Ignazio Scimemi (UCM)

Exploring hadron structures at twist-3

Results in collaboration with
Alexey Vladimirov
Daniel Gutierrez Reyes
arXiv: 1804.08148…

TMD factorization in a nutshell

.. for DY and heavy boson production we have (Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012)

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2,\mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q_T}\cdot\mathbf{b}} \Phi_{q/A}(x_A,\mathbf{b},\zeta_A,\mu)\Phi_{q/B}(x_B,\mathbf{b},\zeta_B,\mu)$$

$$\sqrt{\zeta_A\zeta_B} = Q^2$$

…and similar formulas are valid for SIDIS (EIC) and hadron production in e+e- colliders

The renormalization of the rapidity divergences is responsible for the a new resummation scale

We have new non-perturbative effects which cannot be included in PDFs.

The case of unpolarized TMDs:
the perturbative calculable part of unpolarized TMDs is known at NNLO!
What about polarized TMD's?

Status of unpolarized TMDs in perturbation theory

Perturbative Calculations

- Evolution to N3LO Y. Li, H.X. Zhu, arXiv:1604.01404 A. Vladimirov, arXiv:1610.05791
- Soft function at NNLO M.G. Echevarría, I.S., A. Vladimirov, arXiv:1511.05590.
- NNLO coefficients for TMDPDFs M.G. Echevarría, I.S., A. Vladimirov, arXiv1604.07869, T. Lübbert, J. Oredsson, M. Stahlhofen, arXiv:1602.01829, T. Gehrmann, T. Lübbert, Li Lin Yang arXiv: 1403.6451
- NNLO coefficients for TMD Fragmentation Functions M.G. Echevarría, I.S., A. Vladimirov, arXiv:1509.06392, arXiv:1604.07869

Phenomenology

- Global Fits (SIDIS+DY) A. Bacchetta et al. arxiv:1703.10157,
- DY and Z-boson fits (ResBos, D'Alesio et al. arXiv:1410.4522 up to NNLL)
- Implementation of standard CSS (DYres/DyqT, Cute)
- LHC data
- TMD extraction using higher order corrections (ARTEMIDE) arXiv:1706.01473 and 1803.11089

ζ-prescription

IT IS POSSIBLE TO MAKE A COMPLETE ANALYSIS OF UNPOLARIZED TMD IN DRELL-YAN AND SIDIS USING **NNLO** RESULTS (SEE ALEXEY VLADIMIROV TALK)

THE STUDY OF TRANSVERSELY POLARIZED TMDS AT THE SAME PRECISION IS JUST STARTED (SEE DANIEL GUTIERREZ REYES TALK):

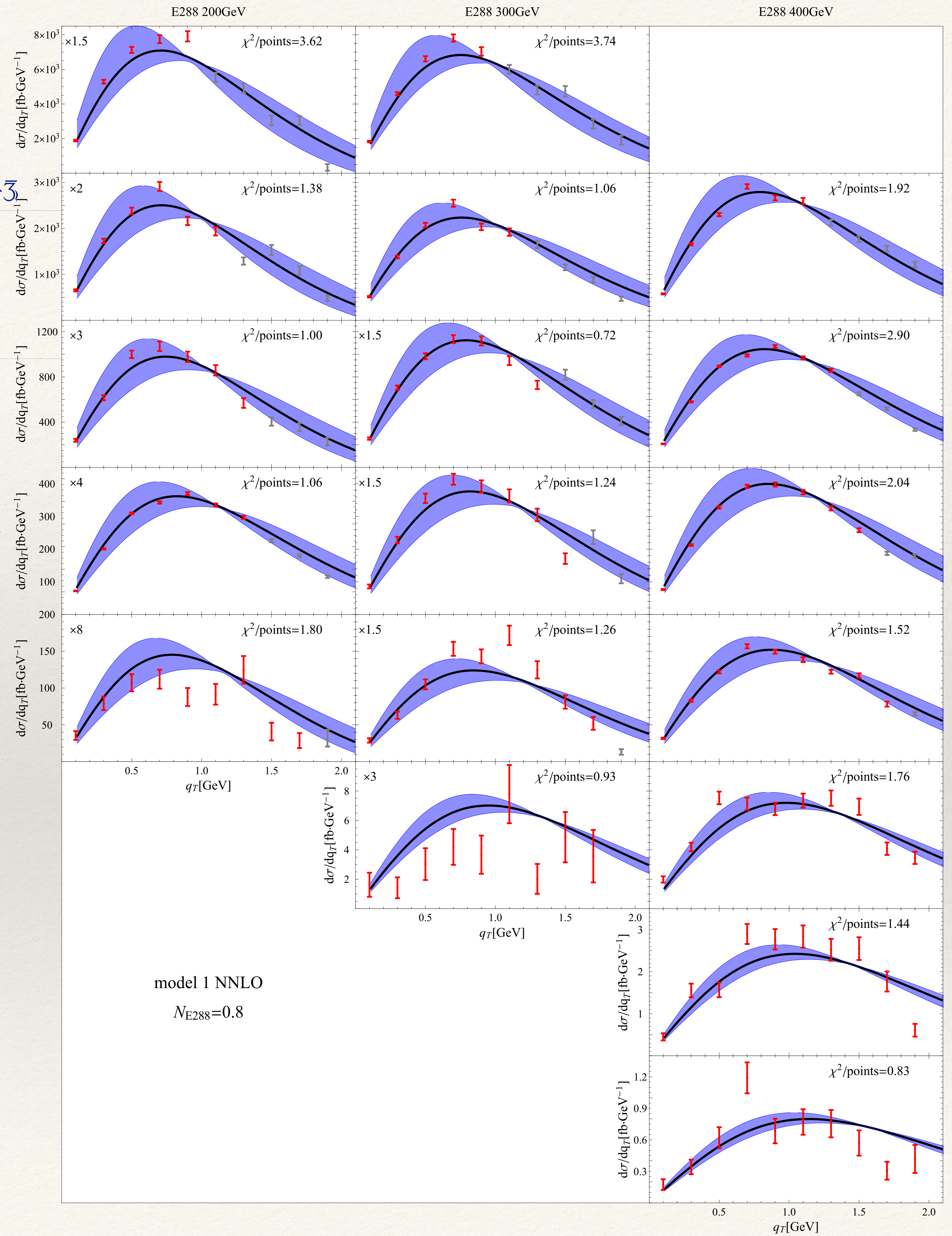
D. Gutierrez-Reyes, I.S., A. Vladimirov, arXiv:1702.06558 and 1805.07243

