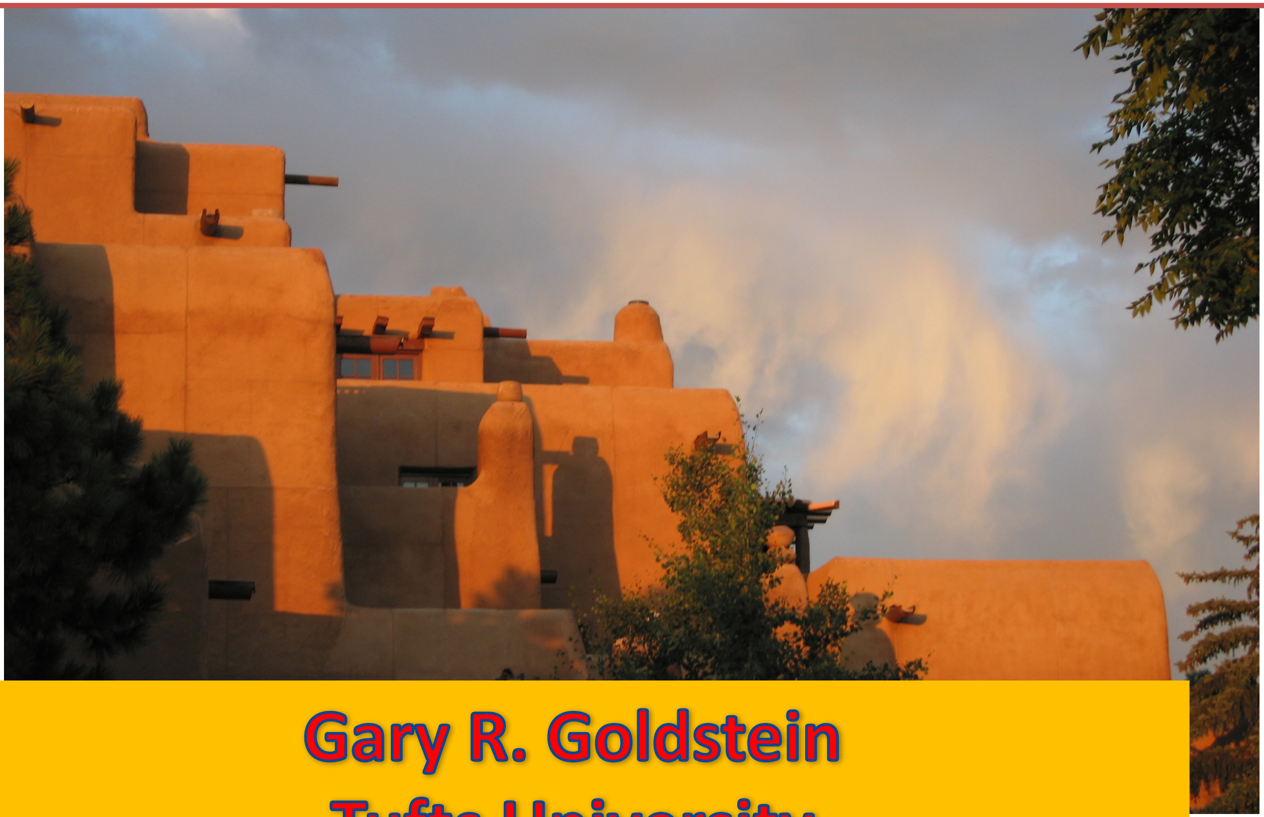
Gluon Transversity and Top Pair Production Spin Correlations



Gary R. Goldstein
Tufts University

QCD Evolution, May, 2018 –Sante Fe, NM

Gluon Distributions

Transversity

Top quarks

Gluon Transversity → Top Pair Spin Correlations

Collaborators: Gluons

Simonetta Liuti[2], Osvaldo Gonzalez Hernandez[3], Jon Poage[1]

- GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- GRG, Liuti, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
- J.Poage, Tufts U. dissertation (2016)
- GRG & Liuti, Hernandez, PoS QCDEV2017, 037 (2017)

Collaborators: Tops

Richard Dalitz,

Discussions: Krzysztof Sliwa, Hugo Beauchemin Tufts and Atlas

- Dalitz, R.H., and GRG, Phys. Rev. D45, 1531 (1992); Phys.Lett.B287, 225 (1992);
- GRG, Sliwa, K., Dalitz, R.H., Phys. Rev. D47, 967 (1993).

Collaborators: Transversity

Micheal J. Moravcsik,

- GRG & M.J. Moravcsik, Ann. Phys. 98, 128 (1976); ibid. 142, 219 (1982);
- Ibid. 195, 213 (1989).

See also K. Chen, GRG, R.L. Jaffe, X.-D. Ji, Nucl Phys B 445 (1995) 380-396.

OUTLINE – getting to gluons

- GPD's in Models– e.g. Reggeized spectator "flexible parameterization"
- electroproduction
  - Gluon GPDs Polarized Gluons? Transversity
- t+t-bar production & decay to measure Gluon polarization in p+p @ LHC. Inclusive → TMDs
- Top spin correlations & Observable quantities

GPD definitions – 8 quark + 8 gluon (twist 2)

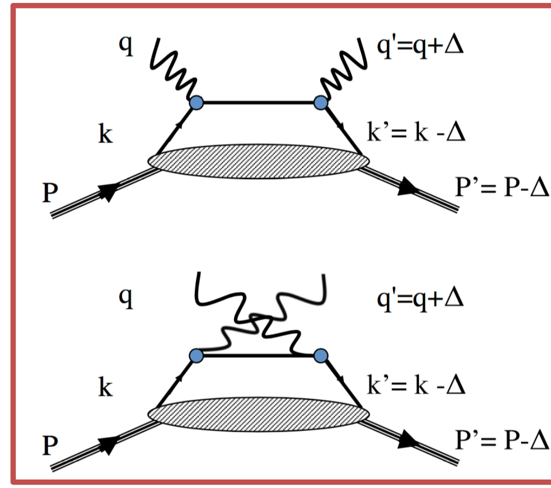
Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

Chiral even GPDs
-> Ji sum rule

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0} = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H^q\gamma^+ + E^q\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0} = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\tilde{H}^q\gamma^+\gamma_5 + \tilde{E}^q\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$



$$\left\langle J_q^x \right\rangle = \tfrac{1}{2}\int dx\left[H(x,0,0)+E(x,0,0)\right]x$$

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,i\sigma^{+i}\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0} = \frac{1}{2P^+}\bar{u}(p',\lambda')\Big[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\frac{P^+\Delta^i-\Delta^+P^i}{m^2} + E_T^q\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m} + \tilde{E}_T^q\frac{\gamma^+P^i-P^+\gamma^i}{m}\Big]u(p,\lambda$$

Chiral odd GPDs
-> transversity
How to measure and/or parameterize them?

Gluon GPDs

$$\frac{1}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)G^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[H^g(x,\xi,t)\gamma^+ + E^g(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}]U(P,\Lambda)$$

Even t-channel parity & Gluon helicity conserving

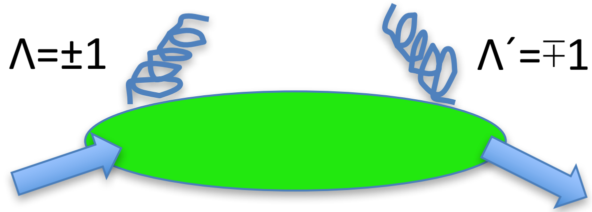
$$\frac{-i}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-}\langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)\tilde{G}^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[\tilde{H}^g(x,\xi,t)\gamma^+\gamma_5 + \tilde{E}^g(x,\xi,t)\frac{\gamma_5(-\Delta^+)}{2M}]U(P,\Lambda)$$

Odd t-channel parity & Gluon helicity conserving

Must have 4 more Gluon helicity NONconserving

Extension to Gluon “Transversity”



$$
\begin{aligned}
&-\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \mathbf{S} F^{+i}(-\tfrac{1}{2}z)\, F^{+j}(\tfrac{1}{2}z)\, |p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0} \\
&= \mathbf{S}\, \frac{1}{2P^+}\, \frac{P^+\Delta^j - \Delta^+P^j}{2mP^+} \\
&\times\ \bar{u}(p',\lambda') \Bigg[H_T^g\, i\sigma^{+i} + \tilde{H}_T^g\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} \\
&\qquad + E_T^g\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^g\, \frac{\gamma^+P^i - P^+\gamma^i}{m} \Bigg] u(p,\lambda).
\end{aligned}
$$

4 GPDs: see M.Diehl, EPJC19, 485 (2001)

4 Gluon helicity NONconserving Double flip

Gluon "transversity" Double helicity flip *does not mix* with quark distributions

Transversity for on-shell gluons or photons : no |0> helicity

$$|+1)_{trans} = \{|+1> + |-1>\}/2 = |-1)_{trans}$$

$$|0\ )_{trans} = \{|+1> - |-1>\}/\sqrt{2}$$

$$\text{helicity}\ \ |\pm 1> = \{-/+\hat{x} - i\hat{y}\}/\sqrt{2}$$

$$\hat{x} = -|0)_{trans} = P_{parallel}$$

Linear polarization in the plane

$$\hat{y} = i\sqrt{2}\,|+1)_{trans} = P_{normal}$$

Linear polarization normal to the plane

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).

