Transverse Force Tomography

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- GPDs $\longrightarrow$ 3D imaging of the nucleon
- twist-3 PDFs $g_2(x) \longrightarrow \perp$ force

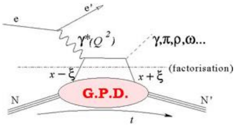
$\hookrightarrow$ twist-3 GPDs $\longrightarrow \perp$ force tomography

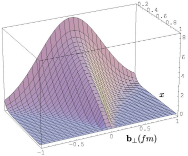
Motivation: why twist-3 GPDs

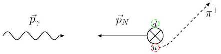
  - twist-3 GPD $G_2^q \longrightarrow L^q$
  - twist 3 PDF $g_2(x) \longrightarrow \perp$ force
  - twist 2 GPDs $\longrightarrow \perp$ imaging (of quark densities)

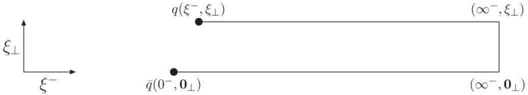
  $\hookrightarrow$ twist 3 GPDs $\longrightarrow \perp$ imaging of $\perp$ forces

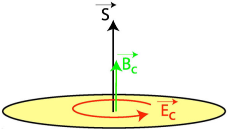
- Summary
- Outlook











MB, PRD62, 071503 (2000)

- form factors: $\stackrel{FT}{\longleftrightarrow} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction x

$\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction x

Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} GPD(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

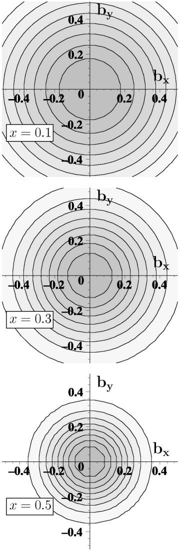
$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i \in q,g} \mathbf{r}_{\perp,i} x_i$

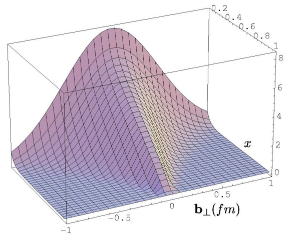
- probabilistic interpretation!
- no relativistic corrections: Galilean subgroup! (MB,2000)

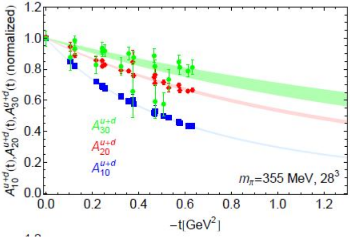
$\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections (MB,2003;G.A.Miller, 2007)

Impact parameter dependent quark distributions

$q(x, \mathbf{b}_\perp)$ for unpol. p



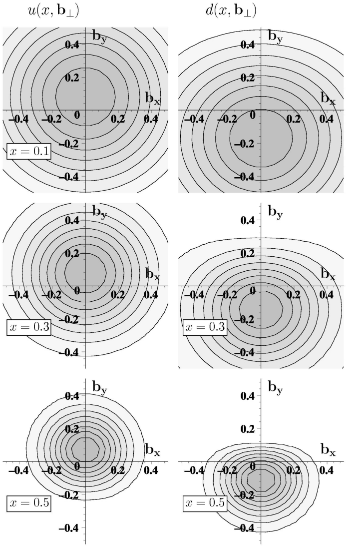




unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- $F_1(-\boldsymbol{\Delta}_\perp^2) = \int dx H(x, 0, -\boldsymbol{\Delta}_\perp^2)$
- x = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$



proton polarized in $+\hat{x}$ direction

no axial symmetry!

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

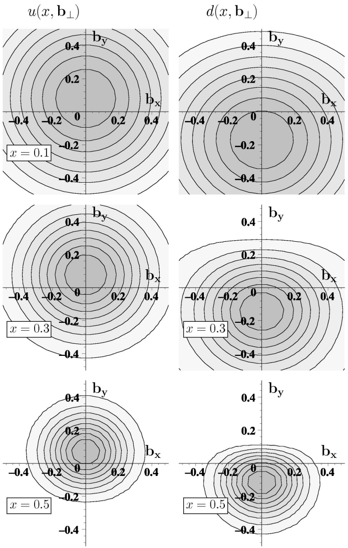
$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

