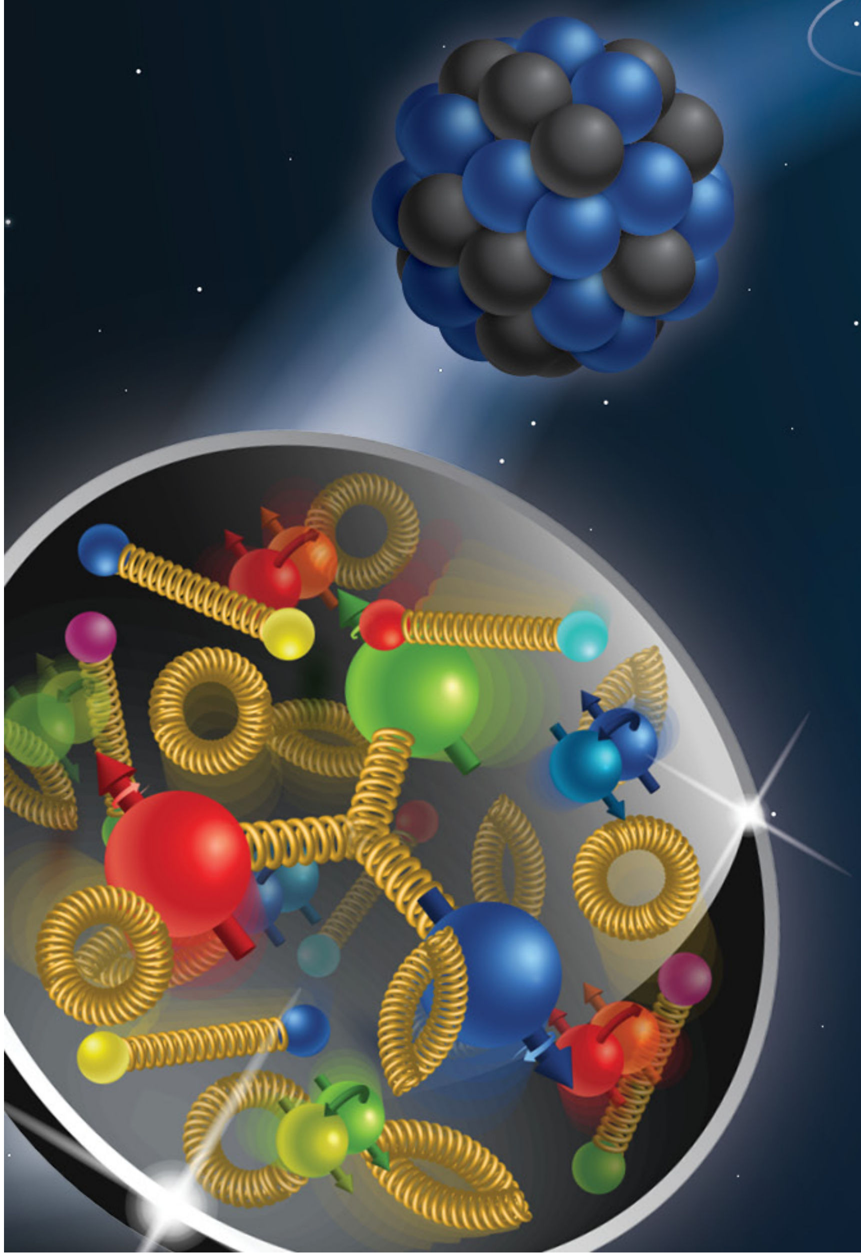

# Spin matching of interaction region with solenoidal spin rotators


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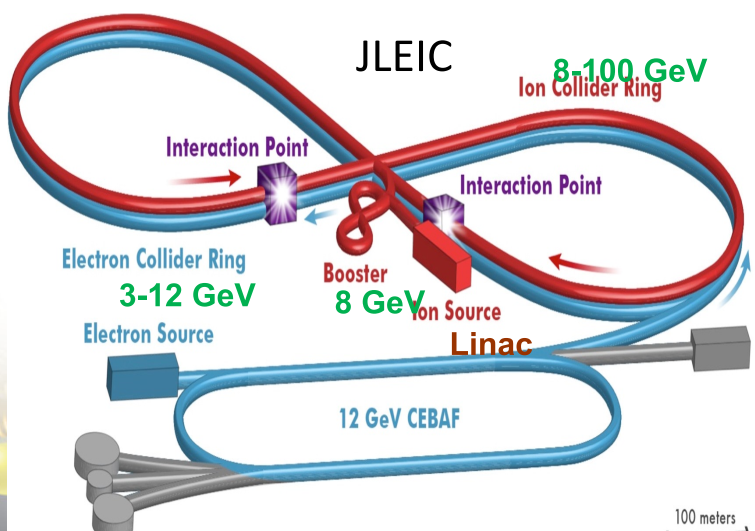
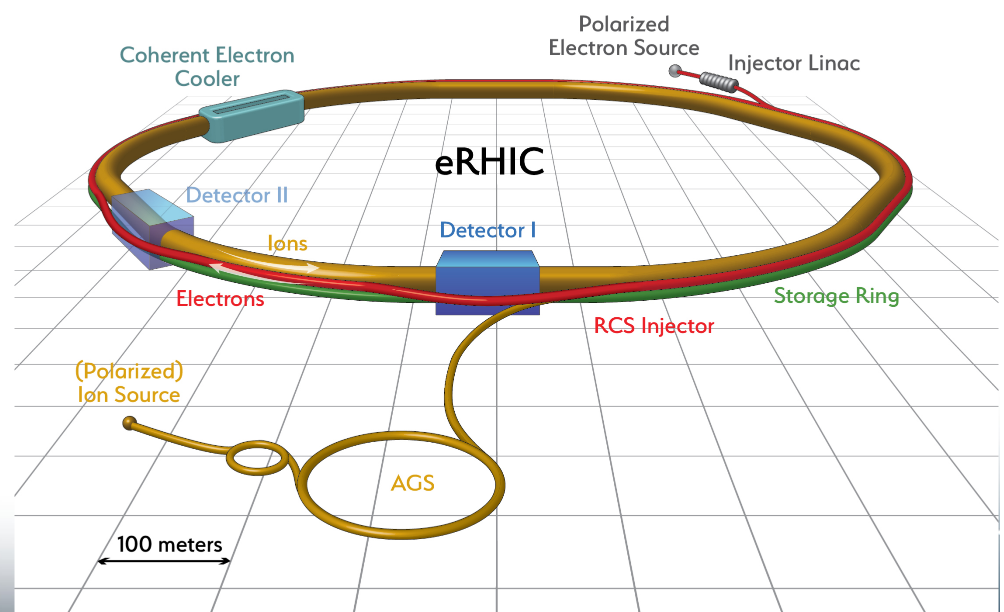
# Longitudinally polarized electrons in EIC

- Two EIC designs have been developed by Jlab (JLEIC) and BNL (eRHIC)
- In both designs highly polarized electron beam is injected into a storage ring.
- Depolarization happens due to Sokolov-Ternov and spin diffusion processes. Depolarization time has to be maximized as much as possible (Ideally to one defined by only ST process.
