Lepton Angular Distributions in Drell-Yan Process

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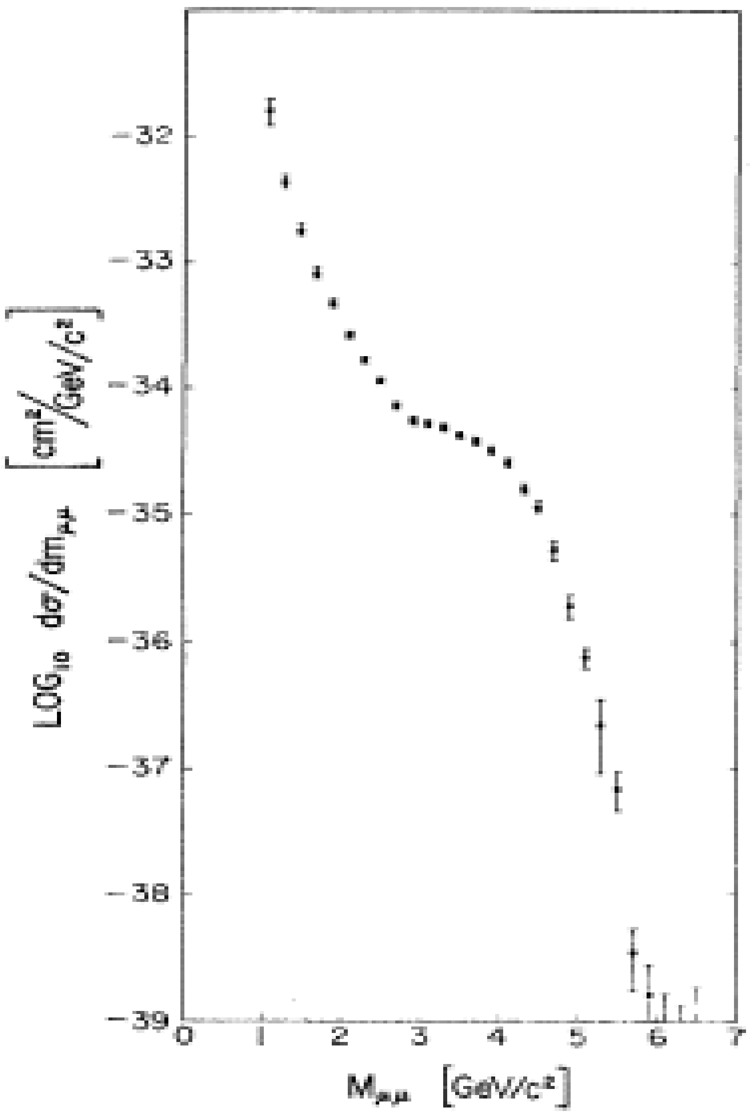
Based on the paper of JCP, Wen-Chen Chang, Evan McClellan, Oleg Teryaev, Phys. Lett. B758 (2016) 384, PRD 96 (2017) 054020, and preprints

First Dimuon Experiment



$p + U \rightarrow \mu^{+} + \mu^{-} + X$ 29 GeV proton

Lederman et al. PRL 25 (1970) 1523

Experiment originally designed to search for neutral weak boson (Z^0)

Missed the J/Ψ signal !

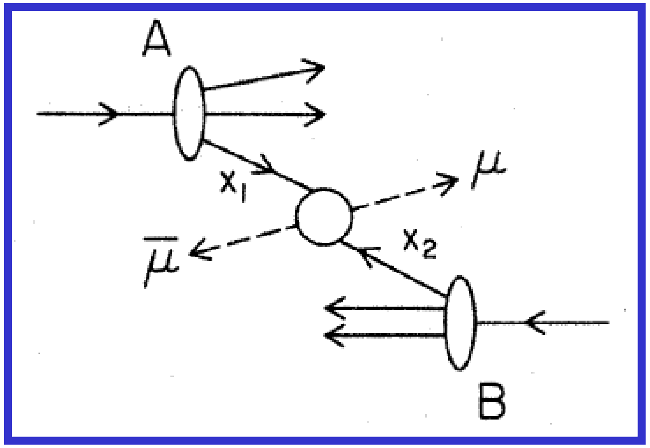
"Discovered" the Drell-Yan process

The Drell-Yan Process

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s\to\infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s\to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function νW_2 near threshold.



$$\left(\frac{d^2\sigma}{dx_1dx_2}\right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2}\sum_a e_a^2\left[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)\right]$$

Angular Distribution in the “Naïve” Drell-Yan

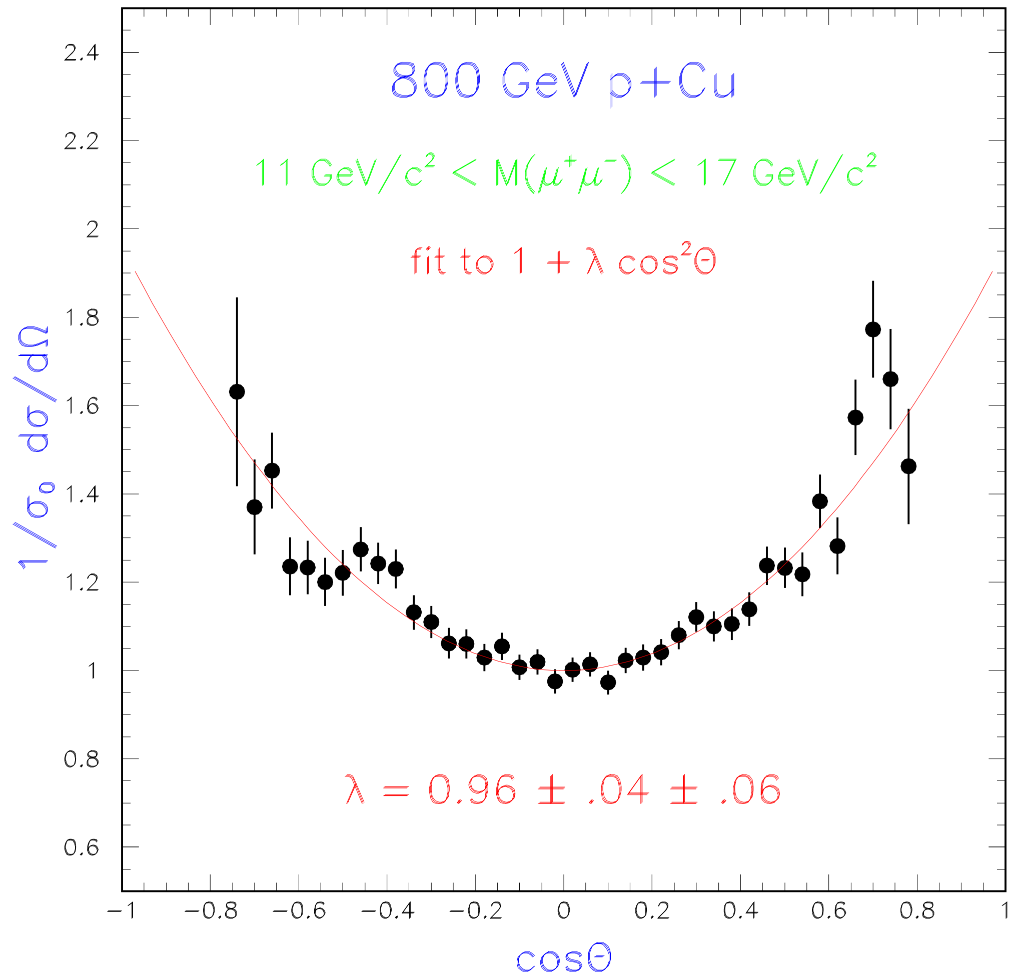
VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying $\boxed{\text{as } (1+\cos^2\theta) \text{ rather than } \sin^2\theta}$ as found in Sakurai's[10] vector-dominance model, where θ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

Drell-Yan angular distribution

Lepton Angular Distribution of “naïve” Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0 (1 + \lambda \cos^2 \theta); \quad \lambda = 1$$

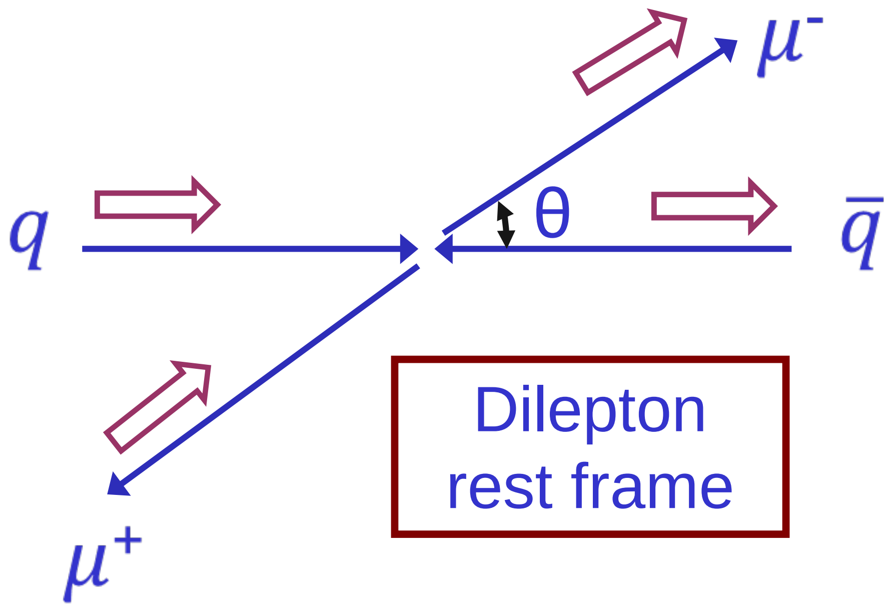


Data from Fermilab E772

(Ann. Rev. Nucl. Part. Sci. 49 (1999) 217-253)

Why is the lepton angular distribution $1+\cos^2\theta$?

Helicity conservation and parity



$RL \to RL$

$$d\sigma \sim (1+\cos\theta)^2$$

$RL \to LR$

$$d\sigma \sim (1-\cos\theta)^2$$

$LR \to LR$

$$d\sigma \sim (1+\cos\theta)^2$$

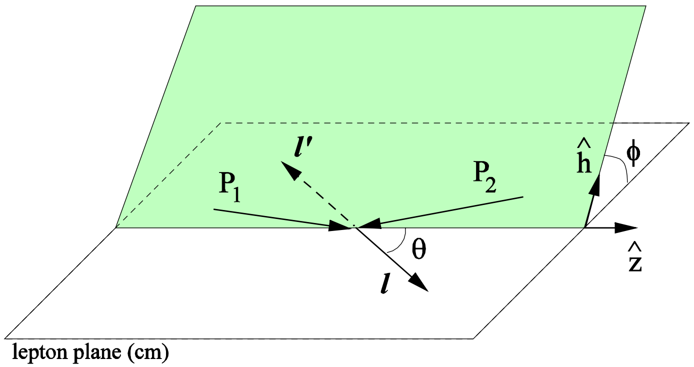
$LR \to RL$

$$d\sigma \sim (1-\cos\theta)^2$$

Adding all four helicity configurations:

$$d\sigma \sim 1+\cos^2\theta$$

Drell-Yan lepton angular distributions



Θ and Φ are the decay polar and azimuthal angles of the μ^- in the dilepton rest-frame

Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

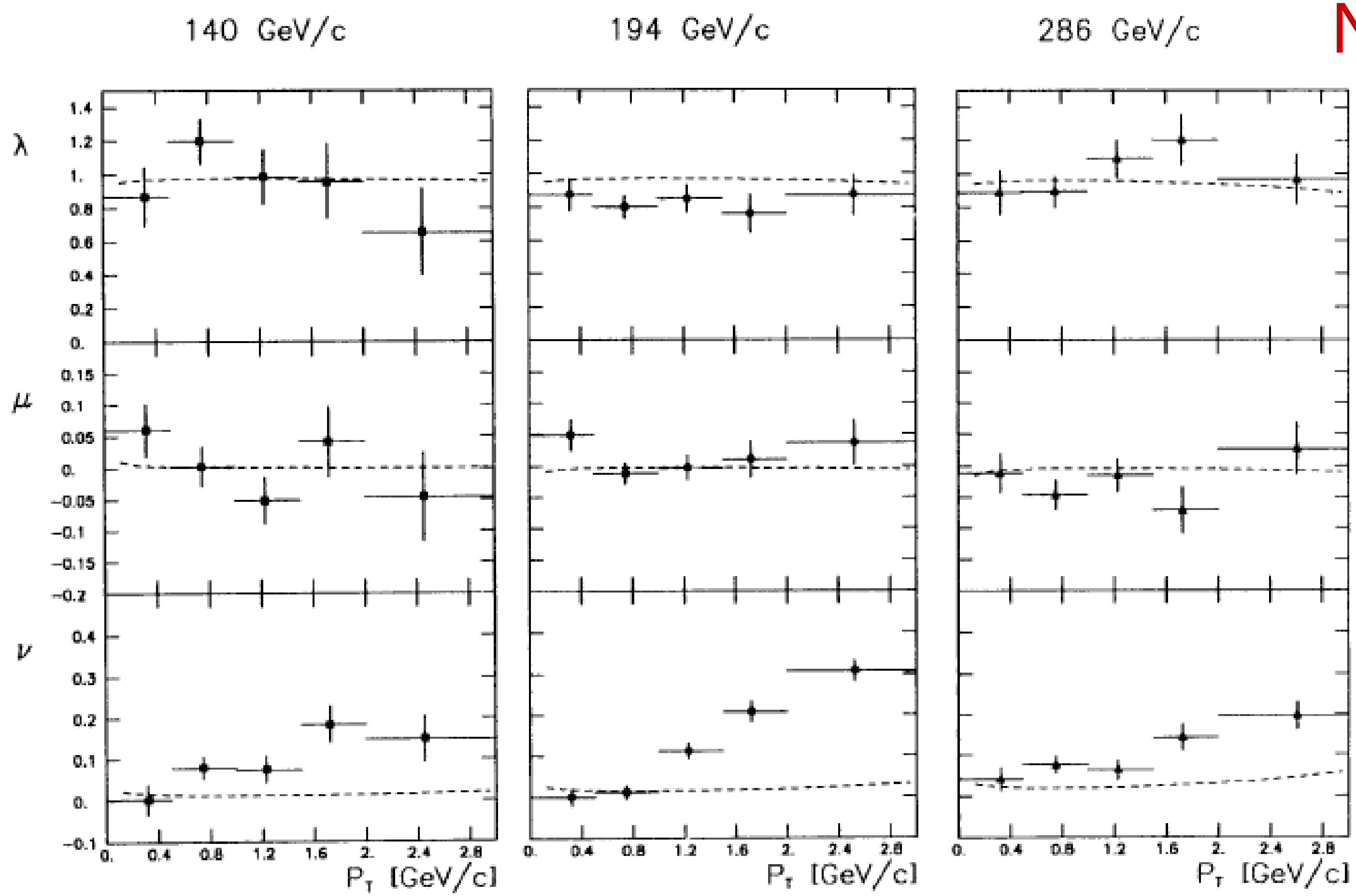
Lam-Tung relation: $1-\lambda=2\nu$

- Reflect the spin-1/2 nature of quarks (analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

Decay angular distributions in pion-induced Drell-Yan

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right)=\left[\frac{3}{4\pi}\right]\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

NA10 $\pi^- + W$



Z. Phys. 37 (1988) 545

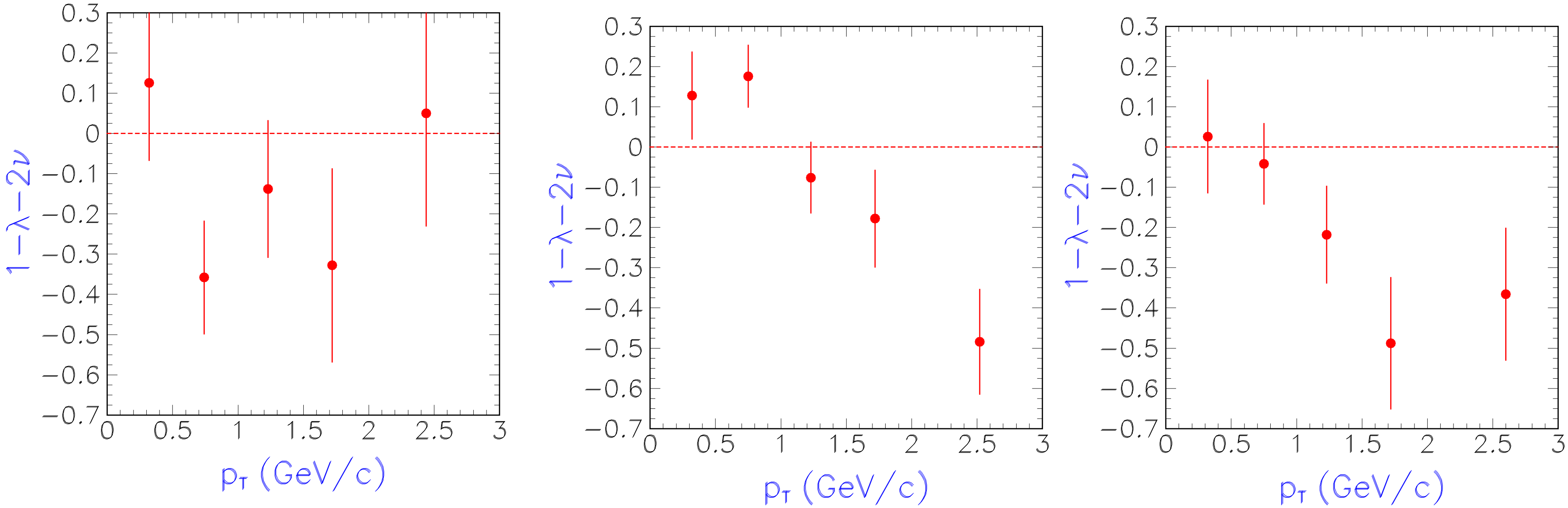
Dashed curves are from pQCD calculations

$\nu \neq 0$ and ν increases with p_T

Decay angular distributions in pion-induced Drell-Yan

Is the Lam-Tung relation (1-λ-2ν=0) violated?

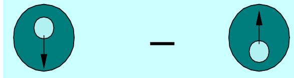
140 GeV/c 194 GeV/c 286 GeV/c



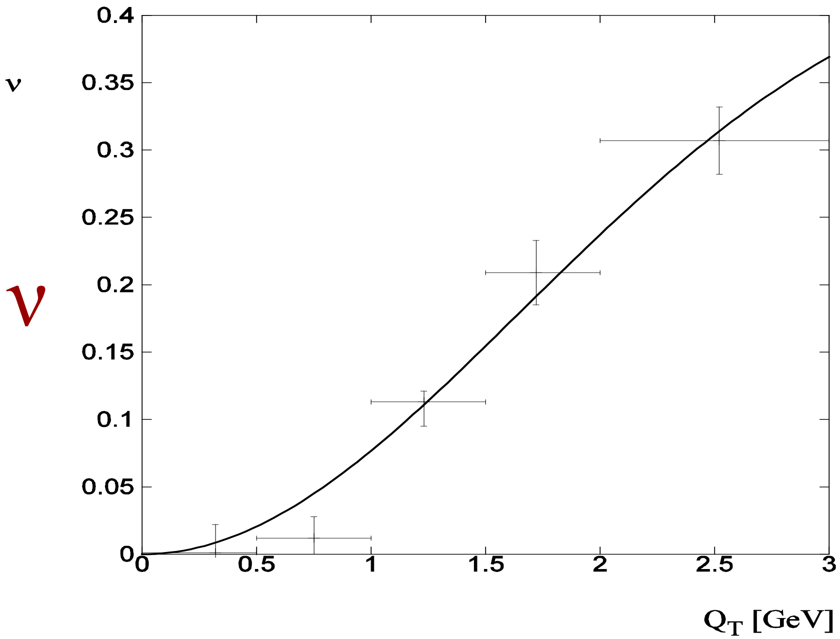
Data from NA10 (Z. Phys. 37 (1988) 545)

Violation of the Lam-Tung relation suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer,Väntinnen, Vogt, etc.)

Boer-Mulders function $h_1^{\perp}$



- Boer pointed out that the cos2ϕ dependence can be caused by the presence of the Boer-Mulders function.
- $h_1^{\perp}$ can lead to an azimuthal dependence with $\nu \propto \left(\frac{h_1^{\perp}}{f_1}\right)\left(\frac{\overline{h}_1^{\perp}}{\overline{f}_1}\right)$



$$h_1^{\perp}(x,k_T^2) = \frac{\alpha_T}{\pi} c_H \frac{M_C M_H}{k_T^2 + M_C^2} e^{-\alpha_T k_T^2} f_1(x)$$

$$\nu = 16\kappa_1 \frac{Q_T^2 M_C^2}{(Q_T^2 + 4M_C^2)^2}$$

Boer, PRD 60 (1999) 014012

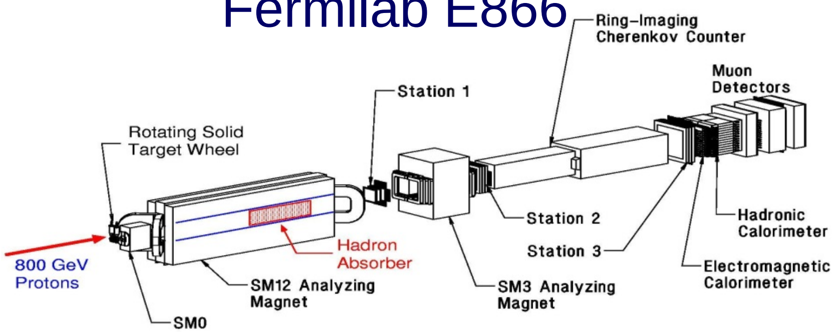
κ_1=0.47, M_C=2.3 GeV

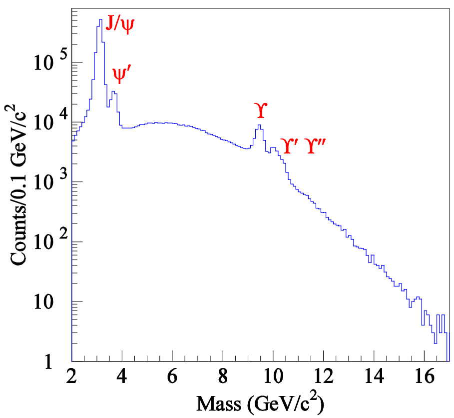
ν>0 implies valence BM functions for pion and nucleon have same signs

