Strong Hadron Cooling with Micro-bunched Electron Beam

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2nd EIC Accelerator Collaboration Meeting, Oct. 29 - Nov. 1, 2018
Thomas Jefferson National Accelerator Facility
Newport News, VA

Outline of the talk

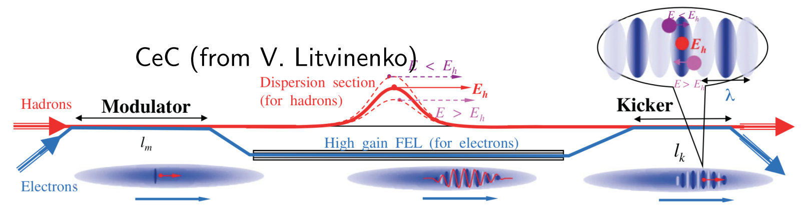
- Physics of the microbunched electron cooling (MBEC)
- 1D theory of longitudinal MBEC without amplification, comparison with simulations
- Longitudinal cooling with one and two sections of amplification
- Transverse cooling of beams
- Summary

Our approach is to develop simplified models of the MBEC that allow to quickly estimate an optimized cooling rate and its scaling with the beam parameters. At a later stage of this study we plan to address (by combination of analysis and computer simulations) other effects that are missing in these simplified models. The analytical models will guide us in the developing of the codes.

Cooling options for EIC

Traditional stochastic cooling is two slow for EIC.

- Conventional electron cooling (adopted for JLEIC)
- Coherent electron cooling (CeC) with an FEL amplifier [Proposed in Refs.[1].]



An FEL is a narrow-band amplifier and the gain is limited by the saturation effects.

- Microbunched coherent electron cooling (MBEC). Proposed in Ref.[2].
- Optical stochastic cooling (see V. Lebedev's presentation).

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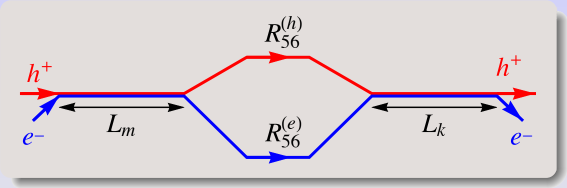
[1] Derbenev, AIP Conf. Proc. **253**, 103 (1992); Litvinenko, Derbenev. PRL, **102**, 114801 (2009).

[2] D. Ratner, PRL, **111**, 084802 (2013).

Microbunched electron cooling (MBEC)



- In microbunched electron cooling, electrons of the cooler beam with $\gamma_e = \gamma_h = \gamma$ first interact with the hadron beam in the modulator.
- The energy perturbations in the electron beam due to the hadrons are then converted to density modulation in the chicane $R_{56}^{(e)}$.
- The longitudinal electric field of these density perturbations - typically after amplification in a separate plasma stage - acts back on hadrons in the kicker.
- High-energy hadrons passing through $R_{56}^{(h)}$ move ahead and get a negative kick, while low-energy hadrons move back and get a positive kick.
- Over many passages, this decreases the energy spread of the hadron beam.

Representative set of parameters for eRHIC MBEC

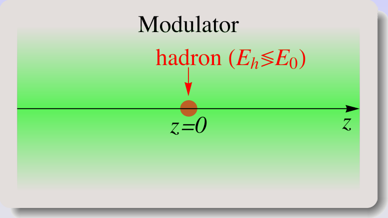
In numerical estimates we assume the following set of parameters for the hadron and electron cooler beams:

Proton energy [GeV]	275
Proton relative energy spread, $\sigma_{\eta h}$	4.6×10^{-4}
Electron energy [MeV]	150
Electron relative energy spread, $\sigma_{\eta e}$	1×10^{-4}
Electron beam charge [nC]	1
Electron beam peak current [A]	30
Repetition rate [MHz]	112
RMS beam size in mod. and kicker, Σ, [mm]	0.7
L_m, L_k [m]	40

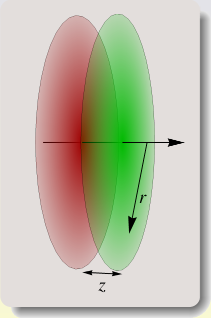
The electron bunch length, $\sigma_{ze} \approx 4$ mm, is much shorter than the proton bunch length, $\sigma_{ze} \lesssim \sigma_{zh} = 5$ cm.

