Exploring the Pion Structure in Minkowski Space

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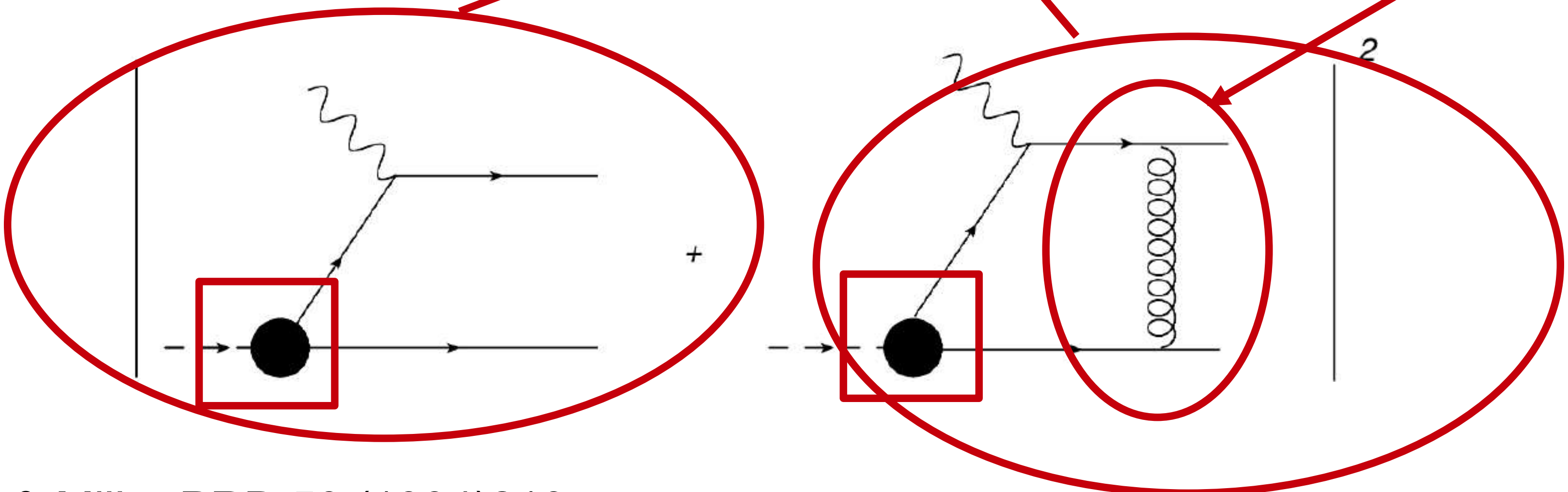
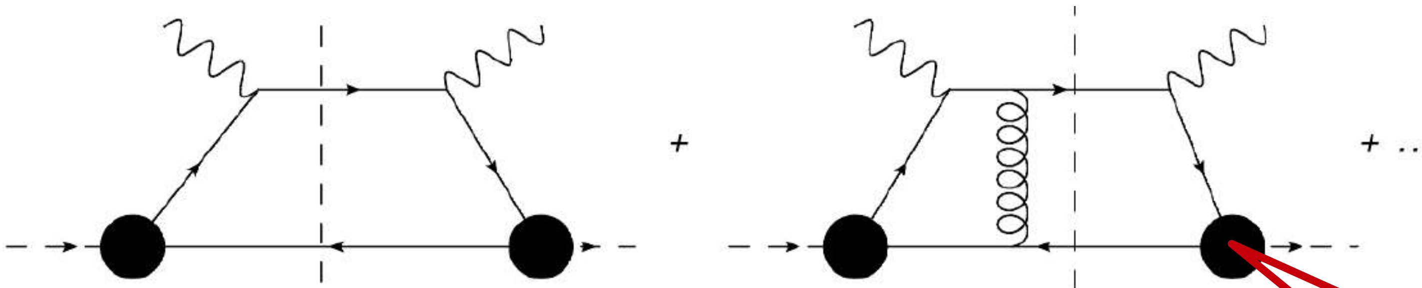
Correlations in Partonic and Hadronic Interactions 2018
Alikhanyan National Laboratory of Armenia, Yerevan, Sept.24-28, 2018

TMDs & PDFs

FSI gluon exchange: T-odd



TF & Miller PRD 50 (1994)210



$q^2 = q^+q^- - q^2{}_T$

$q^+ = q^0+q^3 \quad q^- = q^0-q^3$

$q^- \longrightarrow$ *infty*
DIS

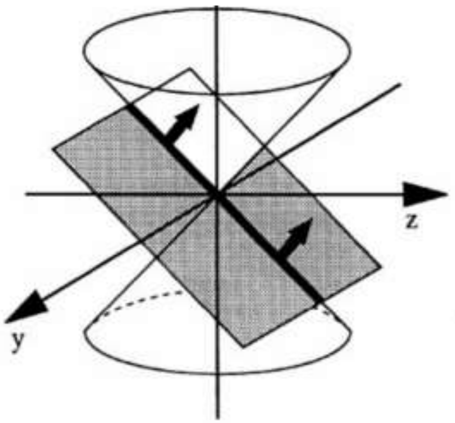
Bethe-Salpeter Amplitude @ $x^+=0$

Bethe-Salpeter Amplitude →Light-Front WF (LFWF)

- **basic ingredient in PDFs, GPDs and TMDs**

$$\tilde{\Phi}(x,p) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\Phi(k,p)$$

$$p^\mu = p_1^\mu + p_2^\mu \qquad k^\mu = \frac{p_1^\mu - p_2^\mu}{2}$$



$$\tilde{\Phi}(x,p) = \langle 0|T\{\varphi_H(x^\mu/2)\varphi_H(-x^\mu/2)\}|p\rangle$$

$$= \theta(x^+)\langle 0|\varphi(\tilde{x}/2)e^{-iP^- x^+/2}\varphi(-\tilde{x}/2)|p\rangle e^{ip^- x^+/4} + \cdots$$

$$= \theta(x^+)\sum_{n,n'} e^{ip^- x^+/4}\langle 0|\varphi(\tilde{x}/2)|n'\rangle\langle n'|e^{-iP^- x^+/2}|n\rangle\langle n|\varphi(-\tilde{x}/2)|p\rangle + \cdots$$

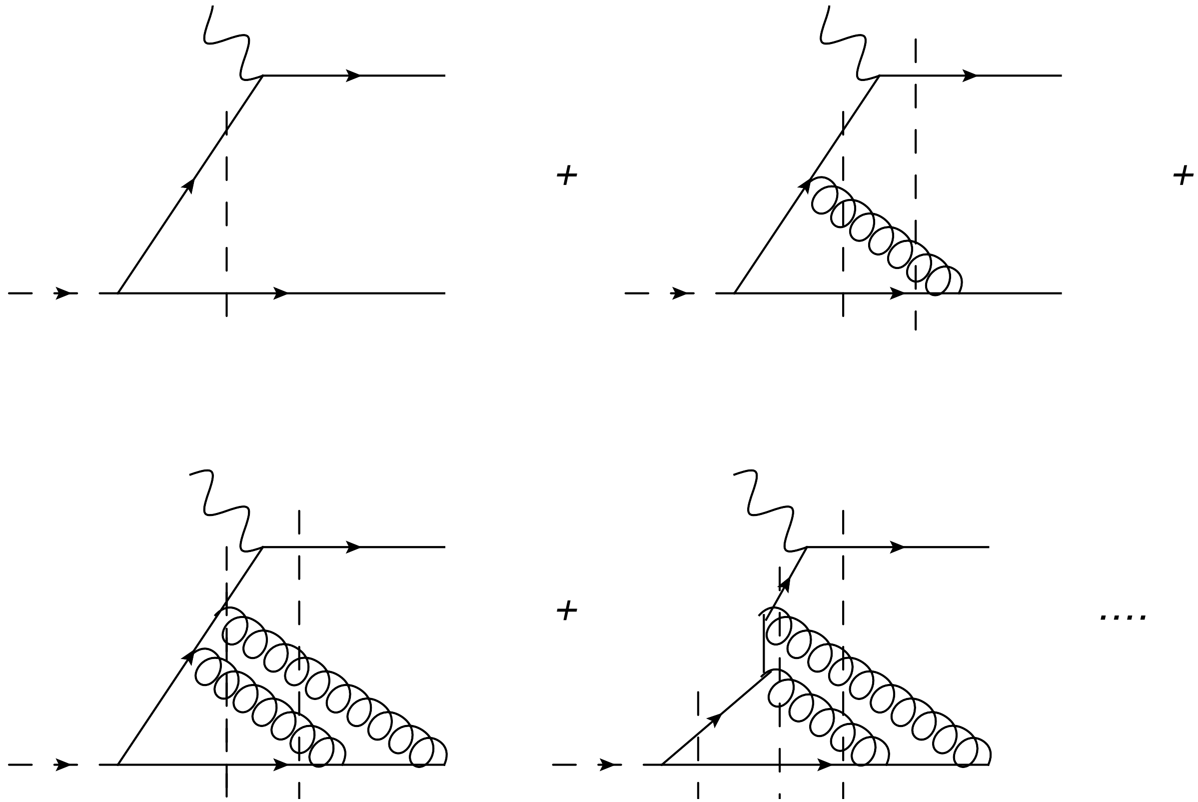
$x^+ = 0$ only valence state remains! How to rebuilt the full BS amplitude?

