

# eRHIC: Electron Beam Polarization in the Storage Ring

## Outline

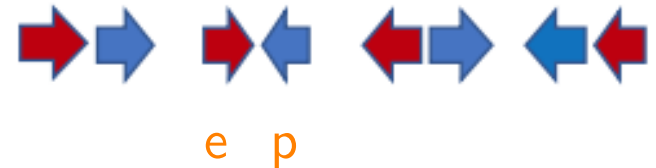
- Radiative polarization and the eRHIC storage ring.
- Simulations of polarization in the eRHIC storage ring.
  - Effect of mis-alignments.
- Summary and Outlook.

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EIC Meeting, JLab October 31, 2018

## Radiative polarization and the eRHIC storage ring

Experiments require

- Large proton and electron polarization ( $\gtrsim 70\%$ )
- Longitudinal polarization at the IP with *both* helicities within the *same* store
- Energy
  - protons: between 41 and 275 GeV
  - electrons: between 5 and 18 GeV

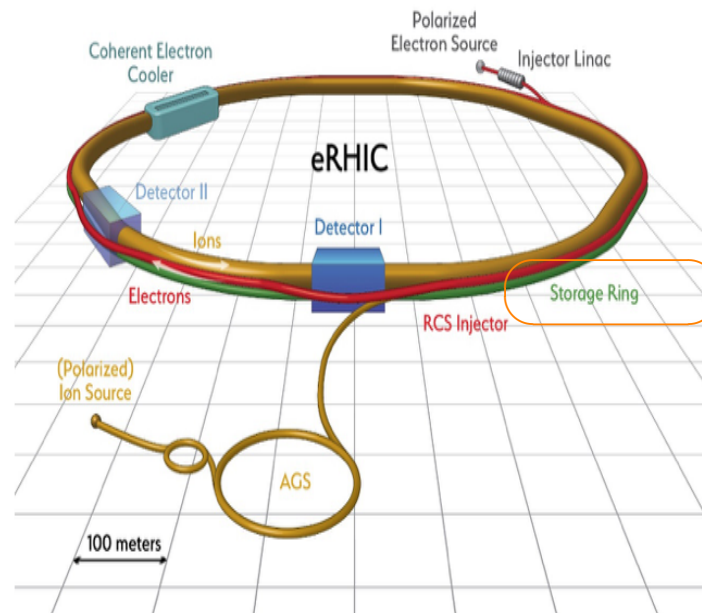


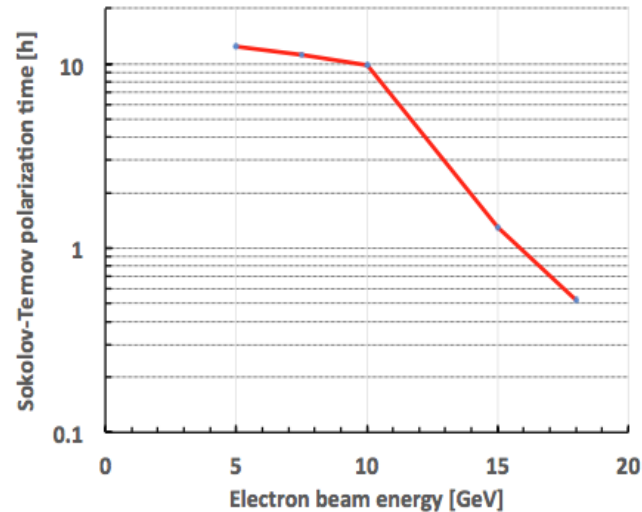
High proton polarization is already routinely achieved in RHIC.

Studies are needed instead for the electron beam polarization.

Because the experimenters call for storage of electron bunches with both spin helicities Sokolov-Ternov effect is not an option but rather a *nuisance*!

- A full energy polarized electron injector is needed: electron bunches are injected into the storage ring with high *vertical* polarization ( $\approx 85\%$ ) and the desired spin direction (up/down).
- In the storage ring the polarization is brought into the longitudinal direction at the IP by a couple of solenoidal spin rotators left and right of the IP.





In the eRHIC energy range the minimum polarization time *nominally* is  $\tau_p \simeq 30'$  at 18 GeV. At first sight a large time before Sokolov-Ternov effect reverses the polarization of the down-polarized electron bunches...

However the machine imperfections may quickly depolarize the whole beam.

