TWIST-3 GENERALIZED PARTON DISTRIBUTIONS

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OUTLINE

- What is and why study twist-3
- Discontinuities in twist-3 GPDs
- Discontinuities and Factorization
- Twist-3 quasi-pdfs and pdfs
- Conclusions
- Outlook

What is TWIST-3 ?

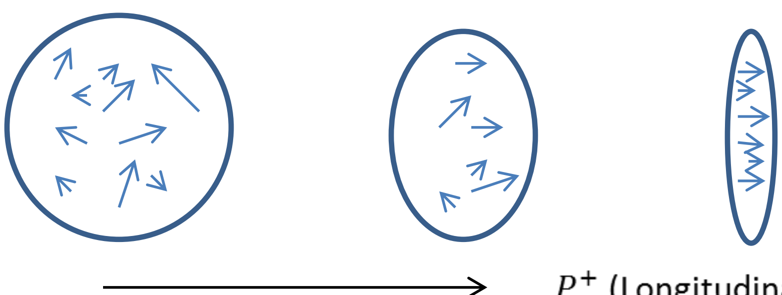
Twist=Dimension-Spin

Twist→ A classification which orders the parton distributions according to their to their dominance for a given hard process.

Leading order → Twist 2
Next to leading order → Twist 3

2 -particle correlations → Twist 2
3-particle correlations (such as quark-gluon-quark) → Twist 3

Twist → Behavior under longitudinal momentum boost in the IMF



Twist 2→ Does not depend on P^+
Twist 3→ $1/P^+$

P^+ (Longitudinal nucleon momentum)

Why study TWIST-3?

- At 12 GeV twist-3 effects are not negligible.

- They are related to the (average) transverse force acting on a quark in a polarized nucleon.

M. Burkardt, Transverse Force on Quarks in DIS (2008)

$$\int dx\, x^2 g_2\,,\ \int dx\, x^2\, e \ \rightarrow \perp force$$

Talk:11:30 am on Wednesday

- There is a relation between one particular twist-3 GPD and the orbital angular momentum of quarks.

$$L^q_{\text{kin}} = \int_{-1}^{1} dx\, x\, G_2^q(x,\xi,t=0)$$

Penttinen, Polyakov, Shuvaev and Strikman, DVCS amplitude in the parton model (2000)

$F^{[\gamma^j]} \longrightarrow G_2\ (twist-3)$

$\Delta = 0$ pdf: X

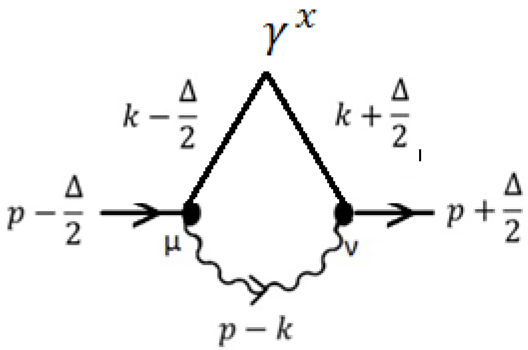
$\int dx$ ff: 0

$F^{[\gamma^j\gamma_5]} \longrightarrow \widetilde{G_2}\ (twist-3)$

$\Delta = 0$ pdf: g_2

$\int dx$ ff: 0

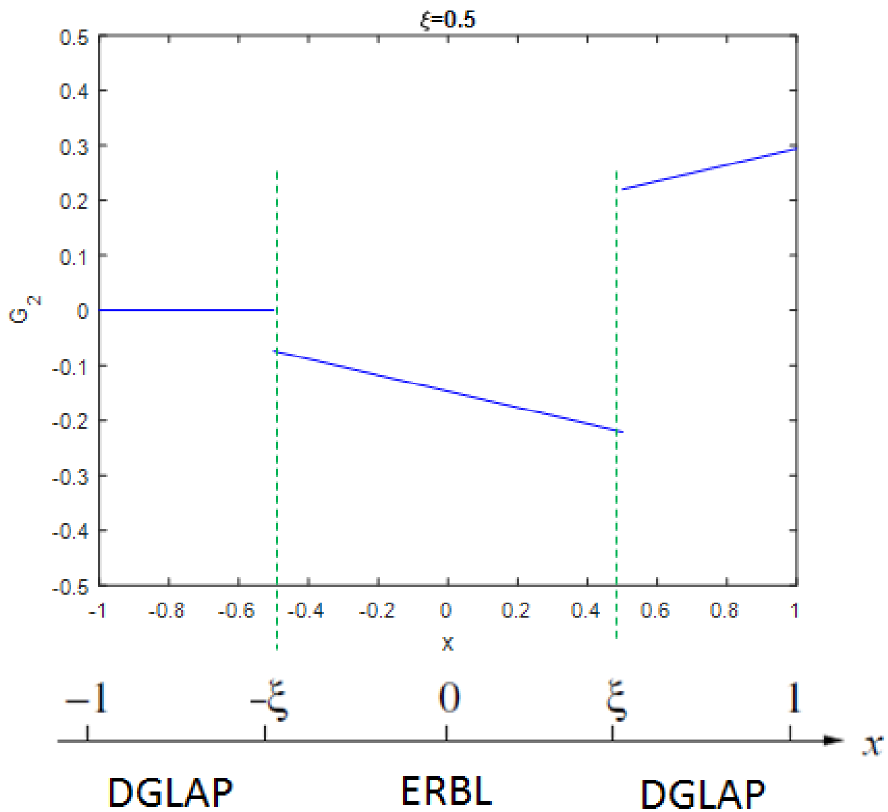
G_2 in quark target model



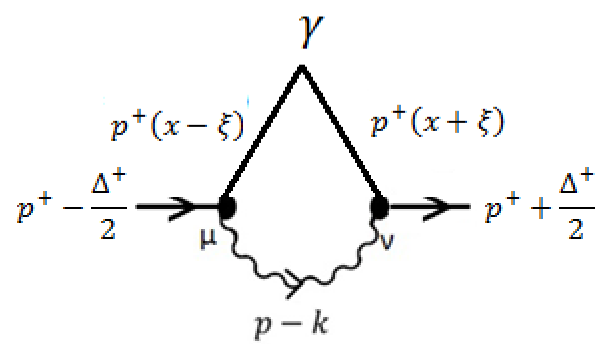
twist-3 GPDs (Polyakov & Kitpily)

$$\int dz^- e^{ixz^-\bar{p}^+}\langle p'|\bar{q}(z^-/2)\gamma^x q(-z^-/2)|p\rangle$$

$$= \frac{1}{2\bar{p}^+}\bar{u}(p')\left[\frac{\Delta^x}{2M}G_1 + \gamma^x(H+E+G_2) + \frac{\Delta^x\gamma^+}{\bar{p}^+}G_3 + \frac{i\Delta^y\gamma^+\gamma_5}{\bar{p}^+}G_4\right]u(p)$$

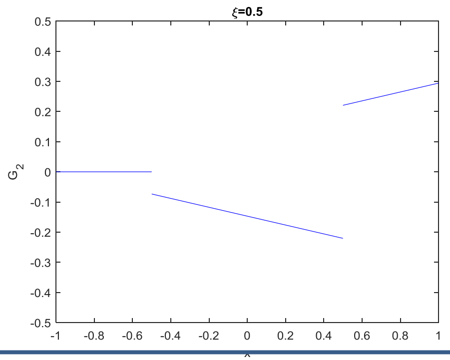


G_2 has discontinuities



$$x = \frac{k^+}{p^+} \qquad \xi = \frac{\Delta^+}{2p^+}$$

- *There are discontinuities in G_2.*



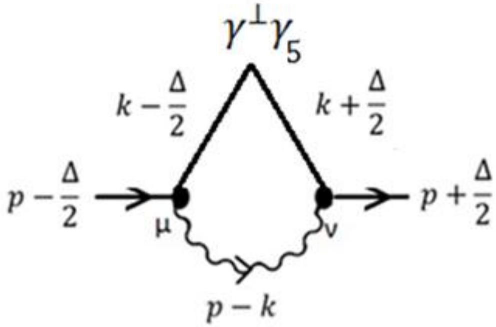
- *Factorization?* $$\int_{-1}^{1} dx \frac{GPD}{x \pm \xi + i\varepsilon}$$

