Two hadron correlations in SIDIS and e^+e^- annihilation reactions

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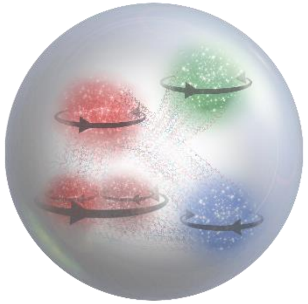
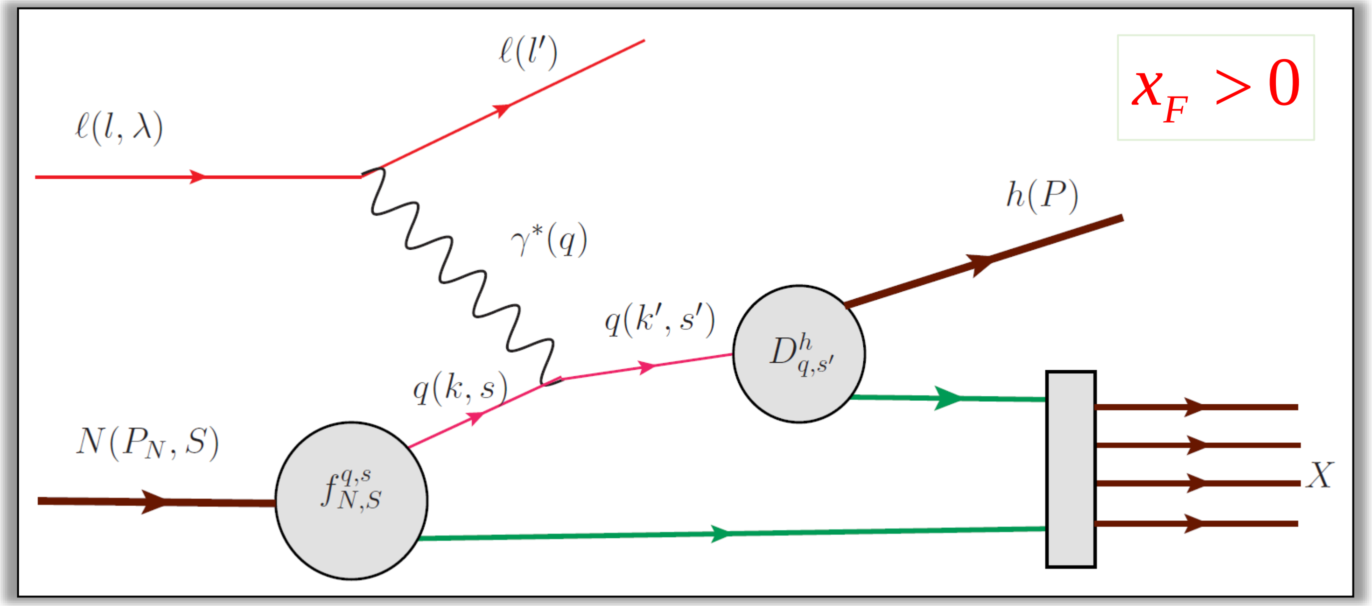


- **Introduction**
  - **accessing TMD PDFs and FFs with electromagnetic probe**
  - **1h production in SIDIS (current fragmentation region, CFR) and semi-inclusive e^+e^- annihilation (SIA)**
- **Two hadron production in CFR of SIDIS and SIA**
- **Target fragmentation region (TFR) of SIDIS**

SIDIS: CFR



$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s'}_T}{m_h} H_1(z,p_T^2)$$

H_1 was measured by BABAR and BELLE to 2 back-to-back jets $e^+e^- \to h_1h_2 + X$

Twist-2 TMD qDFs

	Quark polarization		
Nucleon Polarization	**U**	**L**	**T**
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij} k_T{}^j}{M} h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2) + \frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x,k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\Big[F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+$$
$$S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+$$
$$S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\right)\right)+$$
$$\lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\Big]$$

Virtual photon depolarization factors

$$D_{ll}(y)=\frac{y(2-y)}{1+(1-y)^2},\qquad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}$$

8 terms out of 18 Structure Functions, 6 azimuthal modulations

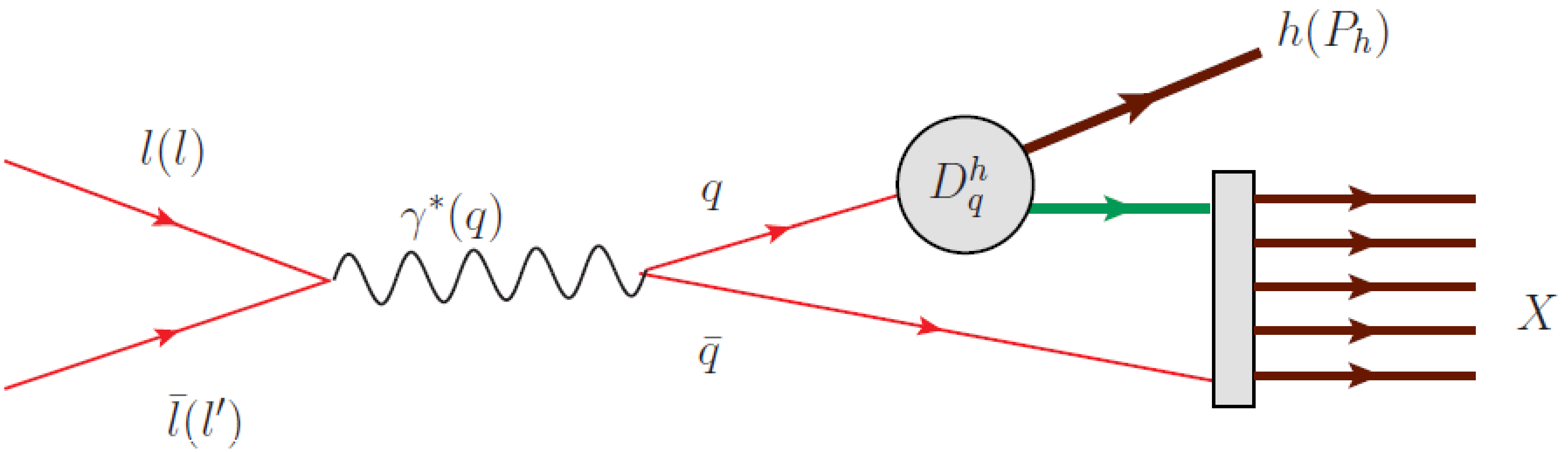
4 terms are generated by Collins effect in fragmentation

8 structure functions $F_{AB}^{f(\phi_h,\phi_S)}$

$$F_{UU,T}\propto f_1^q\otimes D_{1q}^h,\quad F_{UU}^{\cos(2\phi_h)}\propto h_1^{\perp q}\otimes H_{1q}^{\perp h},\quad F_{UL}^{\sin(2\phi_h)}\propto h_{1L}^{\perp q}\otimes H_{1q}^{\perp h},\quad F_{LL}\propto g_1^q\otimes D_{1q}^h$$

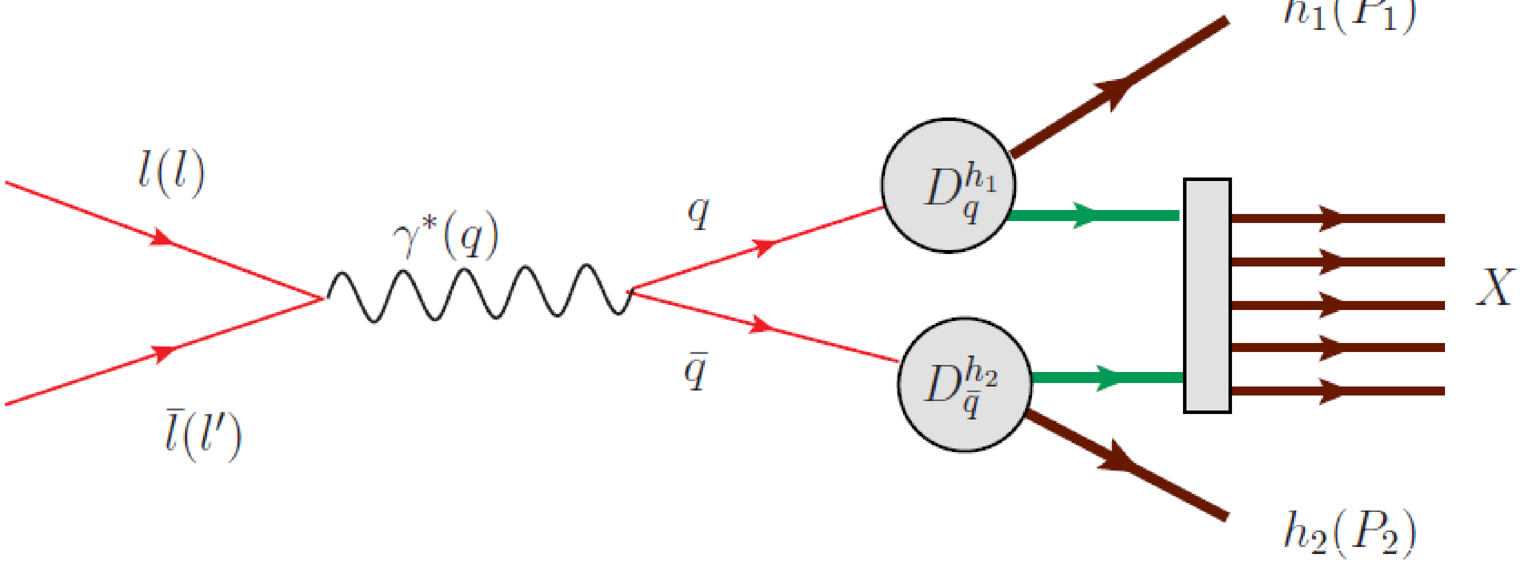
$$F_{UT}^{\sin(\phi_h-\phi_s)}\propto f_{1T}^{\perp q}\otimes D_{1q}^h,\quad F_{UT}^{\sin(\phi_h+\phi_s)}\propto h_1^q\otimes H_{1q}^{\perp h},\quad F_{UT}^{\sin(3\phi_h-\phi_s)}\propto h_{1T}^{\perp q}\otimes H_{1q}^{\perp h},\quad F_{LT}^{\cos(\phi_h-\phi_s)}\propto g_{1T}^{\perp q}\otimes D_{1q}^h$$