- Spin rotators are used around IP(s) for longitudinal polarization

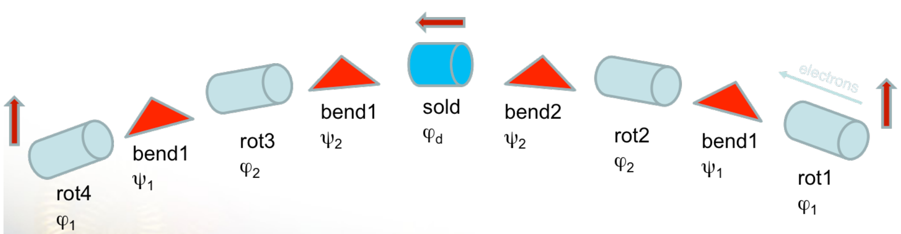


# Electron spin rotators for EIC designs

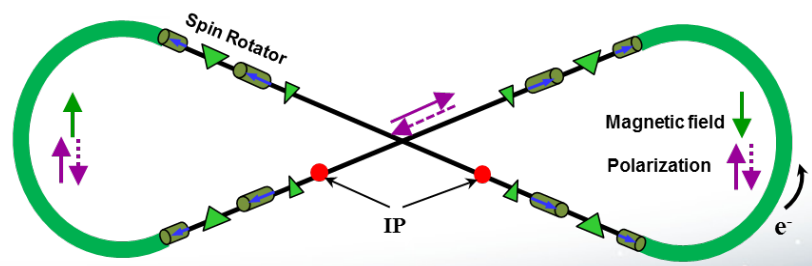
- Electron energy range: 3-12 GeV (JLEIC), 5-18 GeV (eRHIC)
- A HERA-type rotator (based on sequence of vertical and horizontal bend) creates meter scale orbit excursion at lower energies.
- The rotator design capable to operate in all energy range is based on the combination of solenoidal and horizontal bending magnets.