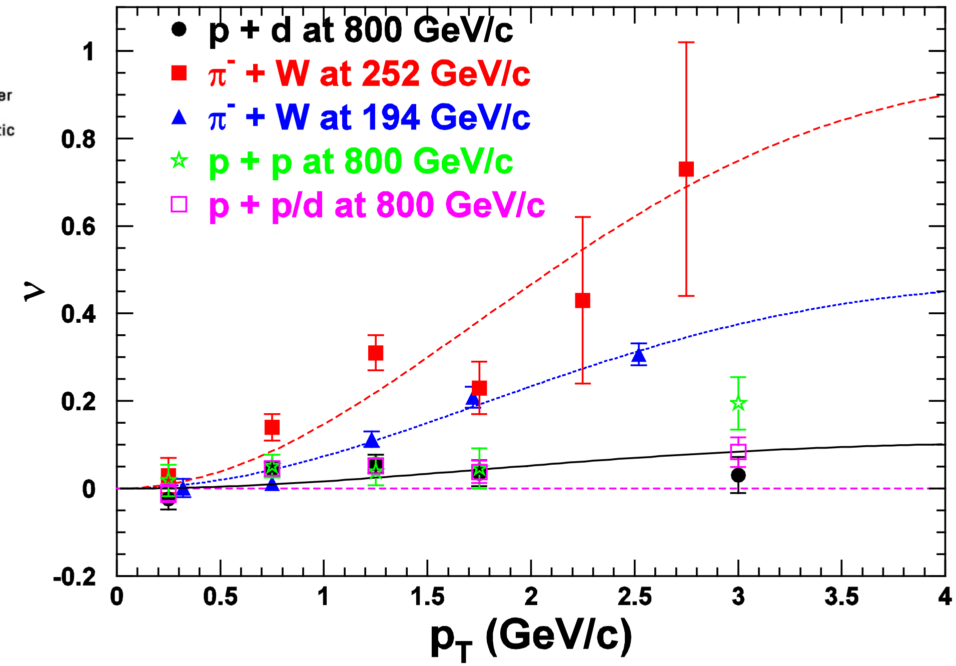
Azimuthal cos2Φ Distribution in p+d Drell-Yan

Lingyan Zhu et al., PRL 99 (2007) 082301;
PRL 102 (2009) 182001

Fermilab E866







With Boer-Mulders function $h_1^{\perp}$:

$\nu(\pi^- W \to \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(\pi)$] * [valence $h_1^{\perp}(p)$]

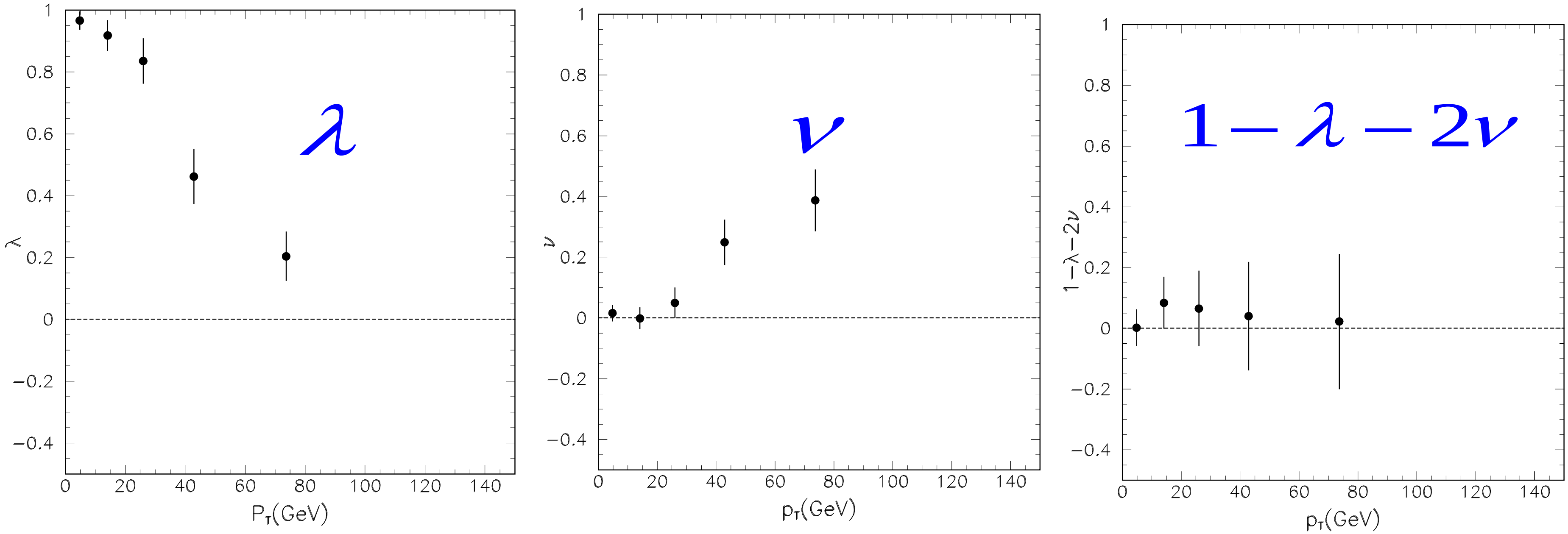
$\nu(pd \to \mu^+\mu^- X) \sim$ [valence $h_1^{\perp}(p)$] * [sea $h_1^{\perp}(p)$]

Sea-quark BM function is much smaller than valence BM function

Lam-Tung relation from CDF Z-production

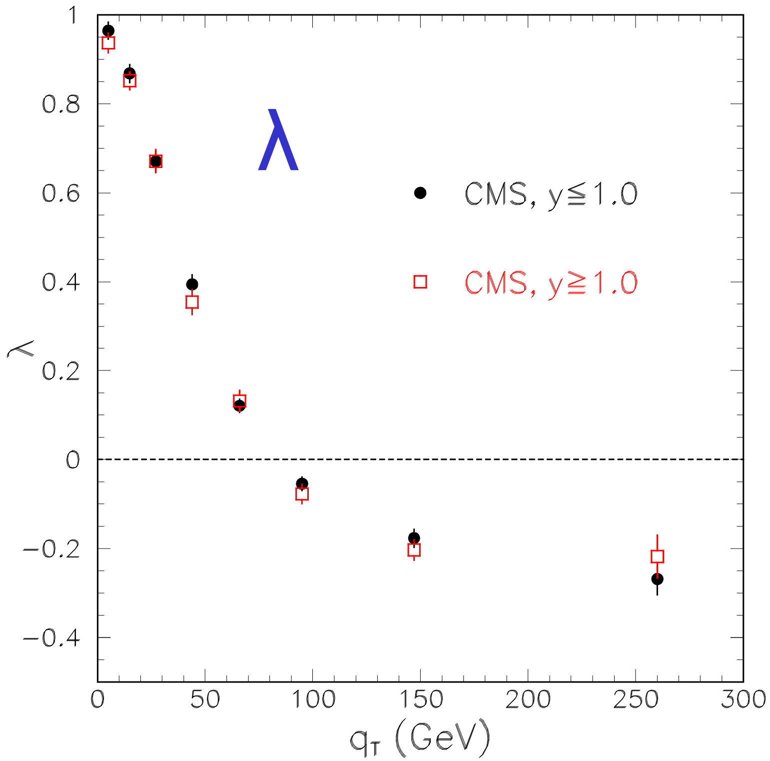
$$p+\overline{p}\rightarrow e^{+}+e^{-}+X \quad \text{at} \sqrt{s}=1.96\,\text{TeV}$$

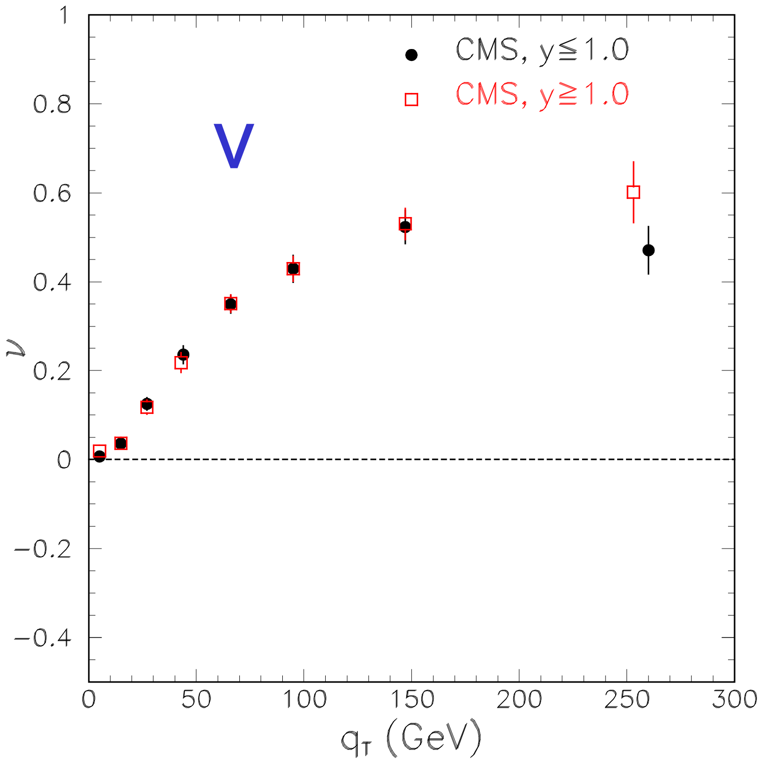
arXiv:1103.5699 (PRL 106 (2011) 241801)



- Strong p_T (q_T) dependence of λ and ν
- Lam-Tung relation (1-λ = 2ν) is satisfied within experimental uncertainties (TMD is not expected to be important at large p_T)

Recent CMS (ATLAS) data for *Z*-boson production in *p*+*p* collision at 8 TeV

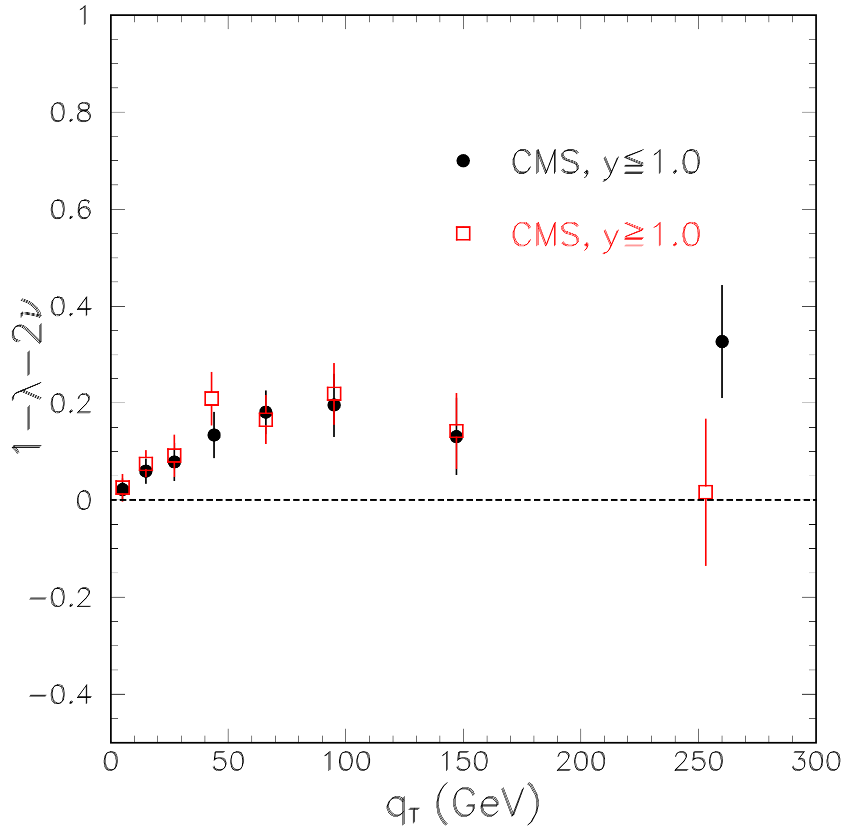




(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking q_T dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

Recent data from CMS for *Z*-boson production in *p*+*p* collision at 8 TeV



- Yes, the Lam-Tung relation is violated ($1-\lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation?

