Transverse Force Tomography

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April 11, 2019

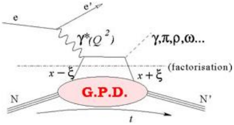
- GPDs $\longrightarrow$ 3D imaging of the nucleon
- twist-3 PDFs $g_2(x) \longrightarrow \perp$ force

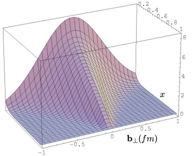
$\hookrightarrow$ twist-3 GPDs $\longrightarrow \perp$ force tomography

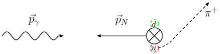
Motivation: why twist-3 GPDs
  - twist-3 GPD $G_2^q \longrightarrow L^q$
  - twist 3 PDF $g_2(x) \longrightarrow \perp$ force
  - twist 2 GPDs $\longrightarrow \perp$ imaging (of quark densities)

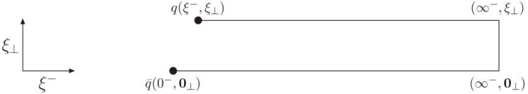
  $\hookrightarrow$ twist 3 GPDs $\longrightarrow \perp$ imaging of $\perp$ forces

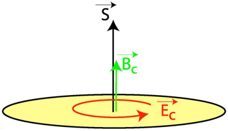
- Summary











MB, PRD62, 071503 (2000)

- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$ — relativistic corrections!!!
- $GPDs(x, \xi, \vec{\Delta})$: form factor for quarks with momentum fraction x

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction x

Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} GPD(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i \in q,g} \mathbf{r}_{\perp,i} x_i$

- probabilistic interpretation!
- no relativistic corrections: Galilean subgroup! (MB,2000)

$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections (MB, 2003; G.A.Miller, 2007)

$\perp$ localized state

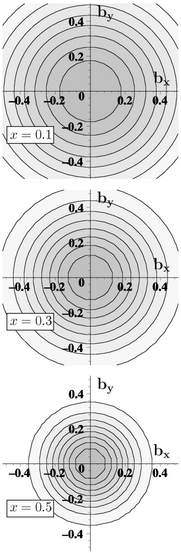
$$|\mathbf{R}_\perp = 0, p^+, \Lambda\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp |\mathbf{p}_\perp, p^+, \Lambda\rangle$$

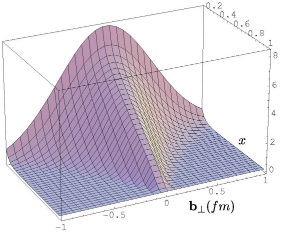
$\perp$ charge distribution (unpolarized quarks)

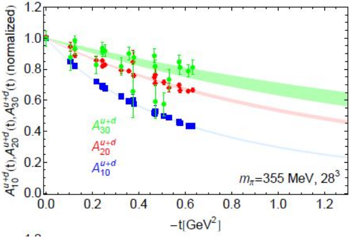
$$\begin{aligned}
\rho_{\Lambda'\Lambda}(\mathbf{b}_\perp) &\equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle \\
&= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)} \\
&= \int d^2\mathbf{\Delta}_\perp F_{\Lambda'\Lambda}(-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}
\end{aligned}$$

- crucial: $\langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle$ depends only on $\mathbf{\Delta}_\perp$
- $F_{\Lambda'\Lambda}(-\mathbf{\Delta}_\perp^2)$ some linear combination of F_1 & F_2 - depending on Λ, Λ'
- similar for various polarized quark densities
- similar for x-dependent densities $\longrightarrow$ GPDs

$q(x, \mathbf{b}_\perp)$ for unpol. p



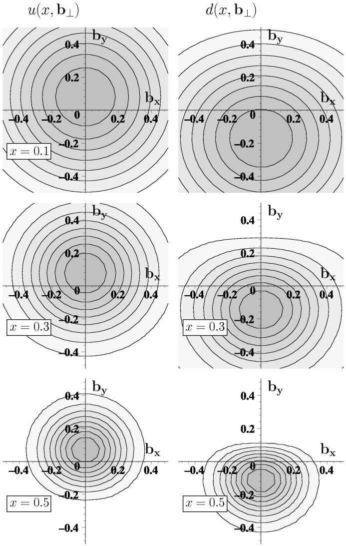




unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- $F_1(-\boldsymbol{\Delta}_\perp^2) = \int dx H(x, 0, -\boldsymbol{\Delta}_\perp^2)$
- x = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$



proton polarized in $+\hat{x}$ direction

no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

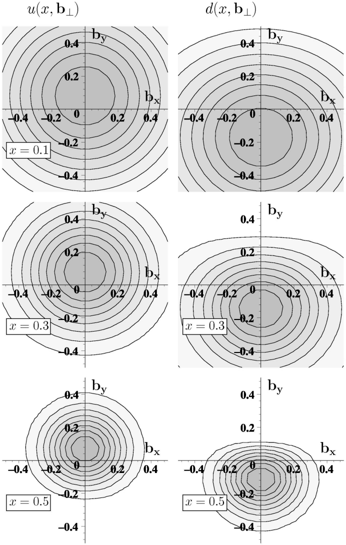
Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