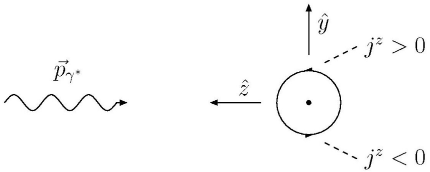
Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

intuitive explanation

- moving Dirac particle with anomalous magnetic moment has electric dipole moment $\perp$ to $\vec{p}$ and $\perp$ magnetic moment

$\hookrightarrow$ γ^* 'sees' flavor dipole moment of oncoming nucleon





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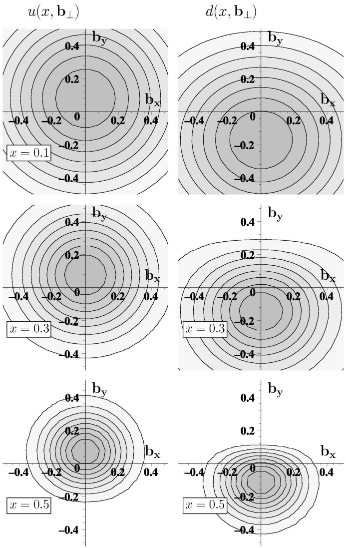

proton polarized in $+\hat{x}$ direction

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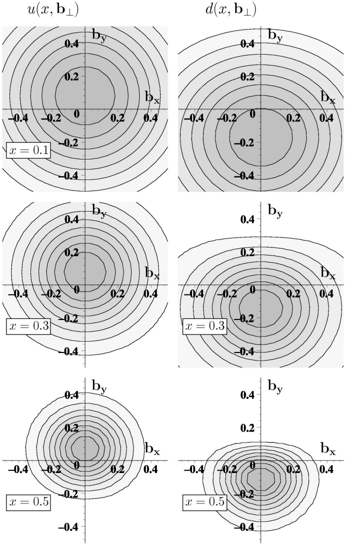
sign & magnitude of the average shift

model-independently related to p/n anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$



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$$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

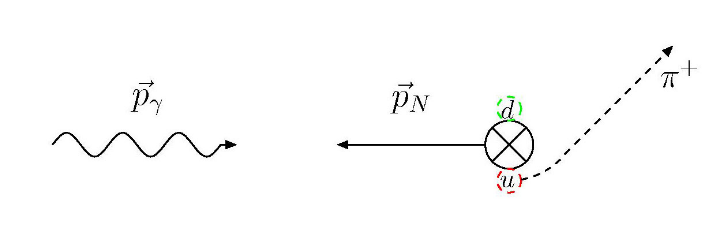
↪ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_u^p = 2\kappa_n + \kappa_p = -2.033$

↪ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}\left(\pm 0.2 fm\right)$!!!!

example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\rightarrow$ chromodynamic lensing

$\Rightarrow$ $\kappa_p\,,\kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

- confirmed by HERMES & COMPASS data

$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ gF^{+y}(0)q(0) \right| P,S \right\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \quad \text{for } \vec{v} = (0,0,-1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral

$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

sign of d_2

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of d_2^q: opposite Sivers

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)

Twist-3 PDFs $\longrightarrow \perp$ Force on Quarks in DIS

chirally even spin-dependent twist-3 PDF $g_2(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 g_2(x) \Rightarrow \perp$ force on unpolarized quark in $\perp$ polarized target

$\hookrightarrow$ 'Sivers force'

scalar twist-3 PDF $e(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 e(x) \Rightarrow \perp$ force on $\perp$ polarized quark in unpolarized target

$\hookrightarrow$ 'Boer-Mulders force'

chirally odd spin-dependent twist-3 PDF $h_2(x)$

M.Abdallah & MB, PRD94 (2016) 094040

- $\int dx\, x^2 h_2(x) = 0$

$\hookrightarrow$ $\perp$ force on $\perp$ pol. quark in long. pol. target vanishes due to parity

- $\int dx\, x^3 h_2(x) \Rightarrow$ long. gradient of $\perp$ force on $\perp$ polarized quark in long. polarized target