Iterated Resolvents: Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998)

Beyond the valence ....

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001)

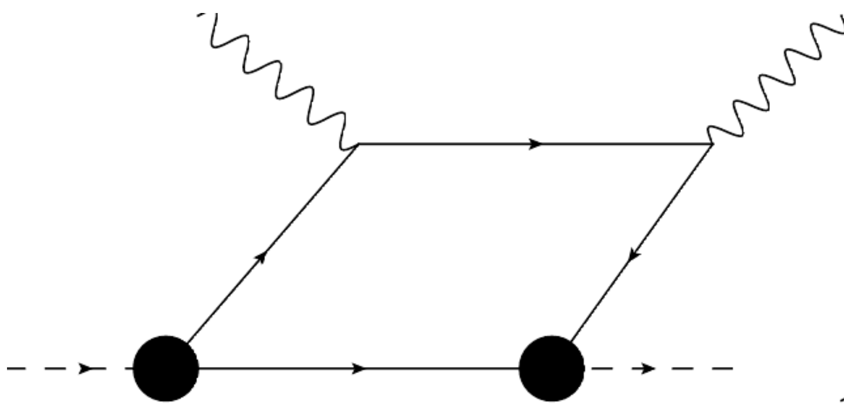
Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010 (2008)

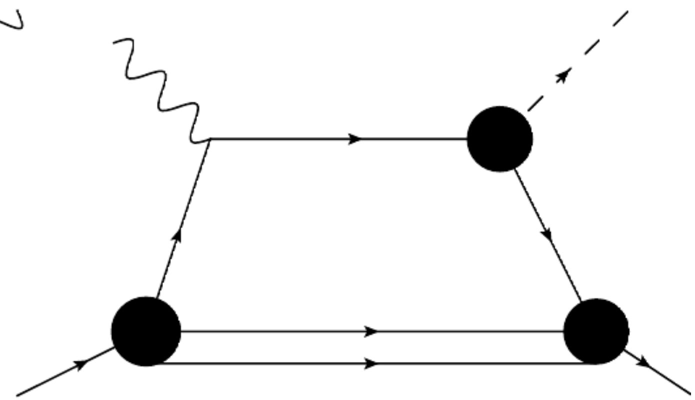


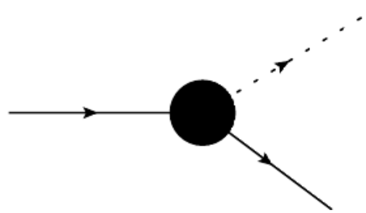
- **Population of lower x, due to the gluon radiation!**
- **Evolution?**

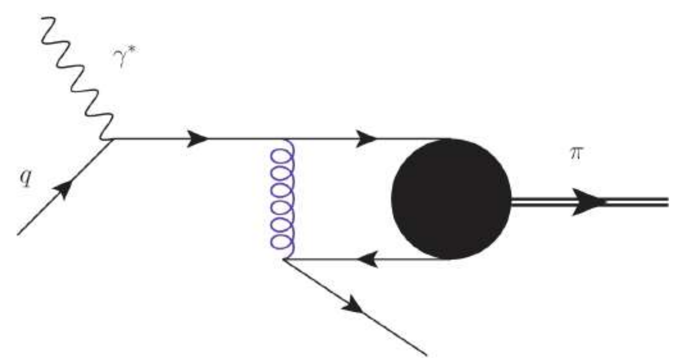
Beyond the valence ….

ERBL – DGLAP regions









Fragmentation function

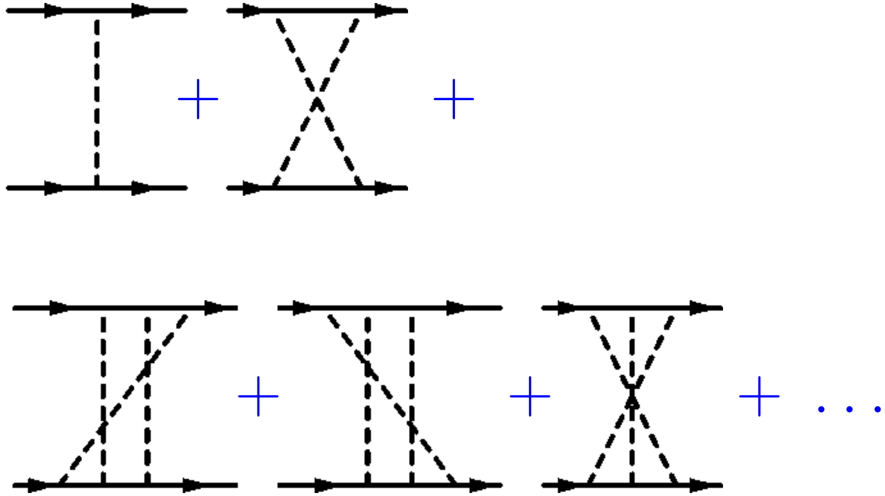
Reminder…

Bethe-Salpeter Bound-State Equation (2 bosons)

$$\Phi(k,p) = G_0^{(12)}(k,p)\int \frac{d^4k'}{(2\pi)^4}\mathrm{i}K(k,k';p)\Phi(k',p)$$

$$G_0^{(12)}(k,p) = \frac{\mathrm{i}}{\left[(p/2+k)^2-m^2+\mathrm{i}\epsilon\right]}\,\frac{\mathrm{i}}{\left[(p/2-k)^2-m^2+\mathrm{i}\epsilon\right]}$$

Kernel: sum 2PI diagrams



- **Valence LF wave function → BSA ?**
- **Valence→ full Fock Space w-f ?**

Sales, et al. PRC61, 044003 (2000)

Main Tool: Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space" [Nakanishi PR130(1963)1230]

Bethe-Salpeter amplitude

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

$$\kappa^2 = m^2 - \frac{M^2}{4}$$

BSE in Minkowski space with NIR for bosons

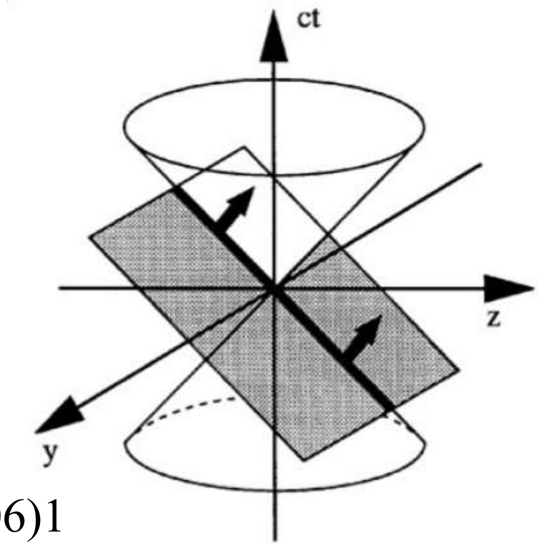
Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k^-

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD85(2012)036009;PRD89(2014) 016010,EPJC75(2015)398 (application to scattering)

LF wave function



& NAKANISHI INTEGRAL REPRESENTATION

Carbonell&Karmanov EPJA27(2006)1

$$\psi_{LF}(\gamma, z) = \frac{1}{4}(1 - z^2) \int_0^\infty \frac{g(\gamma', z) d\gamma'}{\left[\gamma' + \gamma + z^2 m^2 + \kappa^2 (1 - z^2)\right]^2}$$

$$\gamma = k_\perp^2 \qquad z = 2x - 1$$

Solution Method of the Bethe-Salpeter eq.:

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11

$$\Phi(k,p) = G_0(k,p) \int d^4k' \, \mathcal{K}_{BS}(k,k',p) \, \Phi(k',p)$$

$\Rightarrow$

$$\int_0^\infty d\gamma' \frac{g_b(\gamma',z;\kappa^2)}{[\gamma'+\gamma+z^2m^2+(1-z^2)\kappa^2-i\epsilon]^2} =$$

$$= \int_0^\infty d\gamma' \int_{-1}^{1} dz' \; V_b^{LF}(\gamma,z;\gamma',z') g_b(\gamma',z';\kappa^2).$$

with $V_b^{LF}(\gamma,z;\gamma',z')$ determined by the irreducible kernel $\mathcal{I}(k,k',p)$!