Construct helicity flip amps Spectator Model, then GPDs

$$A_{++,+-} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\Big(\tilde{H}_T^g + (1-\xi)\frac{E_T^g+\tilde{E}_T^g}{2}\Big)$$

$$A_{-+,--} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\Big(\tilde{H}_T^g + (1+\xi)\frac{E_T^g-\tilde{E}_T^g}{2}\Big)$$

$$A_{++,--} = +e^{-i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}}{2M}\Big(H_T^g + \frac{t_0-t}{M^2}\tilde{H}_T^g - \frac{\xi^2}{1-\xi^2}E_T^g + \frac{\xi}{1-\xi^2}\tilde{E}_T^g\Big)$$

$$A_{-+,+-} = -e^{i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}^3}{8M^3}\tilde{H}_T^g,$$

Compare to spectator model results

$$\tilde{H}_T^g = 0$$

$$(1-X)A^0_{-+.--} = (1-X')A^0_{++,+-}$$

$$\tilde{E}_T^g = 0.$$

As in Hoodbhoy & Ji, PRD58, 054006 (1998)

Using Reggeized Spectators Model
Many other models & recently

How to Measure? What Processes? Long standing question.

M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, Phys. Lett. B411, 193 (1997).
X. Ji and J. Osborne, UMD PP#98-074, hep-ph/9801260.
P. Kroll, M. Schurmann, and P. A. M. Guichon, Nucl. Phys. A598, 435 (1996).
P. Hoodbhoy & X. Ji, PRD58, 054006 (1998).

TMDs

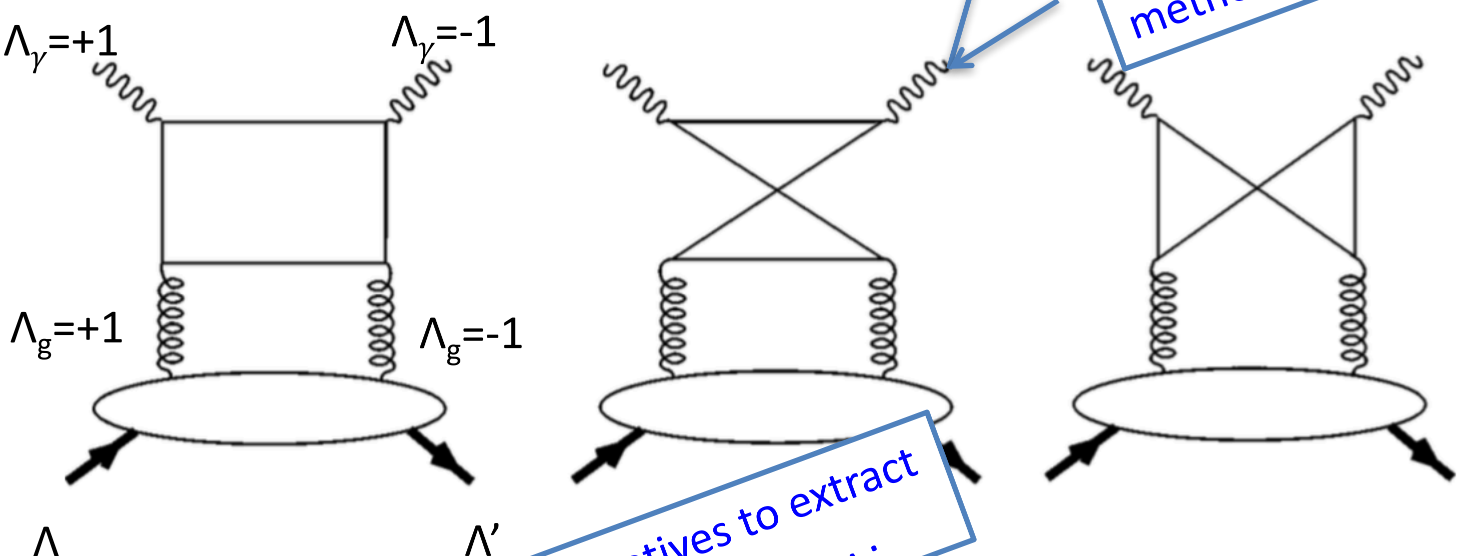
P. Mulders, J. Rodrigues,PRD 63, 094021 (2001).
D. Boer, Few-Body Syst. (2017); C. Pisano, et al., JHEP 10, 024 (2013);
D. Boer, et al., PRL 106, 132001 (2011); . . . .

Helicity flip $A_{\Lambda',-1;\Lambda,+1}$ contributes to DVCS ~ α_S

$$M_{\Lambda',\Lambda'\gamma=-1;\Lambda,\Lambda\gamma=+1} = -\frac{\alpha s}{2\pi}\sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda',\Lambda' g=-1;\Lambda,\Lambda g=+1}(x,\xi,t)}{(\xi - x - i\varepsilon)(\xi + x - i\varepsilon)} \; C'(x,\xi,Q^2)$$

Interference with Bethe-Heitler contains *cos*3φ modulation to distinguish from (leading twist) quark contribution

$\mu^- \mu^+$ alternative method to extract

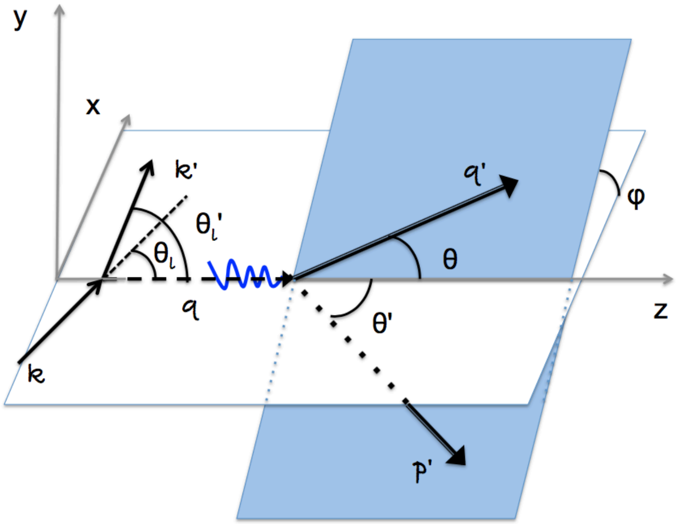


other alternatives to extract J/Ψ production; jets . . . .

See Hoodbhoy & Ji, PRD58, 054006 (1998)

Measuring Gluon GPDs in Nucleons

DVCS



$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}\,.$$

$$\mathcal{I} = T^*_{BH}T_{DVCS} + T^*_{DVCS}T_{BH}.$$

For unpolarized e+p→e'+γ+p' cross section depends on azimuthal angle ϕ.
cos3ϕ modulation in interference dσ measures gluon transversity GPDs (CFF's)

$$\frac{\sqrt{t_0-t}^3}{8M^3}\left[H_T^g F_2 - E_T^g F_1 - 2\tilde{H}_T^g\left(F_1 + \frac{t}{4M^2}F_2\right)\right]\cos 3\phi$$

$\mathcal{H}_T^{\,g} \sim \int dx\, H^g_T/(x-\xi)(x+\xi)$ CFF's

But $\mathcal{H}_T^{\,g} \sim$ may need EIC

See Diehl, *et al*. PLB411, 193 (1997);
Diehl, EPJC25, 223 (2002);
Belitsky, Mueller, PLB486, 369 (2000).

Gluon GPDs from DVCS – Hall A

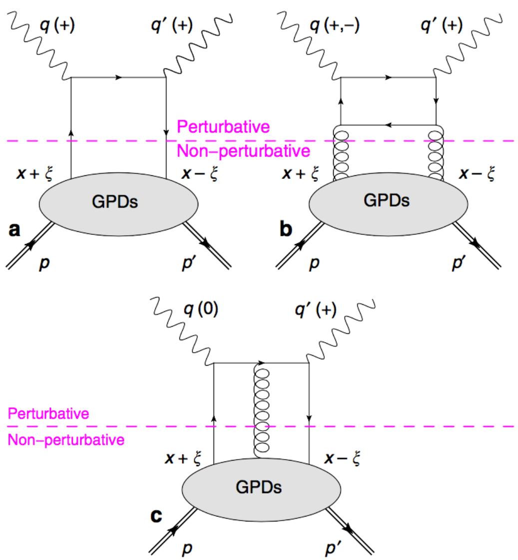
Polarized & unpolarized beam measurements

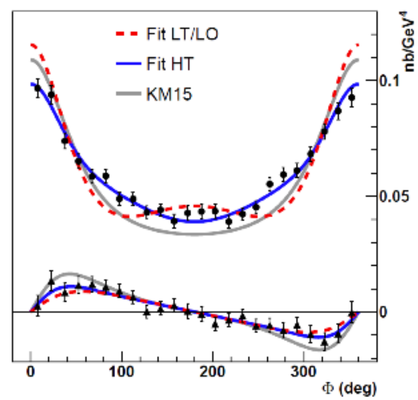
$$\frac{d^4\sigma(h)}{dQ^2 dx_B dt d\phi} = \frac{d^2\sigma_0}{dQ^2 dx_B} \times \left[\left|\mathcal{T}^{BH}\right|^2 + \left|\mathcal{T}^{DVCS}(h)\right|^2 - \mathcal{I}(h) \right]$$

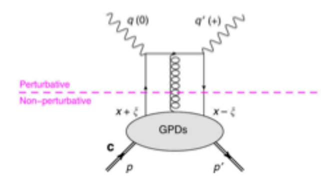
Evidence of gluon transversity

Fitting ϕ distribution requires F_{++} and both F_{+-} gluon transversity and F_{0+} higher twist

A glimpse of gluons through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3







Analysis of 6 GeV Hall A DVCS data on the proton.