Preliminary

arTeMiDe version 1.3

E288

The case of E288 experiment shows that if we want to fit low energy experiments with a stable perturbative matching and theoretical errors under control we need to know higher perturbative orders



Operator expansion on the light cone

Quark TMD's can be defined starting from operators:

$$\mathcal{U}^{\Gamma}_{\rm DY}(z,\mathbf{b}) = \bar{q}(zn+\mathbf{b})[zn+\mathbf{b},-\infty n+\mathbf{b}]\Gamma[-\infty n-\mathbf{b},-zn-\mathbf{b}]q(-zn-\mathbf{b})$$

and

$$\mathcal{U}^{\Gamma}_{\rm DIS}(z,\mathbf{b}) = \bar{q}(zn+\mathbf{b})[zn+\mathbf{b},+\infty n+\mathbf{b}]\Gamma[+\infty n-\mathbf{b},-zn-\mathbf{b}]q(-zn-\mathbf{b})$$

$$\Gamma = \{\gamma^+, \gamma^+\gamma_5, i\sigma_T^{\alpha+}\gamma_5\}$$

Wilson lines differ from DY and SIDIS

TMD DISTRIBUTION DEFINITION

$$\Phi^{[\Gamma]}_{q\leftarrow h}(x,\mathbf{b}) = \int \frac{dz}{2\pi} e^{-2ixzp^+} \langle P,S|\mathcal{U}^{\Gamma}\left(z,\frac{\mathbf{b}}{2}\right)|P,S\rangle$$

Renormalization Universality of TMD's

Formal definition of TMD operator

Applying these operators to the hadron states we obtain unsubtracted TMDs

$$\Phi_{q\leftarrow h}(x,b_T) = \langle h|O_q^{bare}(x,b_T)|h\rangle$$
$$\Delta_{q\to h}(z,b_T) = \langle h|\mathbb{O}_q^{bare}(z,b_T)|h\rangle$$

To define individual TMD we have to take into account rapidity divergences, UV divergences and overlap regions

$$F_{q\leftarrow h}(x,b_T;\zeta,\mu) = \sqrt{S(b_T;\zeta)}\langle h|Z_q(\mu)O_q^{bare}(x,b_T)|h\rangle\Big|_{zero-bin}$$
$$D_{q\to h}(x,b_T;\zeta,\mu) = \sqrt{S(b_T;\zeta)}\langle h|Z_q(\mu)\mathbb{O}_q^{bare}(x,b_T)|h\rangle\Big|_{zero-bin}$$

- μ is scale of UV renormalization.
- ζ is scale of rapidity-divergences separation.

The renormalization factors for rapidity divergences are universal

Mulders-Tangerman '96,
Boer Mulders ' 98
Mulders, 2001 (gluons)
Boer, Mulders, Collins
Mulders, Buffing, Mukherjee 2013
....

