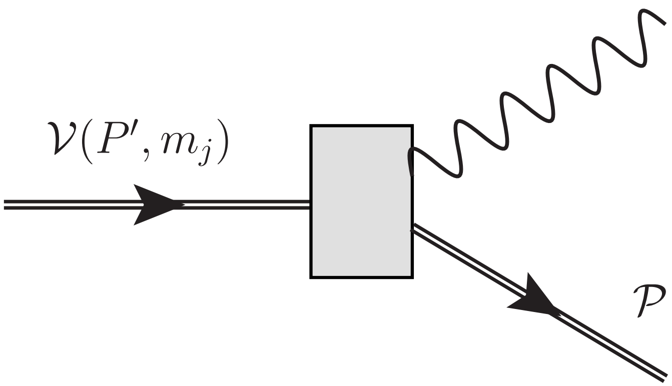


The process is governed by the hadron matrix

$$\langle \psi_B(P) | J^\mu | \psi_A(P') \rangle,$$

where J^μ is the electromagnetic current.

The magnetic dipole transition



The Dalitz decay:
Time-like photon → lepton pair

$\mathcal{V} \to \mathcal{P} l^+ l^-$(or $\mathcal{P} \to \mathcal{V} l^+ l^-$)

$\mathcal{V} \to \mathcal{P}\gamma$ (or $\mathcal{P} \to \mathcal{V}\gamma$)

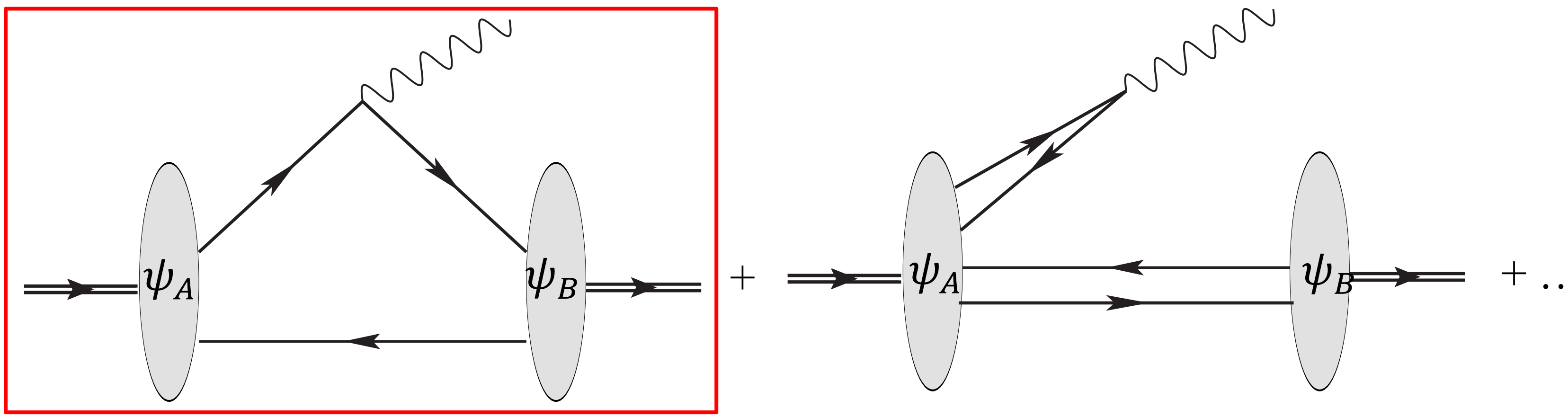
- The transition form factor $V(q^2)$

$$\langle \mathcal{P}(P)| J^\mu(0) |\mathcal{V}(P', m_j)\rangle = \frac{2V(q^2)}{m_{\mathcal{P}} + m_{\mathcal{V}}} \epsilon^{\mu\alpha\beta\sigma} P_\alpha P'_\beta e_\sigma(P', m_j)$$

- $V(0)$ can be determined from the partial decay width

Hadron matrix element

$< \psi_B | J^\mu | \psi_A >=$



(a) $n \to n$ (b) $n + 2 \to n$

(a) and (b) are the leading light-front time-ordered diagrams. Their separate contributions depend on[1]

- the current component (J^+, J^x, J^y, J^-)
- reference frames

In $|q\bar{q}\rangle$, is there a preferred current, or a preferred frame?

[1] M. Sawicki, Phys. Rev. D46, 474 (1992). S. J. Brodsky and D. S. Hwang, Nucl. Phys. B543, 239 (1999).