Polarization builds-up exponentially

$$P(t) = P_{\infty}(1 - e^{-t/\tau_p}) + P(0)e^{-t/\tau_p}$$

In the presence of depolarizing effects it is

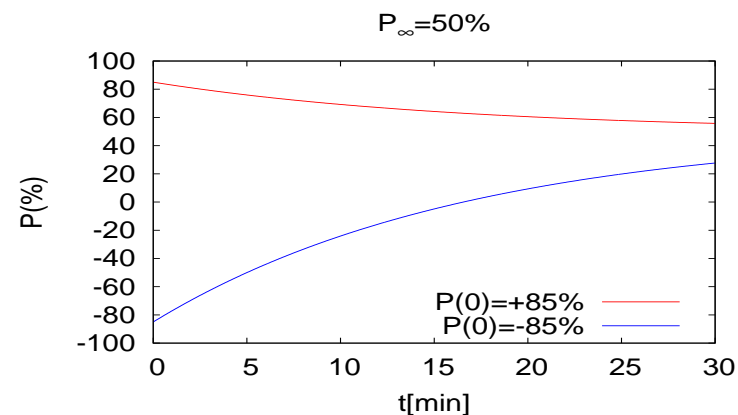
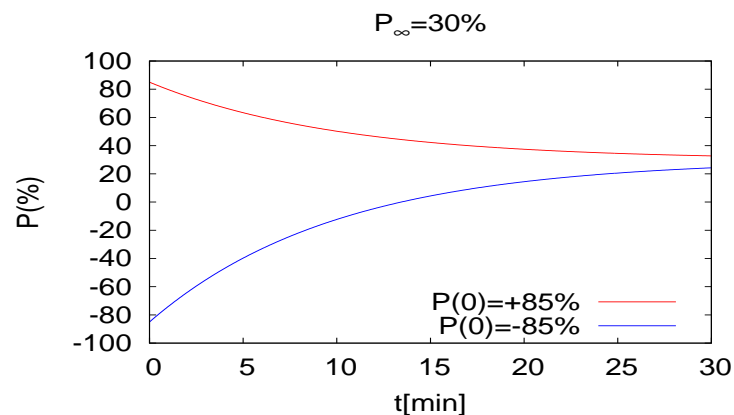
$$P_{\infty} \simeq \frac{\tau_p}{\tau_{\text{BKS}}} P_{\text{BKS}} \quad \text{and} \quad \frac{1}{\tau_p} \simeq \frac{1}{\tau_{\text{BKS}}} + \frac{1}{\tau_d}$$

$P_{\text{BKS}}$  and  $\tau_{\text{BKS}}$  are the Baier-Katkov-Strakhovenko generalization of the Sokolov-Ternov quantities when  $\hat{n}_0$  is not everywhere perpendicular to the velocity.

They may be computed “analytically”; for eRHIC storage ring at 18 GeV it is

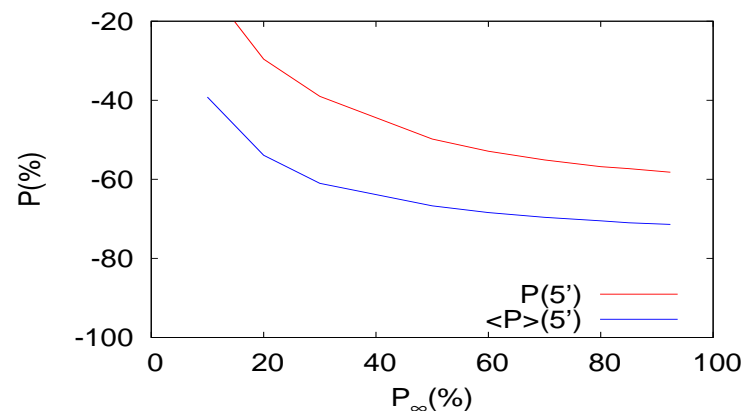
- $P_{\text{BKS}} \simeq 90\%$
- $\tau_{\text{BKS}} \simeq 30$  minutes.

## $P$ for bunches polarized parallel or anti-parallel to the bending field



For instance, with  $P_\infty = 30\%$ , after 5 minutes  $P$  decays from 85% to

- 60% for *up* polarized bunches  
 $\rightarrow \langle P \rangle = 73\%$
- -39% for *down* polarized bunches  
 $\rightarrow \langle P \rangle = -61\%$



$\rightarrow$  No much gain pushing  $P_\infty$  above  $\approx 50\%$ .

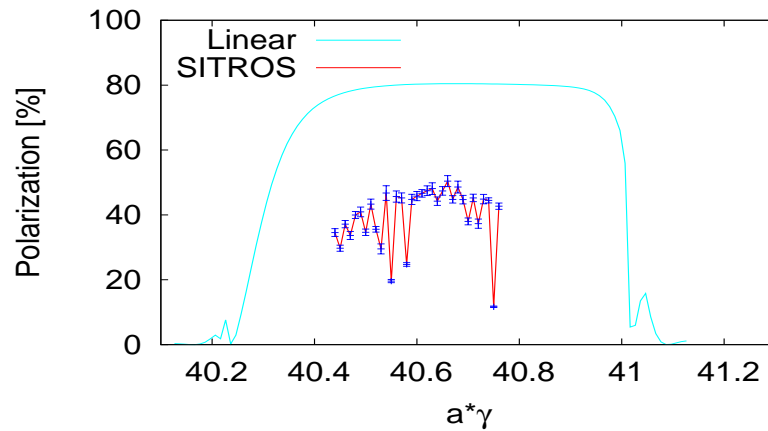
## Simulations for the eRHIC storage ring

- Energy: 18 GeV, the most challenging.
- Simulations shown here are for the “ATS” optics with
  - $90^\circ$  FODO for both planes;
  - $\beta_x^* = 0.7$  m and  $\beta_y^* = 9$  cm.
- Working point for luminosity:  $Q_x = 60.12$ ,  $Q_y = 56.10$ ,  $Q_s = 0.046$

### Tools:

- [MAD-X](#) used for simulating quadrupole misalignments and orbit correction.
- [SITROS](#) (by J. Kewisch) used for computing the resulting polarization.
  - Tracking code with 2nd order orbit description and fully non-linear spin motion.
  - Used for HERA-e in the version improved by M. Böge and M. Berglund.
  - It contains SITF (6D) for analytical polarization computation with *linearized* spin motion.

