

Heavy quark jet substructure

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In collaboration with
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Probing substructure of QCD jets

- Jet substructure observables allow us a deeper understanding of QCD dynamics in a jet and are designed to uncover specific features of a jet 1609.07483, I. Moult et. al.
- Grooming of jets reduces jet contamination from initial state radiation, underlying event and pile up .
- First analytic calculation for groomed massless parton jets , 1603.09338, Larkoski et. al.
- For massive quarks, see recent work on top quark jet mass 1608.01318 I.Stewart et. al.
- We wish to develop the factorization theorem for analytical calculation of groomed jet substructure observables for heavy quark jets, with a specific interest in b quark jets .
- First step of the theoretical effort towards utilizing b quark jets as probes of QGP.

Energy correlators for groomed jets

Simplest jet substructure observables are the class of energy correlator functions

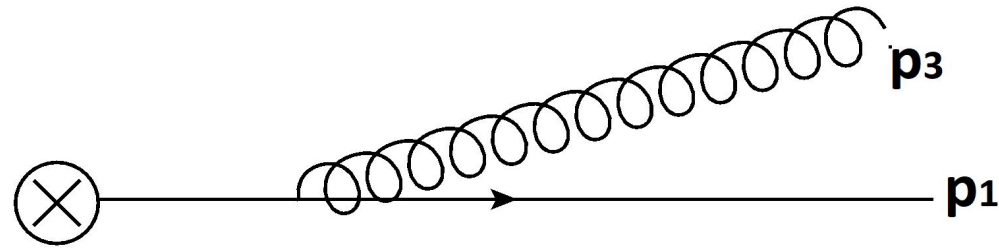
$$e_2^{(\alpha)} \Big|_{e^+e^-} = \frac{1}{E_J^2} \sum_{i < j \in J} E_i E_j \left(\frac{2p_i \cdot p_j}{E_i E_j} \right)^{\alpha/2} \quad z_i \equiv \frac{E_i}{E_J}, \quad \theta_{ij}^2 \equiv \frac{2p_i \cdot p_j}{E_i E_j},$$

Soft drop grooming algorithm

$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\text{cut}} \left(\sqrt{2} \frac{\sin \frac{\theta_{ij}}{2}}{\sin \frac{R}{2}} \right)^\beta$$

We will set $\beta = 0$. This will remove any radiation with energy fraction less than z_{cut} .