- *The relevant DVCS amplitude involves* $G_2 \pm \frac{\tilde{G}_2}{\xi}$

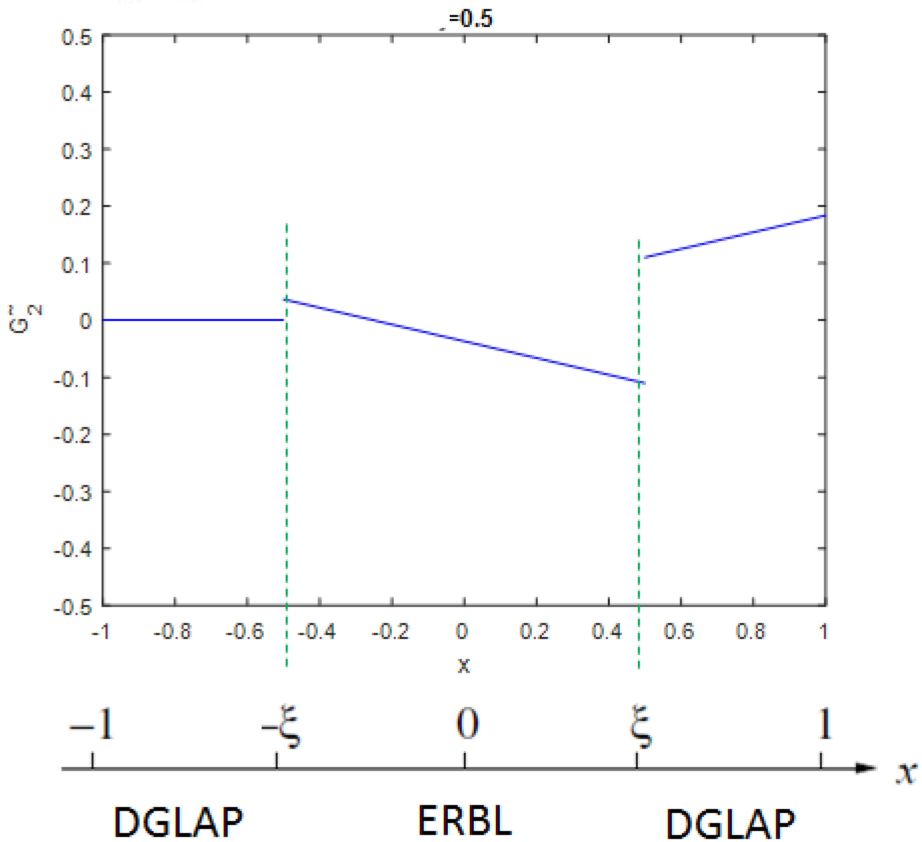
$$\int_{-1}^{1} dx \, \frac{G_2 + \frac{1}{\xi}\tilde{G}_2}{x + \xi + i\varepsilon} \qquad \int_{-1}^{1} dx \, \frac{G_2 - \frac{1}{\xi}\tilde{G}_2}{x - \xi + i\varepsilon}$$

- ***So lets calculate $\tilde{G}_2$***

$\widetilde{G}_2$ in quark target model



$$\int dz^- e^{ixz^-p^+}\langle p'|\bar{q}(z^-/_2)\gamma^x\gamma_5\bar{q}(-z^-/_2)|p\rangle =$$
$$\frac{1}{2p^+}\bar{u}(p')\left[\frac{\Delta^x}{2M}\gamma_5(\tilde{E}+\tilde{G}_1)+\gamma^x\gamma_5(\tilde{H}+\tilde{G}_2)+\frac{\Delta^x\gamma^+}{p^+}\gamma_5\tilde{G}_3+\frac{i\Delta^y\gamma^+}{p^+}\tilde{G}_4\right]u(p)$$

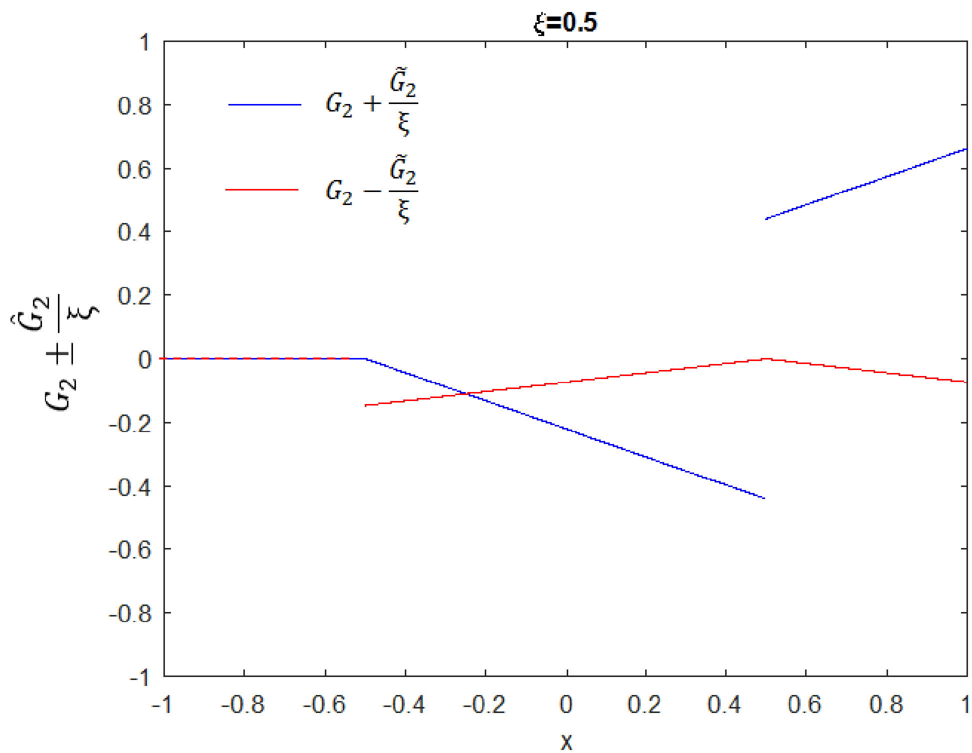


$\widetilde{G}_2$ too has discontinuities

FACTORIZATION

$$\int_{-1}^{1} dx \, \frac{G_2 + \frac{1}{\xi}\tilde{G}_2}{x + \xi + i\varepsilon} \qquad\qquad \int_{-1}^{1} dx \, \frac{G_2 - \frac{1}{\xi}\tilde{G}_2}{x - \xi + i\varepsilon}$$

- $G_2 + \frac{1}{\xi}\tilde{G}_2$ continuous at $x = -\xi$
- $G_2 - \frac{1}{\xi}\tilde{G}_2$ continuous at $x = \xi$



Twist-3 DVCS factorization is safe.

Twist-3 GPDs in Deeply-Virtual Compton Scattering, Aslan, Burkardt, Lorce, Metz, Pasquini, (2018)

Factorization is fine,
but what about the discontinuities?