eRHIC spin rotator
C-type bending configuration



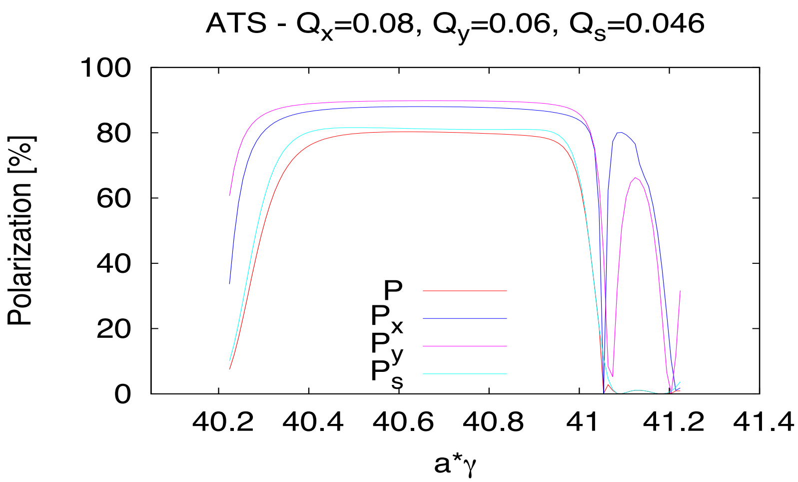

$\varphi_j$ – spin rotation angle in solenoids
$\psi_i$ – spin rotation angle in bends

JLEIC spin rotator
S-type bending configuration

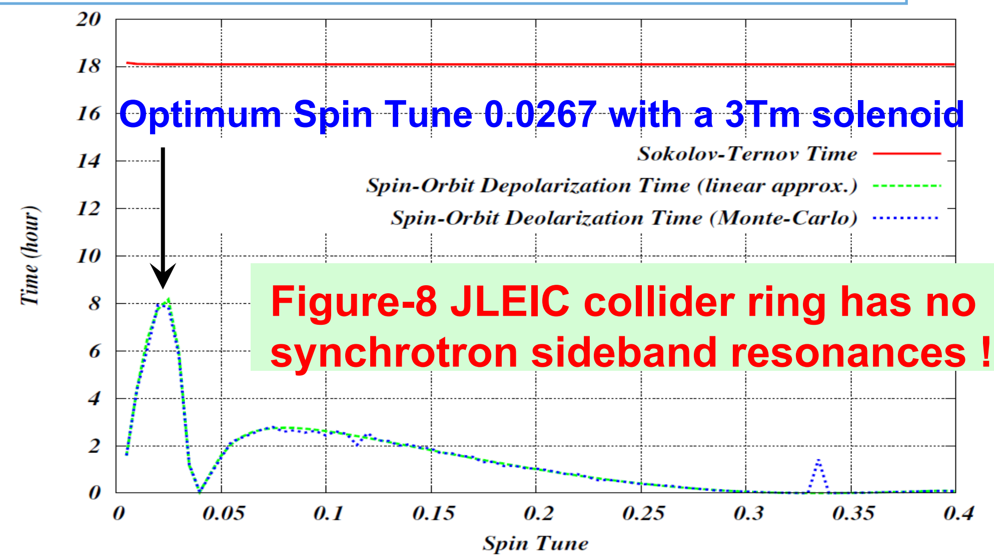


# First-order spin resonances are seen in both EIC designs with spin rotators

Even without misalignment and magnet errors the spin rotators create a pattern of depolarizing resonances.
The spin matching has to be done to minimize depolarization.




eRHIC first-order calculation in 18 GeV area (E.Gianfelice-Wendt).
No longitudinal matching of spin rotators
Not perfect betatron spin matching




JLEIC polarization evaluation for 5 GeV (F.Lin) .
No betatron spin matching

# Polarization evolution

Synchrotron radiation determines the polarization evolution through Sokolov-Ternov spin-flip emission and spin diffusion caused by quantum emission of S photons. Both processes combined define the equilibrium polarization $P_{eq}$ and polarization relaxation time t.