UNIQUENESS OF THE NAKANISHI WEIGHT FUNCTION?

PERTURBATIVE PROOF BY NAKANISHI

NON-PERTURBATIVE PROOF?

Generalized Stietjes transform and the LF valence wave function

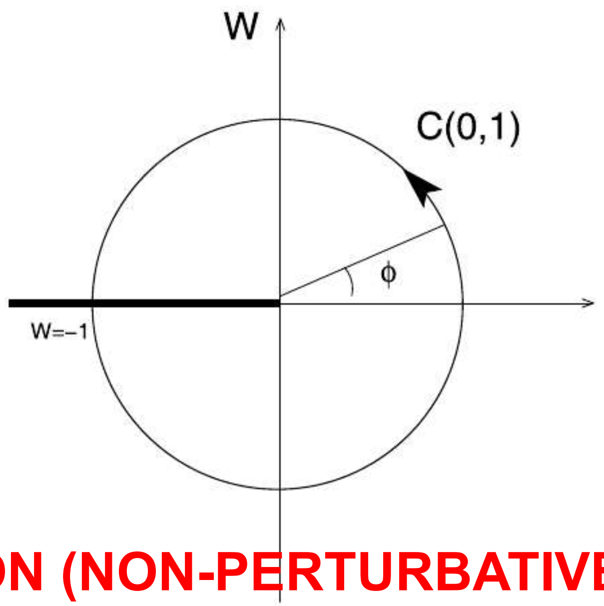
Jaume Carbonell, TF, Vladimir Karmanov PLB769 (2017) 418

$$\psi_{LF}(\gamma,z) = \frac{1-z^2}{4}\int_0^\infty \frac{g(\gamma',z)d\gamma'}{\left[\gamma'+\gamma+z^2m^2+(1-z^2)\kappa^2\right]^2}.$$

$$f(\gamma) \equiv \int_0^\infty d\gamma' \, L(\gamma,\gamma')g(\gamma') = \int_0^\infty d\gamma' \, \frac{g(\gamma')}{(\gamma'+\gamma+b)^2}$$

denoted symbolically as $f = \hat{L}\, g$.

$$g(\gamma) = \hat{L}^{-1} f = \frac{\gamma}{2\pi}\int_{-\pi}^{\pi} d\phi \; e^{i\phi} \; f(\gamma\, e^{i\phi} - b).$$



J.H. Schwarz, J. Math. Phys. 46 (2005) 014501,

- **UNIQUENESS OF THE NAKANISHI REPRESENTATION (NON-PERTURBATIVE)**
- **PHENOMENOLOGICAL APPLICATIONS from the valence wf → BSA!**

(I) Valence LF wave function in impact parameter space

Miller ARNPS 60 (2010) 25

$$F(Q^2) = \int d^2\mathbf{b}\, \rho(\mathbf{b})\, e^{-i\mathbf{b}\cdot\mathbf{q}_\perp}$$

$$\rho(\mathbf{b}) = \rho_{\text{val}}(\mathbf{b}) + \text{ higher Fock states densities} \cdots$$

$$\rho_{\text{val}}(\mathbf{b}) = \frac{1}{4\pi} \int_0^1 \frac{d\xi}{\xi(1-\xi)^3} |\phi(\xi, \mathbf{b}/(1-\xi))|^2$$

» Burkardt IJMPA 18 (2003) 173

$$\phi(\xi, \mathbf{b}) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \psi(\xi, \mathbf{k}_\perp) e^{i\mathbf{k}_\perp\cdot\mathbf{b}}$$

$$\phi(\xi, b) = \frac{\xi(1-\xi)}{4\pi\sqrt{2}} F(\xi, b)$$

$$F(\xi, b) = \int_0^\infty d\gamma\, J_0(b\sqrt{\gamma}) \int_0^\infty d\gamma' \frac{g(\gamma', 1-2\xi; \kappa^2)}{[\gamma + \gamma' + \kappa^2 + (1/2-\xi)^2 M^2]^2}$$

(II) Valence LF wave function in impact parameter space

$$F(\xi, b)|_{b\to\infty} \to \mathrm{e}^{-b\sqrt{\kappa^2+(\xi-1/2)^2M^2}}\, f(\xi, b)$$

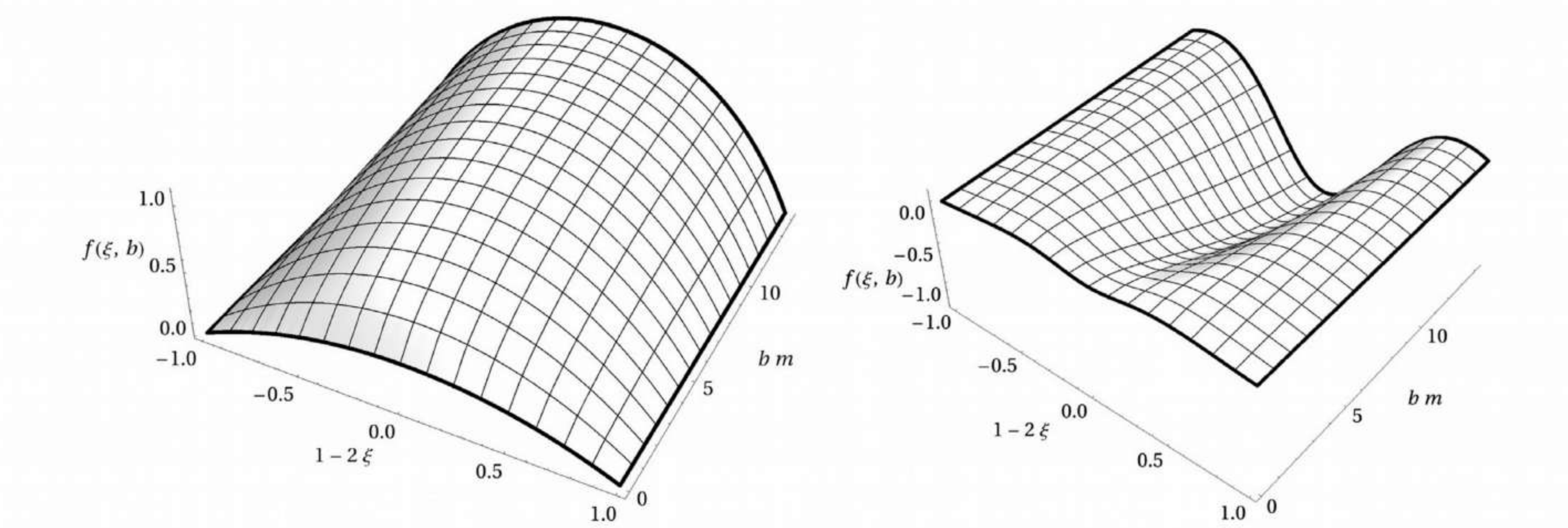


Fig. 7. The valence functions $f(\xi, b)$ in the impact parameter space. Left panel: the ground state, corresponding to $B(0) = 1.9m$, $\mu = 0.1m$ and $\alpha_{gr} = 6.437$. Right panel: first-excited state, corresponding to $B(1) = 0.22m$, $\mu = 0.1m$ and $\alpha_{gr} = 6.437$.

