Other Approaches: Stochastic Cooling

Valeri Lebedev
Fermilab

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U.S. DEPARTMENT OF ENERGY
Fermi Research Alliance LLC

Outline

- Objectives
- IBS and requirements to the beam cooling
- OSC principles
- OSC limitations
- Possible parameters for EIC
- OSC test in IOTA
- Discussion and conclusions

Objectives

- Look how the beam in an electron-ion collider can be cooled with OSC
    - Discussion is aimed at proton cooling (as more demanding in bandwidth)
- Extremely challenging requirements to the proton beam brightness are set by the desired luminosity of ~10^{34} cm^{-2}s^{-1}
- The following parameters are used in further estimates

Tentative parameters of proton bunches used for estimates

	JLEIC [1]	eRHIC [1]	Tevatron[3]	Units
Proton beam energy	100	275	1000	GeV
Bunch frequency	476	112	1.7	MHz
Particles per bunch	0.98	6	22	10^{10}
Rms bunch length	1	4	50	cm
RMS norm.emittances, $\varepsilon_x/\varepsilon_y$	0.5/0.1	2.6/0.5	3.3/3.3	µm
Rms momentum spread	3	5.8	1.16	10^{-4}
Circumference	2.15	3.83	6.28	km
Betatron tune	24/23	28/29	20.5	

[1] https://www.adams-institute.ac.uk/sites/jaidrup.physics.ox.ac.uk/files/slides-Hutton_Andrew_18_01_2018.pdf
[2] F. Willeke, “eRHIC Strong Hadron Cooling”, CAD-MAC-14, 25-27 Oct., 2017
[3] Tevatron Store 4581

IBS Growth Rates

- Cooling rates are determined by IBS - the major heating mechanism
- For ultra-relativistic case

$$\frac{d}{dt}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p^{\,2}\end{pmatrix}=\frac{N r_0^{\,2} c}{4\sqrt{2}\beta^3\gamma^3\sigma_z}\left\langle\frac{L_c\Psi\left(\theta_x,\theta_y\right)}{\sigma_x\sigma_y\theta_\perp}\begin{pmatrix}A_x\\ 0\\ 1\end{pmatrix}\right\rangle_s ,\qquad \Psi\left(\theta_x,\theta_y\right)=1+\frac{\sqrt{2}}{\pi}\ln\left(\frac{x^2+y^2}{2xy}\right)-0.055\left(\frac{x^2-y^2}{x^2+y^2}\right)^2$$

$$A_x=\left(D_x^{\,2}+\left(\beta_x D_x'+\alpha_x D_x\right)^2\right)/\beta_x$$

- For estimate we use a smooth lattice approximation & $\varepsilon_x = \varepsilon_y$ ($\varepsilon=\sqrt{\varepsilon_x\varepsilon_y}$)

$$\frac{d}{dt}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \sigma_p^{\,2}\end{pmatrix}=\frac{N r_0^{\,2} c L_c}{8\beta^3\gamma^3\sigma_z\sigma_x\varepsilon}\begin{pmatrix}A_x\\ 0\\ 1\end{pmatrix},\quad A_x=\frac{D^2}{\beta_x}=\frac{R_0}{Q_x^{\,3}},\quad \sigma_x=\sqrt{\varepsilon\beta_x+D^2\sigma_p^{\,2}},\quad \beta_x=\frac{R_0}{Q_x},\quad D=\frac{R_0}{Q_x^{\,2}}$$

where R_0 and Q_x are the ring circumference and the horizontal tune

- For BNL and JLab proposals the smooth approximation results in:

	$\tau_p=\sigma_p^{\,2}/\left(d\sigma_p^{\,2}/dt\right)$	$\tau_x=\varepsilon_x/\left(d\varepsilon_x/dt\right)$
JLEIC	36 s	33 s
eRHIC	1.77 hour	0.73 hour
Tevatron (averaged over ring)	9.8 (12) hour	15 (14) hour

- JLEIC looks extremely challenging from cooling point of view
- IBS may be mitigated by an increase of σ_p (||) and Q_x ($\perp$)

Stochastic Cooling Bandwidth and Cooling Rate

- For transient-time cooling of bunched beam with rectangular band the emittance cooling rate at the optimal gain is:

$$\boxed{\lambda_{opt} \approx \frac{2\pi^2 W}{N n_\sigma^{\,2}} \frac{\sqrt{\pi}\sigma_s}{C}} \qquad W = \frac{n_{\max} - n_{\min}}{T_0}, \quad n_\sigma = \frac{x_{\max}}{\sigma_p}.$$

For the Gaussian dependence of gain on frequency: $W = 2\sqrt{\pi}\sigma_f$

- We require the cooling rates to be equal to the IBS rates and $n_\sigma = 6$

⇨ **Bandwidth Required to Counteract IBS (ref.λ_0=6 μm)**

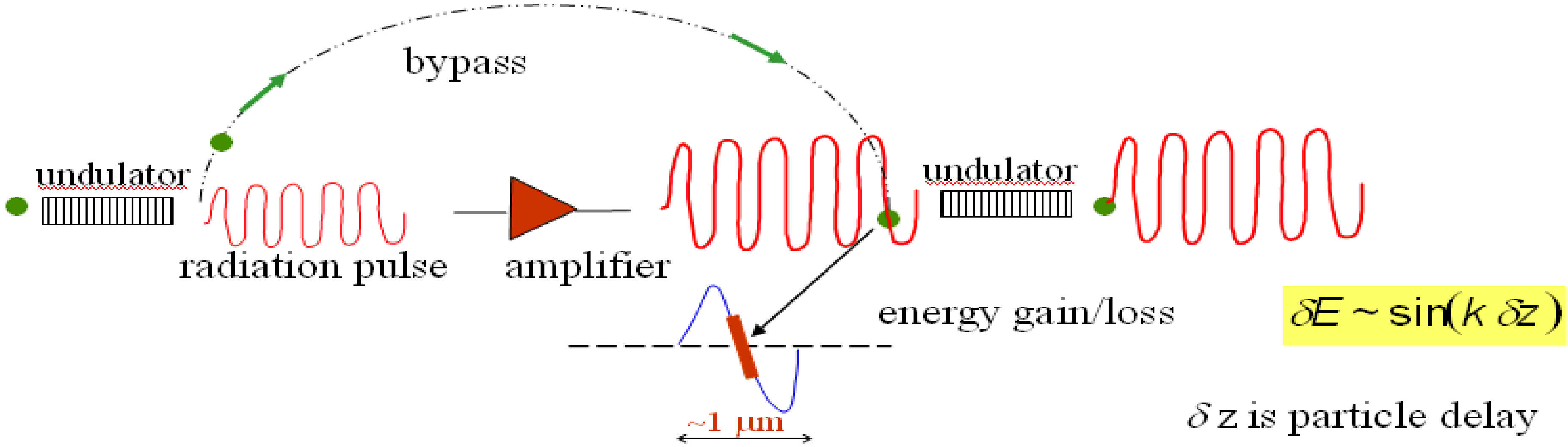
	momentum. cooling		horizontal cooling	
	σ_f [THz]	σ_f / f_L	σ_f [THz]	σ_f / f_L
JLEIC	17	33%	18	36%
eRHIC	0.26	0.524%	0.6	1.26%
Tevatron	0.017	0.035%	0.011%	0.022%

* We account here that both the OSC and heating due to IBS are in the horizontal plane and are absent in the vertical plane

- If bunch length, σ_s, is much longer than the amplification length, σ_g, the cooling rates at optimal gain are additionally reduced as $\sigma_g/2\sigma_s$!!!
- Required bandwidth is in comfortable range for eRHIC & Tevatron

Optical Stochastic Cooling (OSC)

- Suggested by Zolotorev, Zholents and Mikhailichenko (1994)
- Never tested experimentally
- OSC obeys the same principles as the microwave stochastic cooling, but exploits the superior bandwidth of optical amplifiers ~ 10^{13} - 10^{14} Hz
- Pickup and kicker must work in the optical range and support the same bandwidth as the amplifier
  - Microwave pickups and kickers cannot be scaled to μm => Undulators are suggested