SIA



Access to $q+\bar{q}$ unpolarized fragmentation finctions $D^h_{q+\bar{q}}(\mathrm{z}, \mathrm{p}_\perp^2)$

h_1+h_2 SIA

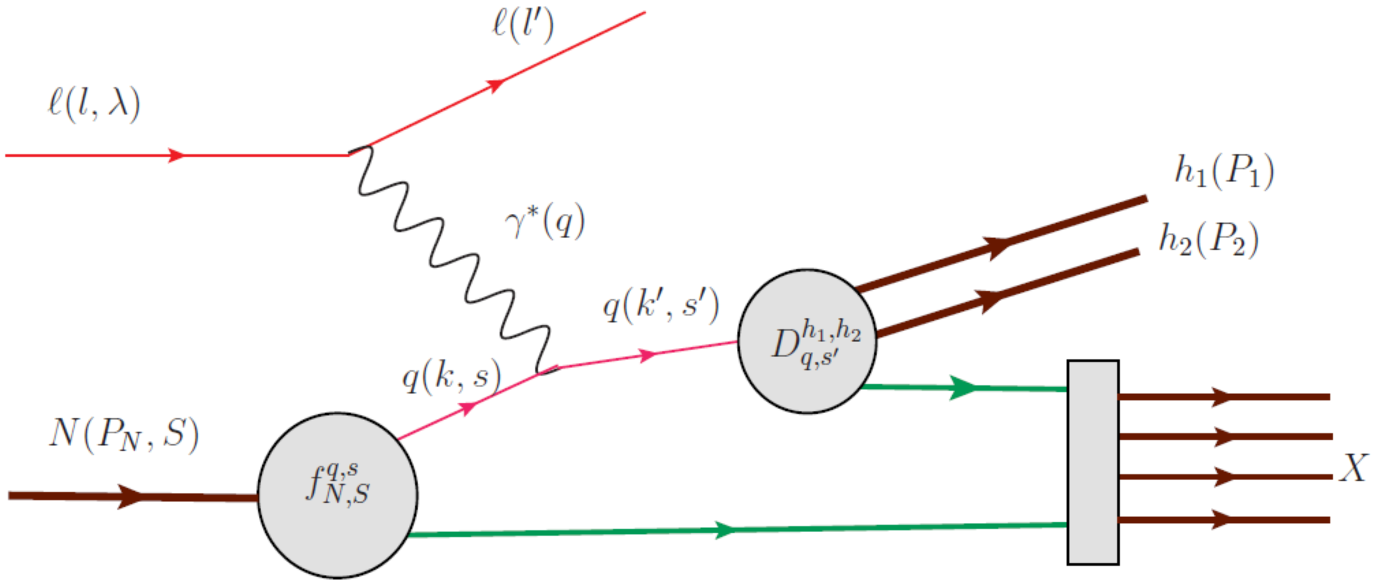


Two hadron production in opposite hemispheres: acces to Collins FF $H_{1q}^{h}(z, p_{\perp}^{2})$.

Quarks are unpolarized, but their transverse polarization are correlated, inducing an azimuthal correlation of produced hadrons in opposite jets.

Obtained $H_{1q}^{h}(z, p_{\perp}^{2})$ FFs are used for transversity $h_1(x, k_T^2)$ extraction from SIDIS data.

2h SIDIS: CFR

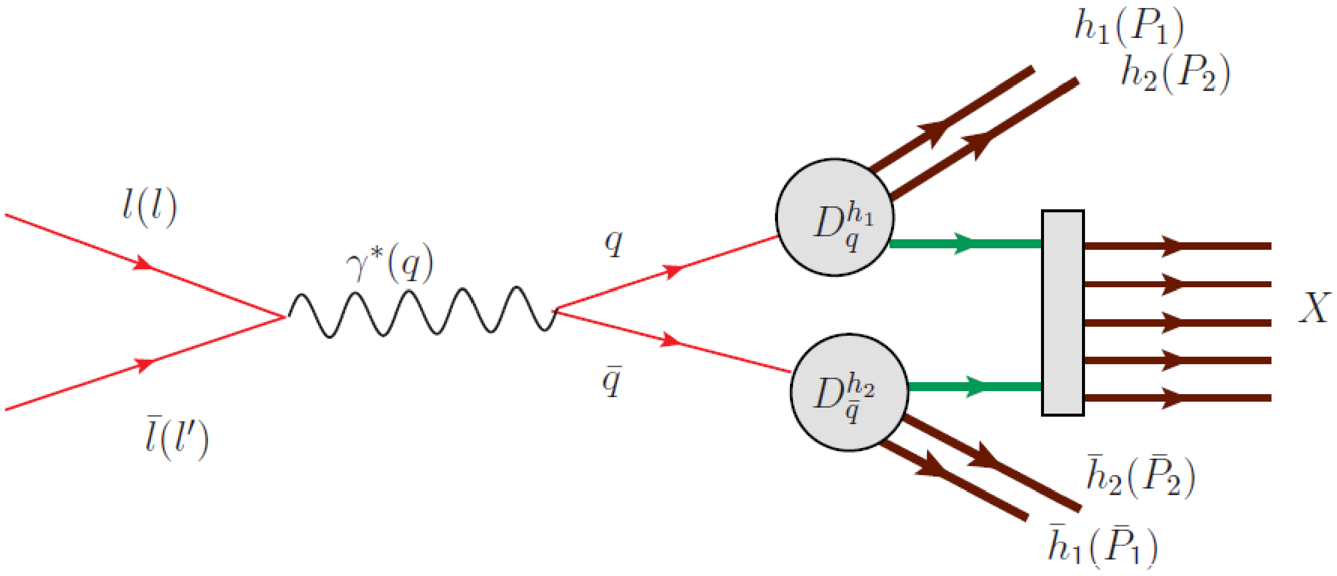


$$x_{F,1} > 0,\ x_{F,2} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz_1\,d^2P_{1T}\,dz_2\,d^2P_{2T}} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1,h_2}_{q,s'}$$

New objects: DiFFs $D^{h_1,h_2}_{q,s'}$

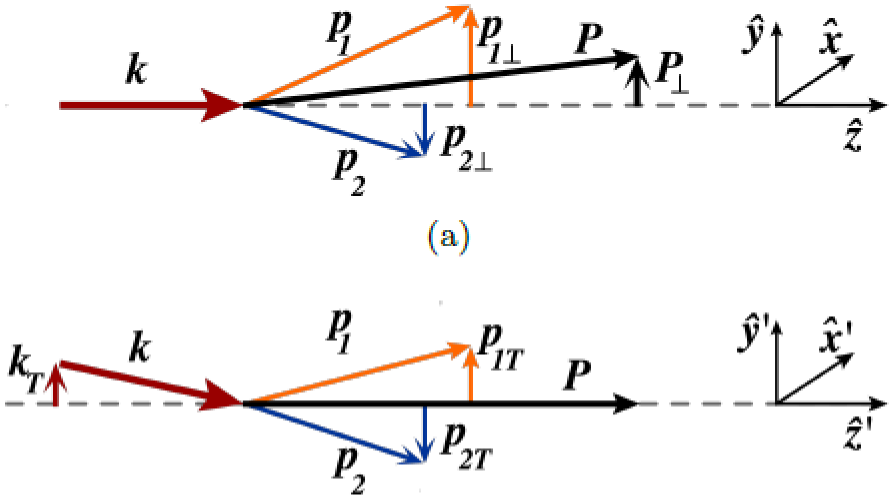
2h+2h SIA



Measured by BELLE: dihadrons production in back-to-back jets in SIA

Access to spin dependent DiFFs $D_{q,s'}^{h_1,h_2}$

Dihadron FFs: pQCD definition



$$P \equiv P_h = P_1 + P_2,$$
$$R = \frac{1}{2}(P_1 - P_2),$$

$$z = z_1 + z_2,$$
$$\xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$

$$z_i = P_i^- / k^-$$

$$P_{1T} = P_{1\perp} + z_1 k_T,$$
$$P_{2T} = P_{2\perp} + z_2 k_T .$$

$$k_T = -\frac{P_\perp}{z},$$
$$R_T = \frac{z_2 P_{1\perp} - z_1 P_{2\perp}}{z} = (1-\xi) P_{1\perp} - \xi P_{2\perp}.$$

$$R_T^2 = \xi(1-\xi) M_h^2 - M_1^2(1-\xi) - M_2^2 \xi$$

$$\Delta_{ij}(k; P_1, P_2) = \sum_X \int d^4\zeta e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)|P_1 P_2, X\rangle \langle P_1 P_2, X|\bar{\psi}_j(0)|0\rangle.$$

$$\Delta^\Gamma(z, \xi, k_T^2, R_T^2, k_T \cdot R_T) = \frac{1}{4z} \int dk^+ \mathrm{Tr}[\Gamma \Delta(k, P_1, P_2)]|_{k^- = P_h^-/z}.$$

$$\Delta^{[\gamma^-]} = D_1(z, \xi, k_T^2, R_T^2, k_T \cdot R_T),$$
$$\Delta^{[\gamma^-\gamma_5]} = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} G_1^\perp(z, \xi, k_T^2, R_T^2, k_T \cdot R_T),$$
$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^\sphericalangle(z, \xi, k_T^2, R_T^2, k_T \cdot R_T)$$
$$+ \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^\perp(z, \xi, k_T^2, R_T^2, k_T \cdot R_T)$$

Number density distribution in quark to 2h fragmentation

q pol.	U	L	T
DiFF	D_1	$G_1^{\perp}$	$H_1^{\sphericalangle}, H_1^{\perp}$

- D_1: Unpolarized DiFF
- $G_1^{\perp}$: Longitudinal handedness
- $H_1^{\sphericalangle}$: Interference DiFF (IFF)
- $H_1^{\perp}$: Collins-like DiFF

$$
\begin{aligned}
F\left(z,\xi,\mathbf{k}_T,\mathbf{R}_T;s\right) &= D_1\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right) \\
&- s_L \frac{R_T k_T \sin\left(\varphi_{RK}\right)}{M_1 M_2} G_1^{\perp}\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right) \\
&+ s_T \frac{R_T \sin\left(\varphi_R-\varphi_S\right)}{M_1+M_2} H_1^{\sphericalangle}\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right) \\
&+ s_T \frac{k_T \sin\left(\varphi_k-\varphi_S\right)}{M_1+M_2} H_1^{\perp}\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right)
\end{aligned}
$$

$$\cos\left(\varphi_{RK}\right) \doteq \cos\left(\varphi_R-\varphi_k\right)$$

$$\mathbf{k}_T = -\frac{\mathbf{P}_{h\perp}}{z}$$

Fourier moments of DiFFs

$$D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{KR})) = \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(n\cdot\varphi_{KR})}{1+\delta_{0,n}}\; D_1^{[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|),$$

$$F^{[n]} = \int d\varphi_{KR}\;\cos(n\varphi_{KR})\; F\Big(\cos(\varphi_{KR})\Big)$$