Fixed order cross section

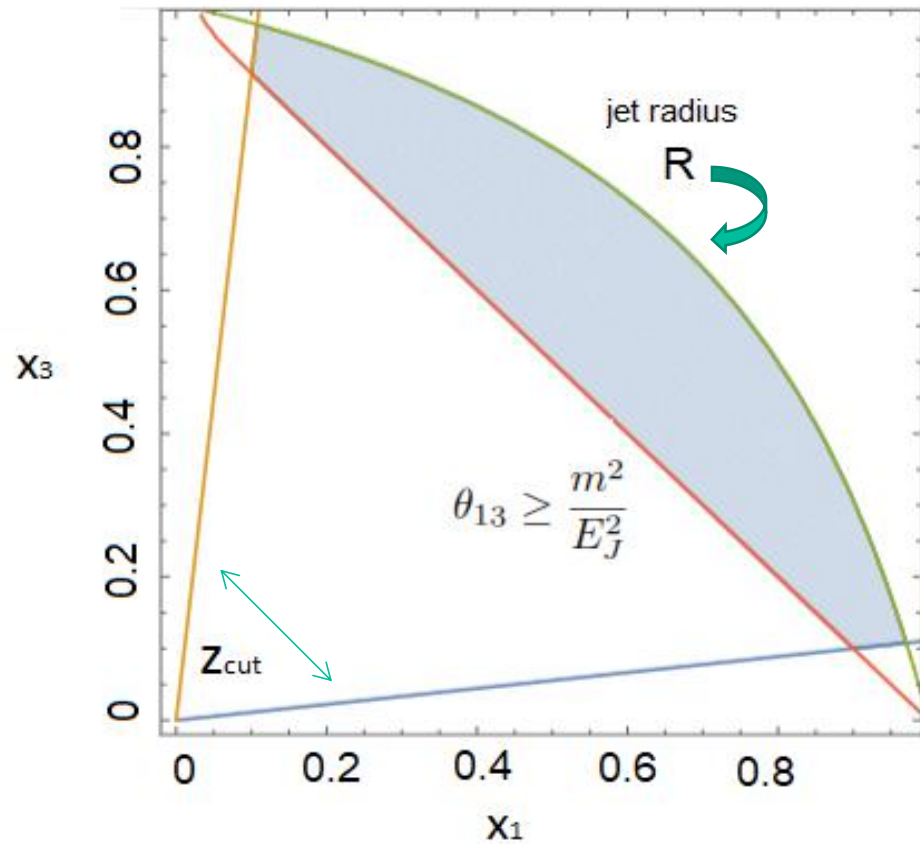


$$x_i = E_i/E_J$$

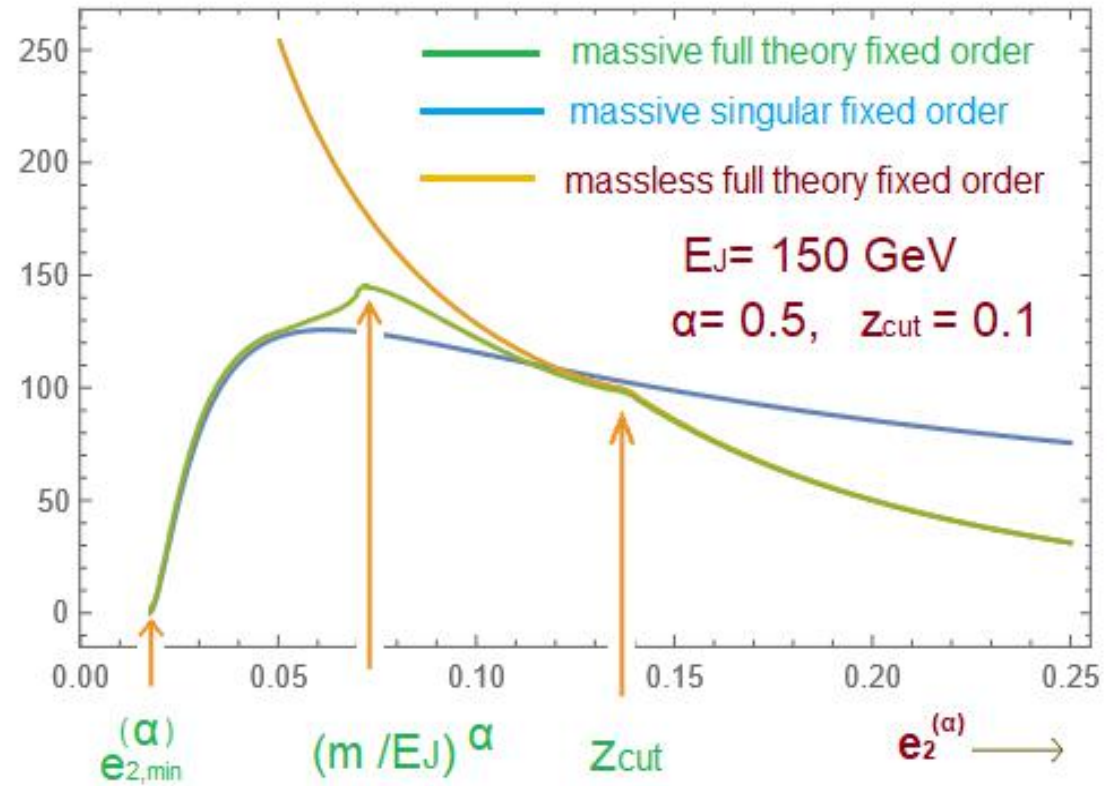
massless integrand

$$\frac{1}{\sigma_0} \frac{d\sigma}{de_2^\alpha} = \frac{\alpha_s}{2\pi} C_F \int_{\text{phase space}} dx_1 dx_3 \left\{ \frac{x_1^2 + (2 - x_1 - x_3)^2}{(1 - x_1)(x_1 + x_3 - 1)} - \frac{m^4}{E_J^4} \frac{x_3^2}{4(x_1 - 1)^2(x_1 + x_3 - 1)^2} \right. \\ \left. + \frac{m^2}{E_J^2} \frac{(1 - 2x_1 - 2x_3)x_1^2 + 2(x_1 + x_3)^2 x_1 - 3(2 - x_1 - x_3)^2 + 8(2 - x_1 - x_3) - 6}{2(x_1 - 1)^2(x_3 + x_1 - 1)^2} \right\} \\ \delta \left(e_2^\alpha - \frac{x_1 x_3}{(x_1 + x_3)^2} \left(4 \frac{(x_1 + x_3 - 1)}{x_1 x_3} \right)^{\alpha/2} \right)$$

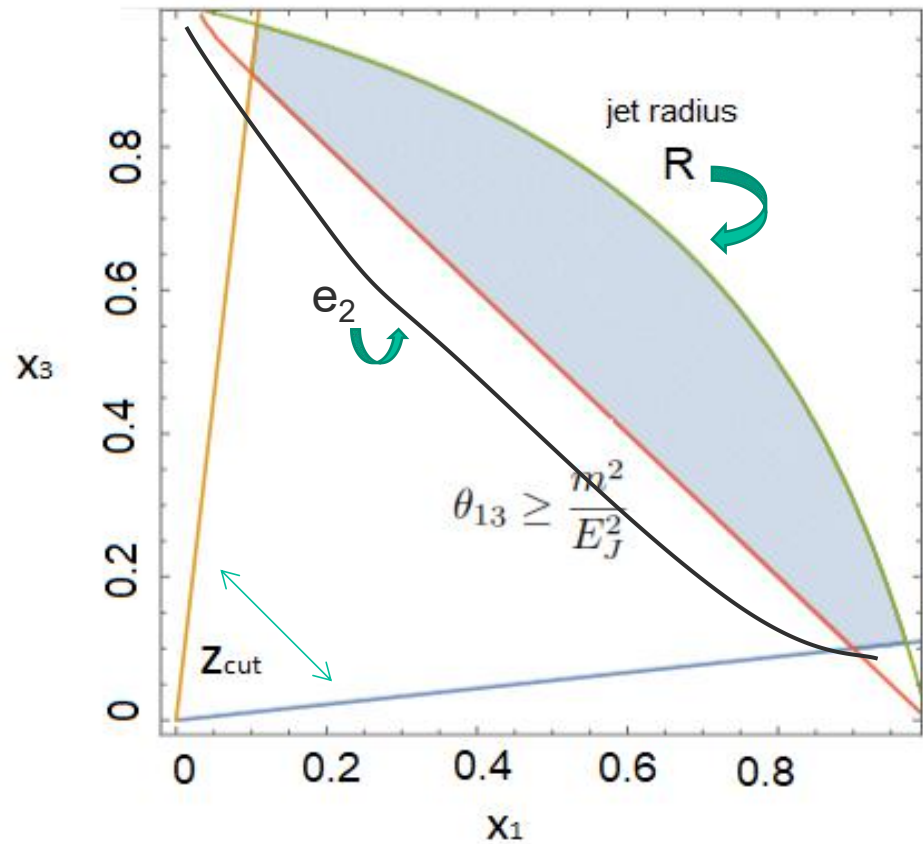
Phase Space



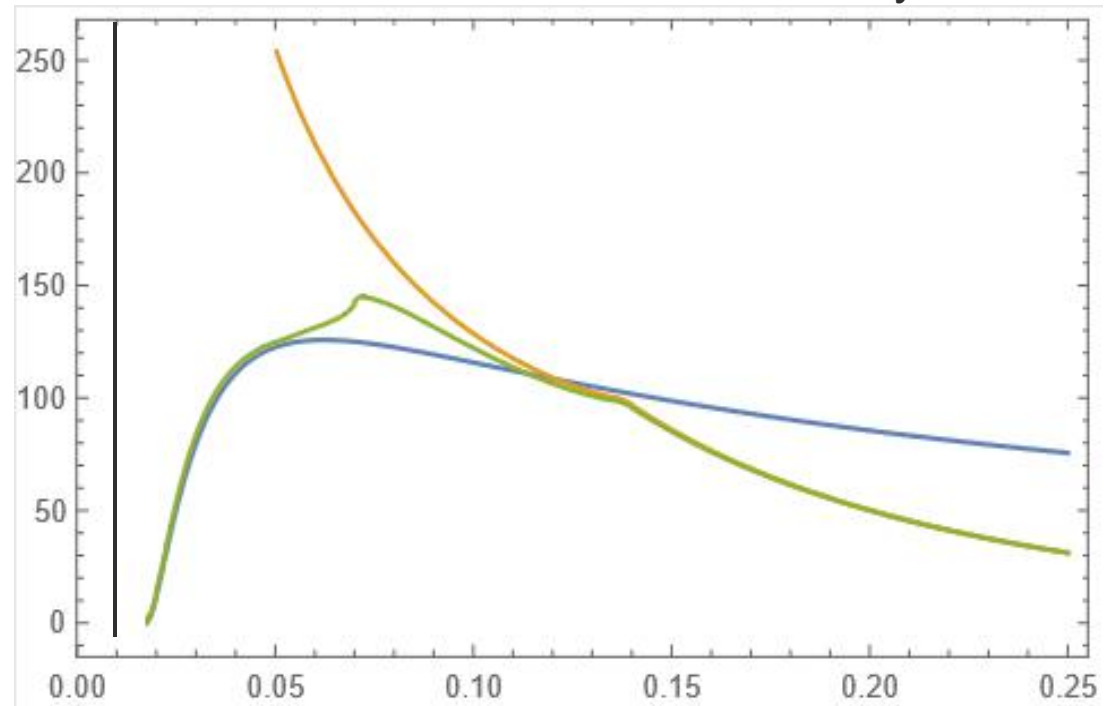
- massive singular
- massive full theory
- massless full theory



