# Azimuthal asymmetries in the pion induced Drell-Yan process within TMD factorization


Zhun Lu

Southeast University (SEU)


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Based on Xiaoyu Wang, **ZL**, Ivan Schmidt, JHEP 1708, 137 (2017)
Xiaoyu Wang , **ZL** , PRD 97, 054005 (2018)
Xiaoyu Wang, **ZL** , EPJC, 78, 643 (2018)

# OUTLINE

1. Introduction

2. Unpolarized πN Drell-Yan process, extracting the pion Sudakov form factor

3.Sivers asymmetry in transversely polarized π-proton Drell-Yan process within TMD factorization

4. Cos2φ asymmetry in the unpolarized process with TMD evolution

5. Summary

# Introduction--Motivation

- Test of the sign change of the Sivers function is a fundamental quest in QCD dynamics. Recent measurement by COMPASS provides hint on this sign change.

- Precise measurement and extraction of the Sivers function (and other TMDs) from the pion-nucleon Drell-Yan process by phenomenological analysis.

- As the unpolarized cross section always appears in the denominator of asymmetries, it is important to acquire the differential cross-section of the unpolarized process with high accuracy.

- The typical energy scale of Drell-Yan process is usually different from those of SIDIS experiments at Jlab, HERMES, COMPASS, therefore, it is also necessary to include the TMD evolution in global analysis

# Introduction-- (TMD)PDF

- **Parton Distribution Functions(PDFs)**
  - Leading twist: $f_1(x), g_1(x), h_1(x)$ describe the quark structure of hadrons
  - Only have one longitudinal freedom $x$, *i.e.*, quarks are perfectly collinear

- **Transverse Momentum Dependent(TMD) PDFs**
  - Admit intrinsic parton transverse momentum
  - Provide 3D internal picture of hadrons
  - Reflect correlation between parton transverse momentum and parton/nucleon transverse spin

| Nucleon \ Quark, Gluon | U | L | T |
|---|---|---|---|
| U | number density $f_1^{q,g}(x,k_T^2)$ | | Boer-Mulders $h_1^{\perp q,g}(x,k_T^2)$ |
| L | | Helicity $g_{1L}^{q,g}(x,k_T^2)$ | worm-gear L $h_{1L}^{\perp q,g}(x,k_T^2)$ |
| T | Sivers $f_{1T}^{\perp q,g}(x,k_T^2)$ | Kotzinian-Mulders worm-gear T $g_{1T}^{\perp q,g}(x,k_T^2)$ | Transversity $h_1^{q,g}(x,k_T^2)$<br>Pretzelosity $h_{1T}^{\perp q,g}(x,k_T^2)$ |

# Introduction--Approach

- TMD factorization and TMD evolutions

- TMD factorization：

  valid in the region $q_\perp \ll Q$

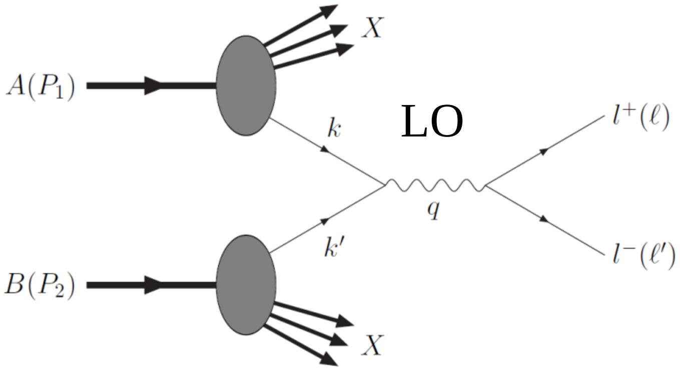
  observables :convolutions of hard factor and well-defined TMD PDFs/FFs

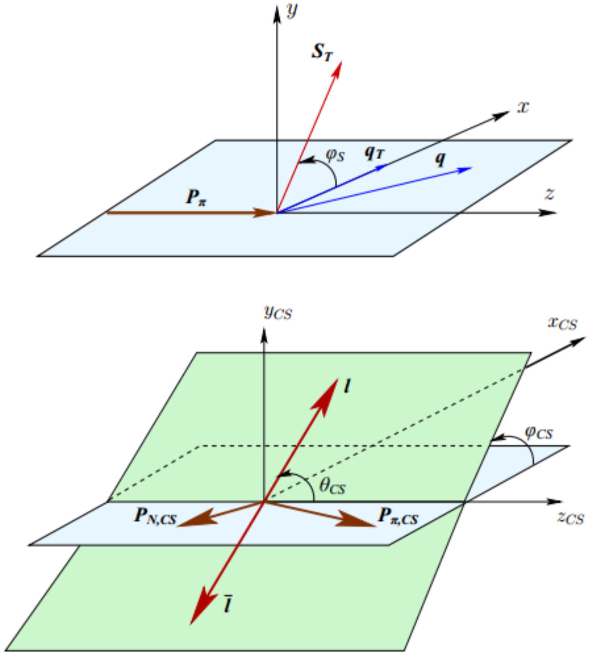
- TMD evolution： (gain more precise results span in different energy scales)

  convenient to perform in b-space (conjugate to $k_\perp$ via FT)

# Introduction—reference frames

- Drell-Yan process $$A(P_1) + B(P_2) \to l^+(\ell) + l^-(\ell') + X\,,$$




Target rest frame (COMPASS convention)

Collins-Soper frame

Azimuthal angles definition

# Introduction-SSAs

- General form of the cross section (Target: unpolarized/transversely polarized)

$$\frac{d\sigma}{d^4 q d\Omega} = \frac{\alpha^2}{Fq^2}\hat{\sigma_U}\Bigg\{ \left(1+\cos^2(\theta)+\sin^2(\theta)A_{UU}^{\cos(2\phi)}\cos(2\phi)\right)$$
$$+S_T\Bigg[(1+\cos^2(\theta))A_{UT}^{\sin(\phi_S)}\sin(\phi_S)$$
$$+\sin^2(\theta)\left(A_{UT}^{\sin(2\phi+\phi_S)}\sin(2\phi+\phi_S)+A_{UT}^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S)\right)\Bigg]\Bigg\}$$

Leading-twist

- The asymmetries

| | | | | Beam | Target |
|---|---|---|---|---|---|
| $A_{UU}^{\cos(2\phi)}$ | $\propto h_{1,\pi}^{\perp q}$ | $\otimes$ | $h_{1,p}^{\perp q}$ | Boer-Mulders | Boer-Mulders |
| $A_{UT}^{\sin(\phi_S)}$ | $\propto f_{1,\pi}^{q}$ | $\otimes$ | $f_{1T,p}^{\perp q}$ | $f_{1,\pi}^{q}$ | Sivers |
| $A_{UT}^{\sin(2\phi-\phi_S)}$ | $\propto h_{1,\pi}^{\perp q}$ | $\otimes$ | $h_{1,p}^{q}$ | Boer-Mulders | Transversity |
| $A_{UT}^{\sin(2\phi+\phi_S)}$ | $\propto h_{1,\pi}^{\perp q}$ | $\otimes$ | $h_{1T,p}^{\perp q}$ | Boer-Mulders | Pretzelosity |