Polarization effects

Quark Polarization

Nucleon Polarization

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T}, h_{1T}^{\perp}$

Gluon Polarization

Nucleon Polarization

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

Time-reversal flip

$$\Phi_{q\leftarrow h}^{[\gamma^+]}(x,\mathbf{b}) = f_1(x,\mathbf{b})$$

$$\Phi_{q\leftarrow h}^{[\gamma^+\gamma_5]}(x,\vec{b}) = \lambda g_{1L}(x,\mathbf{b})$$

$$\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma_5]}(x,\mathbf{b}) = s_T^{\alpha} h_1(x,\mathbf{b})$$

Mulders-Tangerman '96,
Boer Mulders ' 98
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Gluon Polarization

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L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

Time-reversal flip

$$\Phi_{q\leftarrow h}^{[\gamma^+]}(x,\mathbf{b}) = f_1(x,\mathbf{b}) + i\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M f_{1T}^\perp(x,\mathbf{b}),$$

$$\Phi_{q\leftarrow h}^{[\gamma^+\gamma_5]}(x,\vec{b}) = \lambda g_{1L}(x,\mathbf{b}) + i b_\mu s_T^\mu M g_{1T}(x,\mathbf{b}),$$

$$\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma_5]}(x,\mathbf{b}) = s_T^\alpha h_1(x,\mathbf{b}) - i\lambda b^\alpha M h_{1L}^\perp(x,\mathbf{b})$$

$$+ i\epsilon_T^{\alpha\mu} b_\mu M h_1^\perp(x,\mathbf{b}) + \frac{M^2\mathbf{b^2}}{2}\left(\frac{g_T^{\alpha\mu}}{2} + \frac{b^\alpha b^\mu}{\mathbf{b}^2}\right) s_{T\mu} h_{1T}^\perp(x,\mathbf{b})$$

The polarization effects on TMDs appear expanding in the transverse coordinate

See Daniel Gutierrez talk

Collinear and geometrical twist and OPE

$$\mathcal{U}^{\Gamma}(z,\mathbf{b}) = \mathcal{U}^{\Gamma}(z,\mathbf{0}) + b^{\mu}\frac{\partial}{\partial b^{\mu}}\mathcal{U}^{\Gamma}(z,\mathbf{b})\Big|_{\mathbf{b}=\mathbf{0}} + O(\mathbf{b}^2).$$

The impact parameter expansion must be re-phrased into a dimensional-spin physical
TWIST EXPANSION

$$\mathcal{U}(z,\mathbf{b}) = \sum_i \left[C_i * \mathcal{O}_i^{\mathrm{tw2}}\right](z) + \mathbf{b}^{\mu}\sum_i \left[\tilde{C}_i * \mathcal{O}_{\mu,i}^{\mathrm{tw3}}\right](z) + O(\mathbf{b}^2)$$

and the coefficients match onto transverse momentum integrated functions of definite twist.

Collinear and geometrical twist and OPE

$$\mathcal{U}^{\Gamma}(z,\mathbf{b}) = \mathcal{U}^{\Gamma}(z,\mathbf{0}) + b^{\mu}\frac{\partial}{\partial b^{\mu}}\mathcal{U}^{\Gamma}(z,\mathbf{b})\Big|_{\mathbf{b}=\mathbf{0}} + O(\mathbf{b}^2).$$

The impact parameter expansion must be re-phrased into a dimensional-spin physical
TWIST EXPANSION

$$\mathcal{U}^{\Gamma}_{\text{DY}}(z,\mathbf{0}) = \bar{q}(zn)[zn,-\infty n]\Gamma[-\infty n,-zn]q(-zn) = \bar{q}(zn)[zn,-zn]\Gamma q(-zn)$$
$$\mathcal{U}^{\Gamma}_{\text{DIS}}(z,\mathbf{0}) = \bar{q}(zn)[zn,+\infty n]\Gamma[+\infty n,-zn]q(-zn) = \bar{q}(zn)[zn,-zn]\Gamma q(-zn)$$

The distributions differ only for the orientation of Wilson lines.. no practical effects

Collinear and geometrical twist and OPE

$$\mathcal{U}^{\Gamma}(z,\mathbf{b}) = \mathcal{U}^{\Gamma}(z,\mathbf{0}) + b^{\mu}\frac{\partial}{\partial b^{\mu}}\mathcal{U}^{\Gamma}(z,\mathbf{b})\Big|_{\mathbf{b}=\mathbf{0}} + O(\mathbf{b}^2).$$

The impact parameter expansion must be re-phrased into a dimensional-spin physical
TWIST EXPANSION

$$\frac{\partial}{\partial b^{\mu}}\mathcal{U}^{\Gamma}_{\mathrm{DY}}(z,\mathbf{b})\Big|_{\mathbf{b}=0} = \bar{q}(zn)\left(\overleftarrow{D_{\mu}}[zn,-zn] - [zn,-zn]\overrightarrow{D_{\mu}}\right)\Gamma q(-zn)$$
$$+ ig\left(\int_{-\infty}^{z} + \int_{-\infty}^{-z}\right) d\tau\ \bar{q}(zn)[zn,\tau n]\Gamma F_{\mu+}(\tau n)[\tau n,-zn]q(-zn)$$

The first power correction has a more structured form…

Collinear and geometrical twist and OPE

$$\mathcal{U}^\Gamma(z,\mathbf{b}) = \mathcal{U}^\Gamma(z,\mathbf{0}) + b^\mu \frac{\partial}{\partial b^\mu}\mathcal{U}^\Gamma(z,\mathbf{b})\Big|_{\mathbf{b}=\mathbf{0}} + O(\mathbf{b}^2).$$

