SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

CPHI 2018

24-28 September 2018, Yerevan

“Novel measurements of quark fragmentation functions in e^+e^-”

Հրայր Մաթևոսյան

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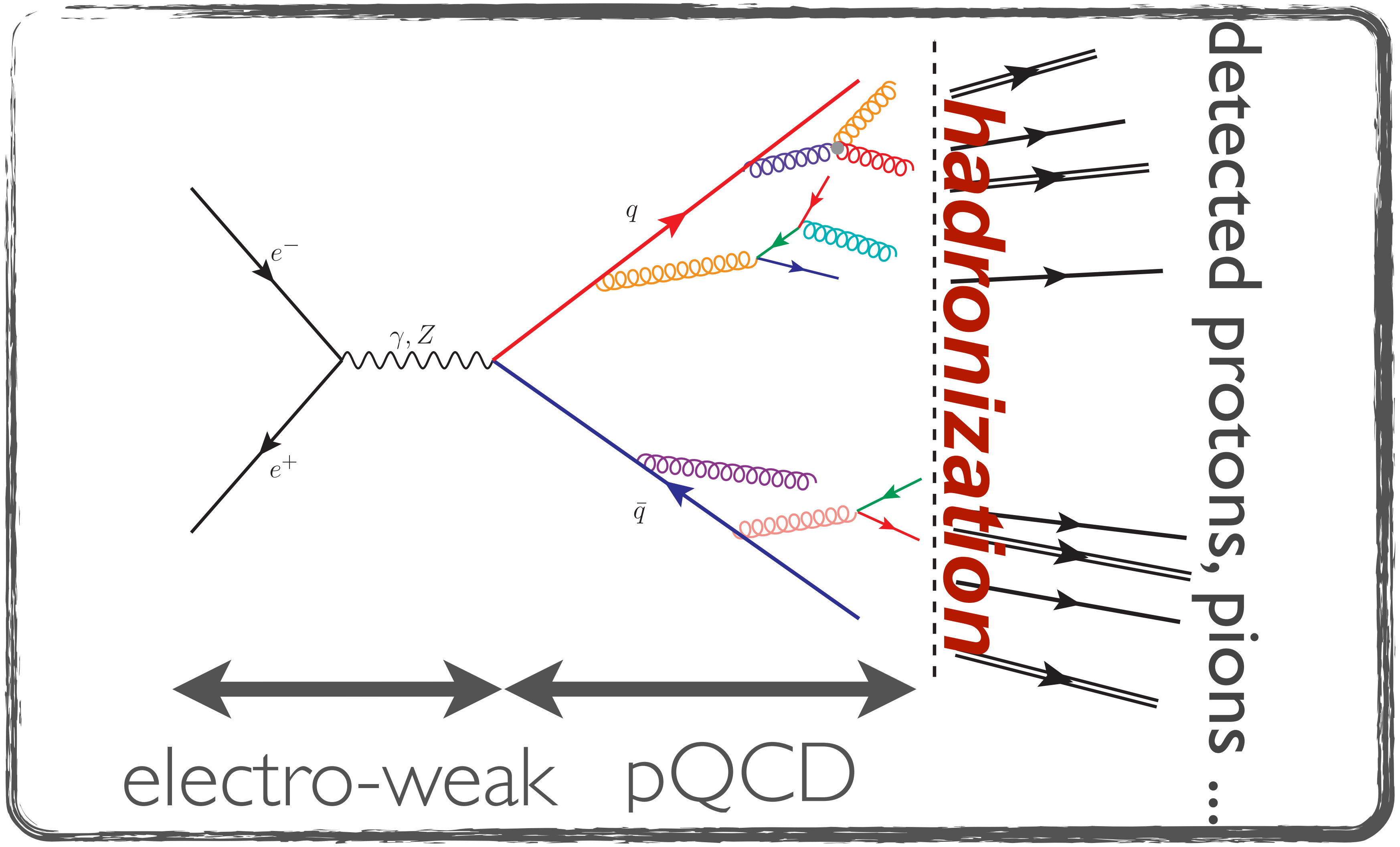
THE UNIVERSITY of ADELAIDE

SUB CRUCE LUMEN

Hadronization: $e^- e^+ \to hX$

- **The conjecture of Confinement:**

✦ ***NO free quarks or gluons have been directly observed: only* HADRONS.**



✦ **Hadronization:** describes the process where colored quarks and gluons form colorless hadrons (in deep inelastic scattering).

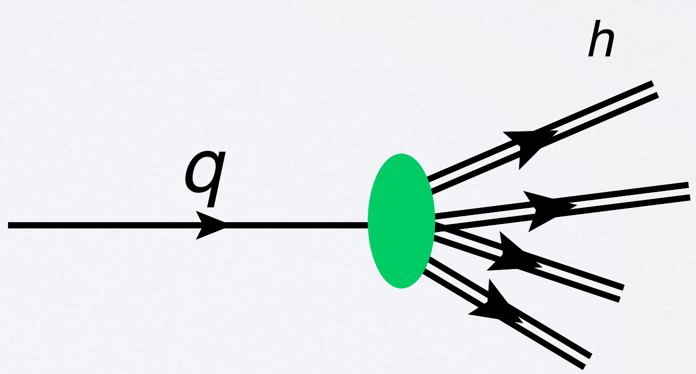
Fragmentation Functions

- The non-perturbative, universal functions encoding parton hadronization are the: *Fragmentation Functions (FF)*.

$$\frac{1}{\sigma}\frac{d}{dz}\sigma(e^-e^+ \to hX) = \sum_i \mathcal{C}_i(z,Q^2) \otimes D_i^h(z,Q^2)$$

- Unpolarized FF is the number density for parton *i* to produce hadron *h* with LC momentum fraction **z**.

$$D_i^h(z,Q^2)$$



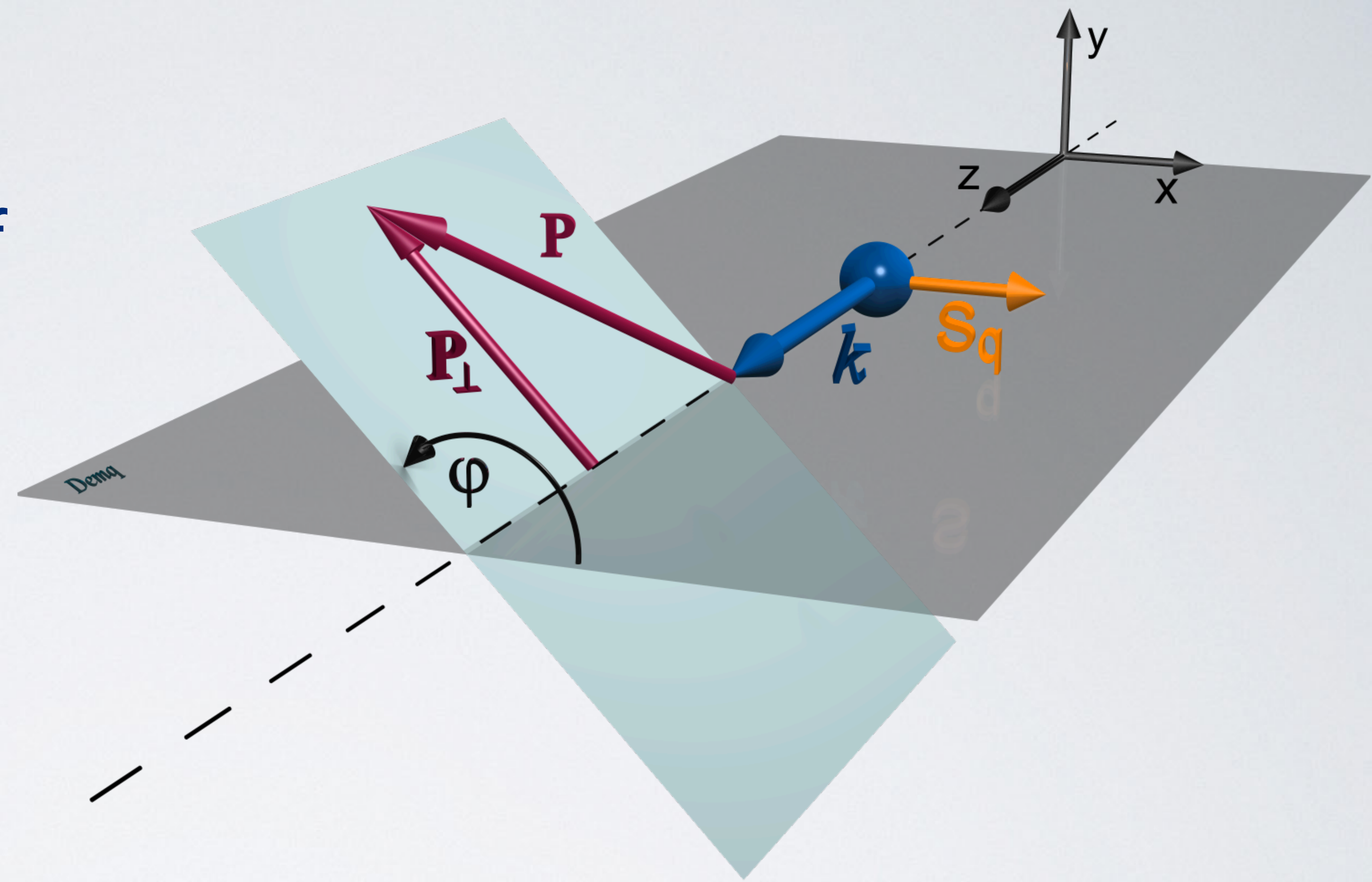
- **z** is the light-cone mom. fraction of the parton carried by the hadron

$$z = \frac{p^-}{k^-} \approx z_h = \frac{2E_h}{Q} \qquad a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$$

COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarize

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

Collins

- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

TMD FFS AND PDFS

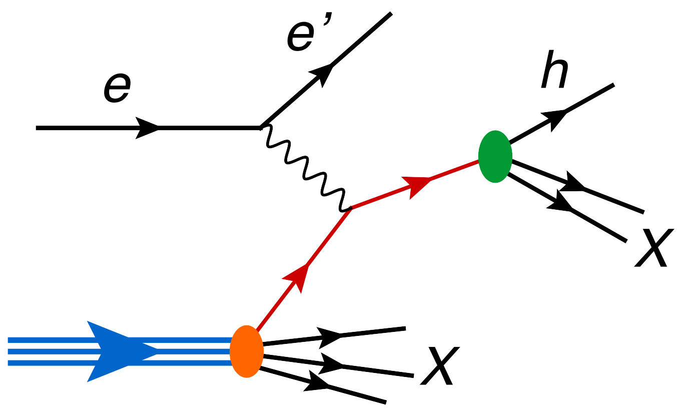
❖ ***Leading Order TMD FFs***

h/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

❖ ***Leading Order TMD PDFs***

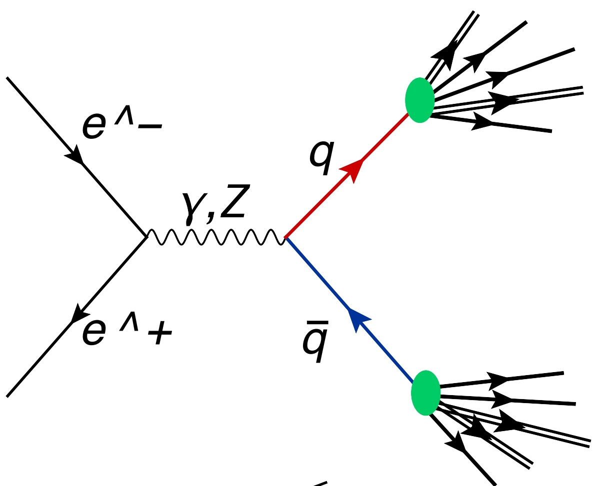
N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\, h_{1T}^\perp$

FACTORIZATION AND ***UNIVERSALITY***



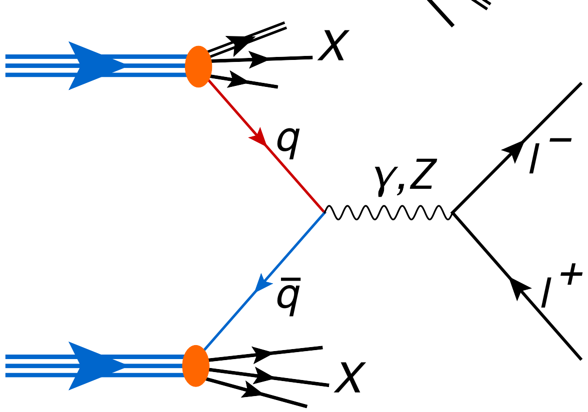
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h$$



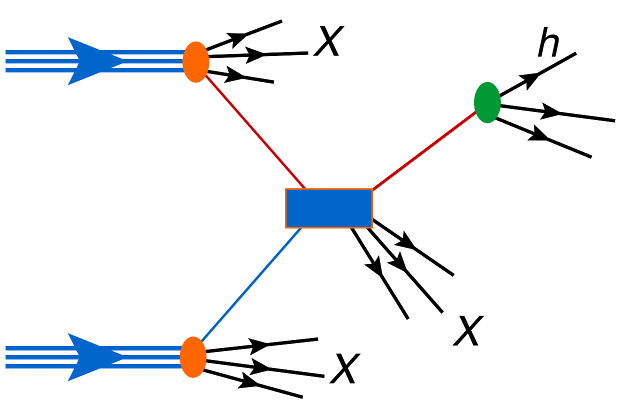
- e^+e^-

$$\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



- DRELL-YAN (DY)

$$\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}$$



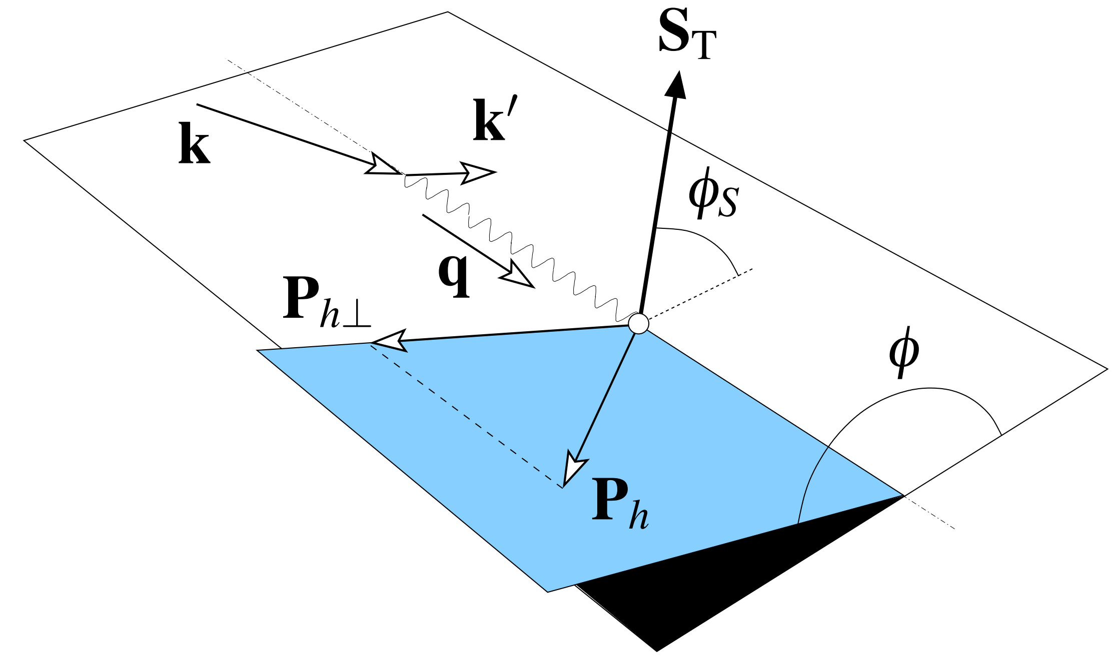
- Hadron Production

$$\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h$$

TMDs from SIDIS $e\,N \rightarrow e\,h\,X$

A. Bacchetta et al., JHEP08 023 (2008).

- **For polarized SIDIS cross-section there are *18 terms* in leading twist expansion (unpolarized final hadron):**



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \ldots$$

$$+ |\boldsymbol{S}_\perp| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon\, \sin(\phi_h + \phi_S)\, F_{UT}^{\sin(\phi_h + \phi_S)} + .. \right]$$

Sivers Effect

Collins Effect

- Access the structure functions via *specific* modulations.
- LO Matching to *convolutions* of PDFs and FFs: $P_T^2 \ll Q^2$

$$F_{UU,T} \sim \mathcal{C}[f_1\ D_1] \qquad F_{UT,T}^{\sin(\phi_h - \phi_S)} \sim \mathcal{C}[f_{1T}^{\perp q}\ D_1]$$

$$F_{UU}^{\cos(2\phi_h)} \sim \mathcal{C}[\mathcal{G}(\vec{k}_T, \vec{P}_T)\ h_1^\perp\ H_1^\perp] \qquad F_{UT}^{\sin(\phi_h + \phi_S)} \sim \mathcal{C}[h_1\ H_1^\perp]$$

TMDs from SIDIS $e\,N \rightarrow e\,h\,X$

D. Boer et al., NPB 564, 471-485 (2000).

- For polarized target *and polarized final hadron* SIDIS cross-section contains ~*30 terms !*