Cross section

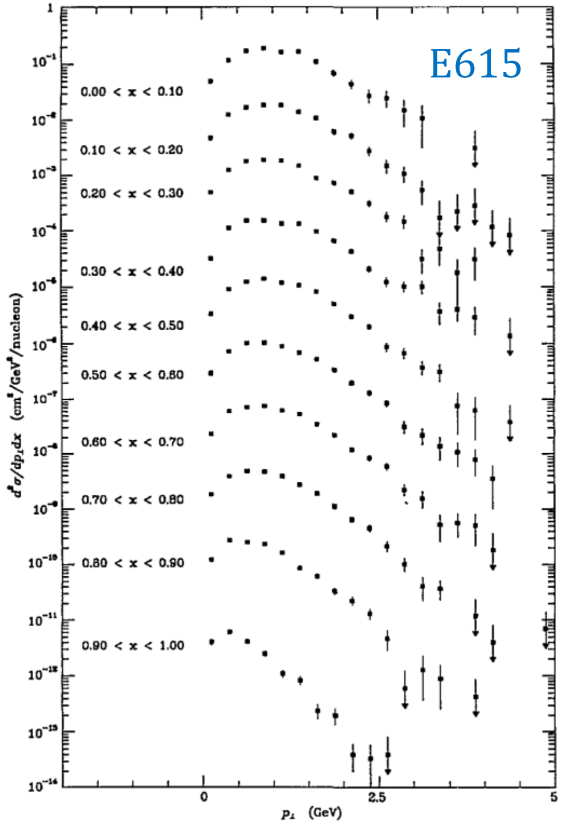
# Introduction--measurements

- Pion-N Drell-Yan: Experimental measurements (Unpolarized)

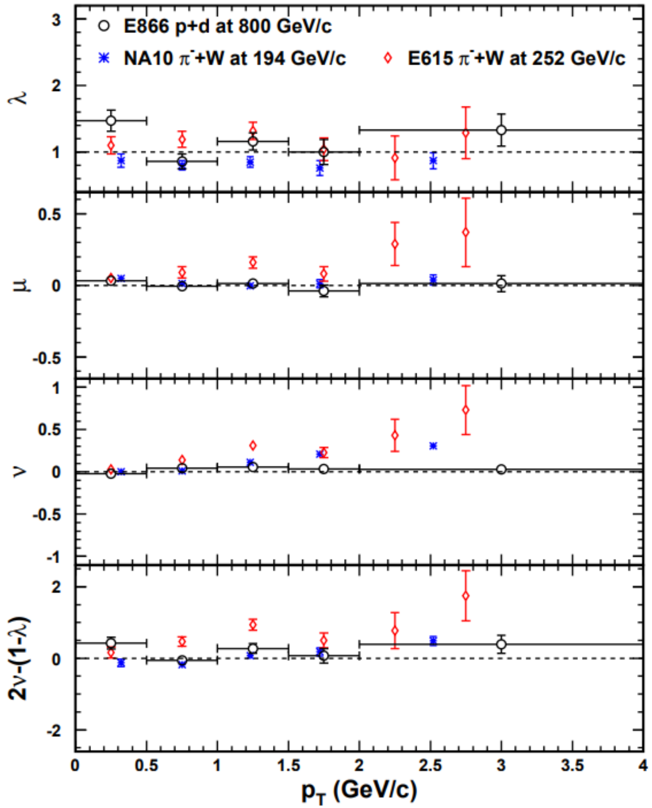
  - E615
  - NA10

$$\frac{dN}{d\Omega} \equiv \frac{d\sigma}{d^4q\, d\Omega} \bigg/ \frac{d\sigma}{d^4q}$$

$$= \frac{3}{4\pi} \frac{1}{\lambda+3} \Big(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi\Big)$$




$q_T$ distribution of dilepton

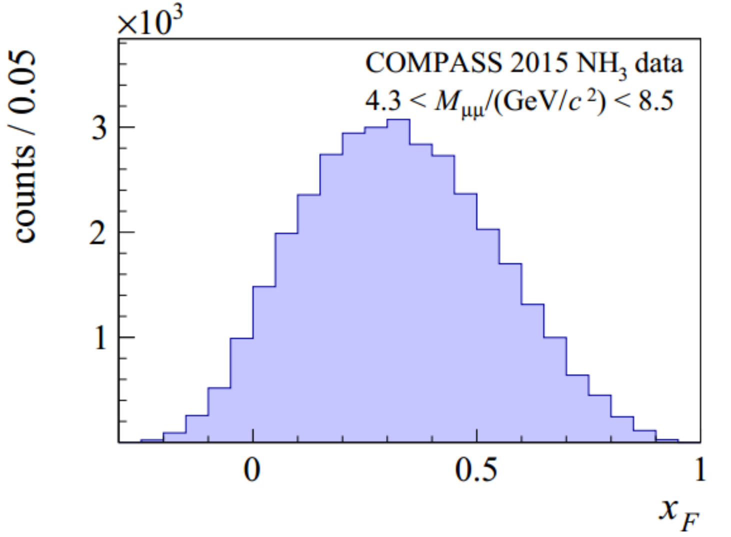



Angular distribution

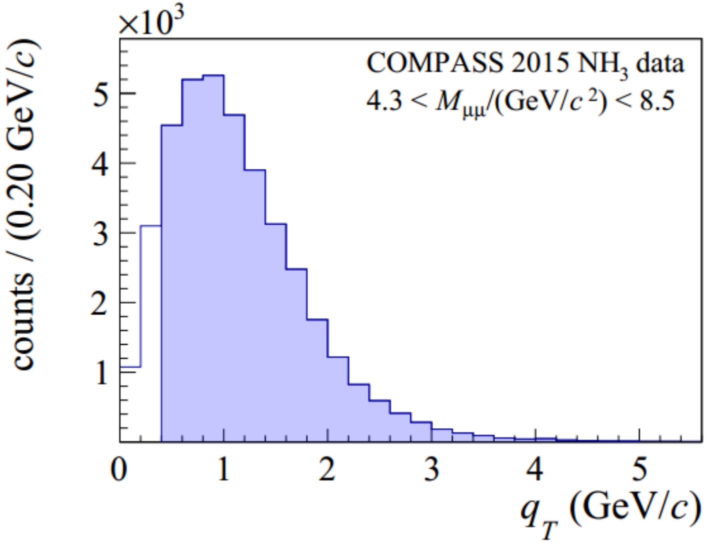
# Introduction--measurements

◆ Pion-N Drell-Yan: Experimental measurements (Unpolarized)

PRL119, 112002 (2017)




$x_F$ distribution of dilepton events



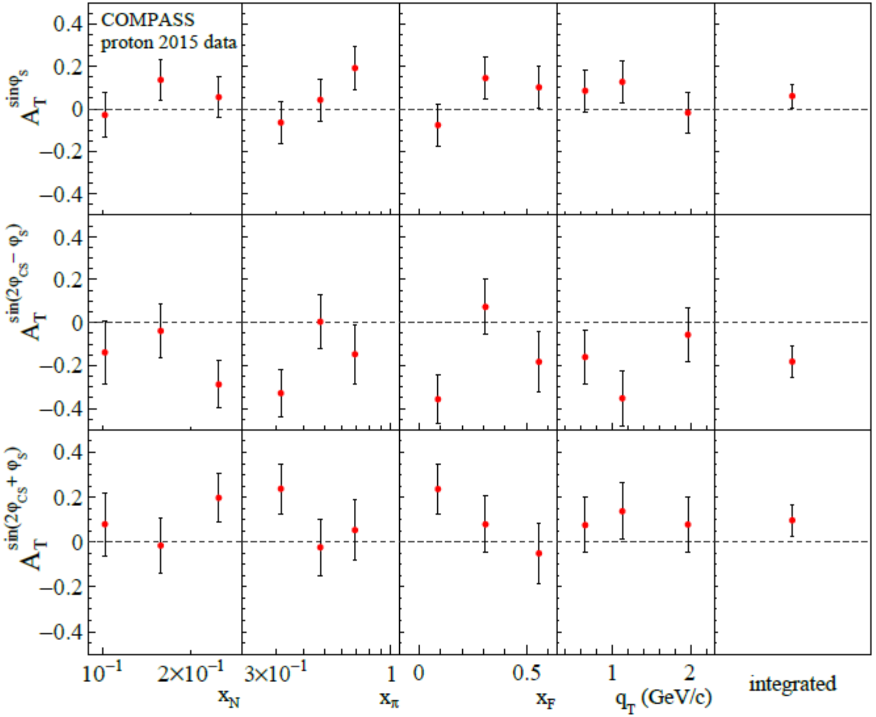

$q_T$ distribution

COMPASS measurement

# Introduction--measurements

- Experimental measurements (transversely polarized target)

