WHEN GLUONS GO ODD: HOW CLASSICAL GLUON FIELDS GENERATE ODD AZIMUTHAL HARMONICS

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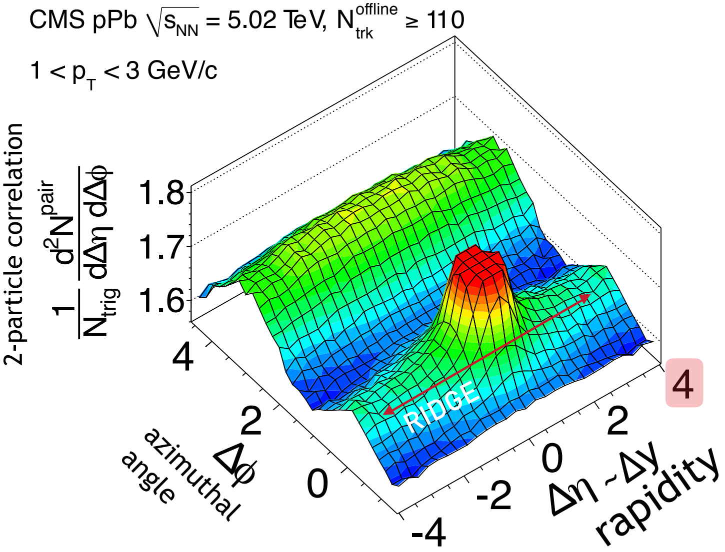
Yuri Kovchegov & V.S. arXiv:1802.08166
Larry Mclerran & V.S. arXiv:1611.09870

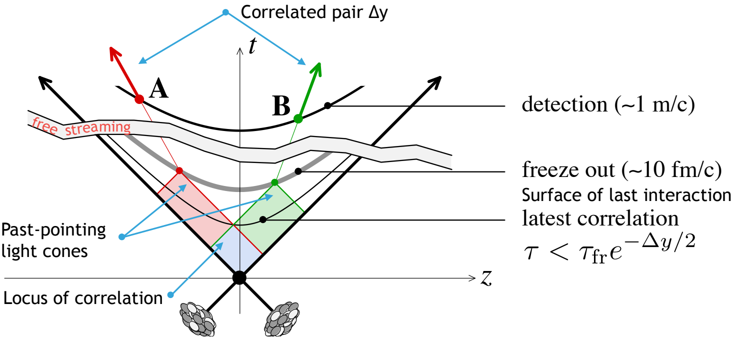
Effect of small-x evolution is beyond the scope of this talk: Alex Kovner, Michael Lublinsky, & V.S. arXiv:1612.07790

First comparison to actual data: Mark Mace, V. S., Prithwish Tribedy, & Raju Venugopalan, in preparation



CMS, Phys. Lett. B 718 (2013) 795

Long-range correlations



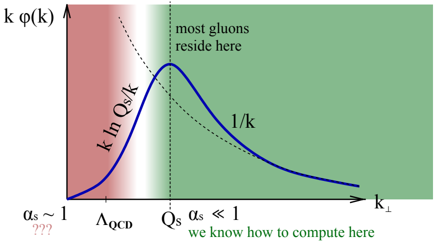
Suppress final state effects in order to probe initial state physics

- Regardless of nature of the ridge
  - long-range rapidity correlations either pre-exist in initial wave function or develop very early after collision
  - understanding initial/early stage is of paramount importance for understanding p-A and p-p.

figure adapted from A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, arXiv:0804.3858

Saturation regime/CGC

- High energy $\rightsquigarrow$ high gluon density $\rightsquigarrow$ formation of perturbative scale, Q_s
- Particle production is dominated by $k_\perp \sim Q_s$
- Weak coupling methods can be applied $\alpha_s(Q_s) \ll 1$



- Still non-perturbative, as fields are strong, $A \sim \frac{1}{g}$ $\rightsquigarrow$ non-linearity is important

What do we know analytically?

Asymmetric collisions, when Q_s of the projectile $\neq$ Q_s of the target, is the easiest case.

Single inclusive production

- In general

$$\frac{dN}{d^3k} = \frac{1}{\alpha_s} f\left(\frac{Q_{sp}^2}{k_\perp^2}, \frac{Q_{sA}^2}{k_\perp^2}\right)$$

$f\left(\frac{Q_{sp}^2}{k_\perp^2}, \frac{Q_{sA}^2}{k_\perp^2}\right)$ is known only numerically; for large $k_1 \gg Q_{sA}^2$: $\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2}\frac{Q_{sA}^2}{k_\perp^2} f^{(1,1)}$

A. Krasnitz, R. Venugopalan, arXiv:9809433 *E. Kuraev, L. Lipatov, V. Fadin, 77*

- If $k_\perp > Q_{sp}$,

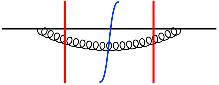
$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$

Functions $f^{(n)}$ are calculable!

Asymmetric collisions, when Q_s of the projectile $\neq Q_s$ of the target, is the easiest case.

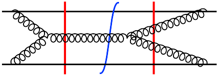
$$\frac{dN}{d^3k} = \frac{1}{\alpha_s}\frac{Q_{sp}^2}{k_\perp^2} f^{(1)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \frac{1}{\alpha_s}\left(\frac{Q_{sp}^2}{k_\perp^2}\right)^2 f^{(2)}\left(\frac{Q_{sA}^2}{k_\perp^2}\right) + \cdots$$

- $f^{(1)}$ is known since '98



Y. V. Kovchegov and A. H. Mueller, arXiv:hep-ph/9802440
A. Dumitru and L. D. McLerran, arXiv:hep-ph/0105268
J.-P. Blaizot, F. Gelis, R. Venugopalan, arXiv:0402256

- $f^{(2)}$: no complete result yet



I. Balitsky, arXiv:hep-ph/0409314
G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

Double inclusive production

Asymmetric collisions, when Q_s of the projectile $\neq Q_s$ of the target, is the easiest case.