- ***Unpolarized target and beam:***

$$
\begin{aligned}
L_{\mu\nu} W_U^{\mu\nu} = \sum_a e_a^2 A(y) \mathcal{F}\left[f_1^a D_1^{a\to\Lambda} \right] \\
- |\boldsymbol{S}_{\Lambda T}| A(y) \mathcal{F}\left[\frac{|\boldsymbol{p}_T| \sin(\varphi_p - \varphi_{S_\Lambda})}{M_\Lambda} f_1^a D_{1T}^{\perp,a\to\Lambda} \right] \\
+ |\boldsymbol{S}_{\Lambda T}| B(y) \mathcal{F}\left[\frac{|\boldsymbol{k}_T| \sin(\varphi_k + \varphi_{S_\Lambda})}{M_N} h_1^{\perp,a} \left(H_{1T}^{a\to\Lambda} + \frac{|\boldsymbol{p}_T|^2}{2M_\Lambda^2} H_{1T}^{\perp,a\to\Lambda} \right) \right] \\
+ |\boldsymbol{S}_{\Lambda T}| B(y) \mathcal{F}\left[\frac{|\boldsymbol{p}_T|^2 |\boldsymbol{k}_T| \sin(\varphi_k + 2\varphi_p - \varphi_{S_\Lambda})}{2M_\Lambda^2 M_N} h_1^{\perp,a} H_{1T}^{\perp,a\to\Lambda} \right] \\
- B(y) \mathcal{F}\left[\frac{|\boldsymbol{p}_T||\boldsymbol{k}_T| \cos(\varphi_k + \varphi_p)}{M_\Lambda M_N} h_1^{\perp,a} H_1^{\perp,a\to\Lambda} \right] \\
+ \lambda_\Lambda B(y) \mathcal{F}\left[\frac{|\boldsymbol{p}_T||\boldsymbol{k}_T| \sin(\varphi_k + \varphi_p)}{M_\Lambda M_N} h_1^{\perp,a} H_{1L}^{\perp,a\to\Lambda} \right]
\end{aligned}
$$

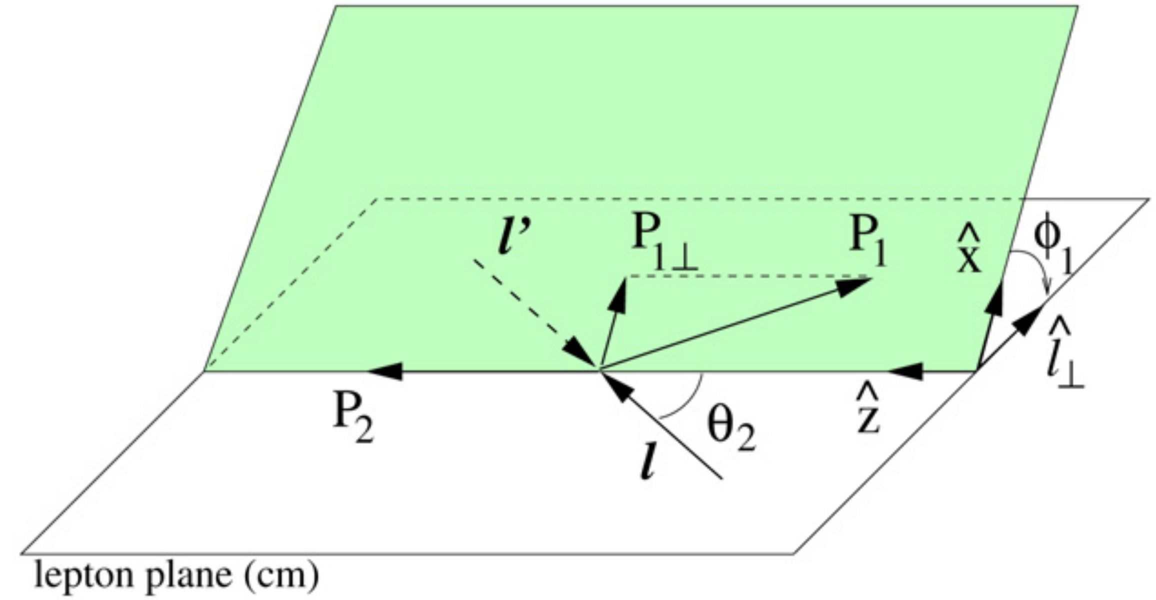
TMD FFs from e⁺e⁻

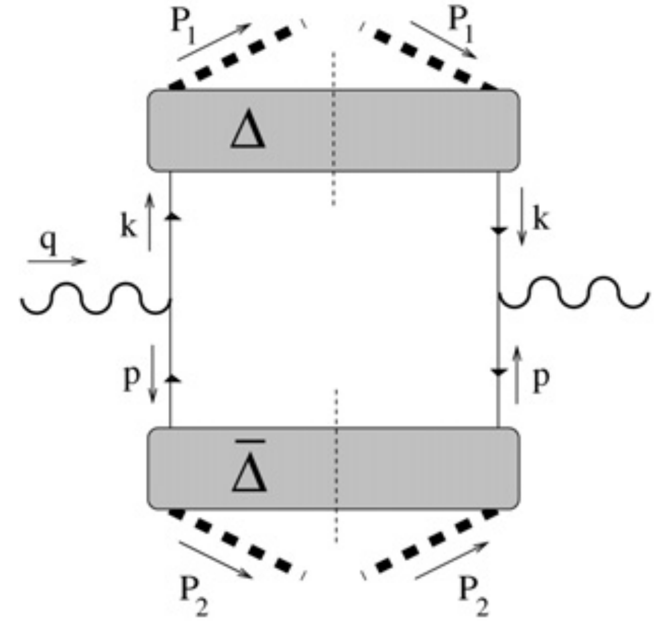
D. Boer et al., NPB 504, 345-380 (1997).

✦ Two back-to-back hadrons in 2-jet events:

$$e^+ + e^- \to h_1 + h_2 + X$$

✦ For polarized final state hadrons there are **28 terms** in the leading twist expansion.





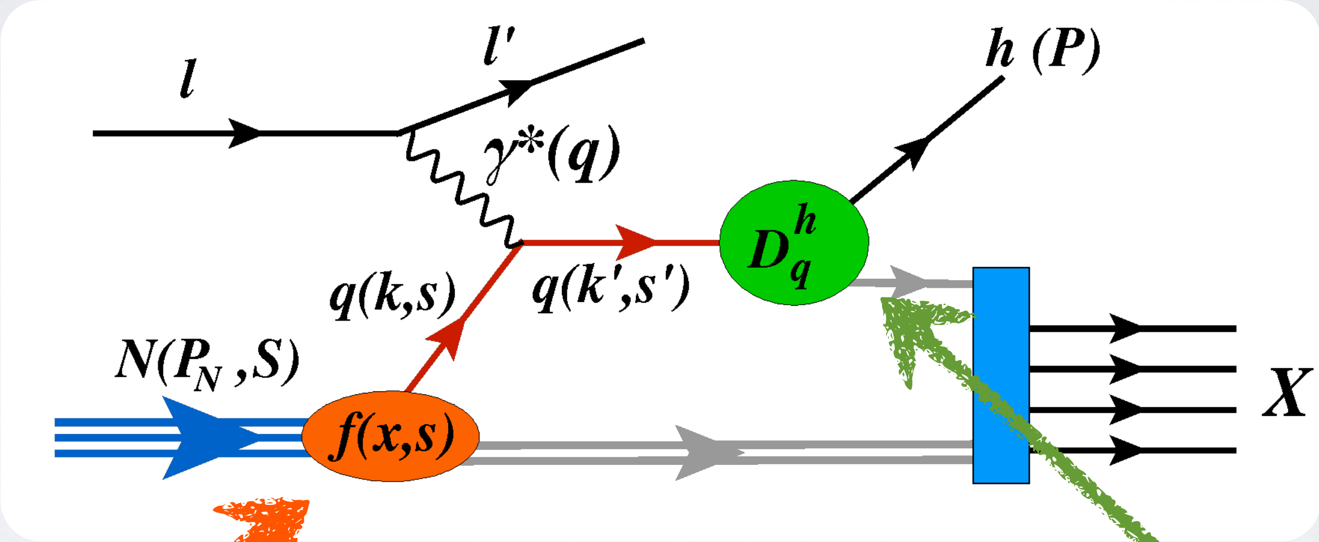
- ***Unpolarized final hadrons:***

$$d\sigma\left(e^+e^- \to h_1 + h_2 + X\right)$$
$$\sim \sum_a e_a^2 \left\{ A(y)\ \mathcal{F}\left[D_1^{a\to h_1} D_1^{\bar{a}\to h_2}\right] \right.$$
$$\left. + B(y)\mathcal{F}\left[\frac{k_T \bar{k}_T}{M_{h_1} M_{h_2}} \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp,a\to h_1} H_1^{\perp,\bar{a}\to h_2}\right]\right\},$$

DIHADRON FRAGMENTATION FUNCTIONS

SIDIS with <u>*one*</u> measured hadron

- ***Measurement of the transverse momentum of the produced hadron in SIDIS provides access to TMD PDFs/FFs.***
  - ***SIDIS Process with TM of hadron measured.***



- ***TMD PDFs***

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

- ***TMD FFs***

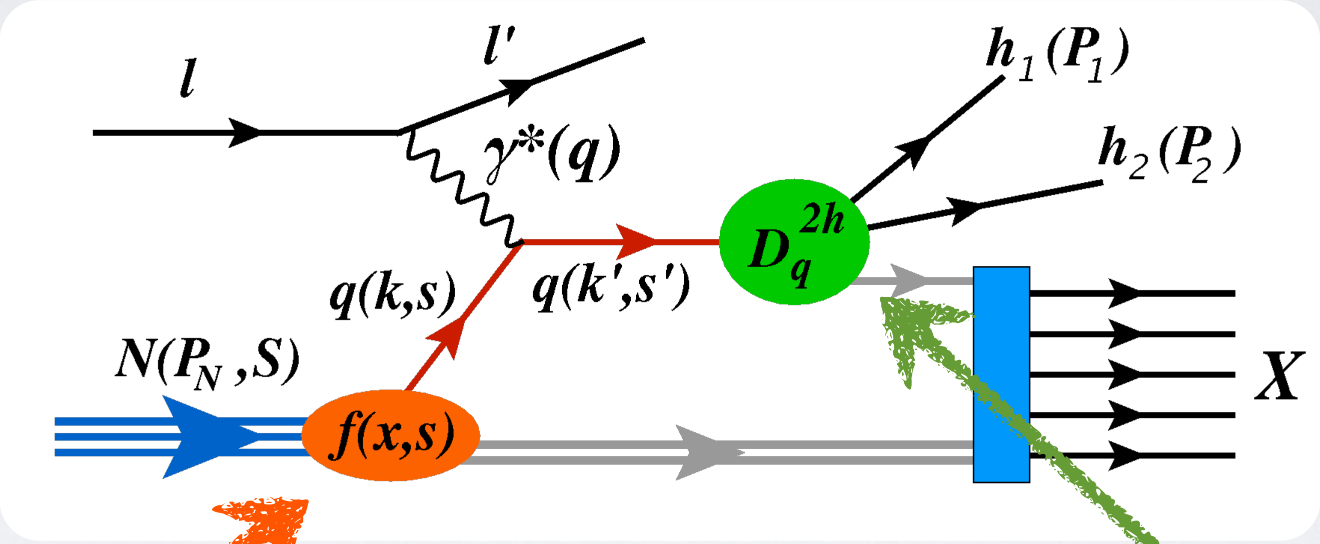
q/h	U
U	D_1
L	
T	$H_1^\perp$

*unpol/spinless h!

SIDIS with two measured

- *Measuring **two-hadron** semi-inclusive DIS: an additional method for accessing TMD PDFs.*

  - ***SIDIS Process with TM of hadrons measured.***



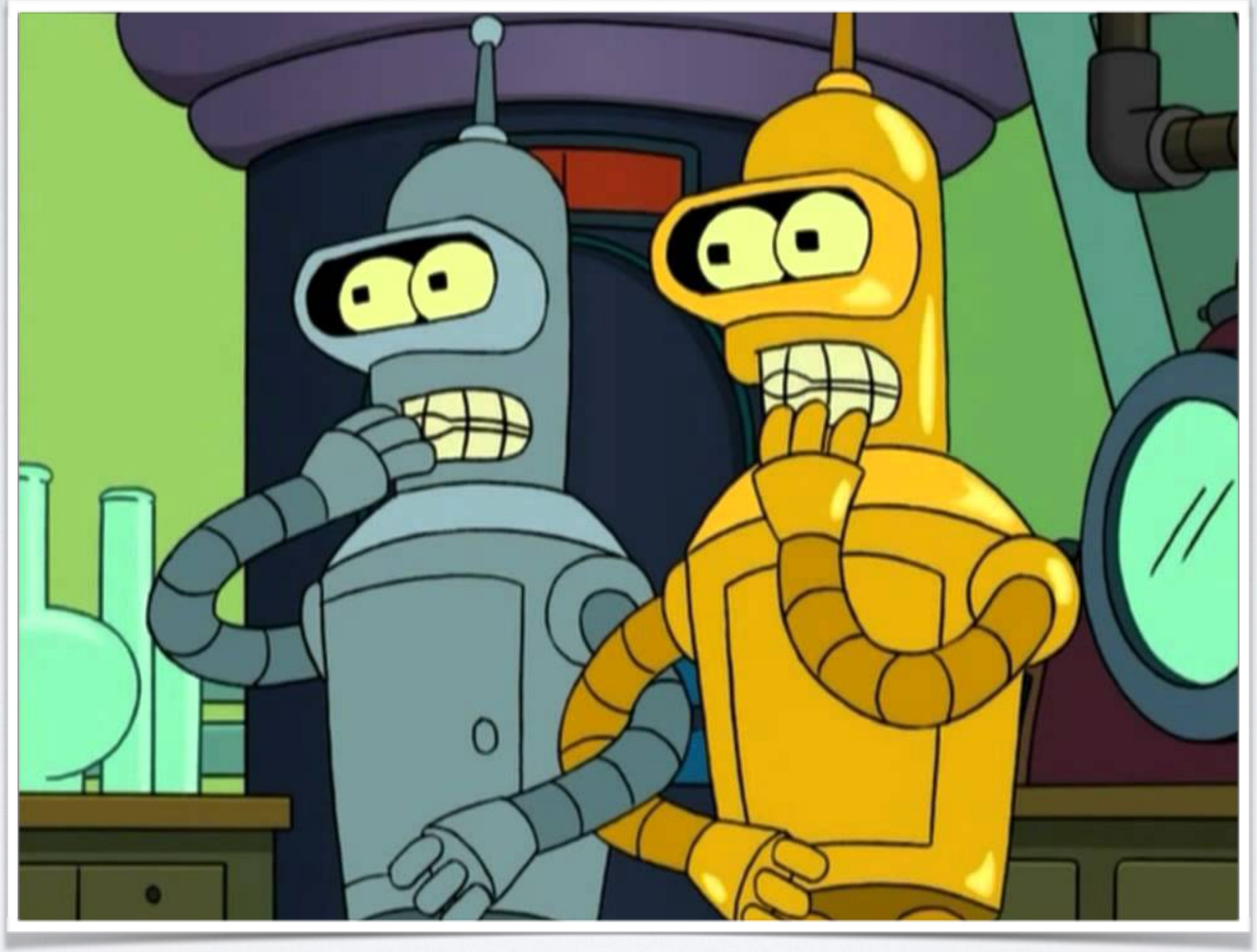
- ***TMD PDFs***

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

- ***TMD DiFFs***

q/h_1h_2	U
U	D_1
L	$G_1^\perp$
T	$H_1^\perp \quad H_1^\sphericalangle$

*unpol/spinless h!



UNIVERSALITY OF FRAGMENTATIONS

Universality of FFs: e^+e^- and SIDIS

- ***Universality was proven explicitly for <u>all</u> the TMD FFs [Gamberg et. al, PRD.83, 071503 (2011)]***

$$D_{1T}^{\perp SIDIS} = D_{1T}^{\perp e^+e^-} \qquad H_1^{\perp SIDIS} = H_1^{\perp e^+e^-}$$

- ***Similar arguments should apply in the case of DiFFs.***

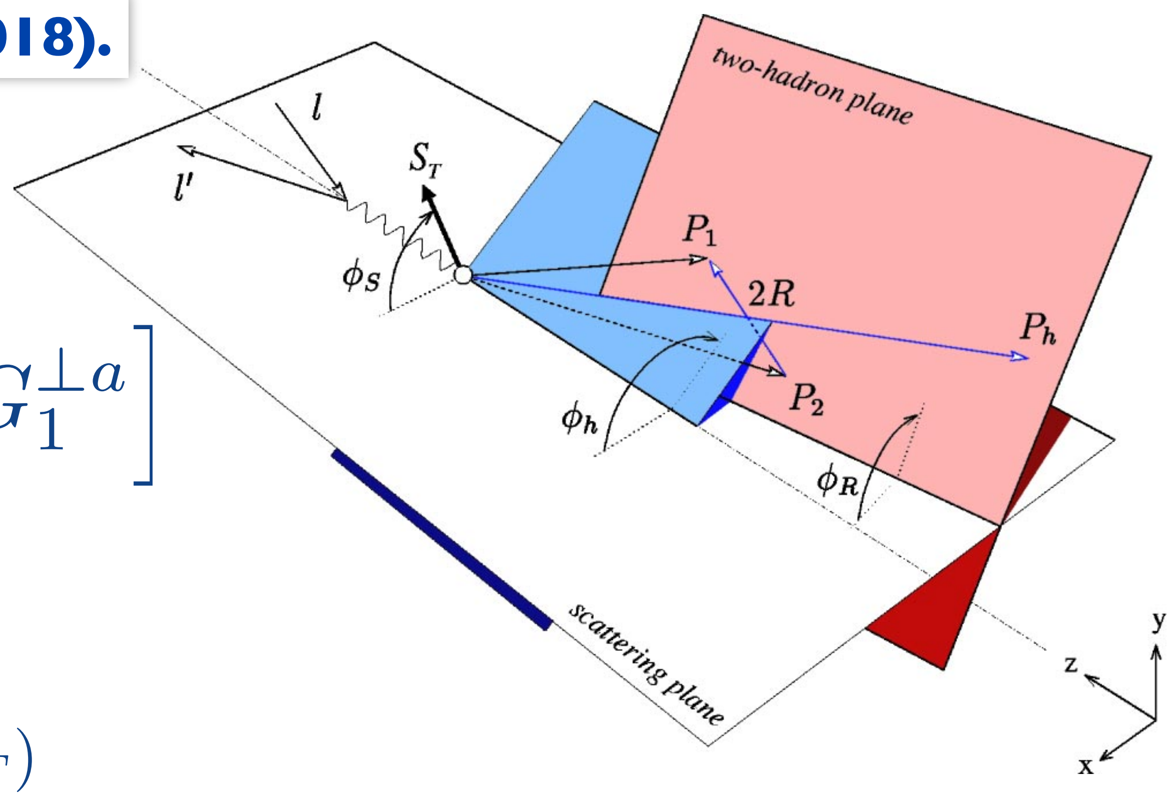
$$H_1^{\sphericalangle SIDIS} = H_1^{\sphericalangle e^+e^-} \qquad G_1^{\perp SIDIS} = G_1^{\perp e^+e^-}$$

- ***Naive-time-reversal-odd Sivers and Boer-Mulders PDFs are predicted to change sign from SIDIS to Drell-Yan [Collins, PLB. 536, 43 (2002)]***

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY} \qquad h_1^{\perp SIDIS} = -h_1^{\perp DY}$$

Accessing $G_1^\perp$ DiFF in *SIDIS*

H.M., Kotzinian, Thomas: PRL. 120 no.25, 252001 (2018).