The cooler-beam current is ~ 100 mA.

The model

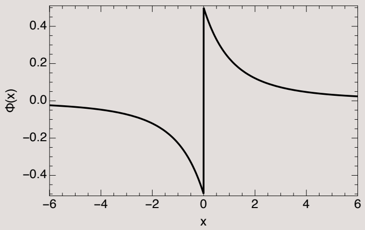


A hadron in the modulator interacts with the nearby electrons and changes their energy.



Each hadron and electron are treated as infinitely thin slices of charge Ze or $-e$ respectively with a Gaussian transversed charge distribution over the surface of the slice, $\sim (Ze/2\pi\Sigma^2)e^{-r^2/2\Sigma^2}$ (this is justifiable if hadrons and electron execute several betatron oscillations during interaction). Here Σ is the rms transverse size of the beam.

1D Model of hadron-electron interaction



The interaction force of two slices

$$F_z(z) = -\frac{Ze^2}{\Sigma^2}\Phi\left(\frac{z\gamma}{\Sigma}\right)$$

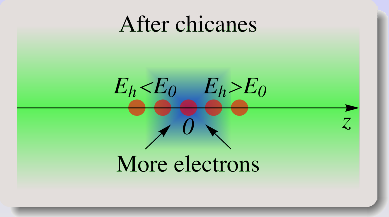
The interaction force extends over the distance $\Delta z_{\rm int} \sim \Sigma/\gamma \approx 2.4\,\mu$m. The number of electrons within $\Delta z_{\rm int}$ is $N_e \sim 1.5 \times 10^6$.

We neglect the longitudinal shift of the particles on the length L_m, L_k. The electrons within the distance $\sim \Delta z_{\rm int}$ change their energy by $\delta E = F_z(z)L_m$,

$$\delta\eta_e = \frac{\delta E}{\gamma m_e c^2} \sim \frac{Ze^2 L_m}{\Sigma^2 \gamma m_e c^2} \sim 10^{-9}$$

(we use the notation $\eta = \Delta E/E_0$). In absolute units $\delta E \sim 0.15$ eV. Electrons are decelerated ahead of the hadron and accelerated behind the it.

Passage through the chicanes



Due to the interaction with the hadron, electrons ahead of the hadron (with smaller energy) will shift less, and electrons behind the hadron (with higher energy) shift more by $\delta z \sim R_{56}^{(e)} \delta\eta_e$. The optimal value $R_{56}^{(e)} \sim \Delta z_{\mathrm{int}}/\sigma_{\eta e}$ (larger $R_{56}^{(e)}$ smears out perturbations on the scale Δz_{int}).

This will cause a density perturbation

$$\delta n_e \sim \frac{\delta z}{\Delta z_{\mathrm{int}}} n_e \sim \frac{\delta \eta_e}{\sigma_{\eta e}} n_e \sim 10^{-5} n_e$$

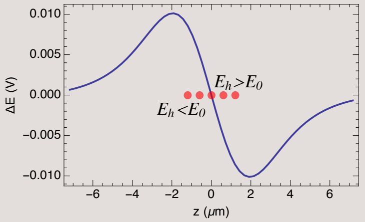
where n_e is the number of electrons (slices) per unit length. For our parameters the excess of electrons created by one hadron is $\delta n_e \Delta z_{\mathrm{int}} \sim 1.5$.