Preferred current for $|q\bar{q}\rangle$

$J^R = J^x + iJ^y$ provides reliable results for the M1 transition in heavy systems[1]

$$\langle \mathcal{P}(P)|\, J^R(0)\, |\mathcal{V}(P', m_j)\rangle = \frac{2V(q^2)}{m_{\mathcal{P}} + m_{\mathcal{V}}} i m_{\mathcal{V}} P^+ \left[\frac{P^R}{P^+} - \frac{P'^R}{P'^+} \right]$$

- J^- always associates with pair annihilations[2]
- J^+ echoes *charge density* J^0, while J^R echoes the 3D spatial *current density* $\vec{J}$
- J^R employs the dominant spin components of wavefunctions, and could restore the non-relativistic limit of the transition form factor

[1] M. Li, Y. Li, P. Maris, and J. P. Vary, Phys. Rev. D98, 034024 (2018)
[2] J. Carbonell, B. Desplanques, V.A. Karmanov and J.F.Mathiot, Phys. Rept. 300, 215 (1998); J.P.B.C de Melo, J.H.O. Sales, T. Frederico and P.U. Sauer, Nucl. Phys. A631, 574C (1998); S. J. Brodsky and D. Hwang, Nucl. Phys. B543, 239(1999).

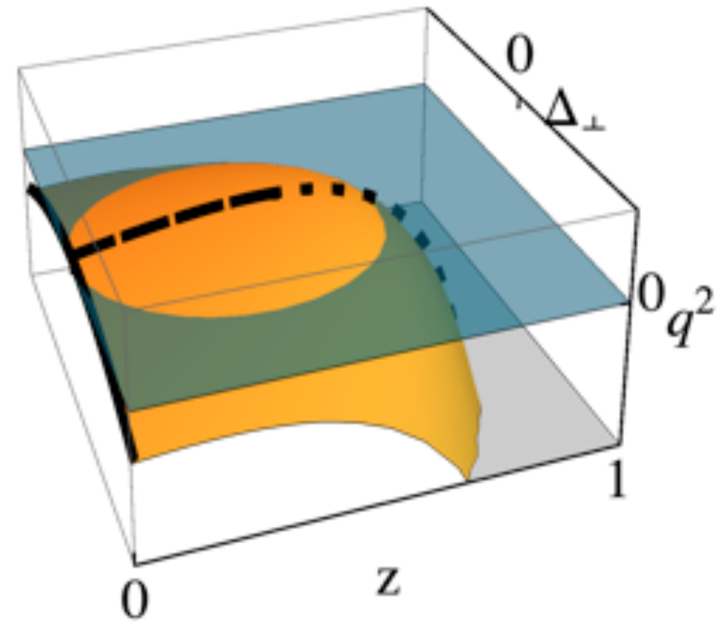
Preferred frame

For the transition $\psi_A(P') \to \psi_B(P)\,\gamma(q = P' - P)$, the momentum transfer squared is

$$q^2 = zm_A^2 - \frac{z}{1-z}m_B^2 - \frac{1}{1-z}\vec{\Delta}_\perp^2 .$$

where

$$z \equiv (P'^+ - P^+)/P'^+ \text{ and } \vec{\Delta}_\perp \equiv \vec{q}_\perp - z\vec{P}'_\perp$$