The impact parameter expansion must be re-phrased into a dimensional-spin physical
TWIST EXPANSION

$$\frac{\partial}{\partial b^\mu}\mathcal{U}^\Gamma_{\text{DIS}}(z,\mathbf{b})\Big|_{\mathbf{b}=0} = \bar{q}(zn)\left(\overleftarrow{D}_\mu[zn,-zn] - [zn,-zn]\overrightarrow{D}_\mu\right)\Gamma q(-zn)$$
$$-ig\left(\int_z^{+\infty} + \int_{-z}^{+\infty}\right) d\tau\ \bar{q}(zn)[zn,\tau n]\Gamma F_{\mu+}(\tau n)[\tau n,-zn]q(-zn)$$

The first power correction has a more structured form…

Intermediate result

$$\mathcal{O}_\Gamma(z) = \bar{q}(zn)[zn, -zn]\Gamma q(-zn)$$

$$\mathcal{T}^\mu_\Gamma(z_1, z_2, z_3) = g\bar{q}(z_1 n)[z_1 n, z_2 n]\Gamma F^{\mu+}(z_2 n)[z_2 n, z_3 n]q(z_3 n)$$

The twist expansion can be done taking the light-cone limit of the (twist-) expansion: both twist-2 and twist-3 structure come out

$$\mathcal{U}^\Gamma_{\text{DY}}(z, \mathbf{b}) = \mathcal{O}_\Gamma(z) + b_\mu\Big\{\lim_{y\to zn}\frac{\partial}{\partial y_\mu}\mathcal{O}_\Gamma(y) - i\int_{-1}^{1} dv\, vz\,\mathcal{T}^\mu_\Gamma(z, vz, -z)$$
$$+i\left(\int_{-\infty}^{z} + \int_{-\infty}^{-z}\right) d\tau \mathcal{T}^\mu_\Gamma(z, \tau, -z)\Big\} + O(\mathbf{b}^2)$$

$$\mathcal{U}^\Gamma_{\text{DIS}}(z, \mathbf{b}) = \mathcal{O}_\Gamma(z) + b_\mu\Big\{\lim_{y\to zn}\frac{\partial}{\partial y_\mu}\mathcal{O}_\Gamma(y) - i\int_{-1}^{1} dv\, vz\,\mathcal{T}^\mu_\Gamma(z, vz, -z)$$
$$-i\left(\int_{z}^{\infty} + \int_{-z}^{\infty}\right) d\tau \mathcal{T}^\mu_\Gamma(z, \tau, -z)\Big\} + O(\mathbf{b}^2)$$

Tw-3 functions

The matching on collinear functions involves several types of collinear twist-3 functions

$$\langle P,S|\mathcal{T}^{\mu}_{\gamma^+}|P,S\rangle = 2(p^+)^2\tilde{s}^{\mu}_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}T(x_1,x_2,x_3)$$

$$\langle P,S|\mathcal{T}^{\mu}_{\gamma^+\gamma^5}|P,S\rangle = 2i(p^+)^2 s^{\mu}_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\Delta T(x_1,x_2,x_3)$$

$$\langle P,S|\mathcal{T}^{\mu}_{i\sigma^{\alpha+}\gamma^5}|P,S\rangle = 2(p^+)^2\epsilon^{\mu\alpha}_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\delta T_\epsilon(x_1,x_2,x_3)$$
$$+ 2i(p^+)^2\lambda g^{\mu\alpha}_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\delta T_g(x_1,x_2,x_3)$$

WE IDENTIFY 4 DIFFERENT TYPES TW-3 FUNCTIONS

Time reversal and symmetries of collinear distributions

Using hermiticity and time-reversal all matching integrals are simplified

$$
\begin{aligned}
T(x_1,x_2,x_3) &= T(-x_3,-x_2,-x_1),\\
\Delta T(x_1,x_2,x_3) &= -\Delta T(-x_3,-x_2,-x_1),\\
\delta T_\epsilon(x_1,x_2,x_3) &= \delta T_\epsilon(-x_3,-x_2,-x_1),\\
\delta T_g(x_1,x_2,x_3) &= -\delta T_g(-x_3,-x_2,-x_1).
\end{aligned}
$$

Useful notation

One can use these functions as PDF's with the same border conditions

$$
\begin{aligned}
T^{(n)}(x) &= \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)+(-1)^n\delta(x+x_1)\right)T(x_1,x_2,x_3)\\
\Delta T^{(n)}(x) &= \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)-(-1)^n\delta(x+x_1)\right)\Delta T(x_1,x_2,x_3)\\
\delta T_\epsilon^{(n)}(x) &= \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)+(-1)^n\delta(x+x_1)\right)\delta T_\epsilon(x_1,x_2,x_3)\\
\delta T_g^{(n)}(x) &= \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)-(-1)^n\delta(x+x_1)\right)\delta T_g(x_1,x_2,x_3)
\end{aligned}
$$

$$
\begin{aligned}
T^{(n)}(\pm 1) &= 0\\
\Delta T^{(n)}(\pm 1) &= 0\\
\delta T_\epsilon^{(n)}(\pm 1) &= 0\\
\delta T_g^{(n)}(\pm 1) &= 0
\end{aligned}
$$