We define Fourier moments of integrated over pair total momentum weighted DiFFs and

$$D_1^a(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\; D_1^{a,[0]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2)$$

$$G_1^{\perp a,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\left(\frac{\boldsymbol{k}_T^2}{2M_h^2}\right)\frac{|\boldsymbol{R}_T|}{M_h}\; G_1^{\perp a,[n]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2)$$

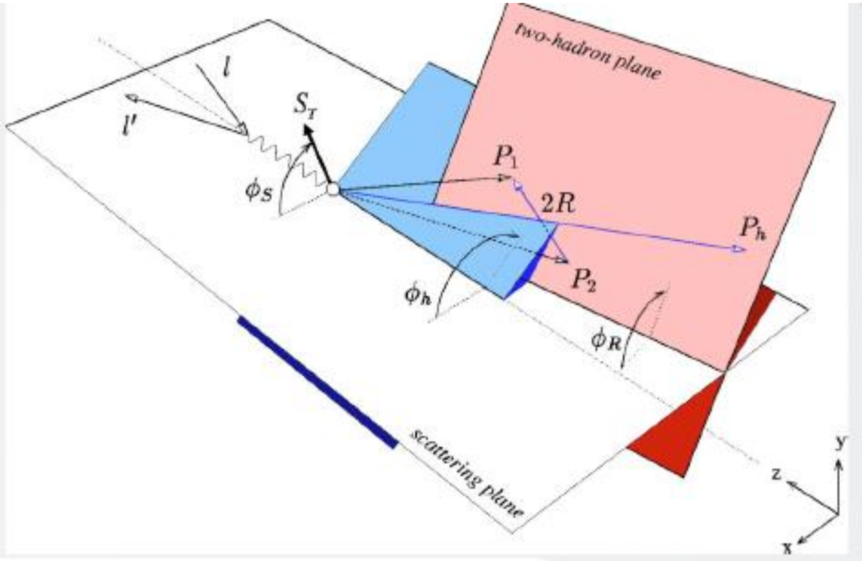
$$H_1^{\sphericalangle,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\,\frac{|\boldsymbol{R}_T|}{M_h}H_1^{\sphericalangle,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

$$H_1^{\perp,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\,\frac{|\boldsymbol{k}_T|}{M_h}H_1^{\perp,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

Access to transversity h_1 in 2h SIDIS

M. Radici, Jakob and Bianconi: PRD 65, 074031 (2002).

- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!



$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 h_1^q(x) H_1^{\sphericalangle q}\left(z, M_h^2\right)}{\sum_q e_q^2 f_1^q(x) D_1^q\left(z, M_h^2\right)}$$

$$H_{1,SIDIS}^{\sphericalangle}\left(z, M_h^2\right) = H_1^{\sphericalangle[0]}\left(z, M_h^2\right)$$

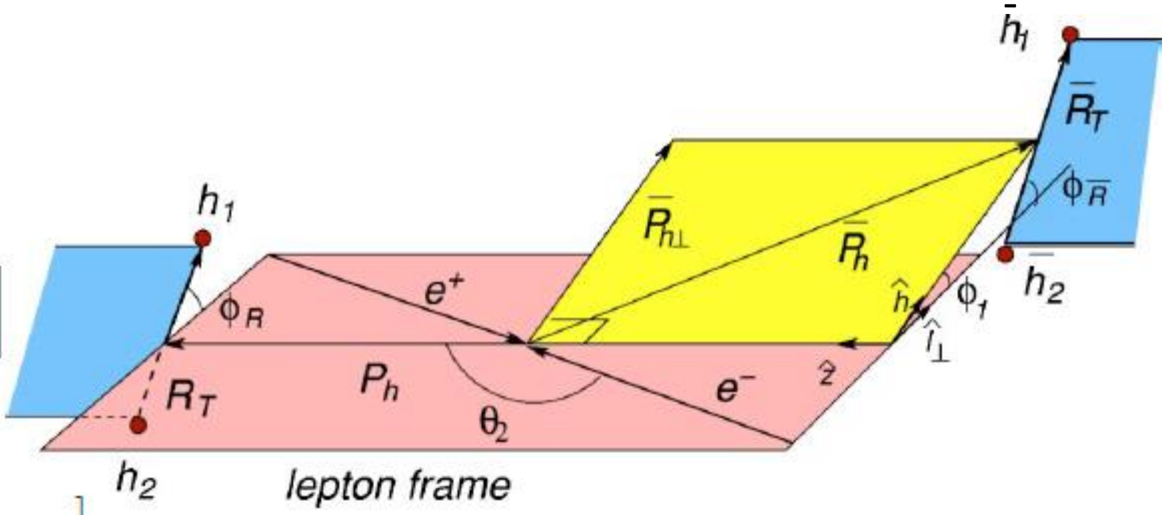
Corrected by A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

$$H_{1,SIDIS}^{\sphericalangle}\left(z, M_h^2\right) = H_1^{\sphericalangle[0]}\left(z, M_h^2\right) + H_1^{\perp[1]}\left(z, M_h^2\right)$$

DiFFs in 2h,2h SIA

D. Boer et al: PRD 67, 094003 (2003).

$$
\begin{aligned}
&\frac{d\sigma(e^+e^-\to(h_1h_2)(\bar h_1\bar h_2)X)}{d\boldsymbol{q}_T\,dz\,d\xi\,dM_h^2\,d\phi_R\,d\bar z\,d\bar\xi\,d\bar M_h^2\,d\phi_{\bar R}\,dy\,d\phi^l}\\
&=\sum_{a,\bar a} e_a^2\frac{6\alpha^2}{Q^2}z^2\bar z^2\Bigg\{A(y)\mathcal{F}[D_1^a\bar D_1^a]+\cos(2\phi_1)B(y)\mathcal{F}\left[(2\hat{\boldsymbol h}\cdot\boldsymbol k_T\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T-\boldsymbol k_T\cdot\bar{\boldsymbol k}_T)\frac{H_1^{\perp a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\right]\\
&-\sin(2\phi_1)B(y)\mathcal{F}\left[(\hat{\boldsymbol h}\cdot\boldsymbol k_T\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T+\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\hat{\boldsymbol g}\cdot\boldsymbol k_T)\frac{H_1^{\perp a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\right]+\cos(\phi_R+\phi_{\bar R}-2\phi^l)\\
&\times B(y)|\boldsymbol R_T||\bar{\boldsymbol R}_T|\mathcal{F}\left[\frac{H_1^{\measuredangle a}\bar H_1^{\measuredangle a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\right]+\cos(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol R_T|\mathcal{F}\left[\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\frac{H_1^{\measuredangle a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\right]\\
&-\sin(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol R_T|\mathcal{F}\left[\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T\frac{H_1^{\measuredangle a}\bar H_1^{\perp a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\right]+\cos(\phi_1+\phi_{\bar R}-\phi^l)B(y)|\bar{\boldsymbol R}_T|\\
&\times\mathcal{F}\left[\hat{\boldsymbol h}\cdot\boldsymbol k_T\frac{H_1^{\perp a}\bar H_1^{\measuredangle a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\right]-\sin(\phi_1+\phi_{\bar R}-\phi^l)B(y)|\bar{\boldsymbol R}_T|\mathcal{F}\left[\hat{\boldsymbol g}\cdot\boldsymbol k_T\frac{H_1^{\perp a}\bar H_1^{\measuredangle a}}{(M_1+M_2)(\bar M_1+\bar M_2)}\right]+A(y)|\boldsymbol R_T||\bar{\boldsymbol R}_T|\\
&\times\Bigg(\sin(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar R}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol h}\cdot\boldsymbol k_T\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\right]+\sin(\phi_1-\phi_R+\phi^l)\cos(\phi_1-\phi_{\bar R}+\phi^l)\\
&\times\mathcal{F}\left[\hat{\boldsymbol h}\cdot\boldsymbol k_T\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\right]+\cos(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar R}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol g}\cdot\boldsymbol k_T\hat{\boldsymbol h}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\right]+\cos(\phi_1-\phi_R+\phi^l)\\
&\times\cos(\phi_1-\phi_{\bar R}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol g}\cdot\boldsymbol k_T\hat{\boldsymbol g}\cdot\bar{\boldsymbol k}_T\frac{G_1^{\perp a}\bar G_1^{\perp a}}{M_1M_2\bar M_1\bar M_2}\right]\Bigg)\Bigg\},
\end{aligned}
\tag{19}
$$



Access to IFF and handedness DiFF in SIA: weighted asymmetries

Boer, Jacob and Radici: PRD 67, 094003 (2003).

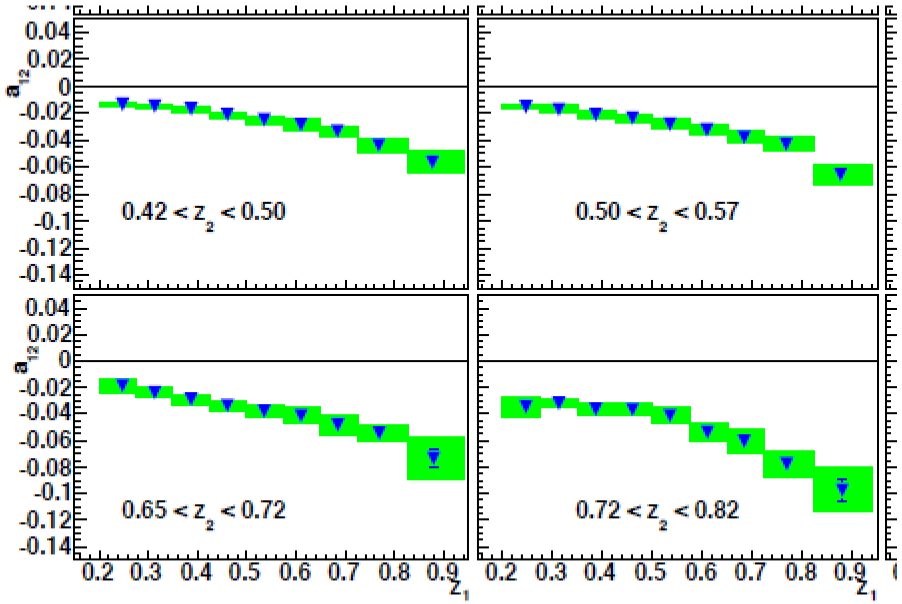
$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