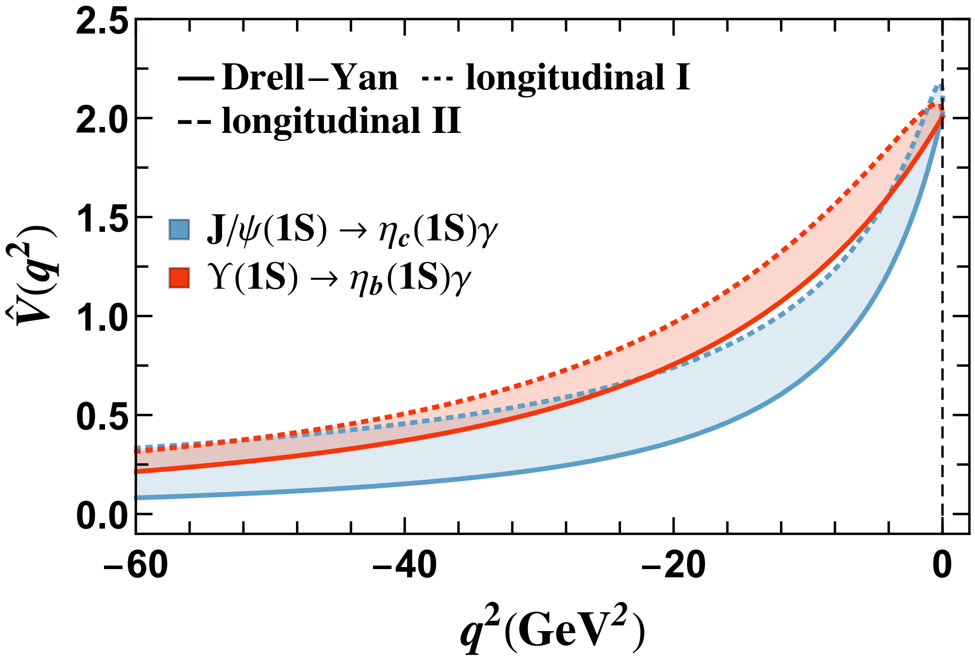


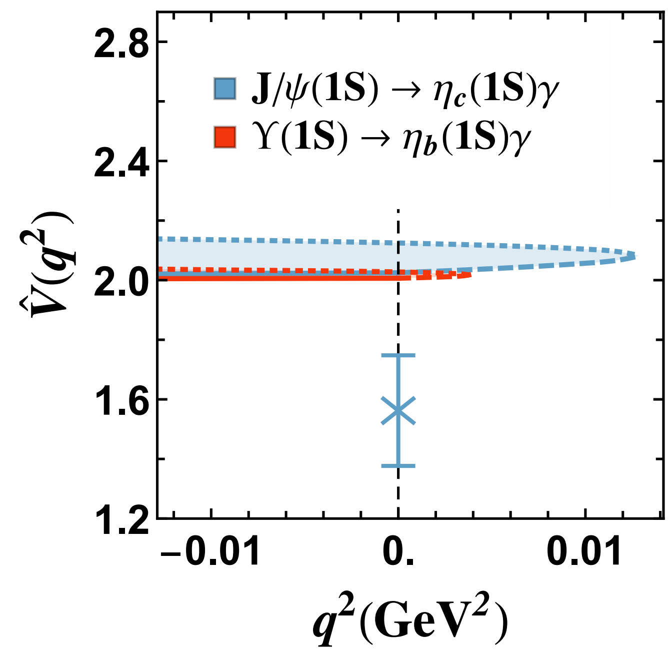
- Different $(z, \Delta_\perp)$ for the same q^2 characterize different frames:
  The Drell-Yan frame ($z = 0$): ———
  The longitudinal frame ($\Delta_\perp = 0$): ········· I, — — II
- Small z could suppress the $n + 2 \to n$ contribution, thus preferred for calculation in $|q\bar{q}\rangle$.

Result: $V(q^2)$ from $|q\bar{q}\rangle$

Allowed transitions: $\psi_i(nS) \to \psi_f(nS)\,\gamma$

initial and final states have the same radial and angular quantum numbers





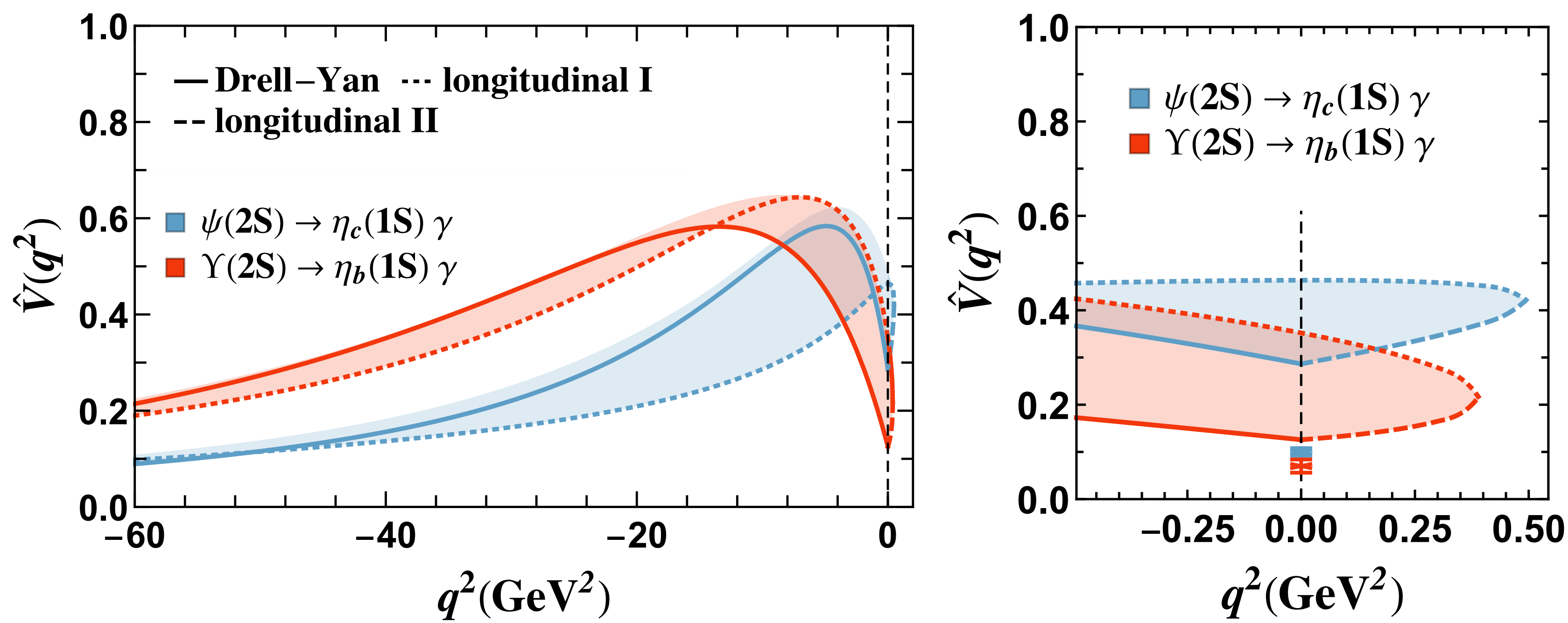
Shaded areas represent the results from all other frames

× **PDG:** M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D98, 030001 (2018)