Vector operator

Notation: $\tilde{F}(z_1, z_2, z_3) = \int [dx] e^{-ip^+(x_1 z_1 + x_2 z_2 + x_3 z_3)} F(x_1, x_2, x_3)$

An example of calculation:

$$\langle P, S | \mathcal{U}^{\gamma^+}_{\mathrm{DY}}(z, \frac{\mathbf{b}}{2}) | P, S \rangle = 2p^+ \int dx e^{2ixzp^+} f_1(x) + 2(p^+)^2 M \tilde{s}^\mu \frac{b_\mu}{2} \Big[- i \int_{-1}^{1} dv v z \tilde{T}(z, vz, -z) + i \left(\int_{-\infty}^{z} + \int_{-\infty}^{-z} \right) d\tau \tilde{T}(z, \tau, -z) \Big] + O(\mathbf{b}^2),$$

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An example of calculation:

$$\langle P, S|\mathcal{U}^{\gamma^+}_{\mathrm{DY}}(z, \frac{\mathbf{b}}{2})|P, S\rangle = 2p^+ \int dx e^{2ixzp^+} f_1(x) + 2(p^+)^2 M \tilde{s}^\mu \frac{b_\mu}{2} \Big[- i \int_{-1}^{1} dv vz \tilde{T}(z, vz, -z) + i \left(\int_{-\infty}^{z} + \int_{-\infty}^{-z} \right) d\tau \tilde{T}(z, \tau, -z) \Big] + O(\mathbf{b}^2),$$

Null because of symmetries

$$\left(\int_{-\infty}^{z} + \int_{-\infty}^{-z} \right) d\tau \tilde{T}(z, \tau, -z) = \int_{-\infty}^{\infty} d\tau \tilde{T}(z, \tau, -z).$$

Then we go back to distributions

$$\Phi^{[\Gamma]}_{q\leftarrow h}(x, \mathbf{b}) = \int \frac{dz}{2\pi} e^{-2ixzp^+} \langle P, S|\mathcal{U}^{\Gamma}\left(z, \frac{\mathbf{b}}{2}\right)|P, S\rangle.$$

Vector operator

$$\Phi^{[\Gamma]}_{q\leftarrow h}(x,\mathbf{b}) = \int \frac{dz}{2\pi} e^{-2ixzp^+} \langle P,S|\mathcal{U}^\Gamma\left(z,\frac{\mathbf{b}}{2}\right)|P,S\rangle.$$

Results

$$\text{(DY)} \qquad \Phi^{[\gamma^+]}_{q\leftarrow h}(x,\mathbf{b}) = f_1(x) + ib_\mu \tilde{s}^\mu_T M\, \pi T(-x,0,x) + O(\mathbf{b}^2)$$

$$\text{(SIDIS)} \qquad \Phi^{[\gamma^+]}_{q\leftarrow h}(x,\mathbf{b}) = f_1(x) - ib_\mu \tilde{s}^\mu_T M\, \pi T(-x,0,x) + O(\mathbf{b}^2)$$

We discover that unpolarized and polarized distributions have a relative normalization!!

$$\text{(DY)} \qquad f_{1T}^\perp(x,\mathbf{b}) = \pi T(-x,0,x) + O(\mathbf{b}^2),$$

$$\text{(SIDIS)} \qquad f_{1T}^\perp(x,\mathbf{b}) = -\pi T(-x,0,x) + O(\mathbf{b}^2).$$

Axial operator

Notation: $\tilde{F}(z_1, z_2, z_3) = \int [dx] e^{-ip^+(x_1 z_1 + x_2 z_2 + x_3 z_3)} F(x_1, x_2, x_3)$

An more involved example of calculation:

$$\langle P, S | \mathcal{U}_{\text{DY}}^{\gamma^+\gamma^5}(z, \frac{\mathbf{b}}{2}) | P, S \rangle = 2\lambda p^+ \int dx e^{2ixzp^+} g_1(x) + 2M s_T^\mu \frac{b_\mu}{2} \Big[\int du e^{2iuzp^+} \frac{g_1(u) - g_T(u)}{z}$$
$$+ (p^+)^2 \int_{-1}^{1} dv v z \Delta\tilde{T}(z, vz, -z) - (p^+)^2 \left(\int_{-\infty}^{z} + \int_{-\infty}^{-z} \right) d\tau \Delta\tilde{T}(z, \tau, -z) \Big] + O(\mathbf{b}^2)$$

The simplifications now are non-trivial

Axial operator

The moments of the 3-point functions enter …

$$\int \frac{dz}{2\pi} e^{-2ixzp^+} \int_{-1}^{1} dv vz \Delta\tilde{T}(z, vz, -z) = \frac{i}{(p^+)^2}\Big[\frac{\Delta T^{(1)}(x)}{2} + \int_{-1}^{1} du \int_0^1 dy\, u \Delta T^{(2)}(u)\delta(x - yu)\Big],$$

$$\int \frac{dz}{2\pi} e^{-2ixzp^+} \int du e^{2iuzp^+} \frac{g_1(u) - g_T(u)}{z} = i \int_{-1}^{1} du \int_0^1 dy\, u(g_1(u) - g_T(u))\delta(x - uy).$$