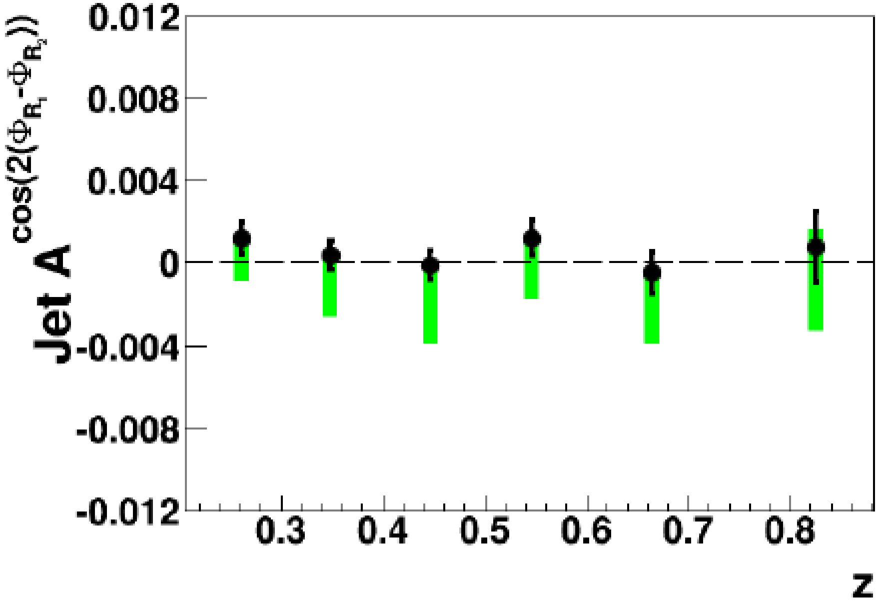
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

$$H_{1,e^+e^-}^{\sphericalangle}\left(z,M_h^2\right)=H_1^{\sphericalangle[0]}\left(z,M_h^2\right)\neq H_{1,SIDIS}^{\sphericalangle}\left(z,M_h^2\right)$$

$$G_{1,e^+e^-}^{\perp}\left(z,M_h^2\right)=G_1^{\perp[0]}\left(z,M_h^2\right)$$

PRL 107 (2011) 072004 (IFF) **BELLE results** **arXiv:1505.08020 (handedness)**





Model calculation of FFs and DiFFs

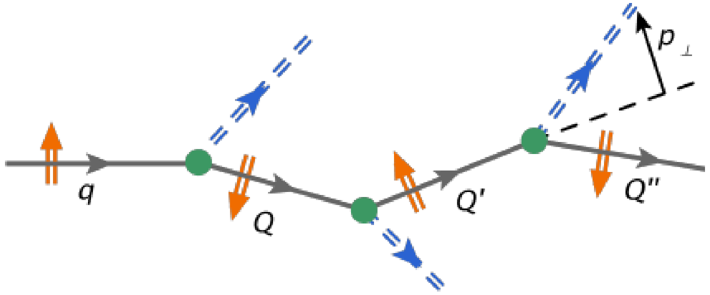
Complete self consistent formalism for spin dependent TMD FFs:

Bentz, *AK*, Matevosyan, Ninomiya, Thomas, Yazaki: PR D94, 034004 (2016)

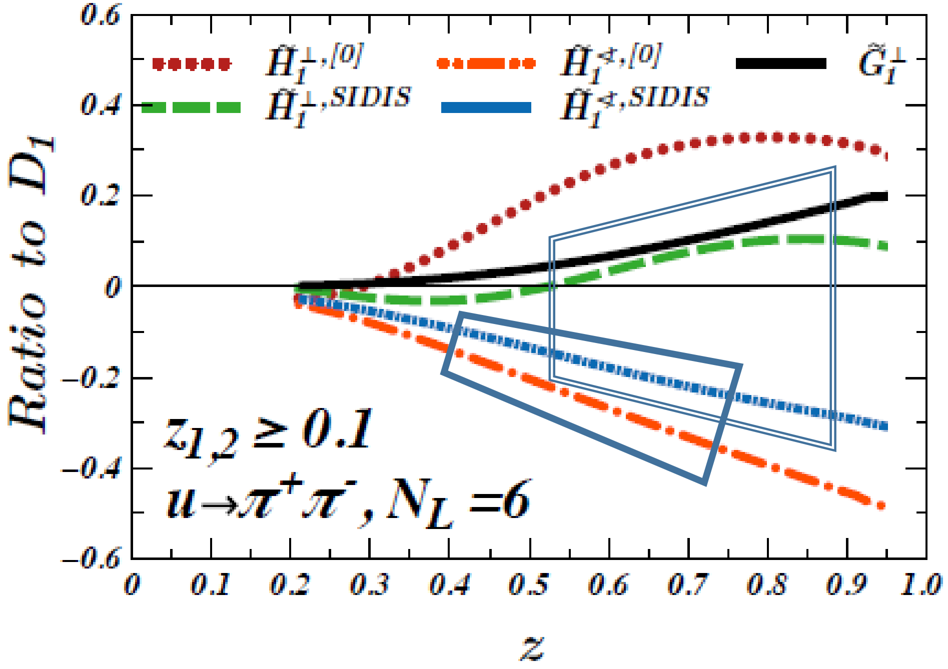
MC implementation: **Matevosyan, *AK*, Thomas: One hadron production, PRD 95, 014021 (2017)**

Two hadron production: Longitudinally polarized quark, PRD 96, 074010 (2017)

Two hadron production: Transversely polarized quark, PRD 97, 014019 (2018)



All 8 elementary quark-to-quark spin-dependent TMD FFs are taken into account



$$H_{1,SIDIS}^{\sphericalangle} \approx 2H_{1,e^+e^-}^{\sphericalangle}\left(z, M_h^2\right)$$

$$G_1^{\perp} \approx \frac{1}{2} H_{1,e^+e^-}^{\sphericalangle}\left(z, M_h^2\right)$$

Rederiving dihadron production cross-sections in e^+e^- and SIDIS

Matevosyan , *AK*, Thomas: PRL 120, 252, 001 (2018), : arXiv:1712.06384.

Matevosyan, Bacchetta, Boer, Courtoy, *AK*, Radici, Thomas: Phys. Rev. D 97, 074019 (2018), arXiv:1802.01578

Fully differential cross section

$$\frac{d\sigma(e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)X)}{d^2\boldsymbol{q}_T dz d\xi d\varphi_R dM_h^2 d\bar{z} d\bar{\xi} d\varphi_{\bar{R}} d\bar{M}_h^2 dy}$$
$$= \frac{3\alpha^2}{\pi Q^2} z^2 \bar{z}^2 \sum_{a,\bar{a}} e_a^2 \Big\{ A(y)\mathcal{F}[D_1^a \bar{D}_1^{\bar{a}}] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar{M}_h}\cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp a} \bar{H}_1^{\perp \bar{a}}\right]$$
$$+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\cos(\varphi_R + \varphi_{\bar{R}}) H_1^{\sphericalangle a} \bar{H}_1^{\sphericalangle \bar{a}}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar{M}_h}\cos(\varphi_k + \varphi_{\bar{R}}) H_1^{\perp a} \bar{H}_1^{\sphericalangle \bar{a}}\right]$$
$$+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar{M}_h}\cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle a} \bar{H}_1^{\perp \bar{a}}\right] - A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T||\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T||\bar{\boldsymbol{k}}_T|}{\bar{M}_h^2}\sin(\varphi_k - \varphi_R)\sin(\varphi_{\bar{k}} - \varphi_{\bar{R}}) G_1^{\perp a} \bar{G}_1^{\perp \bar{a}}\right]\Big\}$$

$$\mathcal{F}[wD^a\bar{D}^{\bar{a}}] = \int d^2\boldsymbol{k}_T d^2\bar{\boldsymbol{k}}_T \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) w(\boldsymbol{k}_T, \bar{\boldsymbol{k}}_T, \boldsymbol{R}_T, \bar{\boldsymbol{R}}_T) D^a(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{R}_T) D^{\bar{a}}(\bar{z}, \bar{\xi}, \bar{\boldsymbol{k}}_T^2, \bar{\boldsymbol{R}}_T^2, \bar{\boldsymbol{k}}_T\cdot\bar{\boldsymbol{R}}_T)$$

IFFs in e⁺e⁻ and SIDIS

- *The asymmetry now involves **exactly the same** integrated IFF as in SIDIS!*

$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} = \frac{1}{2}\frac{B(y)}{A(y)}\frac{\sum_{a,\bar{a}} e_a^2 H_1^{\sphericalangle a}(z, M_h^2)\bar{H}_1^{\sphericalangle \bar{a}}(\bar{z}, \bar{M}_h^2)}{\sum_{a,\bar{a}} e_a^2 D_1^a(z, M_h^2)\bar{D}_1^{\bar{a}}(\bar{z}, \bar{M}_h^2)}$$

$$D_1(z, M_h^2) \equiv z^2 \int d^2\boldsymbol{k}_T \int d\xi\, D_1^{[0]}(z, \xi, |\boldsymbol{k}_T|, |\boldsymbol{R}_T|)$$

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = H_1^{\sphericalangle,[0]} + H_1^{\perp,[1]} \equiv H_{1,SIDIS}^{\sphericalangle}(z, M_h^2)$$

- ***All the previous extractions of the transversity are valid !***

Handedness DiFF in e^+e^-

Matevosyan , *AK*, Thomas: PRL 120, 252, 001 (2018), : arXiv:1712.06384.

- *The relevant terms involving* $G_1^{\perp}$ *:*

$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\Big]$$

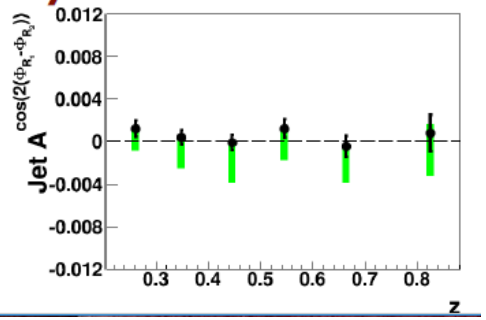
Weighting break-up the convolution

$$\langle \mathcal{I} \rangle \equiv \int d\xi \int d\bar{\xi} \int d\varphi_R \int d\varphi_{\bar{R}} \int d^2\boldsymbol{q}_T \mathcal{I} \frac{d\sigma(e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)X)}{d^2\boldsymbol{q}_T dz d\xi d\varphi_R dM_h^2 d\bar{z} d\bar{\xi} d\varphi_{\bar{R}} d\bar{M}_h^2 dy}$$

$$\langle f(\varphi_R, \varphi_{\bar{R}}) \rangle_L = 0$$

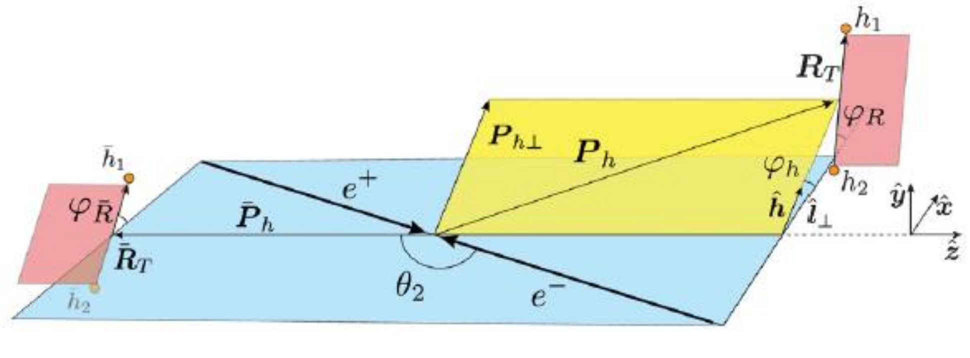
- *The old asymmetry by Boer et. al. exactly vanishes!*
- *Explains the BELLE results.*

$$A^{\Rightarrow} = \frac{\langle \cos(2(\varphi_R - \varphi_{\bar{R}})) \rangle}{\langle 1 \rangle} = 0!$$



New weight to access handedness DiFF in e+e-

Matevosyan , *AK*, Thomas: PRL 120, 252, 001 (2018), : arXiv:1712.06384.