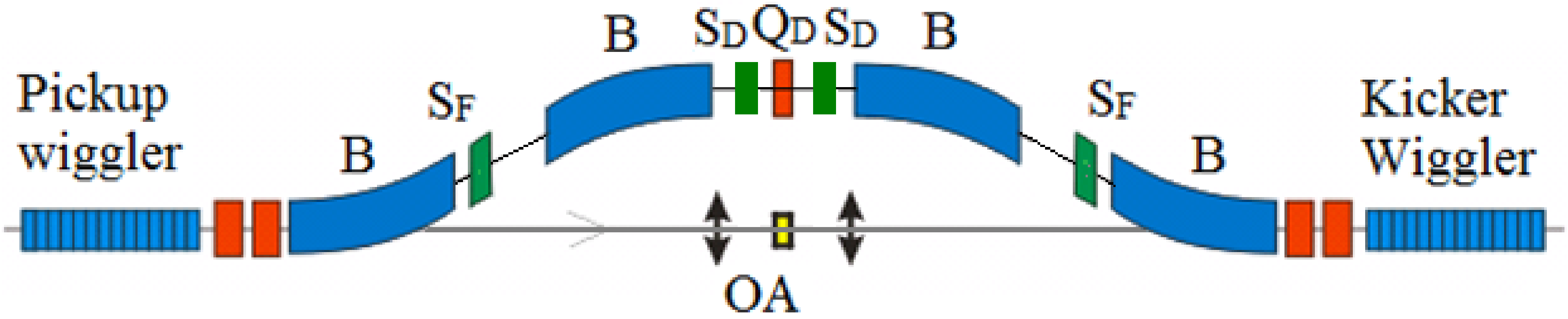


$\delta E \sim \sin(k\,\delta z)$

δz is particle delay

Optical Stochastic Cooling Implementation

- OSC can operate only with ultra-relativistic particles
  - Slow particles do not radiate at optical frequencies
- Only longitudinal kicks are effective
  - Requires s-x coupling for L cooling and x-y coupling for ⊥ cooling
- Detailed analysis yielded that a magnetic chicane with defocusing quad in its center represents the most effective and reliable choice



- Sextupoles are required to correct non-linear lengthening
  - Major contribution to the lengthening:

$$\delta L \approx L\frac{\theta^2}{2} \approx \frac{\varepsilon L}{2\beta_x^*}$$

"Other Approaches: Stochastic Cooling", Valeri Lebedev, FNAL, EIC Accelerator Collaboration Meeting, Jefferson Lab, Oct. 29 – Nov. 1, 2018

Basics of OSC: Damping Rates

- Linearized longitudinal kick in pickup undulator

$$\frac{\delta p}{p} = k_0\xi_0\,\delta s = k_0\xi_0\left(M_{51}x_1 + M_{52}\theta_{x1} + M_{56}\frac{\Delta p}{p}\right)$$

$$\xrightarrow[\text{betatron motion}]{\text{in the absence of}} k_0\xi_0\left(M_{51}D_x + M_{52}D'_x + M_{56}\right)\frac{\Delta p}{p}$$

- Partial slip factor (pickup-to-kicker) describes a longitudinal particle displacement at travel from pickup to kicker

$$\tilde{M}_{56} = M_{51}D_1 + M_{52}D'_1 + M_{56}$$

- Emittance cooling rates (per turn)

$$\boxed{\begin{aligned}\lambda_x &= k_0\xi_0\left(M_{56} - \tilde{M}_{56}\right)\\ \lambda_s &= k_0\xi_0\tilde{M}_{56}\end{aligned}} \Leftrightarrow \boxed{\lambda_x + \lambda_s = k_0\xi_0 M_{56}}$$

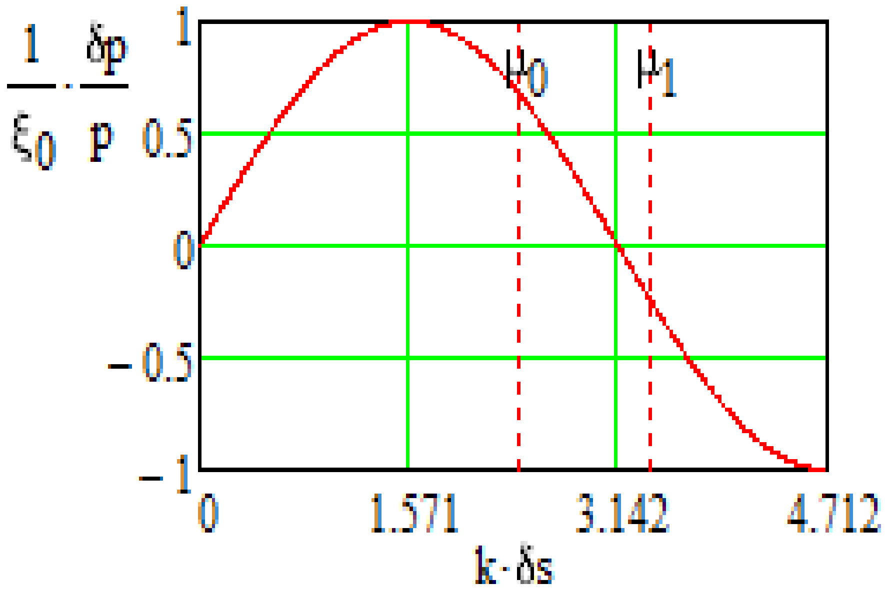
Basics of OSC: Cooling Range

- Cooling force depends on δs nonlinearly

$$\frac{\delta p}{p} = k_0 \xi_0 \, \delta s \quad \Rightarrow \quad \frac{\delta p}{p} = \xi_0 \sin\left(k_0 \delta s\right)$$

where $k_0 \delta s = a_x \sin(\psi_x) + a_p \sin(\psi_p)$

and a_x & a_p are the amplitudes of longitudinal displacements in cooling chicane due to ⊥ and L motions measured in units of laser phase