Result: $V(q^2)$ from $|q\bar{q}\rangle$

Hindered transition: $\psi_i(nS) \to \psi_f(n'S)\,\gamma, (n \neq n')$

initial and final states have different radial quantum numbers



Shaded areas represent the results from all other frames

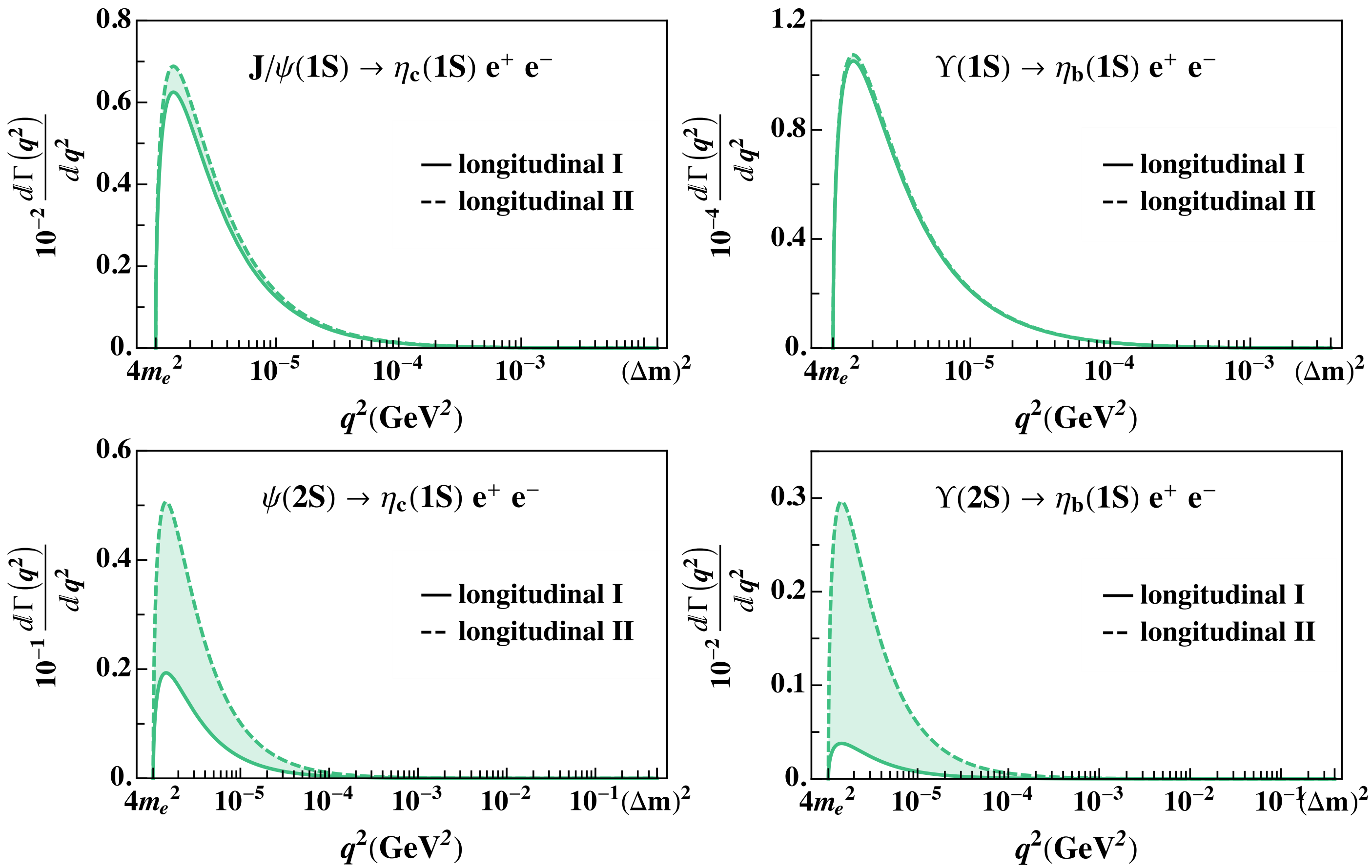
✕ **PDG:** M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D98, 030001 (2018)

The Dalitz decay

Predicted mass spectrum of the leptonic pair through the transition form factor:

Allowed transitions: Less sensitive to frames

Hindered Transitions: More sensitive to frames



Summary

1. We have calculated the M1 transition form factors in different frames for the leading Fock sector

2. Preferred frames for calculation in $|q\bar{q}\rangle$
   Spacelike region: the Drell-Yan frame
   Timelike region: the frame with minimal longitudinal momentum transfer

3. The frame dependence could serve as a measure for the Lorentz symmetry violation due to Fock space truncation
   a. The frame dependence is smaller for heavier systems
   b. Transitions between states w different radial quantum numbers are more sensitive to the choice of frame