ATS -  $Q_x=0.12, Q_y=0.10, Q_s=0.046$

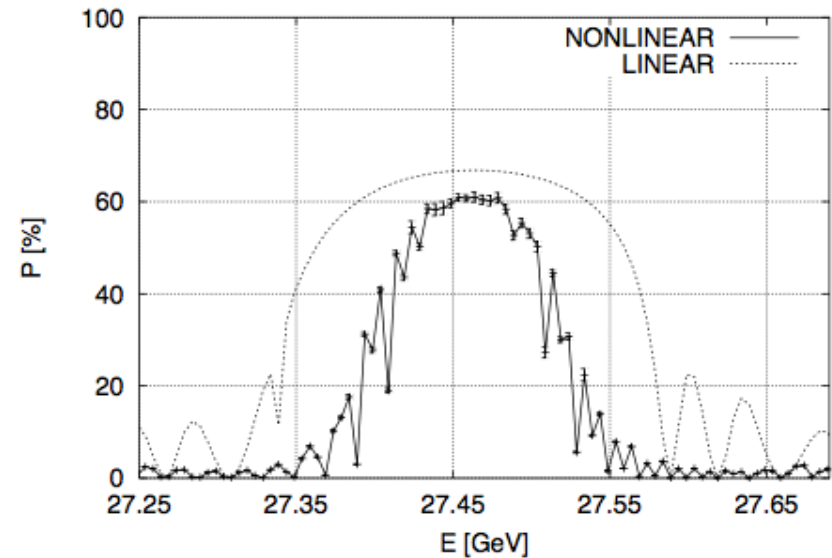


## Beam size at IP

	$\sigma_x$	$\sigma_y$	$\sigma_l$
	[mm]	[ $\mu\text{m}$ ]	[mm]
SITF	0.121	0.588	6.967
SITROS	0.135	1.776	7.046

## Two problems

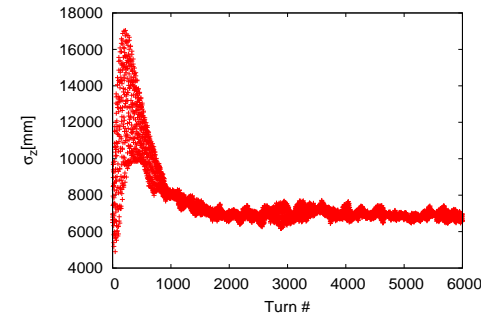
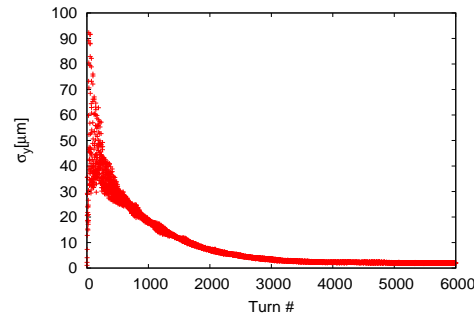
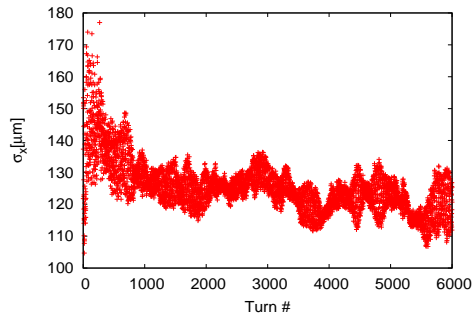
- large equilibrium  $\epsilon_y$ ;
- unusual large difference between linearized calculation and tracking.



For comparison: Hera-e with 3 rotators



Bmad (by D. Sagan) implemented on a MAC laptop for cross-checking SITROS results. 300 particles tracked over 6000 turns (typical SITROS parameters) with SR and stochastic emission with Bmad “standard” tracking.



Beam size

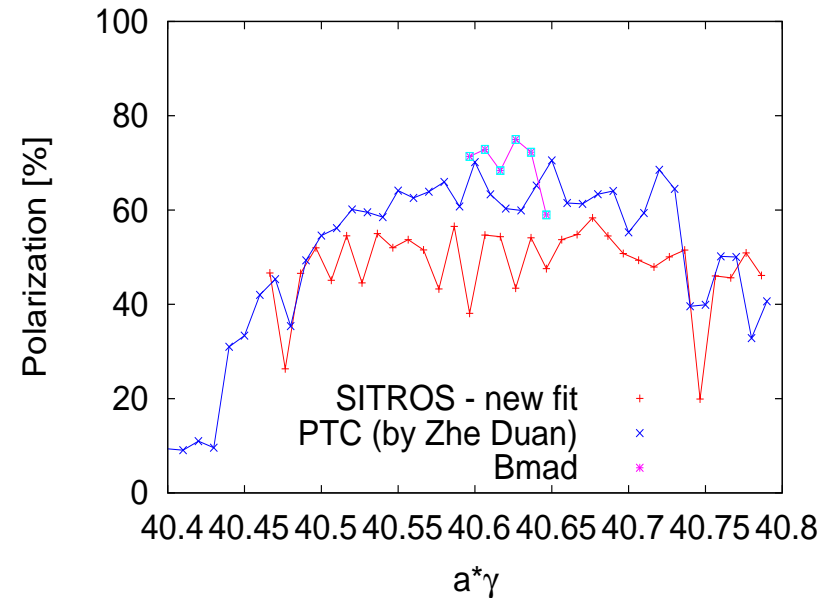
	$\sigma_x$	$\sigma_y$	$\sigma_l$	$\sigma_E$
	[ $\mu\text{m}$ ]	[ $\mu\text{m}$ ]	[mm]	[%]
analytical (Bmad)	123	0.4	7.0	0.1
Bmad tracking	120	2.0	6.7	0.1
SITROS	136	1.8	7.0	0.1

The large  $\epsilon_y$  is not a SITROS artifact.