$$P(t) = (P_0 - P_{eq})\, e^{-t/\tau} + P_{eq}$$

Derbenev-Kondratenko: (1973)

$$P_{eq} = -\frac{8}{5\sqrt{3}} \frac{\alpha_-}{\alpha_+}$$

$$\tau^{-1} = \frac{5\sqrt{3}}{8} \frac{\hbar\, r_0}{m} \gamma^5 \alpha_+$$

$$\alpha_- = \left\langle \oint \frac{d\theta}{|\rho|^3} \hat{\boldsymbol{b}} (\boldsymbol{n} - \boldsymbol{d}) \right\rangle$$

$$\alpha_+ = \left\langle \oint \frac{d\theta}{|\rho|^3} \left[ 1 - \frac{2}{9} (\boldsymbol{n}\hat{\boldsymbol{v}})^2 + \frac{11}{18} |\boldsymbol{d}|^2 \right] \right\rangle$$

Depolarization caused by spin diffusion is defined by a derivative of invariant spin field over $\delta = \frac{\Delta E}{E} = \frac{\Delta \gamma}{\gamma}$ :

$$\boldsymbol{d} = \gamma \frac{\partial \boldsymbol{n}}{\partial \gamma}$$ (taken at const x, x', y, y')

$$\left( \frac{\partial \mathbf{n}}{\partial \delta} \right)_{A_x, A_y} \qquad \frac{\partial \mathbf{n}}{\partial A_x}$$

# First-order spin perturbation consideration

The magnetic fields on design orbit define the periodical spin solution $\mathbf{n_0}$ and two others spin eigenvectors $\mathbf{k_0} = \mathbf{l_0} + i\mathbf{m_0}$, $\mathbf{k_0}^*$.

$\mathbf{l_0}, \mathbf{m_0}, \mathbf{n_0}$ form normalized triad convenient for considering spin motion perturbations.

In the first order spin perturbation $\alpha_0$ by momentum deviation or betatron motion is described by the following equation:

$$\frac{d\alpha_0}{ds} = -i\mathbf{w} \cdot \mathbf{k}_0$$

where components perturbation precession vector $\mathbf{w}$ (neglecting terms of order of anomalous magnetic moment $a$ ) are:

$$w_x = \left(1+\gamma_0 a\right) y'' + K_s x'; \quad w_s = K_y' y - K_s \frac{\Delta E}{E_0} - \gamma_0 a K_y y'; \quad w_y = -\left(1+\gamma_0 a\right) x'' + \gamma_0 a K_y \frac{\Delta E}{E_0} + K_s y'$$

$$K_y = \frac{B_y}{B\rho} \quad ; \qquad K_s = \frac{Bs}{B\rho}$$

With proper periodical conditions the solution of this equation gives the invariant spin field in first order.

# Derivation of spin matching conditions

**Spin matched spin rotator system:**
the spin invariant field (alpha_0) dependence on horizontal betatron amplitude $A_x$ and energy deviation $\delta$ is not allowed outside the rotator system. → $\partial n / \partial \delta = 0 \quad \partial n / \partial A_x = 0$

Thus avoiding any spin dynamics distortion by synchrotron radiation in the arc bends.

The following integral over the whole spin rotator system must be made 0 for terms proportional to $A_x$ and $\delta$:

$$\int_{s_{in}}^{s_{out}} \left( w_x k_{0x} + w_s k_{0s} + w_y k_{0y} \right) ds = 0$$

The orbital motion is considered in a standard form through components of betatron motion eigen-vectors $f_I$ *and* $f_{II}$ and dispersion functions $D_x$, $D_y$:

$$x = f_{Ix} A_x + f_{Ix}^* A_x^* + f_{IIx} A_y + f_{IIx}^* A_y^* + D_x \delta$$

$$y = f_{Iy} A_x + f_{Ix}^* A_x^* + f_{IIy} A_y + f_{IIy}^* A_y^* + D_y \delta$$

# Spin matching conditions for solenoidal rotators

We assume following reasonable optics conditions:

-betatron coupling is fully compensated individually for each of four solenoidal insertions by dividing each solenoid in two parts and using set of quadrupoles/skew quadrupoles between and around them

-the vertical dispersion function $D_y$ does not leak into the horizontal bends

Then, using integration by parts one gets following set of spin matching conditions:

$$\sum_{rot:j=1,4} H_j\left(f_I\right)=0\,; \quad \sum_{rot:j=1,4} H_j\left(f_I^*\right)=0\,;$$