Jefferson Lab

See Latifa Elouadrhiri talk

A glimpse of gluons through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3

Evidence of gluon transversity

Fitting ϕ distribution requires
F_{++} and both F_{+-} gluon transversity
and F_{0+} higher twist

Table 2 Results of the cross-section fits

Fit description	**LO/LT**	**Higher twist**	**NLO**
Helicity states	**++**	**++/0+**	**++/−+**
$t = -0.18$ GeV^2	250	204	206
$t = -0.24$ GeV^2	367	206	208
$t = -0.30$ GeV^2	415	189	190

Values of χ^2 (ndf = 208) obtained in the leading-order, leading-twist (++); higher-twist (++/0+); and next-to-leading-order (++/−+) scenarios. The fit is not performed at the highest value of $-t$ because of the lack of full acceptance in ϕ, resulting in a large statistical uncertainty. The fits include statistical and point-to-point systematic uncertainties

LHC – many opportunities for studying gluons
p+p unpolarized → jets, hadrons, leptons
Interactions via g+g→ Q+Qbar +X
gluon TMDs in some kinematics
Extension to Gluon "Transversity"

c.f. TMDs $h_1^{\perp g}(x, p_T^2)$ Mulders & Rodrigues (2001), Gluon Boer-Mulders function
see D. Boer, Frascati talk (Nov.2016) & many references for measurements at EIC, RHIC, LHC

Gluon TMDs

$$
\begin{pmatrix}
G+\Delta G_L & \frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T-iG_T] & -e^{-2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -i\frac{|\boldsymbol{k}_T|e^{-3i\phi}}{M}\boxed{\Delta H_T^{\perp(1)}} \\
\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T+iG_T] & G-\Delta G_L & -i\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}\Delta H_T & -e^{-2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] \\
-e^{2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] & i\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}\Delta H_T & G-\Delta G_L & -\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T+iG_T] \\
i\frac{|\boldsymbol{k}_T|e^{3i\phi}}{M}\boxed{\Delta H_T^{\perp(1)}} & -e^{2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T-iG_T] & G+\Delta G_L
\end{pmatrix}
$$

Mulders & Rodrigues, PRD63, 94021 (2001)

The matrix representation is also convenient to find the physical meaning of the distributions. Well known is *G* which measures the number of gluons with momentum (*x*,***k****T*) in a hadron. The functions *GL* (*GT*) represents the difference of the numbers of gluons with opposite circular polarizations in a longitudinally transversely polarized nucleon. The off- diagonal function *H* also is a difference of densities, but in this case of linearly polarized gluons in an unpolarized hadron. Using the circular polarizations, *H* flips the polarization.

Corresponding GTMDs generalize GPDs & TMDs. Unintegrated **models** connect all

Other notation $\Delta H_T^{\perp(1)}$, $h_1^{g\perp}$ gluon Boer-Mulders function

Unpolarized Nucleon → polarized gluon | factorization & evolution

Gluon TMDs

TMD Color gauge invariance

Small x gluons, Kharzeev, Kovchegov, Tuchin, saturation, WW vs. DP, MV model, QGP. . .?

See Mulders in this meeting: small x DP is pure gauge link

$$\Gamma^{\mu\nu\,[\mathcal{U},\mathcal{U}']}(x, \boldsymbol{k}_T) \equiv \int \frac{d(\xi \cdot P)\, d^2\xi_T}{(P \cdot n)^2 (2\pi)^3} e^{i(xP+k_T)\cdot\xi} \left\langle P \left| \mathrm{Tr}_c \left[F^{n\nu}(0)\, \mathcal{U}_{[0,\xi]}\, F^{n\mu}(\xi)\, \mathcal{U}'_{[\xi,0]} \right] \right| P \right\rangle \Big|_{\xi\cdot n=0} .$$

Gauge link
$\xi=[0^+, \xi^-, \boldsymbol{\xi}_T]$

$$\mathcal{U}_C[0,\xi] = \mathcal{P} \exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s) \right)$$

Can be forward light front pointing link $+\infty$ FSI. Weizsacker-Williams
Or mixed light front pointing link $-\infty$ ISI Dipole

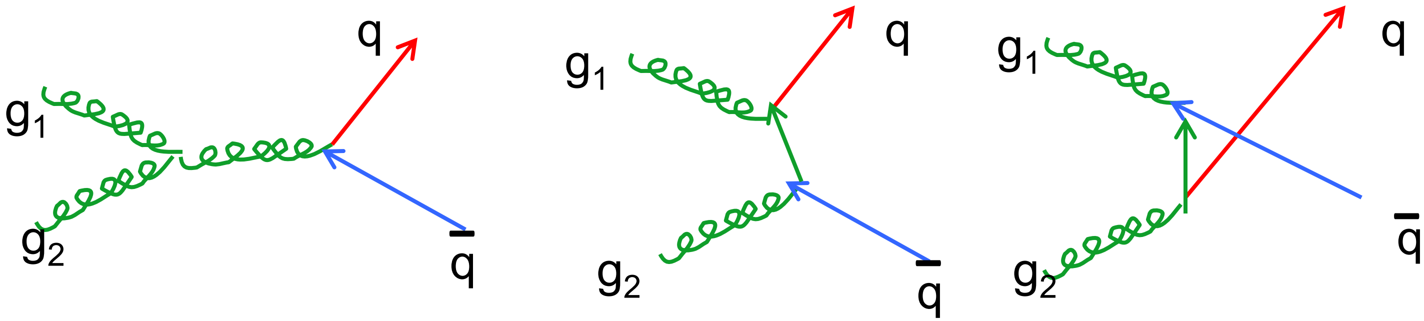
For *U* and *U'* *have* [+ +] or [+ -] (& parity opposites)

The Model for valence quarks– Reggeized Diquarks

The Model – first for Chiral Even – then Odd
Reggeized Diquark Spectator
Diquark: Color anti-3, scalar & axial vector

Gluon GPDs, TMDs, GTMDs

From p+p to gluon TMDs to quark pairs

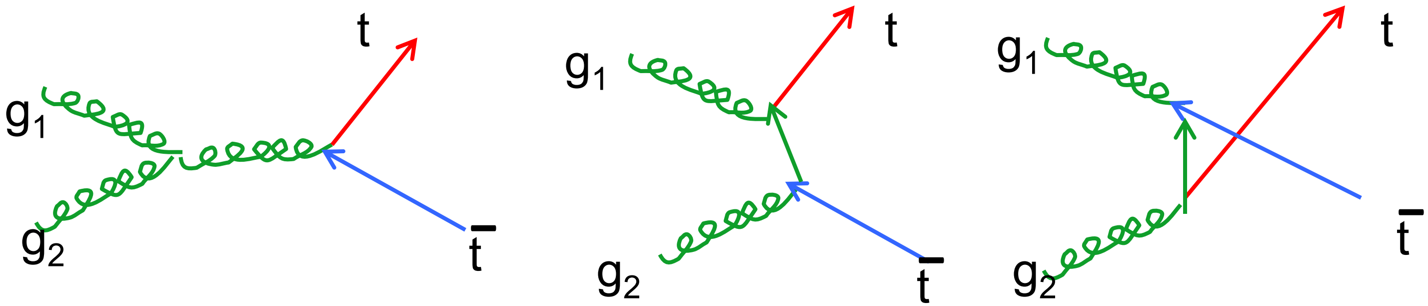


Form quarkonia & different possibilities for *gg*

Complications from f.s.i. & jets – hadronization

Factorization and evolution

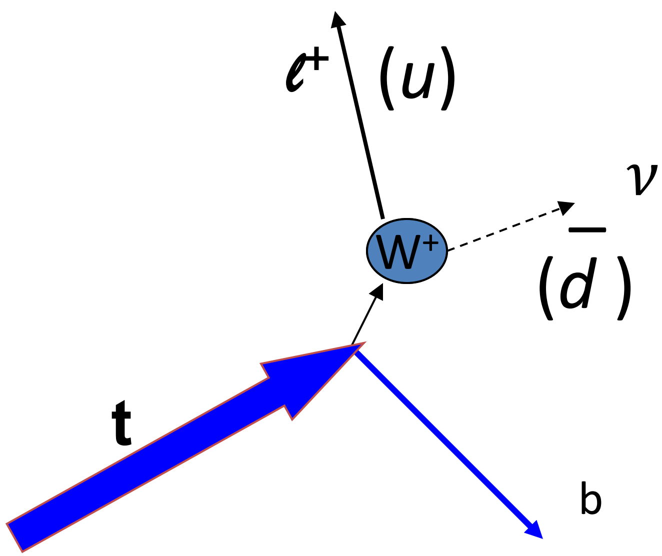
For Gluon fusion top production



- g_1 & g_2 carry helicity $\Lambda_1\ \Lambda_2\ = \pm 1$ & color 1, 8. . . & C=+ or -
- *t* & *t*-bar carry helicity λ_t , λ_{tbar} = ± ½ & color 1 or 8
- *t & tbar decay before hadronizing* => no toponia & large scale

How is top polarization determined?