Phys. Rev. Lett. 119, 112002 (2017)




$$\propto f_{1,\pi}^{q} \otimes f_{1T,p}^{\perp q}$$

$$\propto h_{1,\pi}^{\perp q} \otimes h_{1,p}^{q}$$

$$\propto h_{1,\pi}^{\perp q} \otimes h_{1T,p}^{\perp q}$$

SSAs at COMPASS

# OUTLINE

2. Extraction of pion Sudakov form factor from unpolarized πN Drell-Yan process

# Unpolarized process — cross section

- **General form of the differential cross section (CSS resummation)**

Collins, Soper and Sterman, NPB 250 (1985) 199

$$\frac{d^4\sigma}{dQ^2dyd^2\boldsymbol{q}_\perp} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{\boldsymbol{q}}_\perp\cdot\vec{\boldsymbol{b}}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q,q_\perp),$$

Studied by Collins et. al., PRD94,034014, 16'

- $\sigma_0 = \frac{4\pi\alpha_{em}^2}{3N_C sQ^2}$ **tree level cross section**
- $\widetilde{W}_{UU}(Q;b)$ **dominates in the region** $q_\perp \ll Q$**, all-order resummation**
- $Y_{UU}(Q,q_\perp)$ **provides corrections at** $q_\perp \sim Q$**, ignored here**

General form of differential cross section

# Unpolarized process

- The structure function $\widetilde{W}_{UU}$ can be written as

$$\widetilde{W}_{UU}(Q;b) = H_{UU}(Q;\mu) \sum_{q,\bar{q}} e_q^2 \tilde{f}^{\rm sub}_{1\,\bar{q}/\pi}(x_\pi, b; \mu, \zeta_F)\, \tilde{f}^{\rm sub}_{1\,q/p}(x_p, b; \mu, \zeta_F),$$

- $\tilde{f}^{\rm sub}_{1\,q/H}$ is the subtracted distribution function in the b-space and universal.
- $H_{UU}(Q;\mu)$is the factor associated with hard scattering and scheme-dependent.
- **The way to regularize light-cone singularity in TMD definition and subtract soft gluon contribution defines the scheme for the TMD factorization**

# Unpolarized process -- evolution

- **TMD evolution for the $\zeta_F$ -dependence (energy evolution)**

$$\frac{\partial \ln \tilde{f}^{\rm sub}(x,b;\mu,\zeta_F)}{\partial \sqrt{\zeta_F}} = \tilde{K}(b;\mu)$$

Collins, Soper 81'
Idilbi, Ji, Ma, Yuan 04'

- **TMD evolution for the $\mu$-dependence**

$$\frac{d\,\tilde{K}}{d\,\ln\mu} = -\gamma_K(\alpha_s(\mu)),$$

$$\frac{d\,\ln \tilde{f}^{\rm sub}(x,b;\mu,\zeta_F)}{d\,\ln\mu} = \gamma_F(\alpha_s(\mu);\frac{\zeta_F^2}{\mu^2}),$$

- **General structure of the solution**

$$f(x,b,Q) = \mathcal{F} \times e^{-S} \times f(x,b,\mu_b)$$

# Unpolarized process—solution

- **Solution in *b* space** Collins, Soper 81' Collins, Soper, Sterman 85' Collins, 11' Collins, Rogers 15' Ji,Ma,Yuan 04'

$$\tilde{f}_1^{u/p}(x,b;Q) = e^{-\frac{1}{2}S_{\text{pert}}(Q,b_*)-S_{\text{NP}}^{f_1^{q/p}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/p}(x,\mu_b)$$

**CSS prescription** $b_* = b/\sqrt{1+b^2/b_{\text{max}}^2}$ $b_* \approx b$ at low $b$; $b_* \approx b_{\text{max}}$ at large $b$ $\mu_b = c_0/b_*$

- $S = S_{\text{pert}} + S_{\text{NP}}$ **Sudakov -like form factor**
- $\mathcal{F}(\alpha_s(Q))$ , $C_{q\leftarrow i}^{f_1}$ , $H_{UU}(Q;\mu)$ : scheme-dependent coefficients/factors

Prokudin, Sun, Yuan 15'

- **Ji-Ma-Yuan (JMY) scheme:** Ji,Ma,Yuan, PRD71, 034005; PLB 597,299
- **Collins(JCC) scheme:** J. C. Collins, Foundations of perterbative QCD
- **Lattice (LAT) scheme:** Ji,Ma,Yuan, PRD91, 074009

Proton TMD

# Sudakov-like form factor

- The Sudakov-like form factor

$$S = S_{\rm pert} + S_{\rm NP}.$$

- The perturbative part of S (we adopt $A$ and $B$ up to NLL accuracy )

$$S_{\rm pert}(Q,b) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln\frac{Q^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu}))\right].$$

- non-perturbative form factor of S from pp DY

$$S_{\rm NP} = g_1 b^2 + g_2 \ln\frac{b}{b_*}\ln\frac{Q}{Q_0} + g_3 b^2\left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda\right)$$

Sun, Isaacson, Yuan, Yuan 14'

**Parameters:** $g_1 = 0.212, \quad g_2 = 0.84, \quad g_3 = 0$

$$S_{\rm NP}^{f_1^{q/p}}(Q,b) = \frac{g_2}{2}\ln\frac{b}{b_*}\ln\frac{Q}{Q_0} + \frac{g_1}{2}b^2,$$

# Sudakov form factor of the pion

- Propose a similar non-perturbative Sudakov form factor $S_{\rm NP}^{f_1^{q/\pi}}(Q,b)$ for pion TMD

$$S_{\rm NP}^{f_1^{q/\pi}} = g_1^\pi b^2 + g_2^\pi \ln\frac{b}{b_*}\ln\frac{Q}{Q_0}.$$

Wang, Lu, Schmidt, JHEP 1708, 137

$$g_1^\pi(b) = g_1^\pi b^2$$

contains information on the nonperturbative transverse motion of partons inside pion

- The unpolarized TMD distribution for the pion

$$f_1^{i/\pi}(x,b;Q) = e^{-\frac{1}{2}S_{\rm pert}(Q,b_*)-S_{\rm NP}^{f_1^{q/\pi}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/\pi}(x,\mu_b)$$

$$f_{1q/\pi}(x,k_\perp;Q) = \int_0^\infty \frac{dbb}{2\pi}J_0(k_\perp b)\tilde{f}_{1q/\pi}^{\rm sub}(x,b;Q).$$