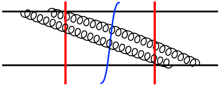
$$\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 \; h^{(1)}(Q_{sA}) + \frac{1}{\alpha_s^2} Q_{sp}^6 \; h^{(2)}(Q_{sA}) + \cdots$$

Momenta dependence is omitted to simplify notation

- $\frac{d^2N}{d^3kd^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 Q_{sA}^4 \; h^{(1,1)}$

A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, arXiv:0804.3858

- $h^{(1)}$ is known since '12 ; invariant under $(k_\perp \to -k_\perp)$



A. Kovner and M. Lublinsky, Int. J. Mod. Phys. E22, 1330001 (2013), 1211.1928
Y. V. Kovchegov and D. E. Wertepny, Nucl. Phys. A906, 50 (2013), 1212.1195

- $h^{(2)}$: no complete result yet

This talk
L. McLerran and V. S., Nucl. Phys. A959, 83 (2017), 1611.09870;
Yu. Kovchegov and V. S., arXiv:1802.08166

What does presence of odd harmonics mean?

- Double inclusive production

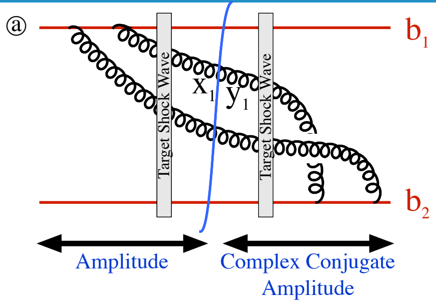
$$\frac{d^2N}{d^2k_1 dy_1 d^2k_2 dy_2} = \frac{d^2N}{k_1 dk_1 dy_1 k_2 dk_2 dy_2}$$
$$\times \left(1 + 2v_2^2\{2\} \cos 2(\phi_1 - \phi_2) + 2v_3^2\{2\} \cos 3(\phi_1 - \phi_2) + \dots\right)$$

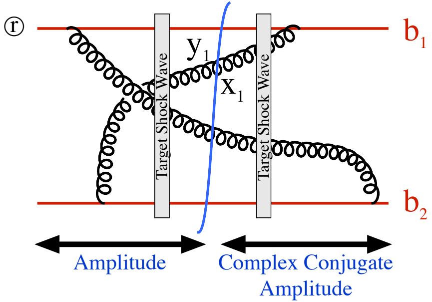
- A non-vanishing $v_3^2\{2\}$

$$\int_0^{2\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\delta\phi\right) = \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\delta\phi\right) - \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \frac{d^2N}{d^2k_1 d^2k_2}\left(\delta\phi + \pi\right)$$
$$= \int_0^{\pi} d\Delta\phi \cos 3\Delta\phi \left[\frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, \underline{k}_2\right) - \frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, -\underline{k}_2\right)\right]$$

- Therefore

$$\frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, \underline{k}_2\right) \neq \frac{d^2N}{d^2k_1 d^2k_2}\left(\underline{k}_1, -\underline{k}_2\right)$$





$$\sigma_{ⓐ}(k,p) = \int_{x_1,y_1,x_2,y_2} e^{-ik\cdot(x_1-y_1)} e^{-ip\cdot(x_2-y_2)} \underbrace{M_1(x_1,b_1)M_1(x_2,b_1)}_{\text{ampl.}} \underbrace{M_1^*(y_1,b_2)M_1^*(y_2,b_2)}_{\text{c.c.ampl.}}$$

$$\sigma_{ⓡ}(k,p) = \int_{x_1,y_1,x_2,y_2} e^{-ik\cdot(y_1-x_1)} e^{-ip\cdot(x_2-y_2)} \underbrace{M_1(y_1,b_2)M_1(x_2,b_1)}_{\text{ampl.}} \underbrace{M_1^*(x_1,b_1)M_1^*(y_2,b_2)}_{\text{c.c.ampl.}}$$

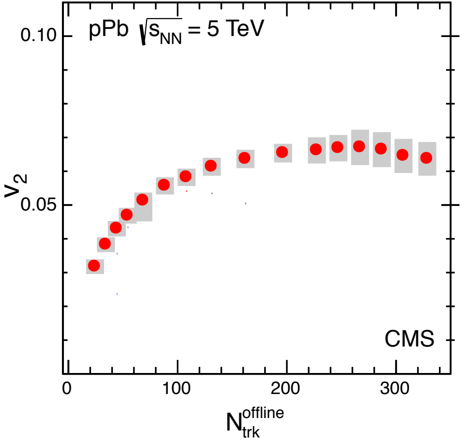
But $M_1(x_1,b_1)M_1^*(y_1,b_2) = M_1^*(x_1,b_1)M_1(y_1,b_2) \rightsquigarrow$

$$\sigma_{ⓐ}(k,p) + \sigma_{ⓡ}(k,p) = \sigma_{ⓐ}(k,p) + \sigma_{ⓐ}(-k,p)$$

$$\rightsquigarrow v_3 = 0$$

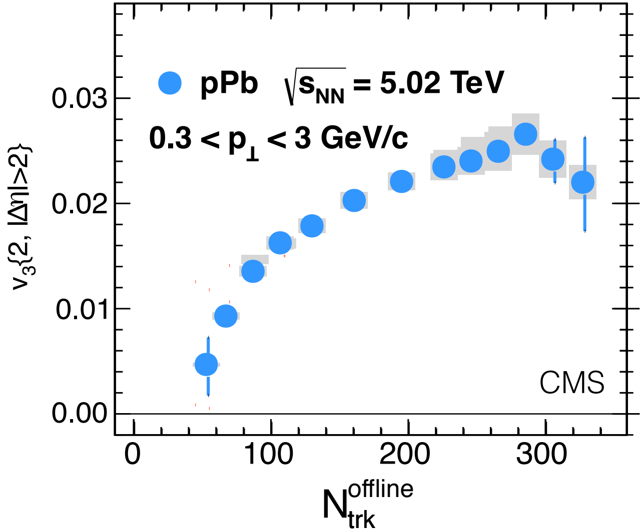
A. Kovner & M. Lublinsky, arXiv:1012.3398,
Yu. Kovchegov & D. Wertepny, arXiv:1212.1195,
Yu. Kovchegov & V.S., arXiv:1802.08166