Backup: future works

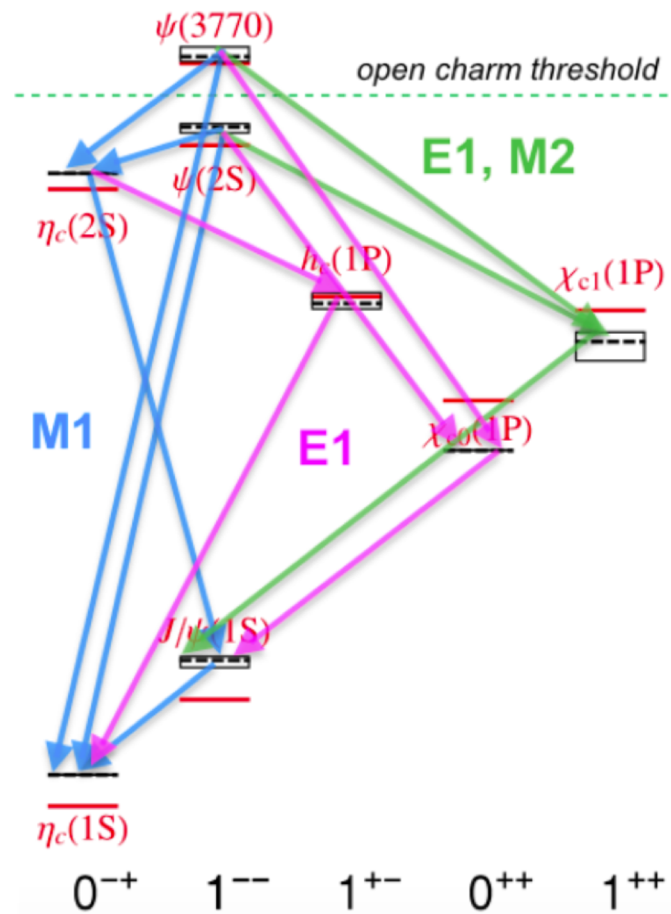
- $\mathcal{P} \rightarrow \gamma\gamma$ or $\gamma\gamma^* \rightarrow \mathcal{P}$ transitions.
- Other transitions,

  vector to scalar

  $(J/\psi \rightarrow \chi_{c0}\gamma)$

  axial vector to vector

  $(\chi_{c1} \rightarrow J/\psi e^+e^-)$
- Higher Fock sector contributions.

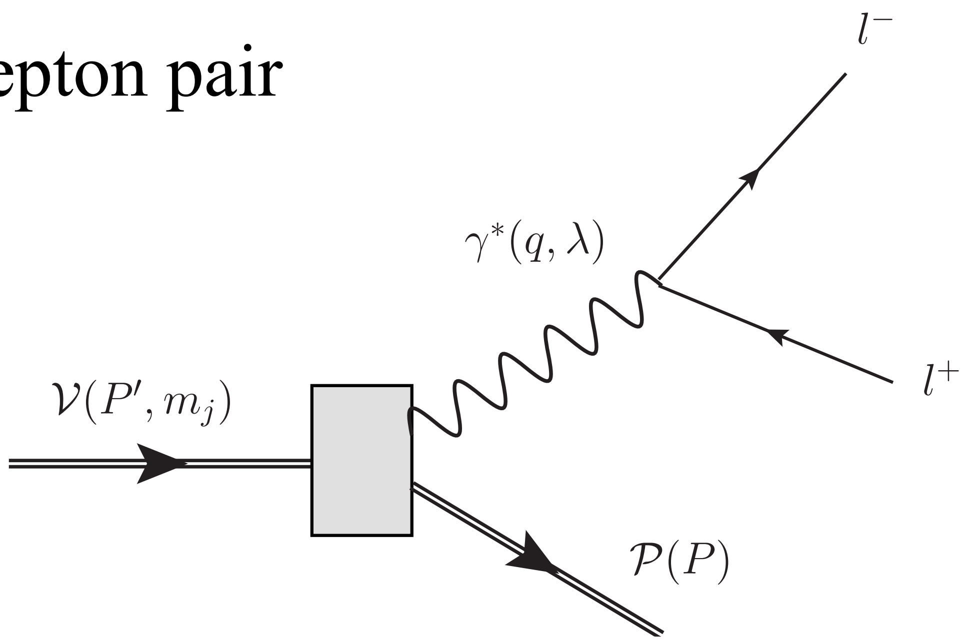


Backup: the Dalitz decay

$\mathcal{V} \to \mathcal{P} l^+ l^-$ (or $\mathcal{P} \to \mathcal{V} l^+ l^-$)

Time-like photon ⟶ lepton pair

$$q^2 = m^2_{l^+ l^-}$$



Decay amplitude:

$$\mathcal{M}_{m_j} = \underbrace{eQ_f \frac{2V(Q^2)}{m_{\mathcal{P}} + m_{\mathcal{V}}} \epsilon^{\mu\alpha\beta\sigma} P'_\alpha P_\beta e_\sigma(P, m_j)}_{\mathcal{V} \to \mathcal{P} + \gamma^*} \times \underbrace{\frac{1}{q^2}}_{\text{photon propagator}} \times \underbrace{\bar{u}\gamma_\mu u}_{\text{leptonic current}}$$