Interpretation of the CMS Z-production results

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi$$

$$+ \frac{A_2}{2}\sin^2\theta \cos 2\phi + A_3 \sin\theta \cos\phi + A_4 \cos\theta$$

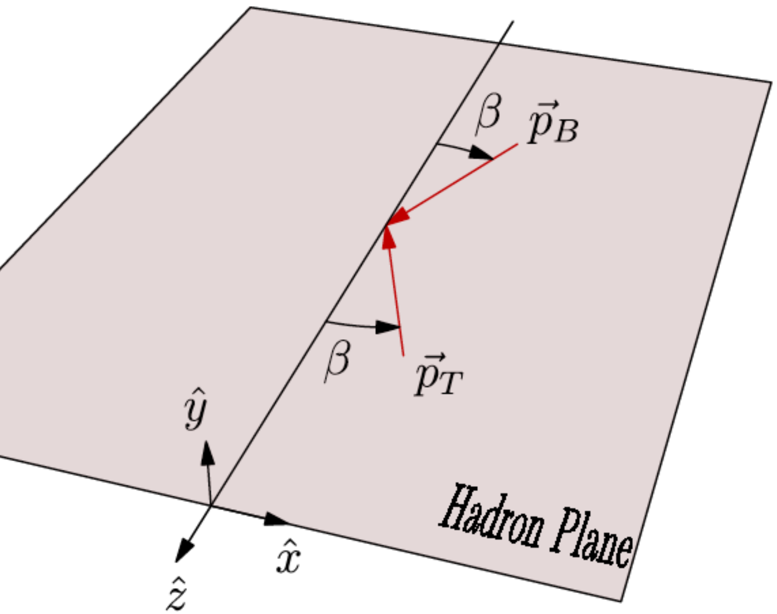
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi$$

Questions:

- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the q_T dependence of $A_0, A_1, A_2,$ etc?
- Can one understand the origin of the violation of Lam-Tung relation?

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

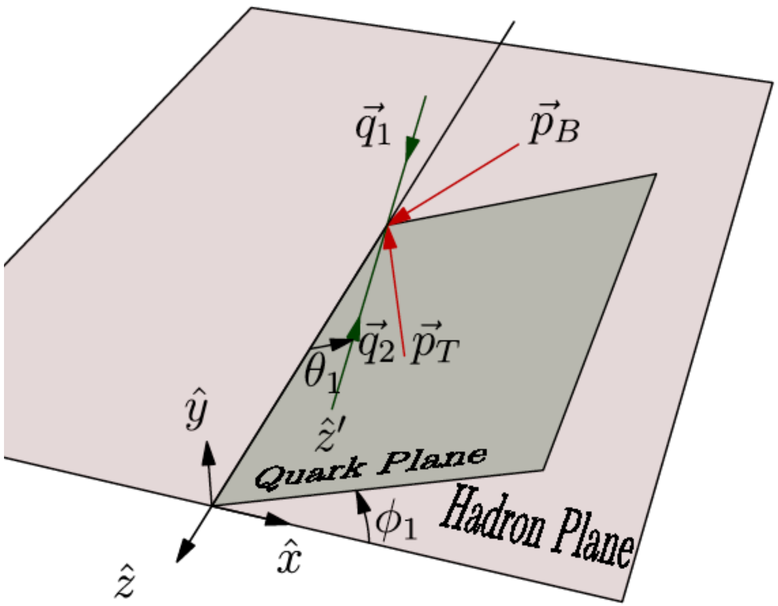
- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

- Q is the mass of the dilepton (Z)
- when $q_T \to 0,\ \beta \to 0^\circ$;

  when $q_T \to \infty,\ \beta \to 90^\circ$

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

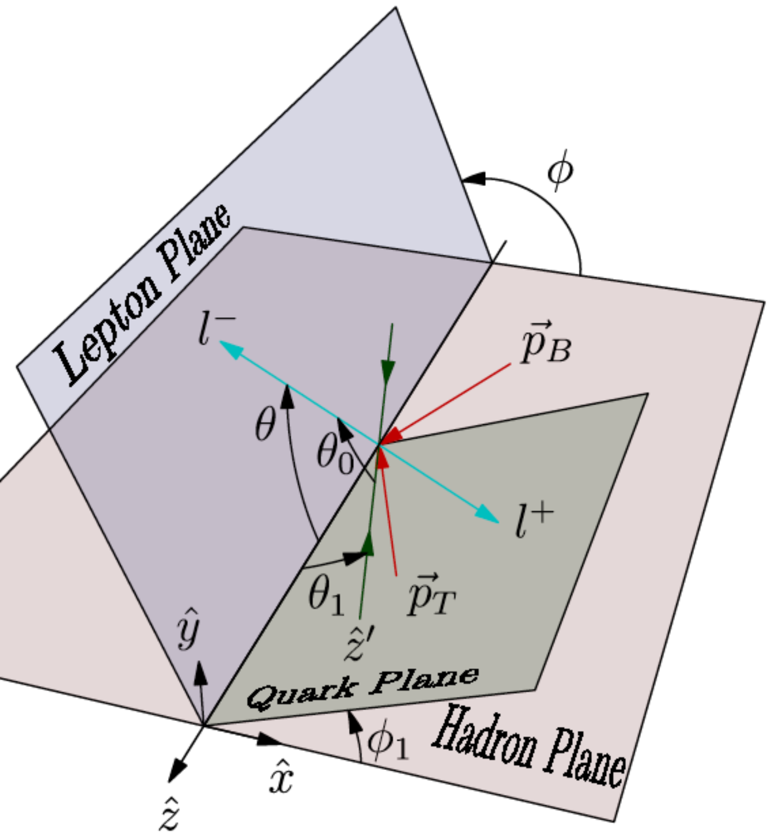
- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ and $\hat{z}$ axes form the quark plane
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

How is the angular distribution expression derived?

Define three planes in the Collins-Soper frame



1) Hadron Plane

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

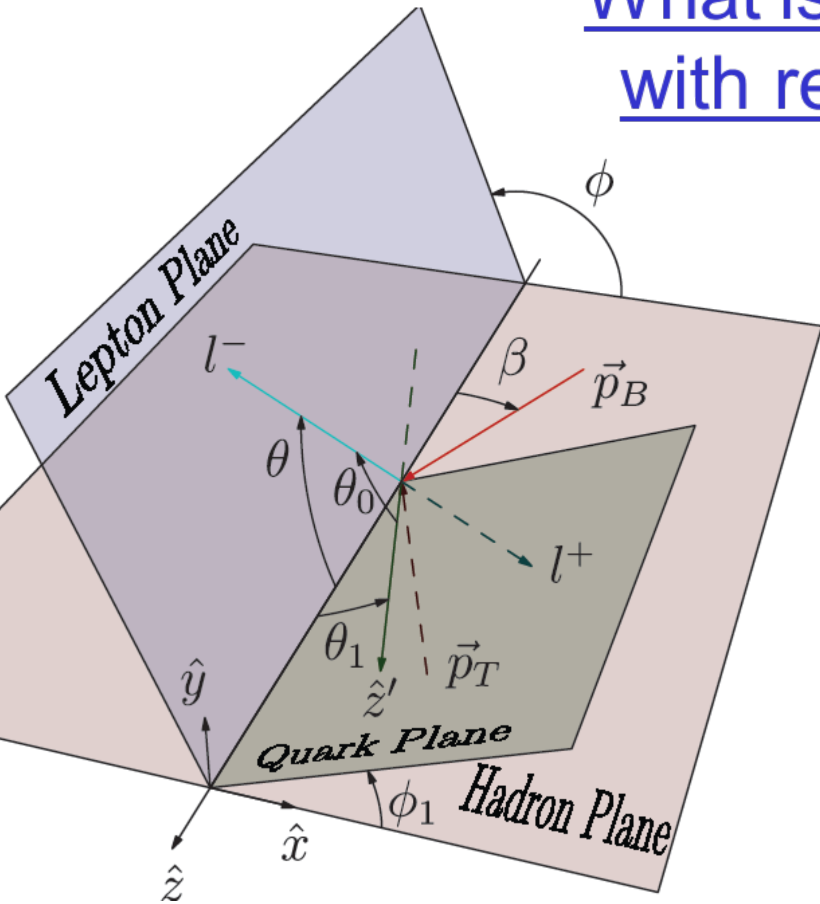
2) Quark Plane

- q and $\overline{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ axis has angles θ_1 and ϕ_1 in the C-S frame

3) Lepton Plane

- l^- and l^+ are emitted back-to-back with equal $|\vec{P}|$
- l^- and $\hat{z}$ form the lepton plane
- l^- is emitted at angle θ and ϕ in the C-S frame

How is the angular distribution expression derived?



<u>What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?</u>

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

Azimuthally symmetric !

<u>How to express the angular distribution in terms of θ and φ?</u>

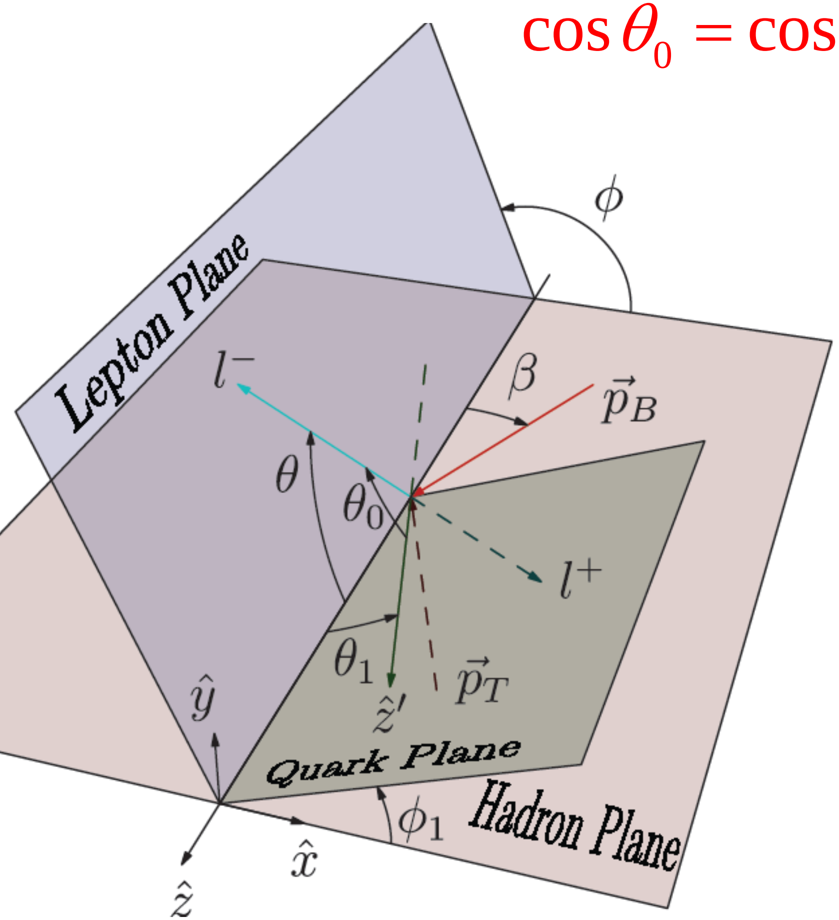
Use the following relation:

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

How is the angular distribution expression derived?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$



$$\begin{aligned}
\frac{d\sigma}{d\Omega} \propto\ & (1 + \cos^2\theta) + \frac{\sin^2\theta_1}{2}(1 - 3\cos^2\theta) \\
& + (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi \\
& + (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi \\
& + (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta \\
& + (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta\sin 2\phi \\
& + (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta\sin\phi \\
& + (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi.
\end{aligned}$$