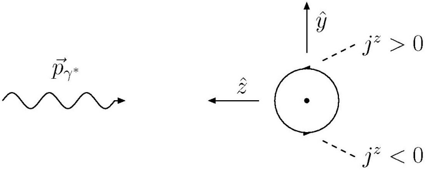
intuitive explanation

- moving Dirac particle with anomalous magnetic moment has electric dipole moment $\perp$ to $\vec{p}$ and $\perp$ magnetic moment

$\hookrightarrow$ γ^* 'sees' flavor dipole moment of oncoming nucleon

Impact parameter dependent quark distributions





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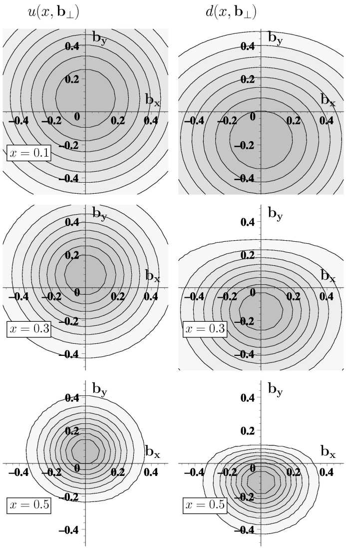

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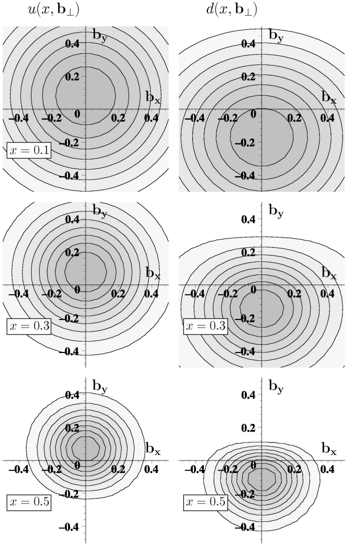
sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$



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$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + ...$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

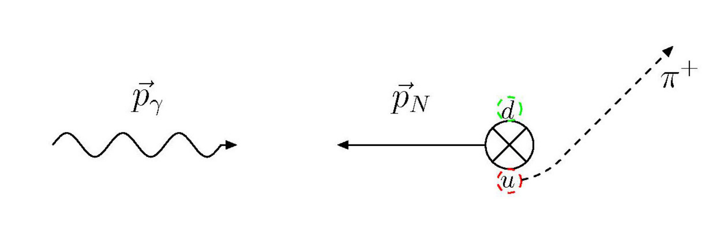
$\hookrightarrow$ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_u^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}\left(\pm 0.2 fm\right)$!!!!

example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\to$ chromodynamic lensing

$\Rightarrow$ $\kappa_p \,, \kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

- confirmed by Hermes & Compass data

This is not stamp collecting

- twist 3 may have to be included to fit JLab data
- twist 3 necessary to understand what makes nucleon structure

↪ need to understand 'the force'

- alternative angular momentum sum rule (M.Polyakov)
- transverse force tomography (this talk)
- there's really weird stuff going on at twist 3
  (F.Aslan + M.B. arXiv:1811.00938)

yes, this will be hard, but...

- lattice QCD can provide (genuine) twist 3 info much sooner

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2} S^x} \left\langle P, S \left| \bar{q}(0) \gamma^+ g F^{+y}(0) q(0) \right| P, S \right\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2} F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \quad \text{for } \vec{v} = (0, 0, -1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral

$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

sign of d_2

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of d_2^q: opposite Sivers

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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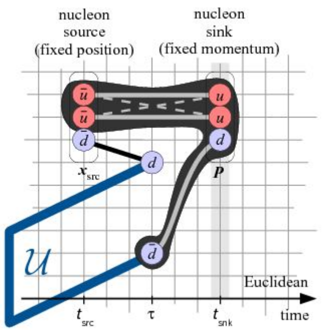
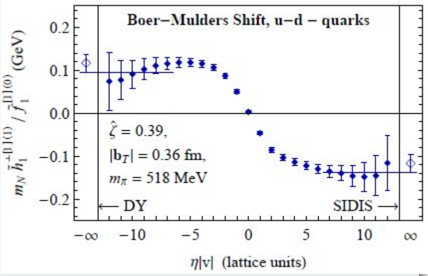
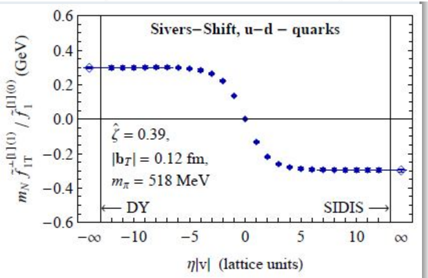
consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)