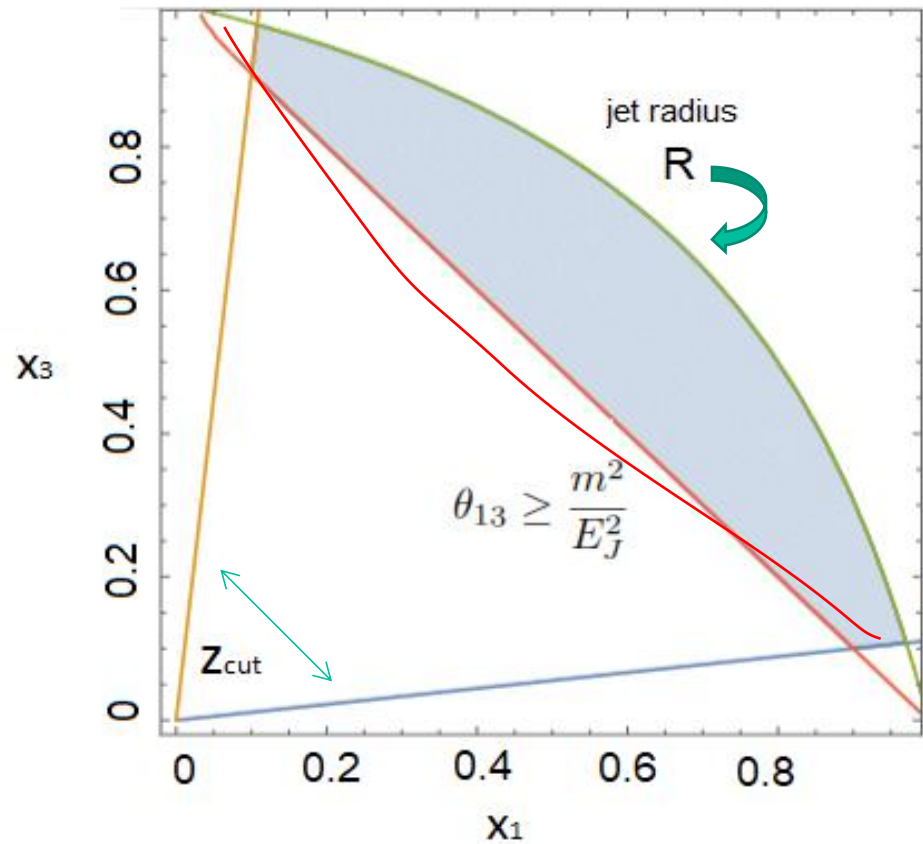
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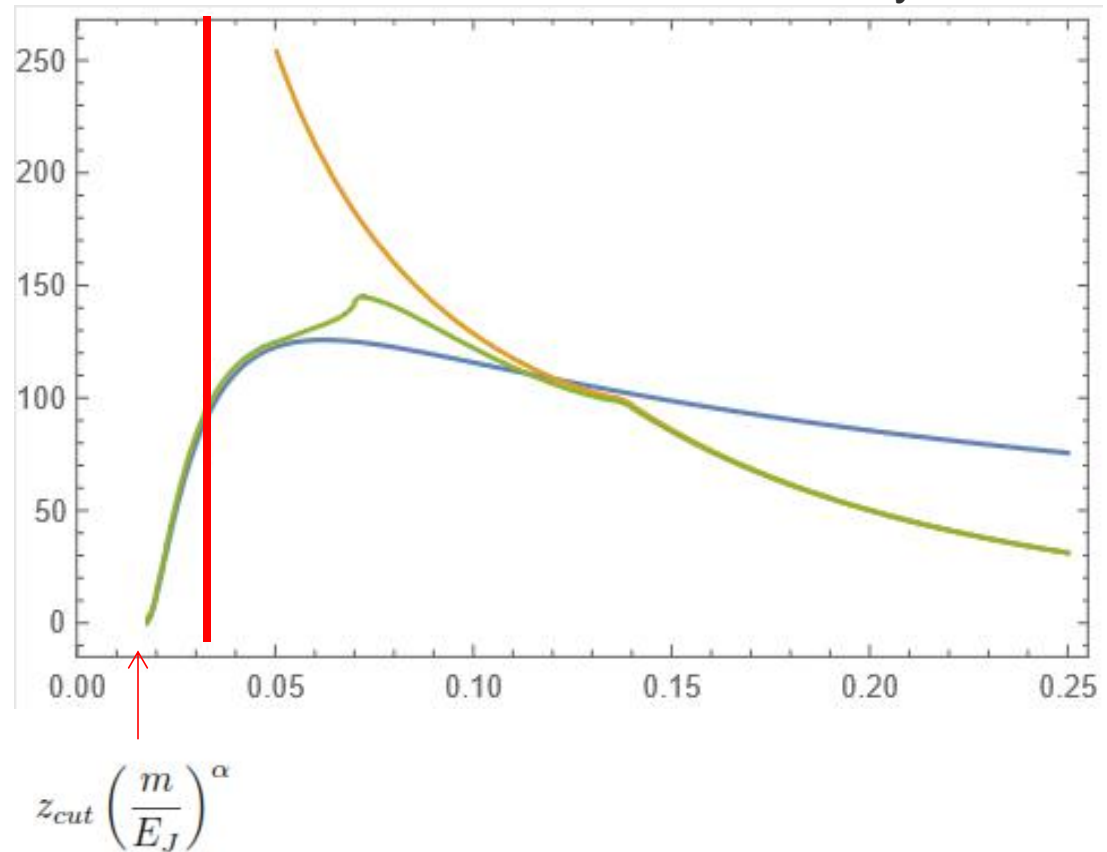
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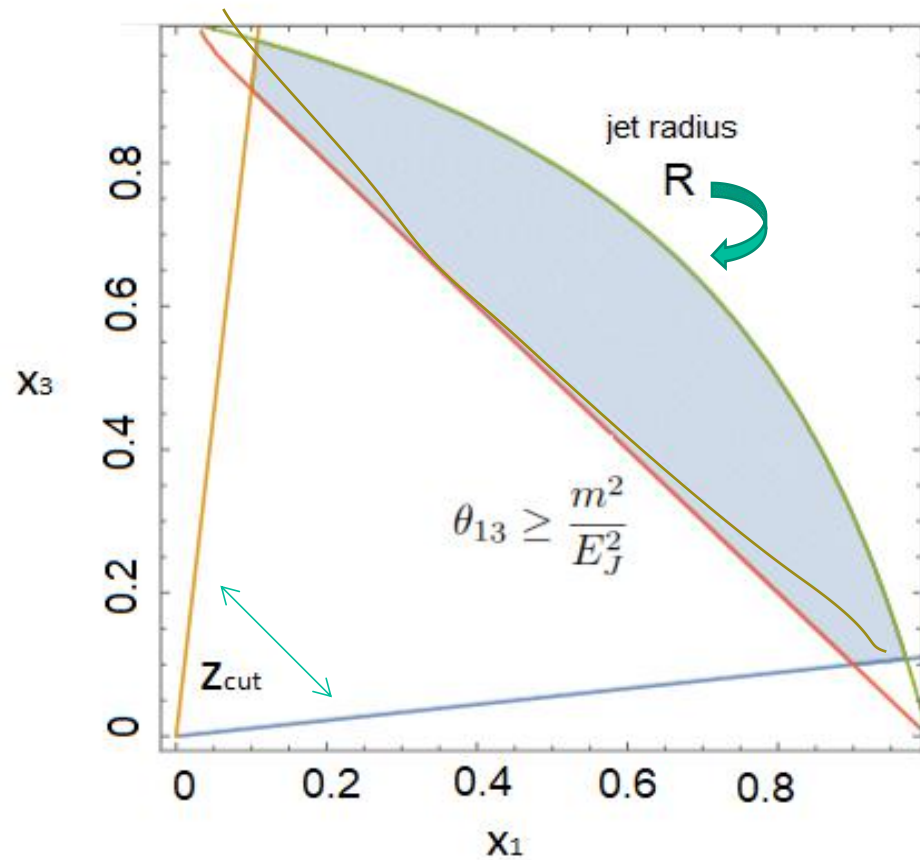