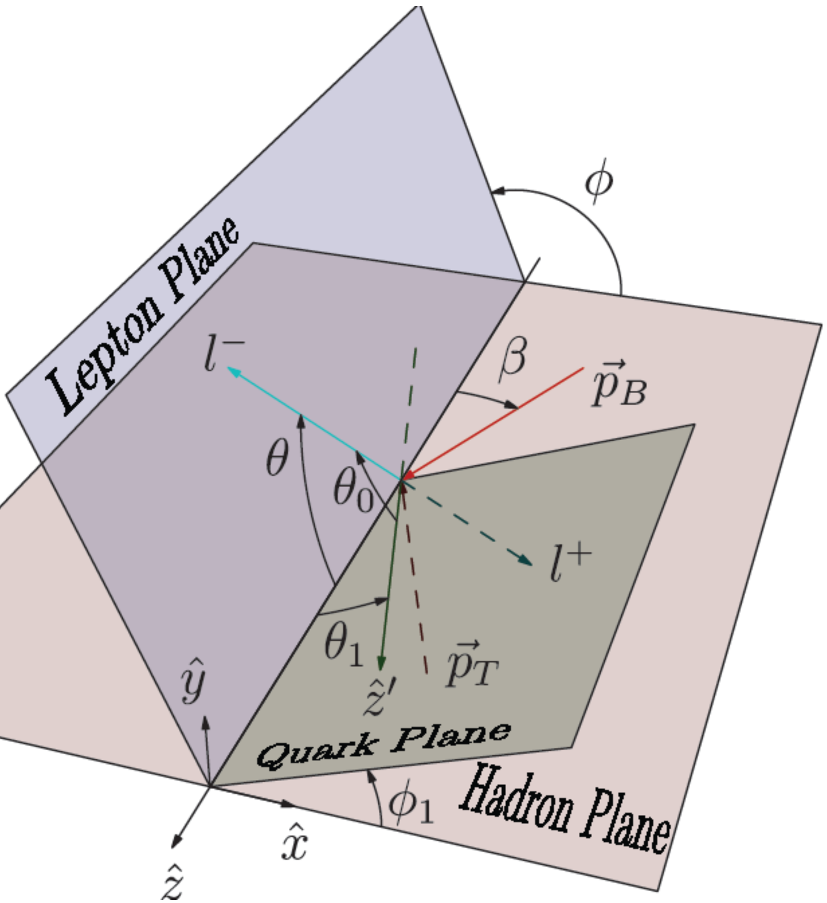
All eight angular distribution terms are obtained!

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta)$$
$$+ (\frac{1}{2}\sin 2\theta_1 \cos\phi_1) \sin 2\theta \cos\phi$$
$$+ (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1) \sin^2\theta \cos 2\phi$$
$$+ (a \sin\theta_1 \cos\phi_1) \sin\theta \cos\phi + (a\cos\theta_1)\cos\theta$$
$$+ (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1) \sin^2\theta \sin 2\phi$$
$$+ (\frac{1}{2}\sin 2\theta_1 \sin\phi_1) \sin 2\theta \sin\phi$$
$$+ (a \sin\theta_1 \sin\phi_1) \sin\theta \sin\phi.$$

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta)$$
$$+ A_1 \sin 2\theta \cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta \cos 2\phi$$
$$+ A_3 \sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi$$
$$+ A_6 \sin 2\theta \sin\phi$$
$$+ A_7 \sin\theta \sin\phi$$

$A_0 - A_7$ are entirely described by θ_1, ϕ_1 and a

Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2 \theta_1 \rangle$$

$$A_1 = \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle$$

$$A_2 = \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a \langle \sin \theta_1 \cos \phi_1 \rangle$$

$$A_4 = a \langle \cos \theta_1 \rangle$$

$$A_5 = \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2} \langle \sin 2\theta_1 \sin \phi_1 \rangle$$

$$A_7 = a \langle \sin \theta_1 \sin \phi_1 \rangle$$

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, a, is reduced by a factor of $\langle \cos\theta_1 \rangle$ for A_4
- A_5, A_6, A_7 are odd function of ϕ_1 and must vanish from symmetry consideration
- Some equality and inequality relations among $A_0 - A_7$ can be obatined

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \left\langle \sin^2\theta_1 \right\rangle$$

$$A_1 = \frac{1}{2}\left\langle \sin 2\theta_1 \cos\phi_1 \right\rangle$$

$$A_2 = \left\langle \sin^2\theta_1 \cos 2\phi_1 \right\rangle$$

$$A_3 = a\left\langle \sin\theta_1 \cos\phi_1 \right\rangle$$

$$A_4 = a\left\langle \cos\theta_1 \right\rangle$$

$$A_5 = \frac{1}{2}\left\langle \sin^2\theta_1 \sin 2\phi_1 \right\rangle$$

$$A_6 = \frac{1}{2}\left\langle \sin 2\theta_1 \sin\phi_1 \right\rangle$$

$$A_7 = a\left\langle \sin\theta_1 \sin\phi_1 \right\rangle$$

Some bounds on the coefficients can be obtained

$$0 < A_0 < 1$$

$$-1/2 < A_1 < 1/2$$

$$-1 < A_2 < 1$$

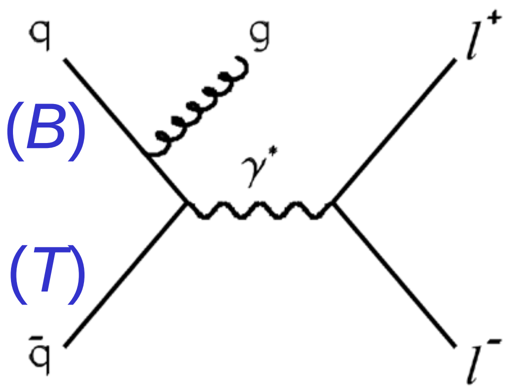
$$-a < A_3 < a$$

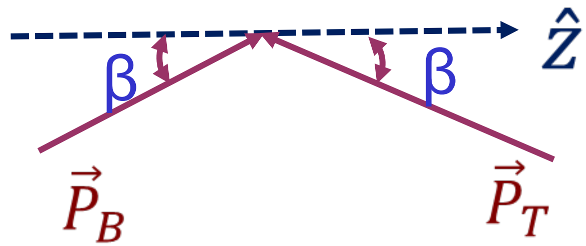
$$-a < A_4 < a$$

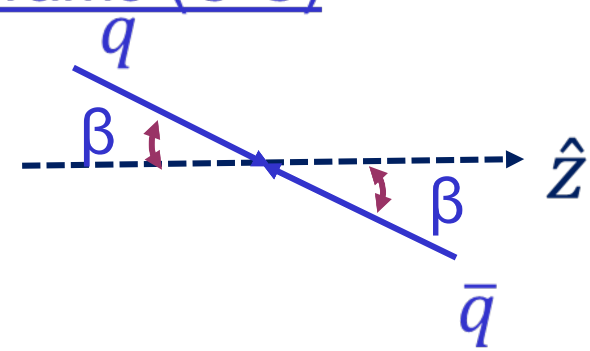
What are the values of θ_1 and ϕ_1 at order α_s?

1) $q\bar{q} \rightarrow \gamma^*(Z^0)g$

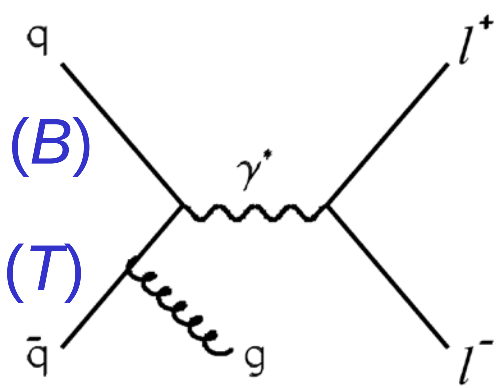
In γ^* rest frame (C-S)

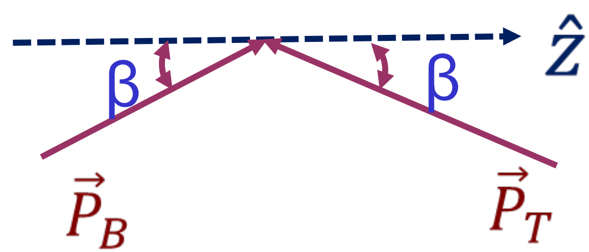


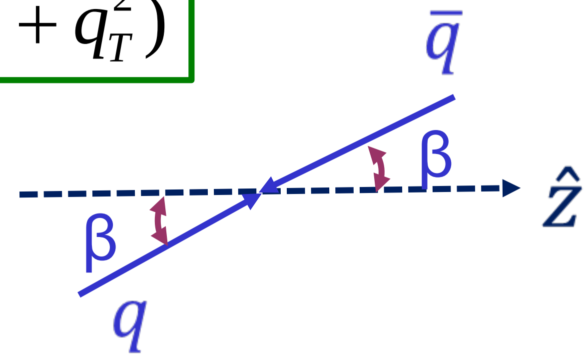




$$\boxed{\sin^2\beta = q_T^2/(Q^2 + q_T^2)}$$







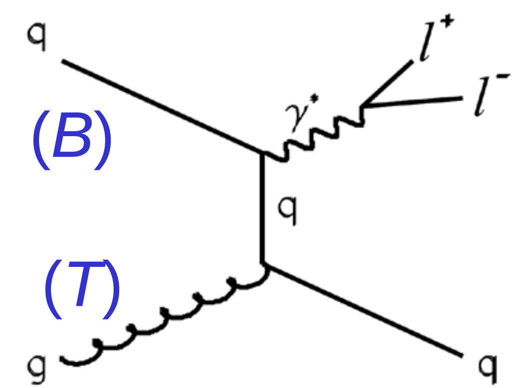
$$\boxed{\theta_1 = \beta \quad \text{and} \quad \phi_1 = 0;} \quad A_0 = A_2 = \sin^2\beta$$

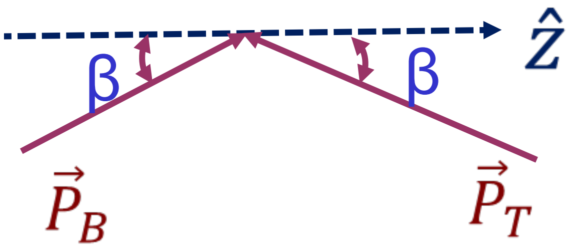
$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

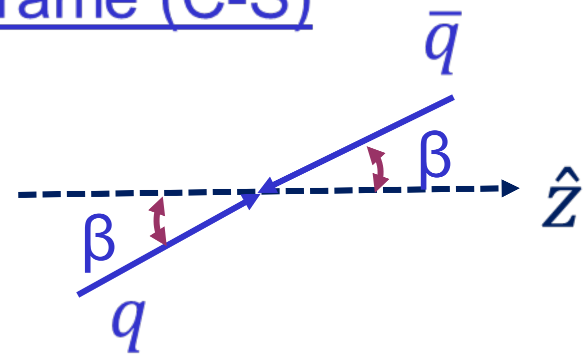
What are the values of θ_1 and ϕ_1 at order α_s?