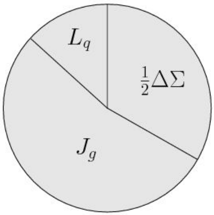
$\hookrightarrow$ chirally odd 'wormgear force'

force distributions

F.Aslan & MB: *work in progress*

- use FT of twist-3 GPDs to map these forces in the $\perp$ plane

Ji decomposition



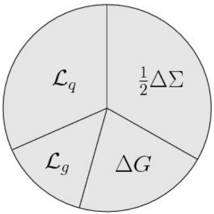
'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

$$\vec{L}_q = \vec{r} \times \left(\vec{p} - g\vec{A}\right)$$

- manifestly gauge inv. & local
- DVCS $\longrightarrow$ GPDs $\longrightarrow L^q$

Jaffe-Manohar decomposition



'pizza quattro stagioni'

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta q + \mathcal{L}_q\right) + \Delta G + \mathcal{L}_g$$

$$\vec{\mathcal{L}}_q = \vec{r} \times \vec{p}$$

- manifestly gauge inv. $\rightarrow$ nonlocal
- $\overrightarrow{p}\,\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i\in q,g} \mathcal{L}^i$

How large is difference $\mathcal{L}_q - L_q$ in QCD and what does it represent?

Ji

$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$

$L_q = \vec{r} \times \left(\vec{p} - g\vec{A}\right)$

Jaffe-Manohar

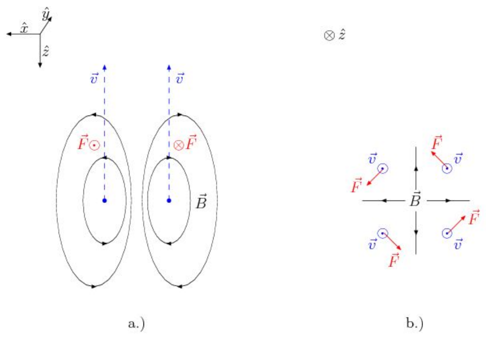
$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$

$\vec{\mathcal{L}}^q = \vec{r} \times \vec{p}$

difference $\mathcal{L}^q - L^q$ (MB, PRD 88 (2013) 014014)

$\mathcal{L}^q - L^q = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

example: torque in magnetic dipole field



difference $\mathcal{L}^q - L^q$

$\mathcal{L}^q_{JM} - L^q_{Ji} = \Delta L^q_{FSI}$ = change in OAM as quark leaves nucleon

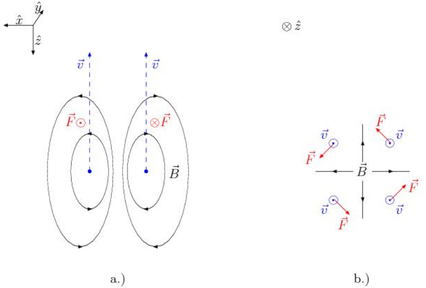
$$\mathcal{L}^q_{JM} - L^q_{Ji} = -g \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+ \left[\vec{x}\times \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)\right]^z q(\vec{x})|P,S\rangle$$

e^+ moving through dipole field of e^-

- consider e^- polarized in $+\hat{z}$ direction
- ↪ $\vec{\mu}$ in $-\hat{z}$ direction (Figure)
- e^+ moves in $-\hat{z}$ direction
- ↪ net torque negative

sign of $\mathcal{L}^q - L^q$ in QCD

- color electric force between two q in nucleon attractive
- ↪ same as in positronium
- spectator spins positively correlated with nucleon spin
- ↪ expect $\mathcal{L}^q - L^q < 0$ in nucleon



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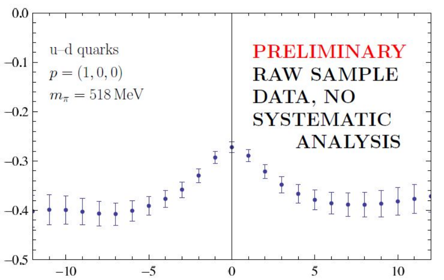
lattice QCD (M.Engelhardt)

- L_{staple} vs. staple length
- ↪ L^q_{Ji} for length $= 0$
- ↪ $\mathcal{L}^q_{JM}$ for length $\to \infty$



- shown $L^u_{staple} - L^d_{staple}$
- similar result for each ΔL^q

twist-3 GPDs (Meissner, Metz, Schlegel) – example $\Gamma = \gamma^x\gamma_5$

$$\int dz^- e^{ixz^-\bar{p}^+}\langle P'|\bar{q}(z^-/2)\gamma^x\gamma_5 q(-z^-/2)|p\rangle = \frac{-i}{2\left(P^+\right)^2}\times$$

$$\bar{u}(p')\left[i\sigma^{+y}H'_{2T} + \frac{\gamma^+\Delta^i - \Delta^+\gamma^y}{2M}E'_{2T} + \frac{P^+\Delta^y - \Delta^+P^y}{M^2}\tilde{H}'_{2T} + \frac{\gamma^+P^y - P^+\gamma^y}{2M}\tilde{E}'_{2T}\right]u(p)$$