… and we have non-trivial relations…

$$\int \frac{dz}{2\pi} e^{-2ixzp^+} \Big(\int_{-\infty}^{z} + \int_{-\infty}^{-z}\Big) d\tau \Delta\tilde{T}(z, \tau, -z) = \frac{i}{(p^+)^2} \frac{\Delta T^{(1)}(x)}{2},$$

$$\int \frac{dz}{2\pi} e^{-2ixzp^+} \Big(\int_{z}^{\infty} + \int_{-z}^{\infty}\Big) d\tau \Delta\tilde{T}(z, \tau, -z) = \frac{-i}{(p^+)^2} \frac{\Delta T^{(1)}(x)}{2},$$

… and a final twist expansion…

$$g_T(x) = \int_0^1 dy \int_{-1}^{1} du \delta(x - yu)\Big[g_1(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}(1 - \delta(\bar{y})) + \Delta T^{(2)}(u)\Big]$$

Axial operator

Final result

$$g_{1T}(x,\mathbf{b}) = x\int_x^1 \frac{du}{u}\left(g_1(u) + \Delta T^{(2)}(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}\right) + O(\mathbf{b}^2) \quad \text{for } x > 0$$

$$g_{1T}(x,\mathbf{b}) = x\int_{-1}^x \frac{du}{|u|}\left(g_1(u) + \Delta T^{(2)}(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}\right) + O(\mathbf{b}^2) \quad \text{for } x < 0$$

In agreement with results based on Lorentz invariant constraints by K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, and M. Schlegel, Phys. Rev. D93, 054024 (2016), arXiv. 1512.07233

Compendium

Name	Function	Leading matching function	Twist of leading matching	Maximum known order of coef.function
unpolarized	$f_1(x,\boldsymbol{b})$	f_1	tw-2	NNLO (a_s^2)
Sivers	$f_{1T}^{\perp}(x,\boldsymbol{b})$	T	tw-3	LO (a_s^0)
helicity	$g_{1L}(x,\boldsymbol{b})$	g_1	tw-2	NLO (a_s^1)
worm-gear T	$g_{1T}(x,\boldsymbol{b})$	$g_1,\ T,\ \Delta T$	tw-2/3	LO (a_s^0)
transversity	$h_1(x,\boldsymbol{b})$	h_1	tw-2	NNLO(a_s^2)
Boer-Mulders	$h_1^{\perp}(x,\boldsymbol{b})$	δT_ϵ	tw-3	LO (a_s^0)
worm-gear L	$h_{1L}^{\perp}(x,\boldsymbol{b})$	$h_1,\ \delta T_g$	tw-2/3	LO (a_s^0)
pretzelosity**	$h_{1T}^{\perp}(x,\boldsymbol{b})$	–	tw-4	–

See Daniel Gutierrez Reyes talk:
we have only 7 leading twist quark TMDs (and not 8)

The normalization of all leading twist TMDs with respect to the unpolarized TMDs are calculated

$$f_{1T}^{\perp}(x,\mathbf{b}) = \pm\pi T(-x,0,x) + O(\mathbf{b^2})$$

$$g_{1T}(x,\mathbf{b}) = x\int_{-1}^{1} du \int_0^1 dy\delta(x-uy)\left(g_1(u) + \Delta T^{(2)}(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u)}{2u}\right) + O(\mathbf{b}^2)$$

$$h_1^{\perp}(x,\mathbf{b}) = \mp\pi\delta T_\epsilon(-x,0,x) + O(\mathbf{b}^2)$$

$$h_{1L}^{\perp}(x,\mathbf{b}) = -x\int_{-1}^{1} du \int_0^1 dy\delta(x-uy)y\left(h_1(u) + \delta T_g^{(2)}(u) - \frac{\delta T_g^{(1)}(u)}{u}\right) + O(\mathbf{b}^2)$$

Comparison with lattice

Ratios of TMDs in b-space can be computed on the lattice (B. Yoon et al. Phys. Rev. D96, 094508 (2017),1706.03406)