Experimental data: $v_2\{2\}$



$$\frac{d^2N}{d^2k_1dy_1d^2k_2dy_2} = \frac{d^2N}{k_1dk_1dy_1k_2dk_2dy_2}\left(1 + 2v_2^2\{2\}\cos 2(\phi_1 - \phi_2) + 2v_3^2\{2\}\cos 3(\phi_1 - \phi_2) + \ldots\right)$$

Experimental data: $v_3\{2\}$



- Suppressed compared to v_2, but non-zero!
- Naturally present in ad hoc hydro models; but their applicability is questionable.

Can saturation dynamics account for observed long-range rapidity correlations with non-zero odd azimuthal harmonics?

Odd contribution is buried somewhere in multiple rescattering i.e. in high order $h^{(N\gg 1)}$ $\Downarrow$

$$\frac{d^2N}{d^3k d^3p} = \frac{1}{\alpha_s^2} Q_{sp}^4 \, h^{(1)}\left(Q_{sA}\right) + \frac{1}{\alpha_s^2} Q_{sp}^6 \, h^{(2)}\left(Q_{sA}\right) + \cdots$$

- Theoretically this is unsatisfactory

- Phenomenologically this is problematic

  $v_3\{2\}$ is observed in p-A

  $v_3\{2\}$ is not much smaller than $v_2\{2\}$

$\rightsquigarrow$ $v_3\{2\}$ must originate from rather low order corrections to the leading order dilute-dense production

- Consider single gluon production

$$\frac{d\sigma}{d^2k} \sim |M(\underline{k})|^2 = \int d^2x\, d^2y\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})} M(\underline{x})\, M^*(\underline{y})$$

- Amplitude may have two contributions

$$M(\underline{x}) = M_1(\underline{x}) + M_3(\underline{x}) + \ldots$$

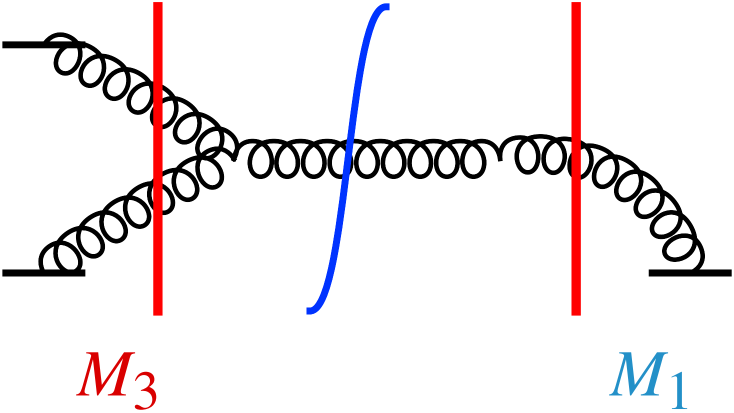
- Asymmetry under $\underline{k} \to -\underline{k}$ would mean that

$$M_1(\underline{x})\, M_3{}^*(\underline{y}) + M_3(\underline{x})\, M_1{}^*(\underline{y}) = -M_1(\underline{y})\, M_3{}^*(\underline{x}) - M_3(\underline{y})\, M_1{}^*(\underline{x})$$

$\rightsquigarrow$ $M_1(\underline{x})\, M_3{}^*(\underline{y})$ is imaginary

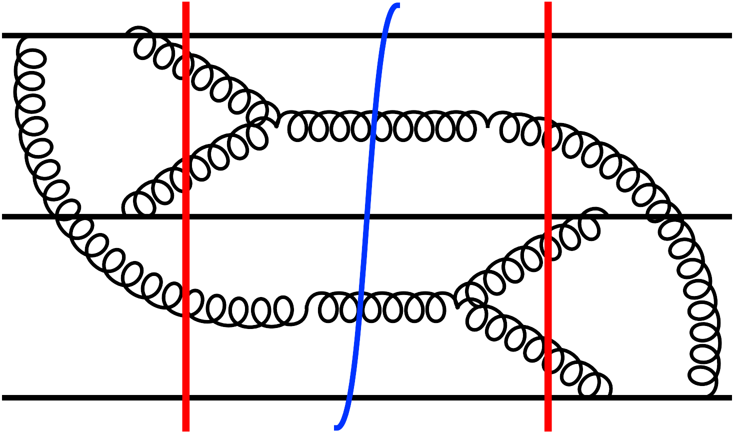
$\rightsquigarrow$ Phase difference between M_1 and M_3 in coordinate space

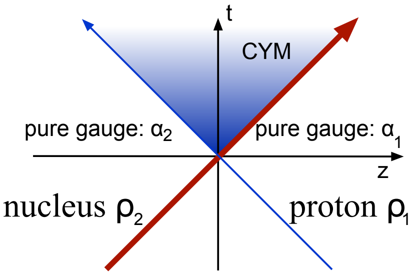
In coordinate space, but not dissimilar from STSA
S. Brodsky, D. S. Hwang, Y. Kovchegov, I. Schmidt, M. Sievert, arXiv:1304.5237



- Vanishes for single-inclusive production…

Double inclusive gluon production





- Before collision, pure gauge soft fields created by "valence" currents

$$\partial_i \alpha^i_{1,2}(\mathbf{x}_\perp) = g\,\rho_{1,2}(\mathbf{x}_\perp)$$
$$\alpha^i_{1,2}(\mathbf{x}_\perp) = -\frac{1}{ig} V_{1,2}(\mathbf{x}_\perp)\partial^i V^\dagger_{1,2}(\mathbf{x}_\perp)$$