The kicker

The excess of electrons creates the longitudinal electric field

$$\mathcal{E}_z \sim \frac{e}{\Sigma^2}\delta n_e \Delta z_{\rm int} \sim e\frac{\delta\eta_e}{\sigma_{\eta e}}\frac{n_e}{\Sigma\gamma}$$

and this field changes the hadron energy by $\Delta E = Ze\mathcal{E}_z L_k$ in the kicker.



Hadrons are shifted by $R_{56}^{(h)}\eta_h$ with the higher energy than the nominal one slipping ahead and the lower energy lagging behind. The cooling time (in revolution periods)

$$N_c \sim \frac{\sigma_{\eta h}}{\Delta E / E_h}$$

This formula has the right scaling with the beam parameters. More accurate calculations (see below) give the cooling time $\sim$ 40 h.

Theoretical analysis (PRAB 2018)

In this analysis we used the Vlasov equation to track the dynamics of microscopic 1D fluctuations in the electron and hadron beams during their interaction and propagation through the system.

Assumptions:

- 1D model: hadrons and electrons are treated as infinitely thin slices of charge Ze ($-e$ for electrons) with a Gaussian transverse charge distribution (round beams).
- Perfect overlap of the electron and hadron beams in the modulator and the kicker.
- Particles (slices) do not shift relative to each longitudinally during the interaction in the modulator and the kicker.
- Chicanes shift particles in the longitudinal direction by $R_{56}\eta$.
- There is a perfect mixing in the hadron beam on the scale Δz_{int} during one revolution in the ring.
- We calculate the cooling time for the longitudinal energy spread.

Kinetic equation for F_h

Evolution of the energy distribution function of hadrons $F_h(\eta, t)$ in the ring over many passages through the cooling system

$$\frac{\partial F_h}{\partial t} = \frac{1}{2t_c}\frac{\partial(\eta F_h)}{\partial \eta} + D\frac{\partial^2 F_h}{\partial^2 \eta}$$

Multiplying this equation by η^2 and integrating it over η we obtain

$$\frac{d\sigma_{\eta h}^2}{dt} = -\frac{\sigma_{\eta h}^2}{t_c} + 2D$$

The cooling time depends on $R_{56}^{(e)}$ and $R_{56}^{(h)}$. The optimal values are: $R_{56}^{(e)} = 0.6\Sigma/\sigma_{\eta e}\gamma$, $R_{56}^{(h)} = 0.6\Sigma/\sigma_{\eta h}\gamma$, with

$$N_c^{-1} \equiv \left(\frac{t_c}{T}\right)^{-1} = \frac{0.1}{\sigma_{\eta h}\sigma_{\eta e}}\frac{1}{\gamma^3}\frac{I_e}{I_A}\frac{r_h L_m L_k}{\Sigma^3}$$

Here, $I_A \approx 17$ kA is the Alfven current and $r_h = (Ze)^2/m_h c^2$ is the classical radius for hadrons ($\approx 1.5 \times 10^{-18}$ m for protons) and T is the revolution period.

Cooling rate

The electron beam overlaps only with a small fraction of the hadron beam. Over many revolutions, hadrons move longitudinally due to the synchrotron oscillations. One needs to average the cooling rate over the length of the electron bunch,

$$N_{\mathrm{c}}^{-1} = \frac{0.1}{\sigma_{\eta h}\sigma_{\eta e}} \frac{1}{\gamma^3} \frac{cQ_e}{\sqrt{2\pi}\sigma_{zh} I_A} \frac{r_h L_m L_k}{\Sigma^3}$$

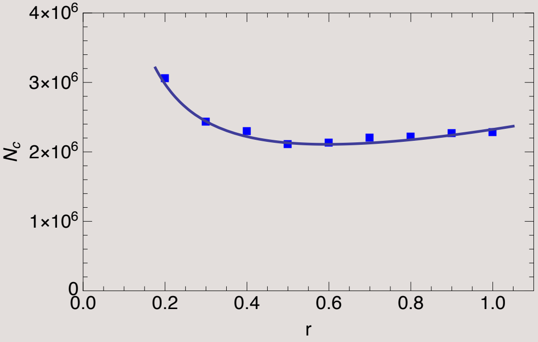
the cooling time is

$$T_{\mathrm{c}} \approx 41 \mathrm{~h}$$

The cooling rate increases for smaller Σ, but we cannot focus both (hadron and electron) beams in the modulator and the kicker. [Currently, it is assumed that the modulator and the kicker are drifts.]