$$\frac{g_{1T}^{(0)}(b \simeq 0.34)}{f_1^{(0)}(b \simeq 0.34)} \approx 0.2.$$

In our case, assuming Wandzura-Wilczeck approximation

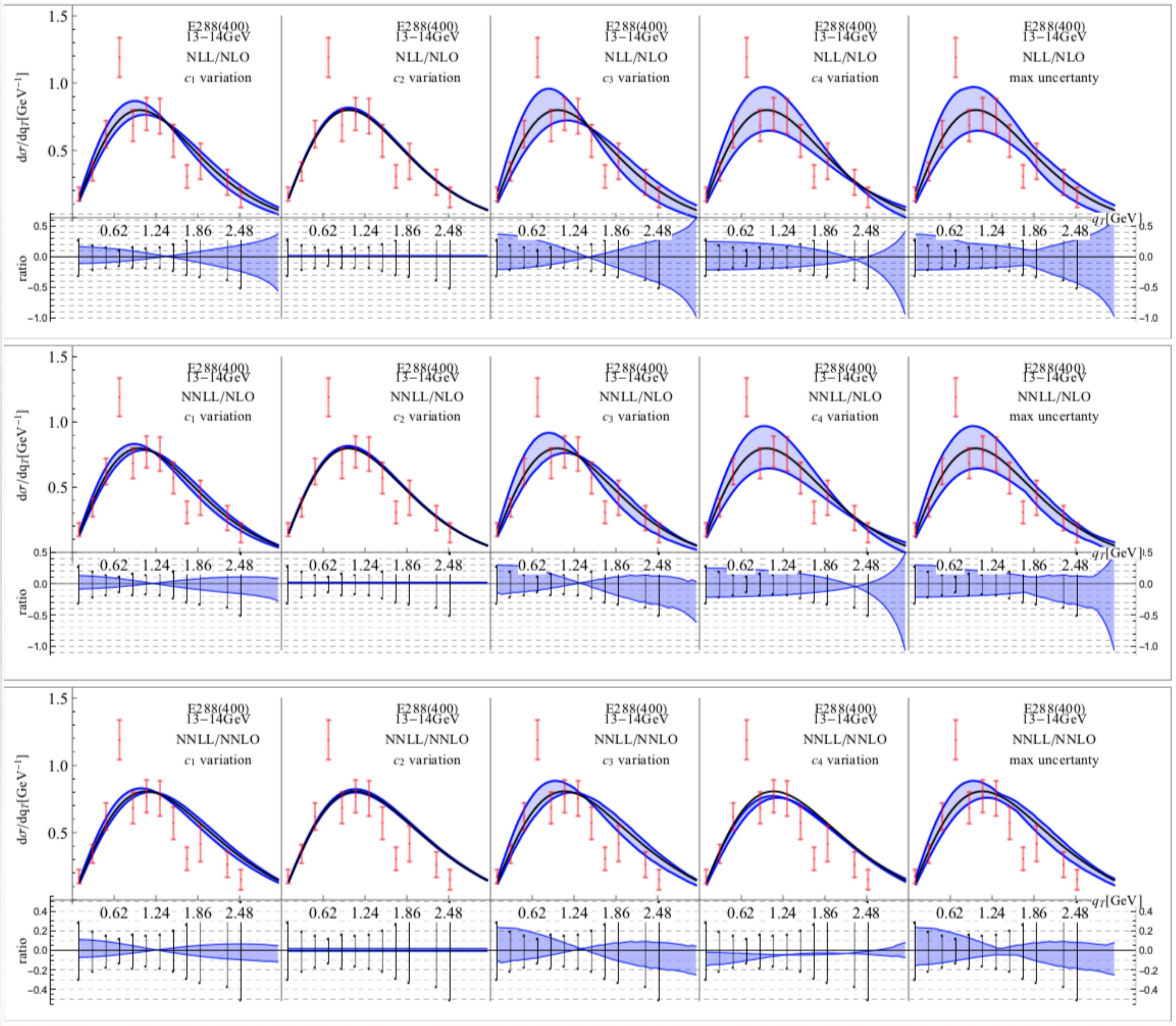
$$\frac{g_{1T}^{(0)}(\mathbf{b})}{f_1^{(0)}(\mathbf{b})} = \frac{g_1^{(1)} + \Delta T^{(2,1)}}{2 f_1^{(0)}} + O(\alpha_s) + O(\mathbf{b}^2) \simeq \frac{g_1^{(1)}}{2 f_1^{(0)}} \simeq 0.13$$

Conclusions

- The twist-3 distributions are theoretically challenging but a perturbative estimation at least at NLO is necessary for phenomenology
- We have presented a formalism to extract the matching to collinear functions using OPE
- For the moment just LO results of quark TMDPDFs: NORMALIZATION OF POLARIZED TMDs
- Some results can be directly compared with lattice, in some approximation