$q_T = -P_{h\perp}/z$

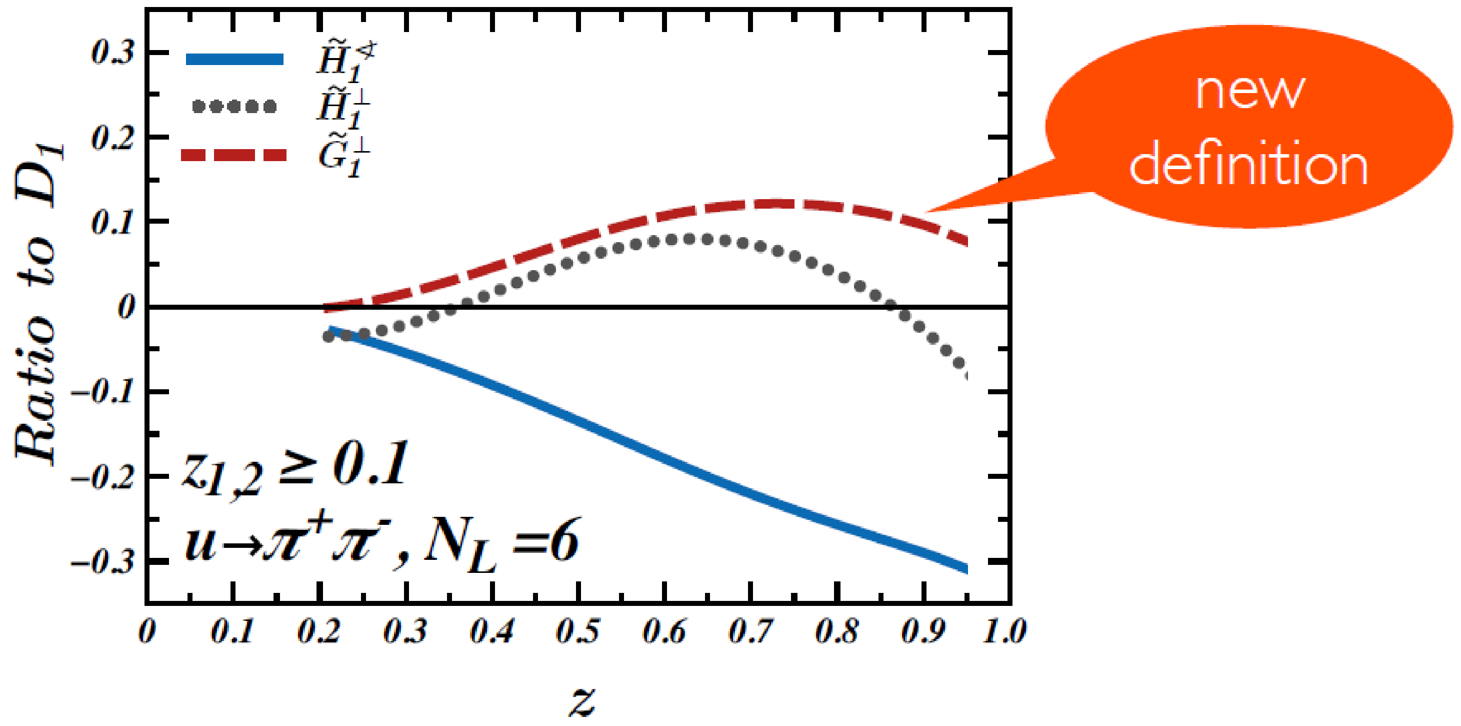
$$\left\langle \frac{q_T^2 (3\sin(\varphi_{qR})\sin(\varphi_{q\bar{R}}) + \cos(\varphi_{qR})\cos(\varphi_{q\bar{R}}))}{M_h \bar{M}_h} \right\rangle = \frac{12\alpha^2 A(y)}{\pi Q^2} \sum_{a,\bar{a}} e_a^2 \left(G_1^{\perp a,[0]} - G_1^{\perp a,[2]} \right)\left(\bar{G}_1^{\perp \bar{a},[0]} - G_1^{\perp \bar{a},[2]} \right)$$

$$A_{e^+e^-}^{\Rightarrow}(z,\bar{z},M_h^2,\bar{M}_h^2) = 4\,\frac{\sum_{a,\bar{a}} G_1^{\perp a}(z,M_h^2) G_1^{\perp \bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^{a}(z,M_h^2) D_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}$$

$$G_1^{\perp a}(z,M_h^2) \equiv G_1^{\perp a,[0]}(z,M_h^2) - G_1^{\perp a,[2]}(z,M_h^2)$$

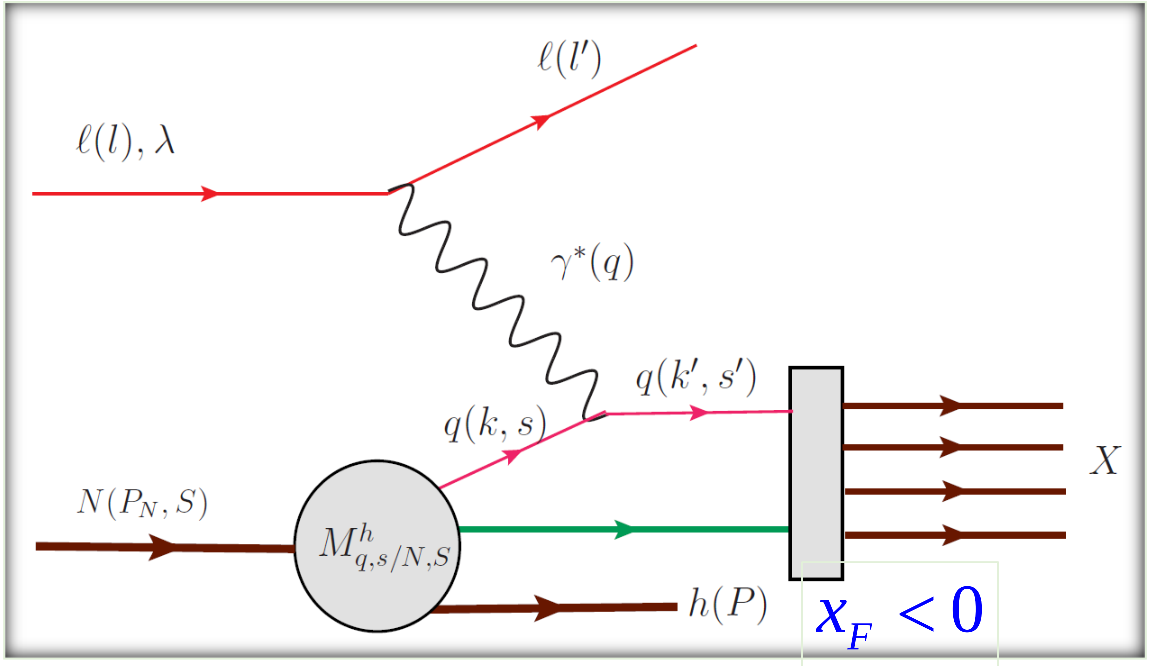
Our MC model results

$G_1^{\perp}$ *naturally smaller than* $H_1^{\sphericalangle}$*, but* ***should be measurable!***



$$G_1^{\perp a}(z, M_h^2) \equiv G_1^{\perp a,[0]}(z, M_h^2) - G_1^{\perp a,[2]}(z, M_h^2)$$

SIDIS: TFR



Trentadue, Veneziano 1994
Graudenz 1994
Collins 1998, 2000, 2002
de Florian, Sassot 1997, 1998
Grazzini, Trentadue, Veneziano 1998
Ceccopieri, Trentadue 2006, 2007, 2008
Sivers 2009
Ceccopieri , Mancusi 2013
Ceccopieri 2013
Applied to HERA data: D. de Glorian, R. Sassot (1998), Shoeibi *et al (2017)*
.....

$$\frac{d\sigma^{\ell(l)+N(P_N)\to\ell(l')+h(P)+X}}{dx dQ^2 d\zeta} = M^h_{q/N}\left(x, Q^2, \zeta\right) \otimes \frac{d\sigma^{\ell(l)+q(k)\to\ell(l')+q(k')}}{dQ^2}$$

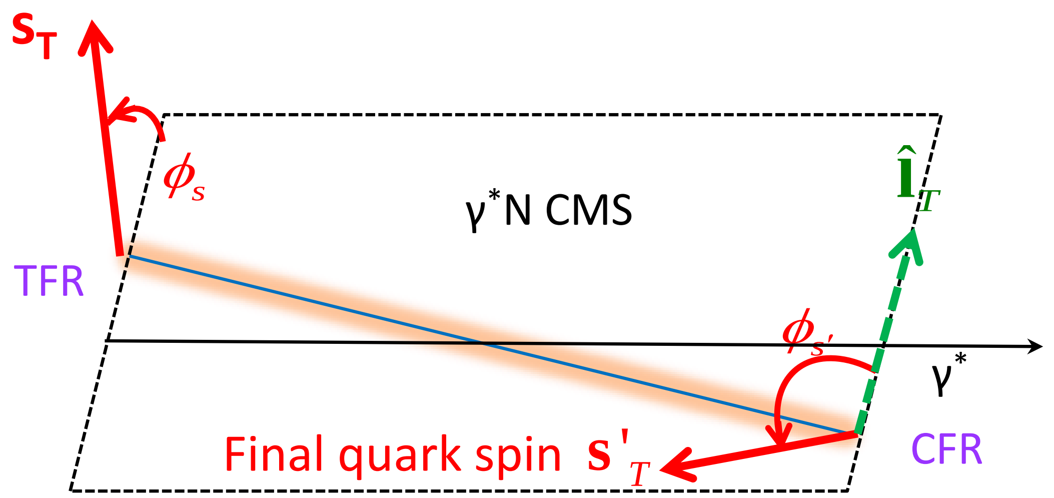
$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

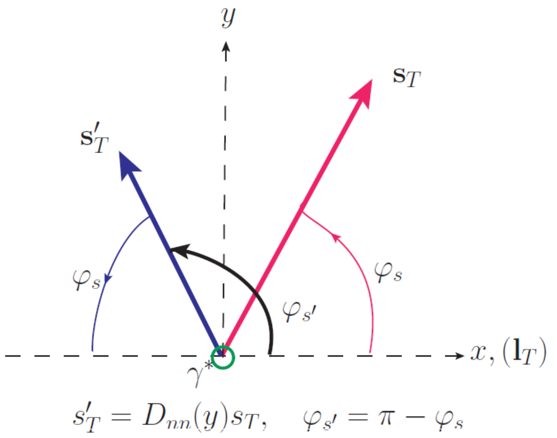
Fracture function *M* is a Conditional Probability Distribution Function (CPDF)
to observe the hadron h produced in nucleon flight direction
when hard probe interacts with parton carrying fraction x of nucleon momentum.