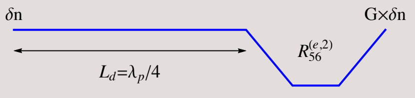
Numerical simulations

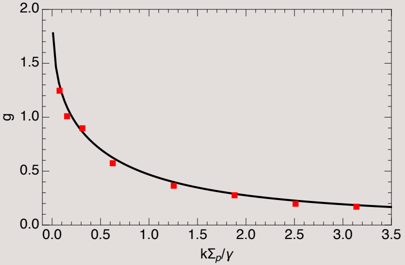
We used $N_e = 10^4$ electron macroparticles and the length of the "electron bunch" $\Delta z = 20\Sigma/\gamma$. The averaging was done over $M = 5 \times 10^6$ runs. The plot of the simulated cooling times as a function of the dimensionless chicane strength $r = R_{56}^{(h)} \sigma_{\eta h} \gamma/\Sigma = R_{56}^{(e)} \sigma_{\eta e} \gamma/\Sigma$.



Our choice of the simulation parameters can be interpreted as if each macroparticle had a charge of approximately 36e.

Amplification of microbunching in the electron beam[3]





In 1D model, the amplification factor $G(k)$ is derived theoretically. For the optimized chicane strength (note the minus sign in G—this is for $R_{56}^{(e,2)} > 0$),

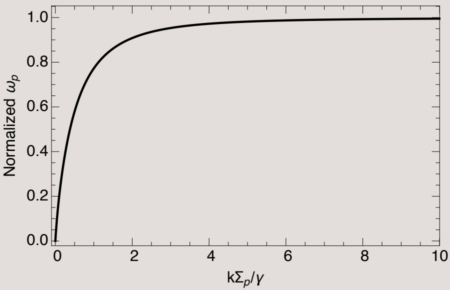
$$G(k) = -\frac{1}{\sigma_{\eta e}}\sqrt{\frac{I_e}{I_A\gamma}}\, g\left(\frac{k\Sigma_p}{\gamma}\right)$$

where Σ_p is the beam radius.
We also simulated g solving equations of motion for electrons in the drift with account of the Coulomb interactions. Red dots—the result of simulations.

This is a broadband amplifier. Unfortunately, small k (long period) plasma oscillations have small plasma frequency.

[3] Schneidmiller and Yurkov, PRSTAB **13**, 110701 (2010); Dohlus, Schneidmiller and Yurkov, PRSTAB **14** 090702 (2011); Marinelli et al., PRL **110**, 264802 (2013).

Plasma oscillations frequency

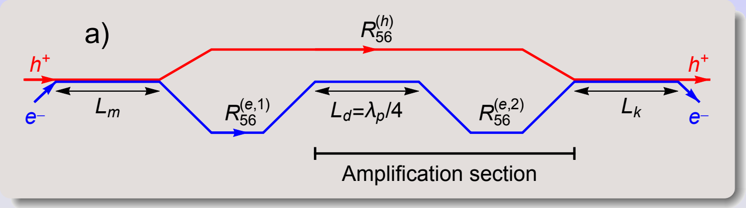


Plasma frequency is normalized by

$$\omega_{p0} = \frac{c}{\Sigma_p}\left(\frac{I_e}{I_A\gamma^3}\right)^{1/2}$$

This makes the optimal length of the amplification section longer than follows from simple estimates.