Add spins following SITROS path:

- Once equilibrium is reached particles coordinates are dumped on file.
- Spins parallel to  $\hat{n}_0(0)$  are added and tracking re-started.

The spin tracking is very slow:  
300 particles and 3000 turns take  
over 24 hours for one single energy  
point!



D. Sagan has speeded up the tracking with spin recently (October 2018 release).

## Machine with misalignments

- 494 BPMs (h+v) added close to each quadrupole.
- 2x494 correctors (h+v) added close to each quadrupole.
- Magnet misalignments and orbit correction simulated by MAD-X.
- Optics with errors and corrections dumped into a SITROS readable file.

### Strategy

#### Assumed quadrupole RMS misalignments

horizontal offset	$\delta x^Q$	200 $\mu\text{m}$
vertical offset	$\delta y^Q$	200 $\mu\text{m}$
roll angle	$\delta \psi^Q$	200 $\mu\text{rad}$

- switch off sextupoles;
- move tunes to 0.2/0.3;
- introduce errors;
- correct orbit (MICADO/SVD);
- turn on sextupoles;
- tunes back to luminosity values.

# MAD-X fails correcting the orbit!

Example with only  $\delta y^Q \neq 0$  and sexts off.  
Large discrepancy between what the correction module promises...

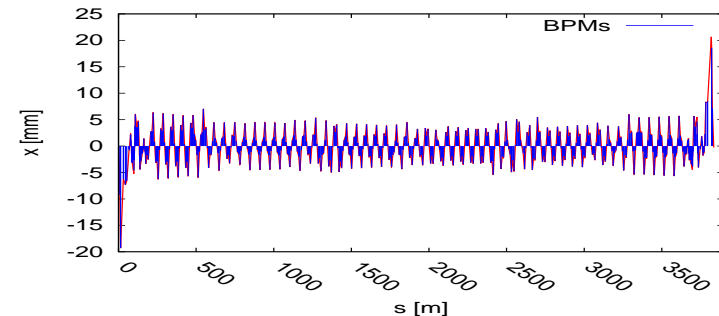
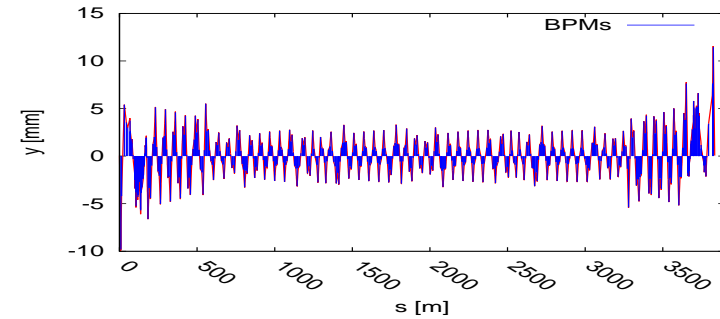
```
Want to correct orbit of a single ring
494 monitors and 494 correctors found in input
494 monitors and 494 correctors enabled
```

```
start SVD correction using      494 correctors
```

### CORRECTION SUMMARY:

	average [mm]	std.dev. [mm]	RMS [mm]	peak-to-peak [mm]
before correction:	0.763587	18.408777	18.424607	216.307978
after correction:	0.000241	0.015902	0.015903	0.213275

...and the actual result!

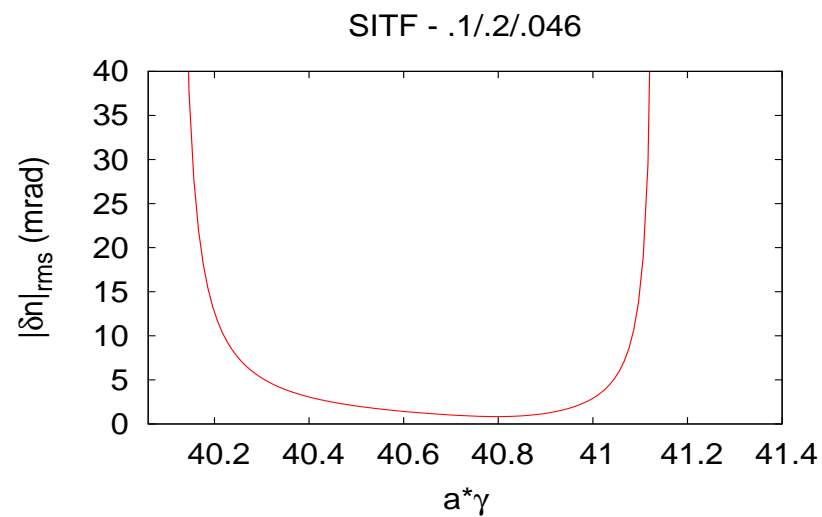
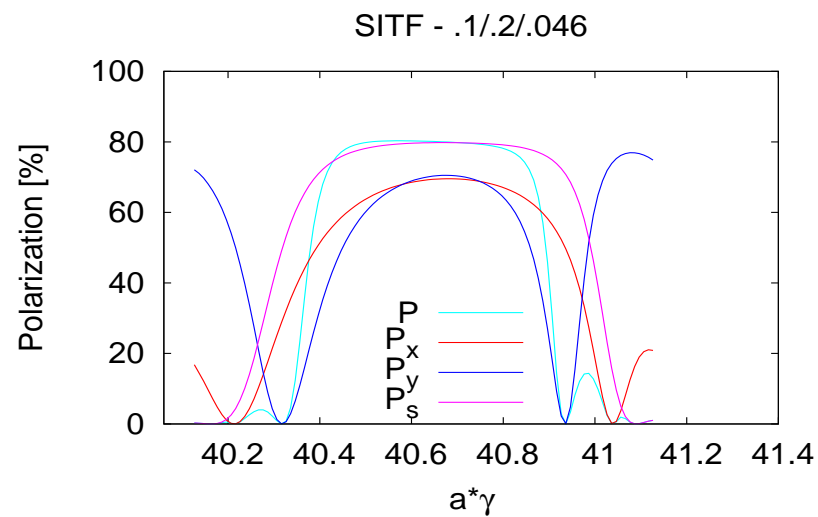