How do they behave?
What do they represent?
What happens in different models?
What about the forward limit?
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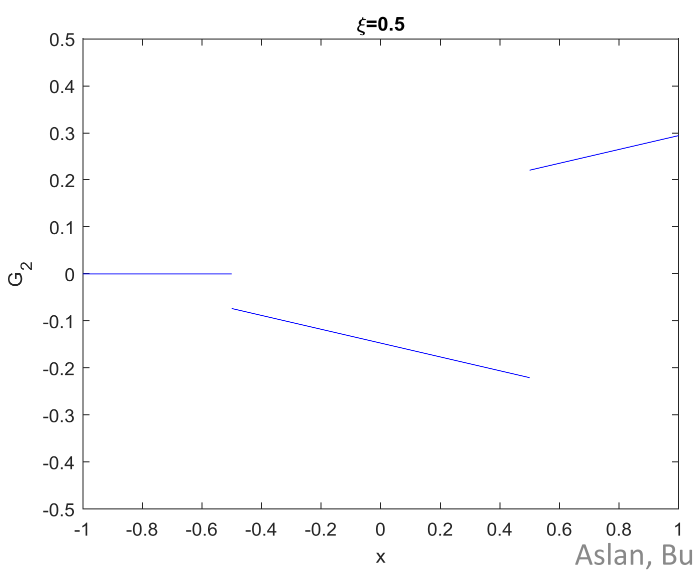
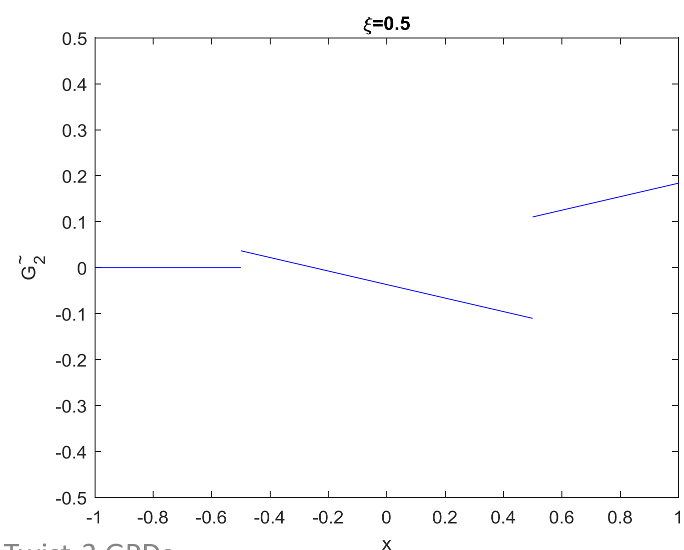
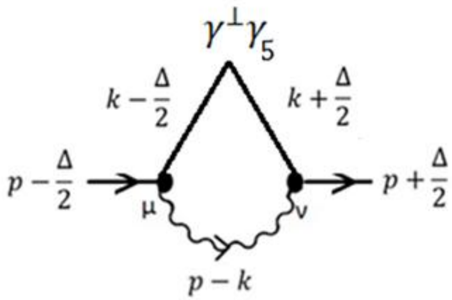
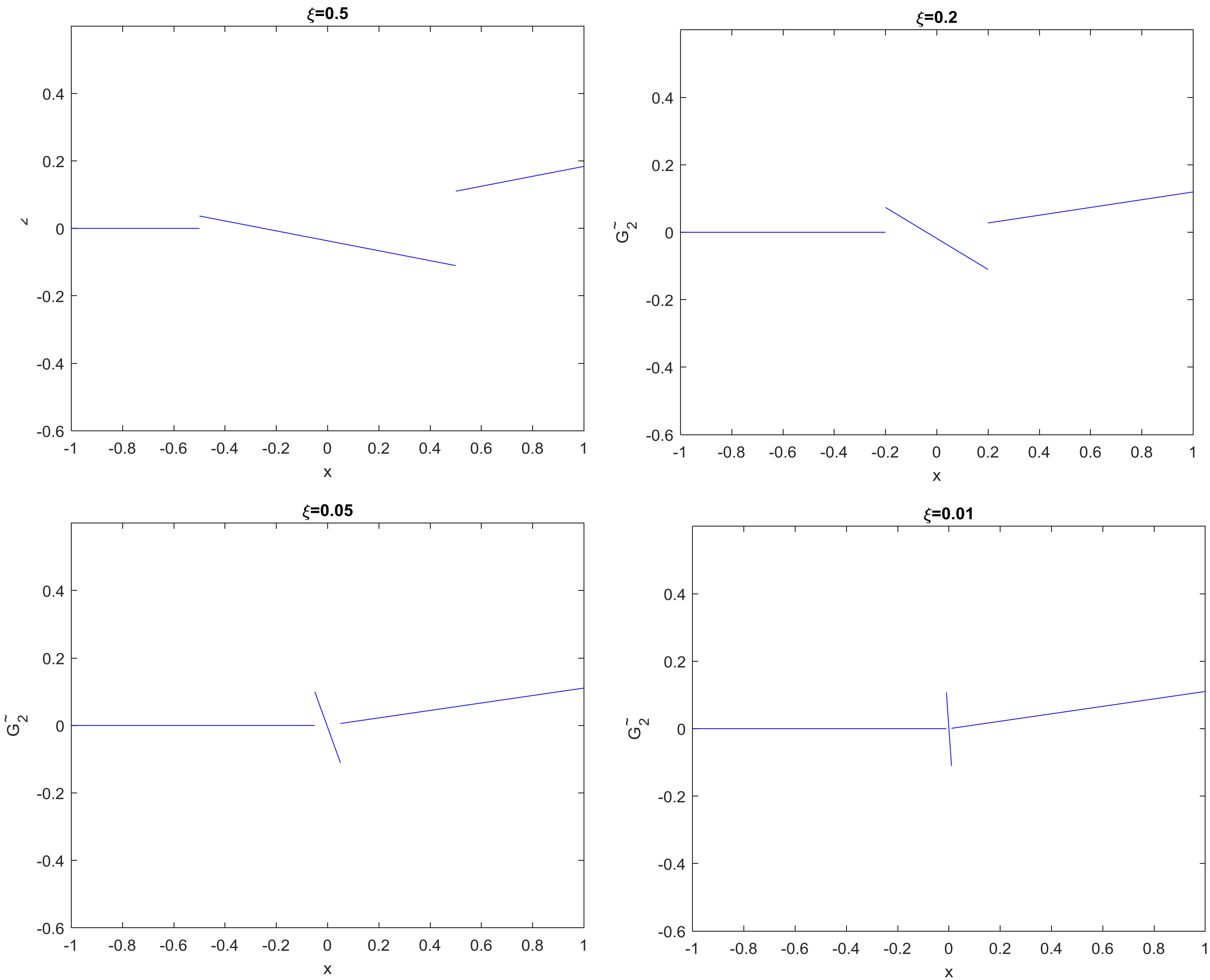