MBEC amplification using plasma oscillations[4]



In this analysis we assumed round beams in the modulator and the kicker. We also assumed $\Sigma_p = 0.14$ mm, $\Sigma_p/\Sigma = 0.2$. Analytic theory predicts for the amplification factor for the beam current I_e

$$G \approx 0.8 \frac{1}{\sigma_{\eta e}} \sqrt{\frac{I_e}{\gamma I_A}}$$

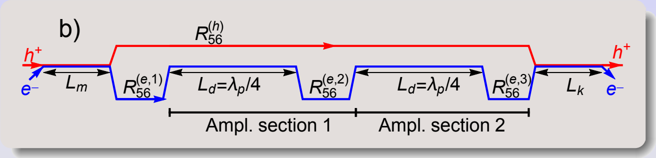
Using $I_e \approx 30$ A and eRHIC parameters we obtain $G \approx 20$.
We estimate the cooling time

$$T_c \approx 2.7 \text{ h}$$

[4] D. Ratner, PRL, **111**, 084802 (2013).

MBEC amplification using plasma oscillations

Two stages of plasma amplification should be enough for eRHIC.



The optimized cooling time is

$$T_{\mathrm{c}} \approx 9 \ \mathrm{min}$$

This would require the length of the amplification drift section $L_d = 84$ m. For $L_d = 30$ m, the cooling time $T_{\mathrm{c}} \approx 23 \ \mathrm{min}$.

It is most likely, the noise amplification will play a role at this large values of the amplification.

MBEC transverse cooling

- Recently, we extended the model by including the transverse motion of the hadrons (but not the electrons!) as they pass from the modulator to the kicker.
- The hadron-electron interaction, as well as the space charge force of the electron beam in the plasma section, are still being treated in the 1D limit.
- Hadron and electron beams consist of slices with a Gaussian profile which can have unequal σ_x and σ_y with $\varepsilon = \sigma_y/\sigma_x$ an ellipticity parameter. This changes the interaction force.
- We begin our analysis with a system without the amplification.

For numerical estimates, we assume the following set of parameters for the hadron and electron cooler beams:

Horizontal/vertical proton emittance [nm]	9.2/1.3
Modulator and kicker lengths L_m, L_k [m]	50
(Effective) electron beam current, I_e [A]	2.4

Assuming $\beta \approx 50$ m, we have $\Sigma = \sqrt{\epsilon_x \beta} \approx 680$ μm and $\Sigma_y = \sqrt{\epsilon_y \beta} \approx 250$ μm (so the ellipticity parameter, in the absence of dispersion, is $\varepsilon_0 = \Sigma_y/\Sigma = 0.37$).

Transverse dynamics

- For the hadron beam, we only consider betatron motion in the vertical (y) direction.
- For the hadron transport line between the modulator and the kicker, including the hadron chicane, the four-dimensional transfer matrix is given by

$$R = \begin{pmatrix} R_{33} & R_{34} & 0 & R_{36} \\ R_{43} & R_{44} & 0 & R_{46} \\ R_{53} & R_{54} & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

  which is supposed to act on the combined vector $\mathbf{y} = (y, P_y, z, \eta)$.
- General expressions exist for the various matrix elements in terms of the lattice functions (R must also be a symplectic matrix).
- We adopt a simplified model in which $\beta_1 = \beta_2 = \beta$, $\alpha_1 = \alpha_2 = 0$, $D_1 = D_2 = D$ and $D_1' = D_2' = 0$ (1 and 2 refer to the modulator and the kicker).