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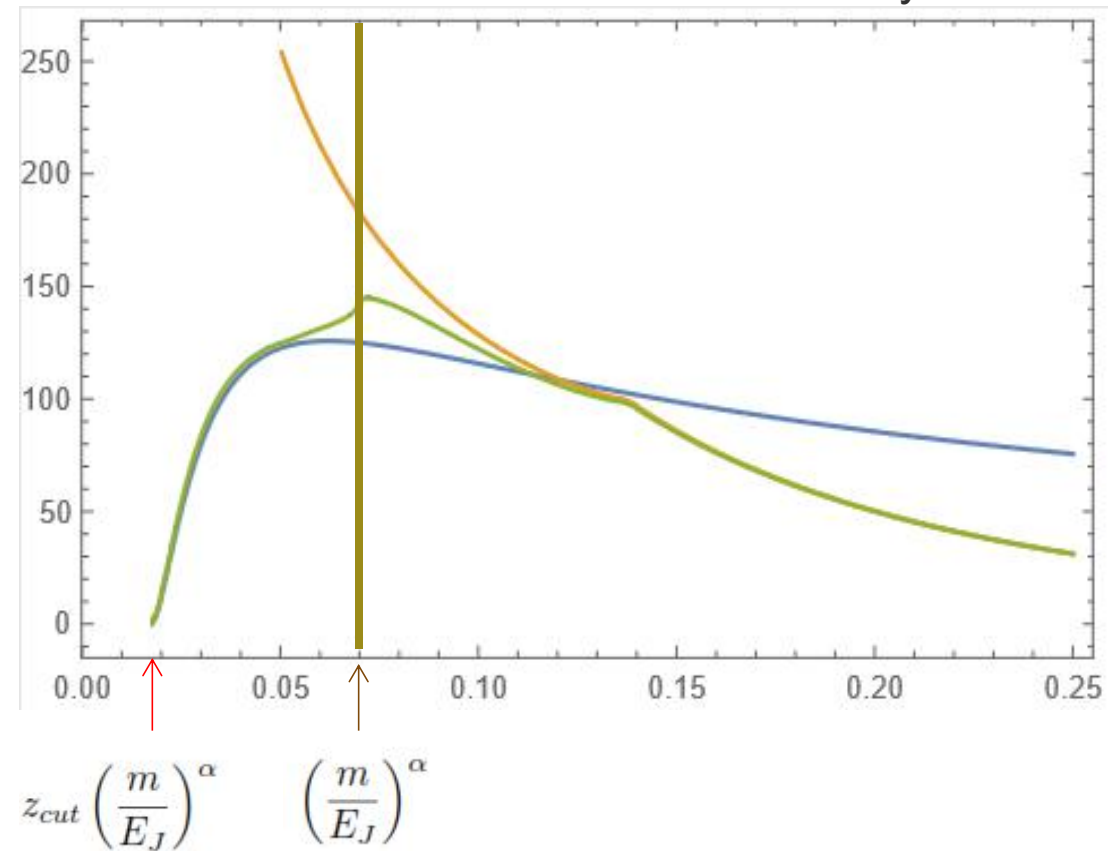
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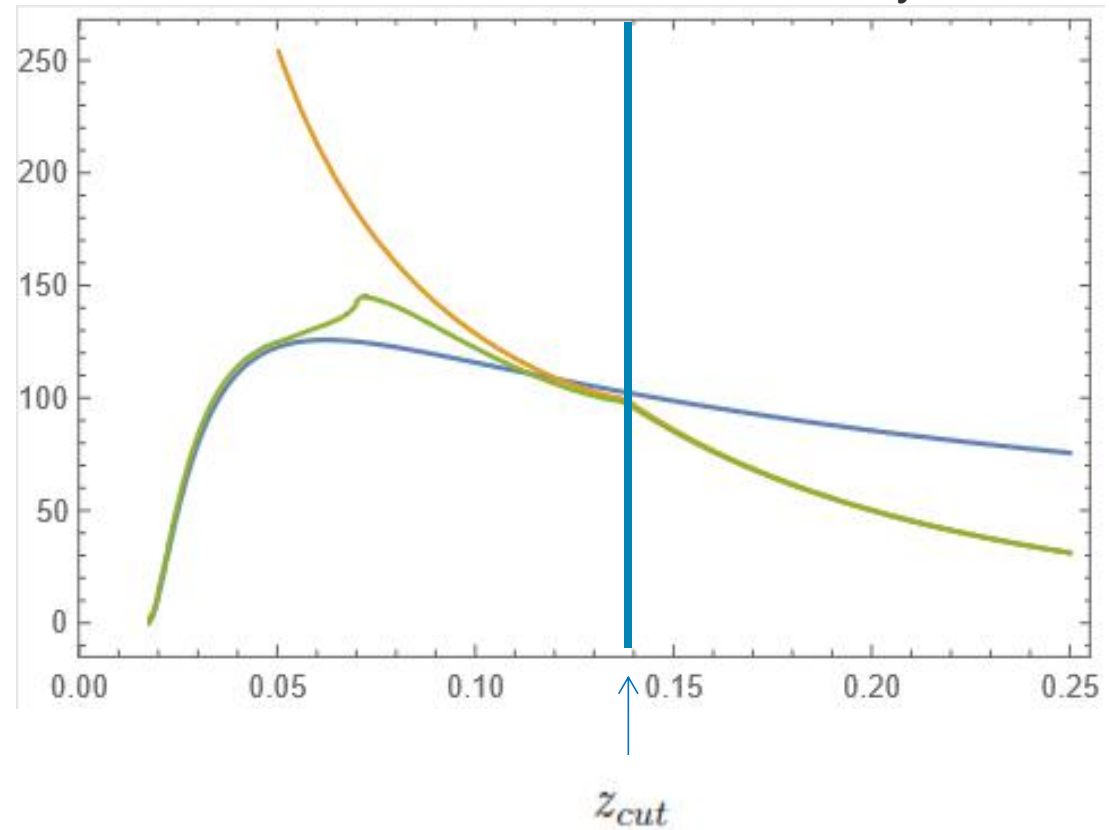
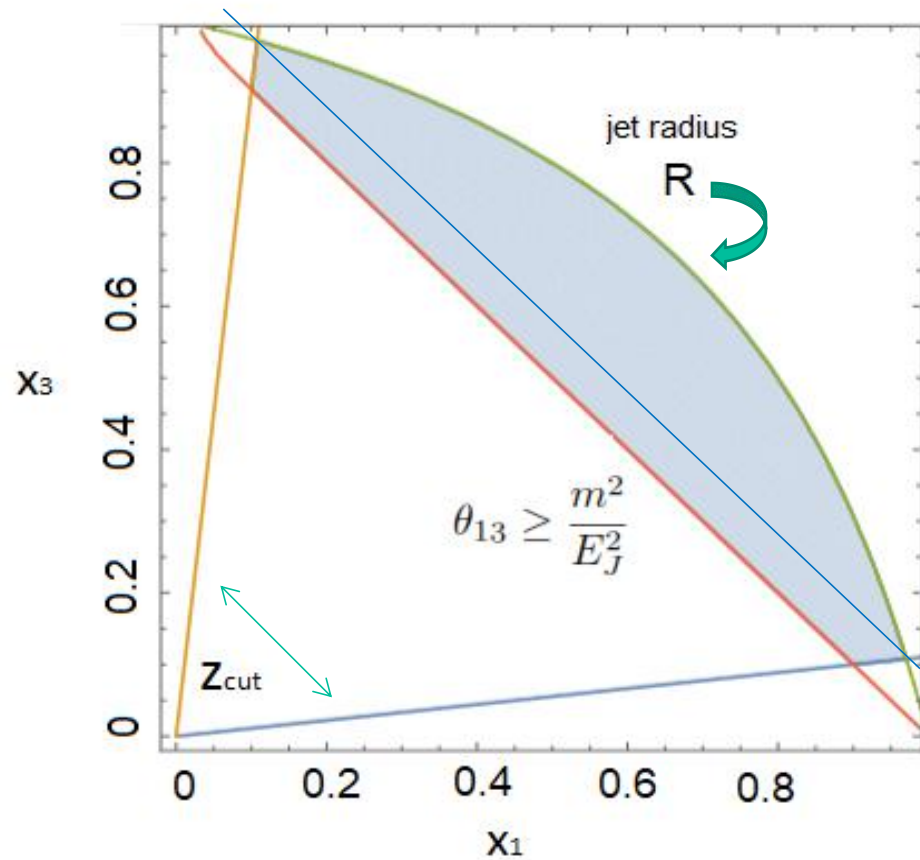
Phase Space