Quark transverse spin in hard *l-q* scattering

Nucleon and initial quark spin

AK, Transversity workshop, Yerevan, 2009



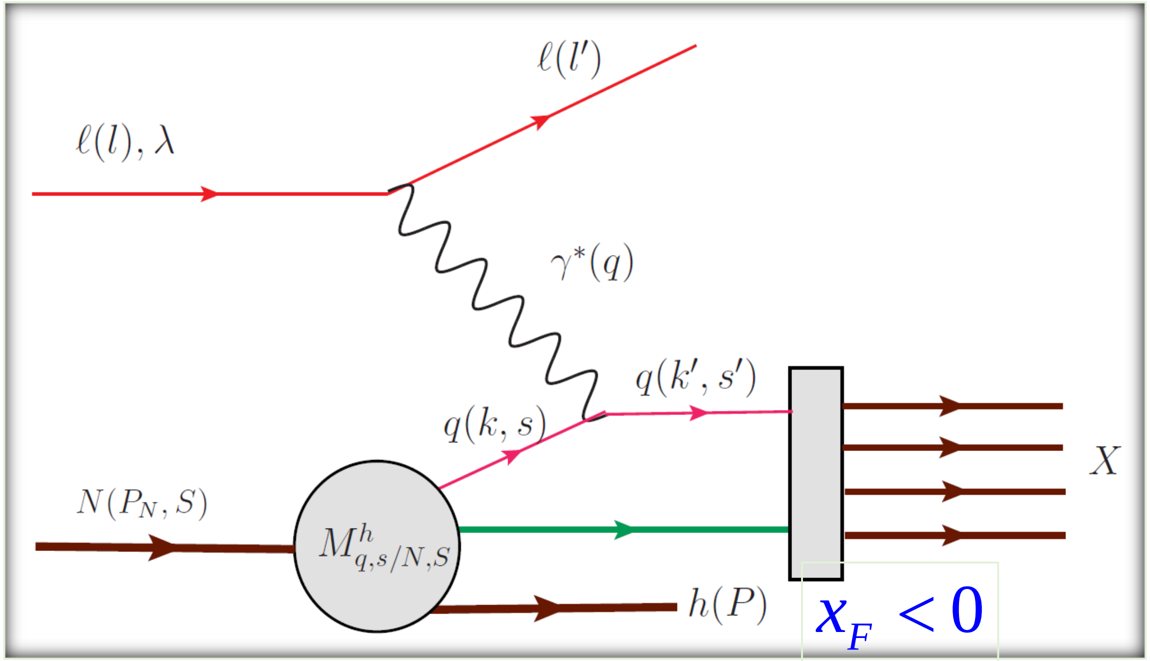


$$\text{QED: } lq \to l'q' \;\Rightarrow\; s'_T = D_{nn}(y) s_T, \quad D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'} = \pi - \phi_s$$

$$[\mathbf{s}'_T \times \mathbf{p}_T] \propto \sin(\phi_h - \phi_{s'}) = -\sin(\phi_h + \phi_s)$$

If only one hadron in TFR of SIDIS is detected there is no final quark polarimetry.
→ No access to quark transverse polarization dependent fracture functions.
No Collins like modulation.

SIDIS TFR. Spin & TMD Fracture Functions



Anselmino, Barone and AK, PL B 699 (2011)108; **706** (2011)46; **713** (2012)317

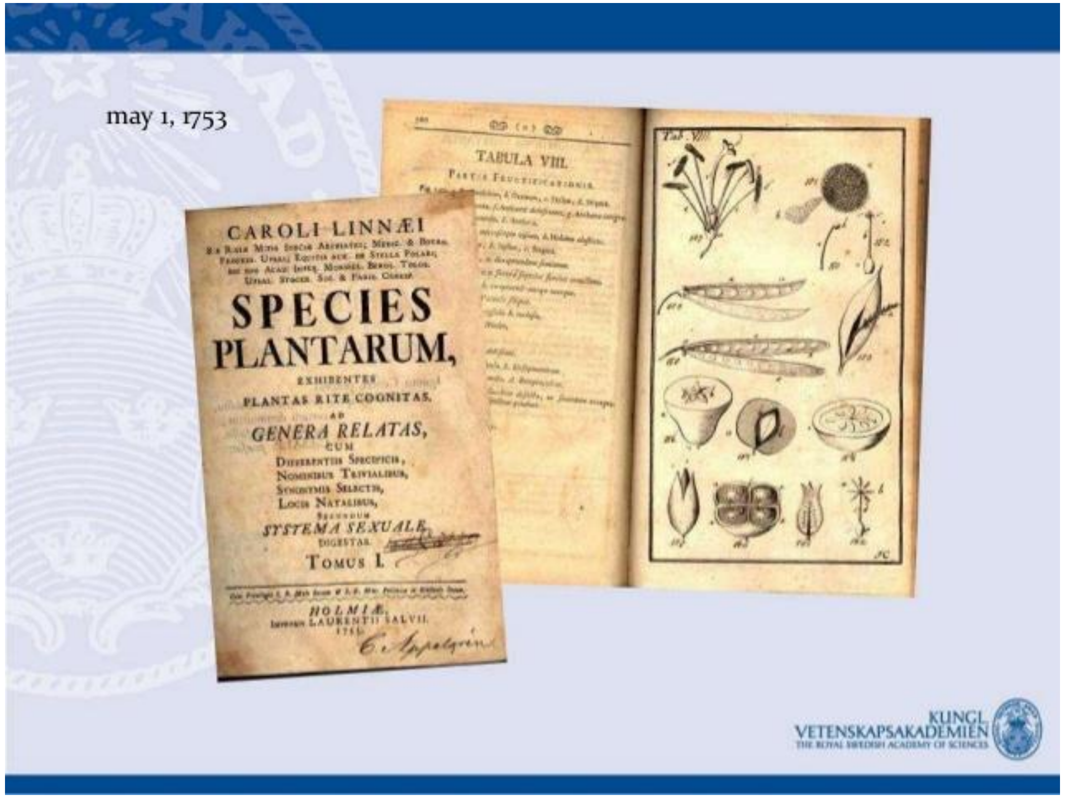
Nucleon and quark polarization are included, produced hadron and quark transverse momentum are not integrated over. Classification of twist-two Fracture Functions and cross sections expressions.

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Karl Linney: plants classification

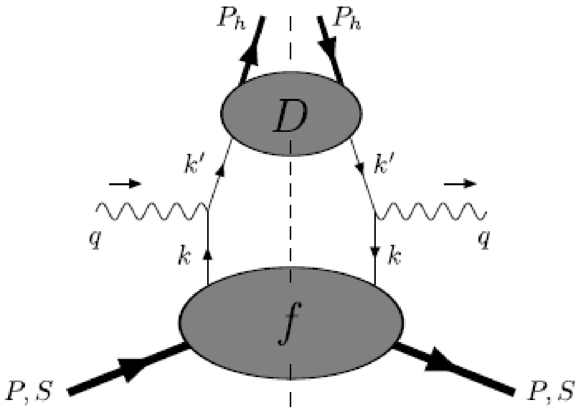
Plants were divided by it into 24 classes and 116 groups on the basis of features of a structure of their reproductive organs.



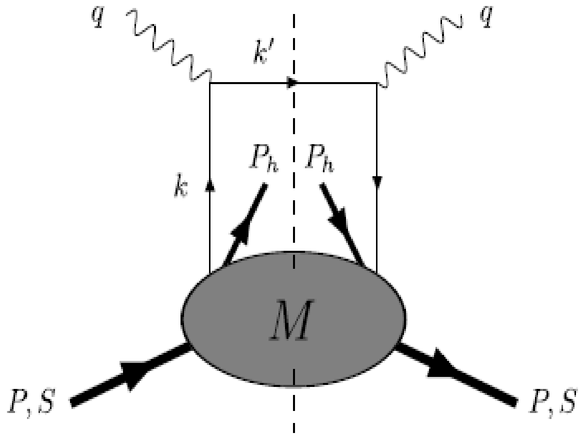
For STMD Fracture Functions I was expecting 32 (Trentadue) independent structures. Fortunately for unpolarized hadron production we end up with only 16 of them at twist-two

Quark correlator

SIDIS



CFR



TFR

$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int\frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6}e^{\mathrm{i}(x_B P^-\xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)}\sum_X\int\frac{d^3P_X}{(2\pi)^3 2E_X}\times$$

$$\times\langle P,S\,|\,\bar{\psi}(0)\Gamma\,|\,P_h,S_h;X\rangle\langle P_h,S_h;X\,|\,\psi(\xi^+,0,\vec{\xi}_\perp)\,|\,P,S\rangle$$

$$\Gamma = \gamma^-,\quad \gamma^-\gamma_5,\quad i\sigma^{i-}\gamma_5$$

At LO 16 independent STMD fracture functions. Probabilistic interpretation at LO: the conditional probabilities to find an unpolarized, a longitudinally polarized or a transversely polarized quark with longitudinal momentum fraction x_B and transverse momentum $\boldsymbol{k}_\perp$ inside a nucleon fragmenting into a hadron carrying a fraction ζ of the nucleon longitudinal momentum and a transverse momentum $\boldsymbol{P}_{h\perp}$.