↑  
Effect on horizontal plane  
with sextupoles off

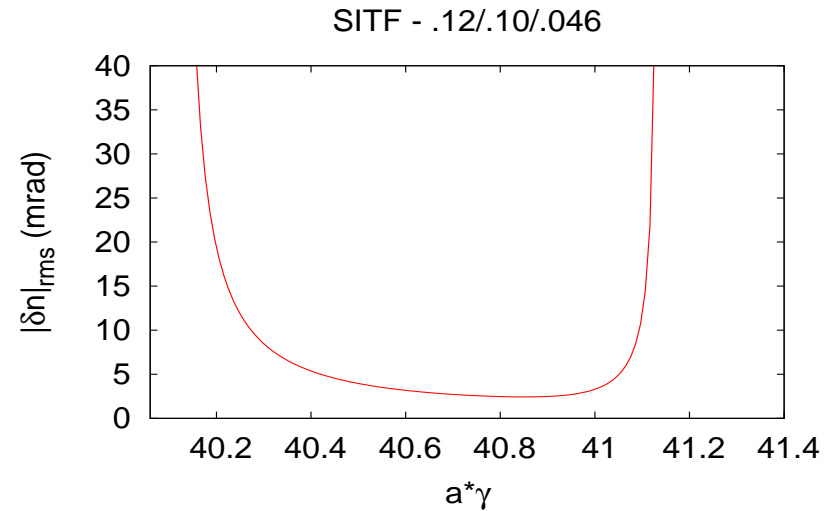
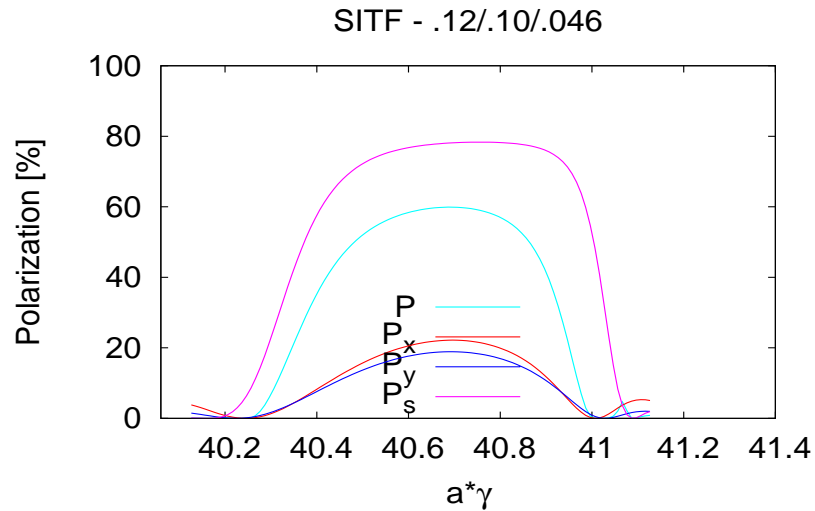
Separate horizontal and vertical orbit correction inadequate in the rotator sections  
→ “external” program used for correcting horizontal and vertical orbits simultaneously.

## One error realization

- after orbit correction
- with  $Q_x=60.10$ ,  $Q_y=56.20$  (HERA-e tunes).



Same error realization, betatron tunes moved to  $Q_x=60.12$ ,  $Q_y=56.10$  for luminosity operation; w/o skew quads,  $|C^-| \approx 0.01$ .



## Coupling and vertical dispersion correction with skew quads

Vertical dispersion due to a skew quad

$$\Delta D_y(s) = \frac{1}{2\pi \sin \pi Q_y} D_x^{skq} \sqrt{\beta_y^{skq} \beta_y(s)} \cos(\pi Q_y - |\mu_y - \mu_y^{skq}|) (K\ell)_{skq}$$

Coupling functions

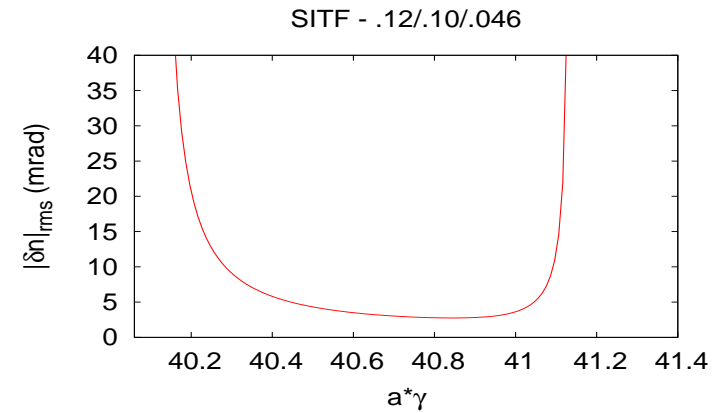
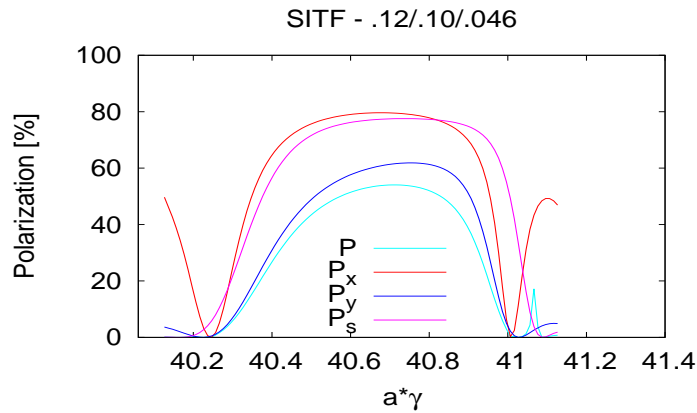
$$w_{\pm}(s) \propto \sqrt{\beta_x^{skq} \beta_y^{skq}(s)}$$