- *The relevant terms involving* $G_1^\perp$ *:*

$$d\sigma_{UL} \sim S_L \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\; g_{1L}^a\; G_1^{\perp a}\Big]$$

$$\mathcal{G}[w f^q D^q] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{k}_T - \boldsymbol{p}_T + \frac{\boldsymbol{P}_{h\perp}}{z}\Big) \times w(\boldsymbol{p}_T, \boldsymbol{k}_T, \boldsymbol{R}_T) f^q(x, \boldsymbol{p}_T^2) D^q(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

- ***Weighted moment accesses same*** $G_1^\perp$ ***as in e⁺e⁻ .***

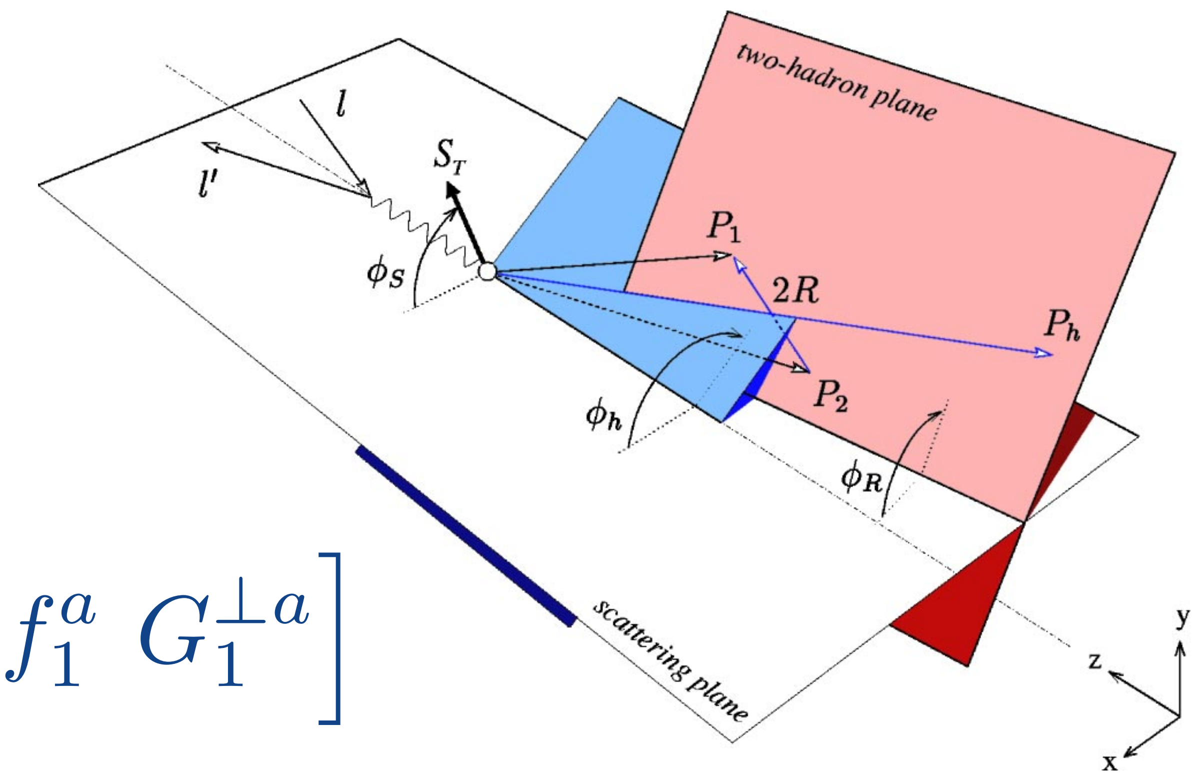
$$\Big\langle \frac{P_{h\perp}\sin(\varphi_h - \varphi_R)}{M_h} \Big\rangle_{UL} \sim S_L \sum_a e_a^2\; g_{1L}^a(x)\; z\; G_1^{\perp a}(z, M_h^2)$$

$$A^{\rightrightarrows}_{SIDIS}(x, z, M_h^2) = S_L \frac{\sum_a g_{1L}^a(x)\;\; z\; G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x)\;\; D_1^a(z, M_h^2)}.$$

Accessing $G_1^\perp$ DiFF in *SIDIS: II*

H.M.: arXiv:1807.11485. POS DIS2018.

- ***The relevant terms involving $G_1^\perp$:***



Consider a polarized beam.

$$d\sigma_{LU} \sim \lambda_e \mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\; f_1^a\; G_1^{\perp a}\Big]$$

- ***Weighted moment accesses same $G_1^\perp$ as in e^+e^- .***

$$\left\langle \frac{P_{h\perp}\sin(\varphi_h - \varphi_R)}{M_h} \right\rangle_{LU} \sim \lambda_e \sum_a e_a^2\; f_1^a(x)\; z\; G_1^{\perp a}(z, M_h^2)$$

$$A_{SIDIS}^{\hookrightarrow}(x, z, M_h^2) \sim \lambda_e \frac{C'(y)}{A'(y)} \frac{\sum_a f_1^a(x)\; z\; G_1^{\perp a}(z, M_h^2)}{\sum_a f_1^a(x)\; D_1^a(z, M_h^2)}.$$

Accessing $G_1^\perp$ DiFF in e^+e^-

H.M., Kotzinian, Thomas: PRL. 120 no.25, 252001 (2018).

- ***The relevant terms involving $G_1^\perp$:***

$$d\sigma_L \sim \mathcal{F}\left[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\right]$$

- ***Need a q_T-weighted asymmetry to get non-zero result***

$$\left\langle \frac{q_T^2\left(3\sin(\varphi_q-\varphi_R)\sin(\varphi_q-\varphi_{\bar{R}})+\cos(\varphi_q-\varphi_R)\cos(\varphi_q-\varphi_{\bar{R}})\right)}{M_h\bar{M}_h}\right\rangle$$

$$= \frac{12\alpha^2 A(y)}{\pi Q^2}\sum_{a,\bar{a}} e_a^2\left(G_1^{\perp a,[0]} - G_1^{\perp a,[2]}\right)\left(\bar{G}_1^{\perp \bar{a},[0]} - G_1^{\perp \bar{a},[2]}\right),$$

- ***A new asymmetry to access*** $G_1^{\perp a} \equiv G_1^{\perp a,[0]} - G_1^{\perp a,[2]}$

$$A_{e^+e^-}^{\Rightarrow}(z,\bar{z}, M_h^2, \bar{M}_h^2) = 4\frac{\sum_{a,\bar{a}} G_1^{\perp a}(z, M_h^2)\ G_1^{\perp \bar{a}}(\bar{z}, \bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^{a}(z, M_h^2)\ D_1^{\bar{a}}(\bar{z}, \bar{M}_h^2)}$$

Accessing $G_1^{\perp}$ DiFF in e$^+$e$^-$

H.M., Kotzinian, Thomas: PRL. 120 no.25, 252001 (2018).

- ***The relevant terms involving $G_1^{\perp}$:***

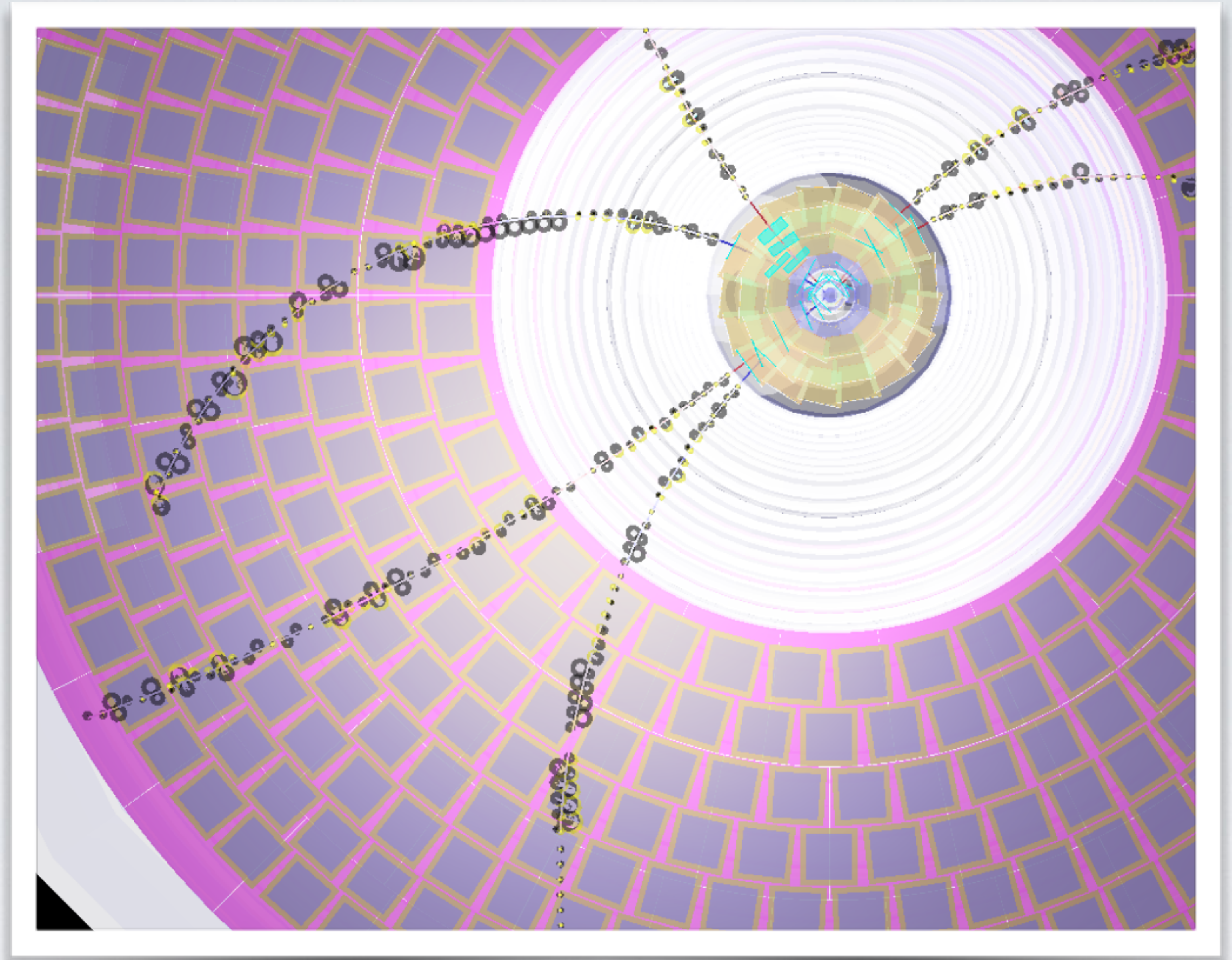
$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T \times \boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T \times \bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2} G_1^{\perp a}(\boldsymbol{R}_T \cdot \boldsymbol{k}_T)\bar{G}_1^{\perp \bar{a}}(\bar{\boldsymbol{R}}_T \cdot \bar{\boldsymbol{k}}_T)\Big]$$

- ***Need a q_T-weighted asymmetry to get non-zero result***

Still cannot test the sign !

- ***A new asymmetry to access*** $G_1^{\perp a} \equiv G_1^{\perp a,[0]} - G_1^{\perp a,[2]}$

$$A^{\Rightarrow}_{e^+e^-}(z,\bar{z}, M_h^2, \bar{M}_h^2) = 4\frac{\sum_{a,\bar{a}} G_1^{\perp a}(z, M_h^2)\ G_1^{\perp \bar{a}}(\bar{z}, \bar{M}_h^2)}{\sum_{a,\bar{a}} D_1^{a}(z, M_h^2)\ D_1^{\bar{a}}(\bar{z}, \bar{M}_h^2)}$$



A NEW MEASUREMENT

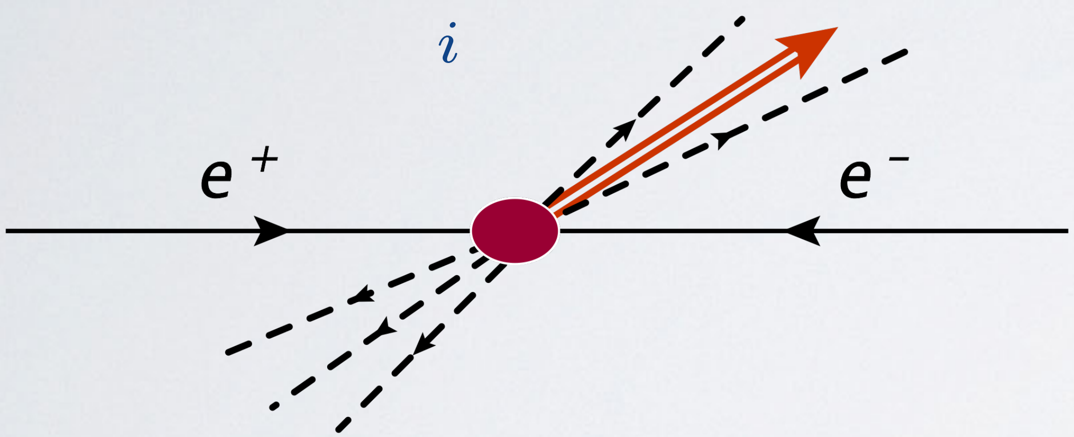
in e^+e^-

FRAGMENTATIONS FROM e+e-

FF

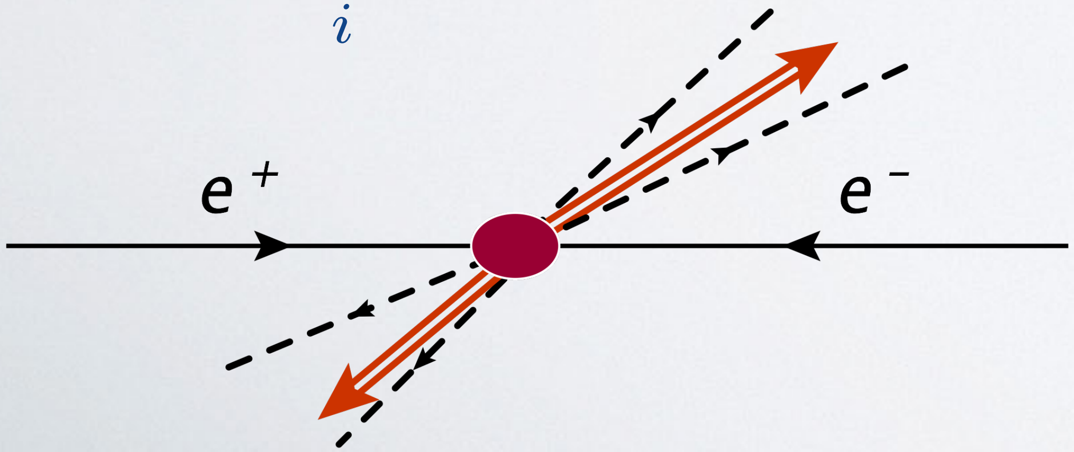
❖ inclusive hadron

$$\sum_i D_i^h$$



❖ back-to-back hadrons

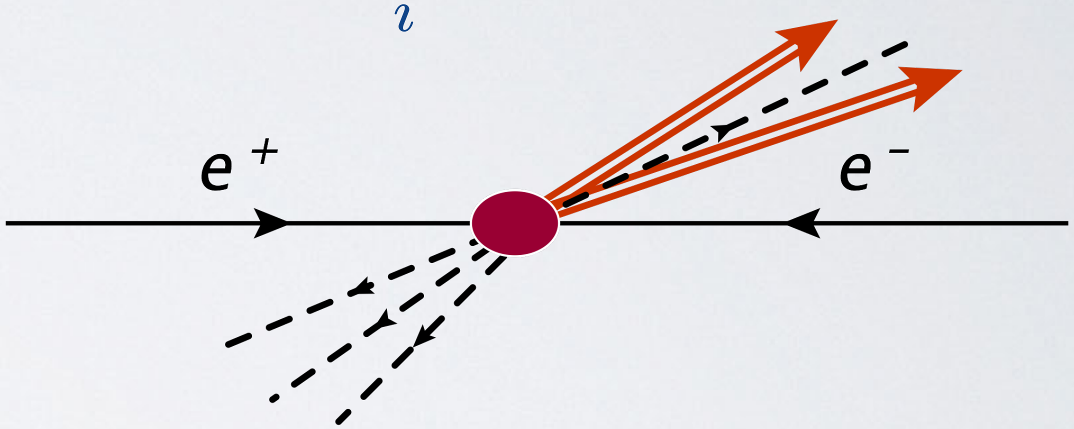
$$\sum_i D_i^{h_1} \otimes D_{\bar{i}}^{\bar{h}_1}$$