Gutierrez, Gigante, TF, Salmè,Viviani, Tomio PLB759 (2016) 131

Light-front valence wave function L+XL

Large momentum behavior

$$\psi_{LF}(\gamma,\xi) \to \alpha\,\gamma^{-2}\,C(\xi)$$

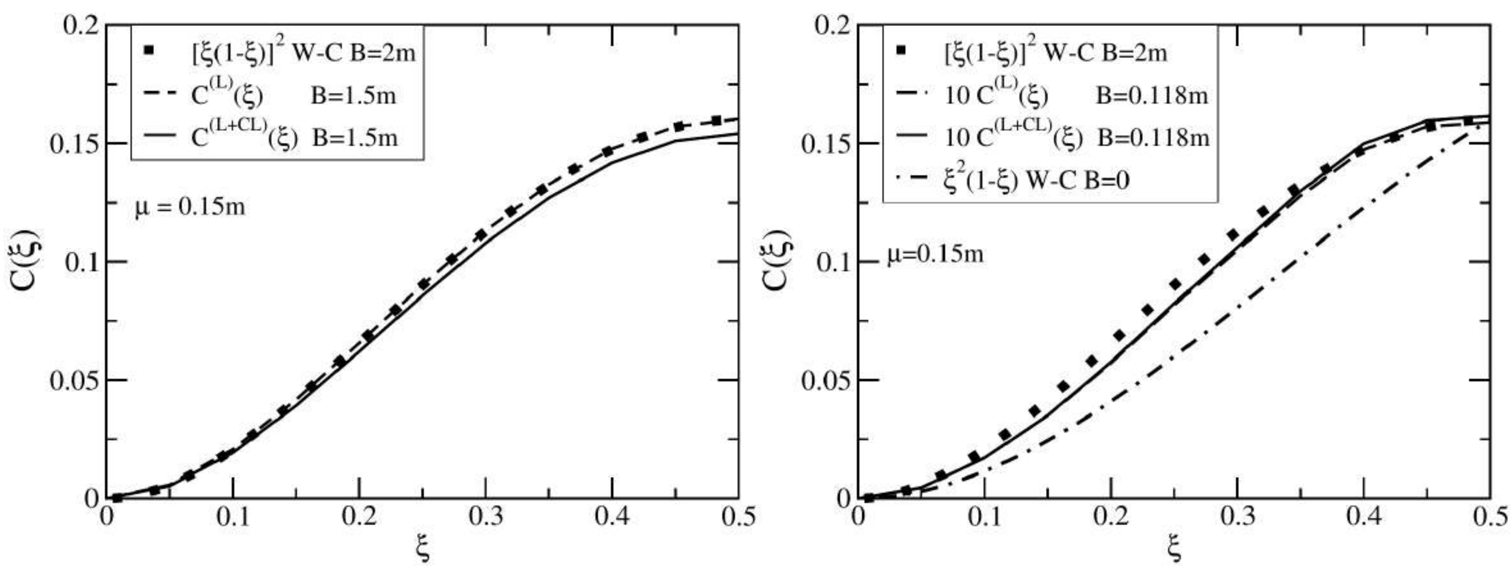


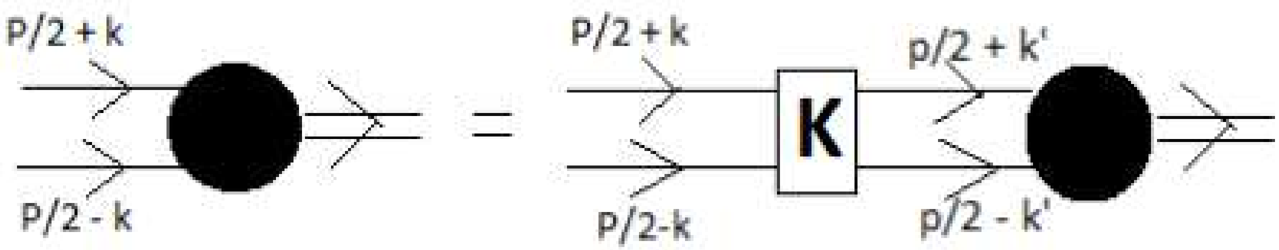
Fig. 2. Asymptotic function $C(\xi)$ defined from the LF wave function for $\gamma \to \infty$ (6) computed for the ladder kernel, $C^{(L)}(\xi)$ (dashed line), and ladder plus cross-ladder kernel, $C^{(L+CL)}(\xi)$ (solid line), with exchanged boson mass of $\mu = 0.15\,m$. Calculations are performed for $B = 1.5\,m$ (left frame) and $B = 0.118\,m$ (right frame). A comparison with the analytical forms of $C(\xi)$ valid for the Wick-Cutkosky model for $B = 2m$ (full box) and $B \to 0$ (dash-dotted line) both arbitrarily normalized.

Gigante, Nogueira, Ydrefors, Gutierrez, Karmanov, TF, PRD95(2017)056012.

BSE for qqbar: pion

Carbonell and Karmanov EPJA 46 (2010) 387;

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;



$$\Phi(k,p) = S(k+p/2) \int \frac{d^4k'}{(2\pi)^4} F^2(k-k') \, i\mathcal{K}(k,k') \, \Gamma_1 \, \Phi(k',p) \, \bar{\Gamma}_2 \, S(k-p/2)$$

Ladder approximation (L): suppression of XL (non-planar diagram) for N_c=3

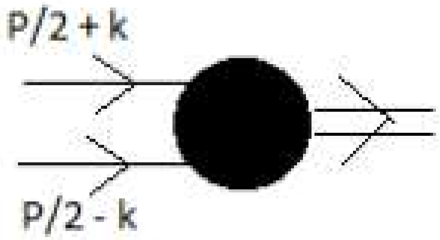
[A. Nogueira, CR Ji, Ydrefors, TF, PLB 777 (2018) 207]

Vector $$i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}$$

Vertex Form-Factor $$F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$$

NIR for fermion-antifermion: 0^- (pion)

BS amplitude



$$\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu\gamma_5$$

$$\phi_i(k,p) = \int_{-1}^{+1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{(k^2 + p\cdot k\ z' + M^2/4 - m^2 - \gamma' + i\epsilon)^3}$$

Light-front projection: integration over k^- (LF singularities)

For the two-fermion BSE, singularities have generic form:

$$\mathcal{C}_j = \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} (k^-)^j \, \mathcal{S}(k^-, v, z, z', \gamma, \gamma') \qquad j = 1, 2, 3$$

with $\mathcal{S}(k^-, v, z, z', \gamma, \gamma')$ explicitly calculable

N.B., in the worst case

$$\mathcal{S}(k^-, v, z, z', \gamma, \gamma') \sim \frac{1}{[k^-]^2} \qquad \text{for} \quad k^- \to \infty$$

End-point singularities: T.M. Yan , Phys. Rev. **D 7**, 1780 (1973)

$$\mathcal{I}(\beta, y) = \int_{-\infty}^{\infty} \frac{dx}{\left[\beta x - y \mp i\epsilon\right]^2} = \pm \frac{2\pi i \; \delta(\beta)}{\left[-y \mp i\epsilon\right]}$$

→ Kernel with delta's and its derivatives!

End-point singularities– more intuitive: can be treated by the pole-dislocation method
de Melo et al. NPA631 (1998) 574C, PLB708 (2012) 87