Introduced 46 independently powered skew quadrupoles in arc locations where

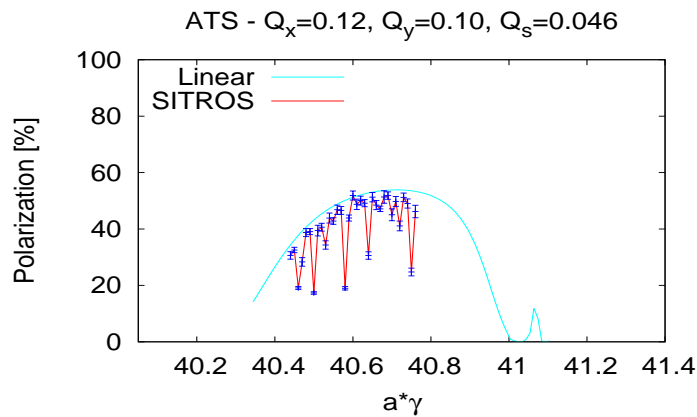
$$D_x^{skq} \sqrt{\beta_y^{skq}} \quad \text{and} \quad \sqrt{\beta_x^{skq} \beta_y^{skq}(s)}$$

are large.

Same error realization, luminosity betatron tunes with optimized skew quads,  $|C^-| \approx 0.002$ .



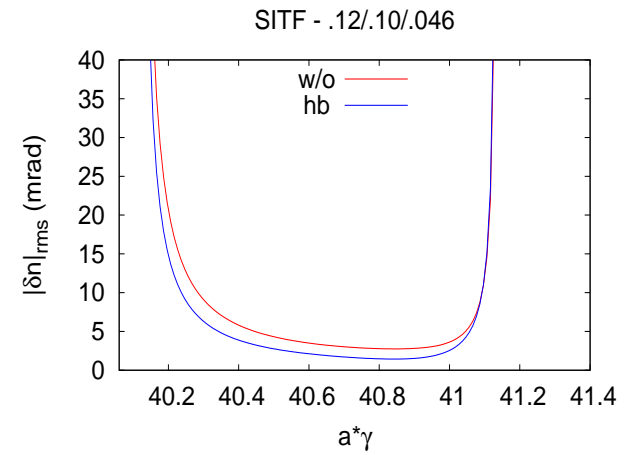
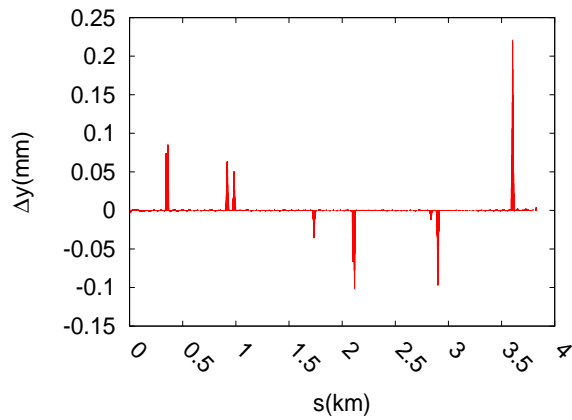
Beam size at IP



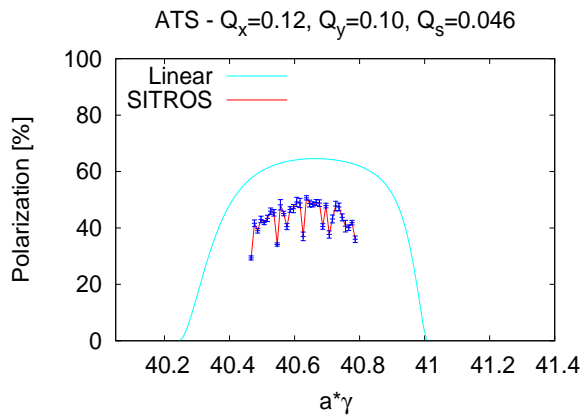
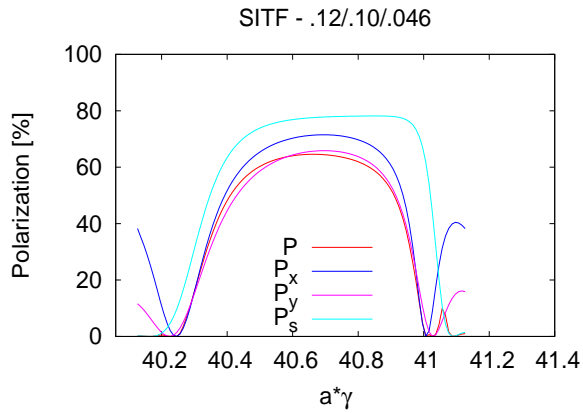
	$\sigma_x$	$\sigma_y$	$\sigma_l$
	[mm]	[ $\mu\text{m}$ ]	[mm]
SITF	0.121	1.718	6.984
SITROS	0.138	3.126	6.969