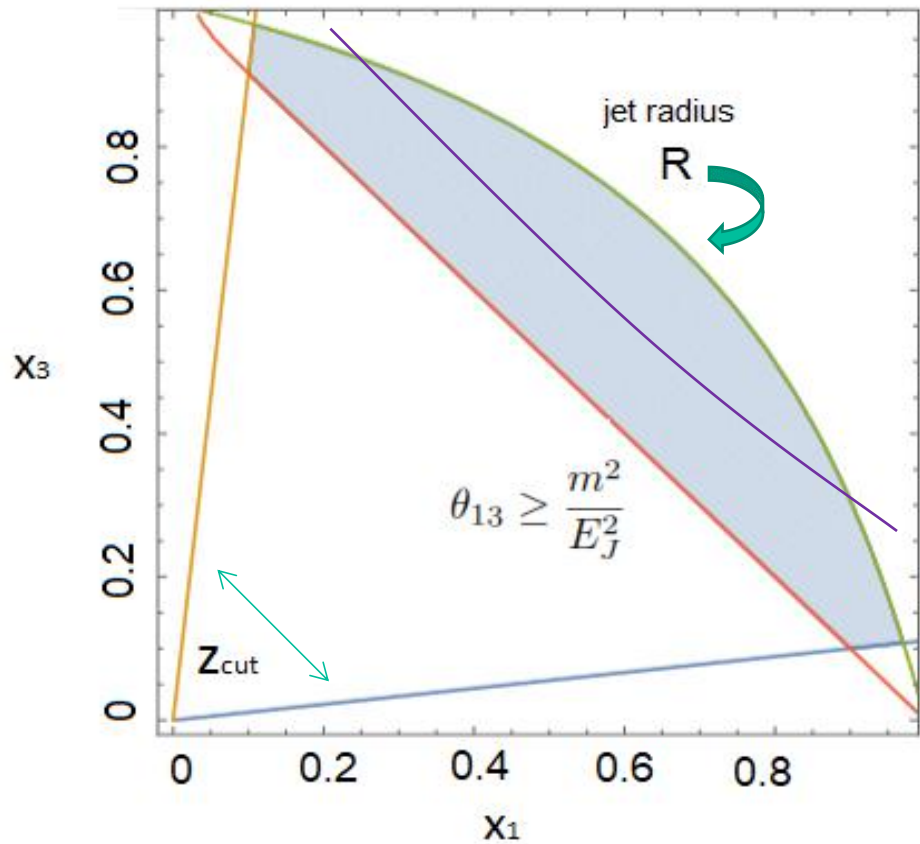
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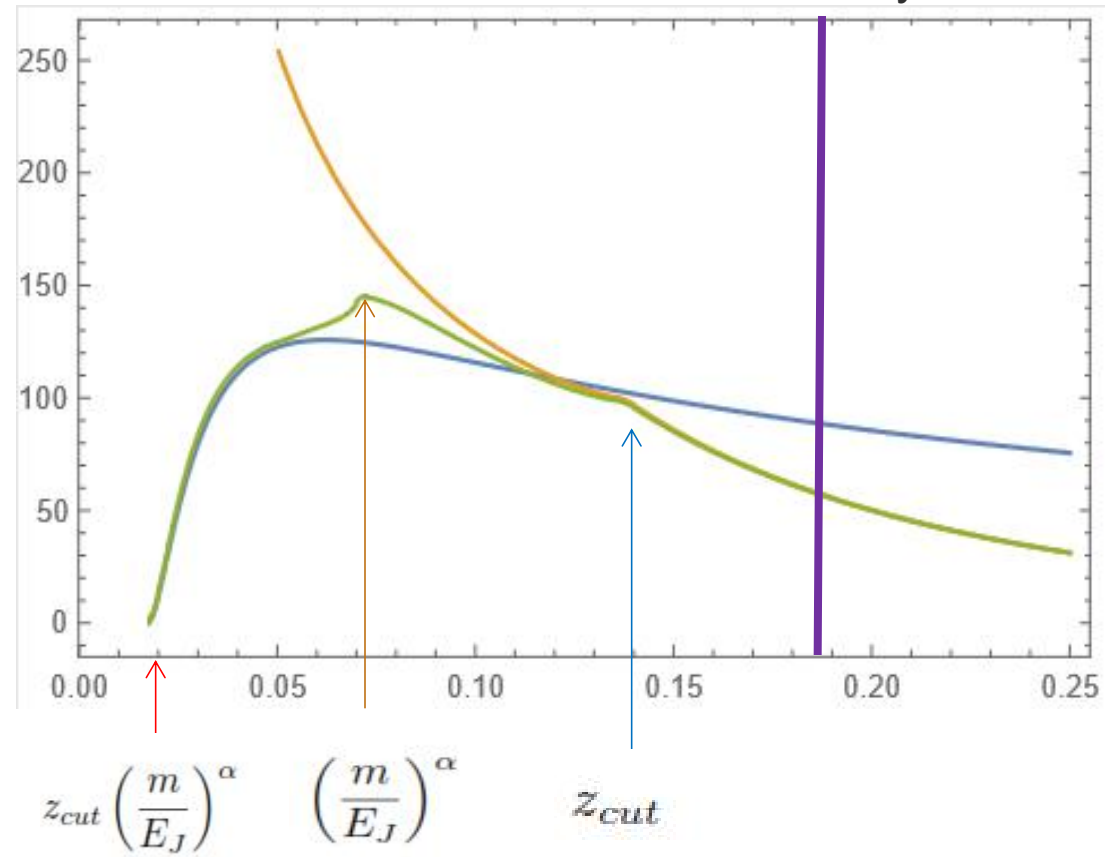
Phase Space