$f_{1T}^{\perp}(x, \mathbf{k}_\perp)$ is $\mathbf{k}_\perp$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target

The Force

slope at length =0

chirally even spin-dependent twist-3 PDF $g_2(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 g_2(x) \Rightarrow \perp$ force on unpolarized quark in $\perp$ polarized target

$\hookrightarrow$ 'Sivers force'

scalar twist-3 PDF $e(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 e(x) \Rightarrow \perp$ force on $\perp$ polarized quark in unpolarized target

$\hookrightarrow$ 'Boer-Mulders force'

chirally odd spin-dependent twist-3 PDF $h_2(x)$

M.Abdallah & MB, PRD94 (2016) 094040

- $\int dx\, x^2 h_2(x) = 0$

$\hookrightarrow$ $\perp$ force on $\perp$ pol. quark in long. pol. target vanishes due to parity

- $\int dx\, x^3 h_2(x) \Rightarrow$ long. gradient of $\perp$ force on $\perp$ polarized quark in long. polarized target

$\hookrightarrow$ chirally odd 'wormgear force'

$\perp$ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Form factors of qgq correlator (F.Aslan, M.B., M.Schlegel arXiv:1904.03494)

$$\langle p', \lambda' | \bar{q}(0)\gamma^+ igF^{+i}(0) q(0) | p, \lambda\rangle = \bar{u}(p', \lambda')\left[\frac{P^+}{M}\gamma^+ \frac{\Delta^i}{M}\Phi_1(t) + \frac{P^+}{M} i\sigma^{+i}\Phi_2(t) + \frac{P^+}{M}\frac{\Delta^i}{M}\frac{i\sigma^{+\Delta}}{M}\Phi_3(t) + \frac{P^+}{M}\frac{\Delta^+}{M}\frac{i\sigma^{i\Delta}}{M}\Phi_4(t)\right] u(p, \lambda).$$

crucial:

- for $p^{+\prime} = p^+$, $\langle p', \lambda' | \bar{q}(0)\gamma^+ igF^{+i}(0) q(0) | p, \lambda\rangle$ only depends on $\mathbf{\Delta}_\perp$

$\hookrightarrow$ similar to $\perp$ charge density ...

Transverse Force Tomography

⊥ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
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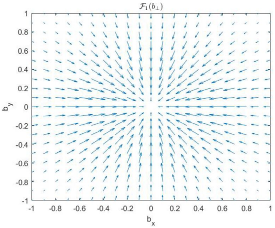
Φ_1

- unpolarized target
- axially symmetric 'radial' force



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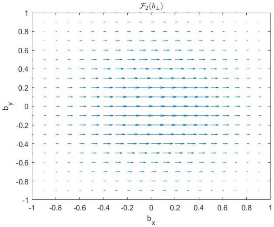

Φ_2

- $\perp$ polarized target; force $\perp$ to target spin

$\hookrightarrow$ spatially resolved Sivers force

$\perp$ force distribution (unpolarized quarks)

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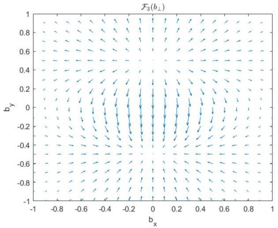
Φ_3

- tensor type force
- similar to charged particle flying through magnetic dipole field



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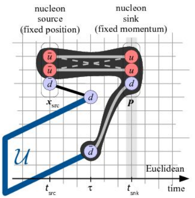
Φ_4

- no conmtribution for $\Delta^+ = 0$

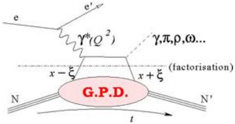
determining Φ_i

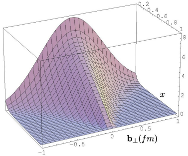
- match with x^2 moments of twist-3 GPDs (minus WW parts) *F.Aslan, M.B. in progress*
- experiments may take a few years, or immediately
- lattice QCD: fit to nonforward matrix elements of 'the force'-operator *in progress (J.Bickerton, R.Young, J.Zanotti)*

the force operator



- form factor with quark denstity involving Wilson line staple
- take derivative w.r.t. staple length at length =0

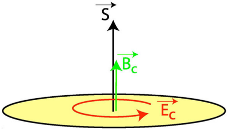


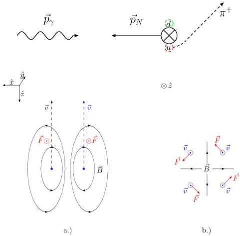


- GPDs $\stackrel{FT}{\longrightarrow} q(x,\mathbf{b}_\perp)$ '3d imaging'
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs

$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp}\Gamma q$ distribution

$\hookrightarrow$ $\perp$ force tomography





Summary

- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs

$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp}\Gamma q$ distribution

$\hookrightarrow$ $\perp$ force tomography