- $H'_{2T}(x,\xi,t)$ etc. all twist 3 GPDs
- similar for $\gamma = \mathbb{1}$, γ_5, $\gamma^\perp$, $\sigma^{ij}\gamma_5$, $\sigma^{+-}\gamma_5$ (16 GPDs)

physics of twist-3 GPDs

- x^2-moment $\Rightarrow \langle p'|\bar{q}(0)\gamma^x\gamma_5 \overset{\leftrightarrow}{D}_-^2 q(0)|p\rangle \quad \rightsquigarrow \quad \langle p'|\bar{q}(0)\gamma^+F^{+y}q(0)|p\rangle$ + twist-2

$\hookrightarrow$ x^2-moment of twist-3 GPDs $\rightsquigarrow$ matrix elements of 'force operator' $\bar{q}\gamma^+F^{+y}q$

$\hookrightarrow$ $\perp$ momentum transfer $\rightsquigarrow$ spatial resolution

$\hookrightarrow$ transverse force tomography ($\perp$ vectorfields of $\perp$ forces)

- $\Gamma = \gamma^x\gamma_5$: force in $\hat{y}$ direction for unpolarized quarks

twist-3 GPDs (Meissner, Metz, Schlegel) – example $\Gamma = \gamma^\perp \gamma_5$

$$\bar{u}(p')\left[i\sigma^{+y} H'_{2T} + \frac{\gamma^+\Delta^i - \Delta^+\gamma^y}{2M} E'_{2T} + \frac{P^+\Delta^y - \Delta^+P^y}{M^2} \tilde{H}'_{2T} + \frac{\gamma^+P^y - P^+\gamma^y}{2M} \tilde{E}'_{2T}\right]u(p)$$

physics of twist-3 GPDs: transverse force tomography

- x^2-moment $\Rightarrow \langle p'|\bar{q}(0)\gamma^j\gamma_5 \overset{\leftrightarrow}{D}{}^2_- q(0)|p\rangle \quad \rightsquigarrow \quad \langle p'|\bar{q}(0)\gamma^+ F^{+i} q(0)|p\rangle$ + twist-2
- What can we learn from

$$\langle p'|\bar{q}(0)\gamma^+ F^{+i} q(0)|p\rangle \leftrightarrow$$

$$\bar{u}(p')\left[i\sigma^{+y} H'_{2T} + \frac{\gamma^+\Delta^y - \Delta^+\gamma^y}{2M} E'_{2T} + \frac{P^+\Delta^y - \Delta^+P^y}{M^2} \tilde{H}'_{2T} + \frac{\gamma^+P^y - P^+\gamma^y}{2M} \tilde{E}'_{2T}\right]u(p)$$

l.h.s.

- similar to twist-2, impact parameter interpretation only for $\Delta^+ = 0$
- Dirac matrix γ^+

↪ no sensitivity to quark pol.

r.h.s.

- different terms depend on nucleon polarization

⊥ localized state

$$|\mathbf{R}_\perp = 0, p^+, \Lambda\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp |\mathbf{p}_\perp, p^+, \Lambda\rangle$$

unpolarized quarks

$$F^i_{\lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda|\bar{q}(\mathbf{b}_\perp)\gamma^+ gF^{+i}(\mathbf{b}_\perp)q(\mathbf{b}_\perp)|\mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}_\perp, p^+, \Lambda|\bar{q}(0)\gamma^+ gF^{+i}(0)q(0)|\mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}}$$

↪ determine using x^2 moments of twist-3 GPDs

- polarized quarks $\gamma^+ \longrightarrow \gamma^+\gamma_5, i\sigma^{+\perp}$

physics of twist-3 GPDs: transverse force tomography

$$\langle p'|\bar{q}(0)\gamma^+ F^{+i} q(0)|p\rangle \leftrightarrow \quad x^2\text{moment of twist-2 part of}$$

$$\bar{u}(p')\left[i\sigma^{+y}H'_{2T} + \frac{\gamma^+\Delta^y}{2M}E'_{2T} + \frac{P^+\Delta^y}{M^2}\tilde{H}'_{2T} - \frac{P^+\gamma^y}{2M}\tilde{E}'_{2T}\right]u(p)$$