Transverse dynamics

- The matrix elements are $R_{33} = R_{44} = \cos\mu$, $R_{34} = \beta\sin\mu$, $R_{43} = -\sin\mu/\beta$, $R_{36} = (1-\cos\mu)D$, $R_{46} = (\beta/D)\sin\mu$, $R_{53} = -(D/\beta)\sin\mu$ and $R_{54} = (\cos\mu - 1)D$, where μ is the phase advance.
- Analysis is greatly facilitated by going to the action-angle coordinates (J, ϕ) via

$$y = \eta D + \sqrt{2\beta J}\cos\phi, \quad P_y = \eta D' - \sqrt{\frac{2J}{\beta}}(\sin\phi + \alpha\cos\phi).$$

- Inside the modulator and the kicker segments, y and P_y are assumed to be *constant*.
- In the kicker section, due to the interaction with the strongly microbunched e-beam, the hadron energy variable η also changes, leading to a corresponding variation in the betatron action J.
- This is an essential feature of the emittance cooling mechanism.
- Electron motion is still being treated in the 1D limit.

Cooling analysis

- Neglecting diffusion effects, the cooling equations for energy spread and emittance are
$$\frac{d\sigma_{\eta h}}{dt} = -\frac{\sigma_{\eta h}}{N_c^{\eta} T}$$
and
$$\frac{d\epsilon}{dt} = -\frac{\epsilon}{N_c^{\epsilon} T},$$
where we recall that T is the revolution period.
- The scaled cooling times N_c^{η} and N_c^{ϵ} are given by $1/N_c^{\eta} = A_0 I_{\eta}$ and $1/N_c^{\epsilon} = A_0 I_{\epsilon}$, where
$$A_0 = \frac{4 I_e L_m L_k r_h}{\pi \Sigma^3 \gamma^3 I_A \sigma_{\eta e} \sigma_{\eta h}}$$
is a dimensionless prefactor (for the eRHIC parameters, $A_0 \approx 2 \times 10^{-9}$). We derived analytical expressions for the cooling integrals I_{η} and I_{ϵ}.

Optimization of the cooling rate

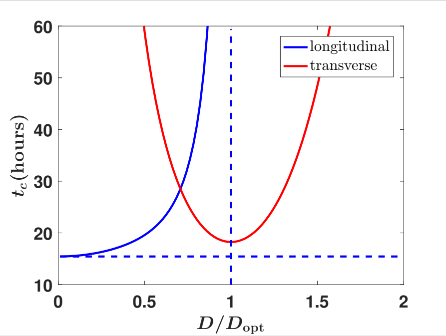
How do we optimize the emittance cooling rate?

- $\varepsilon = \sqrt{\Sigma_y^2 + \sigma_\eta^2 D^2}/\Sigma$ is the ellipticity parameter (or aspect ratio) including the effect of dispersion.
- Zero cooling for the energy spread!
- Then, we need to maximize the modified cooling integral I_ϵ. Optimum $I_\epsilon = 0.103$.

In terms of real parameters, this translates to:

- Optimum dispersion $D \approx 0.87$ m.
- Phase advance $\mu \approx 0.24$.
- Chicane strengths: $R_{56}^h = 3.8$ mm and $R_{56}^e = 1.1$ cm.

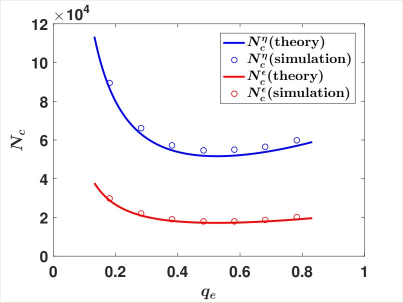
Cooling rates as a function of the dispersion



- We scan the dispersion around the optimum value while keeping everything else constant.
- A minimum is observed for the emittance cooling time (at $D = D_{\mathrm{opt}} = 0.87$ m).
- Minimum cooling time ~ 18 h.

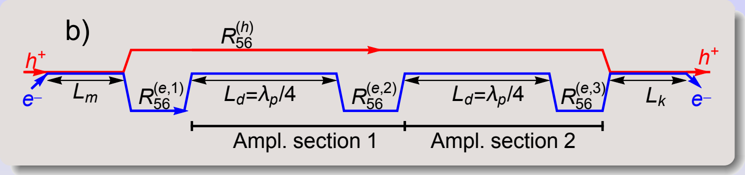
Comparison with simulation

- To benchmark our theory, we compare analytical theory with 1D simulation.
- The actual machine parameters would lead to a very high number of simulation macroparticles ($\sim 10^6$).
- Instead, we use an alternative parameter set for which $A_0 = 6 \times 10^{-4}$. This allows us to use fewer macroparticles ($\sim 10^4$ for electrons and $\sim 10^3$ for hadrons).