# Unpolarized process

$$\tilde{f}_1^{u/p}(x,b;Q) = e^{-\frac{1}{2}S_{\text{pert}}(Q,b_*)-S_{\text{NP}}^{f_1^{q/p}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/p}(x,\mu_b)$$

$$\tilde{f}_1^{i/\pi}(x,b;Q) = e^{-\frac{1}{2}S_{\text{pert}}(Q,b_*)-S_{\text{NP}}^{f_1^{q/\pi}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/\pi}(x,\mu_b)$$

$$\widetilde{W}_{UU}(Q;b) = H_{UU}(Q;\mu)\sum_{q,\bar{q}} e_q^2 \tilde{f}_{1\,\bar{q}/\pi}^{\text{sub}}(x_\pi,b;\mu,\zeta_F)\tilde{f}_{1\,q/p}^{\text{sub}}(x_p,b;\mu,\zeta_F),$$

Structure function

$$\widetilde{W}_{UU}(Q;b) = e^{-S(Q^2,b)}\times\sum_{q,\bar{q}} e_q^2 C_{q\leftarrow i}\otimes f_{i/\pi^-}(x_1,\mu_b)C_{\bar{q}\leftarrow j}\otimes f_{j/p}(x_2,\mu_b)$$

Coefficients(scheme-independent coefficients)

$$C_{q\leftarrow i} = \mathcal{F}(\alpha_S(Q))\times C_{q\leftarrow i}^{f_1}\times\sqrt{H(\mu=Q)}$$

$$C_{q\leftarrow q'}(x,b;\mu_b) = \delta_{qq'}\left[\delta(1-x)+\frac{\alpha_s}{\pi}\left(\frac{C_F}{2}(1-x)+\frac{C_F}{4}(\pi^2-8)\delta(1-x)\right)\right]$$

$$C_{q\leftarrow g}(x,b;\mu_b) = \frac{\alpha_s}{\pi}T_R x(1-x).$$

S. Catani et.al, 01'

Differential cross section

$$\frac{d^4\sigma}{dQ^2dyd^2\boldsymbol{q}_\perp} = \sigma_0\int_0^\infty\frac{dbb}{2\pi}J_0(q_\perp b)\times\widetilde{W}_{UU}(Q;b),$$

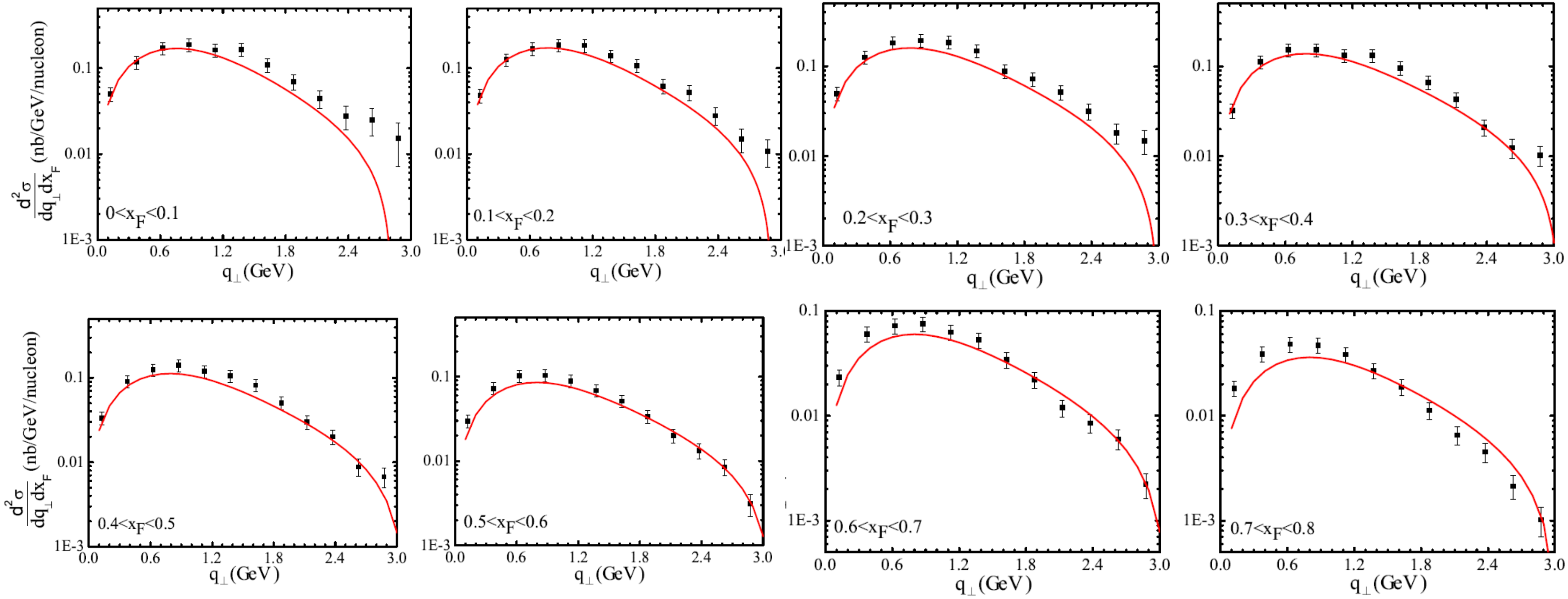
# Unpolarized process

- Fit $S_{\rm NP}^{f_1^{q/\pi}}(Q,b)$ to E615 data,

Wang, Lu, Schmidt, JHEP 1708, 137

$$g_1^\pi = 0.082 \pm 0.022, \quad g_2^\pi = 0.394 \pm 0.103,$$

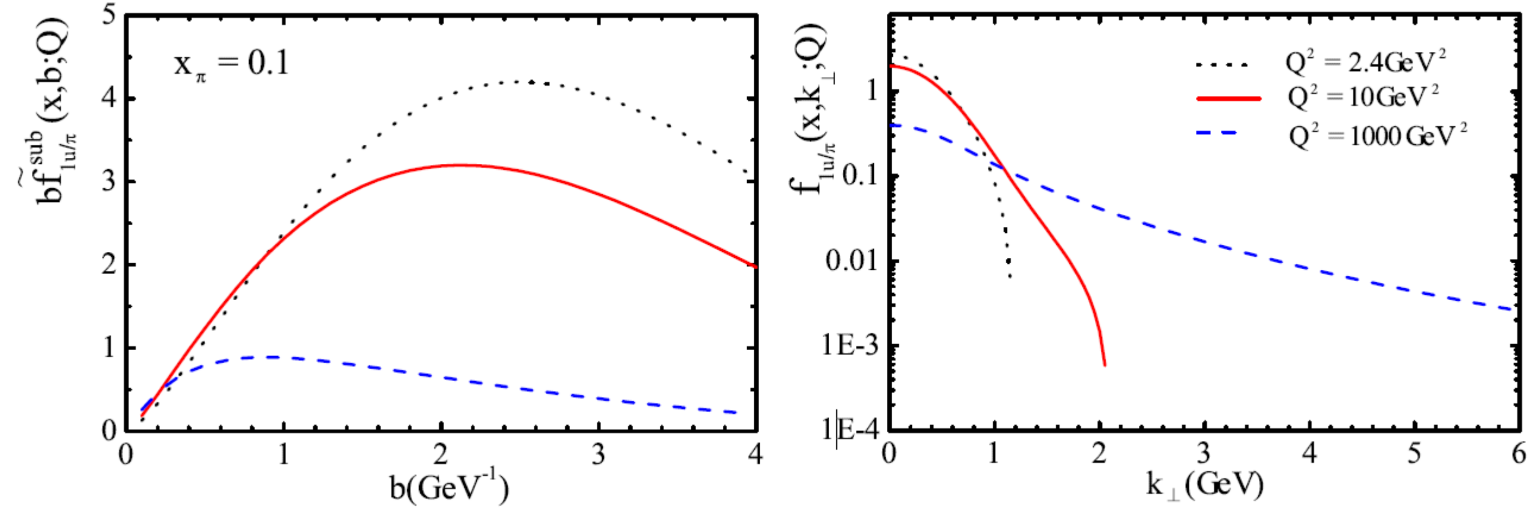
$q_T < 3$ GeV



# Evolution of pion TMD

$$\tilde{f}_1^{i/\pi}(x,b;Q) = e^{-\frac{1}{2}S_{\rm pert}(Q,b_*)-S_{\rm NP}^{f_1^{q/\pi}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/\pi}(x,\mu_b)$$