DiFF

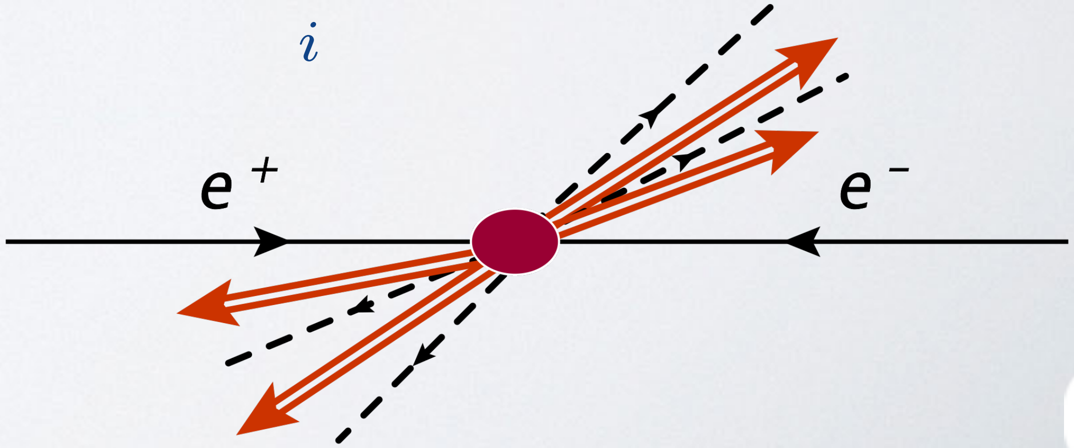
❖ inclusive hadron pair

$$\sum_i D_i^{h_1 h_2}$$



❖ back-to-back hadron pairs

$$\sum_i D_i^{h_1 h_2} \otimes D_{\bar{i}}^{\bar{h}_1 \bar{h}_2}$$

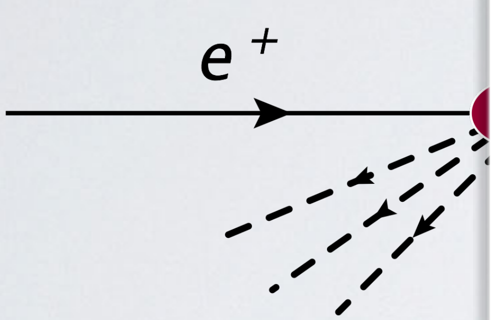


FRAGMENTATIONS FROM e+e-

FF

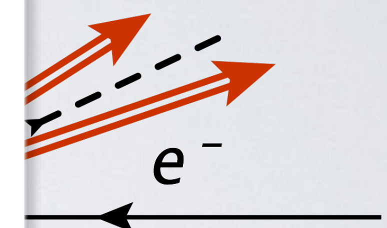
❖ inclusive hadron

$\sum_i$



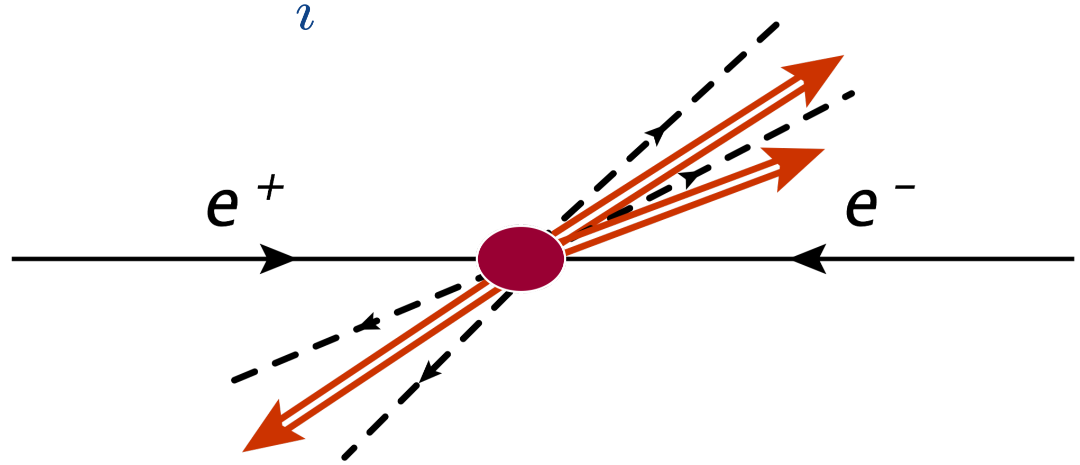
DiFF

❖ inclusive hadron pair



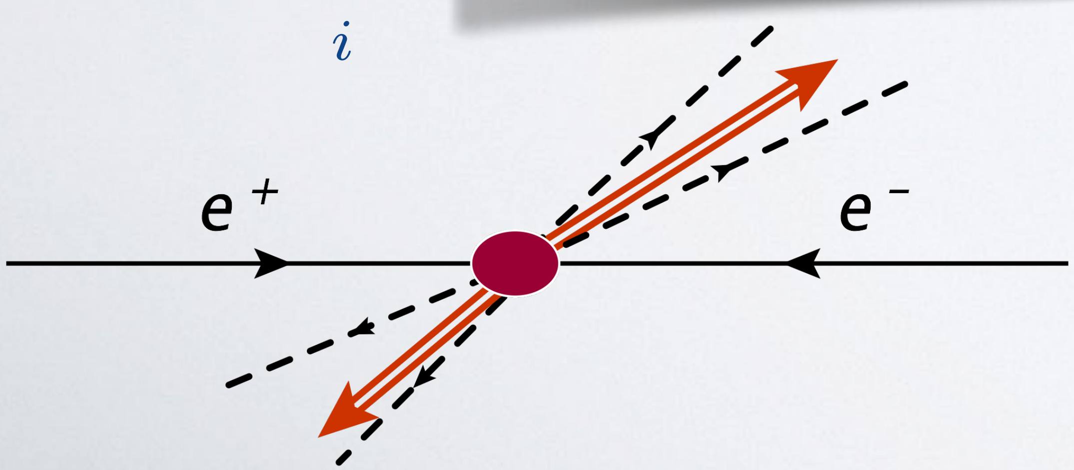
❖ back-to-back hadron pair and a hadron

$$\sum_i D_i^{h_1 h_2} \otimes D_{\bar{i}}^{\bar{h}_1}$$



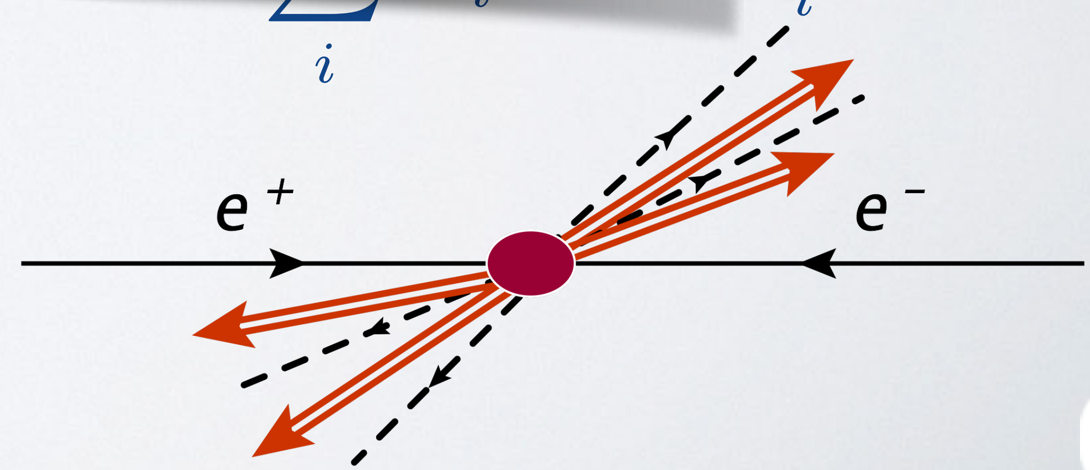
❖ back-to-bac

$\sum_i D$

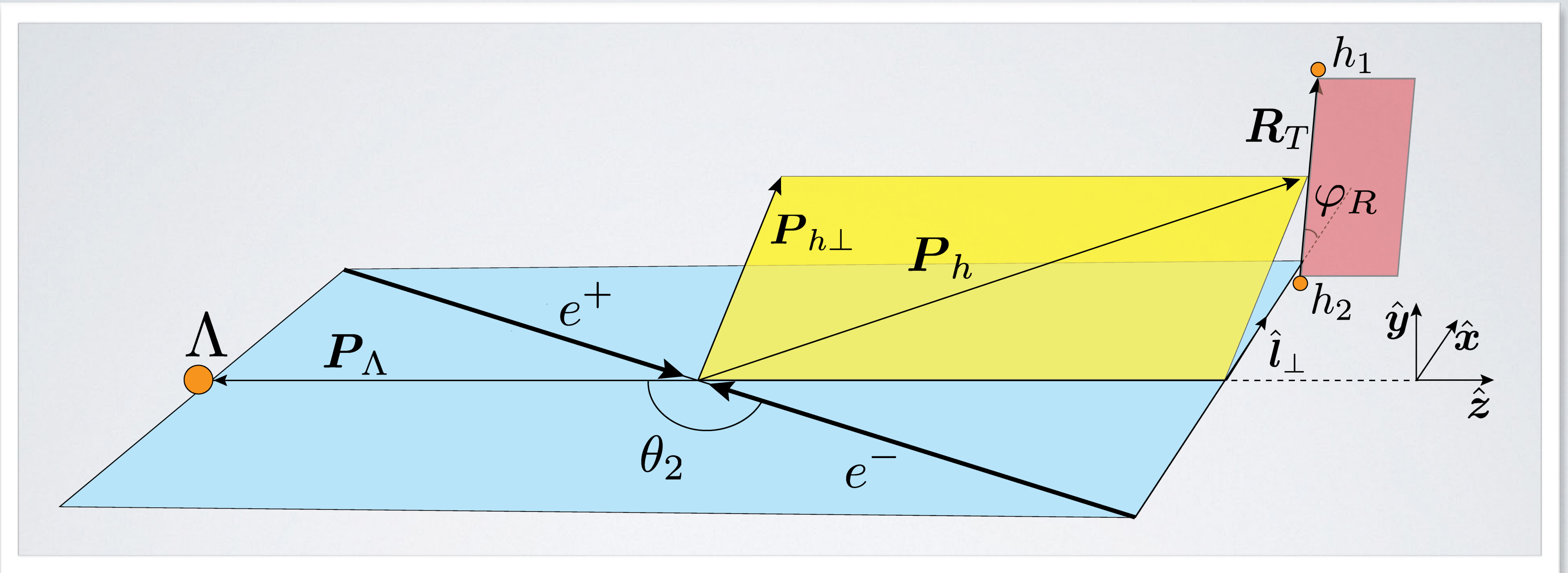


on pairs

$D_{\bar{i}}^{\bar{h}_1 \bar{h}_2}$



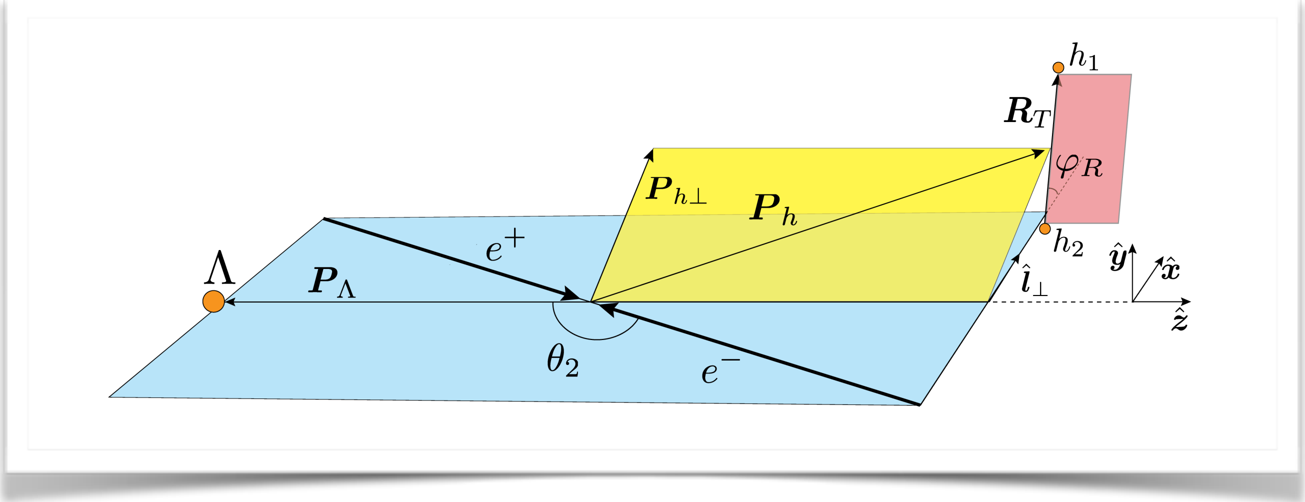
The "usual" kinematics in e^+e^-



The Cross Section

H.M. , Kotzinian, Thomas: arXiv:1808.00954. (in press JHEP)

- *Use the standard kinematics to derive LO x-sec.*



$$
\begin{aligned}
\frac{d\sigma\left(e^+e^- \to (h_1h_2)+\Lambda+X\right)}{d^2\boldsymbol{q}_T\; dz\; d\varphi_R\; dM_h^2\; d\xi\; d\bar{z}\; dy} &= \frac{3\alpha_{em}^2}{(2\pi)^2Q^2} z^2\bar{z}^2 \sum_a e_a^2 \\
\times \Bigg\{ \quad & A(y)\; \mathcal{F}\left[D_1^{a\to h_1h_2} D_1^{\bar{a}\to\Lambda}\right] \\
& - S_T A(y)\; \mathcal{F}\left[\frac{\bar{k}_T}{M_\Lambda}\sin(\varphi_{\bar{k}}-\varphi_S)\; D_1^{a\to h_1h_2} D_{1T}^{\perp,\bar{a}\to\Lambda}\right] \\
& + \lambda_\Lambda A(y)\; \mathcal{F}\left[\frac{k_TR_T}{M_h^2}\sin(\varphi_k-\varphi_R)\; G_1^{\perp,a\to h_1h_2} G_{1L}^{\bar{a}\to\Lambda}\right] \\
& + S_T A(y)\; \mathcal{F}\left[\frac{k_TR_T}{M_h^2}\sin(\varphi_k-\varphi_R)\frac{\bar{k}_T}{M_\Lambda}\cos(\varphi_{\bar{k}}-\varphi_S)\; G_1^{\perp,a\to h_1h_2} G_{1T}^{\bar{a}\to\Lambda}\right] \\
& + S_T B(y)\; \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\sin(\varphi_k+\varphi_S) H_1^{\perp,a\to h_1h_2} \\
& \qquad + \frac{R_T}{M_h}\sin(\varphi_R+\varphi_S) H_1^{\sphericalangle,a\to h_1h_2}\Bigg)\; H_{1T}^{\bar{a}\to\Lambda}\Bigg] \\
& + \lambda_\Lambda B(y)\; \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\sin(\varphi_k+\varphi_{\bar{k}}) H_1^{\perp,a\to h_1h_2} \\
& \qquad + \frac{R_T}{M_h}\sin(\varphi_R+\varphi_{\bar{k}}) H_1^{\sphericalangle,a\to h_1h_2}\Bigg)\; \frac{\bar{k}_T}{M_\Lambda} H_{1L}^{\perp,\bar{a}\to\Lambda}\Bigg] \\
& + S_T B(y)\; \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\sin(\varphi_k+\varphi_{\bar{k}}) H_1^{\perp,a\to h_1h_2} \\
& \qquad + \frac{R_T}{M_h}\sin(\varphi_R+\varphi_{\bar{k}}) H_1^{\sphericalangle,a\to h_1h_2}\Bigg)\; \frac{\bar{k}_T^2}{M_\Lambda^2}\cos(\varphi_{\bar{k}}-\varphi_S) H_{1T}^{\perp,\bar{a}\to\Lambda}\Bigg] \\
& + \quad B(y)\; \mathcal{F}\Bigg[\Bigg(\frac{k_T}{M_h}\cos(\varphi_k+\varphi_{\bar{k}}) H_1^{\perp,a\to h_1h_2} \\
& \qquad + \frac{R_T}{M_h}\cos(\varphi_R+\varphi_{\bar{k}}) H_1^{\sphericalangle,a\to h_1h_2}\Bigg)\; \frac{\bar{k}_T}{M_\Lambda} H_1^{\perp,\bar{a}\to\Lambda}\Bigg] \quad \Bigg\},
\end{aligned}
$$

Flavor Decomposition of DiFFs

❖ Integrated cross section

$$\frac{d\sigma\Big(e^+e^- \to (h_1h_2) + \Lambda + X\Big)}{dz\ dM_h^2\ d\bar{z}\ dy} = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} A(y) \sum_a e_a^2\ D_1^{a\to h_1h_2}(z, M_h^2)\ \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z}),$$

❖ Isospin symmetry

$$D_1^{u\to\pi^+\pi^-} = D_1^{\bar{u}\to\pi^+\pi^-} \approx D_1^{d\to\pi^+\pi^-} = D_1^{\bar{d}\to\pi^+\pi^-},$$

$$D_1^{s\to\pi^+\pi^-} = D_1^{\bar{s}\to\pi^+\pi^-}.$$

❖ One pair inclusive: cannot disentangle the flavor dependence

$$d\sigma(e^+e^- \to (h_1h_2) + X) \sim \sum_q e_q^2\ D_1^{q\to\pi^+\pi^-} \approx \frac{5}{9} D_1^{u\to\pi^+\pi^-}(z) + \frac{1}{9} D_1^{s\to\pi^+\pi^-}(z)$$

❖ New process: use the knowledge of single hadron FFs!

$$d\sigma(e^+e^- \to (h_1h_2) + \pi^+ + X) \sim \frac{5}{9} D_1^{u\to\pi^+\pi^-}(z) D_1^{u^+\to\pi^+}(\bar{z}) + \frac{1}{9} D_1^{s\to\pi^+\pi^-}(z) D_1^{s^+\to\pi^+}(\bar{z}),$$

$$D_1^{q^+\to h}(\bar{z}) \equiv D_1^{q\to h}(\bar{z}) + D_1^{\bar{q}\to h}(\bar{z}).$$

Weighted Asymmetries.