Adding  $\hat{n}_0$  correction by *harmonic bumps*

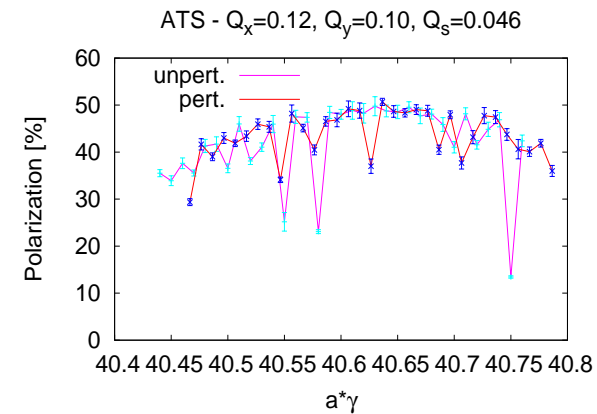


Effect on vertical orbit



## Beam size at IP

	$\sigma_x$	$\sigma_y$	$\sigma_l$
	[mm]	[ $\mu\text{m}$ ]	[mm]
SITF	0.121	3.151	6.985
SITROS	0.139	4.402	7.004



Level of polarizations is as for the *unperturbed* optics. However: BPMs errors must be included and some statistics is needed!

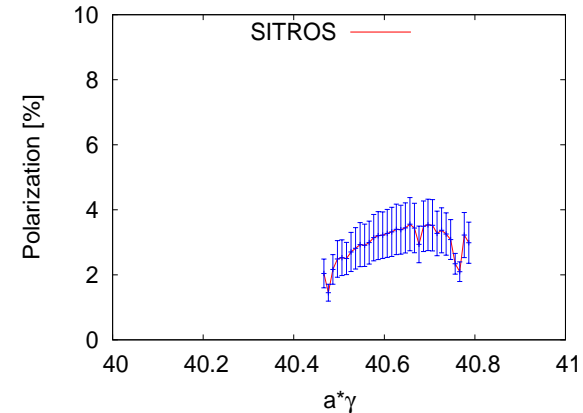
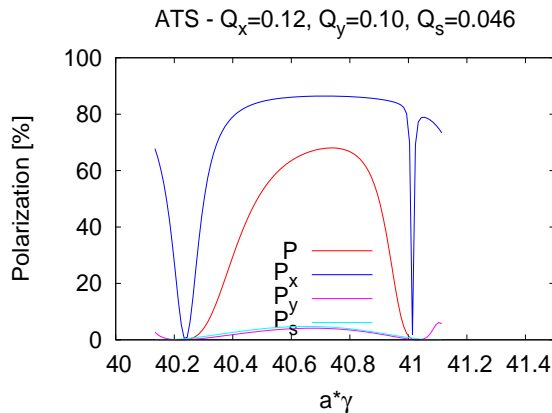
## $\epsilon_y$ bump

The beam vertical emittance is 1.7 pm, corresponding to  $\sigma_y^* \simeq 0.4 \mu\text{m}$ . A larger beam size at the IP may be needed.

The e-beam  $\epsilon_y$  may be efficiently increased by anti-symmetric bumps around low  $\beta_y$  locations.

As a *test* such a bump has been introduced around the IP.

Effect on polarization is detrimental. For  $\epsilon_y = 3 \text{ nm}$  there is no polarization!



## Summary and Outlook

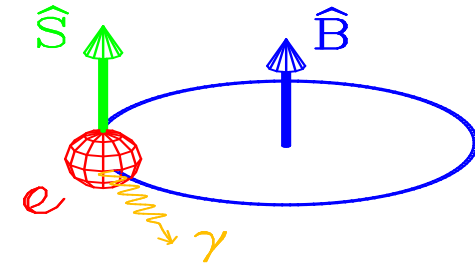
Polarization studies for the eRHIC storage ring are going on.

- With conservative errors  $P_\infty \approx 50\%$  seems within reach:
  - for *upwards* polarized bunches (anti-parallel to the guiding field),  $\langle P \rangle \approx 80\%$ ., over 5 minutes if  $P(0)=85\%$ ;
  - for bunches polarized *downwards* the average polarization drops to 67%.
- Luminosity working point requires linear coupling correction. Here the benefits of a *local correction* using 46 skew quadrupoles have been shown, but
  - the use of correctors for dispersion and of (fewer?) skew quads for betatron coupling correction is an alternative to be tried;
  - implementation of a knob for controlling the vertical beam size at IP w/o affecting polarization is needed: skew quads?
- Comparisons with different codes (Bmad, PTC) is going on.
- Beam-beam effects need to be addressed.

## Back-up slides

## Radiative polarization

Sokolov-Ternov effect in a homogeneous constant magnetic field: a small amount of the radiation emitted by a  $e^\pm$  moving in the field is accompanied by *spin flip*.



Slightly different probabilities  $\rightarrow$  *self polarization!*

- Equilibrium polarization

$$\vec{P}_{\text{ST}} = \hat{y} P_{\text{ST}} \quad |P_{\text{ST}}| = \frac{|n^+ - n^-|}{n^+ + n^-} = \frac{8}{5\sqrt{3}} = 92.4\%$$

$e^-$  polarization is anti-parallel to  $\vec{B}$ , while  $e^+$  polarization is parallel to  $\vec{B}$ .