Phase Space



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Fixed order singular cross section

Real contribution

$$\frac{\alpha_s}{2\pi} C_F \left(\frac{4}{\alpha e_2^{(\alpha)}} \ln \left(\frac{e_2^{(\alpha)}}{e_{2,min}^{(\alpha)}} \right) - \frac{2}{e_2^{(\alpha)}} \left(1 - \left(\frac{e_{2,min}^{(\alpha)}}{e_2^{(\alpha)}} \right)^{2/\alpha} \right) \right)$$

$$e_{2,min}^{(\alpha)} \sim z_{cut} \left(\frac{m}{E_J} \right)^\alpha$$

Virtual contribution

$$\sim \delta(e_2^{(\alpha)}) \left(\ln^2 \frac{E_J}{m}, \ln \frac{E_J}{m} \right)$$

Develop an EFT to resum large logs in e_2 and m

EFT for massive quark jets

- Assumed hierarchy $e_2^{(\alpha)} \ll z_{cut} \ll 1$
- Assume for the heavy quark $z_q \approx 1$
- All modes have a physical constraint $\theta \geq \left(\frac{m^2}{E_J^2}\right)$
- This combined with soft drop imposes a cut-off on $e_2^{(\alpha)}$ $e_2^{(\alpha)} \geq z_{cut} \left(\frac{m}{E_J}\right)^\alpha$

EFT modes

$$e_2^{(\alpha)}|_{e^+e^-} = \sum_{i < j \in J} z_i z_j (\theta_{ij})^\alpha$$

Wide angle radiation

- $\theta \approx 1$ requires $z \approx e_2^{(\alpha)}$ which is groomed away
 $Z \approx Z_{\text{cut}}$,

$$p_s \equiv E_J z_{\text{cut}} (1, 1, 1)$$

Global soft radiation

Collinear radiation

- $z \approx z_{\text{cut}}$ requires $\theta_{\text{cs}} \approx (e_2^{(\alpha)} / z_{\text{cut}})^{1/\alpha} > \theta_{\text{min}}$
- $z \approx 1$ requires $\theta_c \approx (e_2^{(\alpha)})^{1/\alpha}$

This mode exists for $\theta_c > \theta_{\text{min}}$ or $e_2^{(\alpha)} > (m / E_J)^\alpha$

$$p_{\text{cs}} \sim z_{\text{cut}} E_J \left(1, \left(\frac{e_2^{(\alpha)}}{z_{\text{cut}}} \right)^{2/\alpha}, \left(\frac{e_2^{(\alpha)}}{z_{\text{cut}}} \right)^{1/\alpha} \right)$$

Collinear Soft radiation

$$p_c \sim E_J \left(1, (e_2^{(\alpha)})^{2/\alpha}, (e_2^{(\alpha)})^{1/\alpha} \right)$$

Collinear radiation

EFT modes

Collinear radiation

- For $\theta \approx \theta_{\min}$ we need $z \approx e_2^{(\alpha)} (\theta_{\min})^\alpha$

$$p_c \sim E_J e_2^{(\alpha)} / (\theta_{\min})^\alpha (1, \theta_{\min}^2, \theta_{\min})$$

Ultra-collinear radiation

In terms of the velocity of the heavy quark $v^\mu \equiv (E_J/m, m/E_J, 0)$

$$p_c \sim \mathbf{W} (v_+, v_-, 1) \quad \text{with} \quad \mathbf{W} = m_q e_2^{(\alpha)} / (\theta_{\min})^\alpha$$

is the boosted soft mode of HQET

- At $e_2^{(\alpha)} = (\theta_{\min})^\alpha = (m/E_J)^\alpha$ the collinear and ultra-collinear modes are identical