Its decay is good analyzer for transverse polarization.



$$U_{\lambda_t,\lambda'_t} = \sum_{\lambda_b} B^*_{\lambda_b,\lambda'_t} B_{\lambda_b,\lambda'_t}$$

$$\propto (I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})_{\lambda_t,\lambda'_t} (p_b \cdot p_\nu)$$

Calculated in top rest frame

OR

$$U = (p_t - m_t S_t) \cdot p_{\bar{l}} \, (p_b \cdot p_\nu)$$

$$S_t = \left[\frac{\vec{p} \cdot \vec{P}_t}{m_t}, \vec{P}_t + \frac{(\vec{p} \cdot \vec{P}_t)\vec{P}_t}{m_t(E_t + m_t)} \right]$$

Covariant form in any frame

P_t = strength of top polarization

Dalitz & GRG, PLB287,225(1992); PRD45, 1531(1992)

$(I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})$ lepton or u-quark moves parallel to transverse polarization

What is known production of polarized tops?

Top Single Spin Asymmetry and Double Spin Correlations – Measurements

ATLAS PRD93, 012002 (2016) & ref. PRL114, 142001 (2015)

** SSA: B_1 or A_P =-0.035 +\- 0.040. (syst & stat)

*** Double: $C_{helicity}$ =0.315 +\- 0.07 vs. NLO QCD =0.31

(Bernreuther, et al., PRL 87,242002 (2001) QCD corrections but unpolarized gluons)

CMS PRL112, 182001 (2014): Different kinematics & selection criteria

** SSA: A_P=0.005+\- 0.01.

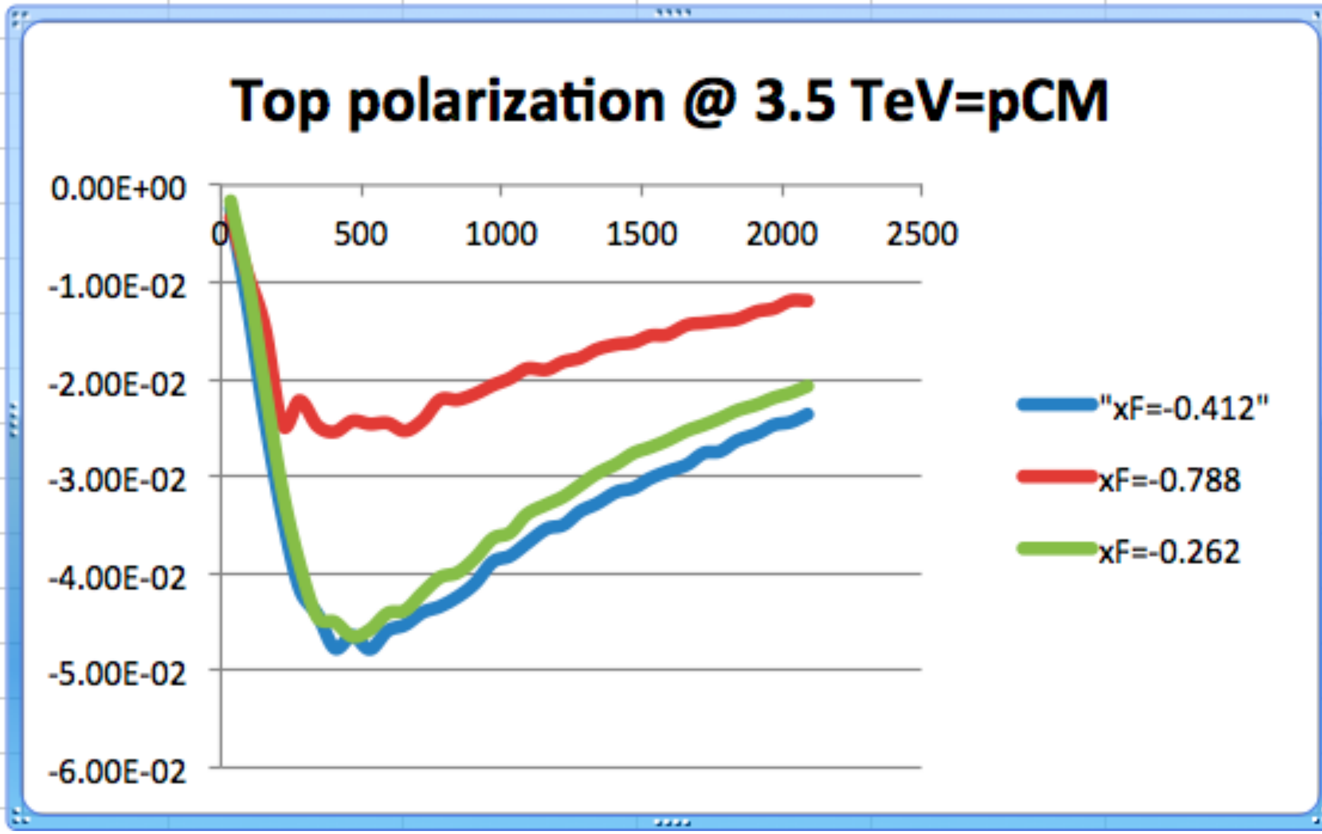
*** Double: $A_{\Delta\phi}$ =0.113 +\- 0.01. vs. 0.110 +\- 0.001 (MC & QCD)

A_{c1c2} = -0.021 +\- 0.03 vs -0.078 +\- 0.001

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4}(1 + B_1\cos\theta_1 + B_2\cos\theta_2 - C_{\text{helicity}}\cos\theta_1\cdot\cos\theta_2)$$

θ_1 θ_2 decay product angles w.r.t. t+tbar CM

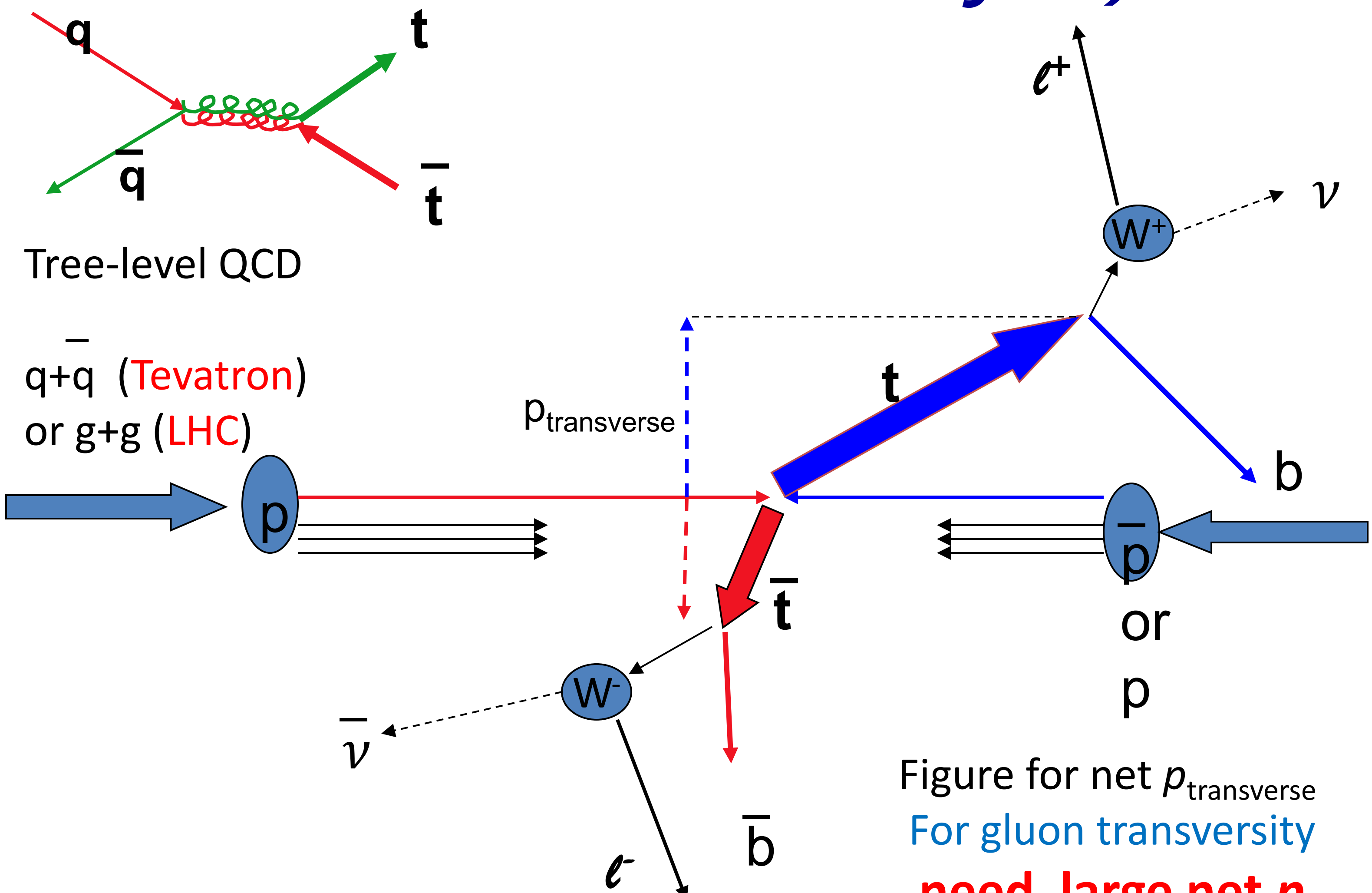
Direct measure of hard process- top polarization
Top decays weakly before hadronizing
$\Rightarrow$ decay "self-analyzing"