- Just after collision, $\tau \to 0+$, (Fock-Schwinger gauge $A_\tau = 0$)
  A. Kovner, L. McLerran, H. Weigert, arXiv:9506320

$$\alpha^i(\tau \to 0, \mathbf{x}_\perp) = \alpha^i_1(\mathbf{x}_\perp) + \alpha^i_2(\mathbf{x}_\perp)$$
$$A_\eta(\tau \to 0, \mathbf{x}_\perp) = \tau^2 \alpha(\tau \to 0, \mathbf{x}_\perp); \quad \alpha(\tau \to 0, \mathbf{x}_\perp) = \frac{ig}{2}[\alpha^i_1(\mathbf{x}_\perp), \alpha^i_2(\mathbf{x}_\perp)]$$

- In forward light-cone $[D_\mu, F^{\mu\nu}] = 0$
- Solve equations perturbatively in ρ_1; use LSZ

Gluon production

- Leading order and the first saturation correction

$$\frac{dN^{\rm even}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_t\right] = \frac{2}{(2\pi)^3}\frac{\delta_{ij}\delta_{lm}+\epsilon_{ij}\epsilon_{lm}}{k^2}\Omega^a_{ij}(\underline{k})\left[\Omega^a_{lm}(\underline{k})\right]^\star$$

$$\frac{dN^{\rm odd}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_T\right] = \frac{2}{(2\pi)^3}{\rm Im}\left\{\frac{g}{\underline{k}^2}\int\frac{d^2l}{(2\pi)^2}\frac{{\rm Sign}(\underline{k}\times\underline{l})}{l^2|\underline{k}-\underline{l}|^2}f^{abc}\Omega^a_{ij}(\underline{l})\Omega^b_{mn}(\underline{k}-\underline{l})\left[\Omega^c_{rp}(\underline{k})\right]^\star\times\right.$$

$$\left.\left[\left(\underline{k}^2\epsilon^{ij}\epsilon^{mn}-\underline{l}\cdot(\underline{k}-\underline{l})(\epsilon^{ij}\epsilon^{mn}+\delta^{ij}\delta^{mn})\right)\epsilon^{rp}+2\underline{k}\cdot(\underline{k}-\underline{l})\epsilon^{ij}\delta^{mn}\delta^{rp}\right]\right\}$$

Here $\delta_{ij}\Omega_{ij}=\Omega_{xx}+\Omega_{yy}$ and $\epsilon_{ij}\Omega_{ij}=\Omega_{xy}-\Omega_{yx}$ and

$$\underbrace{\Omega^a_{ij}(\mathbf{x}_\perp) = g\left[\frac{\partial_i}{\partial^2}\overbrace{\rho^b(\mathbf{x}_\perp)}^{\text{val. sour.}}\right]\partial_j\overbrace{U^{ab}(\mathbf{x}_\perp)}^{\text{target W line}}}_{\text{valence sources rotated by the target}}$$

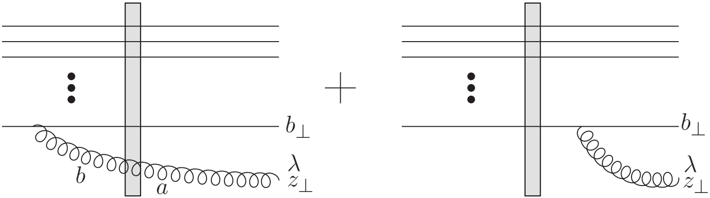
$\frac{dN^{\rm odd}(\underline{k})}{d^2k dy}\left[\rho_p,\rho_T\right]$ is suppressed by extra $\alpha_s\rho_p$

L. McLerran and V. S., Nucl. Phys. A959, 83 (2017), 1611.09870

Alternative approach

- This was obtained in Fock-Schwinger gauge $A_\tau = 0$;
  the gauge is singular; defined in coordinate space.

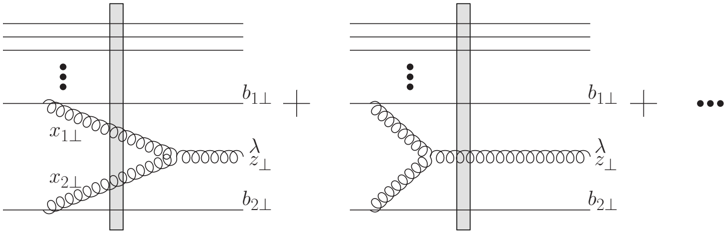
- Motivation to compute in global gauge $A^+ = 0$



$$\underline{\epsilon}^*_\lambda \cdot \underline{M}_1(\underline{z}, \underline{b}) = \frac{{\color{red} i}\, g}{\pi} \frac{\underline{\epsilon}^*_\lambda \cdot (\underline{z} - \underline{b})}{|\underline{z} - \underline{b}|^2} \left[U^{ab}_{\underline{z}} - U^{ab}_{\underline{b}} \right] (V_{\underline{b}}\, t^b)$$

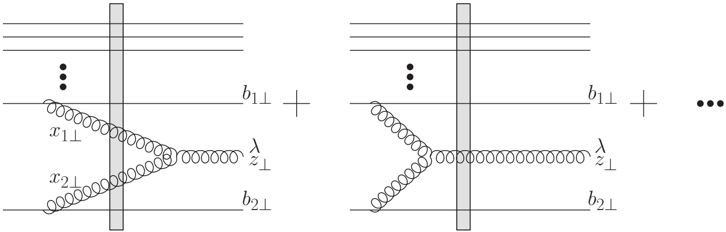
- We have to track the phases ⇑ of the light-cone wave functions