Backup: the decay widths

The decay width of $\psi_A \to \psi_B \gamma$ $(\psi_A, \psi_B = \mathcal{V}, \mathcal{P}$ or $\mathcal{P}, \mathcal{V})$:

$$\Gamma(\psi_A \to \psi_B \gamma) = \frac{(m_A^2 - m_B^2)^3}{(2m_A)^3 (m_A + m_B)^2} \frac{|V(0)|^2}{(2J_A + 1)\pi}$$

The effective mass spectrum of the leptonic pair is[1]

$$\frac{d\Gamma(\psi_A \to \psi_B l^+ l^-)}{dq^2 \cdot \Gamma(\psi_A \to \psi_B \gamma)} = \frac{\alpha}{3\pi} \sqrt{1 - \frac{4m_l^2}{q^2}} \left(1 + \frac{2m_l^2}{q^2}\right) \frac{1}{q^2} \times \left[\left(1 + \frac{q^2}{m_A^2 - m_B^2}\right)^2 - \frac{4m_A^2 q^2}{(m_A^2 - m_B^2)^2}\right]^{3/2} \left|\frac{V(q^2)}{V(0)}\right|^2$$

We could compare with experimental results and make predictions through $V(q^2)$.

[1] L. G. Landsberg, Phys. Rept. 128, 301 (1985).

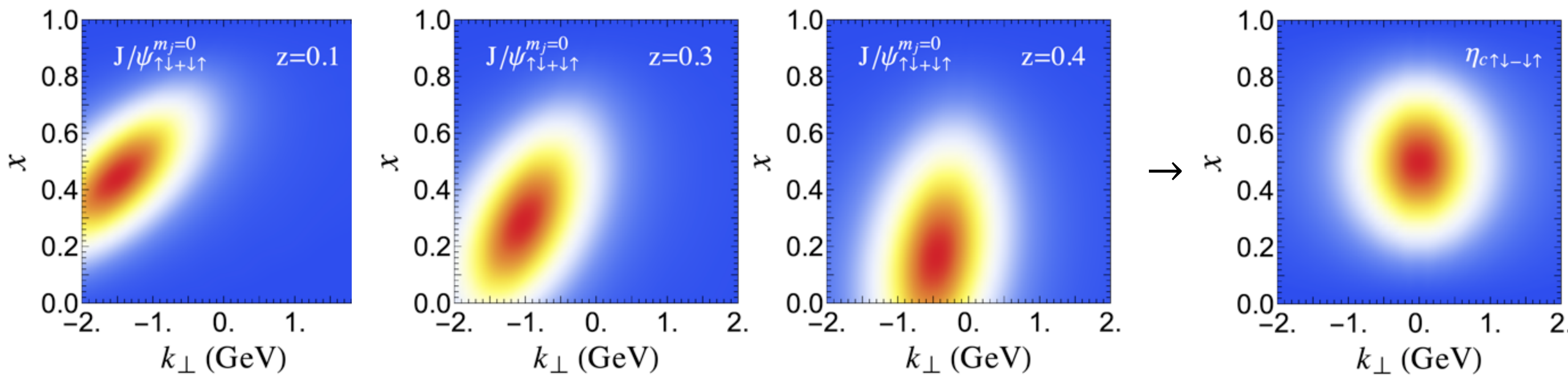
Backup: frames

Matrix element as the convolution of light-front wavefunctions:

$$\langle \psi_{B,q\bar{q}}(P, j, m_j)| J^\mu |\psi'_{A,q\bar{q}}(P', j', m'_j)\rangle$$
$$= \sum_{s,\bar{s}} \int_z^1 \frac{\mathrm{d}x'}{2x'(1-x')} \int \frac{\mathrm{d}^2 \vec{k}'_\perp}{(2\pi)^3} \frac{1}{x} \sum_{s'} \psi^{(m_j)*}_{s\bar{s}/B}(\vec{k}_\perp, x) \psi^{(m'_j)}_{s'\bar{s}/A}(\vec{k}'_\perp, x')$$
$$\times \bar{u}_s(xP^+, \vec{k}_\perp + x\vec{P}_\perp)\gamma^\mu u_{s'}(x'P'^+, \vec{k}'_\perp + x'\vec{P}'_\perp) ,$$

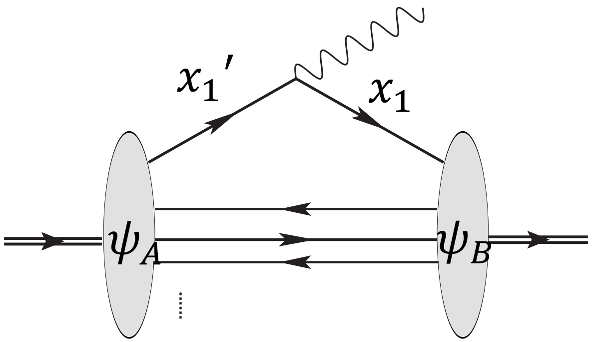
Transition form factors at the same q^2 could be evaluated at different $(z, \Delta_\perp)$.