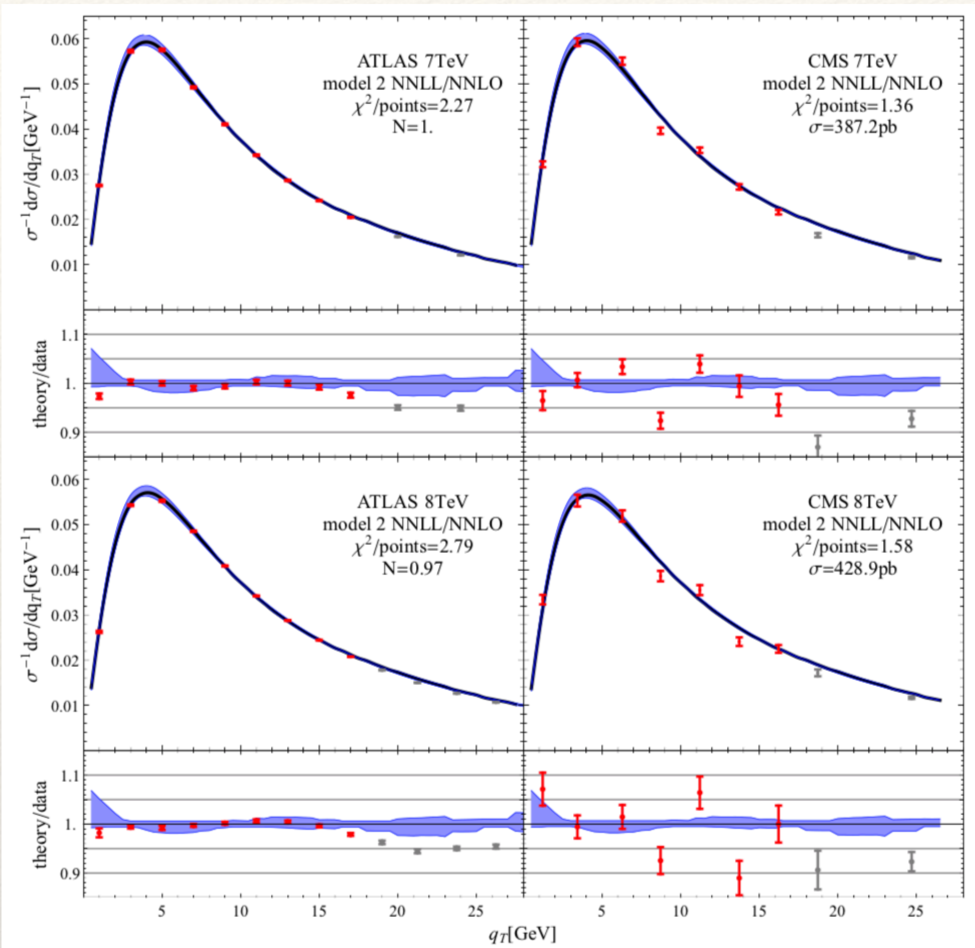
Back up

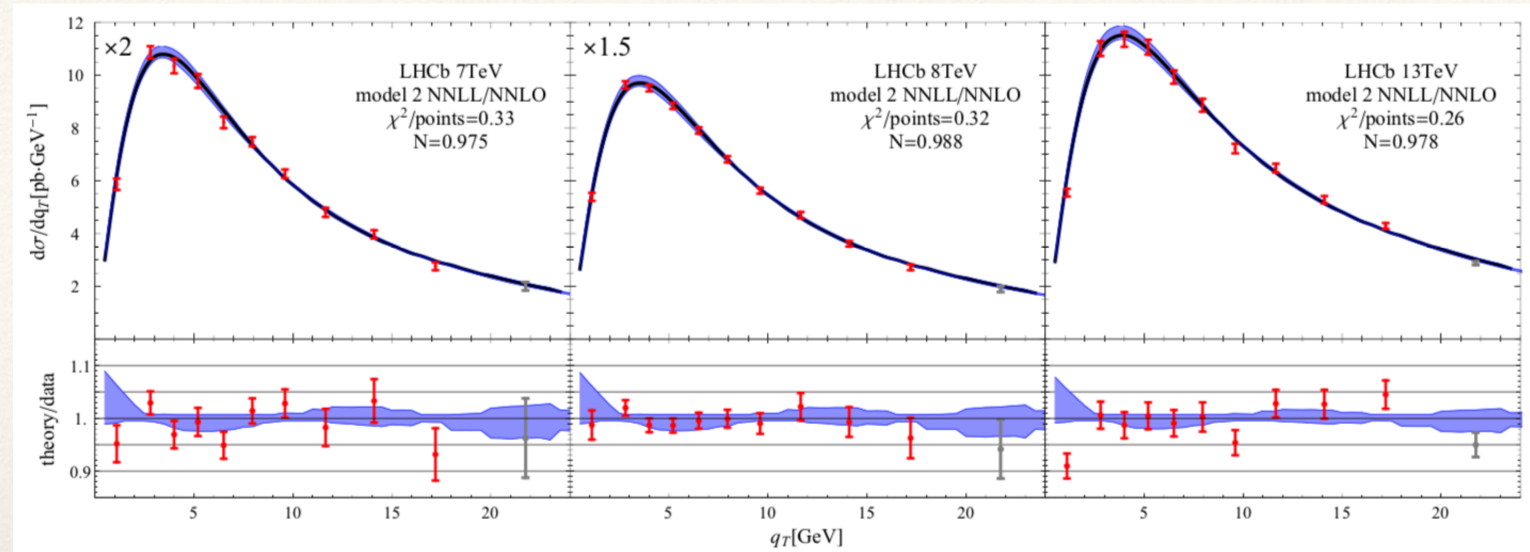
E288 arTeMiDe version 1.1



arTeMiDe version 1.1 (naive zeta-prescription)

Results for LHC in Z-production ….





…and Drell-Yan at NNLO