How do the discontinuities behave as ξ→0

$\tilde{G}_2$ *in quark target model*

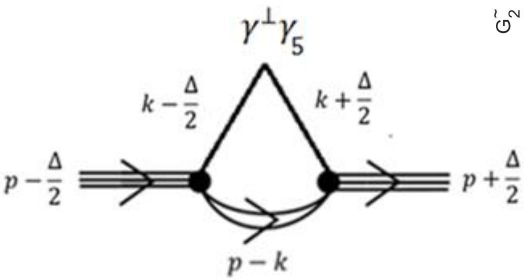


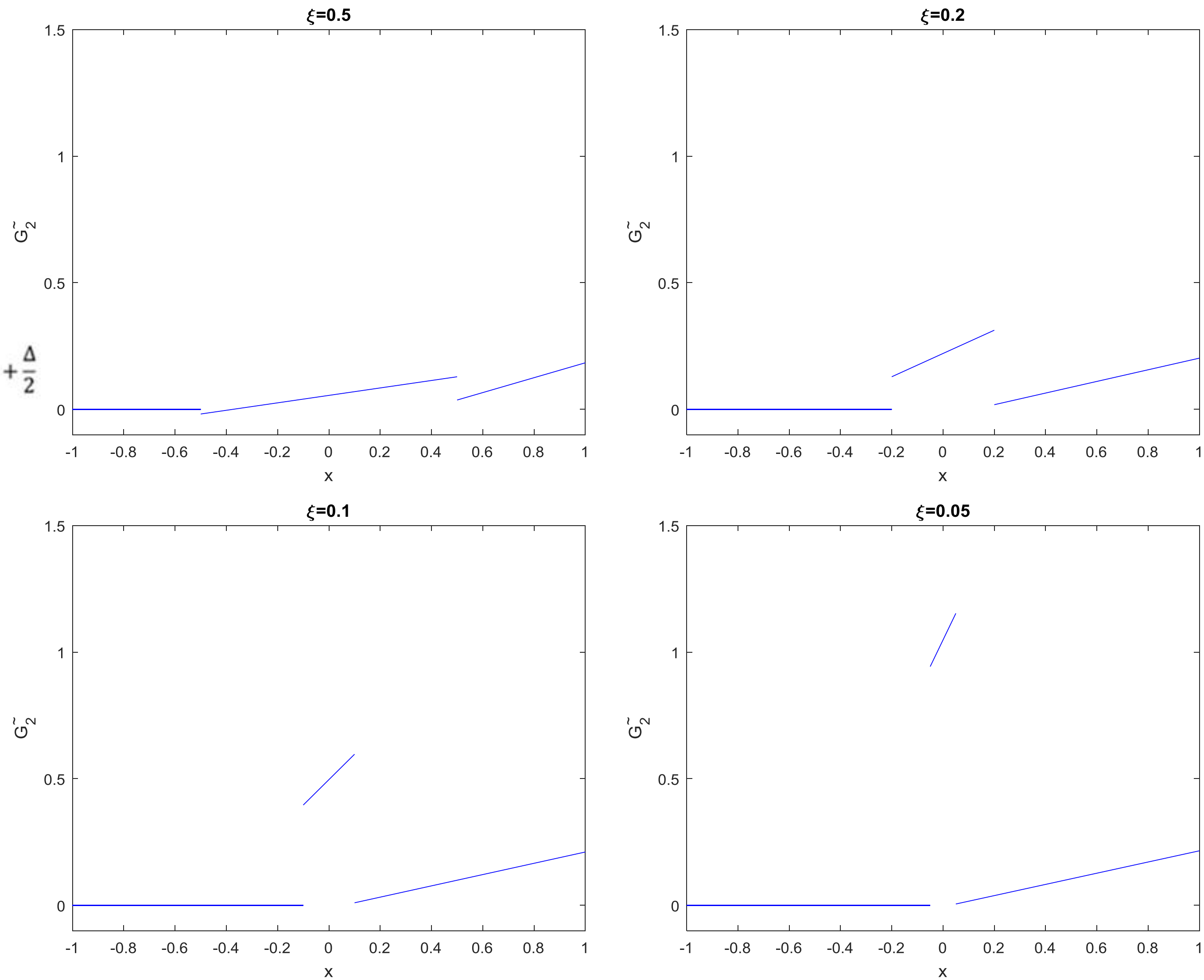


$Discontinuity\ is\ finite\ as\ \xi \to 0$

What happens in different models?

$\tilde{G}_2$ in scalar diquark model





$Discontinuity \rightarrow \infty$ as $\xi \rightarrow 0$

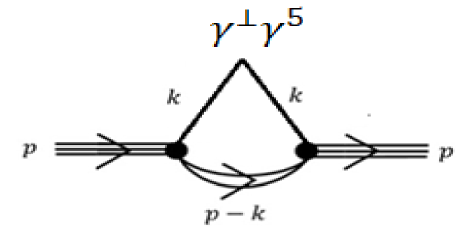
Discontinuities of $\tilde{G}_2$ and G_2 in quark target and scalar diquark model

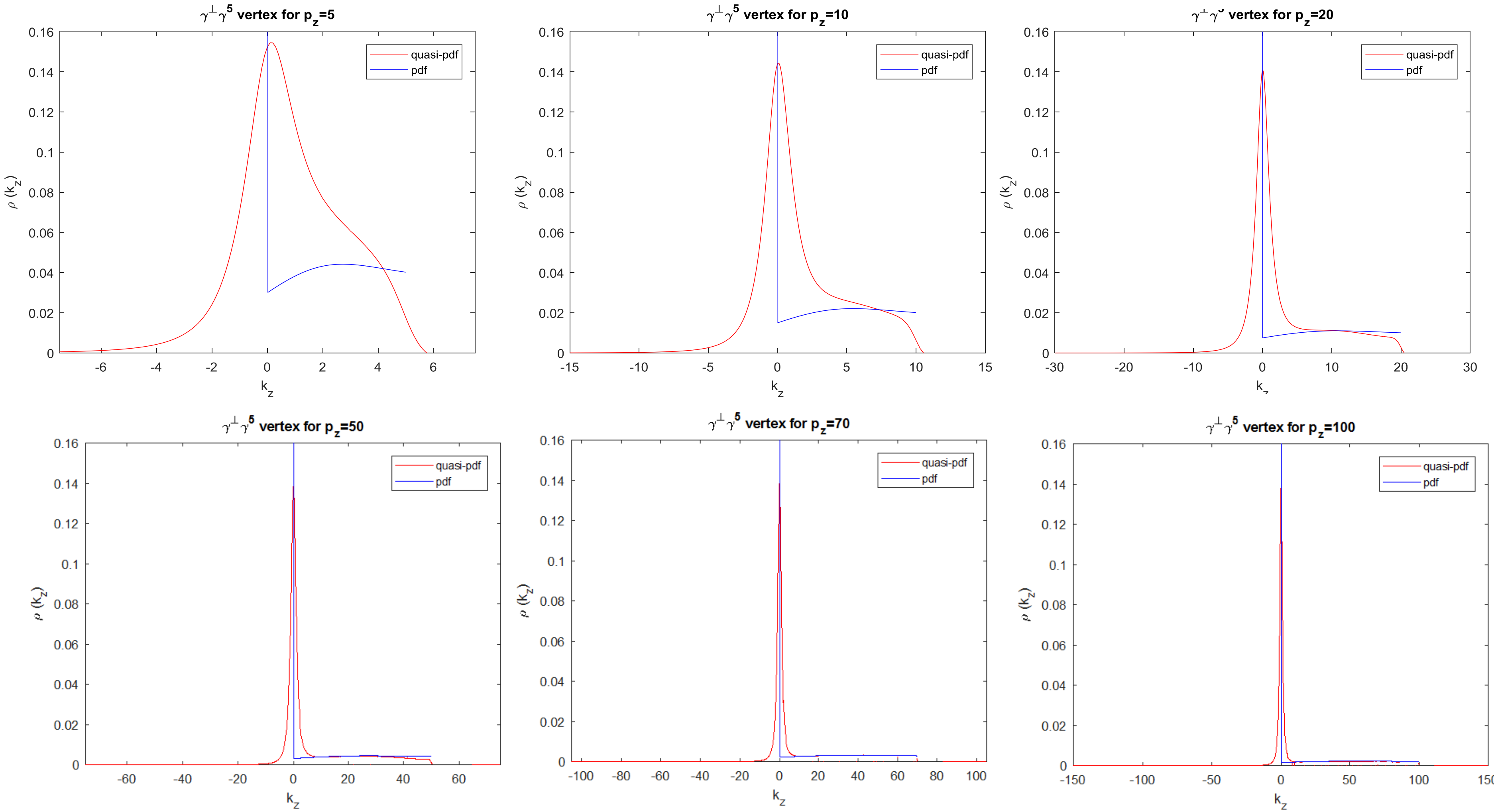
Twist-3 GPD	Scalar diquark model	Quark target model
$\tilde{G}_2$	$Discontinuities \rightarrow \infty$ $as\ \xi \rightarrow 0$	$Discontinuities \rightarrow finite$ $as\ \xi \rightarrow 0$
G_2	$Discontinuities \rightarrow finite$ $as\ \xi \rightarrow 0$	$Discontinuities \rightarrow \infty$ $as\ \xi \rightarrow 0$