$$J/\psi\left(\vec{k}'_\perp = \vec{k}_\perp + (1-x)\vec{\Delta}_\perp, x' = x + z(1-x)\right) \to \eta_c(\vec{k}_\perp, x) \quad (\text{at } q^2 = -3\text{GeV}^2)$$



Backup: hadron matrix

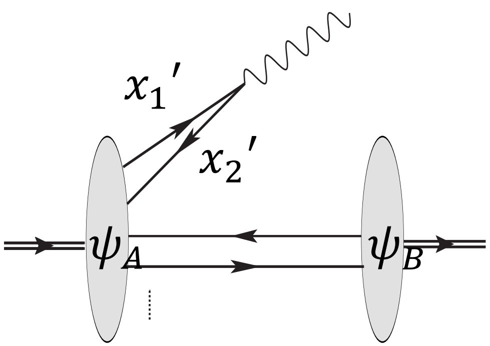
Hadron matrix elements as convolutions of light-front wavefunctions:



$$\langle\psi_{B,n}(P)|\, J^\mu\, |\psi_{A,n}(P')\rangle$$

$$= \int_z^1 \frac{\mathrm{d}x_1'}{2x_1'} \int_0^1 \frac{\mathrm{d}x'_{i(i\neq 1)}}{2x_i'} \int \frac{\mathrm{d}^2 k'_{i\perp}}{(2\pi)^3} \delta_E \psi^*_{n/B} \psi_{n/A} j^\mu_{s_1,s_1'}$$

$$x_1' = x_1 + z(1-x_1),\ \vec{k}'_{1\perp} = \vec{k}_{1\perp} + (1-x_1)\vec{\Delta}_\perp$$



$$\langle\psi_{B,n}(P)|\, J^\mu\, |\psi_{A,n+2}(P')\rangle$$

$$= \int_0^z \frac{\mathrm{d}x_1'}{2x_1'} \int_0^1 \frac{\mathrm{d}x'_{i(i\neq 1,2)}}{2x_i'} \int \frac{\mathrm{d}^2 k'_{i\perp}}{(2\pi)^3} \delta_E \psi^*_{n/B} \psi_{n+2/A} j^\mu_{s_1,s_2}$$

$$x_2' = z - x_1{}' \text{ and } \vec{k}'_{2\perp} = -\vec{k}'_{1\perp} + \vec{\Delta}_\perp.$$

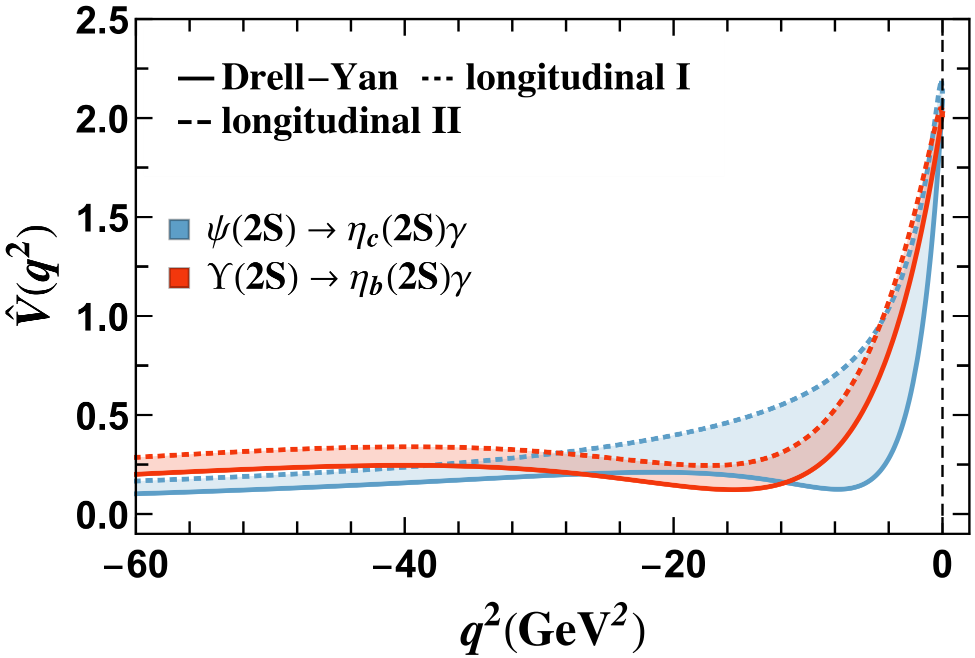
The $n+2 \to n$ contribution could be suppressed by small z, suggesting that small z may be preferred for calculation in $|q\bar{q}\rangle$.