Betatron conditions

$$a\gamma \sum_{rot:j=1,4} H_j\left(D\right)+\sum_{rot:j=1,4}\varphi_j k_{sj}-\sum_{bends:i=1,4}\psi_j k_{yi}=0$$

longitudinal motion condition

where:

$$H_j\left(F\right)=\frac{\varphi_j}{2}\left[\left(k_x\left(F_x'+\frac{K_s}{2}F_y\right)+k_y\left(F_y'-\frac{K_s}{2}F_x\right)\right)_{j,entrance}+\left(k_x\left(F_x'+\frac{K_s}{2}F_y\right)+k_y\left(F_y'-\frac{K_s}{2}F_x\right)\right)_{j,exit}\right]$$

*F* is either $f_I$ or *D*

# Longitudinal spin matching

$$a\gamma \sum_{rot:j=1,4} H_j(D) + \sum_{rot:j=1,4} \varphi_j k_{sj} - \sum_{bends:i=1,4} \psi_j k_{yi} = 0$$

This term can be nullified either by not allowing dispersion function in solenoids or by proper optics

This combination is completely defined by the choice of bending angles of the rotator dipoles and solenoidal fields.

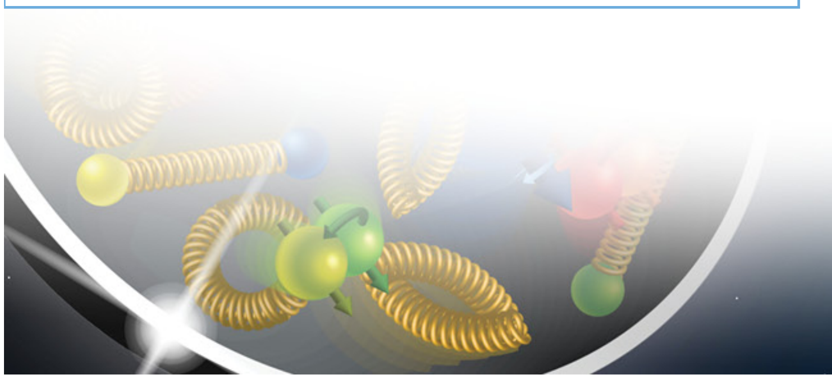
For JLEIC S-type bending configuration and spin-up to spin-up transformation through the whole rotator system: automatically zero .

For eRHIC C-type bending configuration can be nullified at a particular energy with following choice of rotator parameters:

$\varphi_1 = \varphi_4 = 0.524$ rad, $\varphi_2 = \varphi_3 = 2.094$ rad

$\psi_1 = \psi_4 = \pi$ rad, $\psi_2 = \psi_3 = \pi/2$ rad

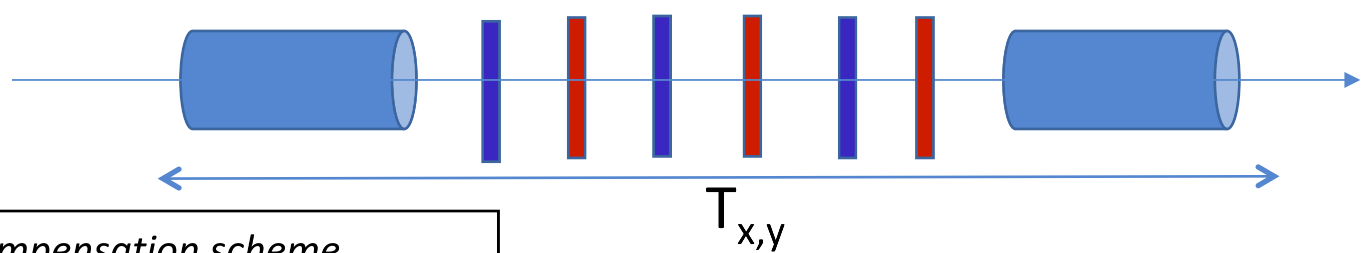
It makes sense to fully match the rotators at eRHIC highest energy, 18 GeV



# Solenoidal insertion with betatron spin matching

Spin matching conditions related with betatron motion can be satisfied for each individual solenoidal insertion, using two solenoid halves and (at least) 6 quadrupoles between them.