- Different models give different results
- Discontinuities in $\tilde{G}_2$ behave like a δ(x) in scalar diquark model

Now let's check the forward limit

Twist -3 pdf & Twist -3 quasi-pdf , g_T , in scalar diquark model

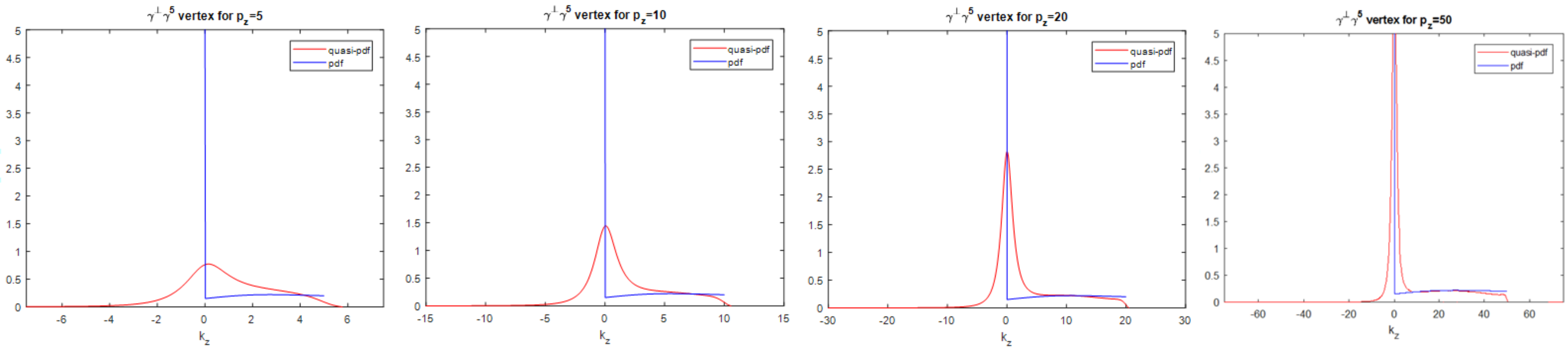




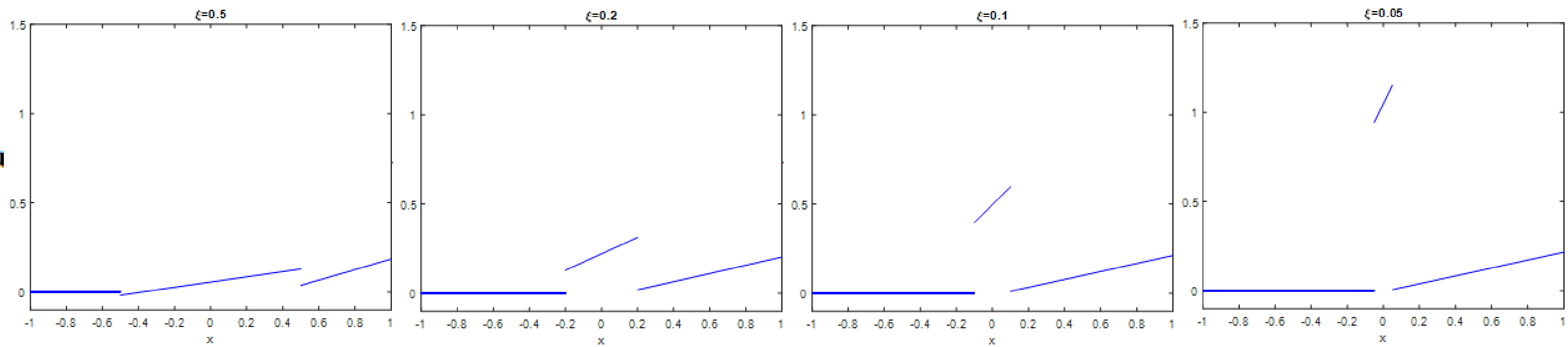
There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.