STMD Fracture Functions for spinless hadron production

Nucleon Polarization	Quark polarization: U	Quark polarization: L	Quark polarization: T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N\, m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^{\,j}}{m_h} \hat{t}_1^{h} + \frac{\epsilon_T^{ij} k_T^{\,j}}{m_N} \hat{t}_1^{\perp}$
L	$\frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N\, m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^{h} + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^{\perp}$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^{h} + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^{\perp}$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^{h} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^{\perp}$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T(\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T(\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T(\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

STMD fracture functions

depend on

$x, k_T^2, \zeta, P_T^2, \mathbf{k}_T \cdot \mathbf{P}_T$

$\mathbf{k}_T \cdot \mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q)$

azimuthal dependence

in fracture functions

LO cross-section for single hadron production in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = \frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2 \times$$

$$\times\left[\tilde{u}_1\left(x,\zeta,P_T^2\right) - S_T\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S) + \lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right) + S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\right]$$

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{u}_1$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{u}_{1T}^h + \frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\,\hat{l}_{1L}$$

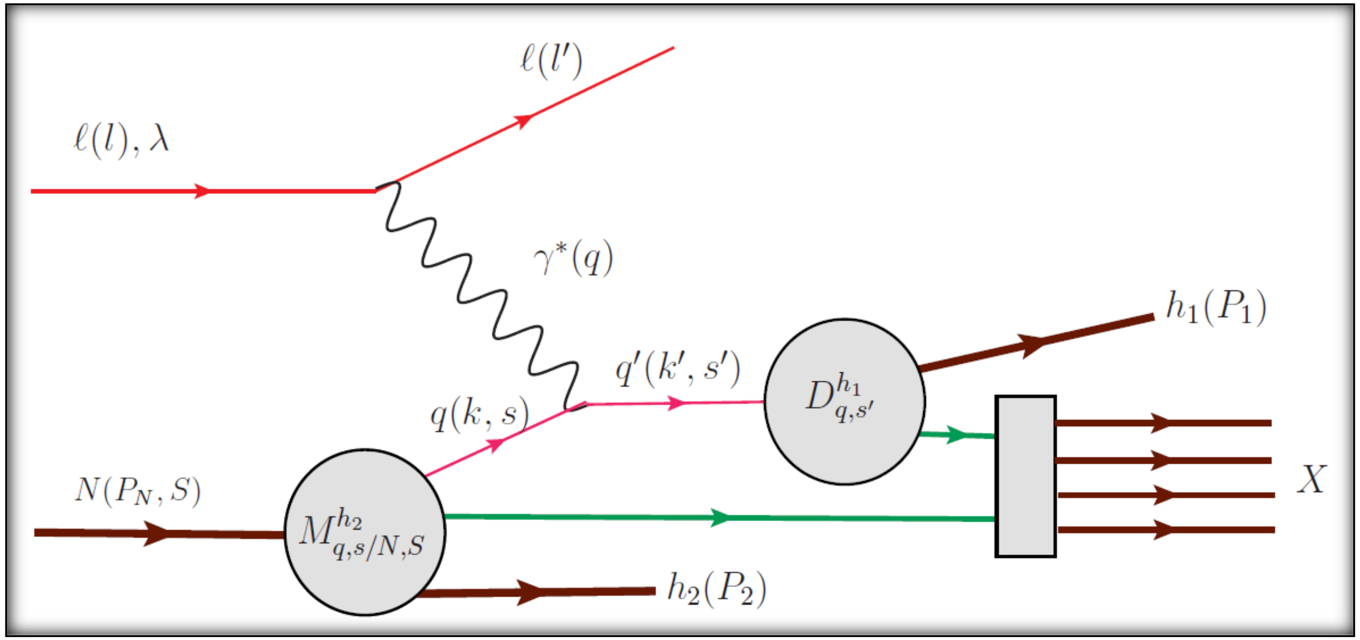
$$\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2) = \int d^2k_T\left\{\hat{l}_{1T}^h + \frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

At LO only 4 terms out of 18 Structure Functions,
Only 2 azimuthal modulations

No Collins-like $\sin(\phi_h+\phi_S)$ modulation

No access to quark transverse polarization

Double hadron production in DIS (DSIDIS): TFR & CFR



$$x_{F2} < 0, \quad x_{F1} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx dQ^2 d\phi_S dz d^2P_{T1} d\zeta d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z, p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s'}_T}{m_h} H_1(z, p_T^2)$$

Unintegrated DSIDIS cross-section: accessing quark polarization

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\hat{u}^{h_2}\otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2}\otimes D_1^{h_1} + \hat{t}^{h_2}\otimes\frac{\mathbf{p}_T\times\mathbf{s}'_T}{m_{h_1}}H_1^{h_1}\right)$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\left(\sigma_{UU}+S_L\sigma_{UL}+S_T\sigma_{UT}+\lambda D_{ll}\left(\sigma_{LU}+S_L\sigma_{LL}+S_T\sigma_{LT}\right)\right)$$

DSIDIS cross section is a sum of polarization independent, single and double spin dependent terms, similarly to 1h SIDIS cross section.

Twist-2 A_{LU} asymmetry in DSIDIS

AK @ DIS2011,

Anselmino, Barone and AK, PLB **713** (2012) 317

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

One can choose as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$

Integrating σ_{UU} and σ_{lU} over ϕ_2 we obtain

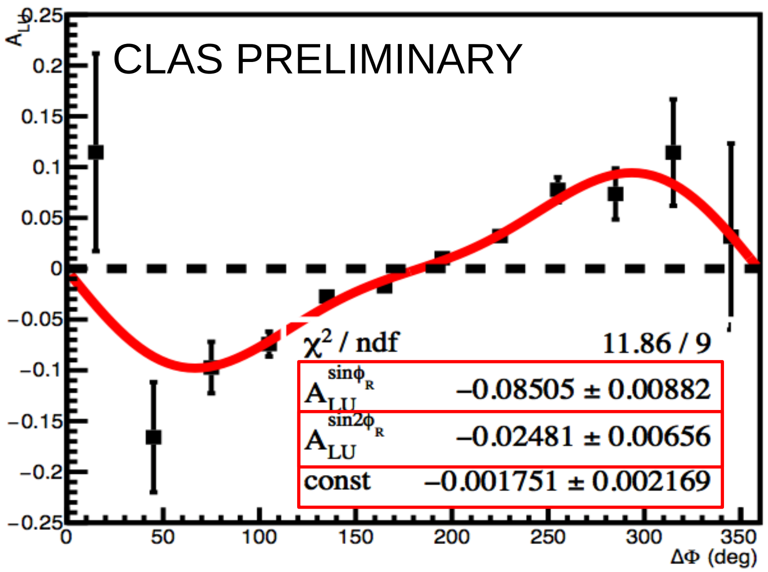
$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} = \frac{-\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)\sin(\Delta\phi)}{F_0^{\hat{u}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)}$$

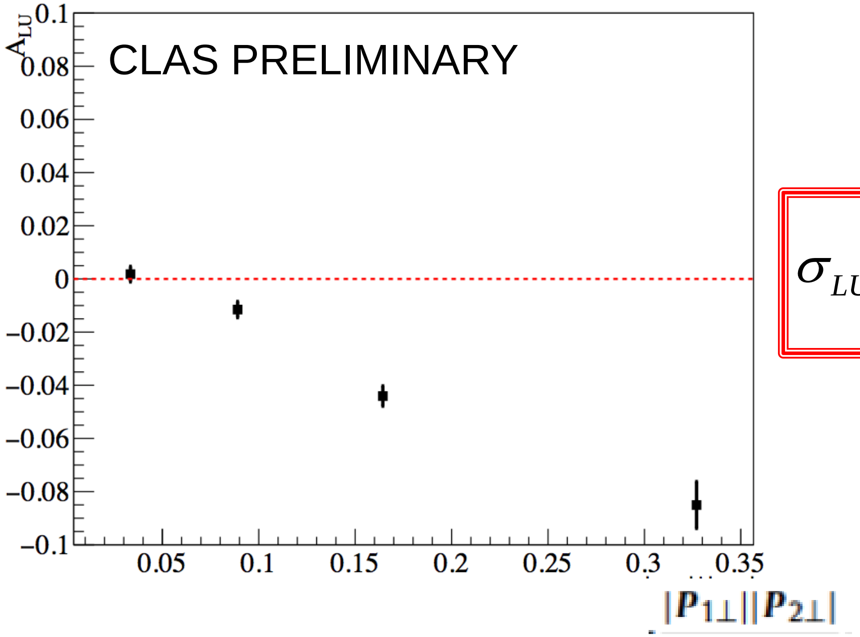
A_{LU} @ CLAS

$$A_{LU} = \frac{\sigma_{LU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{LU1}\cos(\Delta\phi)+a_{LU2}\cos(2\Delta\phi)+\cdots\right)\sin(\Delta\phi)}{\sigma_{UU}(x,z,\zeta,P_{T1}^2,P_{T2}^2)\left(1+a_{UU1}\cos(\Delta\phi)+a_{UU2}\cos(2\Delta\phi)+\cdots\right)} \approx$$

$$\approx p_1\sin(\Delta\phi)+p_2\sin(2\Delta\phi)+\cdots$$

Courtesy of S.Pisano & H.Avakian (unpublished 😟)





$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\sin(\phi_1-\phi_2)$$

Presence of higher harmonics indicate that $\sigma_{LU}(\Delta\Phi) \neq \sigma_{UU}(\Delta\Phi)$

Conclusions

- Azimuthal correlations in dihadron production in SIDIS and SIA provide a new way to study nucleon structure and hadronization process
- In our recent work
  - The inconsistency between IFF definitions in SIDIS and SIA was resolved
  - The BELLE zero result in quark handedness TMD FF study was explained
  - New weighted asymmetries are proposed for measurement of this FFs both in SIDIS and SIA
- To describe TFR of SIDIS 16 LO spin-dependent TMD fracture functions
- For one hadron in TFR SIDIS SSA contains only a Sivers-type modulation.
  - Observation of Collins-type SSA will indicate that LO factorized approach fails
    - Indication of long range correlation between the struck quark polarization and P_T of produced in TFR hadron might be important
  - Preliminary data from JLab show nonzero A_LU
- We expect more news for JLab 12 and EIC

adds

Collinear Frac.Func.: application to HERA data, 1

D. de Glorian, R. Sassot, Leading Proton Structure Function. PRD 58, 054003 (1998)

$$\frac{d^3\sigma^p_{target}}{d\beta dQ^2 dx_P} = \frac{4\pi\alpha^2}{\beta Q^4}\left(1-y+\frac{y^2}{2}\right)M_p^h\left(\beta,Q^2,x_P\right),\quad \beta=\frac{x}{1-\zeta},\quad \zeta=\frac{p_h^+}{p_N^+}\qquad x_P=\zeta$$

$$xM_q^{p/p}(\beta,Q_0^2,x_P)=N_s\beta^{a_s}(1-\beta)^{b_s}\{C_P\beta x_P^{\alpha_P}+C_{LP}(1-\beta)^{\gamma_{LP}}[1+a_{LP}(1-x_P)^{\beta_{LP}}]\}$$