❖ Unpolarized hadrons: Accessing Collins x IFF.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} \frac{B(y)}{M_\Lambda^2 M_h}$$
$$\times \sum_a e_a^2 \int d\xi \int d\varphi_R \int d^2\boldsymbol{q}_T \int d^2\boldsymbol{k}_T \int d^2\bar{\boldsymbol{k}}_T \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) q_T \cos(\varphi_q + \varphi_R)$$
$$\times \left[\left(k_T \bar{k}_T \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp, a\to h_1 h_2} + R_T \bar{k}_T \cos(\varphi_R + \varphi_{\bar{k}}) H_1^{\sphericalangle, a\to h_1 h_2} \right) H_1^{\perp, \bar{a}\to\Lambda} \right],$$

❖ Momentum weighing helps to disentangle TM convolutions.

$$\int d^2\boldsymbol{q}_T \; \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) \; q_T \cos(\varphi_q + \varphi_R) = (k_T \cos(\varphi_k + \varphi_R) + \bar{k}_T \cos(\varphi_{\bar{k}} + \varphi_R)).$$

❖ Resulting moment and the asymmetry.

$$\left\langle \frac{q_T}{M_\Lambda} \cos(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\perp \bar{a},[1]}(\bar{z}),$$

$$A^{Coll} = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\perp \bar{a},[1]}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}.$$

Final State Polarization

❖ X-Sec: conditional probability for a given $\boldsymbol{S}_\Lambda$:

$$\frac{d\sigma}{dV} = \alpha + \boldsymbol{\beta} \cdot \boldsymbol{S}_\Lambda$$

❖ Extract *acquired* polarization $\boldsymbol{s}_\Lambda$, that is measured in experiment:

$$\frac{d\sigma}{dV} = \mathrm{Tr}\left[\rho^{\boldsymbol{s}_\Lambda} \rho^{\boldsymbol{S}_\Lambda}\right] \sim 1 + \boldsymbol{S}_\Lambda \cdot \boldsymbol{s}_\Lambda$$

$$\boldsymbol{s}_\Lambda = \frac{\boldsymbol{\beta}}{\alpha}, \qquad \langle \boldsymbol{s}_\Lambda \rangle = \frac{\langle \boldsymbol{\beta} \rangle}{\langle \alpha \rangle},$$

Acquired polarization integrated over R_T

❖ Using the derived X-Section:

$$\alpha = \frac{3\alpha_{em}^2}{Q^2} z^2 \bar{z}^2 \sum_a e_a^2 \Bigg\{ A(y)\mathcal{F}\Big[D_1^{a\to h} D_1^{\bar{a}\to\Lambda}\Big] + \frac{B(y)}{M_\Lambda M_h}\mathcal{F}\Big[k_T \bar{k}_T \cos(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp,a\to h} H_1^{\perp,\bar{a}\to\Lambda}\Big] \Bigg\},$$

$$\boldsymbol{\beta} = \frac{3\alpha_{em}^2}{Q^2} z^2 \bar{z}^2 \sum_a e_a^2 \Bigg\{ -A(y)\mathcal{F}\Bigg[\frac{\bar{\boldsymbol{k}}_T'}{M_\Lambda} D_1^{a\to h} D_{1T}^{\perp,\bar{a}\to\Lambda}\Bigg] + B(y)\mathcal{F}\Bigg[\frac{\boldsymbol{k}_T''}{M_h} H_1^{\perp,a\to h} H_{1T}^{\bar{a}\to\Lambda}\Bigg] + B(y)\mathcal{F}\Bigg[\frac{\bar{\boldsymbol{k}}_T \; k_T \bar{k}_T}{M_h M_\Lambda^2}\sin(\varphi_k + \varphi_{\bar{k}}) H_1^{\perp,a\to h} H_{1T}^{\perp,\bar{a}\to\Lambda}\Bigg] \Bigg\},$$

$$\boldsymbol{k}_T' \equiv (k_T^2, -k_T^1) \qquad \boldsymbol{k}_T'' \equiv (k_T^2, k_T^1)$$

❖ Only depends in the final state momenta, parametrised in terms of FFs and DiFFs!

Weighted Polarized Asymmetries: L

❖ Accessing Helicity DiFF

$$\langle \beta_L \rangle_{G_1^\perp G_{1L}} = \left\langle \frac{q_T}{M_h} \sin(\varphi_q - \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} A(y) \sum_a e_a^2 \; G_1^{\perp, a\to h_1 h_2}(z, M_h^2) \; G_{1L}^{\bar{a}\to\Lambda}(\bar{z}).$$

$$\langle s_L \rangle^{\sin(\varphi_q - \varphi_R)}(z, M_h^2, \bar{z}, y) = \frac{\sum_a e_a^2 \; G_1^{\perp, a\to h_1 h_2}(z, M_h^2) \; G_{1L}^{\bar{a}\to\Lambda}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})},$$

Nonzero measurements of longitudinal Λ polarization at ALEPH!

❖ Combination of IFF with Kotzinian-Mulders type FF:

$$\langle \beta_L \rangle_{H_1^\sphericalangle H_{1L}^\perp} = \left\langle \frac{q_T}{M_\Lambda} \sin(\varphi_q + \varphi_R) \right\rangle = \frac{3\alpha_{em}^2}{(2\pi)^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_{1L}^{\perp \bar{a}, [1]}(\bar{z}),$$

$$\langle s_L \rangle^{\sin(\varphi_q + \varphi_R)}(z, M_h^2, \bar{z}, y) = \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_{1L}^{\perp \bar{a}, [1]}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}.$$

Weighted Polarized Asymmetries: T

❖ Accessing IFF

$$\langle \boldsymbol{\beta}_x \rangle^{\sin(\varphi_R)}_{H_1^{\sphericalangle} H_1} = \langle \boldsymbol{\beta}_y \rangle^{\cos(\varphi_R)}_{H_1^{\sphericalangle} H_1} = \frac{3\alpha_{em}^2}{8\pi^2 Q^2} B(y) \sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\bar{a}\to\Lambda}(\bar{z}).$$

$$\langle \boldsymbol{s}_T \rangle^{\sin(\varphi_R)}_x = \langle \boldsymbol{s}_T \rangle^{\cos(\varphi_R)}_y = \frac{1}{2} \frac{B(y)}{A(y)} \frac{\sum_a e_a^2 \; H_1^{\sphericalangle, a\to h_1 h_2}(z, M_h^2) \; H_1^{\bar{a}}(\bar{z})}{\sum_a e_a^2 \; D_1^{a\to h_1 h_2}(z, M_h^2) \; \bar{D}_1^{\bar{a}\to\Lambda}(\bar{z})}.$$

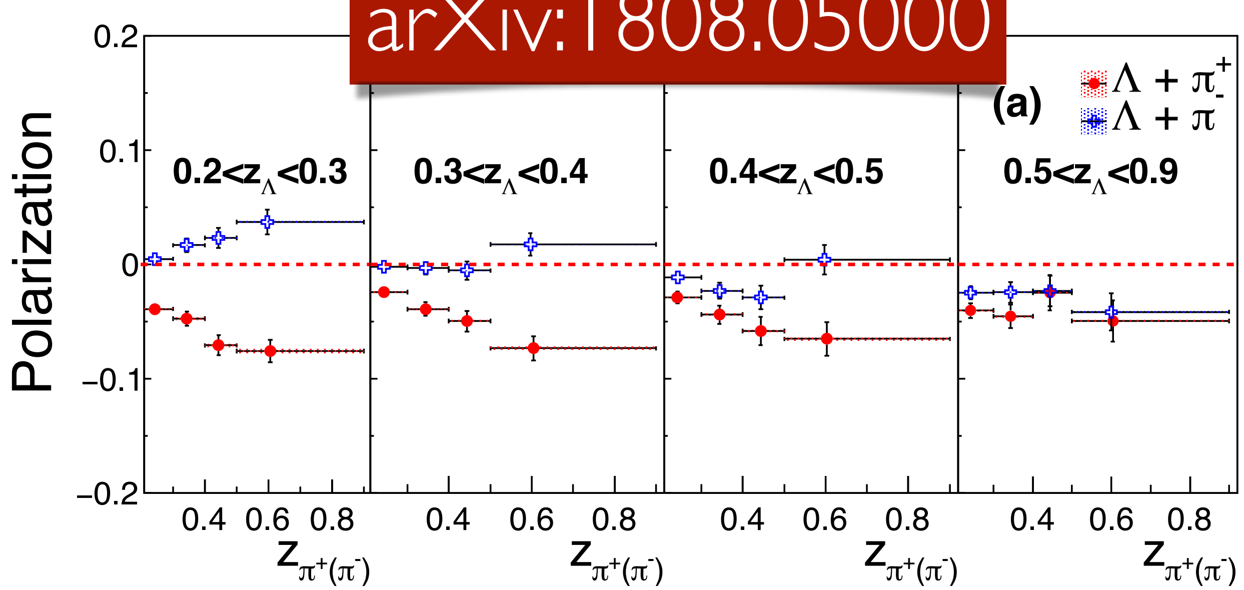
❖ (Self-) polarizing DiFF from normal polarization: $\beta_\perp = \boldsymbol{\beta}_T \cdot \boldsymbol{q}'_T / q_T$

similar to* *Boer et. al. PRL. 105, 202001 (2010).

$$\langle \beta_\perp \rangle \sim \sum_a e_a^2 \; D_1^{a\to h_1 h_2}(z, M_h^2) \; D_{1T}^{\perp, \bar{a}\to\Lambda}(\bar{z})$$

❖ BELLE results for single hadron "associated" production:

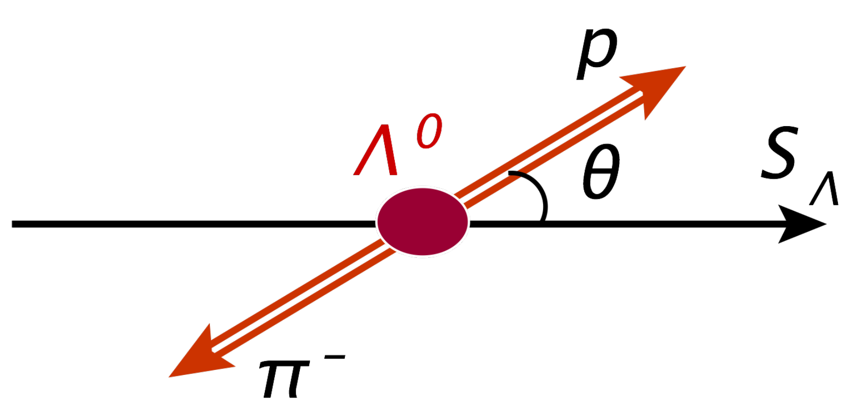
arXiv:1808.05000



Measuring Hyperon Polarization

❖ Measuring polarization of a hyperon using weak decay, ($\Lambda^0 \to p + \pi^-$).

$$\frac{dN}{N d\cos\theta} \sim 1 + \alpha_\Lambda s_\Lambda \cos(\theta),$$



angle between proton mom. and "quantization" axis

❖ Asymmetry for the full final state: *narrow width approximation*.

$$e^+e^- \to (h_1 h_2) + \Lambda^0 + X \to (h_1 h_2) + (p + \pi^-) + X$$

$$\left\langle \cos(\theta_p) \frac{q_T}{M_h} \sin(\varphi_q - \varphi_R) \right\rangle \sim \alpha_\Lambda \; G_1^{\perp, a \to h_1 h_2} \; G_{1L}^{\bar{a} \to \Lambda},$$

CONCLUSIONS

❖ Quark polarization gives access to non-perturbative dynamics in DIS.

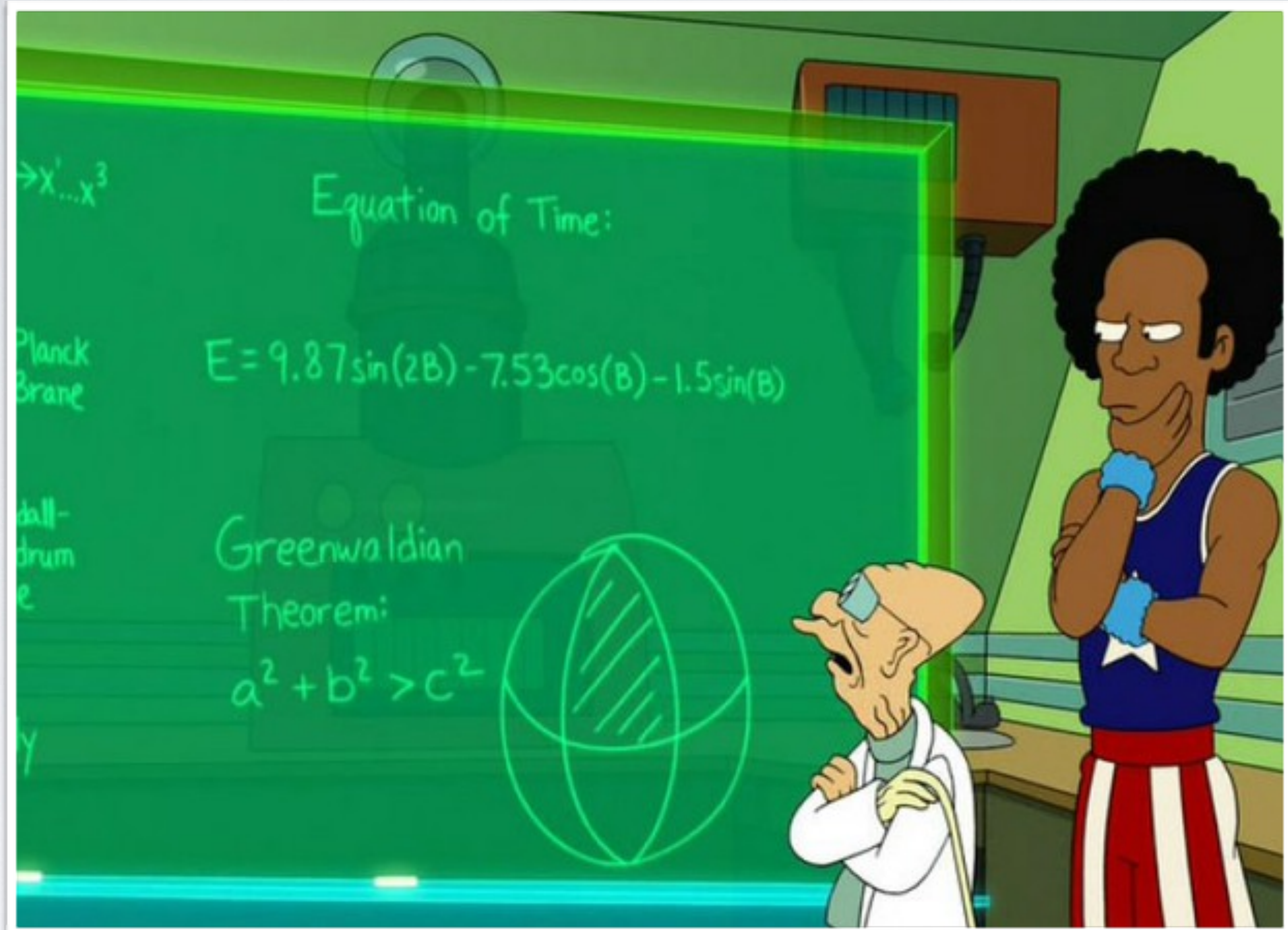
❖ DiFFs provide information on the polarization of the fragmenting quark.

❖ *Universality* of FFs and DiFFs needs to be tested experimentally!

❖ *New Measurements* in e^+e^- to probe FF⊗DiFF.

❖ Employing the extended *weighted asymmetry* method:

- Flavour decomposition of DiFFs.
- Combined global fits for polarized FFs and DiFFs.
- Test universality of DiFFs.