2) $qg \to \gamma^*(Z^0)q$

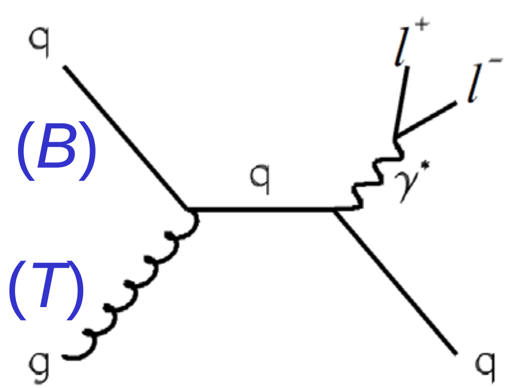
In γ^* rest frame (C-S)

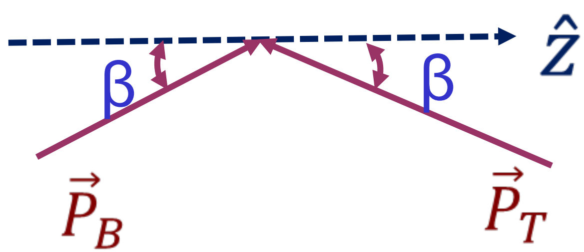


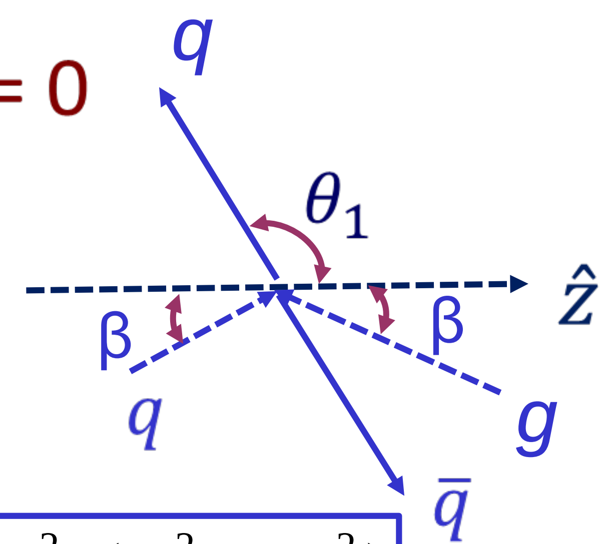




$\theta_1 = \beta$ and $\phi_1 = 0$





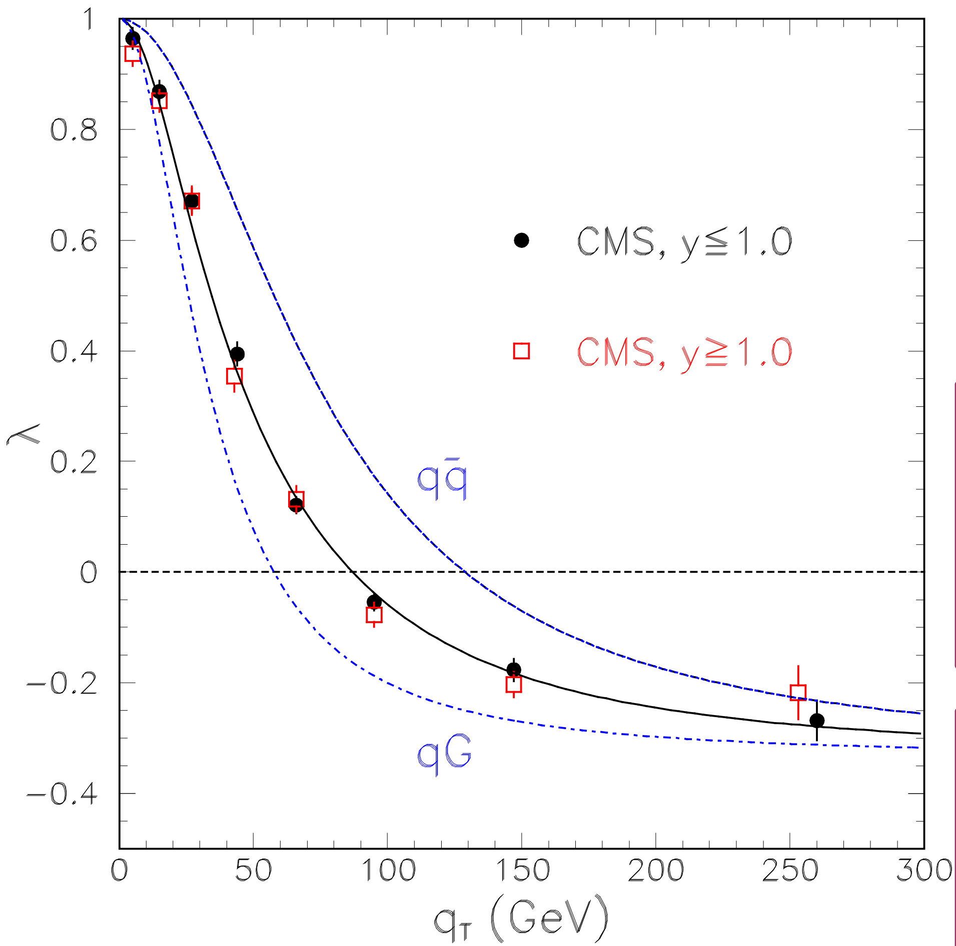


$\boxed{\theta_1 > \beta \text{ and } \phi_1 = 0; \quad A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)}$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

Compare with CMS data on λ

(*Z* production in *p*+*p* collision at 8 TeV)



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

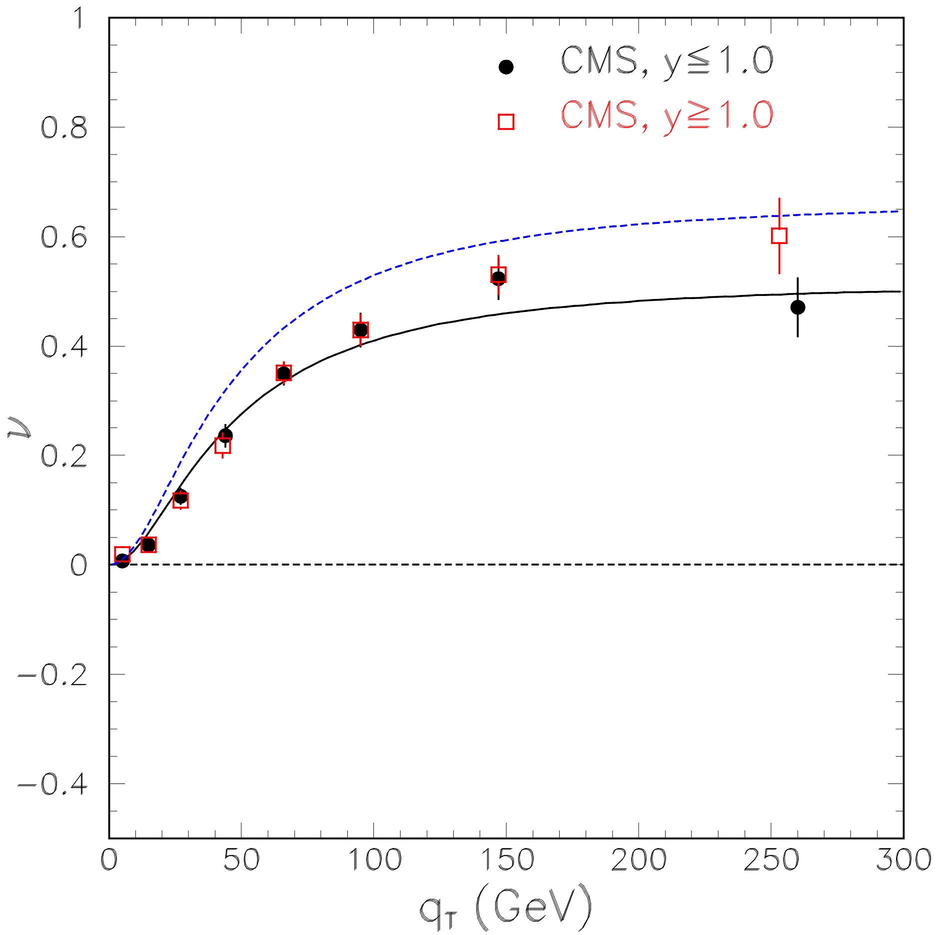
$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

For both processes
$\lambda = 1$ at $q_T = 0$ ($\theta_1 = 0^o$)
$\lambda = -1/3$ at $q_T = \infty$ ($\theta_1 = 90^o$)

Data can be well described with a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

Compare with CMS data on ν

(Z production in p+p collision at 8 TeV)



$$\nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \quad \text{for} \quad q\bar{q} \to Zg$$

$$\nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \quad \text{for} \quad qG \to Zq$$

Dashed curve corresponds to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

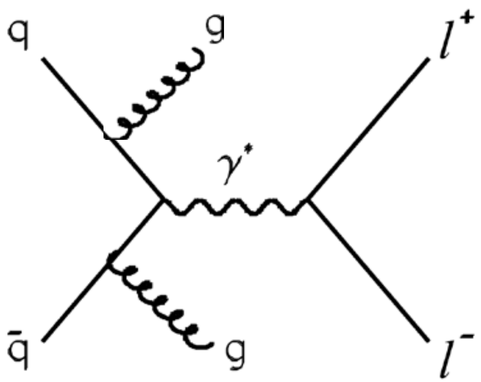
Solid curve corresponds to

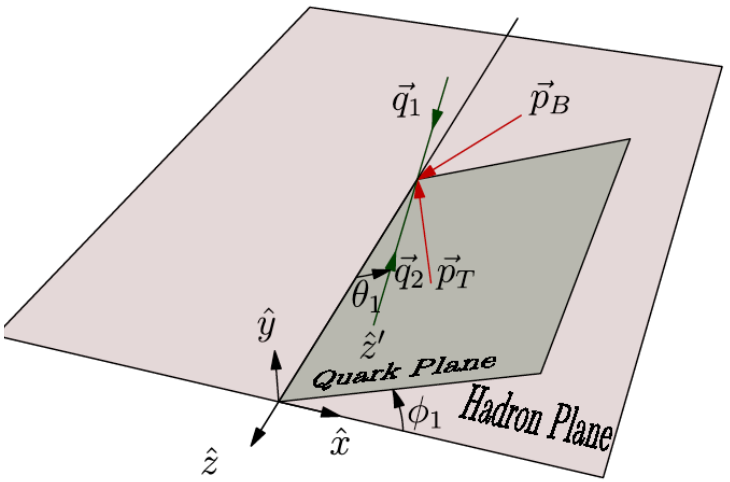
$$\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$$