$i\sigma^{+y}H'_{2T}$

- nucleon polarized in $\hat{x}$ direction
- magnitude axially symmetric
- force in $\hat{y}$ direction

$\hookrightarrow$ forward limit: Sivers $\Rightarrow$ H'_{2T} $\perp$ position dependece of Sivers

unpolarized target

- contributing terms: E'_{2T} & $\tilde{H}'_{2T}$
- Gordon identity: $2P^+ \longrightarrow 2M\gamma^+ + i\sigma^{+j}\Delta^j$

$\hookrightarrow$ unpolarized target: $\frac{\gamma^+}{2M}\Delta^y\left(E'_{2T} + 2\tilde{H}'_{2T}\right)$

$\hookrightarrow$ $\perp$ force on unpolarized quarks in unpolarized target described by $\perp$ gradient of FT of $E'_{2T} + 2\tilde{H}'_{2T}$

physics of twist-3 GPDs: transverse force tomography

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$$\bar{u}(p')\left[i\sigma^{+y}H'_{2T} + \frac{\gamma^+\Delta^y}{2M}E'_{2T} + \frac{P^+\Delta^y}{M^2}\tilde{H}'_{2T} - \frac{P^+\gamma^y}{2M}\tilde{E}'_{2T}\right]u(p)$$

unpolarized target

- contributing terms: E'_{2T} & $\tilde{H}'_{2T}$
- Gordon identity: $2P^+ \longrightarrow 2M\gamma^+ + i\sigma^{+j}\Delta^j$

↪ unpolarized target: $\frac{\gamma^+}{2M}\Delta^y\left(E'_{2T} + 2\tilde{H}'_{2T}\right)$

↪ $\perp$ force on unpolarized quarks in unpolarized target described by $\perp$ gradient of FT of $E'_{2T} + 2\tilde{H}'_{2T}$

$\tilde{H}'_{2T}$

- additional $\Delta^y\frac{\sigma^{+j}\Delta^j}{2M^2}\tilde{H}'_{2T}$ from Gordon identity

↪ $\nabla^y\left(\vec{S}_\perp \times \vec{\nabla}_\perp\right)\text{FT}\left[\tilde{H}'_{2T}\right]$ force

physics of twist-3 GPDs: transverse force tomography

$$\langle p'|\bar{q}(0)\gamma^+ F^{+i} q(0)|p\rangle \leftrightarrow \quad x^2\text{moment of twist-2 part of}$$

$$\bar{u}(p')\left[i\sigma^{+y}H'_{2T} + \frac{\gamma^+\Delta^y}{2M}E'_{2T} + \frac{P^+\Delta^y}{M^2}\tilde{H}'_{2T} - \frac{P^+\gamma^y}{2M}\tilde{E}'_{2T}\right]u(p)$$

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$\hookrightarrow$ 'wormgear' force (g_{1L})

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- sigimer for $\gamma^\perp\gamma_5 \longrightarrow \gamma^\perp, \mathbb{1}, ...$
- x^2 moment of each twist-3 GPD (after subtracting WW and other twist-2) yields femtoimage of force

TMDs

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}, h_{1T}^\perp$

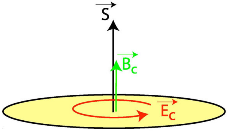
- each TMD affected by final state interactions (force)!
- for each case, twist-3 GPD provides insight about FSI contribution
- Sivers/Boer Mulders ($f_{1T}^\perp / h_1^\perp$) only FSI
- twist-3 GPDs provide local information

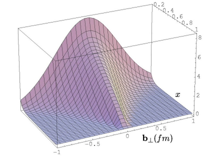
Summary

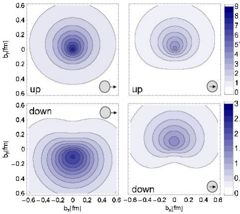
- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs

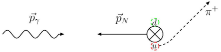
$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp} q$ distribution

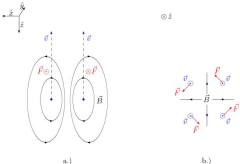
$\hookrightarrow$ $\perp$ force tomography











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