$p_z \cdot g_T$ *vs* k_z *and* $\tilde{G}_2$ *vs* x *in scalar diquark model*

$p_z \cdot g_T$

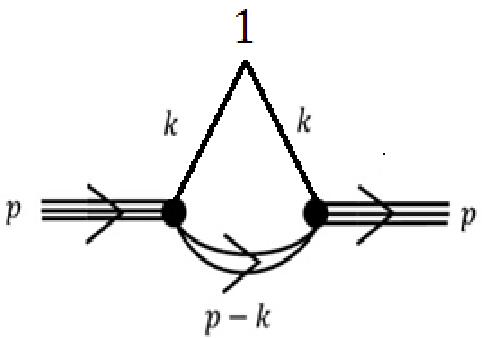


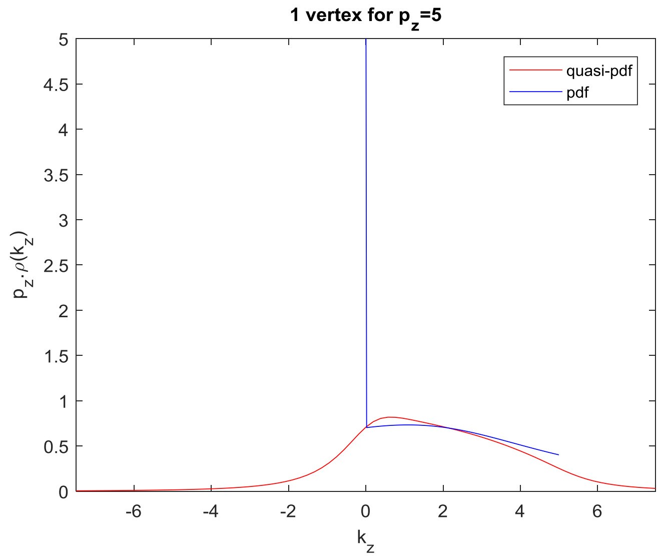
$\tilde{G}_2$

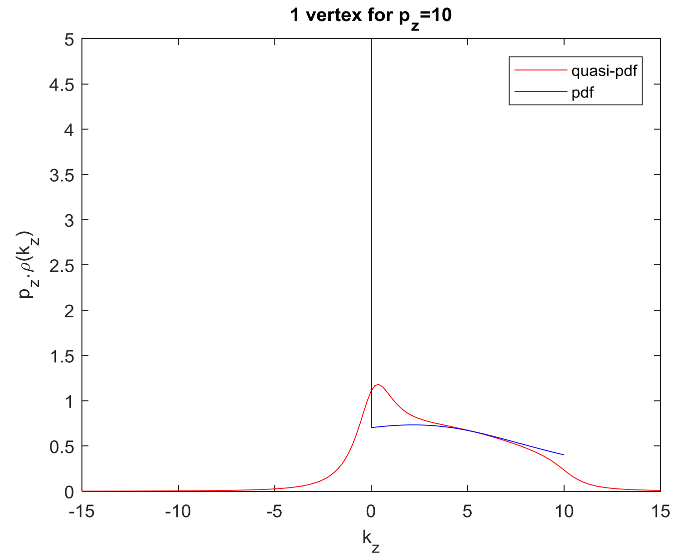


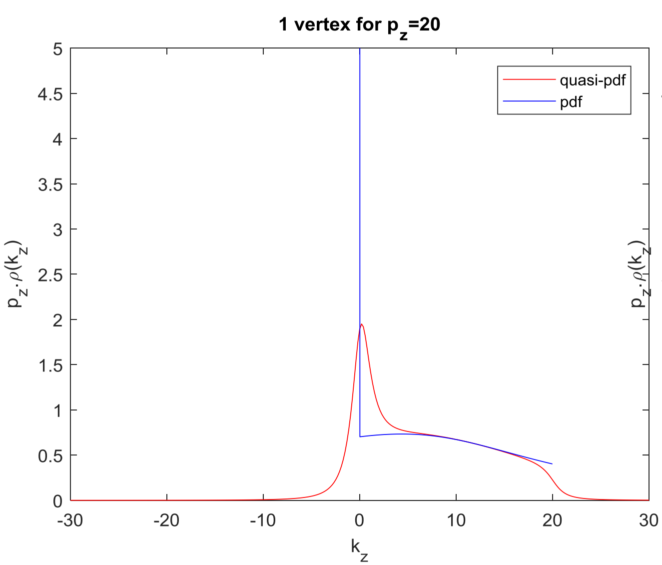
	Scalar diquark model	Quark target model
$\tilde{G}_2$	$Discontinuities \rightarrow \infty\ as\ \xi \rightarrow 0$	$Discontinuities \rightarrow finite\ as\ \xi \rightarrow 0$
$p_z \cdot g_T$	There is a δ(x)	There is no δ(x)

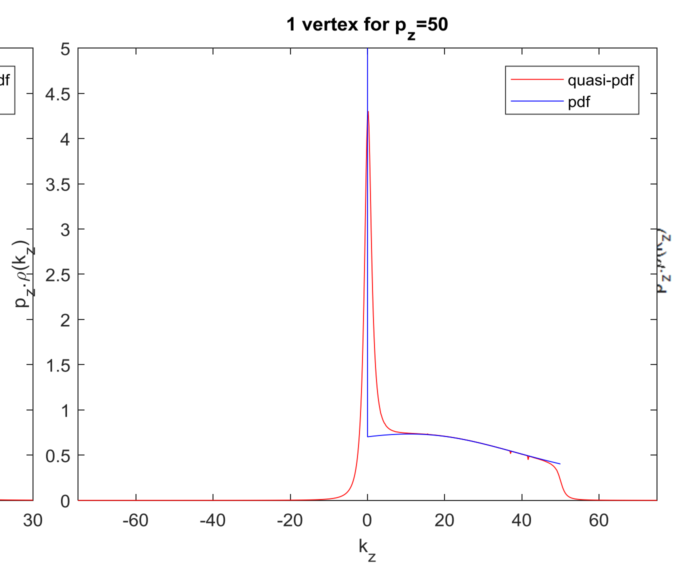
Twist -3 pdf & *Twist -3 quasi-pdf* : *e*