→x'...x³
Equation of Time:
Planck
Brane
E=9.87sin(2B)-7.53cos(B)-1.5sin(B)
Greenwaldian
Theorem:
a²+b²>c²

BACKUPS

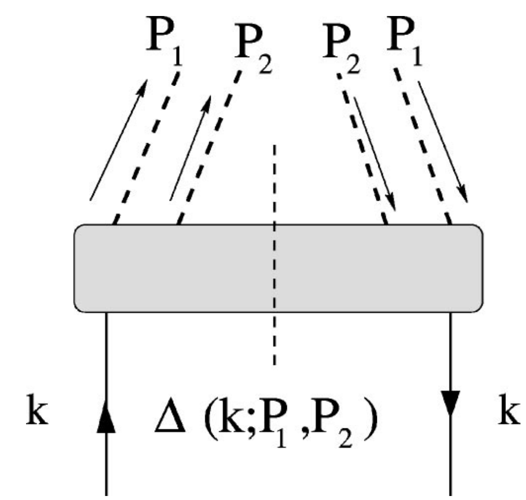
Two-Hadron Kinematics

A. Bianconi et al: PRD 62, 034008 (2000).

✦ **Total and Relative TM of hadron pair.**

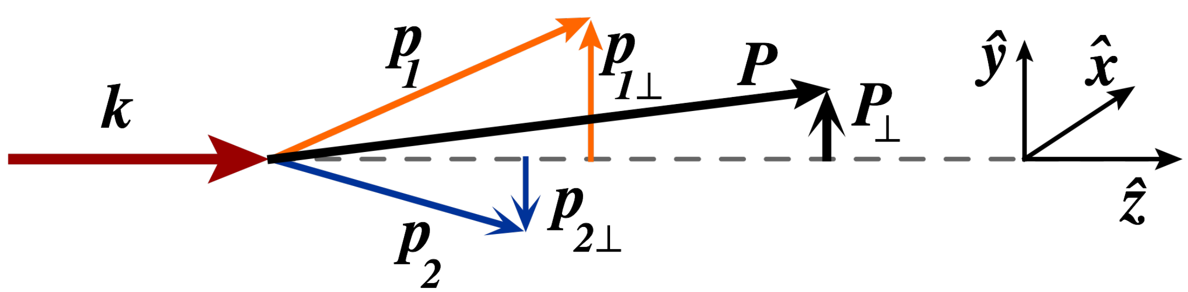
$$P = P_1 + P_2 \qquad z = z_1 + z_2$$

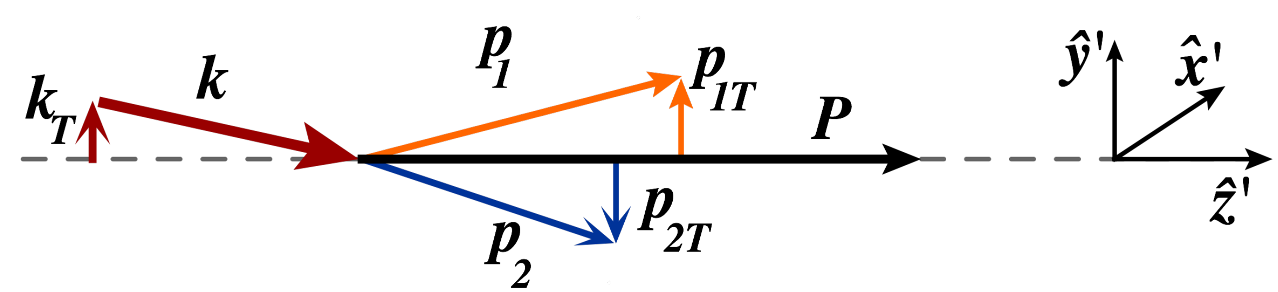
$$R = \frac{1}{2}(P_1 - P_2) \qquad \xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$



✦ **Two Coordinate systems:**

- $\perp$: modelling hadronization
- T: field-theoretical definition of DiFFs





✦ **Lorentz Boost:**

$$\boldsymbol{P}_{1T} = \boldsymbol{P}_{1\perp} + z_1 \boldsymbol{k}_T$$

$$\boldsymbol{P}_{2T} = \boldsymbol{P}_{2\perp} + z_2 \boldsymbol{k}_T$$

$$\boldsymbol{k}_T = -\frac{\boldsymbol{P}_\perp}{z}$$

✤ **Relative TM in two systems**

$$\boldsymbol{R}_\perp = \frac{1}{2}(\boldsymbol{P}_{1\perp} - \boldsymbol{P}_{2\perp})$$

$$\boldsymbol{R}_T = \frac{z_2 \boldsymbol{P}_{1\perp} - z_1 \boldsymbol{P}_{2\perp}}{z}$$

Field-Theoretical Definitions

- ***The quark-quark correlator.***

$$\Delta_{ij}(k;P_1,P_2) = \sum_X \int d^4\zeta e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)|P_1P_2,X\rangle\langle P_1P_2,X|\bar{\psi}_j(0)|0\rangle$$

- ***The definitions of DiFFs from the***

Quark Polarization

$$\Delta^{[\gamma^-]} = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Unpolarised

related to "jet handedness"

$$\Delta^{\gamma^-\gamma_5]} = \frac{\epsilon_T^{ij}R_{Ti}k_{Tj}}{M_h^2} G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Longitudinal

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij}R_{Tj}}{M_h} H_1^{\sphericalangle}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T) + \frac{\epsilon_T^{ij}k_{Tj}}{M_h} H_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

Transverse

Fourier Moments of DiFFs

- ***Expanded dependence on*** $\varphi_{RK} \equiv \varphi_R - \varphi_k$ ***in cos series***

$$D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{KR})) = \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\cos(n\cdot\varphi_{KR})}{1+\delta_{0,n}}\, D_1^{[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|),$$

$$F^{[n]} = \int d\varphi_{KR}\;\cos(n\varphi_{KR})\; F\Big(\cos(\varphi_{KR})\Big)$$

- ***Integrated DiFFs and Fourier moments***

$$D_1^a(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\; D_1^{a,[0]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2)$$

$$G_1^{\perp a,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\left(\frac{\boldsymbol{k}_T^2}{2M_h^2}\right)\frac{|\boldsymbol{R}_T|}{M_h}\; G_1^{\perp a,[n]}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2).$$

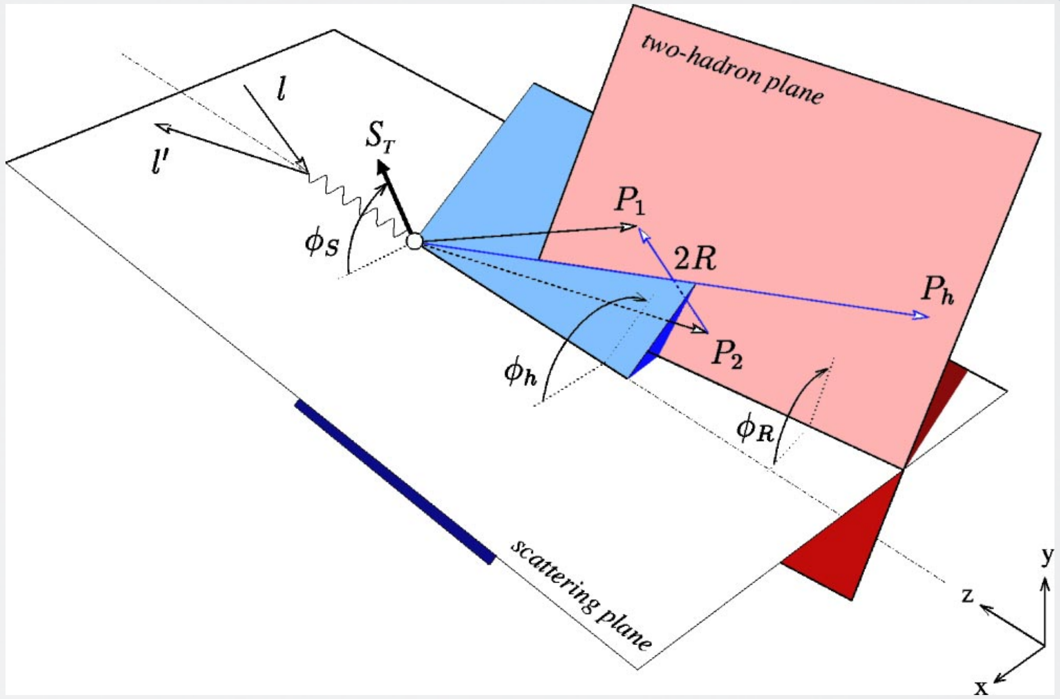
$$H_1^{\sphericalangle,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\;\frac{|\boldsymbol{R}_T|}{M_h}H_1^{\sphericalangle,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

$$H_1^{\perp,[n]}(z,M_h^2) = z^2\int d^2\boldsymbol{k}_T\int d\xi\;\frac{|\boldsymbol{k}_T|}{M_h}H_1^{\perp,[n]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

ACCESS TO TRANSVERSITY PDF From DiFF

M. Radici, et al: PRD 65, 074031 (2002).

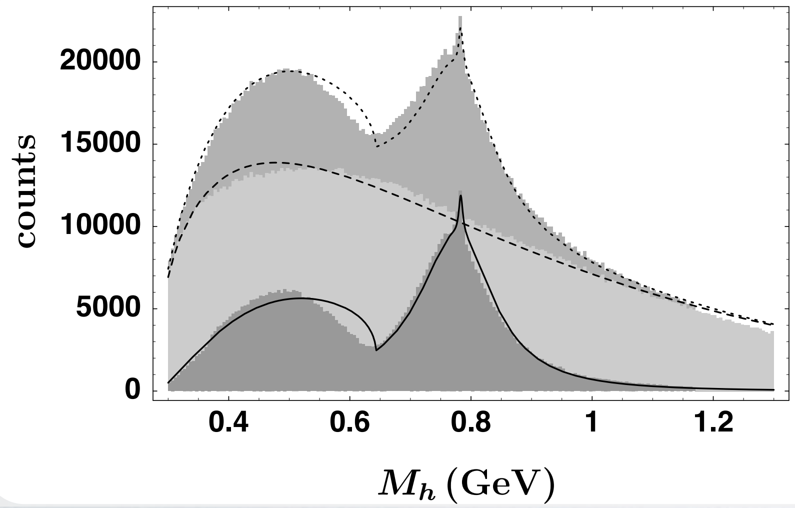
- **In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!**
- **Allows clean access to *transversity*.**

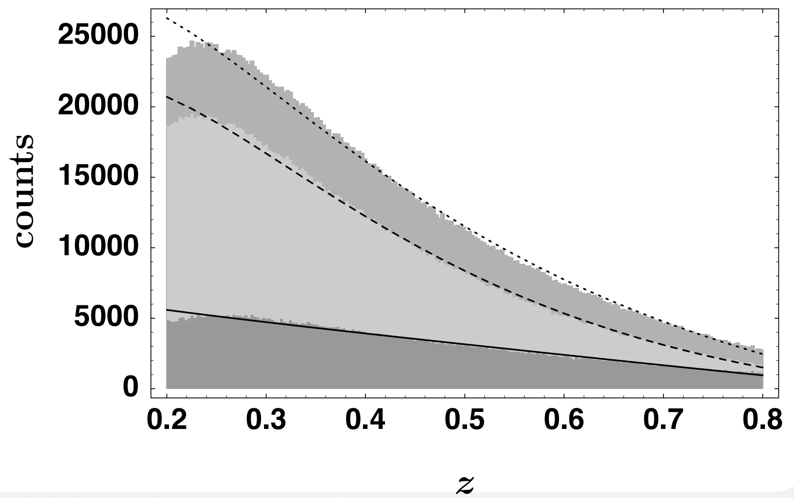


$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \;\; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \;\; D_1^q(z, M_h^2)}$$

- **Empirical Model for D_1^q has been fitted to PYTHIA**

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).





Experiments

SSA: HERMES, COMPASS.

IFFs: BELLE.

Moments of DiFFs in SIDIS

A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

- ***Here transversely polarised DiFFs are admixture of cos Fourier moments of both unintegrated DiFFs:***

$$H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle [0]} + H_1^{\perp [1]}\right]$$

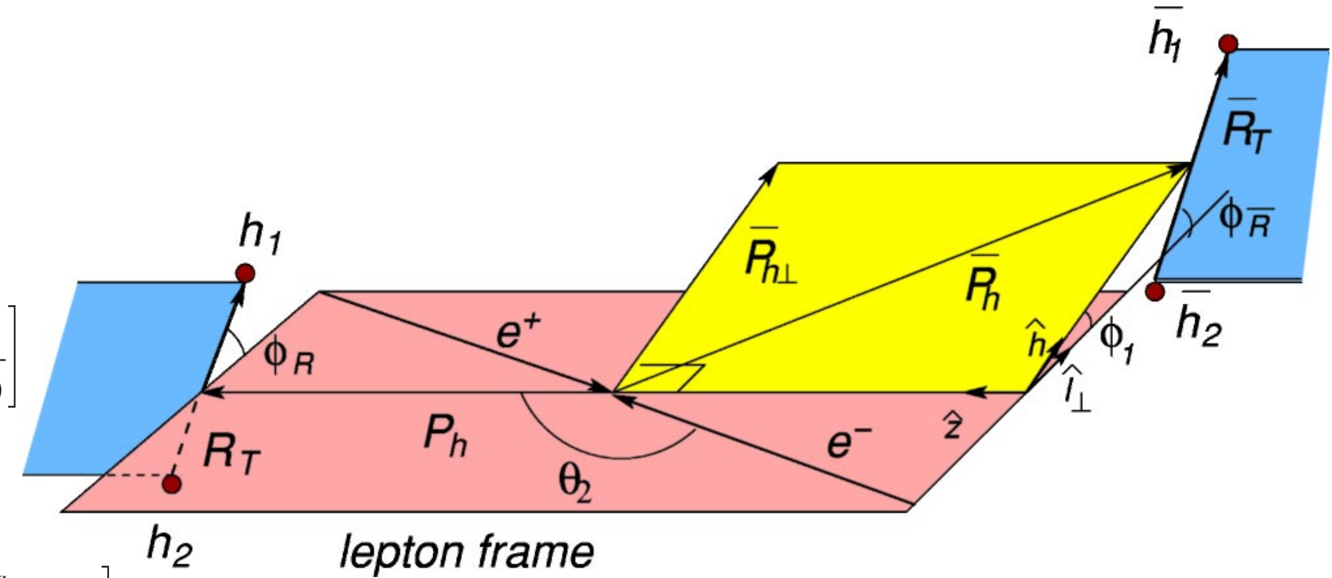
$$H_{1,SIDIS}^{\perp}(z, M_H^2) = \left[H_1^{\perp [0]} + H_1^{\sphericalangle [1]}\right]$$

- ***Generated by*** $\cos(\varphi_{RK})$ ***dependences of unintegrated DiFFs:*** $\varphi_{RK} \equiv \varphi_R - \varphi_k$

$$d\sigma_{UT} \sim \sin(\varphi_R + \varphi_S)\mathcal{C}\left[h_1^{\perp} H^{\sphericalangle}(\cos(\varphi_{RK}))\right]$$
$$+ \sin(\varphi_k + \varphi_S)\mathcal{C}\left[h_1^{\perp} H^{\perp}(\cos(\varphi_{RK}))\right] + ..$$

Back-to-back *two* hadron pairs in e⁺e⁻

D. Boer et al: PRD 67, 094003 (2003).



$$
\begin{aligned}
&\frac{d\sigma(e^+e^-\to(h_1h_2)(\bar{h}_1\bar{h}_2)X)}{d\boldsymbol{q}_T dz\, d\xi\, dM_h^2\, d\phi_R\, d\bar{z}\, d\bar{\xi}\, d\bar{M}_h^2\, d\phi_{\bar{R}}\, dy\, d\phi^l}\\
&=\sum_{a,\bar{a}} e_a^2 \frac{6\alpha^2}{Q^2} z^2\bar{z}^2 \Bigg\{ A(y)\mathcal{F}[D_1^a\bar{D}_1^a] + \cos(2\phi_1)B(y)\mathcal{F}\left[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T - \boldsymbol{k}_T\cdot\bar{\boldsymbol{k}}_T)\frac{H_1^{\perp a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right]\\
&-\sin(2\phi_1)B(y)\mathcal{F}\left[(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T + \hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T)\frac{H_1^{\perp a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + \cos(\phi_R+\phi_{\bar{R}}-2\phi^l)\\
&\times B(y)|\boldsymbol{R}_T||\bar{\boldsymbol{R}}_T|\mathcal{F}\left[\frac{H_1^{\sphericalangle a}\bar{H}_1^{\sphericalangle a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + \cos(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol{R}_T|\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\frac{H_1^{\sphericalangle a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right]\\
&-\sin(\phi_1+\phi_R-\phi^l)B(y)|\boldsymbol{R}_T|\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T\frac{H_1^{\sphericalangle a}\bar{H}_1^{\perp a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + \cos(\phi_1+\phi_{\bar{R}}-\phi^l)B(y)|\bar{\boldsymbol{R}}_T|\\
&\times\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{H_1^{\perp a}\bar{H}_1^{\sphericalangle a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] - \sin(\phi_1+\phi_{\bar{R}}-\phi^l)B(y)|\bar{\boldsymbol{R}}_T|\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\frac{H_1^{\perp a}\bar{H}_1^{\sphericalangle a}}{(M_1+M_2)(\bar{M}_1+\bar{M}_2)}\right] + A(y)|\boldsymbol{R}_T||\bar{\boldsymbol{R}}_T|\\
&\times\Bigg(\sin(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar{R}}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right] + \sin(\phi_1-\phi_R+\phi^l)\cos(\phi_1-\phi_{\bar{R}}+\phi^l)\\
&\times\mathcal{F}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right] + \cos(\phi_1-\phi_R+\phi^l)\sin(\phi_1-\phi_{\bar{R}}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{h}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right] + \cos(\phi_1-\phi_R+\phi^l)\\
&\times\cos(\phi_1-\phi_{\bar{R}}+\phi^l)\mathcal{F}\left[\hat{\boldsymbol{g}}\cdot\boldsymbol{k}_T\hat{\boldsymbol{g}}\cdot\bar{\boldsymbol{k}}_T\frac{G_1^{\perp a}\bar{G}_1^{\perp a}}{M_1M_2\bar{M}_1\bar{M}_2}\right]\Bigg)\Bigg\}, \qquad (19)
\end{aligned}
$$

- ***Can access both helicity and transverse pol.***

$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_h^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_h^2)}$$

$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_h^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_h^2)}$$

Moments of DiFFs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).