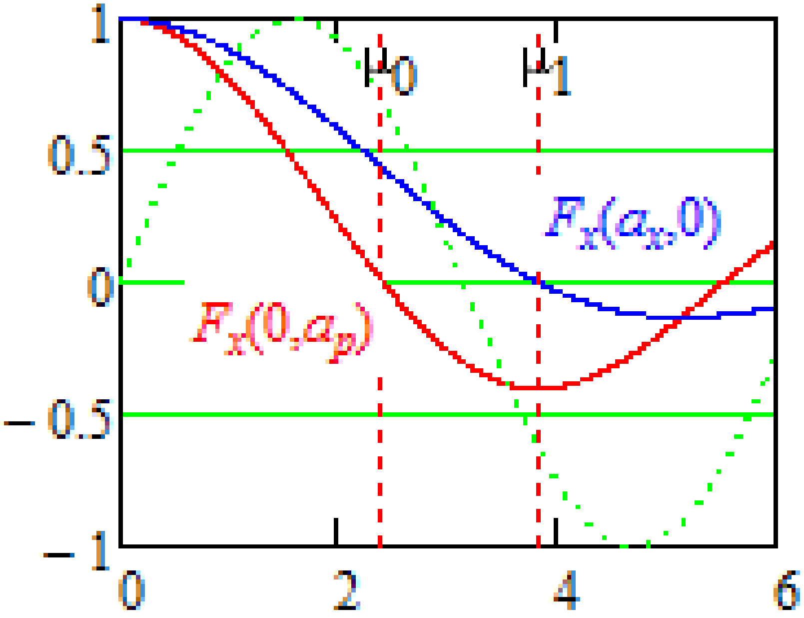


- Averaging yields the form-factors for damping rates

$$\lambda_{s,x}(a_x, a_p) = F_{s,x}(a_x, a_p)\lambda_{s,x}$$

$$F_x(a_x, a_p) = \frac{2}{a_x} \mathrm{J}_0(a_p)\mathrm{J}_1(a_x)$$

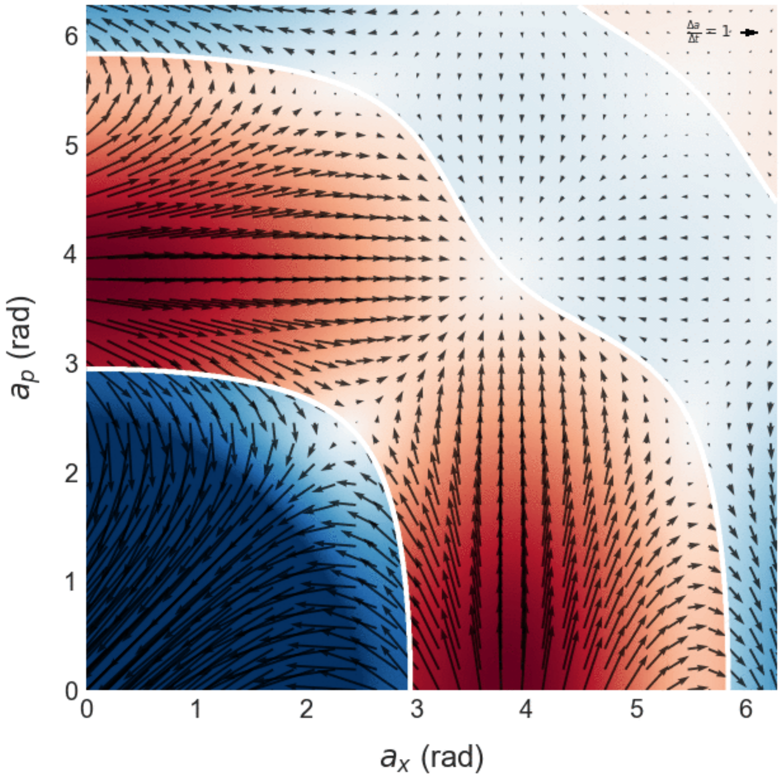
$$F_p(a_x, a_p) = \frac{2}{a_p} \mathrm{J}_0(a_x)\mathrm{J}_1(a_p)$$



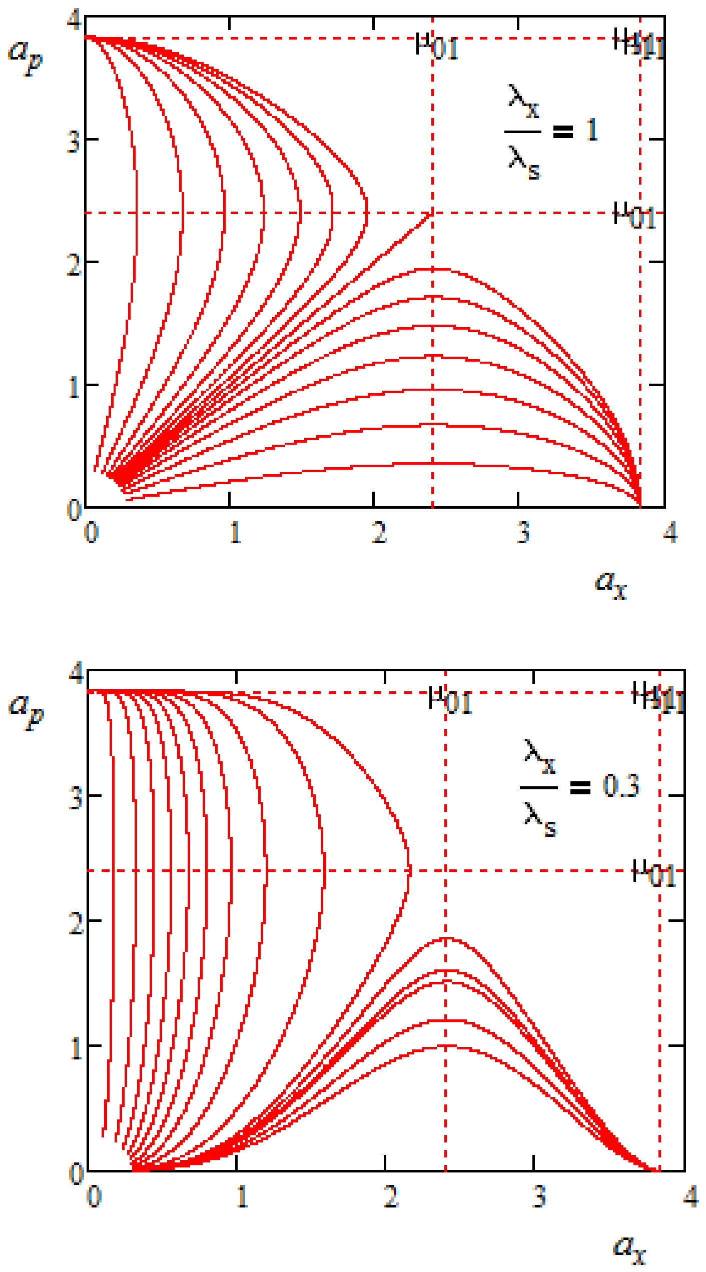
- Damping requires both lengthening amplitudes (a_x and a_p) to be smaller than $\mu_{01} \approx 2.405$

"Other Approaches: Stochastic Cooling", Valeri Lebedev, FNAL, EIC Accelerator Collal

Basics of OSC: Cooling Range (2)



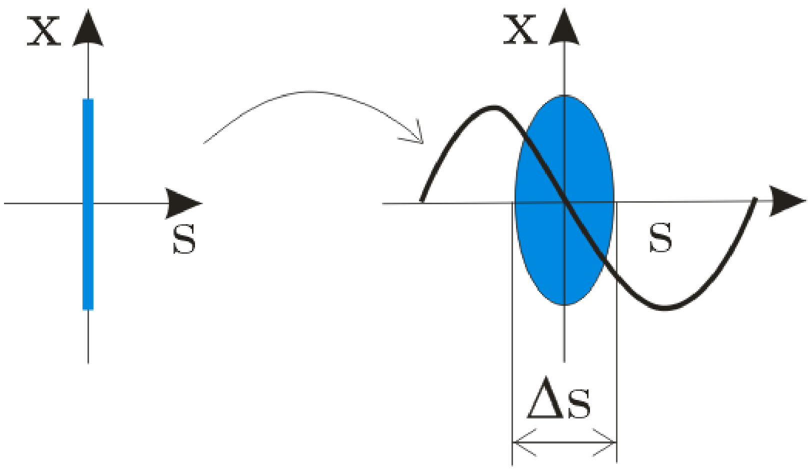
- Particles located outside of cooling range are “cooled” to attractors at large betatron/synchrotron amplitudes and expirience much smaller “cooling” rates



Amplitude trajectories in the course of OSC cooling; top – $\lambda_x/\lambda_s=1$; bottom – $\lambda_x/\lambda_s=0.3$.

Basics of OSC – Sample Lengthening on Pickup-to-Kicker Travel

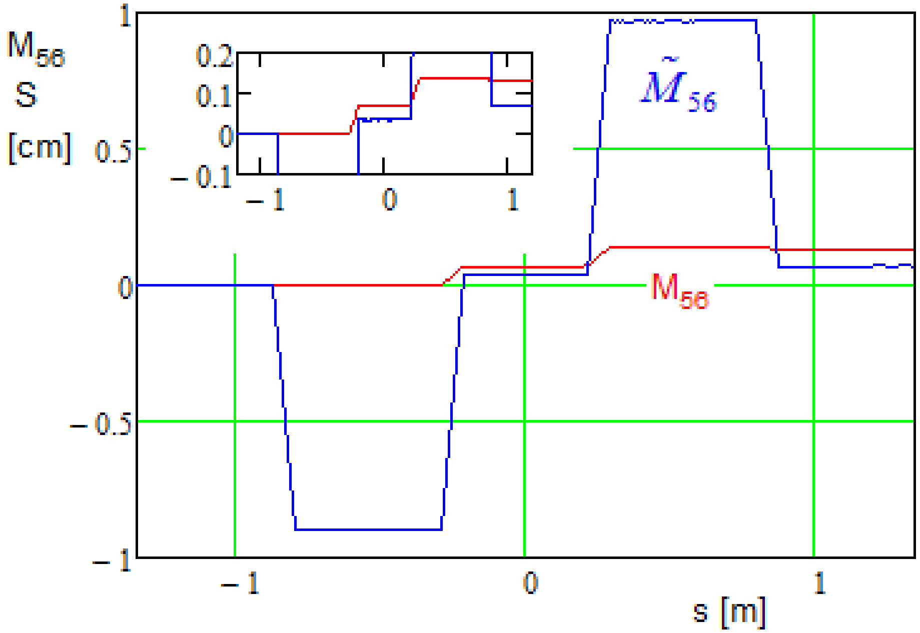
- Zero length sample lengthens on its way from pickup-to-kicker