First saturation correction



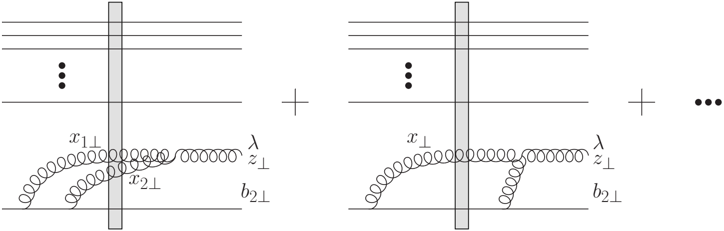
G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106

First saturation correction



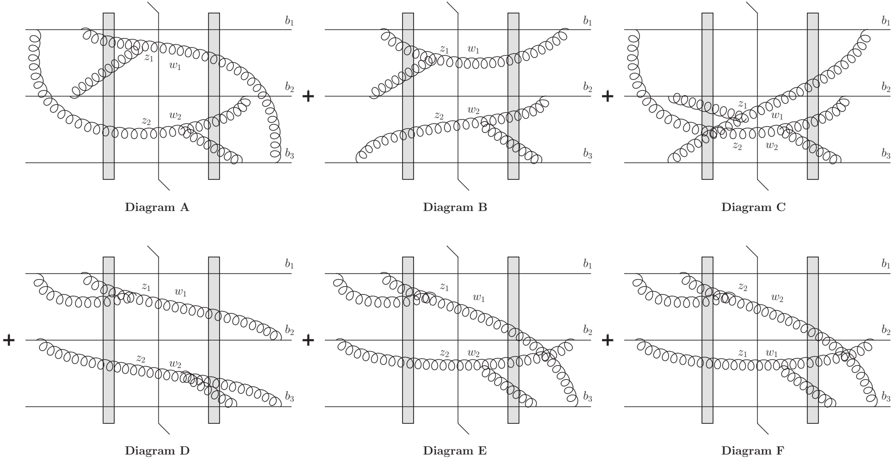
$$
\begin{aligned}
\epsilon_\lambda^* \cdot \underline{M}_3^{ABC} = & -\frac{g^3}{4\pi^4}\int d^2x_1\, d^2x_2\, \delta[(\underline{z}-\underline{x}_1)\times(\underline{z}-\underline{x}_2)] \left[\frac{\epsilon_\lambda{}^* \cdot (\underline{x}_2-\underline{x}_1)}{|\underline{x}_2-\underline{x}_1|^2}\, \frac{\underline{x}_1-\underline{b}_1}{|\underline{x}_1-\underline{b}_1|^2} \cdot \frac{\underline{x}_2-\underline{b}_2}{|\underline{x}_2-\underline{b}_2|^2} - \frac{\epsilon_\lambda^* \cdot (\underline{x}_1-\underline{b}_1)}{|\underline{x}_1-\underline{b}_1|^2}\, \frac{\underline{z}-\underline{x}_1}{|\underline{z}-\underline{x}_1|^2} \cdot \frac{\underline{x}_2-\underline{b}_2}{|\underline{x}_2-\underline{b}_2|^2} \right. \\
& \left. + \frac{\epsilon_\lambda^* \cdot (\underline{x}_2-\underline{b}_2)}{|\underline{x}_2-\underline{b}_2|^2}\, \frac{\underline{x}_1-\underline{b}_1}{|\underline{x}_1-\underline{b}_1|^2} \cdot \frac{\underline{z}-\underline{x}_2}{|\underline{z}-\underline{x}_2|^2} \right] f^{abc} \left[U^{bd}_{\underline{x}_1} - U^{bd}_{\underline{b}_1}\right]\left[U^{ce}_{\underline{x}_2} - U^{ce}_{\underline{b}_2}\right] \left(V_{\underline{b}_1} t^d\right)_1 \left(V_{\underline{b}_2} t^e\right)_2 + \frac{i g^3}{4\pi^3} f^{abc} \left(V_{\underline{b}_1} t^d\right)_1 \left(V_{\underline{b}_2} t^e\right)_2 \\
& \times \int d^2x \left[U^{bd}_{\underline{b}_1}\left(U^{ce}_{\underline{x}} - U^{ce}_{\underline{b}_2}\right) \left(\frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{x})}{|\underline{z}-\underline{x}|^2}\, \frac{\underline{x}-\underline{b}_1}{|\underline{x}-\underline{b}_1|^2} \cdot \frac{\underline{x}-\underline{b}_2}{|\underline{x}-\underline{b}_2|^2} - \frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{b}_1)}{|\underline{z}-\underline{b}_1|^2}\, \frac{\underline{z}-\underline{x}}{|\underline{z}-\underline{x}|^2} \cdot \frac{\underline{x}-\underline{b}_2}{|\underline{x}-\underline{b}_2|^2} - \frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{b}_1)}{|\underline{z}-\underline{b}_1|^2}\, \frac{\underline{x}-\underline{b}_1}{|\underline{x}-\underline{b}_1|^2} \cdot \frac{\underline{x}-\underline{b}_2}{|\underline{x}-\underline{b}_2|^2} \right) \right. \\
& \left. - \left(U^{bd}_{\underline{x}} - U^{bd}_{\underline{b}_1}\right) U^{ce}_{\underline{b}_2} \left(\frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{x})}{|\underline{z}-\underline{x}|^2}\, \frac{\underline{x}-\underline{b}_1}{|\underline{x}-\underline{b}_1|^2} \cdot \frac{\underline{x}-\underline{b}_2}{|\underline{x}-\underline{b}_2|^2} - \frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{b}_2)}{|\underline{z}-\underline{b}_2|^2}\, \frac{\underline{z}-\underline{x}}{|\underline{z}-\underline{x}|^2} \cdot \frac{\underline{x}-\underline{b}_1}{|\underline{x}-\underline{b}_1|^2} - \frac{\epsilon^{\lambda *} \cdot (\underline{z}-\underline{b}_2)}{|\underline{z}-\underline{b}_2|^2}\, \frac{\underline{x}-\underline{b}_1}{|\underline{x}-\underline{b}_1|^2} \cdot \frac{\underline{x}-\underline{b}_2}{|\underline{x}-\underline{b}_2|^2} \right) \right] \\
& - \frac{i g^3}{4\pi^2} f^{abc} \left(V_{\underline{b}_1} t^d\right)_1 \left(V_{\underline{b}_2} t^e\right)_2 \left[(U^{bd}_{\underline{z}} - U^{bd}_{\underline{b}_1}) U^{ce}_{\underline{b}_2} \frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{b}_1)}{|\underline{z}-\underline{b}_1|^2} \ln \frac{1}{|\underline{z}-\underline{b}_2|\Lambda} - U^{bd}_{\underline{b}_1} (U^{ce}_{\underline{z}} - U^{ce}_{\underline{b}_2}) \frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{b}_2)}{|\underline{z}-\underline{b}_2|^2} \ln \frac{1}{|\underline{z}-\underline{b}_1|\Lambda} \right] \\
& - \frac{i g^3}{4\pi^3} \int d^2x \left[U^{ab}_{\underline{x}} - U^{ab}_{\underline{z}}\right] f^{bde} \left(V_{\underline{b}_1} t^d\right)_1 \left(V_{\underline{b}_2} t^e\right)_2 \frac{\epsilon_\lambda^* \cdot (\underline{z}-\underline{x})}{|\underline{z}-\underline{x}|^2}\, \frac{\underline{x}-\underline{b}_1}{|\underline{x}-\underline{b}_1|^2} \cdot \frac{\underline{x}-\underline{b}_2}{|\underline{x}-\underline{b}_2|^2}\, \mathrm{Sign}(b_2^- - b_1^-)
\end{aligned}
$$