$q - \bar{q}$ axis is non-coplanar relative to the hadron plane

Origins of the non-coplanarity

1) Processes at order α_s^2 or higher

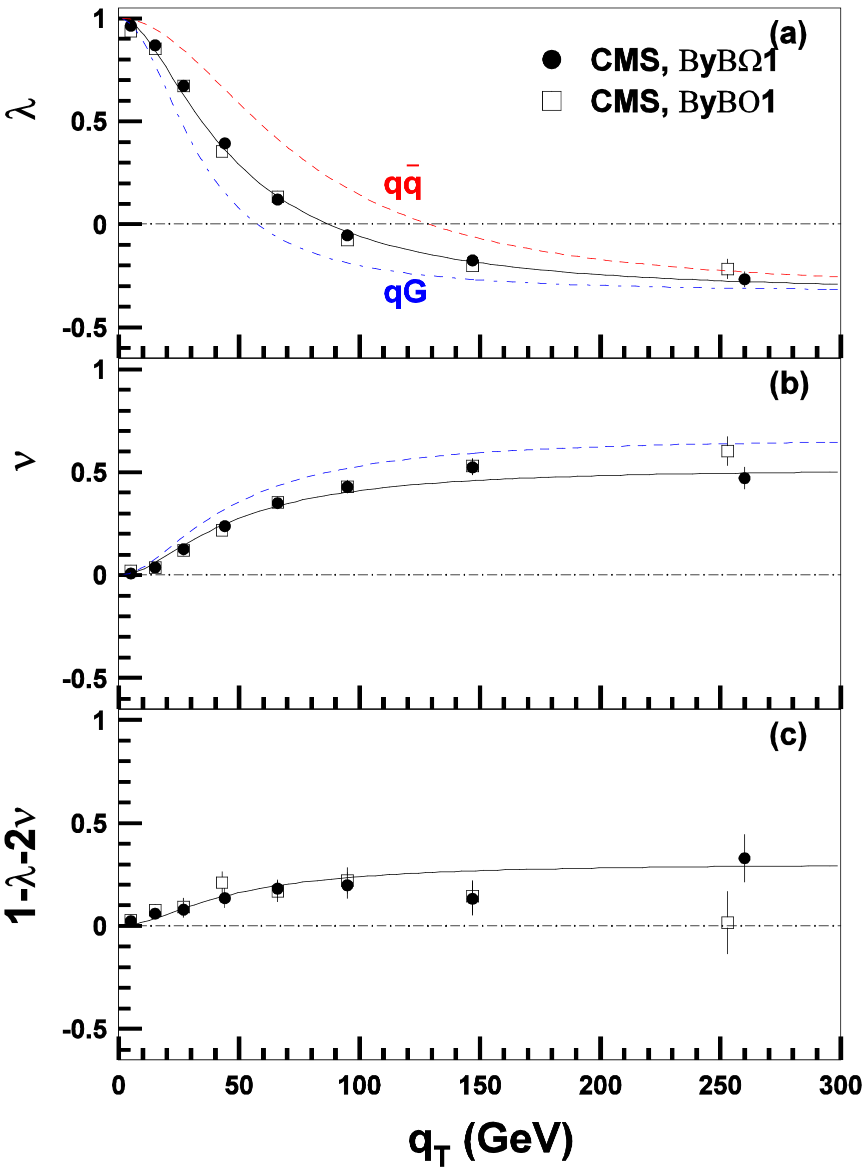




2) Intrinsic k_T from interacting partons

(Boer-Mulders functions in the beam and target hadrons)

Compare with CMS data on Lam-Tung relation

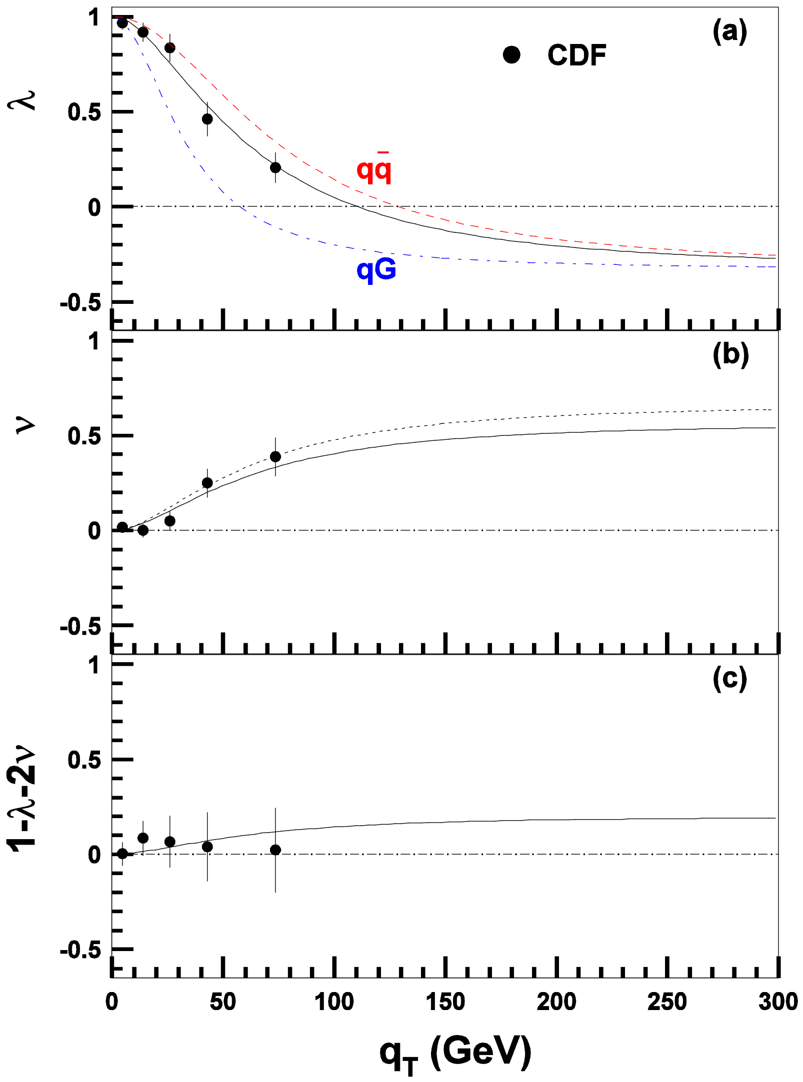


Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes, and $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$

Violation of Lam-Tung relation is well described

Compare with CDF data

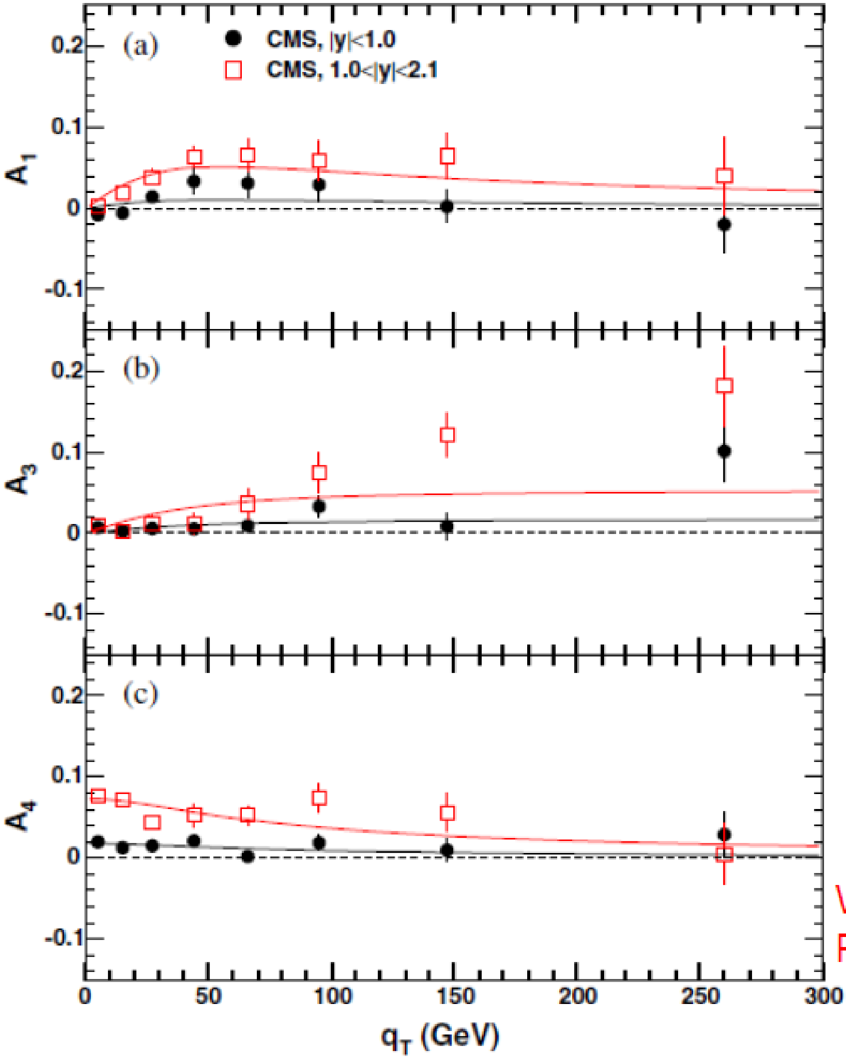
(Z production in $p + \bar{p}$ collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5% qG and 72.5% $q\bar{q}$ processes, and $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.85$

Violation of Lam-Tung relation is not ruled out

Compare with CMS data on A_1, A_3 and A_4



$$A_1 = r_1 \left[f \frac{q_T Q}{Q^2 + q_T^2} + (1 - f) \frac{\sqrt{5} q_T Q}{Q^2 + 5 q_T^2} \right]$$

$$A_3 = r_3 \left[f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

$$A_4 = r_4 \left[f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{Q}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

Rapidity of A_1, A_3 and A_4 are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Phys. Rev. D 96, 054020 (2017)

Future prospects

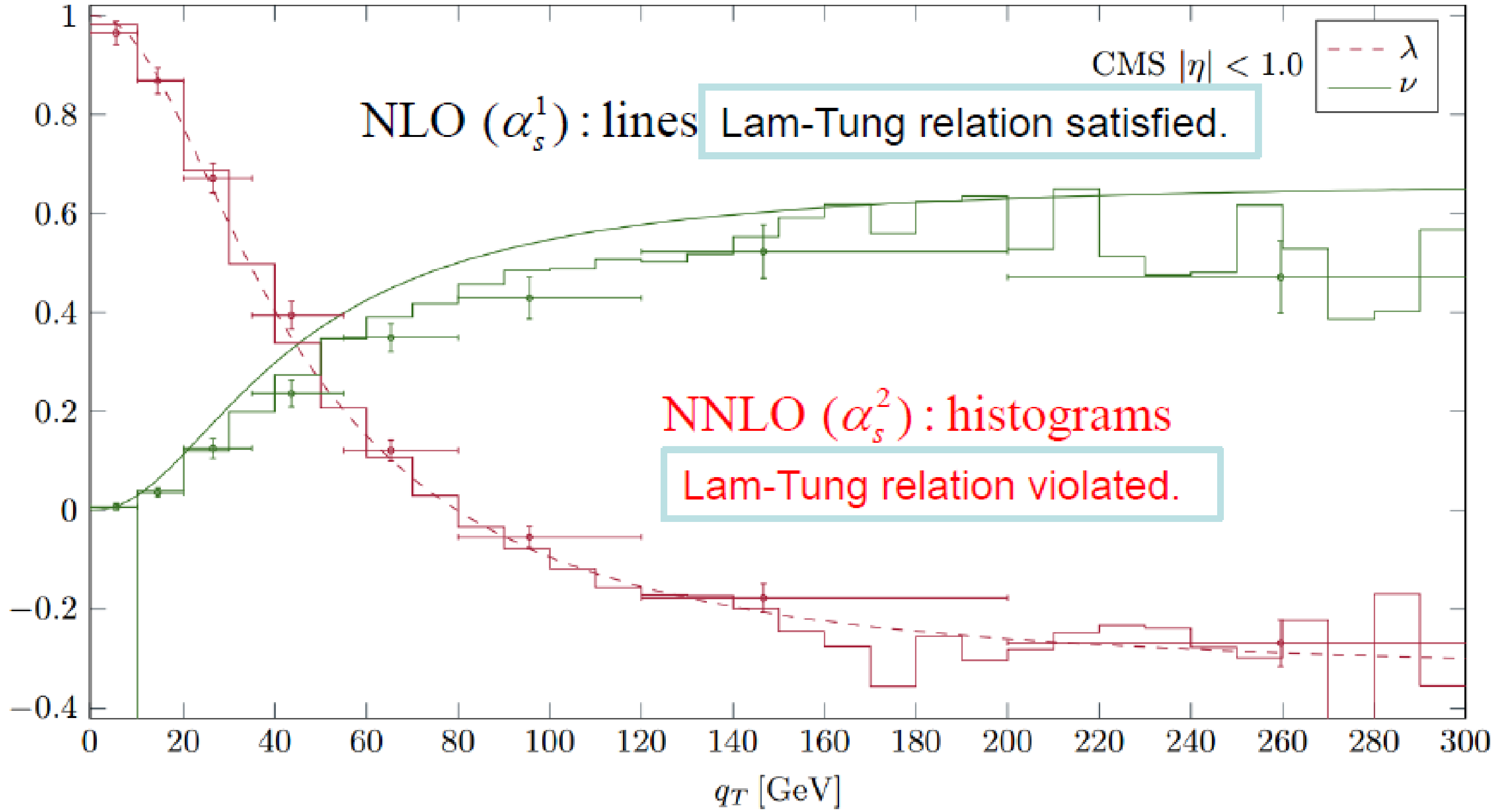
- Extend this study to W-boson production
  - Preliminary results show that the data can be well described
- Extend this study to fixed-target Drell-Yan data
  - Extraction of Boer-Mulders functions must take into account the QCD effects
- Extend this study to dihadron production in $e^- e^+$ collision (inverse of the Drell-Yan)
  - Analogous angular distribution coefficients and analogous Lam-Tung relation