Contributions to order α_S
Imaginary Part (Dharmaratna & GRG 1990,1996)

Analyze t--> W^+ b

Dilepton events or lepton+hadron jets or all Hadron jets)

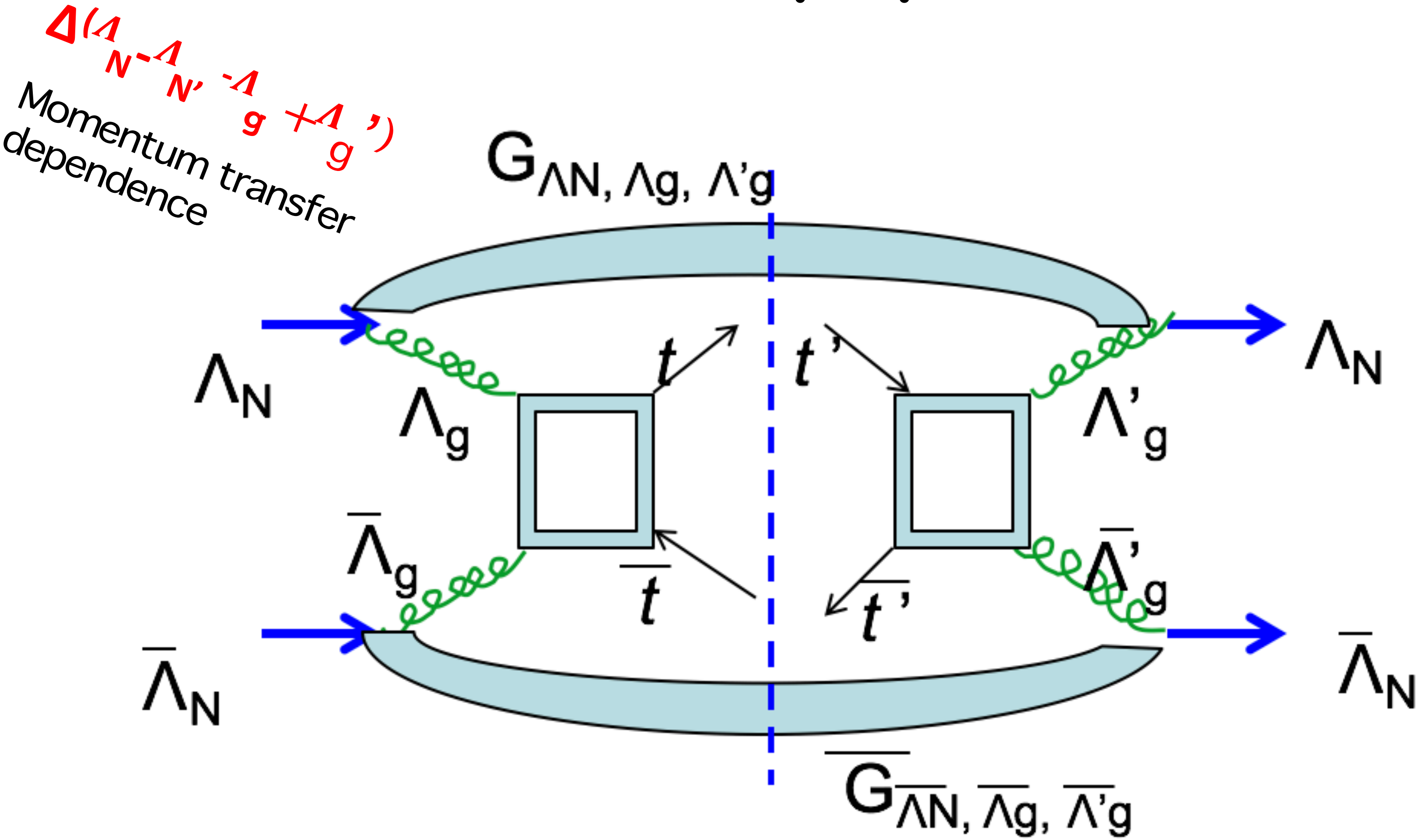


Tree-level QCD

$q+\bar{q}$ (Tevatron)
or g+g (LHC)

Figure for net $p_{transverse}$
For gluon transversity
need large net $p_{transverse}$ to access transversity

For inclusive p+p→t+tbar+X



Gluon linear polarization with like and unlike t-tbar helicities

(work in progress S.Liuti, GRG, Gonzalez-Hernandez, Poage)

$F \sim G_{XX}+G_{YY}$, $H \sim G_{XX}-G_{YY}$ or linear polarization

$\rho_{t',\bar{t}';t,\bar{t}}$	$\bar{F}\,F$	$\bar{H}\,H$	$\bar{F}\,H$	$\bar{H}\,F$
++;++	$\gamma^{-2}(1+\beta^2(1+\sin^4\theta))$	$\gamma^{-2}(-1+\beta^2(1+\sin^4\theta))$	$-2\dfrac{\beta^2}{\gamma^2}\sin^2\theta$	$-2\dfrac{\beta^2}{\gamma^2}\sin^2\theta$
+-;+-	$\beta^2\sin^2\theta(2-\sin^2\theta))$	$-\beta^2\sin^4\theta$	0	0

$q+q$-bar → t + t-bar dilepton channel

- The light quark-antiquark annihilation mechanism gives rise to the **angular distribution between opposite charge lepton pairs**, more information than $C_{helicity}$ or $A_{c1\,c2}$

$$
\begin{aligned}
W(\theta, p, p_{\bar{l}}, p_l) = \frac{1}{4}\{&1 + [\sin^2\theta([p^2+m^2](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + [p^2-m^2](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}}) \\
&- 2mp\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}) + ([p^2-m^2] \\
&+[p^2+m^2]\cos^2\theta)(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}]/[(p^2+m^2)+(p^2-m^2)cos^2\theta]\} \\
= \frac{1}{4} + \frac{1}{4}\{&(2-\beta^2)\sin^2\theta(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + \beta^2(\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
&+[\beta^2+(2-\beta^2)\cos^2\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
&-\frac{2}{\gamma}\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}})\Big\}/[(2-\beta^2)+\beta^2\cos^2\theta]
\end{aligned}
$$

m =top mass, θ = t production angle in q+q-bar CM
p= light quark 3-momentum in CM
Unit vectors p-hat are anti-lepton$^+$ and lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

See G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass'' in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

$g_1+g_2 \to t + t\text{-bar}$
Spin correlations – dilepton channel

Correlations expressed as a weighting factor first ***for unpolarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

$$W(\theta,p,p_{\bar l},p_l) = \frac{1}{4} - \frac{1}{4}\{[p^4\sin^4\theta + m^4](\hat p_{\bar l})_x(\hat p_l)_{\bar x} + [p^2(p^2-2m^2)\sin^4\theta - m^4](\hat p_{\bar l})_y(\hat p_l)_{\bar y}$$
$$+ [p^4\sin^4\theta - 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)](\hat p_{\bar l})_z(\hat p_l)_{\bar z}$$
$$+ 2mp^2\sqrt{p^2-m^2}\cos\theta\sin^3\theta[(\hat p_{\bar l})_x(\hat p_l)_{\bar z} - (\hat p_{\bar l})_z(\hat p_l)_{\bar x}]\}$$
$$/\left[p^2(2m^2-p^2)\sin^4\theta + 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)\right] \quad (20)$$

$$= \frac{1}{4} - \frac{1}{4}\{[(1-\beta^2)^2 + \sin^4\theta)](\hat p_{\bar l})_x(\hat p_l)_{\bar x}$$
$$+[-(1-\beta^2)^2 - (1-2\beta^2)\sin^4\theta](\hat p_{\bar l})_y(\hat p_l)_{\bar y}$$
$$+[(1-\beta^4) - 2\beta^2\sin^2\theta + \sin^4\theta](\hat p_{\bar l})_z(\hat p_l)_{\bar z}$$
$$+2\frac{\beta}{\gamma}\sin^3\theta\cos\theta[(\hat p_{\bar l})_x(\hat p_l)_{\bar z} - (\hat p_{\bar l})_z(\hat p_l)_{\bar x}]\Big\}$$
$$/\left[(1-\beta^4) + 2\beta^2\sin^2\theta + (1-2\beta^2)\sin^4\theta\right] \quad (21)$$

m =top mass, θ = t production angle in g+g CM

p= gluon 3-momentum in CM

p-hat's are lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

Use these to test SM vs. BSM – Integrated version agrees –

with big errors -- GRG *in process* – see also Mahlon & Parke

$g_1+g_2 \rightarrow t + t\text{-}bar$
Spin correlations

Correlations expressed as a weighting factor first ***for polarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

$$
\begin{aligned}
W^{(LP,LP)}(\theta,p,p_{\bar{l}},p_l) = -\frac{1}{4} + \frac{1}{4}\{ & [(1-\beta^4)+\beta^2\sin^2\theta(-2+(2-\beta^2)\sin^2\theta)](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} \\
& +[(1-\beta^4)+\beta^2\sin^2\theta(2-\beta^2\sin^2\theta)](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
& +[-(1-\beta^2)^2+\beta^2(2-\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
& -4\frac{\beta^2}{\gamma}\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}}-(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\Big\} \\
& /\left[(1-\beta^2)^2+\beta^4\sin^4\theta\right]
\end{aligned}
$$