Scalar boson exchange

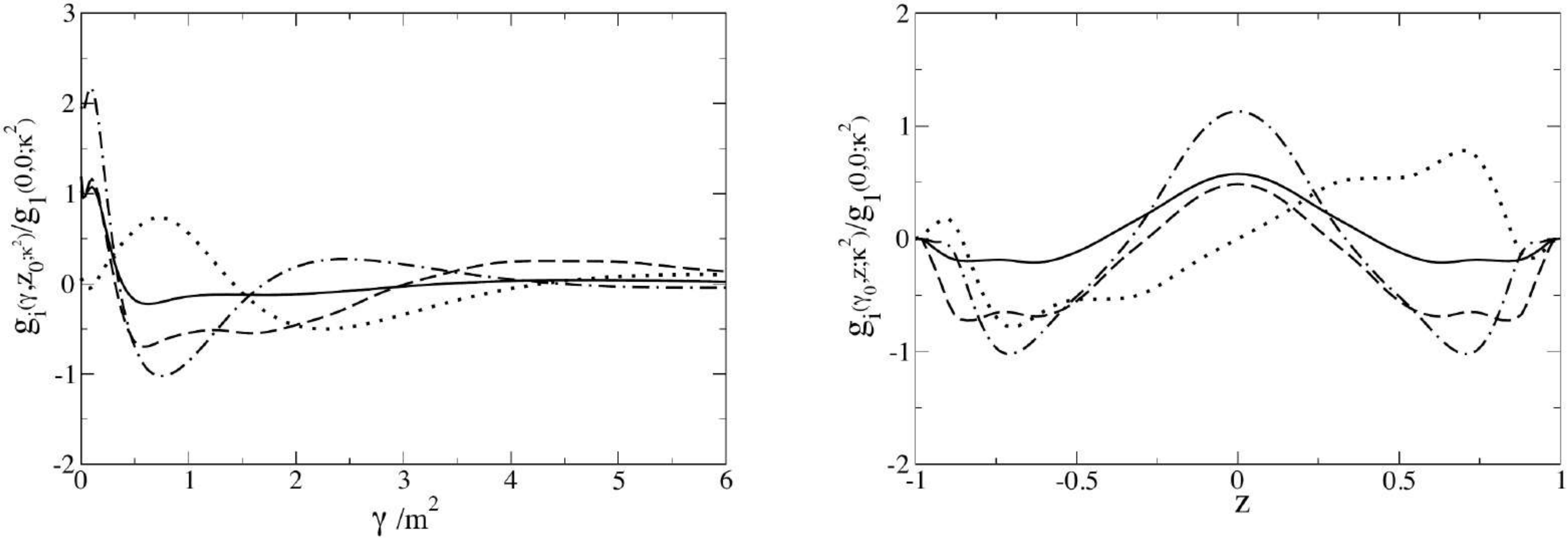
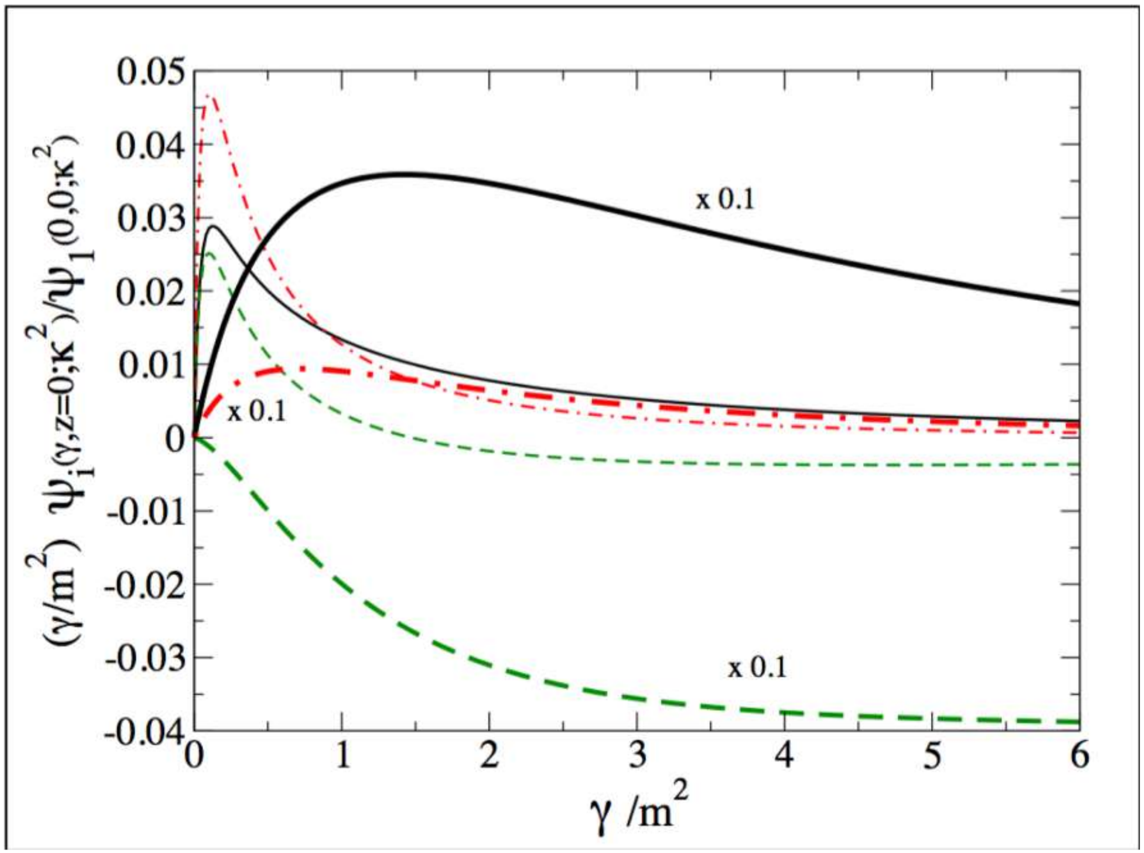


Figure 2. Nakanishi weight-functions $g_i(\gamma, z; \kappa^2)$, Eqs. 3.1 and 3.2 evaluated for the 0^+ two-fermion system with a scalar boson exchange such that $\mu/m = 0.5$ and $B/m = 0.1$ (the corresponding coupling is $g^2 = 52.817$ [17]). The vertex form-factor cutoff is $\Lambda/m = 2$. Left panel: $g_i(\gamma, z_0; \kappa^2)$ with $z_0 = 0.6$ and running γ/m^2. Right panel: $g_i(\gamma_0, z; \kappa^2)$ with $\gamma_0/m^2 = 0.54$ and running z, The Nakanishi weight-functions are normalized with respect to $g_1(0, 0; \kappa^2)$. Solid line: g_1. Dashed line: g_2. Dotted line: g_3. Dot-dashed line: g_4.

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;

Massless vector exchange: high-momentum tails

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;



LF amplitudes ψ_i times γ/m^2 at fixed $z = 0$, for the vector coupling. $B/m = 0.1$ (thin lines) and 1.0 (thick lines).

—— : $(\gamma/m^2)\ \psi_1$.

— —: $(\gamma/m^2)\ \psi_2$.

— • : $(\gamma/m^2)\ \psi_4$.

$\psi_3 = 0$ for $z = 0$

Power one is expected for the pion valence amplitude:

X Ji et al, PRL 90 (2003) 241601.

PION MODEL

W. de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

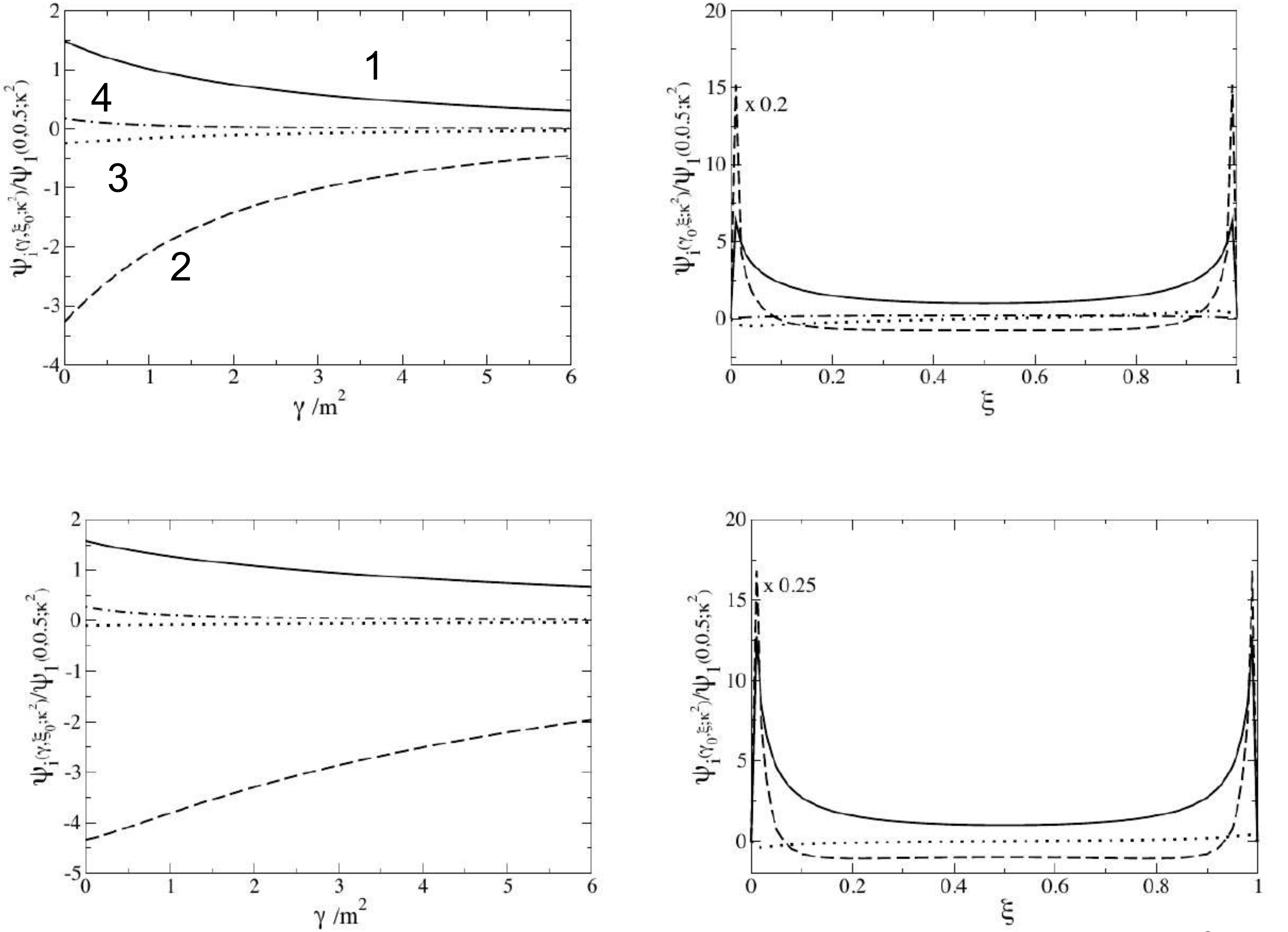
- **Gluon effective mass ~ 500 MeV – Landau Gauge LQCD**