Future prospects

- Extend this study to semi-inclusive DIS at high p_T (involving two hadrons and two leptons)
  - Relevant for EIC measurements
- Rotational invariance, equality, and inequality relations formed by various angular distribution coefficients
- Comparison with pQCD calculations

pQCD NLO and NNLO Calculations

(M. Lambertsen and W. Vogelsang, Phys. Rev. D 93, 114013 (2016))

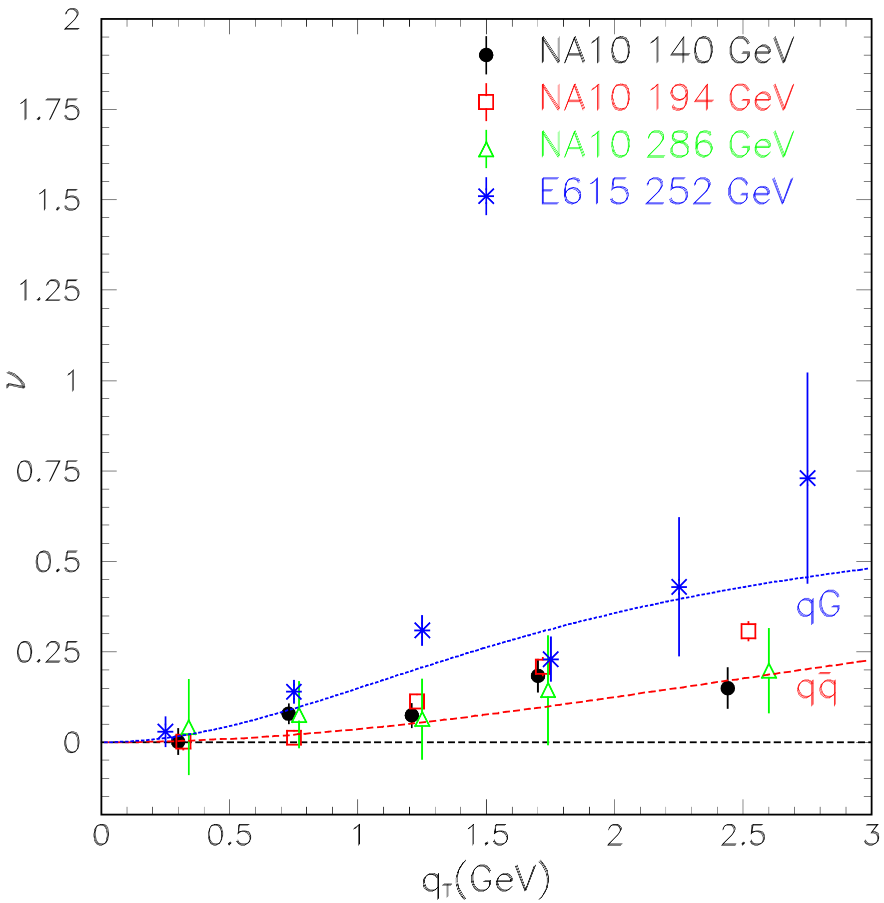


- NNLO pQCD calculation can describe the violation of Lam-Tung relation.

Summary

- The lepton angular distribution coefficients A_0-A_7 are described in terms of the polar and azimuthal angles of the $q-\bar{q}$ axis.
- The striking q_T dependence of A_0 (or equivalently, λ) can be well described by the mis-alignment of the $q-\bar{q}$ axis and the Collins-Soper *z*-axis.
- Violation of the Lam-Tung relation ($A_0 \neq A_2$) is described by the non-coplanarity of the $q-\bar{q}$ axis and the hadron plane. This can come from order α_s^2 or higher processes or from intrinsic k_T.
- This study can be extended to fixed-target Drell-Yan data.

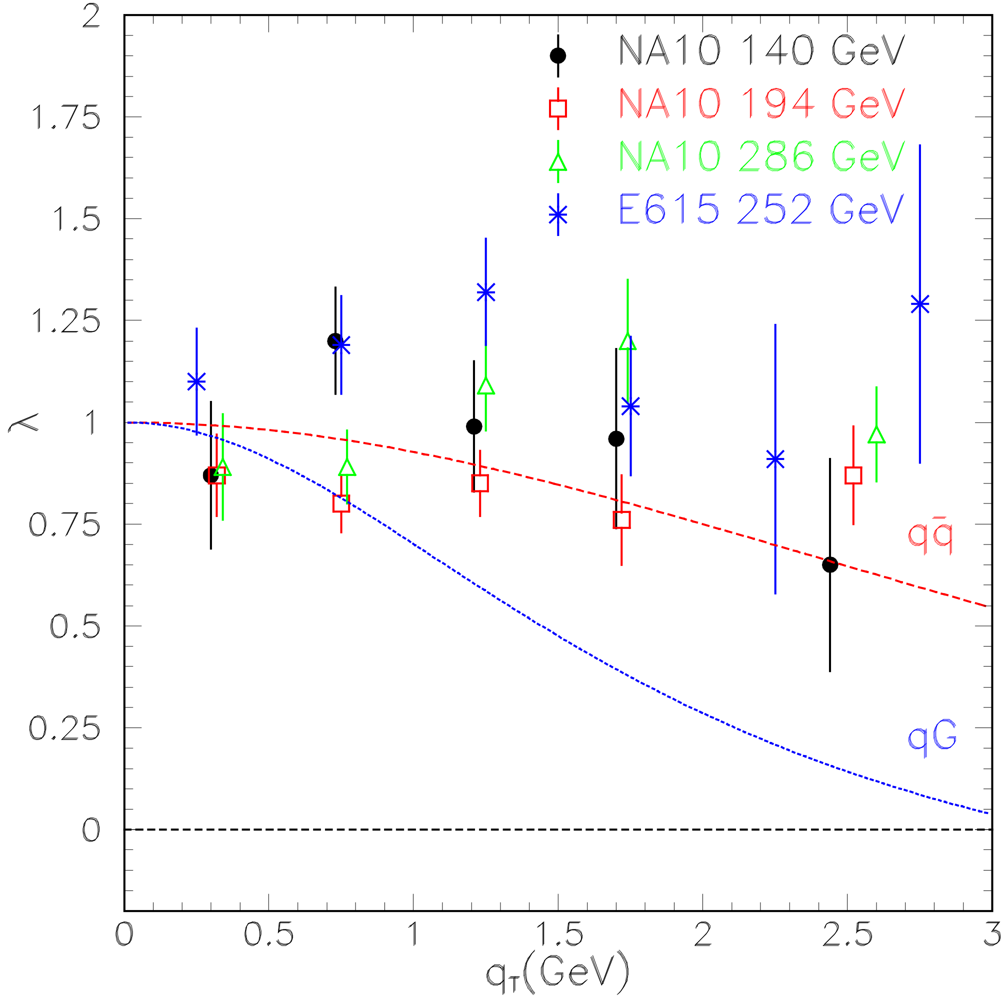
Pion-induced D-Y



See Lambertsen and Vogelsang, arXiv: 1605.02625

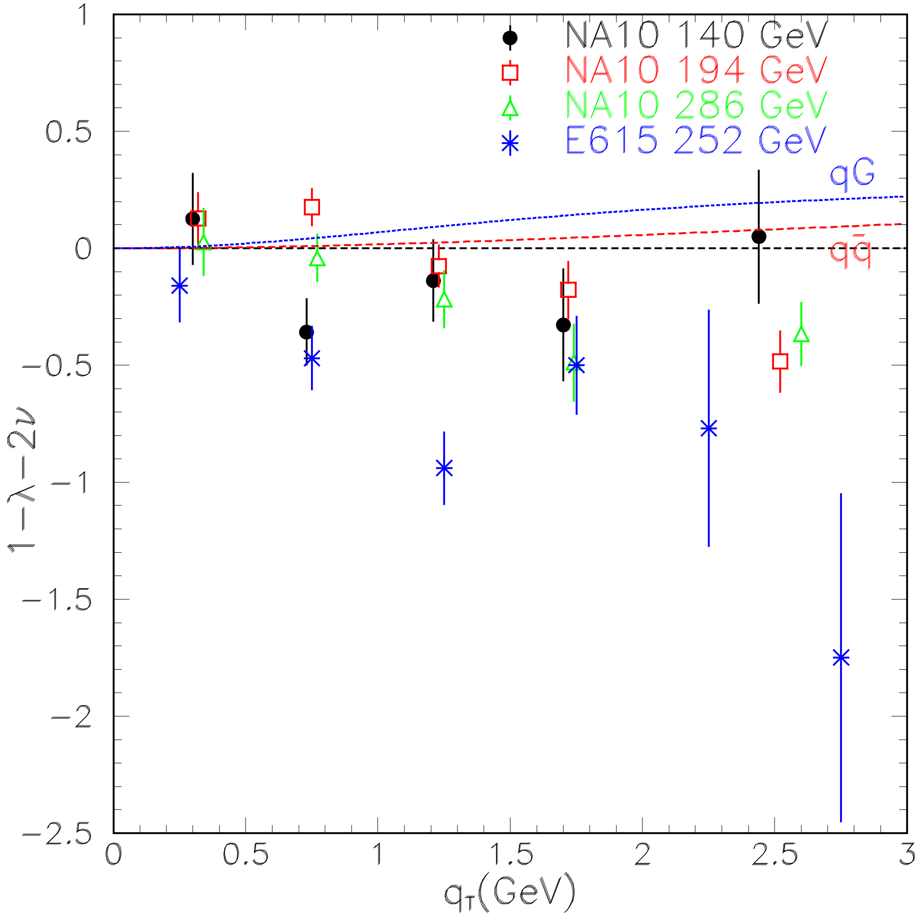
- The ν data should be between the $q\bar{q}$ and qG curves, if the effect is entirely from pQCD. The $q\bar{q}$ process should dominate.
- Surprisingly large pQCD effect is predicted!
- Extraction of the B-M functions must remove the pQCD effect.

Pion-induced D-Y



- The λ data should be between the $q\bar{q}$ and qG curves, if the effect is entirely from pQCD. Also λ must be less than 1 (from positivity)!!
- The data suggest the presence of other effects (or poor data)

Pion-induced D-Y



- The L-T violation should be between the $q\bar{q}$ and qG curves, if the effect is entirely from pQCD (we assume the same non-coplanarity as in the LHC).
- pQCD effect can only be positive, while the data are large and negative!
- Large violation of L-T (due to $\lambda > 1$) cannot be explained by pQCD. Need better data