G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106



$$
\begin{aligned}
\underline{\epsilon}^*_\lambda \cdot \underline{M}_3^{DE} = & -\frac{g^3}{8\,\pi^4}\int d^2x_1\, d^2x_2\, \delta[(\underline{z}-\underline{x}_1)\times(\underline{z}-\underline{x}_2)]\left[\frac{\underline{\epsilon}^*_\lambda\cdot(\underline{x}_2-\underline{x}_1)}{|\underline{x}_2-\underline{x}_1|^2}\,\frac{\underline{x}_1-\underline{b}_2}{|\underline{x}_1-\underline{b}_2|^2}\cdot\frac{\underline{x}_2-\underline{b}_2}{|\underline{x}_2-\underline{b}_2|^2}\right. \\
& \left. -\frac{\underline{\epsilon}^{\lambda *}\cdot(\underline{x}_1-\underline{b}_2)}{|\underline{x}_1-\underline{b}_2|^2}\,\frac{\underline{z}-\underline{x}_1}{|\underline{z}-\underline{x}_1|^2}\cdot\frac{\underline{x}_2-\underline{b}_2}{|\underline{x}_2-\underline{b}_2|^2} + \frac{\underline{\epsilon}^*_\lambda\cdot(\underline{x}_2-\underline{b}_2)}{|\underline{x}_2-\underline{b}_2|^2}\,\frac{\underline{x}_1-\underline{b}_2}{|\underline{x}_1-\underline{b}_2|^2}\cdot\frac{\underline{z}-\underline{x}_2}{|\underline{z}-\underline{x}_2|^2}\right] \\
& \times f^{abc}\left[U^{bd}_{\underline{x}_1}-U^{bd}_{\underline{b}_2}\right]\left[U^{ce}_{\underline{x}_2}-U^{ce}_{\underline{b}_2}\right]\left(V_{\underline{b}_1}\right)_1\left(V_{\underline{b}_2}t^e\,t^d\right)_2 \\
& +\frac{i\,g^3}{4\,\pi^3}\int d^2x\, f^{abc}\,U^{bd}_{\underline{b}_2}\left[U^{ce}_{\underline{x}}-U^{ce}_{\underline{b}_2}\right]\left(V_{\underline{b}_1}\right)_1\left(V_{\underline{b}_2}t^e\,t^d\right)_2\left(\frac{\underline{\epsilon}^*_\lambda\cdot(\underline{z}-\underline{x})}{|\underline{z}-\underline{x}|^2}\,\frac{1}{|\underline{x}-\underline{b}_2|^2}\right. \\
& \left. -\frac{\underline{\epsilon}^*_\lambda\cdot(\underline{z}-\underline{b}_2)}{|\underline{z}-\underline{b}_2|^2}\,\frac{\underline{z}-\underline{x}}{|\underline{z}-\underline{x}|^2}\cdot\frac{\underline{x}-\underline{b}_2}{|\underline{x}-\underline{b}_2|^2}-\frac{\underline{\epsilon}^*_\lambda\cdot(\underline{z}-\underline{b}_2)}{|\underline{z}-\underline{b}_2|^2}\,\frac{1}{|\underline{x}-\underline{b}_2|^2}\right) \\
& +\frac{i\,g^3}{4\,\pi^2}\,f^{abc}\,U^{bd}_{\underline{b}_2}\left[U^{ce}_{\underline{z}}-U^{ce}_{\underline{b}_2}\right]\left(V_{\underline{b}_1}\right)_1\left(V_{\underline{b}_2}t^e\,t^d\right)_2\,\frac{\underline{\epsilon}^*_\lambda\cdot(\underline{z}-\underline{b}_2)}{|\underline{z}-\underline{b}_2|^2}\,\ln\frac{1}{|\underline{z}-\underline{b}_2|\,\Lambda}
\end{aligned}
$$

G. A. Chirilli, Y. V. Kovchegov, and D. E. Wertepny, arXiv:1501.03106



- We reproduced the result obtained in the Fock-Schwinger gauge!
- The structure of the result: six adjoint Wilson lines multiplying a non-trivial function.