- ***In asymmetry: helicity-dependent DiFF in the.***

$\cos(\varphi_R - \varphi_k)$ **moment**

$$G_1^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \, (\boldsymbol{k}_T \cdot \boldsymbol{R}_T) \, G_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- ***In asymmetry: IFF.***

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \, |\boldsymbol{R}_T| H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- ***Differ from SIDIS !***

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = H_1^{\sphericalangle,[0]} \qquad H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle[0]} + H_1^{\perp[1]}\right]$$

Moments of DiFFs in e⁺e⁻

D. Boer et al: PRD 67, 094003 (2003).

- ***In asymmetry: he'... ...endent DiFF in the.***

$-\varphi_k)$ **moment**

$$G_1^{\perp}(z, M_h^2) = \int d\xi \int \ldots {}^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- ***In asymmetry: IFF.***

$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = \int d\xi \int \ldots \xi, k_T^2, R_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

- ***Differ from SIDIS !***

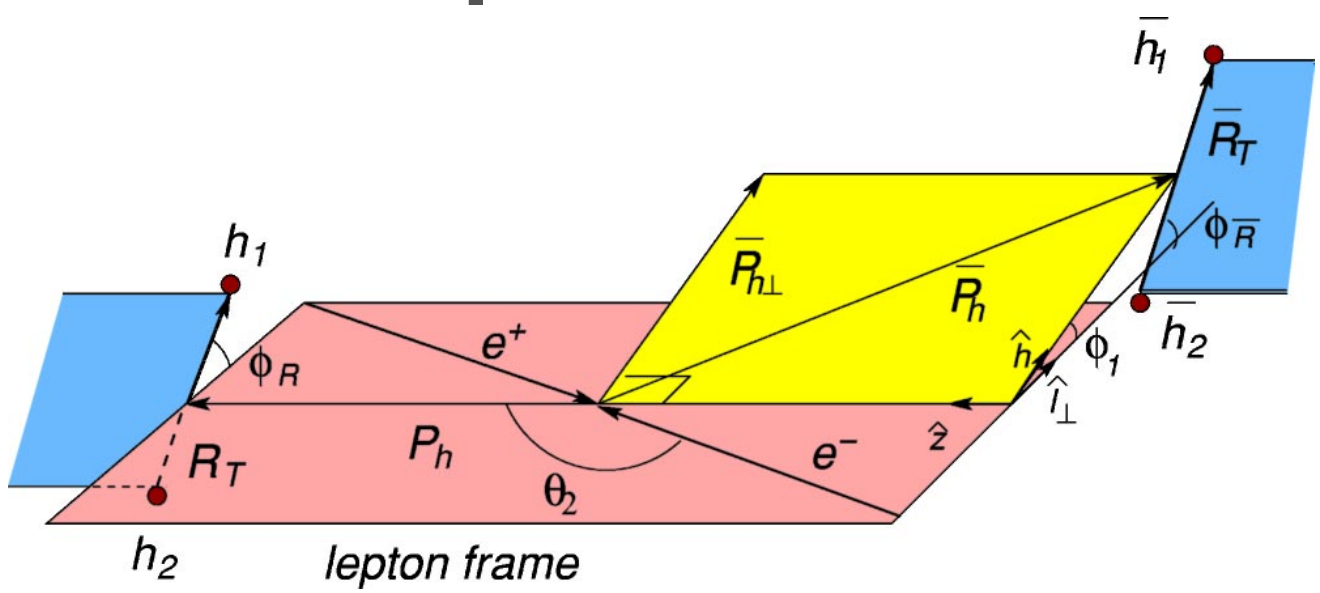
$$H_{1,e^+e^-}^{\sphericalangle}(z, M_h^2) = H_1^{\sphericalangle,[0]} \qquad H_{1,SIDIS}^{\sphericalangle}(z, M_H^2) = \left[H_1^{\sphericalangle[0]} + H_1^{\perp[1]}\right]$$

- ***Might strongly affect combined***

Back-to-back *two* hadron pairs in e^+e^-

D. Boer et al: PRD 67, 094003 (2003).

- ***Can access both helicity and transverse pol. dependent DiFFs:***

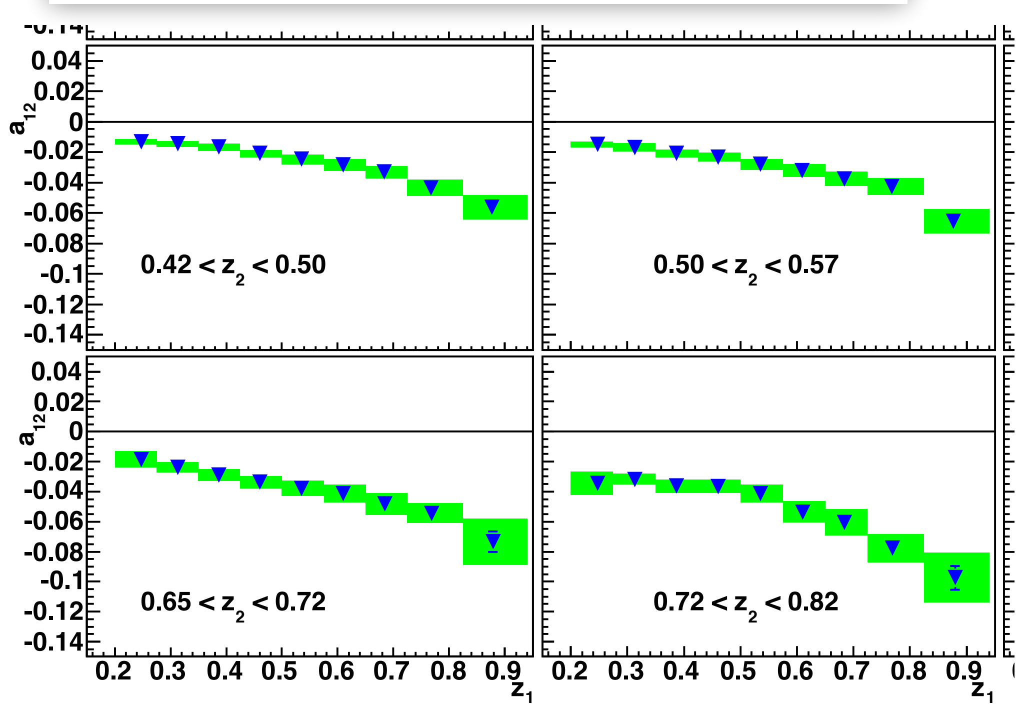


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z,M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

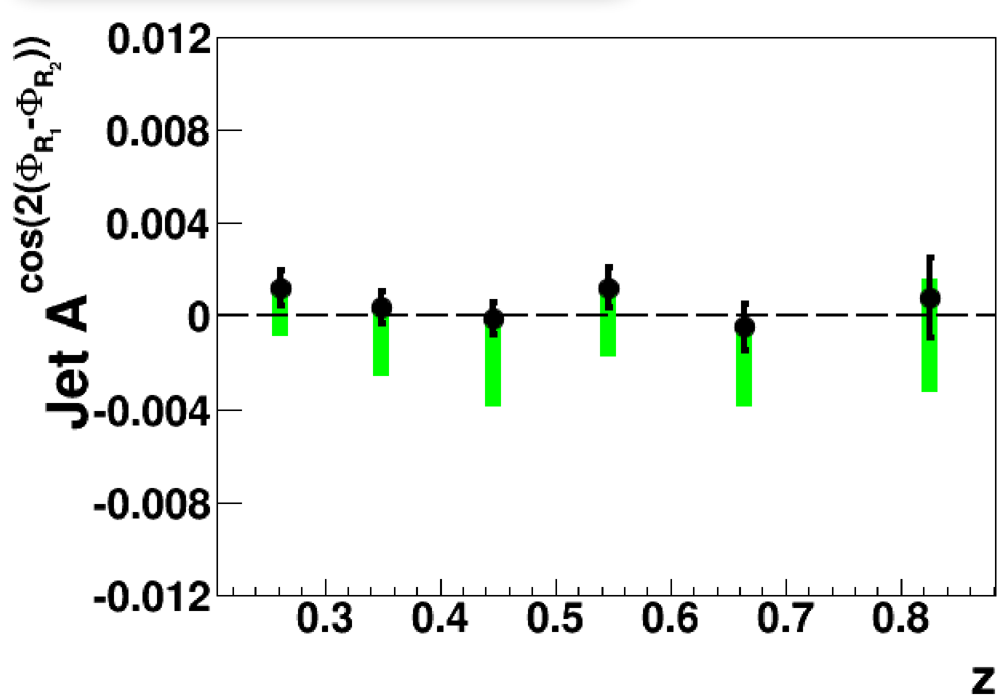
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z,M_h^2)\bar{G}_1^{\perp}(\bar{z},M_{\bar{h}}^2)}{D_1(z,M_h^2)\ \bar{D}_1(\bar{z},M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



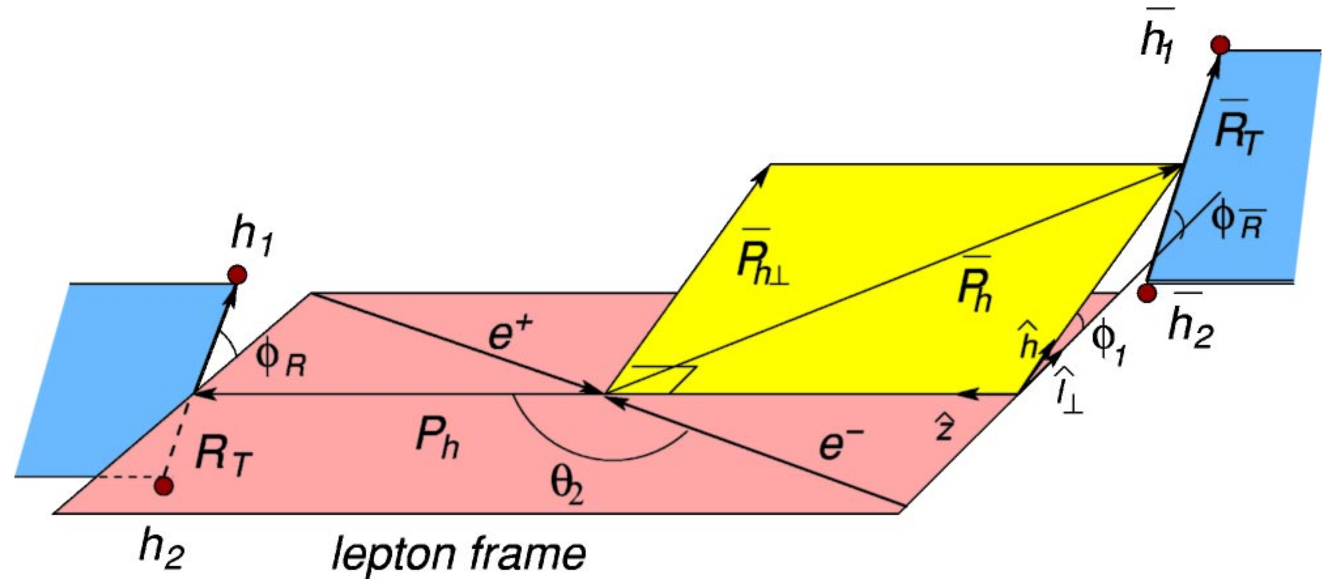
PoS DIS2015 (2015) 216



Back-to-back *two* hadron pairs in e⁺e⁻

D. Boer et al: PRD 67, 094003 (2003).

- ***Can access both helicity and transverse pol. dependent DiFFs:***

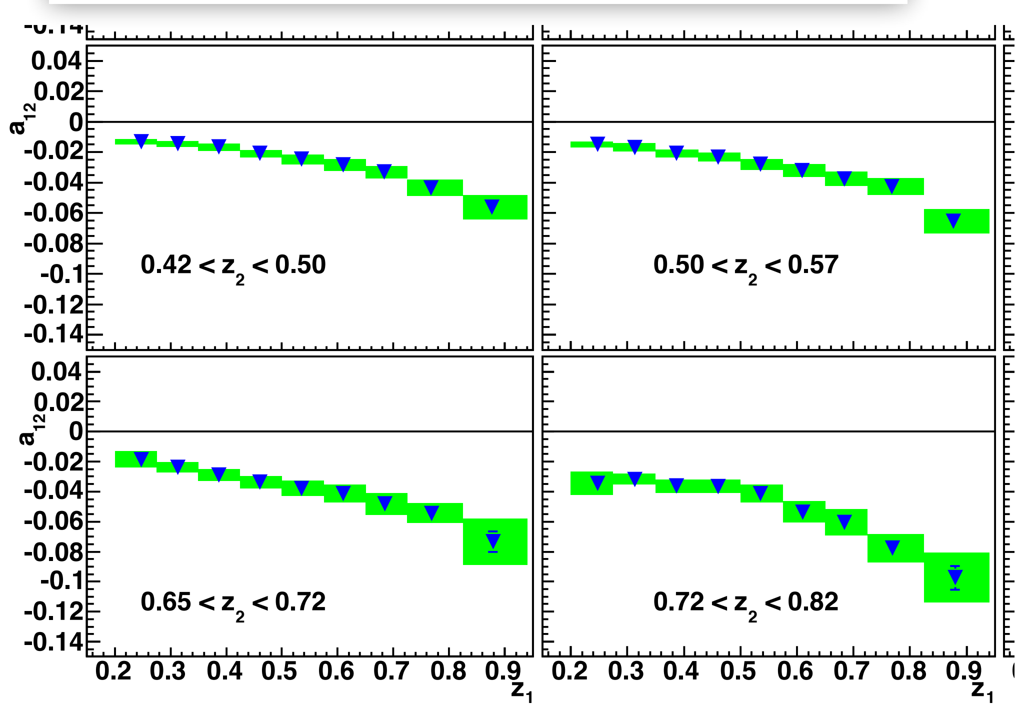


$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} \sim \frac{H_1^{\sphericalangle}(z, M_h^2)\bar{H}_1^{\sphericalangle}(\bar{z}, M_{\bar{h}}^2)}{D_1(z, M_h^2)\ \bar{D}_1(\bar{z}, M_{\bar{h}}^2)}$$

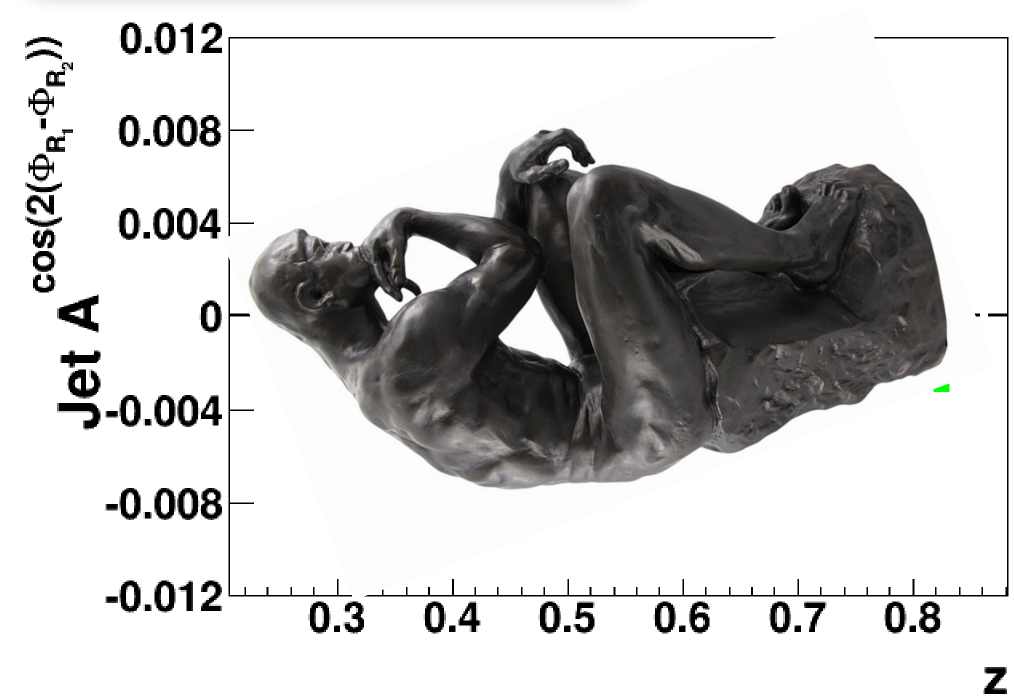
$$A^{\cos(2(\varphi_R-\varphi_{\bar{R}}))} \sim \frac{G_1^{\perp}(z, M_h^2)\bar{G}_1^{\perp}(\bar{z}, M_{\bar{h}}^2)}{D_1(z, M_h^2)\ \bar{D}_1(\bar{z}, M_{\bar{h}}^2)}$$

✦ ***BELLE results.***

Phys.Rev.Lett. 107 (2011) 072004



PoS DIS2015 (2015) 216



How to resolve these?