JCC scheme

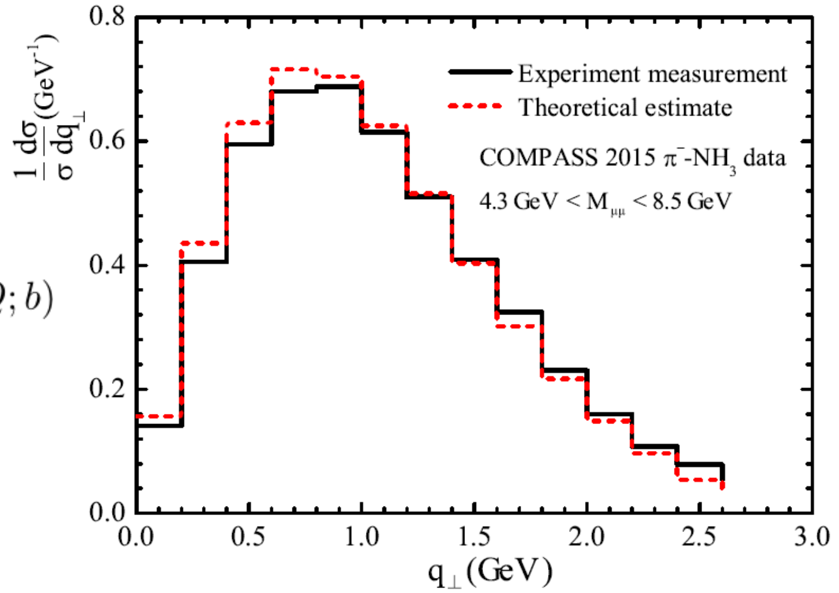



Subtracted unpolarized TMD distribution of the pion meson for valence quarks in $b$-space (left panel) and $k_\perp$-space (right panel), at energies: $Q^2 = 2.4$ GeV$^2$ (dotted lines), $Q^2 = 10$ GeV$^2$ (solid lines) and $Q^2 = 1000$ GeV$^2$ (dashed lines).

Wang, Lu, Schmidt, JHEP 1708 (2017) 137

# Unpolarized process

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int_0^\infty \frac{dbb}{2\pi} J_0(q_\perp b) \times \widetilde{W}_{UU}(Q;b)$$




The transverse spectrum of lepton pair production in the unpolarized pion-nucleon Drell-Yan process, with an $NH_3$ target at COMPASS. The dashed line is our theoretical calculation using the extracted Sudakov form factor for the pion TMD PDF. The solid line shows the experimental measurement at COMPASS.

Wang, ZL, Schmidt, JHEP 1708 (2017) 137

# OUTLINE

3. Sivers asymmetry in transversely polarized πN process

# Transversely polarized process

- The transverse single spin asymmetry can be defined as

$$A_{UT} = \frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} \Bigg/ \frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp},$$

Spin-dependent

Spin-independent(Unpolarized)

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q,q_\perp).$$

$$\frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \epsilon_\perp^{\alpha\beta} S_\perp^\alpha \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \widetilde{W}_{UT}^\beta(Q;b) + Y_{UT}^\beta(Q,q_\perp).$$

Sivers Asymmetry

# Transversely polarized process

- Spin-dependent structure function

$$\widetilde{W}_{UT}^{\alpha}(Q;b) = H_{UT}(Q;\mu) \sum_{q,\bar{q}} e_q^2 \tilde{f}_{1\,\bar{q}/\pi}(x_\pi, b; \mu, \zeta_F) \tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x_p, b; \mu, \zeta_F).$$

$$H_{UT}(Q;\mu) = H_{UU}(Q;\mu)$$

$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x, b; \mu, \zeta_F) = \int d^2\boldsymbol{k}_\perp e^{-i\vec{\boldsymbol{k}}_\perp \cdot \vec{\boldsymbol{b}}} \frac{k_\perp^\alpha}{M_p} f_{1T,q/p}^{\perp(\mathrm{DY})}(x, \boldsymbol{k}_\perp; \mu),$$

Sivers function in b-space

TMDs follows the same evolution equation in the perturbative region.
The evolution for $\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}$ can be written in a similar form.

Echevarria, Idilbi, Kang, and Vitev, 14’

$$f(x,b,Q) = \mathcal{F} \times e^{-S} \times f(x,b,\mu_b)$$

# Transversely polarized process

- $S_{\text{pert}}$ for the Sivers function has the same form as unpolarized PDF
- Nonperturbative Sudakov form factor can be parameterized as:

Echevarria, Idilbi, Kang, and Vitev, 14'

$$S_{\text{NP}}^{\text{Siv}} = \left( g_1^{\text{Siv}} + g_2^{\text{Siv}} \ln \frac{Q}{Q_0} \right) b^2,$$

$$g_1^{\text{Siv}} = \langle k_{s\perp}^2 \rangle_{Q_0}/4 = 0.071\text{GeV}^2$$

$$g_2^{\text{Siv}} = g_2^{f_1} = \frac{g_2}{2} = 0.08\text{GeV}^2$$

$g_2$ is the same for $f_{1T}^{\perp}$ and $f_1$

Sudakov form factor for Sivers function

# Transversely polarized process

- In the small b region, the Sivers function can be also expressed as the convolution of perturbatively calculable hard coefficients and the corresponding collinear twist-3 correlation functions

$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x,b;\mu) = (\frac{-ib^\alpha}{2})\sum_i \Delta C^T_{q\leftarrow i}\otimes f^{(3)}_{i/p}(x',x'';\mu).$$

Qiu-Sterman matrix element $T_{q,F}(x,x)$ is the most relevant one

$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x,b)\big|_{\mathrm{LO}} = \left(\frac{-ib^\alpha}{2}\right)T_{q,F}(x,x)$$

$$T_{q,F}(x,x) = \int d^2k_\perp \frac{|k_\perp^2|}{M_p} f_{1T\,q/p}^{\perp\mathrm{DY}}(x,k_\perp) = 2M_p\, f_{1T\,q/p}^{\perp(1)\mathrm{DY}}(x),$$

small b region

# Transversely polarized process

◆ Evolution of the Sivers function in the b space

$$\tilde{f}_{1T,q/p}^{\perp}(x,b;Q) = \frac{b^2}{2\pi}\sum_i \Delta C_{q\leftarrow i}^{T} \otimes T_{i,F}(x,x;\mu_b)e^{-S_{\rm NP}^{\rm siv}-\frac{1}{2}S_{\rm P}},$$

◆ Sivers function in the transverse momentum space

$$\frac{k_\perp}{M_p} f_{1T,q/p}^{\perp}(x,k_\perp;Q) = \int_0^\infty db \frac{b^2}{2\pi} J_1(k_\perp b)\sum_i \Delta C_{q\leftarrow i}^{T} \otimes f_{1T,i/p}^{\perp(1)}(x,\mu_b)e^{-S_{\rm NP}^{\rm siv}-\frac{1}{2}S_{\rm P}}.$$