Approximations

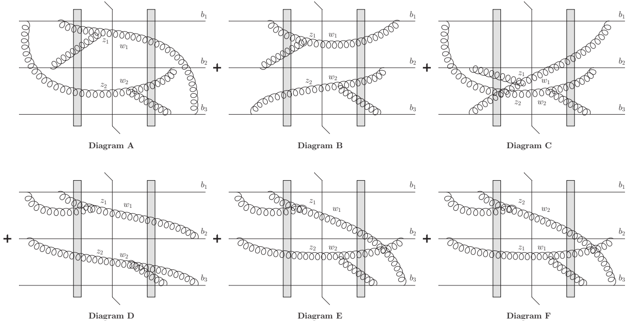
- The sum of all contributions can be computed numerically; relatively low cost
- However, our goal is to obtain an analytical result
- Approximations:

  – Large N_c
  – Golec-Biernat–Wusthoff model

$$S = \exp\left(-\frac{1}{8}Q_s^2 r^2 \ln\frac{1}{r^2\Lambda^2}\right) \to \exp\left(-\frac{1}{8}Q_s^2 r^2\right)$$

  – Only lowest non-trivial order in interaction with the target

$$\frac{Q_s^2}{k^2} \ll 1$$



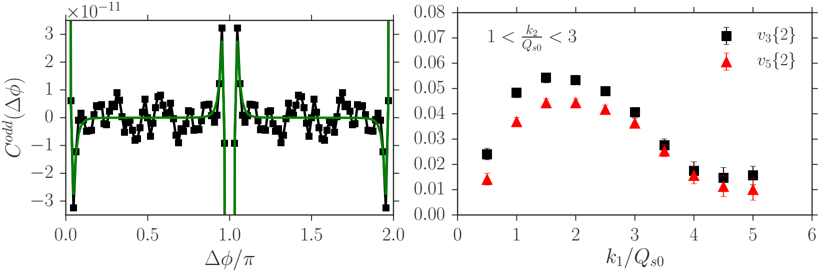
- Under these approximations, non-vanishing contributions from diagrams A, B and C

$$\frac{d\sigma_{odd}}{d^2k_1\, dy_1\, d^2k_2\, dy_2} = \frac{1}{[2(2\pi)^3]^2} \int d^2B\, d^2b \left[T_1(\underline{B}-\underline{b})\right]^3 g^8\, Q_{s0}^6(b)\, \frac{1}{\underline{k}_1^6\, \underline{k}_2^6}$$

$$\times \Bigg\{ \underbrace{\left[\frac{(\underline{k}_1^2+\underline{k}_2^2+\underline{k}_1\cdot\underline{k}_2)^2}{(\underline{k}_1+\underline{k}_2)^6} - \frac{(\underline{k}_1^2+\underline{k}_2^2-\underline{k}_1\cdot\underline{k}_2)^2}{(\underline{k}_1-\underline{k}_2)^6}\right]}_{A} + \underbrace{\frac{10\,c^2}{(2\pi)^2}\,\frac{1}{\Lambda^2}\,\frac{\underline{k}_1\cdot\underline{k}_2}{k_1\,k_2}}_{B}$$

$$+ \underbrace{\frac{1}{4\pi}\,\frac{k_1^4}{\Lambda^4}\left[\delta^2(\underline{k}_1-\underline{k}_2)-\delta^2(\underline{k}_1+\underline{k}_2)\right]}_{C} \Bigg\}$$

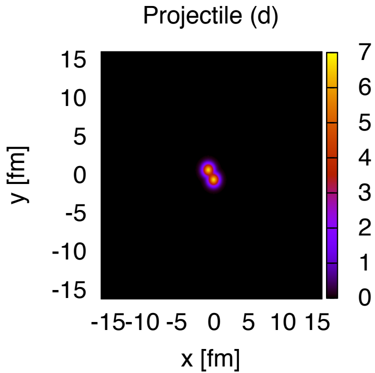
Numerical results in MV model

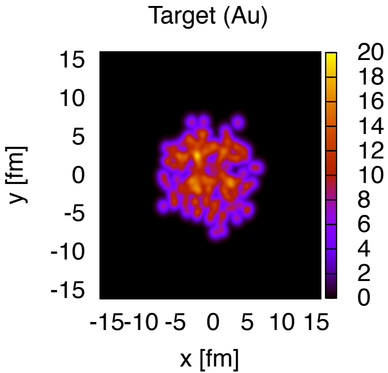


- Prominent HBT peak, diagram C
- Visible contribution from diagram A
- Phenomenologically appropriate numbers for v_3

Yu. Kovchegov & V.S., arXiv:1802.08166

Phenomenological application





- Glauber+IP-Sat for color density distribution

Kowalski, Teaney, Phys.Rev. D68 (2003)
Schenke, Tribedy, Venugopalan PRL 108 (2012)

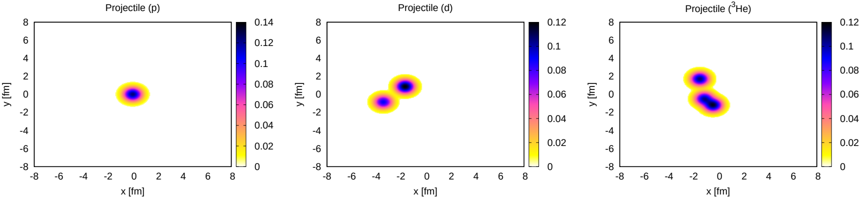
- Negative binomial distribution from first principles – not an input!