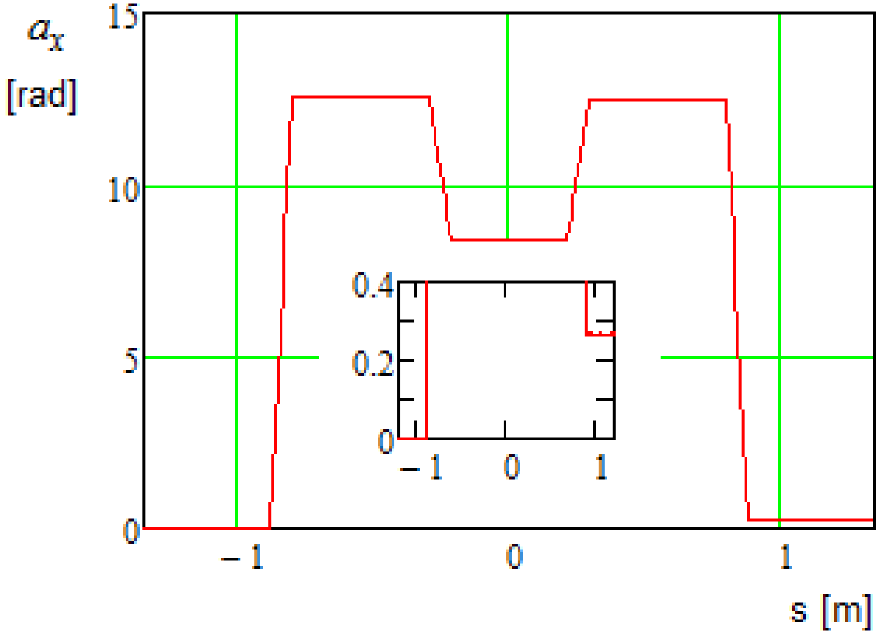


$$a_p = k_0 \tilde{M}_{56}\left(\frac{\Delta p}{p}\right)_m = k_0\left(M_{51}D + M_{52}D' + M_{56}\right)\left(\frac{\Delta p}{p}\right)_m$$

$$a_x = k_0\sqrt{\tilde{\varepsilon}\left(\beta M_{51}{}^2 - 2\alpha M_{51}M_{52} + \left(1+\alpha^2\right)M_{52}{}^2/\beta\right)}$$

- While in linear approximation β_p and α_p do not affect damping rates they affect sample lengthening in *x*

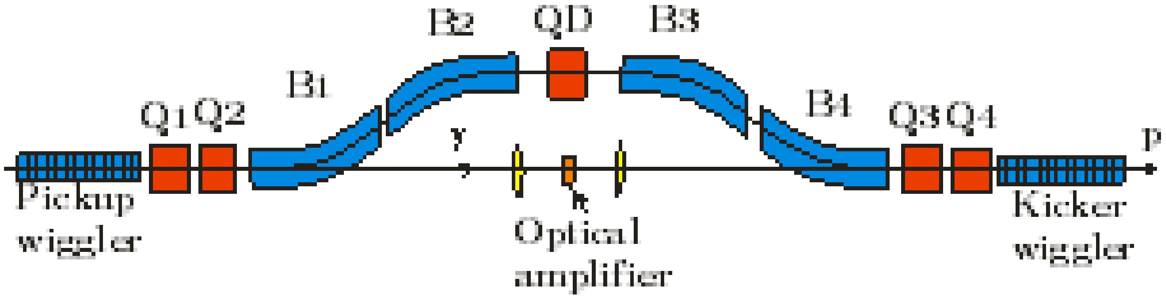




- Sample lengthening gets quite large on the way from pickup to kicker

OSC Limitations on IOTA Optics

- In the first approximation the orbit offset in the chicane (h), the path lengthening in OA (Δs), the defocusing strength of chicane quad (Φ), and dispersion and beta-function in the chicane center (D^*, β^*) determine the entire cooling dynamics
  - Here we accounted that $\beta \approx L_{tot}^2/4\beta^*$
  - $\Phi D^* h$ determines the ratio of cooling rates
  - $k_0 = 2\pi/\lambda_0$ is the wave-number
- Δs is set by delay in amplifier => M_{56}
- For known ε and $n_{\sigma x}$ we chose an appropriate value of dispersion invariant (A^*)
  - Limited by large β-function at chicane ends



$$M_{51} = -\Phi h\,,\ M_{51} = -\Phi h L_{tot}\,,$$

$$M_{56} \approx 2\Delta s\,,$$

$$\tilde{M}_{56} \approx 2\Delta s - \Phi D^* h\,,$$

$$R \equiv \lambda_x / \lambda_s \approx \Phi D^* h / \left(2\Delta s - \Phi D^* h\right),$$

$$n_{\sigma p} \approx \frac{\mu_{01}}{k_0 \Delta s \sigma_p}\,\frac{1+R}{2}\,,$$

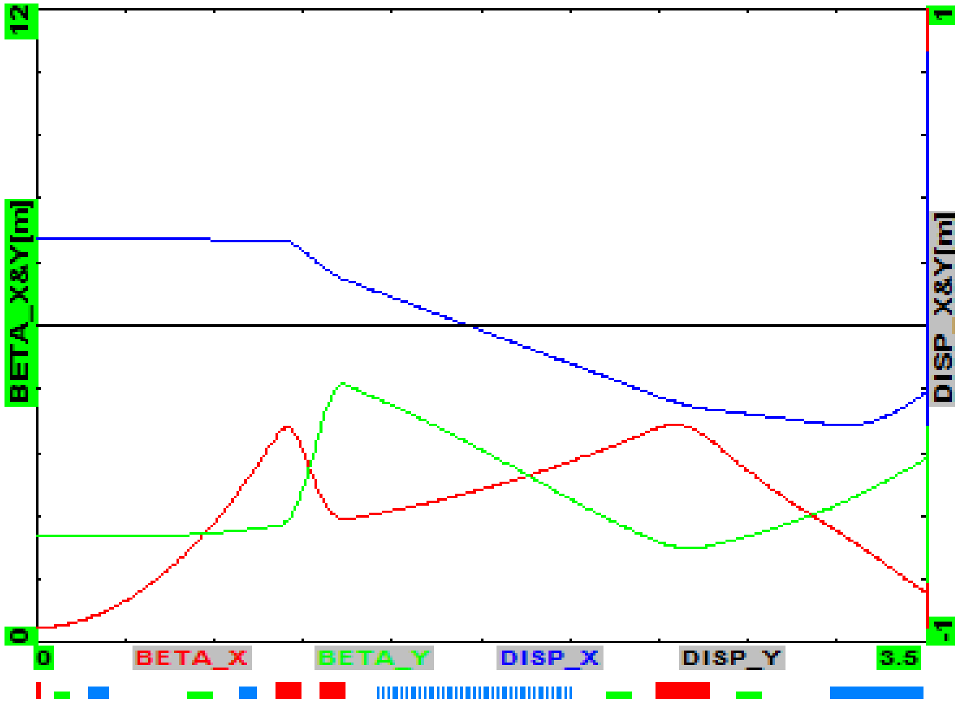
$$n_{\sigma x} \approx \frac{\mu_{01}}{k_0 \Delta s}\sqrt{\frac{1}{\varepsilon}\,\frac{D^{*2}}{\beta^*}}\,\frac{1+R}{2R}$$

Wavelength, Delay in OA and Beam Optics

- For a given wavelength an increase of delay in OA reduces both cooling ranges
- An increase of cooling rates ratio, R, increases the momentum cooling range
- To achieve large horizontal cooling range one needs large dispersion invariant $A^* \equiv D^{*2} / \beta^*$
  - ⇨ Collider type optics for horizontal plane

$$n_{\sigma p} \approx \frac{\mu_{01}}{k_0 \Delta s \sigma_p} \frac{1+R}{2},$$

$$n_{\sigma x} \approx \frac{\mu_{01}}{k_0 \Delta s} \sqrt{\frac{1}{\varepsilon} \frac{D^{*2}}{\beta^*}} \frac{1+R}{2R}$$