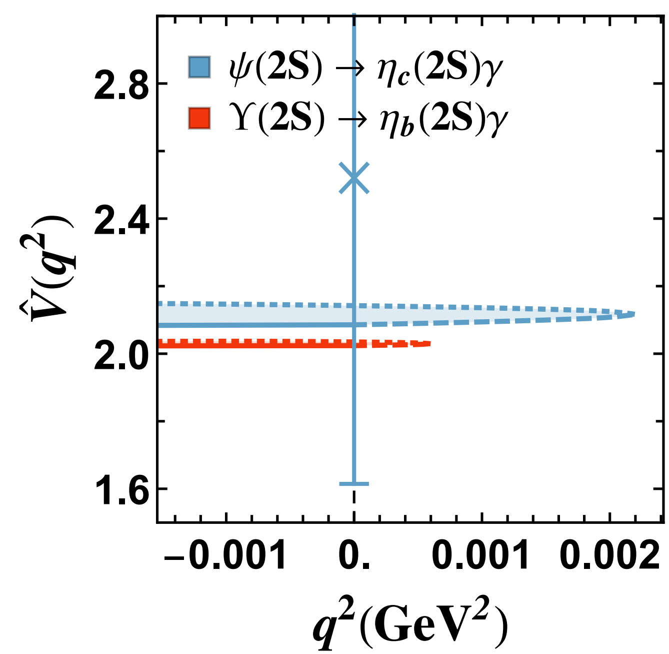
$z \equiv (P'^+ - P^+)/P'^+$ and $\vec{\Delta}_\perp \equiv \vec{q}_\perp - z\vec{P}'_\perp$

Backup: more result

The *allowed* transition: initial and final states have the same radial and angular quantum numbers.

$$\psi_i(nS) \to \psi_f(nS)\,\gamma$$





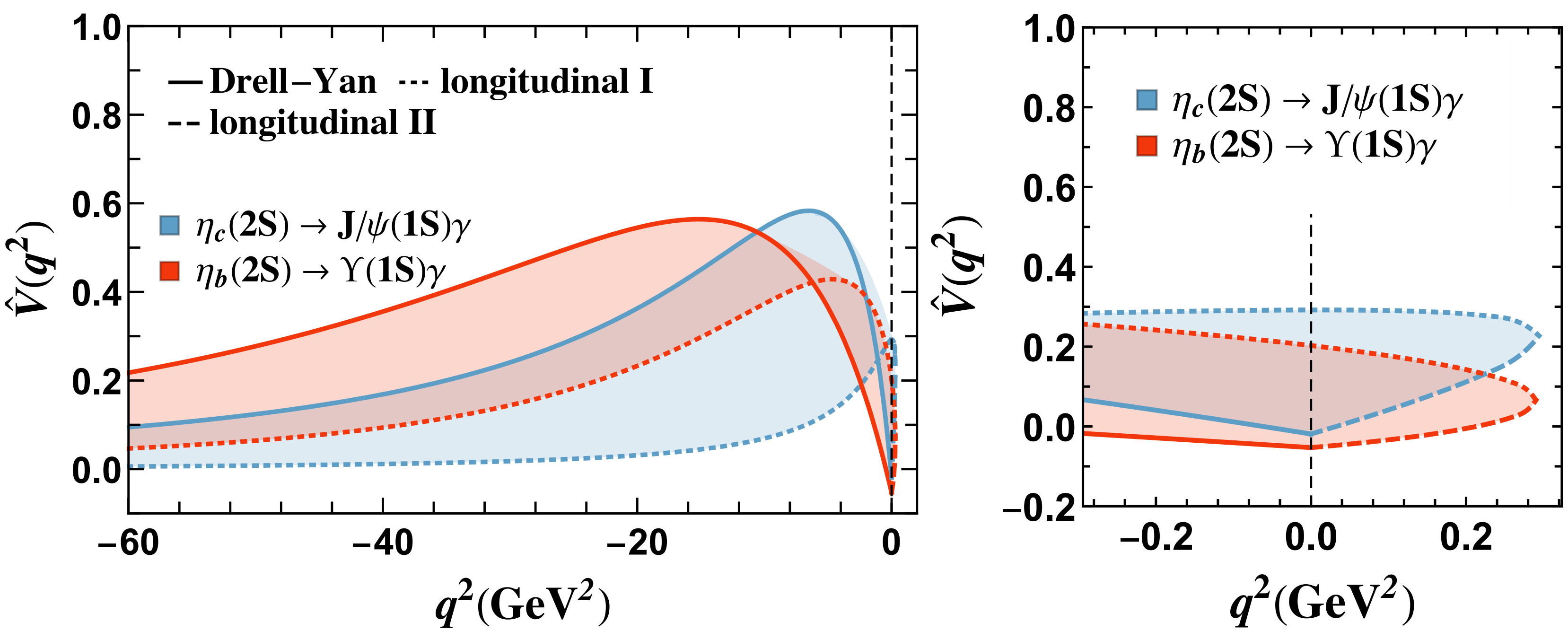
Results from all other frames are in the shaded areas bounded by the Drell-Yan and the longitudinal frames.

✕ **PDG:** M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D98, 030001 (2018)

Backup: more result

The *hindered* transition: initial and final states have different radial quantum numbers.

$$\psi_i(nS) \to \psi_f(n'S)\,\gamma, (n \neq n')$$



Shaded areas represent the results from all other frames.

Backup: basis convergence

Trends toward convergence with increasing basis cutoff are observed in different frames.

Frame dependence remains visible.