# Transversely polarized process

- The spin-dependent differential cross section

$$\frac{d^4\Delta\sigma}{dQ^2dyd^2\boldsymbol{q}_\perp} = \sigma_0\epsilon^{\alpha\beta}S_\perp^\alpha \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}}\widetilde{W}_{UT}^\beta(Q;b)$$
$$= \frac{\sigma_0}{4\pi}\int_0^\infty dbb^2 J_1(q_\perp b)\sum_{q,i,j} e_q^2\Delta C_{q\leftarrow i}^T \otimes T_{i,F}(x_p,x_p;\mu_b)$$
$$\times C_{\bar{q}\leftarrow j}\otimes f_{1,j/\pi}(x_\pi,\mu_b)e^{-\left(S_{\rm NP}^{\rm Siv}+S_{\rm NP}^{f_{1q/\pi}}+S_{\rm P}\right)}.$$

- We adopt the C-coefficient up to NLO

Kang, xiao, Yuan, 11'
Sun, Yuan 13'

$$\Delta C_{q\leftarrow q'}^T(x,b;\mu_b) = \delta_{qq'}\left[\delta(1-x)+\frac{\alpha_s}{\pi}\left(-\frac{1}{4N_c}(1-x)+\frac{C_F}{4}(\pi^2-8)\delta(1-x)\right)\right].$$

Spin-dependent

# Transversely polarized process

- Parameterization of the Qiu-Sterman function (EIKV14)

Echevarria, Idilbi, Kang, and Vitev, 14'

$$T_{q,F}(x,x;\mu) = N_q \frac{(\alpha_q+\beta_q)^{(\alpha_q^{\alpha_q}+\beta_q^{\beta_q})}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}} x^{\alpha_q}(1-x)^{\beta_q} f_{q/p}(x,\mu),$$

scale dependence

Set 1:the same as that of unpolarized PDF

Set 2:adopt an approximate evolution kernel from homogenous terms in the exact solution

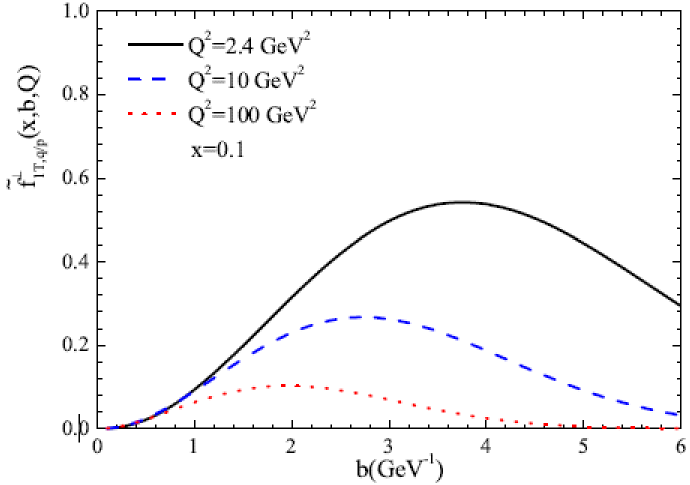
$$P_{qq}^{f_1} = \frac{4}{3}\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right).$$

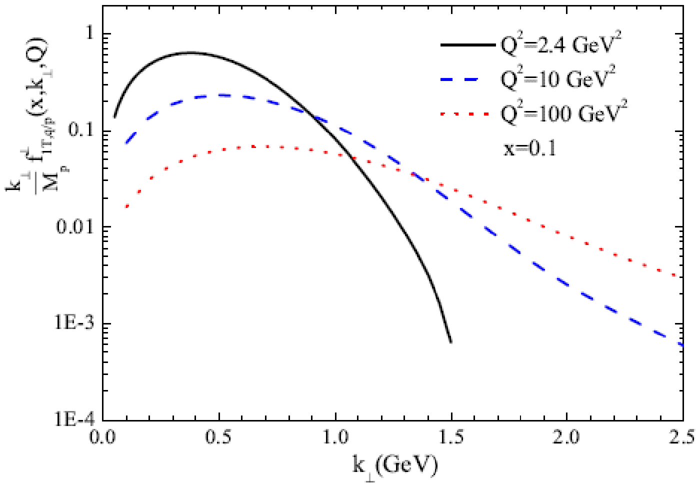
$$P_{qq}^{\mathrm{QS}} \approx P_{qq}^{f_1} - \frac{N_c}{2}\frac{1+z^2}{1-z} - N_c\delta(1-z),$$

Qiu-Sterman

# Transverselypolarized process

◆ TMD evolution of the Sivers function --Set 1



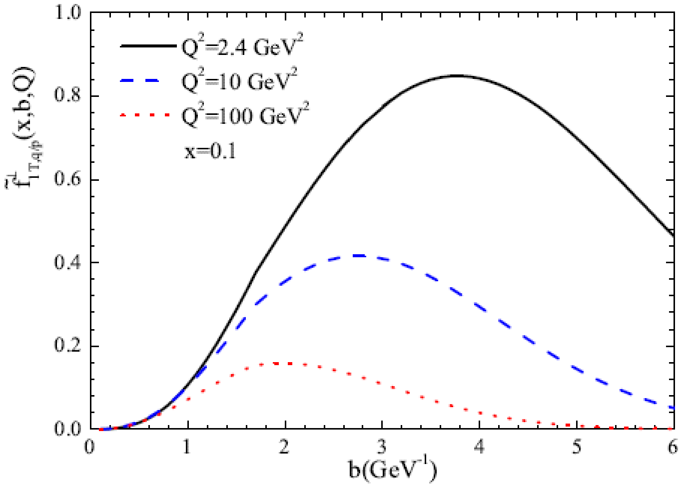



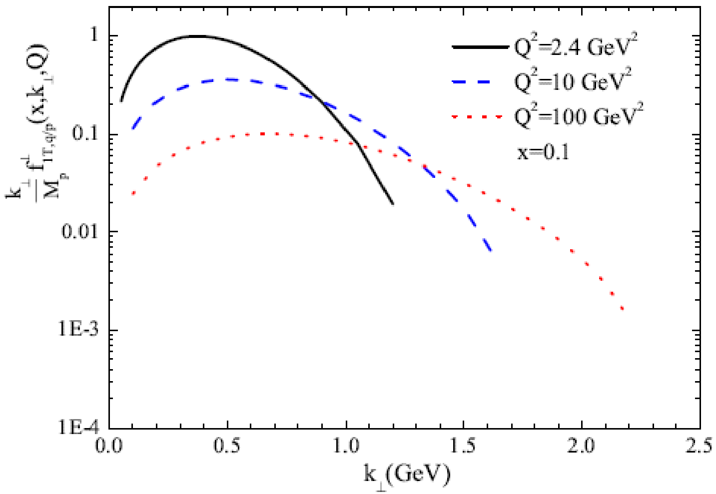

$$P_{qq}^{\mathrm{QS}} \approx P_{qq}^{f_1} - \frac{N_c}{2}\frac{1+z^2}{1-z} - N_c\delta(1-z),$$

Wang, ZL, PRD 97, 054005 (2018)

# Transversely polarized process

- TMD evolution of the Sivers function --Set 2




$$P_{qq}^{\mathrm{QS}} \approx P_{qq}^{f_1} - \frac{N_c}{2}\frac{1+z^2}{1-z} - N_c\delta(1-z),$$