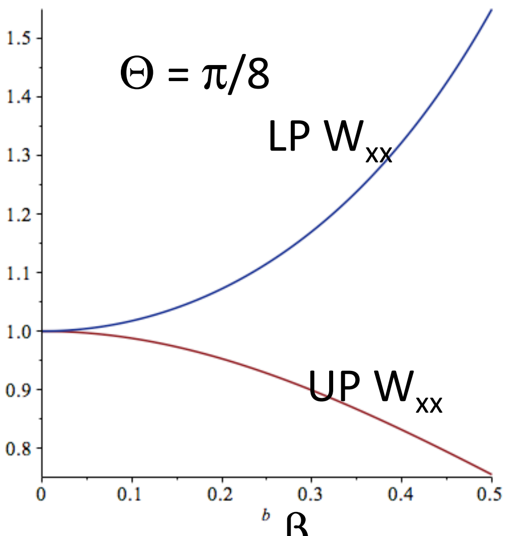
Crucial measurements $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}}$ $=W_{xx}$, $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}}$ $=W_{xz}$, ... **Weighting tensor**

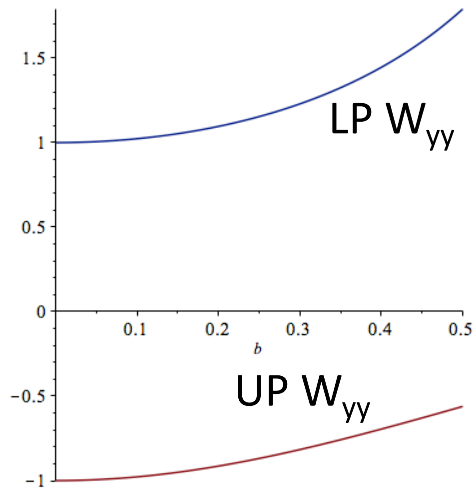
- **Use these to compare with unpolarized to extract the Gluon transversity**
- **or linear polarizations G_{xx}-G_{yy}**
- **Careful about Frames:**
- **Collider LAB, $t+\bar{t}$ pair CM, separate *t* & *t*-bar rest, $W^{+/-}$ rest frames**

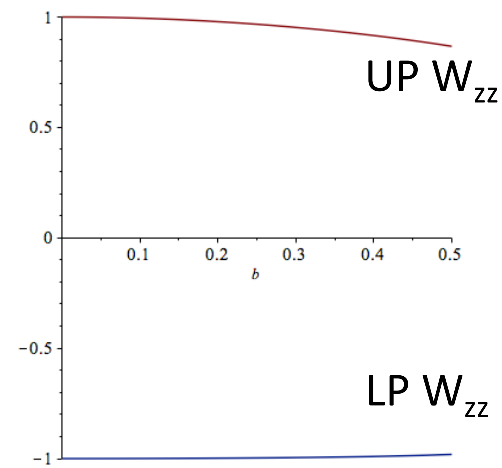
Comparing lepton directional correlations

Weighting tensor for lepton$^+$ lepton$^-$ when $\theta=\pi/8$

or lepton$^+$ d-quark *or* u-quark lepton$^-$







Each event has μ^- μ^+ momenta → $p^\pm$ (x, y, z) as well as θ & β

Probability for given event configuration is given by

G(UP) W(θ,p,$\bar{p}$l,pl) +G(LP) W^{LP} (θ,p,$\bar{p}$l,pl)

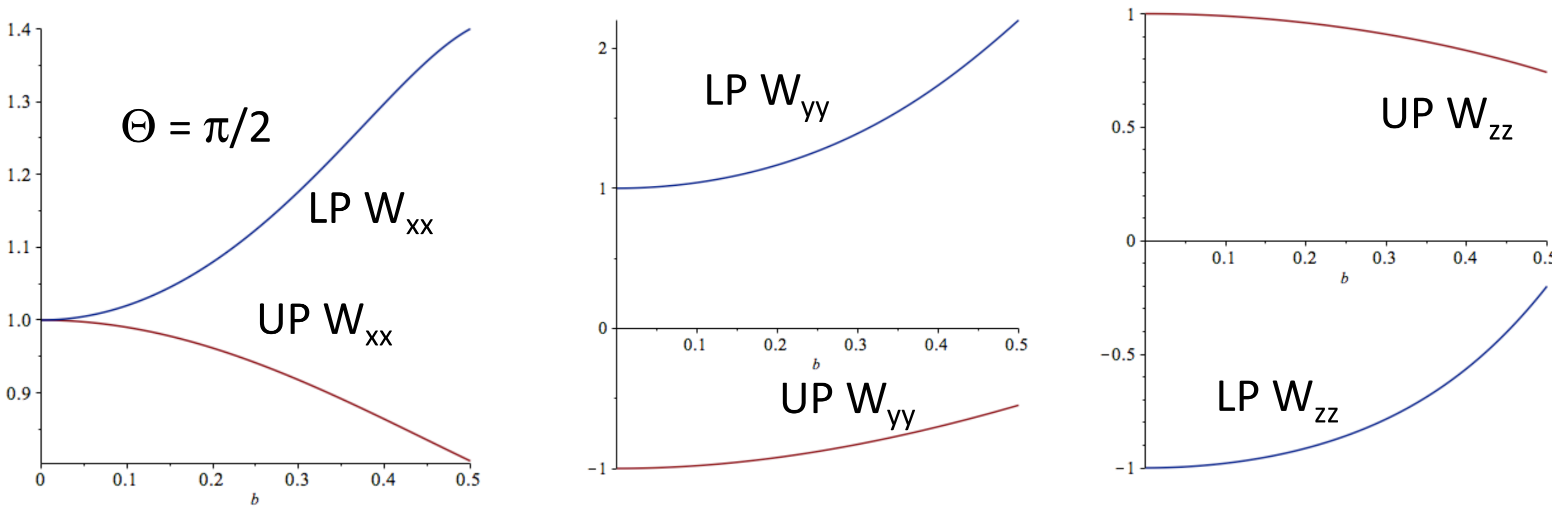
Quite distinct! x & y components are aligned for LP, anti-aligned for UP

Can Diagonalize (with $W_{XY,\,YX}$) to obtain positive ellipsoidal weighting

Comparing lepton directional correlations

Weighting factors for lepton$^+$ lepton$^-$ when $\theta=\pi/2$

$W_{xz}=0$ for the off-diagonal



Each event has μ^- μ^+ momenta → $p^{\pm}$ (x, y, z) as well as θ & β

Probability for given event configuration is given by

G(UP) W(θ,p,$\bar{p}$l,pl) +G(LP) W^{LP} (θ,p,$\bar{p}$l,pl)

Quite distinct! x & y components are aligned for LP, anti-aligned for UP

Diagonalize (with $W_{XY,\ YX}$) to obtain positive ellipsoidal weighting

Separating polarized gluons

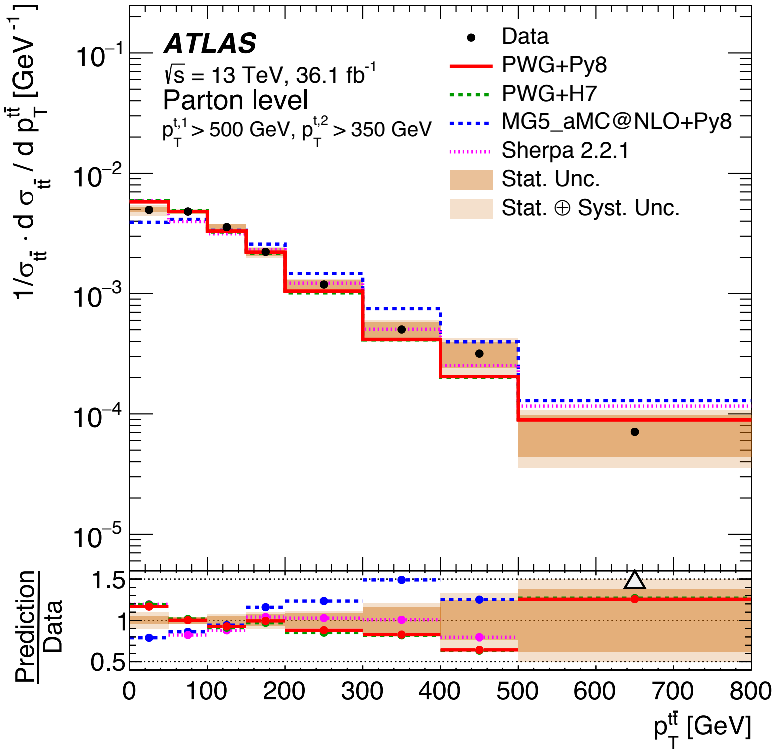
* Each event has μ^- μ^+ momenta → $p^{\pm}$ (x, y, z)
  in t & tbar rest frame
* t+tbar CM determines θ direction as well as β for t & tbar
* Probability for given event configuration is given by

$G(UP)\ W(\theta,p,\bar{p}l,pl)\ + G(LP)\ W^{LP}(\theta,p,\bar{p}l,pl)$ (ignoring light quarks)

- Quite distinct! x & y components are
- aligned for LP, anti-aligned for UP
- G's convoluted with W's all gluon k_T & $\bar{k}_T$ satisfying
- measured $p_t + p_{anti\text{-}t}$ <-> large transverse momenta : transversity

Large transverse momentum

- t-tbar inclusive at 13 TeV



Summary

- Gluon vs. quark GPDs (from spectator & Regge R✖Dq)
- *Helicity* conserving & Helicity flip →gluon ***Transversity***
- Electroproduction & DVCS
- pp→gluons→t+tbar + X
- Gluon TMDs?
- Measurements? Top polarization
- t+tbar spin correlations via lepton decays or hadron jets

  – To Do List
- More phenomenology to come
- Care about evolution, factorization, power counting, . . .