F. Gelis, T. Lappi, L. McLerran arXiv:0905.3234

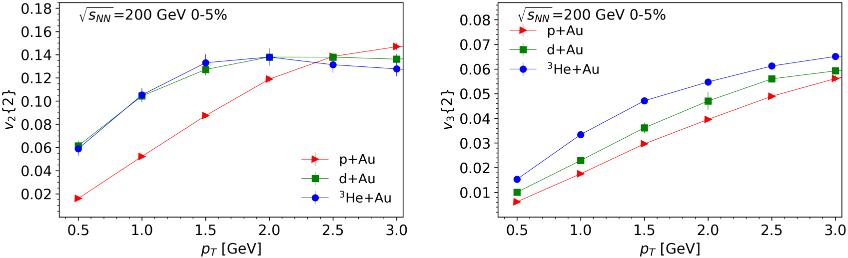
- In good agreement with STAR d+Au multiplicity distribution

M. Mace, V. S., P. Tribedy, & R. Venugopalan, in preparation

Numerical results for p-Au, d-Au and ^{3}He-Au at RHIC



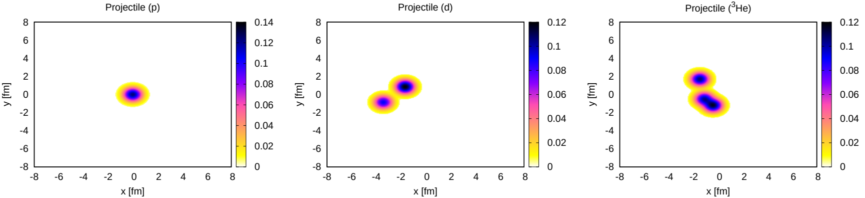
- Hierarchy of anisotropies across systems



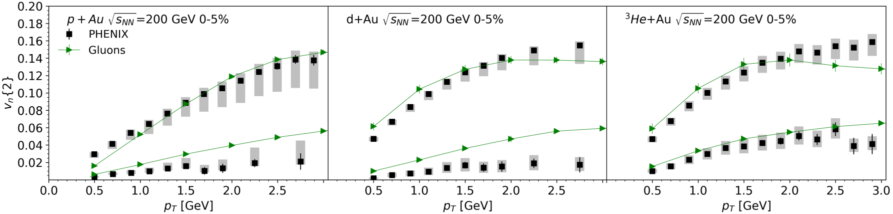
- System size dependence observed at STAR captured in the framework
  no need for hydrodynamic response to initial geometry!

M. Mace, V. S., P. Tribedy, & R. Venugopalan, in preparation

Numerical results for p-Au, d-Au and ^{3}He-Au at RHIC



- Hierarchy of anisotropies across systems



- Goes beyond qualitative description

M. Mace, V. S., P. Tribedy, & R. Venugopalan, in preparation

Conclusions

- Issue is resolved: the source of odd azimuthal asymmetry is identified.
  Odd azimuthal harmonics
  are an inherent property of particle production in the saturation framework

- The result was reproduced in two different guages

- First phenomenological application: p-Au, d-Au, ^{3}He-Au
  – able to describe system size hierarchy of v_2 and v_3 at RHIC

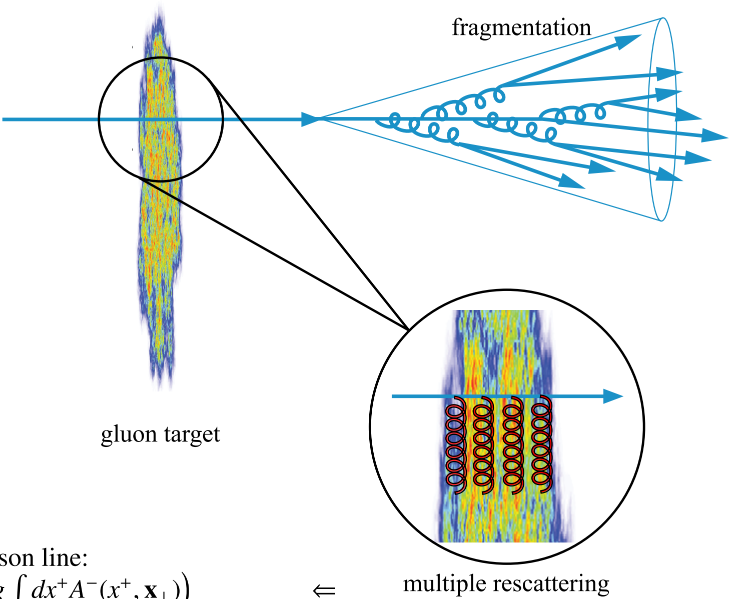
  application to LHC: work in progress

- Check on systematic uncertainties is required

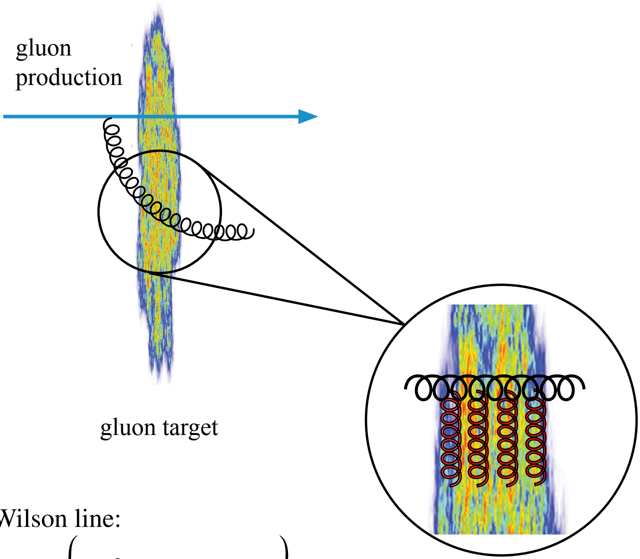
Dilute-dense approximation: high density effects need to be quantified
Fragmentation
. . .


BACKUP

Multiple rescattering



Fundamental Wilson line:
$V(\mathbf{x}_\perp) = \mathcal{P}\exp\left(ig\int dx^+ A^-(x^+, \mathbf{x}_\perp)\right)$ $\Leftarrow$



Adjoint Wilson line:

$$U(\mathbf{x}_\perp) = \mathcal{P}\exp\left(ig\int dx^+ \underbrace{A^-}_{\text{adj.}}(x^+,\mathbf{x}_\perp)\right) \quad \Leftarrow \quad \text{multiple rescattering}$$