[Oliveira, Bicudo, JPG 38 (2011) 045003;
Duarte, Oliveira, Silva, Phys. Rev. D 94 (2016) 01450240]

- **Mquark = 250 MeV**

[Parappilly, et al, PR D73 (2006) 054504]

- **Λ/m =1, 2, 3**



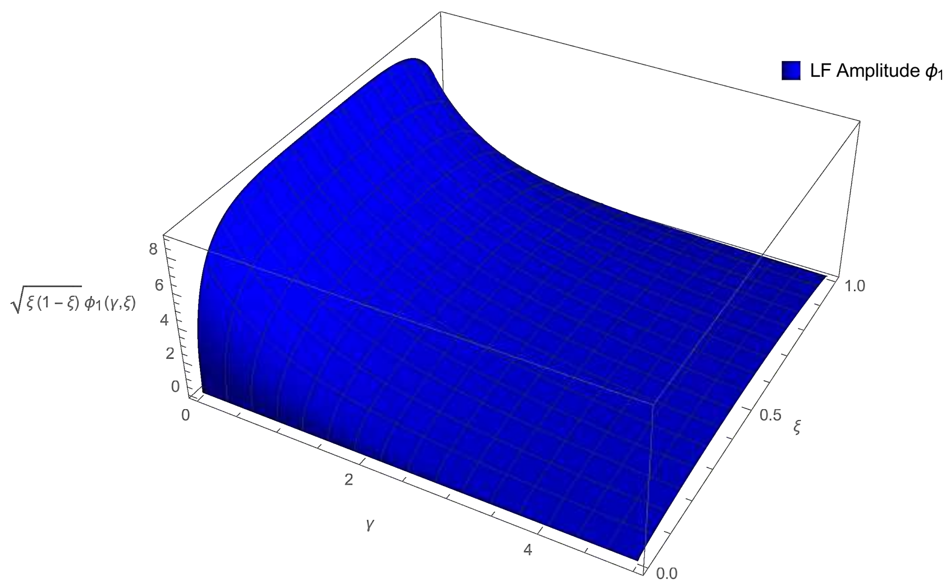
$f_\pi = 150\,\mathrm{MeV}$

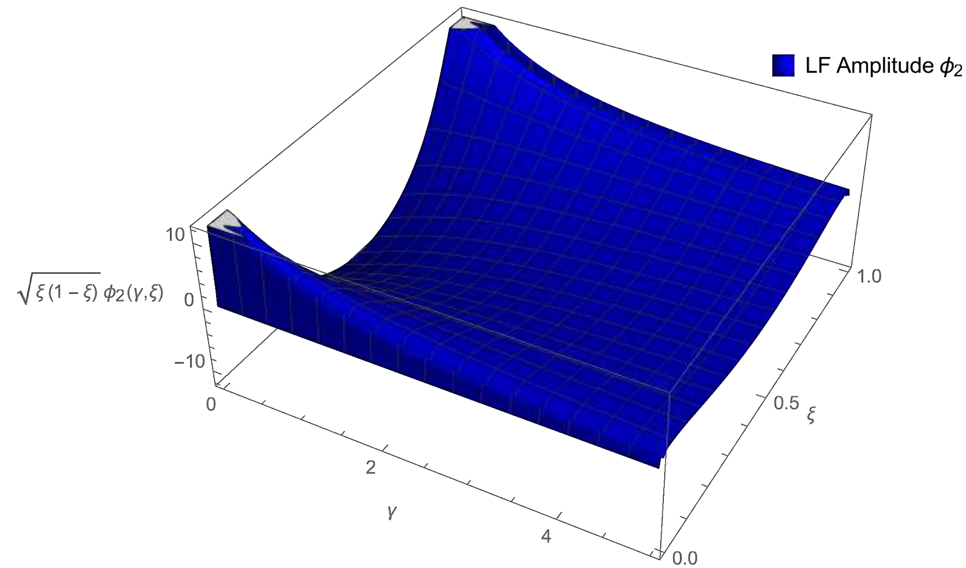
Figure 6. Light-front amplitudes $\psi_i(\gamma, \zeta)$, Eq. 3.11, for the pion-like system with a heavy-vector exchange ($\mu/m = 2$), binding energy of $B/m = 1.44$ and constituent mass $m = 250$ MeV. Upper panel: vertex form-factor cutoff $\Lambda/m = 3$ and $g^2 = 435.0$, corresponding to $\alpha_s = 10.68$ (see text for the definition of α_s). Lower panel: vertex form-factor cutoff $\Lambda/m = 8$ and $g^2 = 53.0$, corresponding to $\alpha_s = 3.71$. The value of the longitudinal variable is $\xi_0 = 0.2$ and $\gamma_0 = 0$. Solid line: ψ_1 . Dashed line: ψ_2. Dotted line: ψ_3. Dot-dashed line: ψ_4