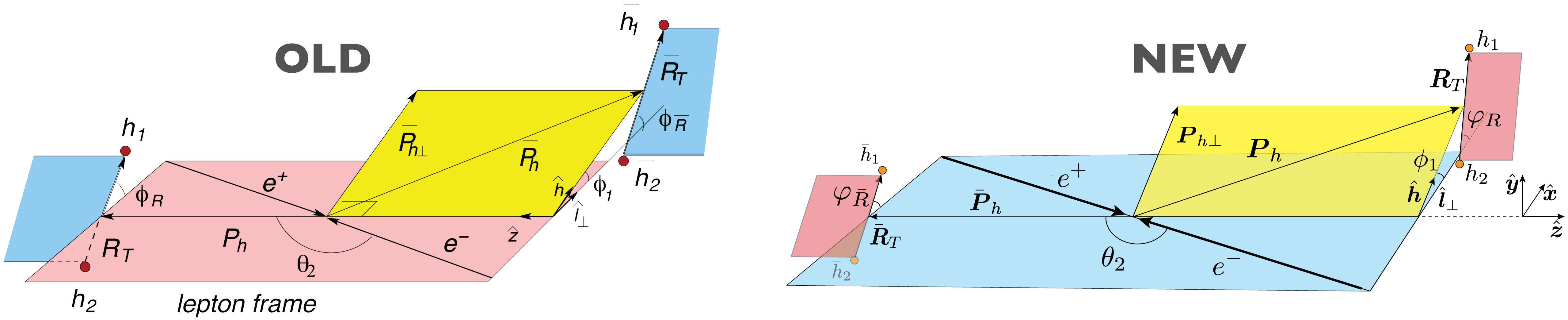
Quote from Anatoly Radyushkin:

‘‘I am old enough to know that if something is published, it is not necessarily correct”

Re-derived e⁺e⁻ Cross Section

H.M. , Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- ***An error in kinematics was found:***



- ***The new fully differential cross-section***

$$\frac{d\sigma\left(e^+e^- \to (h_1h_2)(\bar h_1 \bar h_2)X\right)}{d^2\boldsymbol{q}_T\, dz\, d\xi\, d\varphi_R\, dM_h^2\, d\bar z\, d\bar\xi\, d\varphi_{\bar R}\, d\bar M_h^2\, dy} = \frac{3\alpha^2}{\pi Q^2} z^2 \bar z^2 \sum_{a,\bar a} e_a^2 \Bigg\{ A(y)\mathcal{F}\left[D_1^a \bar D_1^{\bar a}\right]$$

$$+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar M_h}\cos(\varphi_k+\varphi_{\bar k}) H_1^{\perp a}\bar H_1^{\perp \bar a}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar M_h}\cos(\varphi_R+\varphi_{\bar R}) H_1^{\sphericalangle a}\bar H_1^{\sphericalangle \bar a}\right]$$

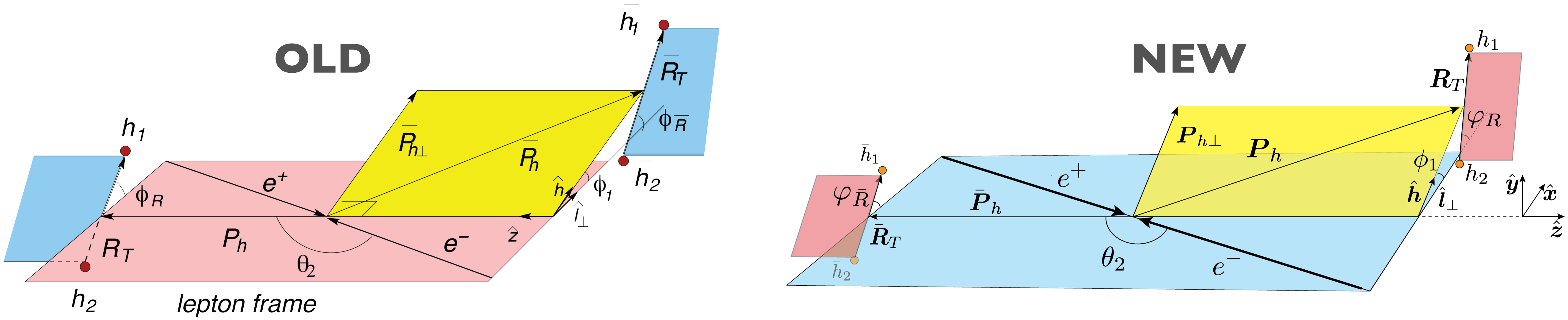
$$+ B(y)\mathcal{F}\left[\frac{|\boldsymbol{k}_T|}{M_h}\frac{|\bar{\boldsymbol{R}}_T|}{\bar M_h}\cos(\varphi_k+\varphi_{\bar R}) H_1^{\perp a}\bar H_1^{\sphericalangle \bar a}\right] + B(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|}{M_h}\frac{|\bar{\boldsymbol{k}}_T|}{\bar M_h}\cos(\varphi_R+\varphi_{\bar k}) H_1^{\sphericalangle a}\bar H_1^{\perp \bar a}\right]$$

$$- A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|\,|\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T|\,|\bar{\boldsymbol{k}}_T|}{\bar M_h^2}\sin(\varphi_k-\varphi_R)\sin(\varphi_{\bar k}-\varphi_{\bar R}) G_1^{\perp a}\bar G_1^{\perp \bar a}\right]\Bigg\}.$$

Re-derived e^+e^- Cross Section

H.M. , Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- ***An error in kinematics was found:***



- ***The new fully differential cross-section***

$$\frac{d\sigma\left(e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)X\right)}{d^2\boldsymbol{q}_T dz d\xi d\varphi_R dM_h^2 d\bar{z} d\bar{\xi} d\varphi_{\bar{R}} d\bar{M}_h^2 dy} = \frac{3\alpha^2}{\pi Q^2} z^2 \bar{z}^2 \sum_{a,\bar{a}} e_a^2 \Big\{ A(y)\mathcal{F}\Big[D_1^a \bar{D}_1^{\bar{a}}\Big]$$

$$\mathcal{F}[wD^a\bar{D}^{\bar{a}}] = \int d^2\boldsymbol{k}_T d^2\bar{\boldsymbol{k}}_T\ \delta^2(\boldsymbol{k}_T + \bar{\boldsymbol{k}}_T - \boldsymbol{q}_T) w(\boldsymbol{k}_T, \bar{\boldsymbol{k}}_T, \boldsymbol{R}_T, \bar{\boldsymbol{R}}_T)\ D^a\ D^{\bar{a}}.$$

$$- A(y)\mathcal{F}\left[\frac{|\boldsymbol{R}_T|\,|\boldsymbol{k}_T|}{M_h^2}\frac{|\bar{\boldsymbol{R}}_T|\,|\bar{\boldsymbol{k}}_T|}{\bar{M}_h^2}\sin(\varphi_k - \varphi_R)\sin(\varphi_{\bar{k}} - \varphi_{\bar{R}}) G_1^{\perp a}\bar{G}_1^{\perp \bar{a}}\right]\Big\}.$$

IFFs in e^+e^- and SIDIS.

H.M., Bacchetta, Boer, Courtoy, Kotzinian, Radici, Thomas: Phys. Rev. D 97, 074019 (2018).

- ***The asymmetry now involves exactly the same integrated IFF as in SIDIS!***

$$A^{\cos(\varphi_R+\varphi_{\bar{R}})} = \frac{1}{2}\frac{B(y)}{A(y)}\frac{\sum_{a,\bar{a}} e_a^2 H_1^{\sphericalangle a}(z,M_h^2)\bar{H}_1^{\sphericalangle \bar{a}}(\bar{z},\bar{M}_h^2)}{\sum_{a,\bar{a}} e_a^2 D_1^a(z,M_h^2)\bar{D}_1^{\bar{a}}(\bar{z},\bar{M}_h^2)}$$

$$D_1(z,M_h^2) \equiv z^2 \int d^2\boldsymbol{k}_T \int d\xi\, D_1^{[0]}(z,\xi,|\boldsymbol{k}_T|,|\boldsymbol{R}_T|)$$

$$H_{1,e^+e^-}^{\sphericalangle}(z,M_h^2) = H_1^{\sphericalangle,[0]} + H_1^{\perp,[1]} \equiv H_{1,SIDIS}^{\sphericalangle}(z,M_h^2)$$

✦ ***All the previous extractions of the transversity are valid !***

Helicity-dependent DiFF in e+e-

H.M. , Kotzinian, Thomas: arXiv:1712.06384.

- ***The relevant terms involving*** $G_1^\perp$:

$$d\sigma_L \sim \mathcal{F}\Big[\frac{(\boldsymbol{R}_T\times\boldsymbol{k}_T)_3}{M_h^2}\frac{(\bar{\boldsymbol{R}}_T\times\bar{\boldsymbol{k}}_T)_3}{\bar{M}_h^2}G_1^{\perp a}(\boldsymbol{R}_T\cdot\boldsymbol{k}_T)\bar{G}_1^{\perp\bar{a}}(\bar{\boldsymbol{R}}_T\cdot\bar{\boldsymbol{k}}_T)\Big]$$

- ***Note:*** ***any azimuthal moment involving only*** $\varphi_R,\ \varphi_{\bar{R}}$ ***is zero.***

Break-up the convolution: $\int d^2\boldsymbol{q}_T\delta^2(\boldsymbol{k}_T+\bar{\boldsymbol{k}}_T-\boldsymbol{q}_T)$

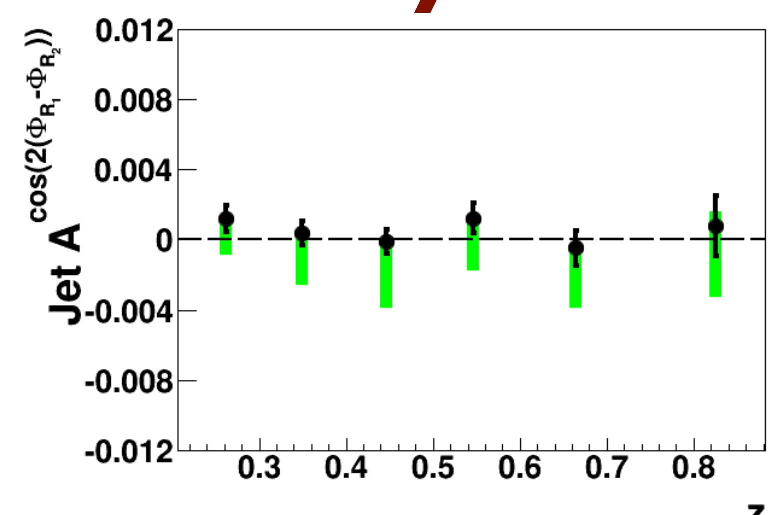
decouple $\boldsymbol{k_T}$ on both sides

Using: $\varphi_k \to \varphi'_k + \varphi_R$, $\int d^2\boldsymbol{k}_T\sin(\varphi_k)\cos(n\varphi_k) = 0$

$$\langle f(\varphi_R,\varphi_{\bar{R}})\rangle_L = 0$$

- ***The old asymmetry by Boer et. al. exactly vanishes!***
- ***Explains the BELLE results.***

$$A^{\Rightarrow} = \frac{\langle\cos(2(\varphi_R-\varphi_{\bar{R}}))\rangle}{\langle 1\rangle} = 0!$$

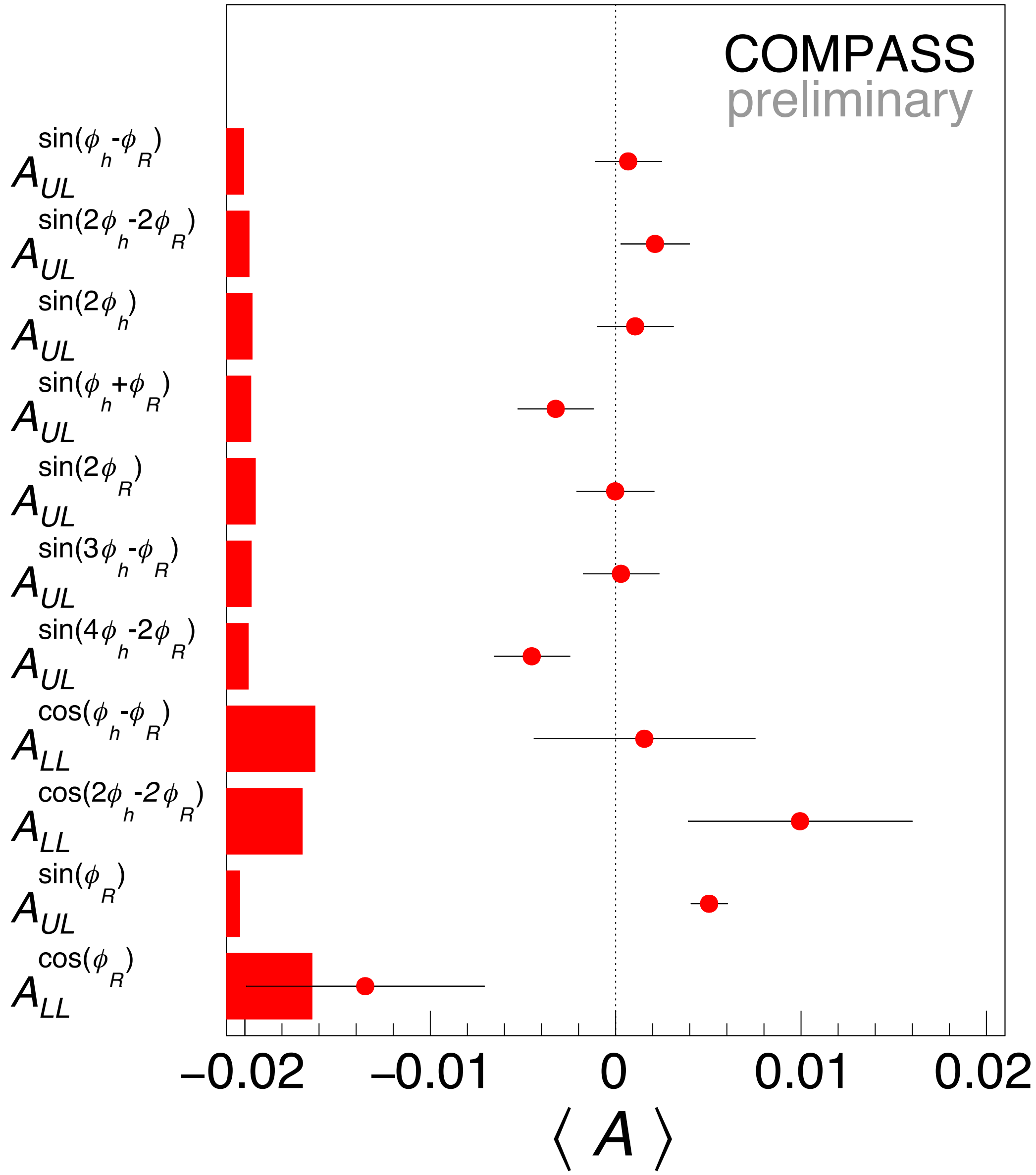


Helicity DiFFs at COMPASS

- ***SIDIS extraction in***

$$d\sigma_{UL} \sim -A(y)\mathcal{G}\Big[\frac{k_T R_T \sin(\varphi_k - \varphi_R)}{M_h^2}\; g_{1L}^a\; G_1^{\perp a}\Big]$$
$$+B(y)\mathcal{G}\Big[\frac{p_T k_T \sin(\varphi_p + \varphi_k)}{M M_h}\; h_{1L}^{\perp a}\; H_1^{\perp a}\Big]$$
$$+B(y)\mathcal{G}\Big[\frac{p_T R_T \sin(\varphi_p + \varphi_R)}{M M_h} h_{1L}^{\perp a}\; H_1^{\sphericalangle a}\Big]$$

$$\mathcal{G}[w f^q D^q] \equiv \int d^2\boldsymbol{p}_T \int d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{k}_T - \boldsymbol{p}_T + \frac{\boldsymbol{P}_{h\perp}}{z}\Big)$$
$$\times\, w(\boldsymbol{p}_T, \boldsymbol{k}_T, \boldsymbol{R}_T) f^q(x, \boldsymbol{p}_T^2) D^q(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$



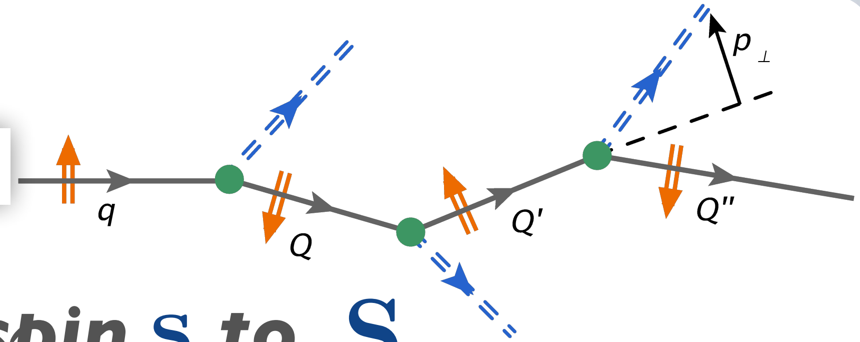
✦ $A^{\sin(n(\varphi_h - \varphi_R))}$ **are *convolutions* of** g_{1L} **and** $G_1^{\perp}$ **!**

- **Low <x> = 0.05 !**
- **Limited statistics.**

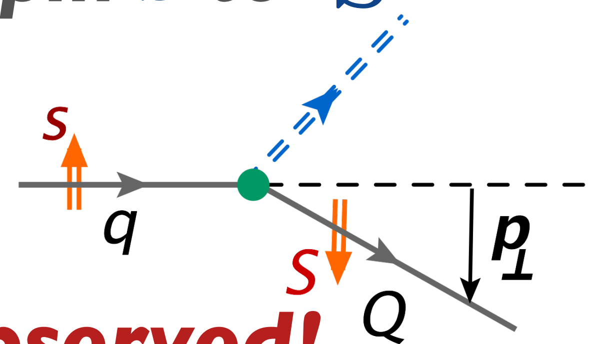
POLARIZATION IN QUARK-JET

✦ Extended quark-jet:

Bentz, Kotzinian, *H.M*, Ninomiya, Thomas, Yazaki: PRD 94 034004 (2016).



- ***The probability for the process*** $q \to Q$, ***initial spin*** $\mathbf{s}$ ***to*** $\mathbf{S}$



$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) = \alpha_\mathbf{s} + \boldsymbol{\beta}_\mathbf{s} \cdot \mathbf{S}$$

- ***Intermediate quarks in quark-jet are <u>unobserved!</u>***

We need the induced final state spin $\mathbf{S}'$.

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'} \rho^{\mathbf{S}}] \sim 1 + \mathbf{S}' \cdot \mathbf{S}$$

- ***Remnant quark's*** $\mathbf{S}'$ ***uniquely determined by*** $z, \mathbf{p}_\perp$ ***and*** $\mathbf{s}$ ***!***

$$\mathbf{S}' = \frac{\boldsymbol{\beta}_\mathbf{s}}{\alpha_\mathbf{s}}$$

- ***Process probability is the same as transition to unpolarized state.***

$$F^{q\to Q}(z, \mathbf{p}_\perp; \mathbf{s}, \mathbf{0}) = \alpha_s$$