Beta-functions and dispersion for IOTA OSC

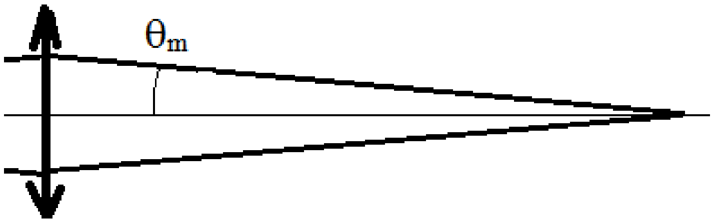
- Small β^* increases non-linearity in the longitudinal motion

  $$\delta s_{NL} \approx L\,\theta^2/2 \approx L\,\varepsilon/\beta^*$$

  The nonlinearity is compensated by sextupoles

Cooling Force

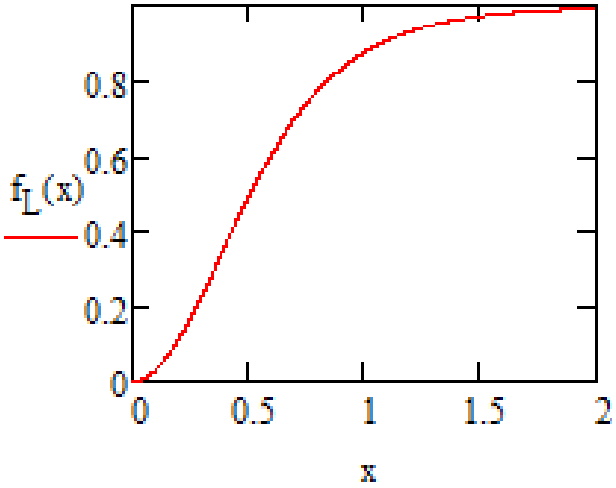
- Undulator parameter: $K \equiv \theta_e \gamma = \dfrac{eB_0 \lambda_w}{2\pi m c^2}$ $\left(\text{for flat undulator: } \lambda_0 = \dfrac{\lambda_w}{2\gamma^2}\left(1 + K^2/2\right)\right)$
- K < 1 for protons in EIC
  => simplified equations for cooling force



$$\Delta \mathrm{E}(s) = -\Delta \mathrm{E}_{tot}\left(1 + G f_L\left(\gamma\theta_m\right)\sin\left(k_0 s\right)\right)$$

$$\Delta \mathrm{E}_{tot} = \frac{2e^4 B_0^{\,2} \gamma^2 L_u}{3m^2 c^4} F \;, \quad F = \begin{cases} 1 - \text{flat und.} \\ 2 - \text{helical und.} \end{cases}$$

$$f_L(x) = 1 - \frac{1}{\left(1 + x^2\right)^3}$$



ΔEtot – energy loss in 2 undulators

G – OA gain in amplitude

L_u – total length of undulator

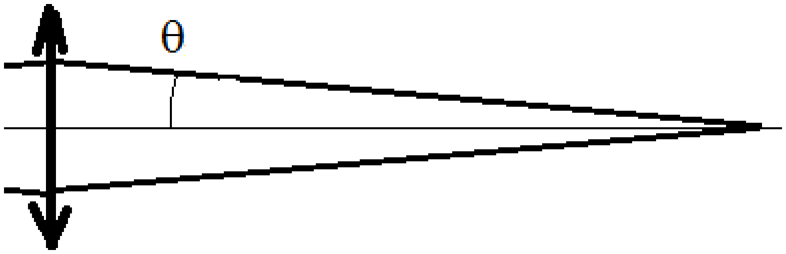
- Helical undulator yields twice larger kick for given peak mag. field, B_0
- The spot size at the focal point is determined by diffraction

$$x_0 \approx \lambda_0 \sqrt[3]{(1.51\gamma)^3 + (0.159 + 0.619/\theta_m)^3} \;,$$

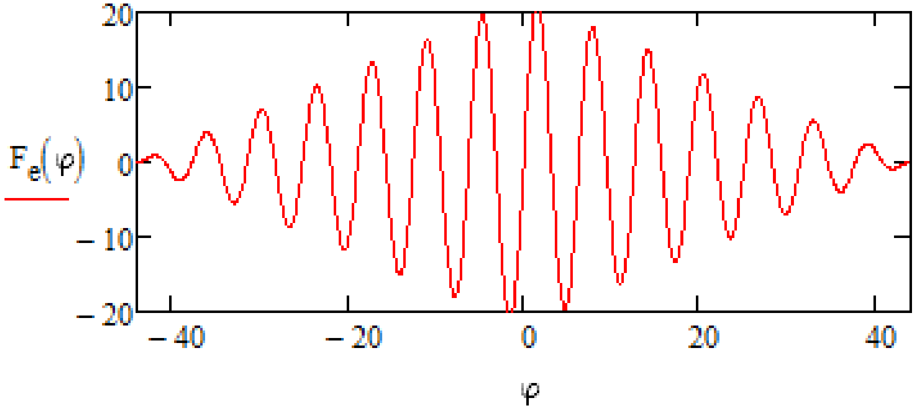
$$y_0 \approx \lambda_0 \sqrt[3]{(1.08\gamma)^3 + (0.619/\theta_m)^3} \;,$$

$$\gamma\theta_m \geq 0.1 \xrightarrow{\theta_m \ll \gamma} x_0 \approx y_0 \approx 0.619 \frac{\lambda_0}{\theta_m}$$

OSC Bandwidth

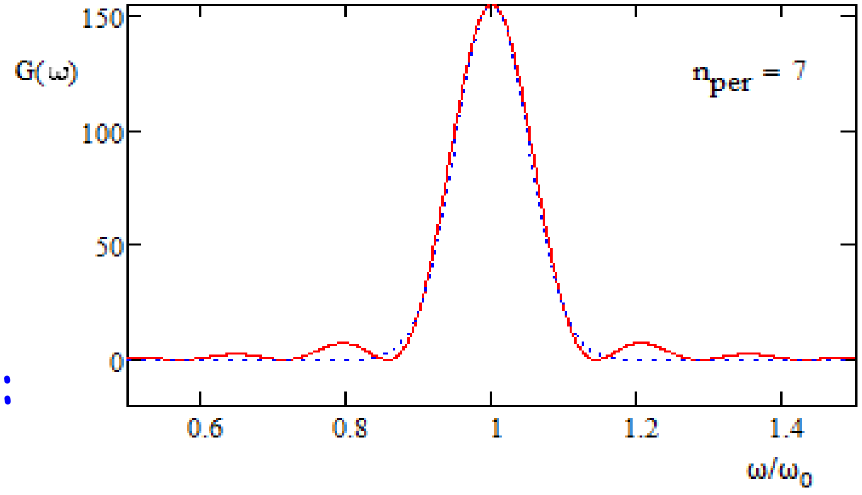


- Radiation wavelength depends on the radiation angle: $\lambda(\theta)=\frac{\lambda_w}{2\gamma^2}\left(1+\gamma^2\theta^2+\frac{K^2}{2}\right)$
- However, after focusing the summation of all waves yields fields oscillating on the basic wavelength ($\lambda_0=\lambda_w\left(1+K^2/2\right)/2\gamma^2$)



- The number of undulator periods results in the bandwidth for the system gain

$$\frac{\sigma_\omega}{\omega_0}=\frac{1}{2\sqrt{2}n_{per}}$$



- In difference to microwave stoch. cooling the OSC has two bandwidths:
  - Due to $\lambda(\theta)$.
    It affects the wave amplitude and, consequently, the required OA gain
  - Due to number of periods.
    It determines the optimal gain and maximum cooling force