Thank you!

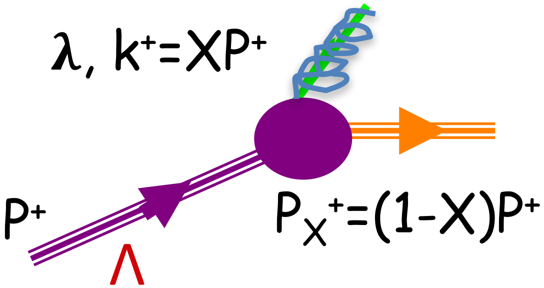
Backup Slides

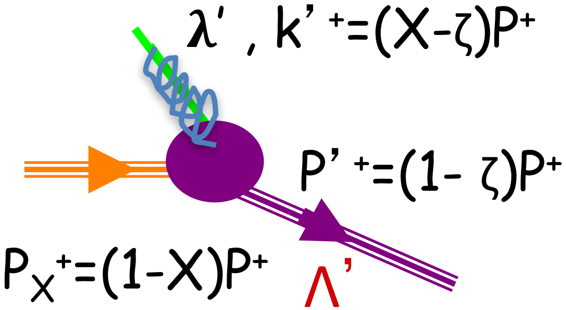
Gluon Double Flip Amps

$$\begin{aligned}
A_{++,+-} &= \frac{\Delta_\perp^2}{4M^2(1-\zeta)(1-\frac{\zeta}{2})}\left(\tilde{H}_T^g + \frac{1-\zeta}{1-\frac{\zeta}{2}}\frac{E_T^g + \tilde{E}_T^g}{2}\right) \\
A_{-+,--} &= \frac{\Delta_\perp^2}{4M^2(1-\zeta)(1-\frac{\zeta}{2})}\left(\tilde{H}_T^g + \frac{1}{1-\frac{\zeta}{2}}\frac{E_T^g - \tilde{E}_T^g}{2}\right) \\
A_{++,--} &= \frac{\Delta_1 - i\Delta_2}{2M\sqrt{1-\zeta}(1-\frac{\zeta}{2})}\left(\frac{1-\zeta}{1-\frac{\zeta}{2}}H_T^g + \frac{\Delta_\perp^2}{M^2(1-\frac{\zeta}{2})}\tilde{H}_T^g - \frac{\zeta}{2}\left[\frac{\zeta}{1-\frac{\zeta}{2}}E_T^g - \tilde{E}_T^g\right]\right) \\
A_{-+,+-} &= -\frac{\Delta_1 + i\Delta_2}{2M\sqrt{1-\zeta}(1-\frac{\zeta}{2})}\frac{\Delta_\perp^2}{4M^2(1-\frac{\zeta}{2})}\tilde{H}_T^g,
\end{aligned}$$

All will involve Δ powers for each net helicity flip
So need t and tbar 3-momenta with Δ non-zero!

Constructing gluon GPDs
Gluon 'vertex functions' $G_{\Lambda x\ ;\ \Lambda g,\ \Lambda}$





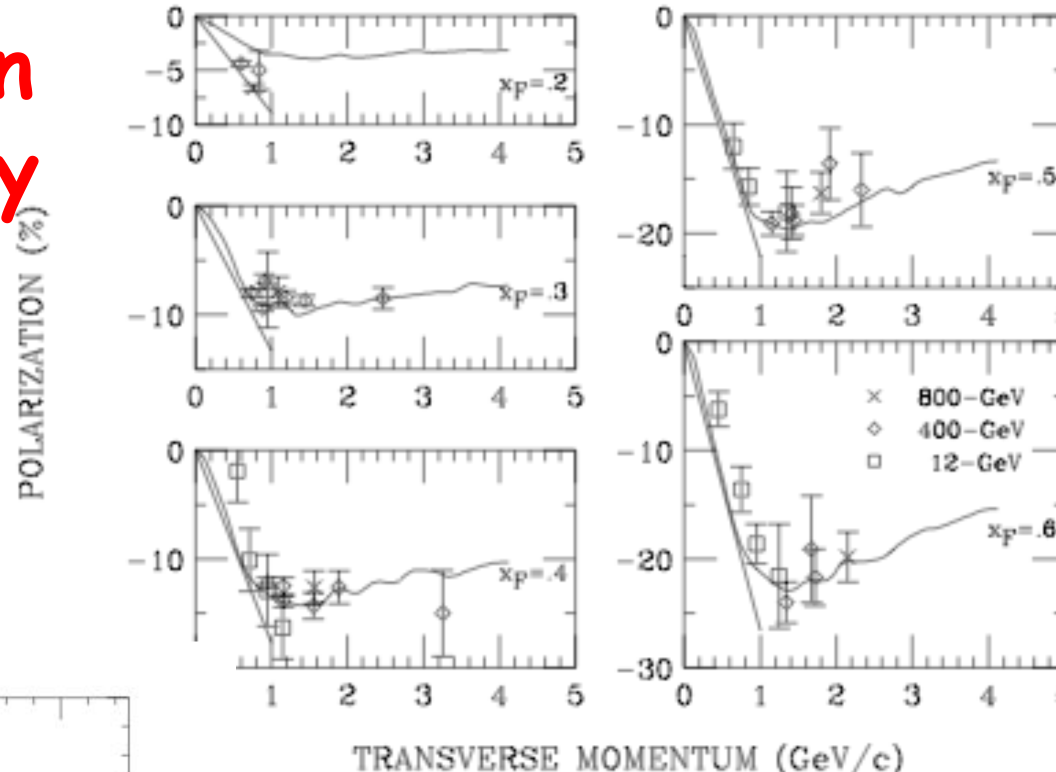
$\mathcal{G}_{+++}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}\frac{(k_x-ik_y)}{X}$
$\mathcal{G}_{-++}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}(M(1-X)-M_x)$
$\mathcal{G}_{++-}(x,\vec{k}_T^2)$	0
$\mathcal{G}_{-+-}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}(1-X)\frac{(k_x-ik_y)}{X}$
$\mathcal{G}^*_{+++}(x,\vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}}\frac{(\tilde{k}_x+i\tilde{k}_y)}{X'}$
$\mathcal{G}^*_{-++}(x,\vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}}(M(1-X')-M_x)$
$\mathcal{G}^*_{++-}(x,\vec{k}_T'^2)$	0
$\mathcal{G}^*_{-+-}(x,\vec{k}_T'^2))$	$-\frac{2}{\sqrt{2(1-X')}}(1-X')\frac{(\tilde{k}_x+i\tilde{k}_y))}{X'}$

$$X'=\frac{X-\zeta}{1-\zeta},\ \tilde{k}_{i=1,2}=k_i-\frac{1-X}{1-\zeta}\Delta_i;$$

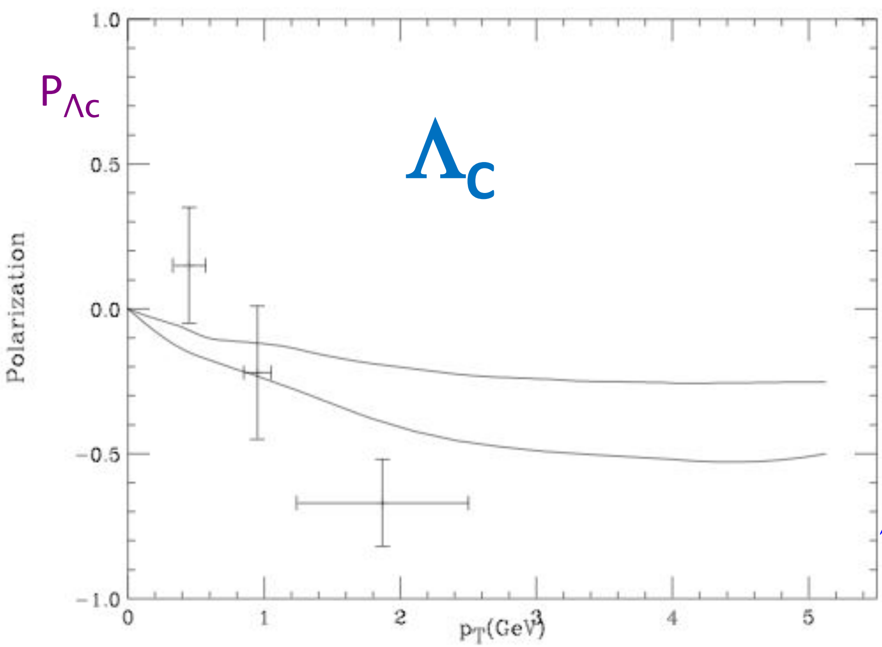
GRG & S. Liuti, QCD Evolution 2014, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
GRG, Gonzalez Hernandez, Liuti, Poage, in progress