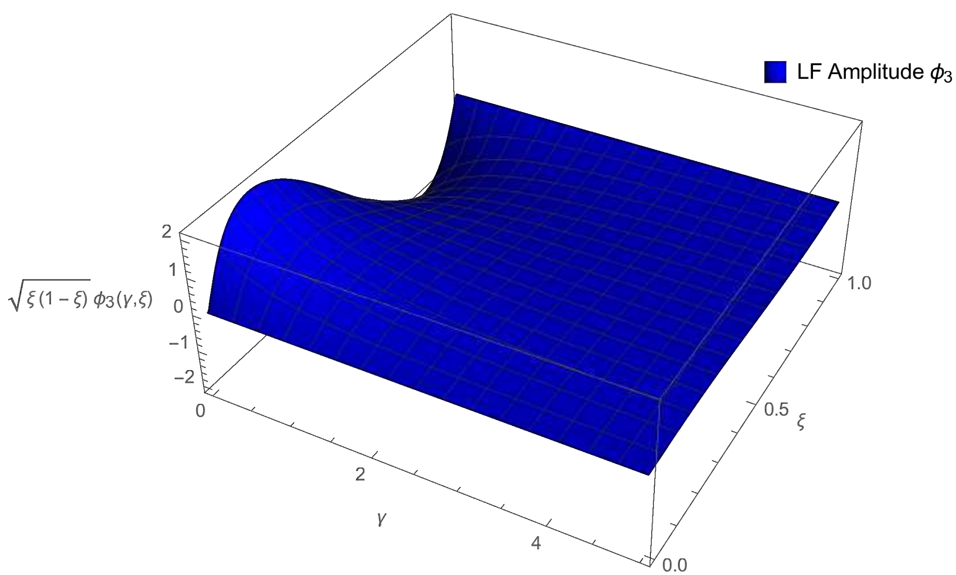
Light-front amplitudes

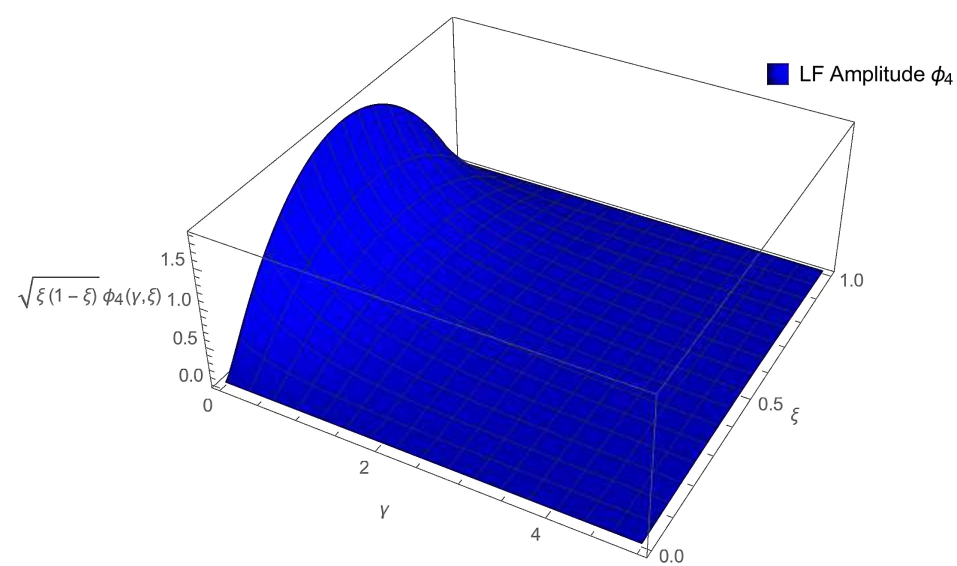
$(B/m = 1.35, \mu/m = 2.0, \Lambda/m = 1.0, \tilde{m}_q\text{=215 MeV}): f_\pi = 96\ \text{MeV}$

$P_{val} = 0.68$









Valence distribution functions

W. de Paula, et. al, in preparation

Valence probability:

$$N_2 = \frac{1}{32\,\pi^2} \int_{-1}^{1} dz \int_0^\infty d\gamma \left\{ \tilde{\psi}_{val}(\gamma, \xi)\ \tilde{\psi}_{val}(\gamma, \xi) + \frac{\gamma}{M^2}\ \psi_{val;4}(\gamma, \xi)\ \psi_{val;4}(\gamma, \xi) \right\}$$

$$\tilde{\psi}_{val}(\gamma, z) = -\frac{i}{M} \int_0^\infty d\gamma' \frac{g_2(\gamma', z)}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2)\kappa^2 - i\epsilon]^2}$$
$$-\frac{i}{M}\frac{z}{2} \int_0^\infty d\gamma' \frac{g_3(\gamma', z)}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2)\kappa^2 - i\epsilon]^2}$$
$$+\frac{i}{M^3} \int_0^\infty d\gamma' \ \frac{\partial g_3(\gamma', z)/\partial z}{[\gamma + \gamma' + z^2 m^2 + (1 - z^2)\kappa^2 - i\epsilon]}$$

$$\psi_{val;4}(\gamma, z) = -\frac{i}{M} \int_0^\infty d\gamma' \frac{g_4(\gamma', z)}{[\gamma + \gamma' + m^2 z^2 + (1 - z^2)\kappa^2 - i\epsilon]^2}.$$

Valence probability

Table 1 Valence probability for a massive vector exchange, with $\mu/m = 0.15$ and a cut-off $\Lambda/m = 2$ for the vertex form-factor. The number of gaussian points is 72.

B/m	Prob.
0.01	0.96
0.1	0.78
1.0	0.68

Table 2 Valence probability for a massive vector exchange, with $\mu/m = 0.5$ and a cut-off $\Lambda/m = 2$ for the vertex form-factor. The number of gaussian points is 72.

B/m	Prob.
0.01	0.96
0.1	0.84
1.0	0.68

Lot of room for the higher LF Fock components of the wave function to manifest!