Cooling Rates

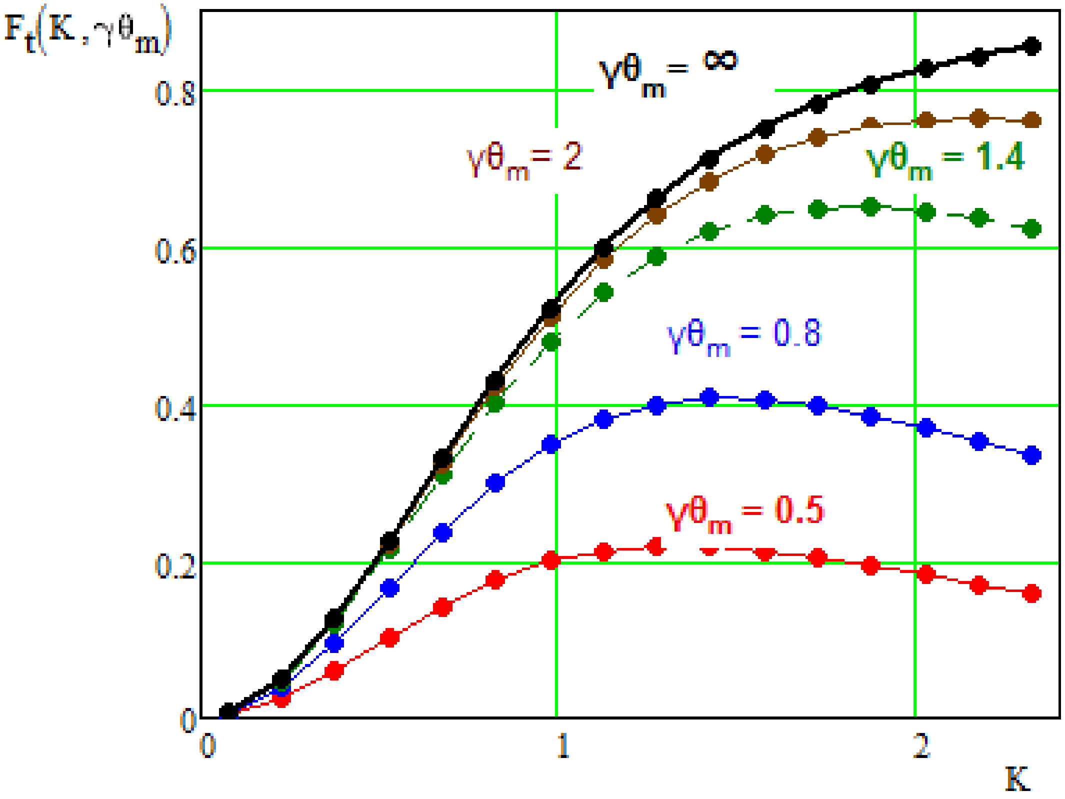
- Cooling rates for flat undulator are:

$$\begin{bmatrix}\lambda_x\\ \lambda_s\end{bmatrix} = \mu_{01}\frac{2e^4 B_0{}^2 L_u}{3m^3c^6}\frac{\gamma G f_L(\theta_{\max}\gamma)}{\left(\Delta p/p\right)_{\max}}\begin{bmatrix}R\\ 1\end{bmatrix}$$

  - For $K << 1$, the rates do not depend on the wavelength
  - The basic wavelength is determined by the delay in OA, the cooling acceptance in momentum and the ratio of cooling rates

$$k_0 \equiv \frac{2\pi}{\lambda_0} \approx \frac{\mu_{01}}{\Delta s n_{\sigma p}\sigma_p}\frac{1+R}{2}$$

  - Larger cooling rates ratio enables the wavelength decrease
  - Cooling rates saturate for K>1



Tentative Parameters for OSC with Flat Undulator

	JLEIC	eRHIC	Tevatron
Peak magnetic field, T	6	12	12
Undulator parameter	0.042	0.8	0.95
Undulator period, m	0.14	1.31	1.56
Undulator length	15.5	15.7	15.6
Number of periods	112	12	10
Basic wavelength, λ_{min}, μm	6	10	1
Bandwidth of OA, $\lambda_{max}/\lambda_{min}$	1.2	1.2	1.5
Angular acceptance of OA, mrad	4.2	1.7	0.8
Ratio of cooling rates, $R=\lambda_x/\lambda_s$	2.1	2.6	2
Delay in OA, mm	2	2	1
Long. cooling range, $n_{\sigma s}=(\Delta p/p)_{max}/\sigma_p$	5.9	5.9	4.8
Long. cooling acceptance, $(\Delta p/p)_{max}$, 10^{-3}	1.78	3.4	0.57
Long. emit. cooling time for unit gain, hour	276	173	10
OA gain (in ampl.) to counteract IBS, G	55,000	197	2.2
Power radiated into the OA band, mW	0.1	2.1	1.9
OA power required to counteract IBS, W	300,000	84	0.01
OSC relative bandwidth, $\sigma_\omega/\omega_0 = 1/(2\sqrt{2}n_{per})$	0.3%	2.9%	3.5%

Comments to Tentative Parameters for OSC

- Transverse cooling rates exceed IBS for all cases
- Cooling in Tevatron could be achieved without optical amplifier
- OSC cannot be useful for JLEC due to too high IBS rates at low energy. That leads to too high gain & too high power of OA
- eRHIC OSC operates well below optimal gain
  - Available bandwidth of 2.9% is well above the bandwidth of 0.873% required to counteract IBS
  - However, the margin is not large and does not allow the gain length to be significantly smaller than the bunch length

Discussion on the Parameter Choice for eRHIC

- OSC for eRHIC looks feasible
- Decrease of basic wavelength to ~6 μm requires reduction of σ_p

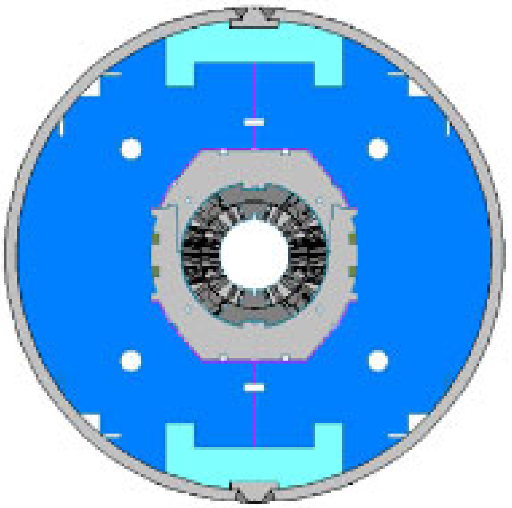
  => increases IBS & OA power
  - 10 μm looks close to optimum

	eRHIC-6μm	eRHIC-10μm
Peak magnetic field, T	12	
Undulator parameter	0.55	0.8
Undulator period, m	0.90	1.31
Undulator length	15.3	15.7
Number of periods	17	12
Basic wavelength, λ_{min}, μm	6	10
Bandwidth, $\lambda_{max}/\lambda_{min}$	1.2	
Ratio of cooling rates, $R=\lambda_x/\lambda_s$	2.6	
Delay in OA, mm	2	
Long. cooling range, $n_{\sigma s}=(\Delta p/p)_{max}/\sigma_p$	5.9	5.9
Long. cooling acceptance, $(\Delta p/p)_{max}$, 10^{-3}	2.1	3.5
Long. emit. cooling time for unit gain, hour	76	173
Transv. emit. cooling time for unit gain, hour	29	66
OA gain (in amplitude) to suppress IBS in x&s, *G*	326	197
Power radiated into the OA band, mW	3	2.1
OA power, W	313	84