- Build-up rate

$$\tau_{\text{ST}}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0} \frac{\gamma^5}{|\rho|^3} \quad \rightarrow \quad \tau_p^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \hbar}{m_0 C} \oint \frac{ds}{|\rho|^3} \quad \text{for an ideal storage ring}$$

In eRHIC electrons (clock-wise rotating) self-polarization is *upwards*.

A perfectly planar machine (w/o solenoids) is always *spin transparent*.

This property is lost in presence of

- spin-rotators
  - spin transparency partially restored by *optical spin-matching*
- mis-alignments

Derbenev-Kondratenko expressions for non-homogeneous constant magnetic field involve averaging across the phase space and along the ring

$$\vec{P}_{\text{DK}} = \hat{n}_0 \frac{8}{5\sqrt{3}} \frac{\oint ds \left\langle \frac{1}{|\rho|^3} \hat{b} \cdot \left( \hat{n} - \frac{\partial \hat{n}}{\partial \delta} \right) \right\rangle}{\oint ds \left\langle \frac{1}{|\rho|^3} \left[ 1 - \frac{2}{9} (\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \right] \right\rangle} \quad \hat{b} \equiv \vec{v} \times \dot{\vec{v}} / |\vec{v} \times \dot{\vec{v}}|$$

periodic solution to T-BMT eq. on c.o.

randomization of particle spin directions due to photon emission ( $\delta \equiv \delta E/E$ )

Polarization rate

$$\tau_{\text{DK}}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \gamma^5 \hbar}{m_0 C} \oint ds \left\langle \frac{1}{|\rho|^3} \left[ 1 - \frac{2}{9} (\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \right] \right\rangle$$

Perfectly planar machine (w/o solenoids):  $\partial\hat{n}/\partial\delta=0$ .

In general  $\partial\hat{n}/\partial\delta \neq 0$  and large when

$$\nu_{spin} \pm mQ_x \pm nQ_y \pm pQ_s = \text{integer} \quad \nu_{spin} \simeq a\gamma$$

- Polarization time may be greatly reduced.
- $P_{DK} < P_{ST}$ .



## Tools

Accurate simulations are necessary for evaluating the polarization level to be expected in presence of misalignments. Evaluation of D-K expressions is difficult.

- [MAD-X](#) used for simulating quadrupole misalignments and orbit correction
- [SITROS](#) (by J. Kewisch) used for computing the resulting polarization.
  - Tracking code with 2nd order orbit description and non-linear spin motion.
  - Used for HERA-e in the version improved by M. Böge and M. Berglund.
  - It contains SITF (fully 6D) for analytical polarization computation with *linearized* spin motion.
    - \* Useful tool for preliminary checks before embarking in time consuming tracking.
    - \* Computation of polarization related to the 3 degree of freedom separately: useful for disentangling problems!

## Polarization evolution formulas

The exponential grow

$$P(t) = P_{\infty}(1 - e^{-t/\tau_p}) + P(0)e^{-t/\tau_p} \quad 1/\tau_p = w_{\mp} + w_{\pm}$$

follows from the fact that

$$\frac{dn^+}{dt} = n^- w_{\mp} - n^+ w_{\pm} \quad \text{and} \quad \frac{dn^-}{dt} = n^+ w_{\pm} - n^- w_{\mp}$$

The Derbenev-Kondratenko polarization rate

$$\tau_{\text{DK}}^{-1} = \frac{5\sqrt{3} r_e \gamma^5 \hbar}{8 m_0 C} \oint \left\langle \frac{1}{|\rho|^3} \left[ 1 - \frac{2}{9} (\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \right] \right\rangle$$

may be written as

$$\tau_{\text{DK}}^{-1} = \tau_p^{-1} \simeq \tau_{\text{BKS}}^{-1} + \tau_d^{-1}$$

with

$$\tau_{\text{BKS}}^{-1} = \frac{5\sqrt{3} r_e \gamma^5 \hbar}{8 m_0 C} \oint ds \frac{1}{|\rho|^3} \left[ 1 - \frac{2}{9} (\hat{n}_0 \cdot \hat{v}_0)^2 \right]$$

and

$$\tau_d^{-1} = \frac{5\sqrt{3} r_e \gamma^5 \hbar}{8 m_0 C} \oint ds \left\langle \frac{1}{|\rho|^3} \left[ \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \right] \right\rangle$$

Similarly for  $P_\infty$

$$\vec{P}_{\text{DK}} = \hat{n}_0 \frac{8}{5\sqrt{3}} \frac{\oint ds \left\langle \frac{1}{|\rho|^3} \hat{\mathbf{b}} \cdot \left( \hat{\mathbf{n}} - \frac{\partial \hat{\mathbf{n}}}{\partial \delta} \right) \right\rangle}{\oint ds \left\langle \frac{1}{|\rho|^3} \left[ 1 - \frac{2}{9} (\hat{\mathbf{n}} \cdot \hat{\mathbf{v}})^2 + \frac{11}{18} \left| \frac{\partial \hat{\mathbf{n}}}{\partial \delta} \right|^2 \right] \right\rangle} \quad \hat{\mathbf{b}} \equiv \vec{v} \times \dot{\vec{v}} / |\vec{v} \times \dot{\vec{v}}|$$

$$P_\infty = P_{\text{DK}} \simeq P_{\text{BKS}} \frac{\tau_d}{\tau_{\text{BKS}} + \tau_d} = P_{\text{BKS}} \frac{\tau_p}{\tau_{\text{BKS}}}$$

Approximations done

- $\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}$  is evaluated on the closed orbit,
- $\hat{\mathbf{b}} \cdot \frac{\partial \hat{\mathbf{n}}}{\partial \delta}$  has been neglected. In general it is small.