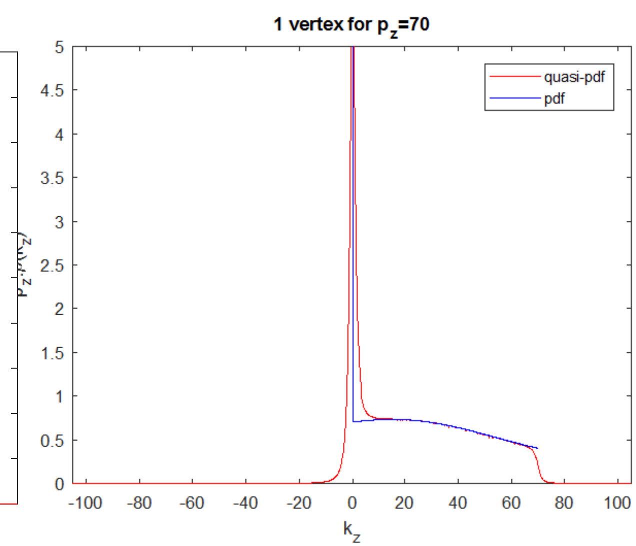




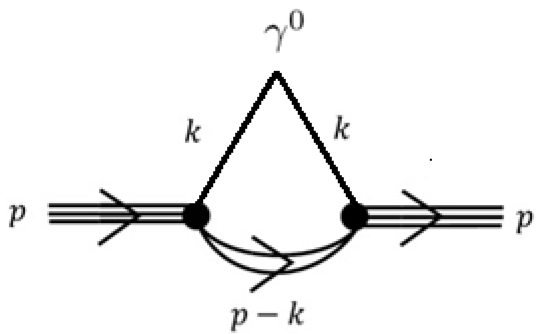


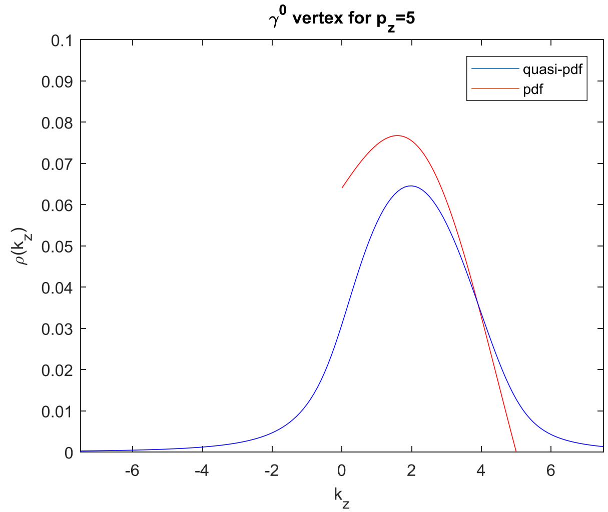


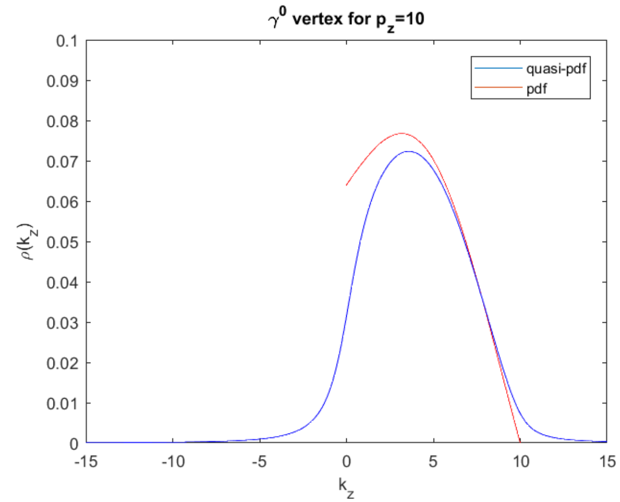


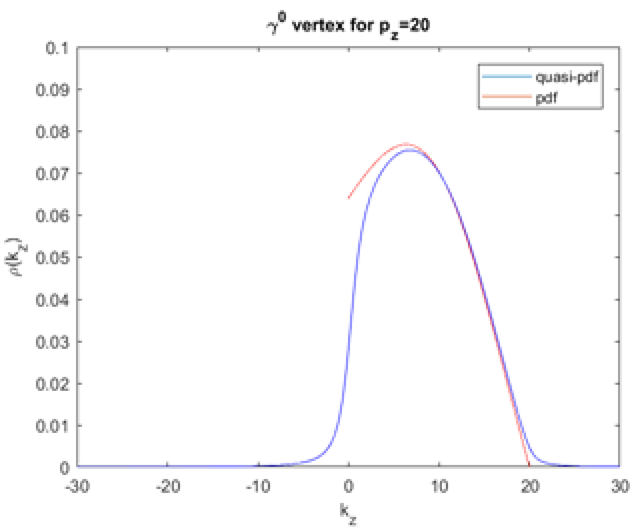


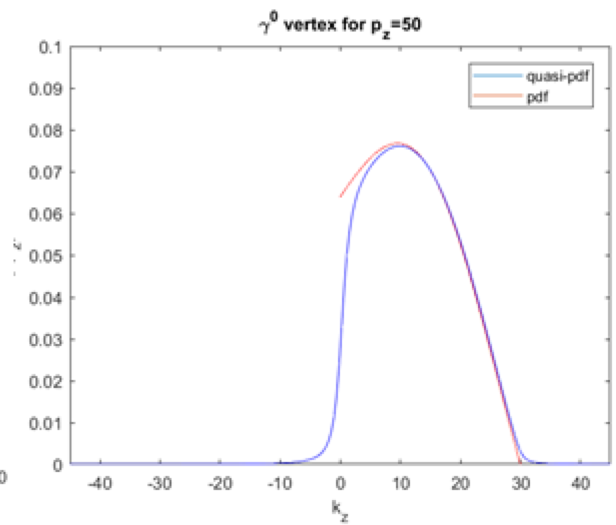
Twist -2 pdf & *Twist -2 quasi-pdf*

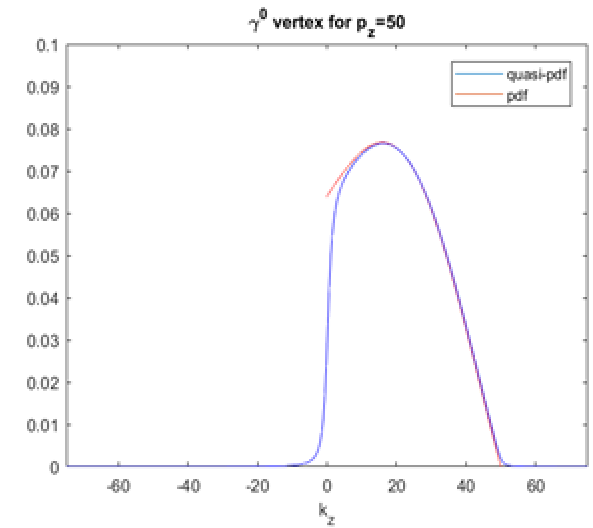












✓: There is $\delta(x)$
✗: There is no $\delta(x)$

Twist-2 pdf	Measurement	Scalar diquark model	Quark target model
f_1	Spin average	✗	✗
g_1	Helicity difference	✗	✗
h_1	Helicity flip	✗	✗

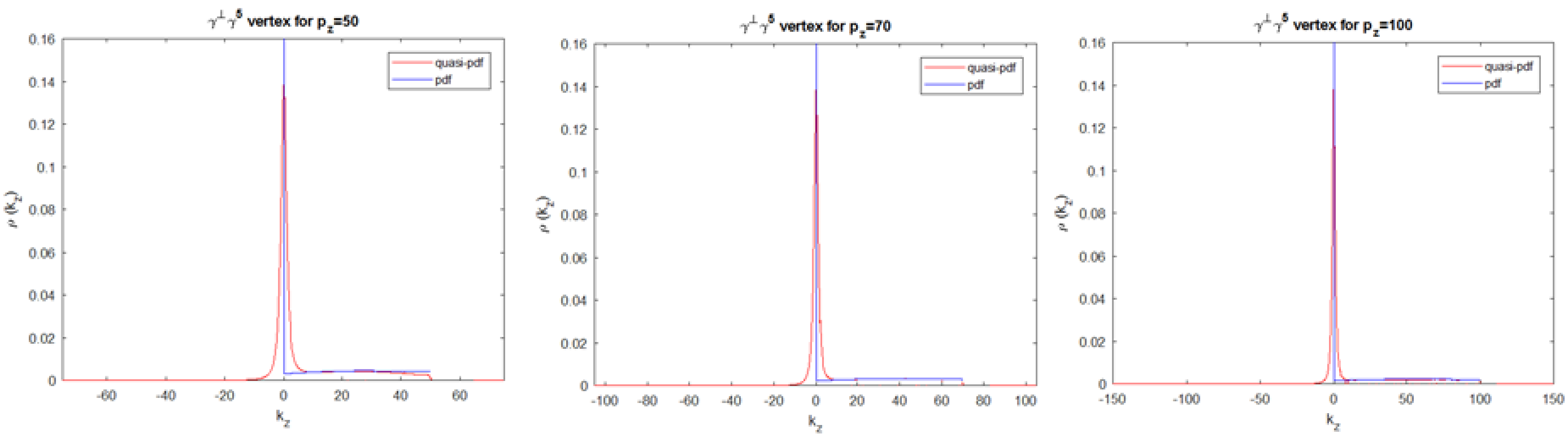
Twist-3 pdf	Measurement	Scalar diquark model	Quark target model
e	Spin average	✓	✓
h_L	Helicity difference	✓	✓
g_T (g_2)	Helicity flip	✓	✗

AT TWIST-3 THERE IS SOMETHING THAT DOES NOT EXIST IN TWIST-2: THERE ARE DELTA FUNCTIONS

✓: *There is δ(x)*
✗: *There is no δ(x)*

Twist-3 pdf	***Measurement***	***Scalar diquark model***	***Quark target model***
e	*Spin average*	✓	✓
h_L	*Helicity difference*	✓	✓
g_T (g_2)	*Helicity flip*	✓	✗

We identify these delta functions with momentum components in the nucleon state that do not scale as the nucleon is boosted to the infinite momentum.



Some sum rules are violated if we do not take the $\delta(x)$ into account.

- *Lorentz invariance*: $\int_{-1}^{1} dx\, G_2(x) = 0$

$$\lim_{\epsilon \to 0} \int_{\epsilon}^{1} dx \,\{G_2(x) + G_2(-x)\} \neq 0$$

- *Burkhardt – Cottingham sum rule*: $\int_{-1}^{1} dx\, g_1(x) = \int_{-1}^{1} dx\, g_T(x)$

$$\text{where } g_\perp(x) = g_1(x) + g_2(x)$$

- *σ – term sum rule*: $\int_{-1}^{1} dx\, e(x) = \frac{1}{2M} \langle p|\bar{\psi}(0)\psi(0)|p\rangle = \frac{d}{dm_q} M_N$

$$\lim_{\epsilon \to 0} \int_{\epsilon}^{1} dx \,\{e(x) + e(-x)\} \neq \frac{d}{dm_q} M_N$$

It is incorrect to draw conclusions from smooth behavior near x=0 about the behavior at x=0

CONCLUSIONS

✓Twist-3 GPDs have discontinuities at $x=\pm\xi$.

✓No issues with DVCS factorization for twist-3.

✓There is a Delta function at $x=0$ in twist-3 PDFs (Therefore it would also be interesting to study twist-3 quasi pdfs in lattice qcd).

✓These $\delta(x)$ functions are related to the wave function components that do not scale when the nucleon is boosted to the infinite momentum frame.

✓Not taking these $\delta(x)$ functions into account imply apparent violations of sum rules.

OUTLOOK 1

- Calculation of twist -3 GPD/PDFs in models with dynamical symmetry breaking: How does non-perturbative dressing affect the discontinuities and δ(x)?

- Decomposition of twist 3. $$g_2(x) = {g_2}^{WW}(x) + {g_2}^{m}(x) + {g_2}^{3}(x)$$

Quark mass term potentially contains a $\delta(x)$ function

Genuine twist-3 term potentially contains a $\delta(x)$ function

Previously

$$h_L(x) = h_L^{WW}(x) + h_L^m(x) + h_L^3(x)$$

We remark that the above calculation indicates that the $\delta(x)$ term appears not only in h_L^m but also in h_L^3. Furthermore they do not cancel but add up to give rise to $-\delta(x)$ in $h_L(x, Q^2)$ itself.

Burkardt & Koike, Violation of Sum Rules for Twist 3 Parton Distributions in QCD, 2001

- Using the x^2 moments of genuine twist-3 GPDs we can map out the transverse force acting on a quark in a polarized nucleon.