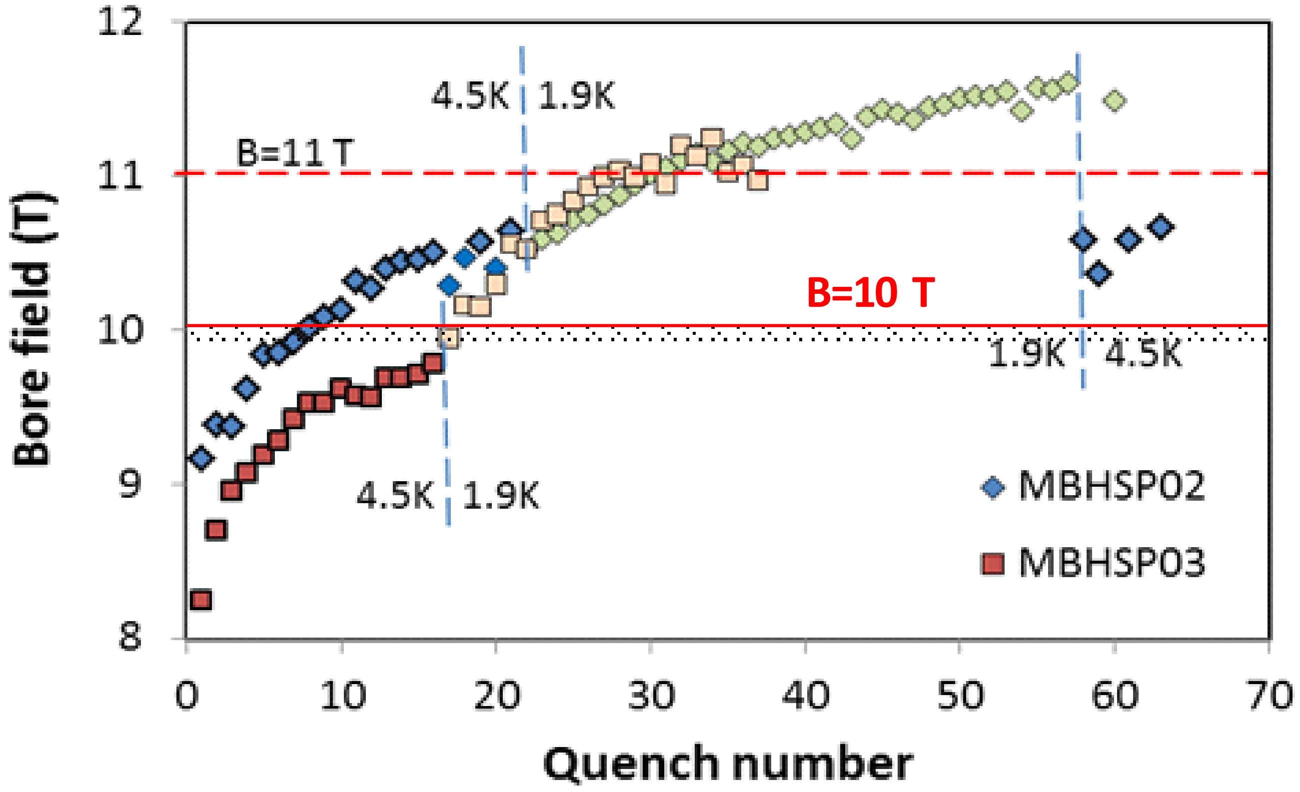
Undulator

- A usage of the present state-of-the-art dipoles developed in Fermilab is proposed for undulators

Nb_3Sn 11 T dipole (FNAL)



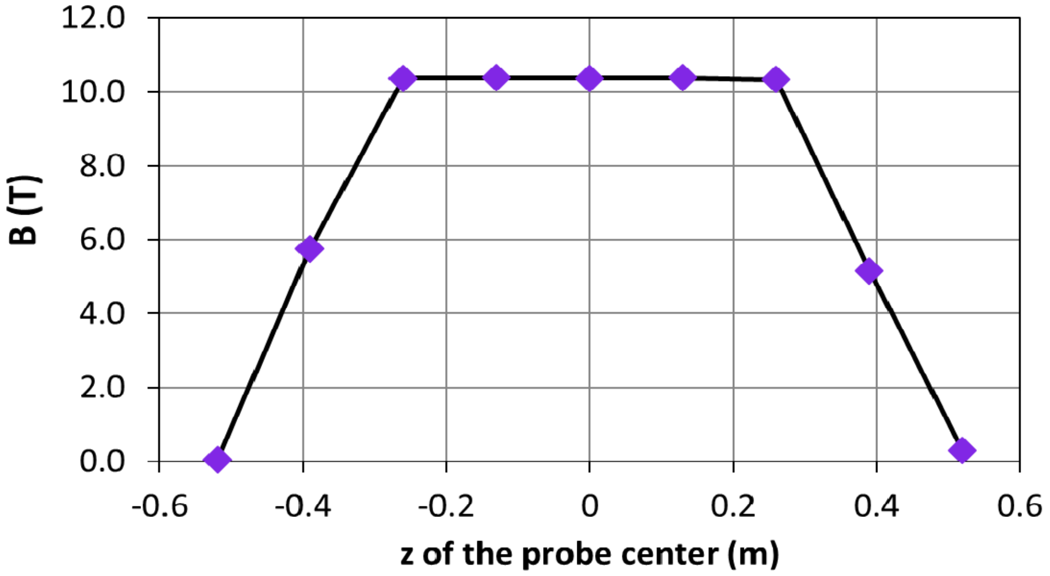
Parameter	Value
Coil aperture	60 mm
Yoke outer diameter	400 mm
Nominal bore field at 11.85 kA	10.9 T
Short sample field B_{SSL} at 1.9 K	13.4 T
Margin B_{nom}/B_{SSL} at 1.9 K	81%
Stored energy at 11.85 kA	424 kT/m
F_x/quadrant at 11.85 kA	2.89 MN/m
F_y/quadrant at 11.85 kA	-1.58 MN/m

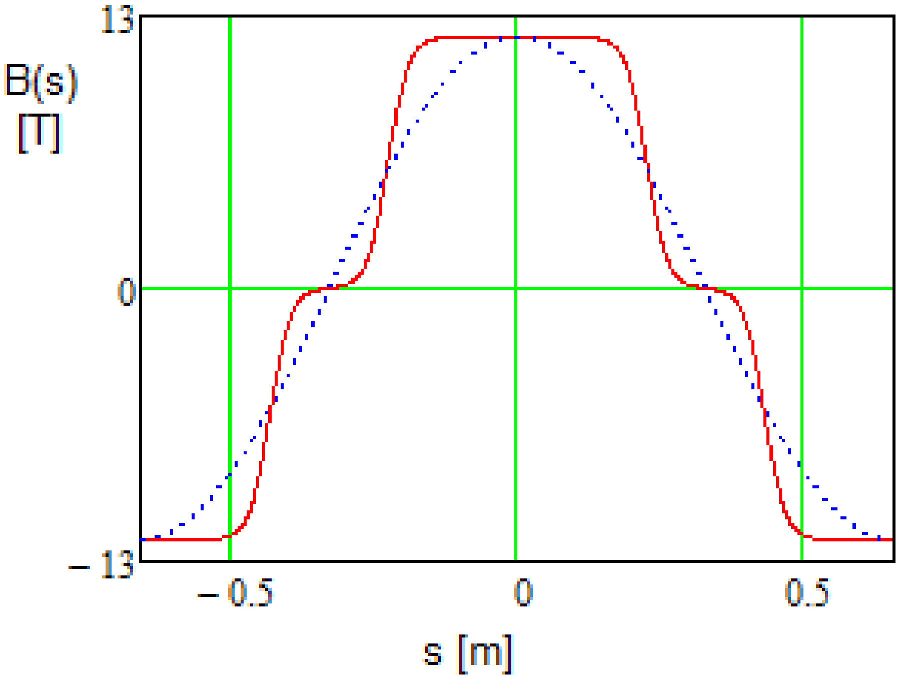


* Curtesy of A. Zlobin, Fermilab

Undulator (continue)

- Undulator -> Wiggler
  - Sequence of sign changing SC dipole magnets
  - Non-harmonic field
- Cooling force ∝ square of 1st harmonic
  - ⇨ Cooling force increases by 1.5 times relative to the harmonic undulator with the same peak field (not accounted above)