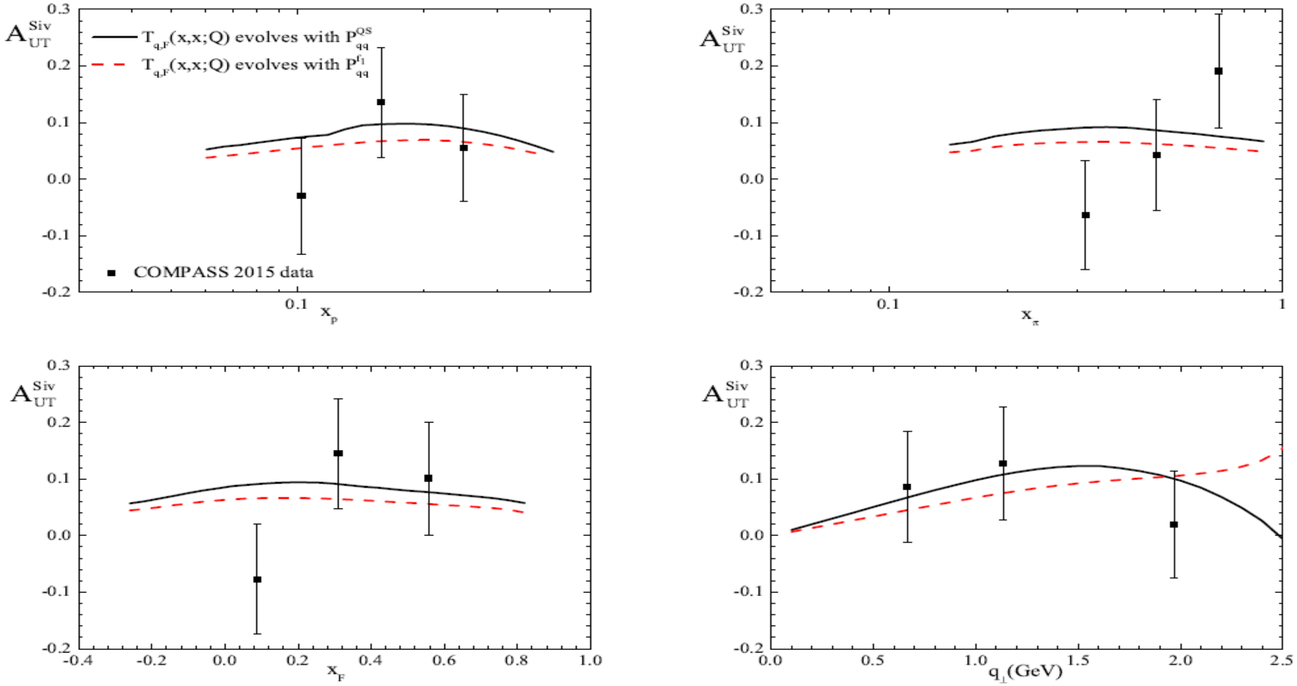
Wang, ZL, PRD 97, 054005 (2018)

# Transversely polarized process

- Sivers asymmetry with the COMPASS measurement

Wang, ZL, PRD 97, 054005 (2018)

$$A_{UT} = \frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} \Big/ \frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp},$$



# OUTLINE

4. Cos2φ asymmetry from BM function in the unpolarized πN DY

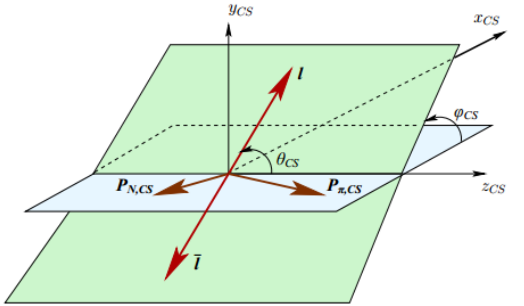
# cos2φ asymmetry in pi-N Drell-Yan

- Cos2φ asymmetry contributed by the Boer-Mulders functions

Boer, 99'

$$\nu_{\rm BM} = \frac{2\sum_q \mathcal{F}\left[\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T - \boldsymbol{k}_T\cdot\boldsymbol{p}_T\right)\frac{h^{\perp}_{1,q/\pi}h^{\perp}_{1,\bar{q}/p}}{M_\pi M_p}\right]}{\sum_q \mathcal{F}\left[f_{1,q/\pi}f_{1,\bar{q}/p}\right]}$$

$$\mathcal{F}[\omega f \bar{f}] = e_q^2 \int d^2\boldsymbol{k}_T d^2\boldsymbol{p}_T \delta^2(\boldsymbol{k}_T + \boldsymbol{p}_T - \boldsymbol{q}_T)\omega f(x_\pi, \boldsymbol{k}_T^2)\bar{f}(x_p, \boldsymbol{p}_T^2)$$




- It might be measured through combination (eliminate pQCD contribution):

$$2\nu_{\rm BM} \approx 2\nu + \lambda - 1$$

- Sudakov effect for Cos2φ asymmetry has been studied by Boer 01'

# cos2φ asymmetry in pi-N Drell-Yan

- In leading order

$$\widetilde{h}^{\alpha\perp}_{1,q/H}(x,b;\mu)=(\frac{-ib^\alpha}{2})T^{(\sigma)}_{q/H,F}(x,x;\mu)$$

- Chiral-odd Twist-3 function

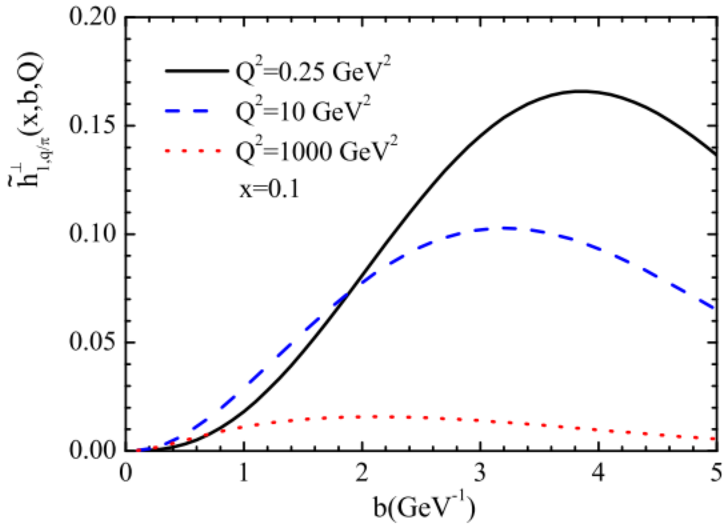
$$T^{(\sigma)}_{q/H,F}(x,x;\mu)=\int d^2\boldsymbol{k}_T\frac{\boldsymbol{k}_T^2}{M_H}h^\perp_{1,q/H}(x,\boldsymbol{k}_T;\mu)=2M_H h^{\perp(1)}_{1,q/H}$$