Valence distribution functions: longitudinal and transverse

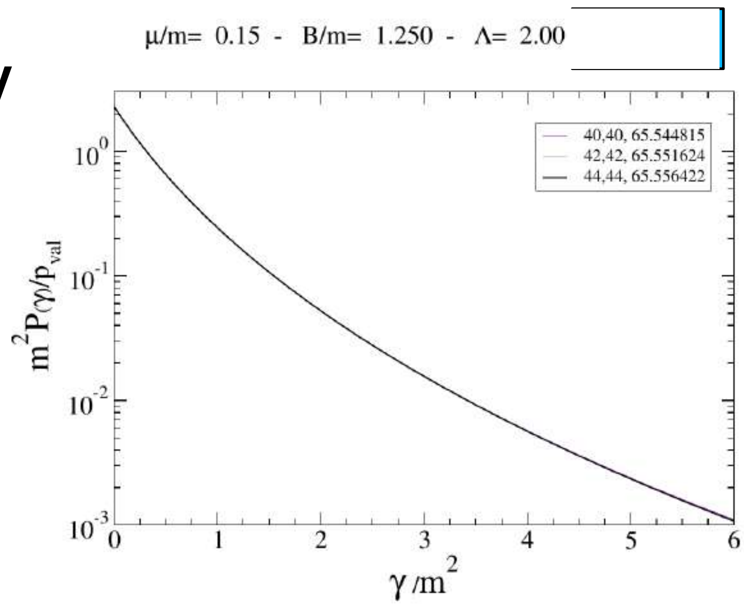
Mquark 187 MeV

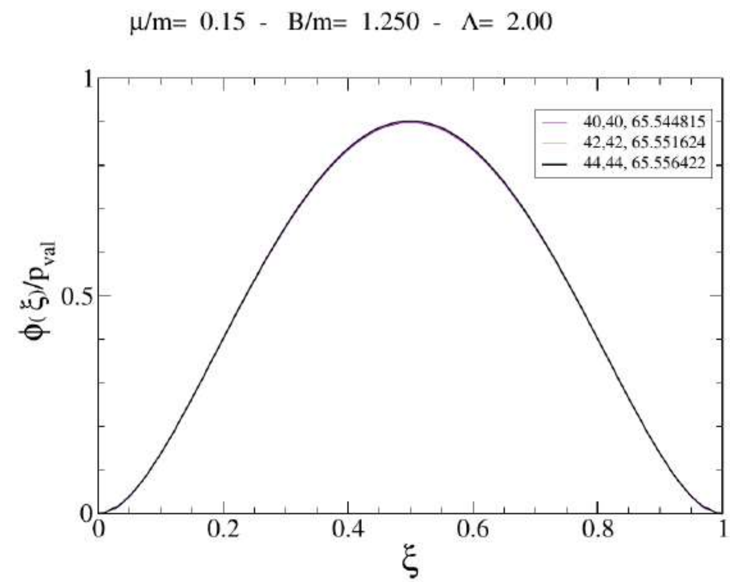
Mgluon 28 MeV

Λ/m =2

Pval=0.64

$f_\pi = 77\,\mathrm{MeV}$





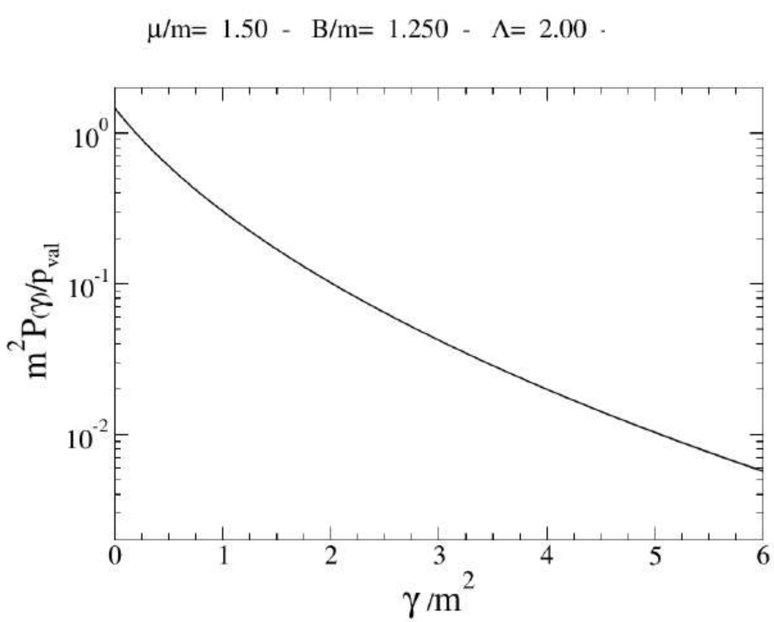
Mquark 187 MeV

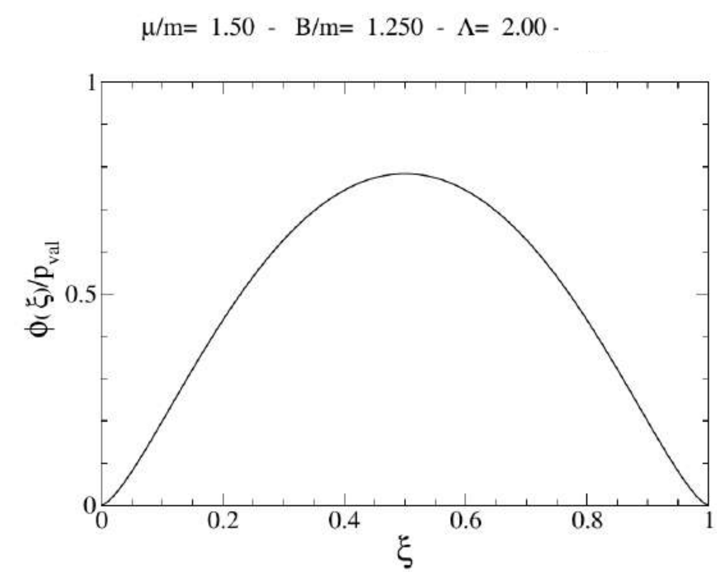
Mgluon 280 MeV

Λ/m =2

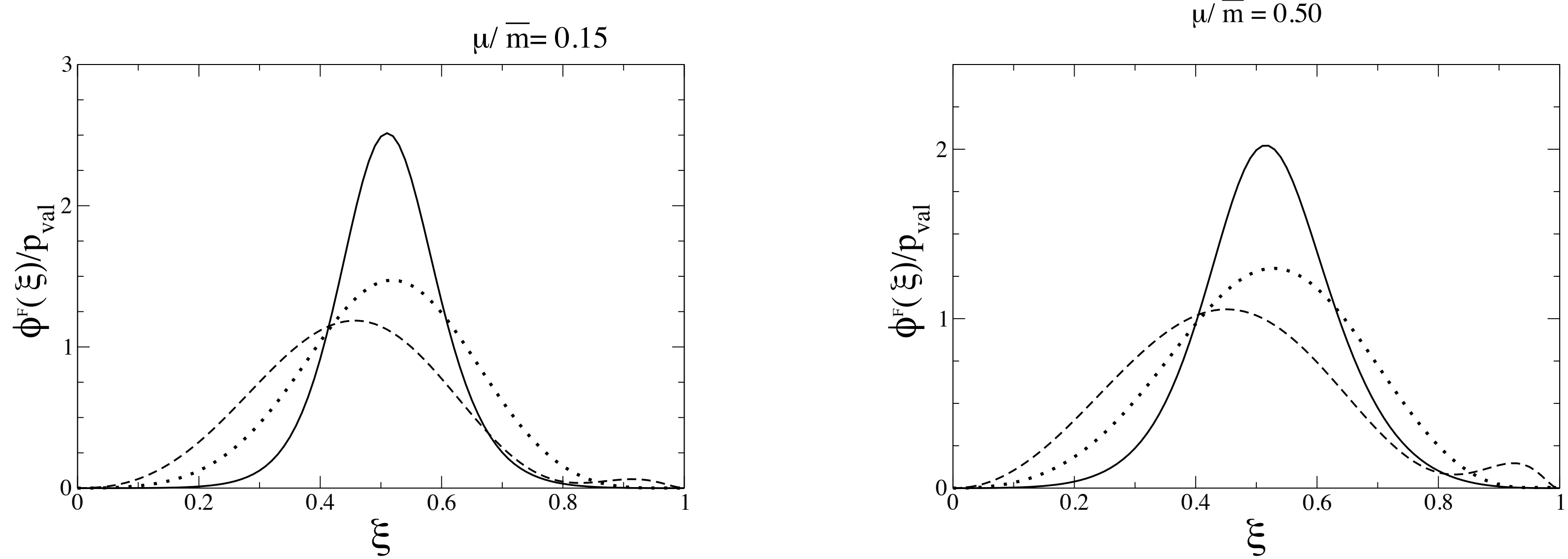
Pval=0.78

$f_\pi = 99\,\mathrm{MeV}$



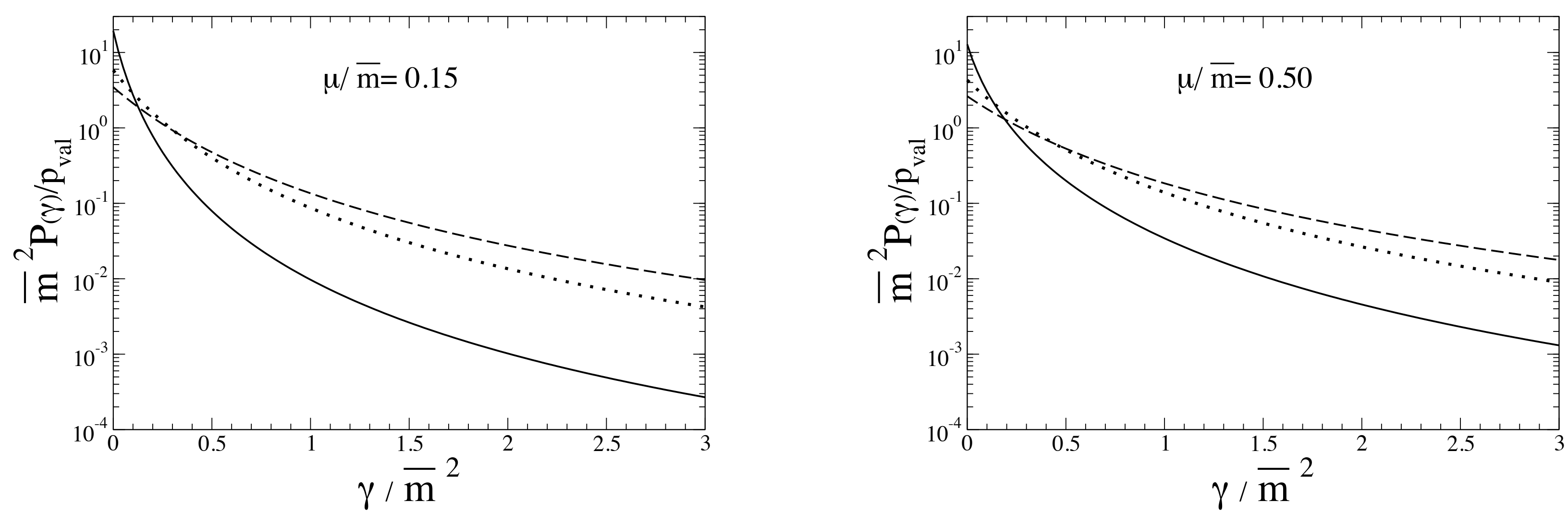


Fermion-scalar system interacting through a massive scalar exchange



Longitudinal light-cone distribution for a fermion in the valence component. Solid line : $B/\bar{m} = 0.1$. Dotted line: $B/\bar{m} = 0.5$. Dotted line: $B/\bar{m} = 1.0$

with A. Nogueira, Salmè and Pace



Transverse light-cone distribution for a fermion in the valence component.

Conclusions and Perspectives

- **Bethe-Salpeter framework Minkowski space: LF wave function & beyond;**
- **Nakanishi Integral Representation and fermions and fermion-boson BSE's;**
- **Plataform for extracting mometum distributions valence and beyond;**
- **Self-energies, vertex corrections, Landau gauge, ingredients from LQCD….**
- **Confinement?**
- **Other applications: kaon, D, B, rho…, and the nucleon**
- **Form-Factors, PDFs, TMDs, Fragmentation Functions...**

THANK YOU!

CAPES 65 anos

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LIA/CNRS - SUBATOMIC PHYSICS: FROM THEORY TO APPLICATIONS

IPNO (Jaume Carbonell)…. + Brazilian Institutions …