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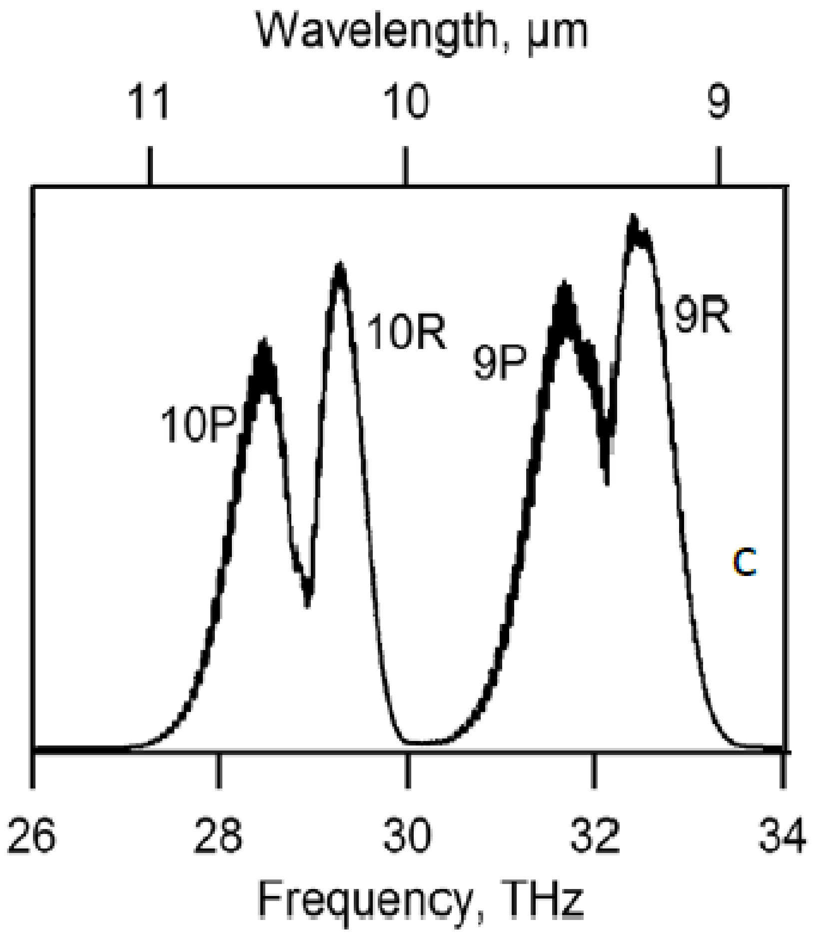
Optical Amplifier

- OA concept requires additional studies
- CO_2 based OA is a possibility
  - 1 kW CW lasers are used in industry
  - BNL TW laser may be considered as "zero step" approximation [1]
    - 10-bar isotopic CO_2 amplifier ($C^{16}O_2$, $C^{18}O_2$ 50:50% mixture) has sufficiently wide band
    - Length=1 m
    - 10^3 net amplification
- Obtaining the 2 mm delay and the required gain in CW regime may present considarable challenge
  - The delay in 1 m CO_2 at P=10 bar is ~4 mm (well above required 2 mm)

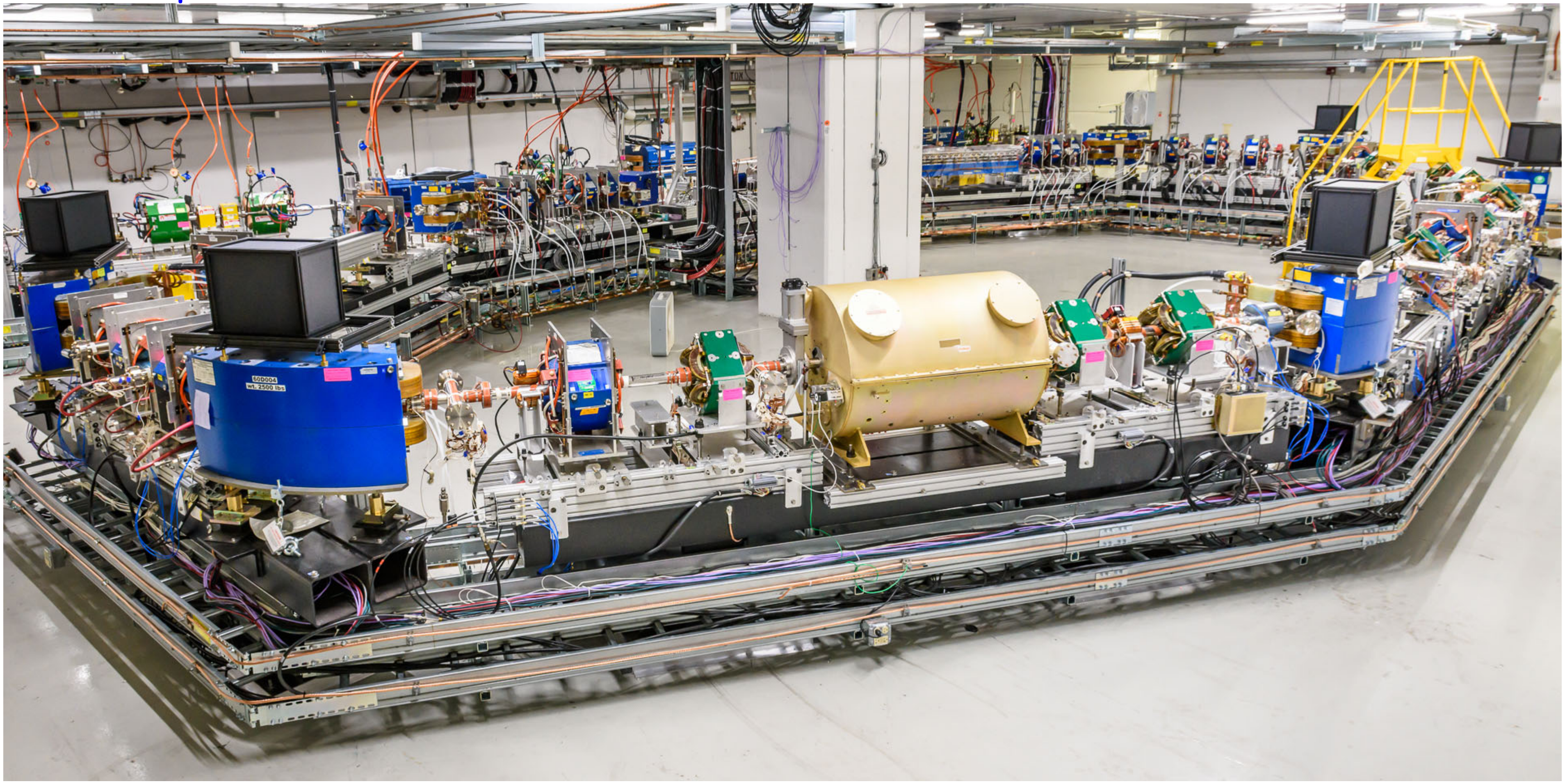


[1] I. Pogorelsky, M. Babzien, I. Ben-Zvi, J. Skaritka, M. Polyanskiy, "BESTIA - the next generation ultra-fast CO_2 laser for advanced accelerator research" BNL-111612-2015-JA

OSC Test in IOTA

- 100 MeV ($\gamma \approx 200$) electrons
- Passive OSC at 0.95 μm (no OA)
  - 16 period undulators
- Active OSC at 2.2 μm (7 dB OA, Cr:ZnSe crystal, 2 mm delay)
  - 7 period undulators

- We plan OSC demonstration
  - with large number of particles
  - and single electron



“Other Approaches: Stochastic Cooling”, Valeri Lebedev, FNAL, EIC Accelerator Collaboration Meeting, Jefferson Lab, Oct. 29 – Nov. 1, 2018

Discussion and Conclusions

- OSC cannot be used in JLEIC because too small energy of proton beam
- OSC in eRHIC looks as a possibility for cooling protons
  - To move forward with the proposal,
    we need a solid concept for OA
  - Usage of parametric OA is limited due to its gain length being much shorter than the bunch length.
    Usage of FEL has the same problem
- Cooling of fully striped ions looks effective only for heavy ions if we assume the same rigidity of the ring and the same gain in OA

$$\lambda_{ions} \approx \frac{Z^4}{A^3}\frac{\gamma}{\gamma_{protons}}\lambda_{protons} \xrightarrow[A=2Z]{Z/(A\gamma)=const} \frac{Z}{16}\lambda_{protons}$$

  - It also requires an undulator with another period
- Energy increase of proton beam would greatly simplify OSC system and requirements to OA
- We plan to look if OSC can be used for electron beam cooling in a ring electron cooler with e-beam energy of ~100 MeV