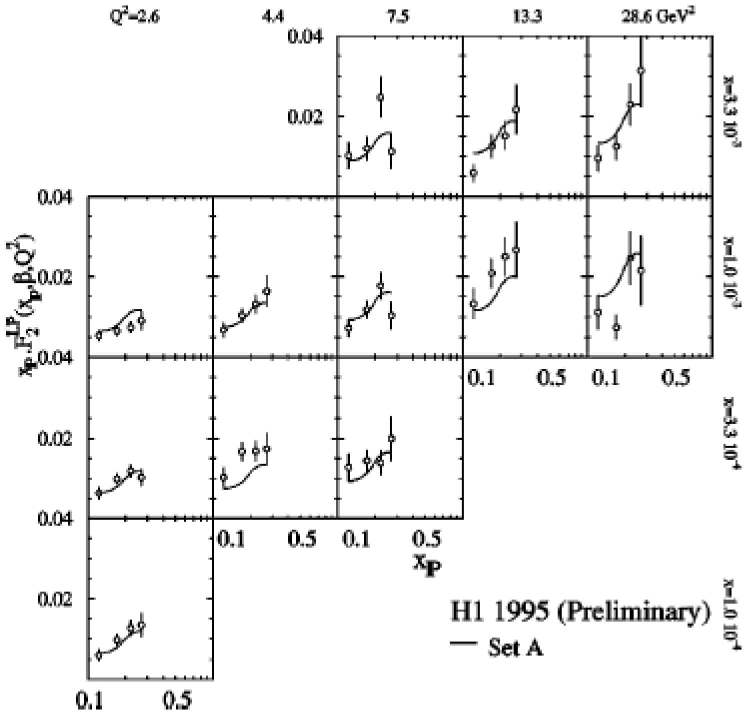


FIG. 2. H1 leading-proton data against the outcome of the fracture function parametrization (solid lines).

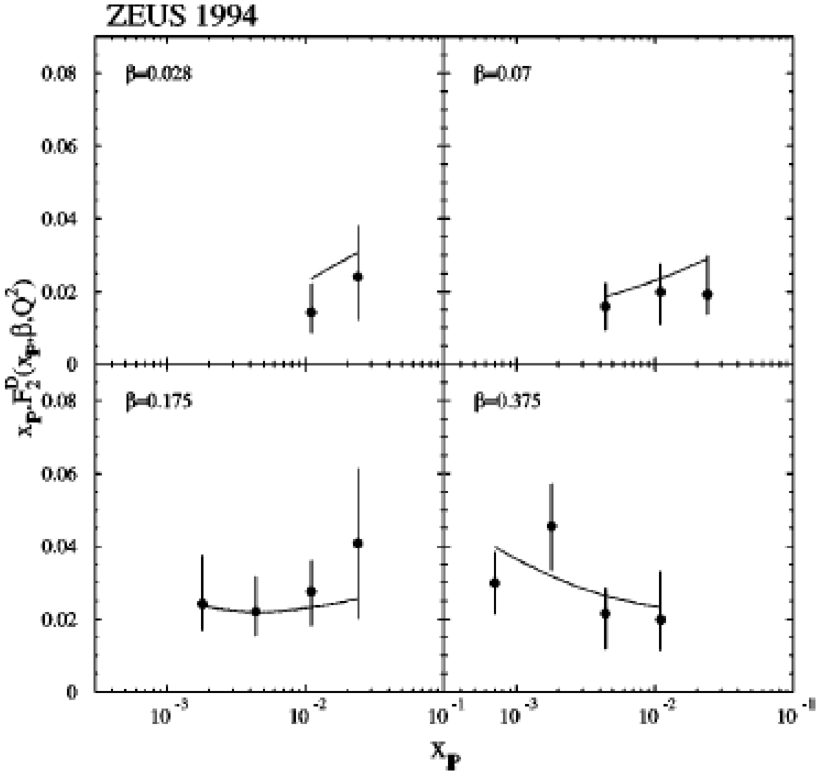
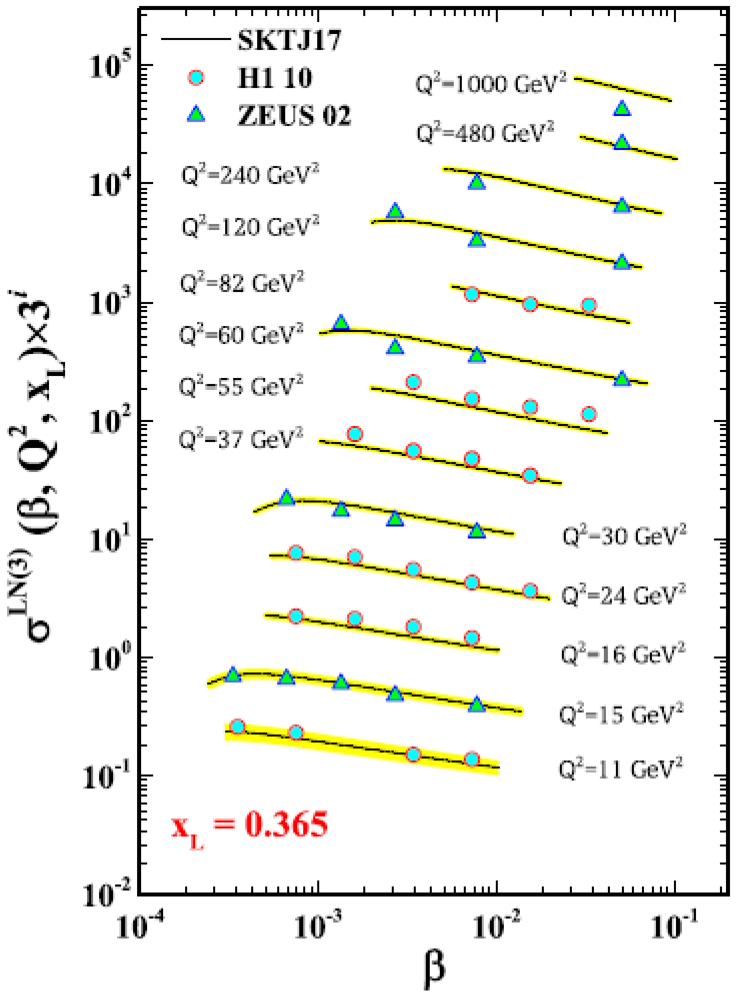
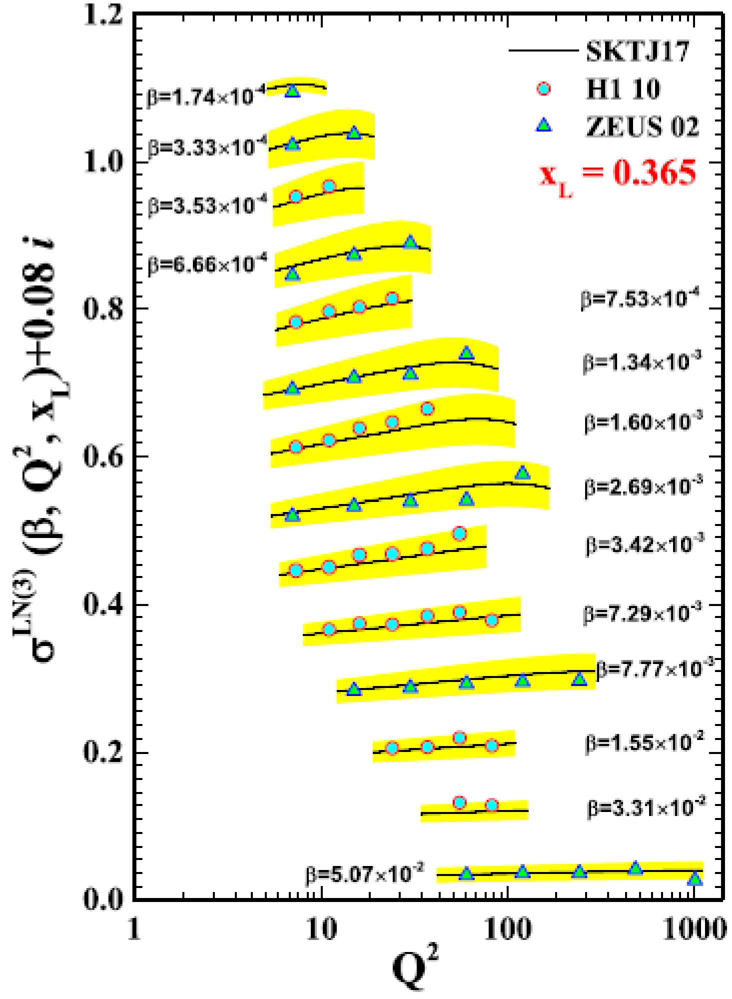


FIG. 8. ZEUS diffractive data, against the expectation coming from the fracture function parametrization (fit A).

Collinear Frac.Func.: application to HERA data, 2

Shoeibi *et al*, Neutron fracture functions. PRD 95, 074011 (2017)





DSIDIS azimuthal modulations

AK @ DIS2011

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1+\phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right) \cos(2\phi_2) \right)$$

$$D_{nn}(y) = \frac{2(1-y)}{1+(1-y)^2}$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

$$C[\hat{M}\cdot Dw] = \sum_a e_a^2 \int d^2k_T\, d^2p_T\, \delta^{(2)}(z\mathbf{k}_T + \mathbf{p}_T - \mathbf{P}_{T1}) \hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2}) D_a(z,p_T^2) w$$

Structure functions $F_{\dots}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi), \text{ with } \Delta\phi = \phi_1 - \phi_2$$

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{LL} = F_0^{\hat{l}_1\cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$
$$+D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

σ_{UT}

$$
\begin{aligned}
\sigma_{UT} =& -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1 - \phi_S) \\
& -\left(\frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1} \right) \sin(\phi_2 - \phi_S) \\
& + D_{nn}(y) \Bigg[\left(\frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1 m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + \phi_S) \\
& \quad + \left(\frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1 m_2 m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \right. \\
& \qquad \left. + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}}{m_1 m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_2 + \phi_S) \\
& \quad + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(3\phi_1 - \phi_S) \\
& \quad + \left(\frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2 - \phi_S) \\
& \quad + \left(\frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + 2\phi_2 - \phi_S) \\
& \quad - \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(2\phi_1 - \phi_2 + \phi_S) \\
& \quad - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(\phi_1 - 2\phi_2 - \phi_S) \\
& \quad + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(2\phi_1 + \phi_2 - \phi_S) \Bigg]
\end{aligned}
$$