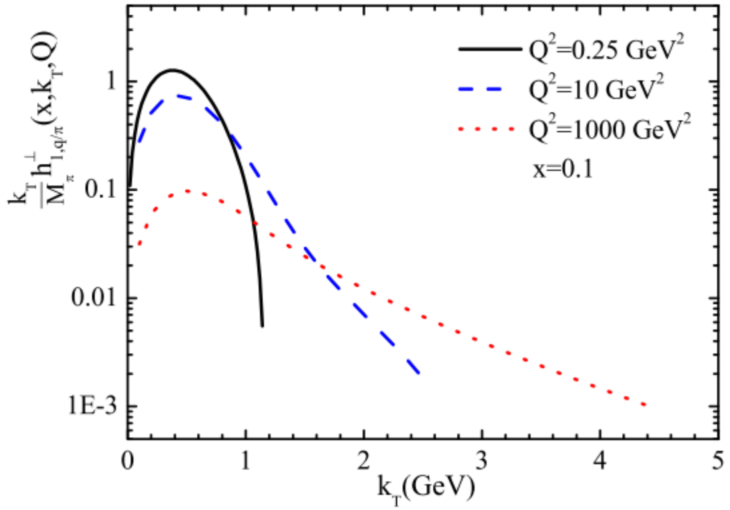
- Assuming the Sudakov form factor associated with the Boer-Mulders function is the same as that for the unpolarized distribution,

$$\widetilde{h}^{\alpha\perp}_{1,q/p}(x,b;Q)=(\frac{-ib^\alpha}{2})e^{-\frac{1}{2}S_{\rm P}(Q,b_*)-S^{f_{1,q/p}}_{\rm NP}(Q,b)}T^{(\sigma)}_{q/p,F}(x,x;\mu_b),$$
$$\widetilde{h}^{\alpha\perp}_{1,q/\pi}(x,b;Q)=(\frac{-ib^\alpha}{2})e^{-\frac{1}{2}S_{\rm P}(Q,b_*)-S^{f_{1,q/\pi}}_{\rm NP}(Q,b)}T^{(\sigma)}_{q/\pi,F}(x,x;\mu_b).$$

# cos2φ asymmetry in pi-N Drell-Yan

- TMD evolution of the pion Boer-Mulders function




Wang , Mao, ZL, EPJC78, 643 (2018)

# cos2φ asymmetry in pi-N Drell-Yan

- Numerator

$$\mathcal{F}\left[\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T-\boldsymbol{k}_T\cdot\boldsymbol{p}_T\right)\frac{h_{1,q/\pi}^{\perp}h_{1,\bar{q}/p}^{\perp}}{M_\pi M_p}\right]$$

$$=\sum_q e_q^2\int\frac{d^2b}{(2\pi)^2}e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}}(2\hat{h}_\alpha\hat{h}_\beta-g_{\alpha\beta}^{\perp})\widetilde{h}_{1,q/\pi}^{\alpha\perp}(x_\pi,b;Q)\widetilde{h}_{1,\bar{q}/p}^{\beta\perp}(x_p,b;Q)$$

$$=\sum_q e_q^2\int\frac{d^2b}{(2\pi)^2}e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}}(2\hat{h}_\alpha\hat{h}_\beta-g_{\alpha\beta}^{\perp})(\frac{-ib^\alpha}{2})T_{q/\pi,F}^{(\sigma)}(x_\pi,x_\pi;\mu_b)(\frac{-ib^\beta}{2})T_{\bar{q}/p,F}^{(\sigma)}(x_p,x_p;\mu_b)e^{-\left(S_{\rm NP}^{f_{1,q/p}}+S_{\rm NP}^{f_{1,q/\pi}}+S_{\rm P}\right)}$$

$$=\sum_q e_q^2\int_0^\infty\frac{dbb^3}{8\pi}J_2(q_Tb)T_{q/\pi,F}^{(\sigma)}(x_\pi,x_\pi;\mu_b)T_{\bar{q}/p,F}^{(\sigma)}(x_p,x_p;\mu_b)e^{-\left(S_{\rm NP}^{f_{1,q/p}}+S_{\rm NP}^{f_{1,q/\pi}}+S_{\rm P}\right)}.$$

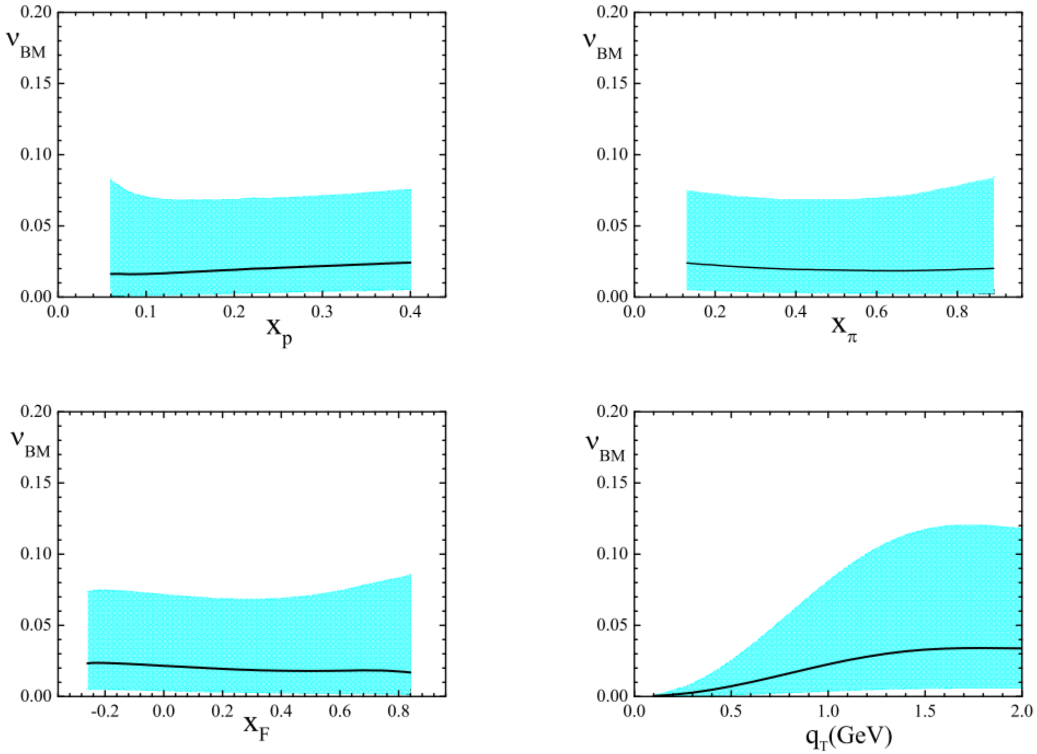
$$\delta^2(\boldsymbol{k}_T+\boldsymbol{p}_T-\boldsymbol{q}_T)=\int\frac{d^2\boldsymbol{b}}{(2\pi)^2}e^{-i\boldsymbol{b}\cdot(\boldsymbol{k}_T+\boldsymbol{p}_T-\boldsymbol{q}_T)}$$

$$\tilde{h}_1^{\alpha\perp}(x,\boldsymbol{b})=\int d^2\boldsymbol{k}_T e^{-i\boldsymbol{b}\cdot\boldsymbol{k}_T}\frac{\boldsymbol{k}_T^\alpha}{M}h_1^{\perp}(x,\boldsymbol{k}_T)$$

# cos2φ asymmetry in pi-N Drell-Yan

- Boer-Mulders asymmetry at COMPASS

Wang , Mao, Lu, EPJC78, 643 (2018)




Boer-Mulders of the proton adopted from PRD81,034023

Bands correspond to the Uncertainty of the proton BM

# Summary

- The Sudakov form factor for the unpolairzed TMD of the pion is extracted for the first time from the E615 DY data within the TMD factorization incorporating TMD evolution.

- The transvere momentum spectrum of the dilepton agrees with the COMPASS measurement at small q_T region, indicating that our approach can be used a first step for precision study of pion-nucleon DY.

- The Sivers asymmetry calculated from the TMD evolution formalism is qualitatitvely consistent with the data at COMPASS. Prediction on the cos2phi asymmetry from the Boer-Mulders function is also presented.

- The framework applied here can be also extended to the study of the other azimuthal asymmetries in the pion-nucleon DY.

# THANK YOU!