OUTLOOK 2

The origin of these Delta functions → $\int dk^- \frac{1}{(2k^+k^- - \mathbf{k}_\perp^2 - m_q^2 + i\varepsilon)^2}$

for $k^+ \neq 0$ $\int dk^- \frac{1}{(2k^+k^- - \mathbf{k}_\perp^2 - m_q^2 + i\varepsilon)^2} = 0$

for $k^+ = 0$ $\int dk^- \frac{1}{(2k^+k^- - \mathbf{k}_\perp^2 - m_q^2 + i\varepsilon)^2} = \frac{i\pi\delta(k^+)}{\mathbf{k}_\perp^2 + m_q^2}.$

S.-J. Chang and T. –M, Phys. Rev. D7, 1147 (1972)

There is a close relationship between these δ(x) terms and the infamous zero modes in LF field theory

(Burkardt & Koike, Violation of Sum Rules for Twist 3 Parton Distributions in QCD, 2001)

Only these zero mode excitations can mix with the trivial LF vacuum.

(Matthias Burkardt, Light Front Quantization, 1995)

OUTLOOK 2

.... Upon boosting the system to infinite momentum the partons would all become very far from $\eta = 0$, where η is the fraction of the particle's longitudinal momentum carried by the parton. Since all the vacuum activity takes place at $\eta = 0$, it seems very curious how these partons (at finite η) could "feel" what is going on at $\eta = 0$.

The right way to think about spontaneous breaking of chiral symmetry on the LC is that it somehow manifests itself through interactions between partons at finite η and $\eta = 0$ (the vacuum). The problem or puzzle with this is that matrixelements connecting states which are separated by a large distance in rapidity[1] are suppressed. So ho how could the valence quarks possibly feel what is going on at $\eta = 0$? .

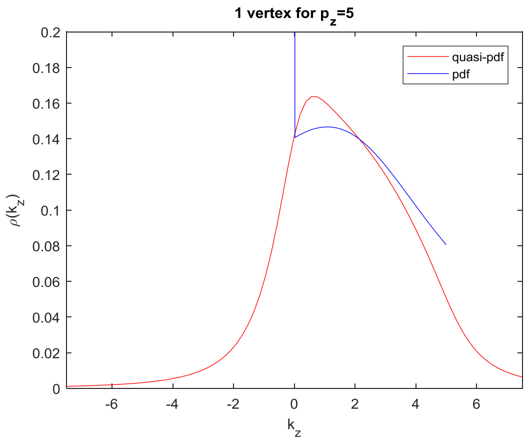
Leonard Susskind, Matthias Burkardt

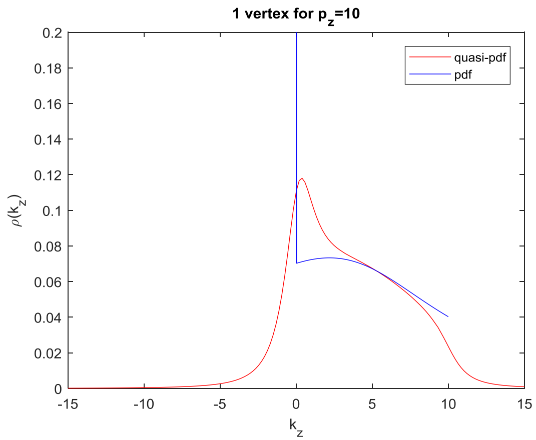
A Model of Mesons based on χSB in the Light-Front Frame (1994)

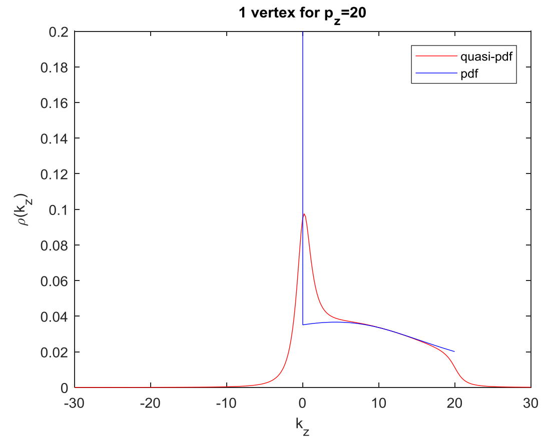
There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.

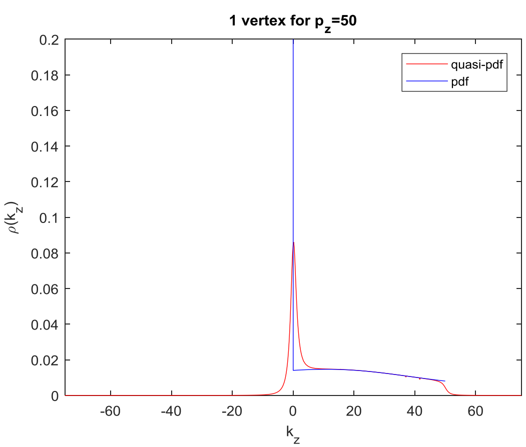
Thank you for listening

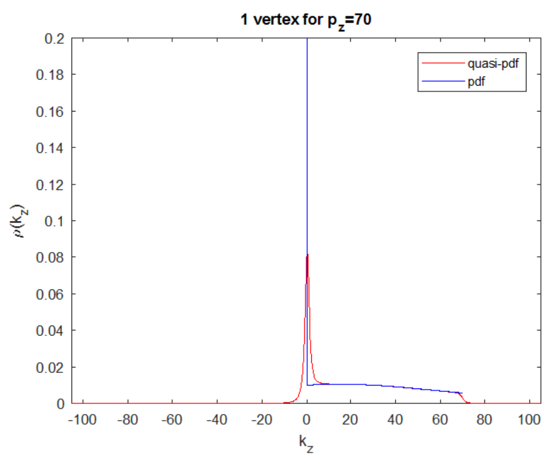
Twist -3 pdf & Twist -3 quasi-pdf : e

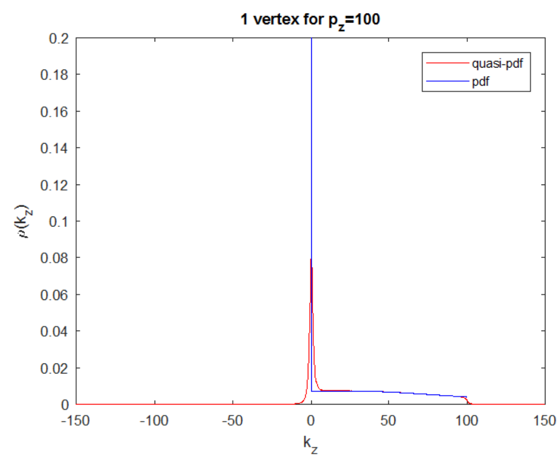












$$\text{for } k^+ \neq 0 \qquad \int dk^- \frac{1}{(2k^+k^- - \mathbf{k}_\perp^2 - m_q^2 + i\varepsilon)^2} = 0$$

$$\int dk^+ dk^- \frac{1}{(2k^+k^- - \mathbf{k}_\perp^2 - m_q^2 + i\varepsilon)^2} = \int d^2k_L \frac{1}{(k_L^2 - \mathbf{k}_\perp^2 - m_q^2 + i\varepsilon)^2} = \frac{i\pi}{\mathbf{k}_\perp^2 + m_q^2}$$

$$f_{sin}(k^+, \mathbf{k}_\perp) = \frac{1}{2p^+} \frac{i\pi\delta(k^+)}{\mathbf{k}_\perp^2 + m_q^2}.$$

there is a close relationship between these $\delta(x)$ terms and the infamous zero-modes in LF field theory