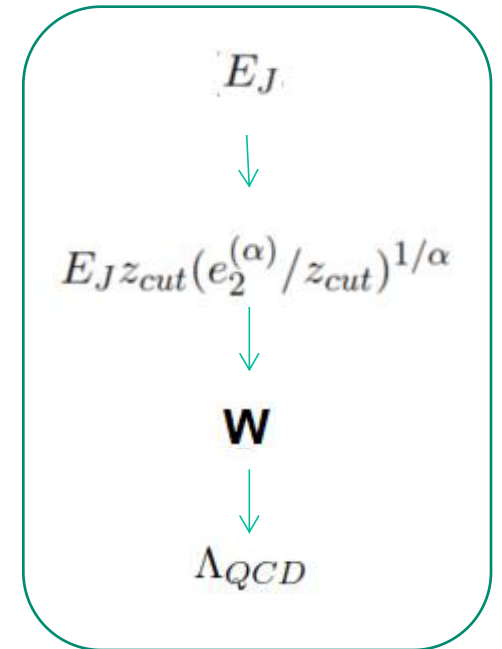
Factorization

- $(m/E_J)^\alpha > e_2^{(\alpha)} > e_2^{(\alpha)}_{\min}$

$$\frac{d\sigma}{de_2^{(\alpha)}} = \sigma_0(Q^2) \times S(E_J z_{\text{cut}}) \times H(m_q) \times S_c(E_J z_{\text{cut}} (e_2^{(\alpha)} / z_{\text{cut}})^{1/\alpha}) \otimes J_q(\Gamma)$$

- $z_{\text{cut}} > e_2^{(\alpha)} > (m/E_J)^\alpha$

$$\frac{d\sigma}{de_2^{(\alpha)}} = \sigma_0(Q^2) \times S(E_J z_{\text{cut}}) S_c(E_J z_{\text{cut}} (e_2^{(\alpha)} / z_{\text{cut}})^{1/\alpha}) \otimes J_q(E_J (e_2^{(\alpha)})^{1/\alpha})$$



Dominant non-perturbative corrections come from the HQET jet function

Factorization

Massive jet function

$$J = \frac{\alpha_s C_F}{\pi} \left(\frac{1}{\alpha - 1} L_C^2 + L_V^2 + L_C + \frac{1}{2} L_V \right)$$

$$L_C = \ln \left(\frac{\mu s e^{\gamma_E}}{E_J} (4\Delta)^{\frac{\alpha-1}{2}} \right)$$

$$\Delta = \frac{m^2}{E_J^2}, \quad L_V = \ln \left(\frac{\mu}{m} \right)$$

Anomalous dimension

$$\gamma_\mu^J = \frac{\alpha_s C_F}{\pi} \left(\frac{\alpha}{\alpha - 1} \ln \left(\frac{\mu^2 (s e^{\gamma_E})^{2/\alpha}}{\omega^2} \right) + \frac{3}{2} \right)$$

- At $1/s = (m/E_J)^\alpha$ the massive and massless jet function are identical.

Matching

to HQET

Wilson coefficient

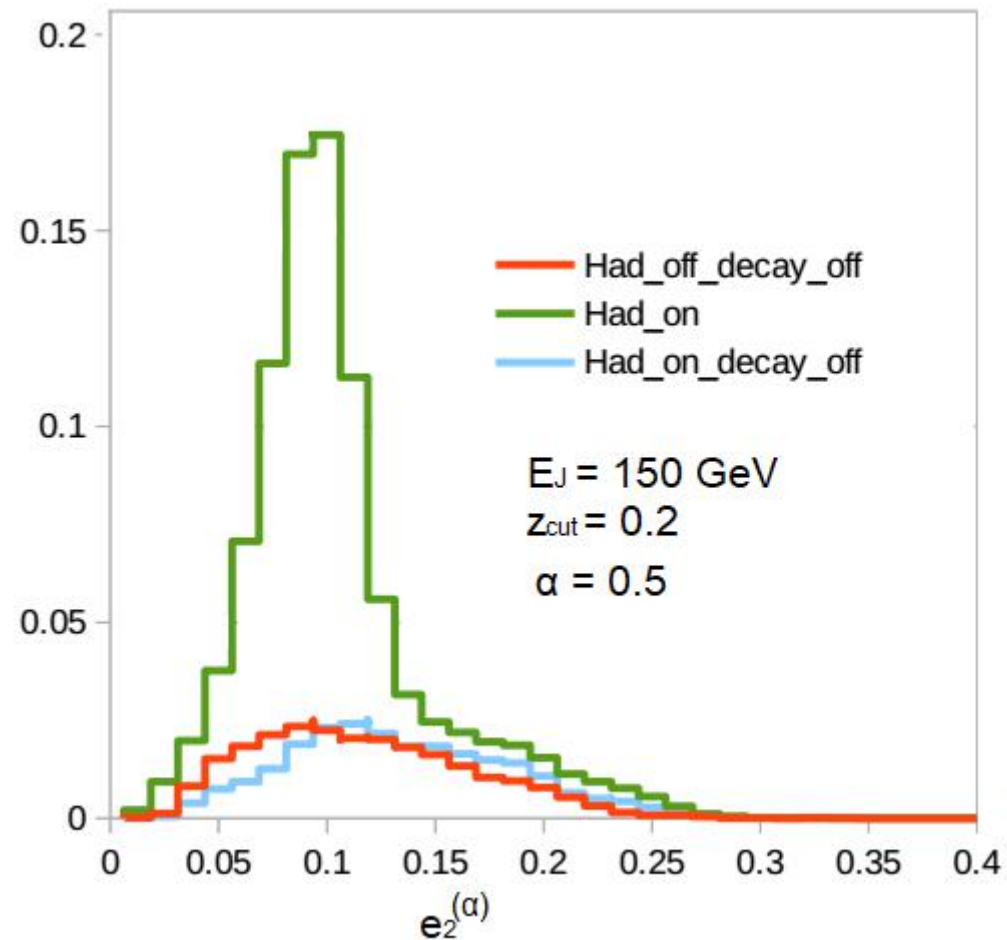
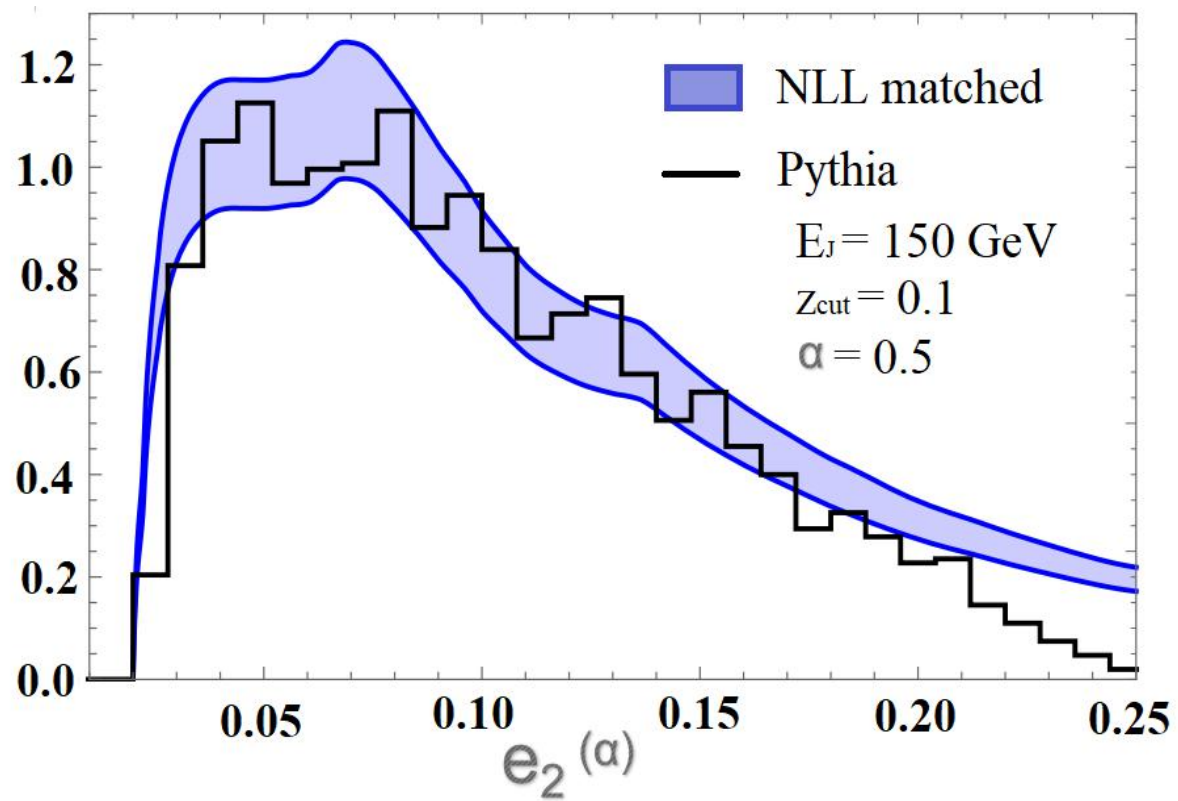
HQET boosted
jet function