That is for each j : $H_j(f_I) = 0$ and $H_j(f_I^*) = 0$




*Coupling compensation scheme by A. Zholents and V. Litvinenko (1984)*

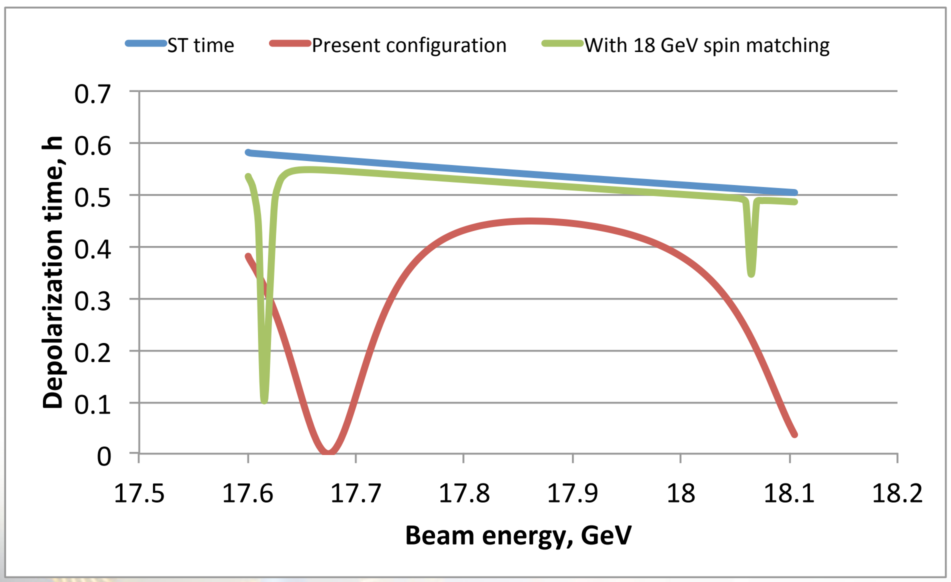
For a betatron spin-matched and fully decoupled solenoidal insertion the horizontal and vertical transport matrices must have following forms:

$$T_X = \begin{pmatrix} -\cos(\varphi) & -\frac{2}{K_s}\sin(\varphi) \\ \frac{K_s}{2}\sin(\varphi) & -\cos(\varphi) \end{pmatrix}; \quad T_Y = -T_X = \begin{pmatrix} \cos(\varphi) & \frac{2}{K_s}\sin(\varphi) \\ -\frac{K_s}{2}\sin(\varphi) & \cos(\varphi) \end{pmatrix}$$

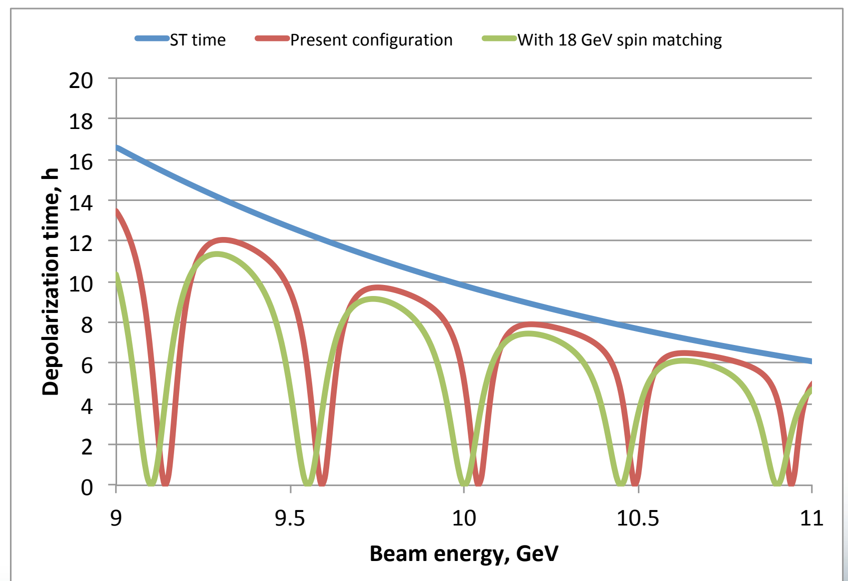
$$K_s = \frac{B_s}{B\rho} \qquad \varphi = (1+a)K_s$$

# eRHIC depolarization time

At high energy area of eRHIC the spin matching provides considerable improvement for depolarization time




At lower energies spin matching, optimized for 18 GeV, is not effective.
But, depolarization time is very large anyway.




Note: since synchrotron motion is not included, there is no split into first-order sidebands on the plots.

# Summary

✧To maximize the depolarization time the spin rotator insertions need spin matching

✧The conditions for spin matching of rotators based on a sequence of solenoidal and bending magnets have been derived from spin-orbital integrals

✧Betatron related spin-matching can be done by using a special transport matrix of  solenoidal insertions

✧eRHIC rotator parameters providing longitudinal spin matching would improve the depolarization time at higher energies.
It needs to be verified beyond first-order with spin simulations.