- We scan the dimentionless electron chicane strength q_e.
- Good agreement is observed between theory and simulation.
- A deviation of a few percent can probably be attributed to diffusion effects.

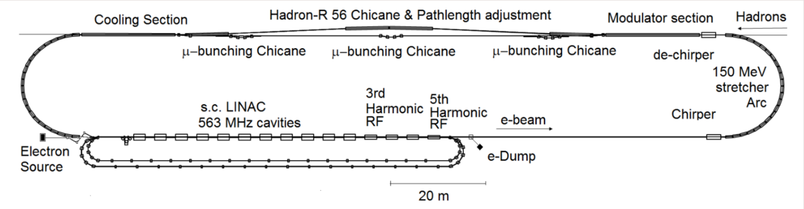
MBEC with two amplification stages

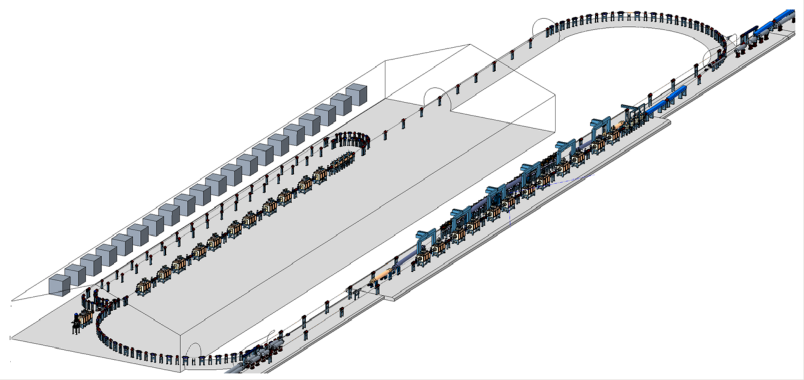


The cooling time can be substantially reduced by adding cascaded plasma stages after the first electron chicane. Here, we consider the case of two stages.

- Squeeze the electron beam to a size $\Sigma_p \approx 135\,\mu\text{m}$.
- The optimized integral I_ϵ is now 0.032 (instead of 0.103). At the same time, the prefactor A_0 is boosted by a factor of $2I_e/(\gamma I_A \sigma_e^2) \approx 96$.
- The optimum cooling time ($\sim (A_0 I_\epsilon)^{-1}$) is now about 0.6 h (30 times lower than without plasma amplification).
- For the optimized machine parameters, we now have $D \approx 1$ m, $\mu \approx 0.45$, $R_{56}^h = 8.2$ mm, $R_{56}^{(e,1)} = R_{56}^{(e,2)} = R_{56}^{(e,3)} = 2.2$ cm.

Sketch of MBEC cooler for eRHIC from pre-CDR





Summary

- We have developed a theoretical model that describes the MBEC process for both the energy spread and the transverse emittance of the hadron beam. Amplification is provided by two sections consisting of $\frac{1}{4}\lambda_p$ drift+chicane.
- Our derivation is based on a one-dimensional (1D) Vlasov technique that tracks the evolution of the beam fluctuations through the MBEC setup.
- Our analysis is benchmarked via comparison with 1D simulation and simple formulas are obtained for the cooling times, allowing for fast optimization studies.
- From a practical point of view, cooling times below 1h appear to be feasible for the eRHIC parameters (both for energy spread and emittance) by making use of two cascaded plasma amplification stages.

Future plans

- Extend the analytical theory to the three-dimensional (3D) regime.
- Develop a 3D simulation tool for MBEC.