$$H_+ = \frac{\alpha_s C_F}{\pi} \left(L_V^2 + \frac{1}{2} L_V \right)$$

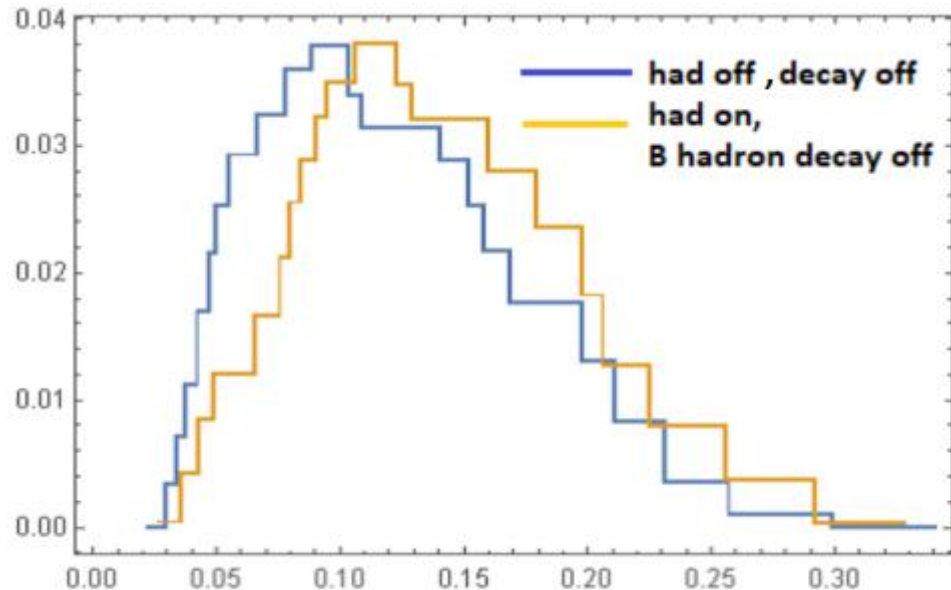
$$B_+^{(1)} = \frac{\alpha_s C_F}{\pi} \left(\frac{1}{\alpha - 1} L_C^2 + L_C \right)$$

Identical to the result in
0711.2079 ,
I. Stewart et. al

Comparison of pythia with matched NLL



Hadronization corrections



- Turning off B hadron decay produces a simple shift in the partonic distribution
- A control over the prediction (with a shape function) with a reconstructed B hadron momentum
- To compare with the experimental result directly, we need to take into account b decays.
- The excess events when decay is turned on are coming from the partonic zero bin.

Hadronization corrections

- Non-perturbative corrections dominate in the HQET jet function $\longrightarrow \frac{\Lambda_{QCD}}{Qe_2^{(\alpha)}} \left(\frac{Q}{m_b}\right)^{1-\alpha}$
- Using the Lorentz transformation properties of the HQET jet function, the leading non perturbative correction in Laplace space

$$P_1(s) = -\frac{s(4\Delta)^{\alpha/2}}{m} A(\alpha)$$

- **Prediction for scaling behaviour with energy and mass of the heavy quark**

$$(e_2^{(\alpha)})^{\text{avg}} = (e_2^{(\alpha)})^{\text{perturbative}} + \frac{(4\Delta)^{\alpha/2}}{m} A(\alpha)$$

$$A(\alpha) = 0.5\text{GeV} \quad \text{for } \alpha = 0.5$$

$$A(\alpha) = \int d\eta F(\eta, \alpha) \langle 0 | \bar{h}_{v_+} W_n \mathcal{E}_T(\eta) W_n^\dagger h_{v_+} | 0 \rangle$$

$$F(\eta, \alpha) = e^\eta \left(1 + e^{-2\eta}\right)^{\alpha/2}$$

$$\Delta = \frac{m^2}{E_J^2}$$



Summary and Outlook

- A first principles calculation of energy correlator observables for heavy quark jets
- An EFT to resum large logarithms in the jet observable and quark mass
- A prediction for the scaling behaviour of the leading non-perturbative corrections with the mass and energy of the jet
- **An analytic understanding of jet substructure as a function of energy and mass of the heavy quark.**
- Long term application for studying medium effects in QGP
- The EFT calculation can be extended by taking into account decay of the heavy quark , which can then be used to tag b jets.

Thank

You