Single Spin Asymmetry

$$pp \rightarrow \Lambda^{\uparrow}(\Lambda_c^{\uparrow})X$$

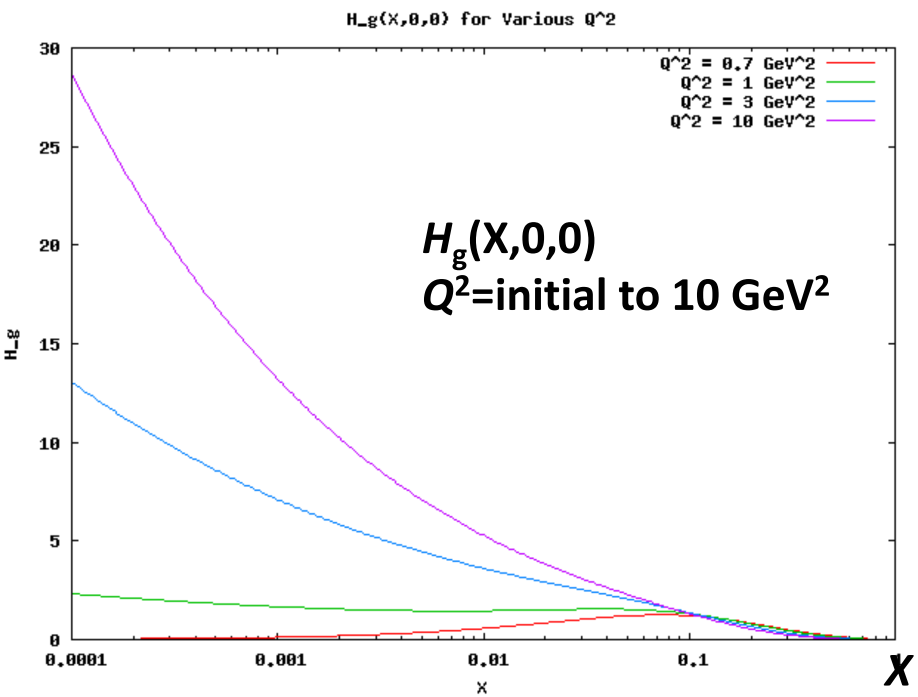


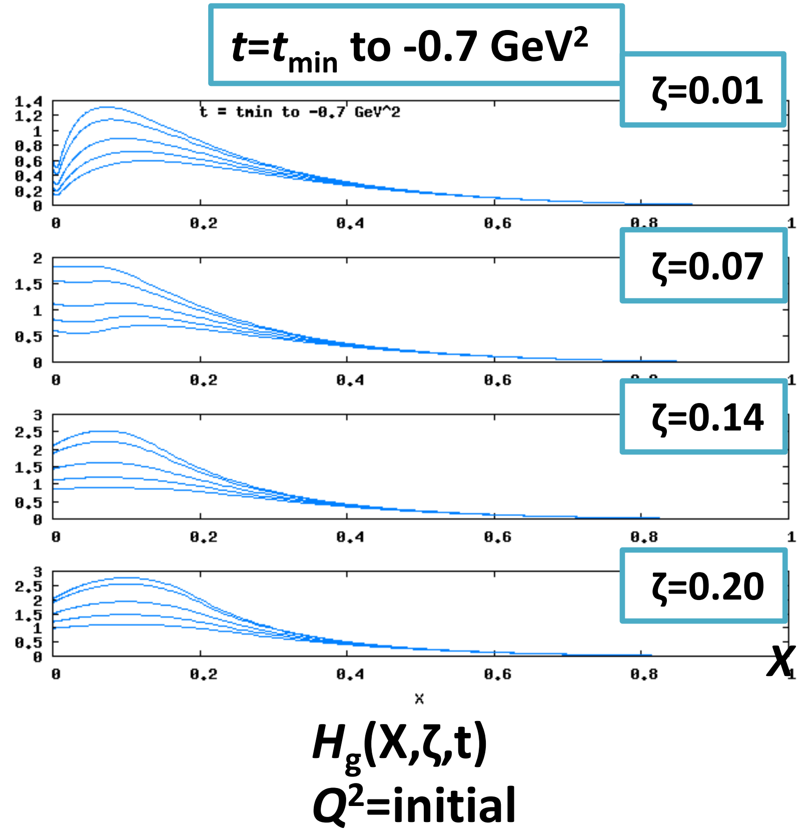
K. Heller, PRD1997
curves from model of
Dharmaratna & GRG PRD '90 & '97



E791, PLB 471, 449 (2000)
$\pi^- + p \rightarrow \Lambda_C + X$
curves from GRG hep-ph/9907573

After pdf's vs. Q^2 → fix x dependence
Regge behavior determines t dependence
Spectator determines ζ dependence





from J. Poage

Gluon & Sea quark distributions
Spectator Model

- N→ ***g*** + "color octet N" spectator ($8 \otimes 8 \supset 1$)
  (could be spin ½ or 3/2)
- (N→ ***anti-u*** + color 3 "tetraquark"uuud)

- How to normalize?
  $H_g(x,\xi,t)_{Q^2} \rightarrow H_g(x,0,0)_{Q^2} = xG(x)_{Q^2}$

Evolution & small x phenomenology

- Sea quark distributions $H_{anti-u}(x,0,0)$ . . .
- Use pdf's to fix x dependence
- Small x ~ Pomeron
- Model generalizes to GTMDs > TMDs . . .

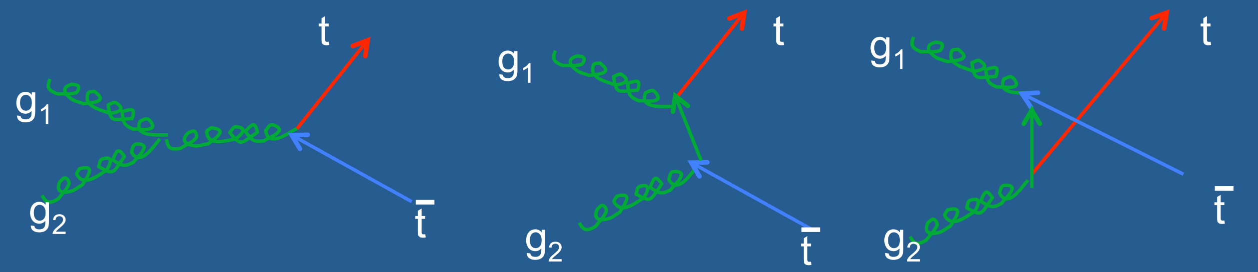
$$\rho_{t',\bar{t}';t,\bar{t}} \propto \sum_{all-helicities-not-tops} \bar{G}_{\bar{\Lambda}_N \bar{\Lambda}_g \bar{\Lambda}'_g} A^{*}_{\Lambda'_g \bar{\Lambda}'_g; t',\bar{t}'} A_{\Lambda_g \bar{\Lambda}_g; t,\bar{t}} G_{\Lambda_N \Lambda_g \Lambda'_g}$$

- The gluon spin correlations are transmitted to (determine the spin of) the decay products.
- The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.
- The gluon fusion mechanism gives rise to a higher order (wrt quark antiquark) angular distribution due to the combination of two spin 1 gluons.

G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass" in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328

At LHC:

Gluon fusion tree level mechanism
(Color gauge invariance)



g_1, g_2 carry helicity $\Lambda_1\ \Lambda_2 = \pm 1$ **OR transversity 1 or 0**

t, *t*-bar carry helicity $\lambda_t\ \lambda_{tbar} = \pm$ ½ OR transversity $\pm 1/2$

Introduced in:

G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass” in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

R.H. Dalitz, G.R. Goldstein and R. Marshall, “Heavy Quark Spin Correlations in e+e- annihilations”, Phys. Lett. B215, 783 (1988);

R.H. Dalitz, G.R. Goldstein and R. Marshall, “On the Helicity of Charm Jets”, Zeits.f. Phys. C42, 441 (1989).

Top spin correlations & gluon polarizations

$\rho_{t',\bar{t}';t,\bar{t}}$	UP,UP	LP,LP	UP,LP + LP,UP
$++,++$	$\gamma^{-2}(1+\beta^2(1+sin^4\theta))$	$\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$	$-4\gamma^{-2}\beta^2 sin^2\theta$
$+-,+-$	$\beta^2 sin^2\theta(2-sin^2\theta))$	$-\beta^2 sin^4\theta$	0
$++,--$	$\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$	$\gamma^{-2}(+1+\beta^2(1+sin^4\theta))$	$+4\gamma^{-2}\beta^2 sin^2\theta$
$+-,-+$	$\beta^2 sin^4\theta$	$-\beta^2 sin^2\theta(2-sin^2\theta))$	0
$++,+-$	$-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$-4\gamma^{-1}\beta^2 sin\theta cos\theta$
$++,-+$	$2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$4\gamma^{-1}\beta^2 sin\theta cos\theta$

TABLE I. Values of double density matrix elements ρ for combinations in Eq. 11 using values of helicity amplitudes from Eq. 13 , evaluated in the $t+\bar{t}$ center of mass.

UP = unpolarized, LP = Linearly polarized gluon distributions
assuming g+g→ t + t-bar in single plane CM
γ & β for top & antitop in CM.
θ = top production angle in CM relative to (t+tbar) momentum direction in lab

Taking X-Z plane for p+p→$(t+tbar)_{CM}$+X gives ϕ dependence to t+tbar plane for opposite helicities: Re($e^{\pm(1or2)i\phi} \cdot e^{\pm(-i(1or2)\phi)}$) leading to cos2$\phi$ for UP,LP and LP,UP and cos4ϕ modulations for LP,LP.