Inclusive prompt photon production in e+A DIS at small x as a probe of gluon saturation

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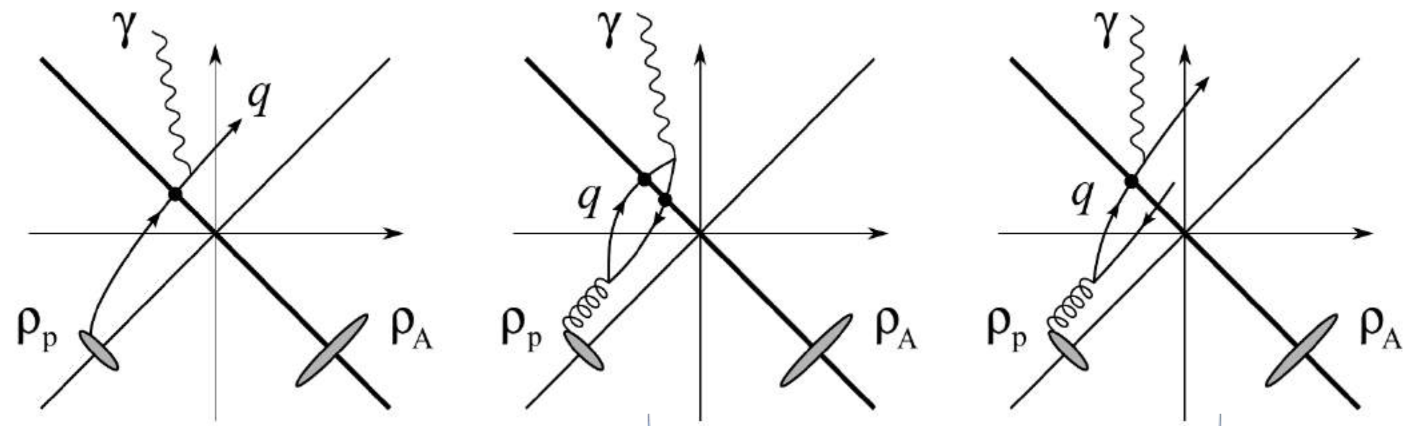
QCD Evolution 2018, Santa Fe, May 20-24, 2018

KR, Venugopalan, JHEP 1805 (2018) 013 [arXiv:1802.09550]; and work in preparation

Outline of the talk

- Background and motivation: Saturation physics overview, CGC essentials, power counting and all that..
- Inclusive photon production at LO: Kinematics, allowed diagrams, shockwave fermion propagator, key analytical results, interesting limits..
- Progress towards NLO computation: gluon shockwave propagator, Wilsonian RG ideology (LO and NLO JIMWLK), NLO inclusive photon impact factor..
- Summary

Background and Motivation



LO in CGC power counting

NLO in CGC power counting

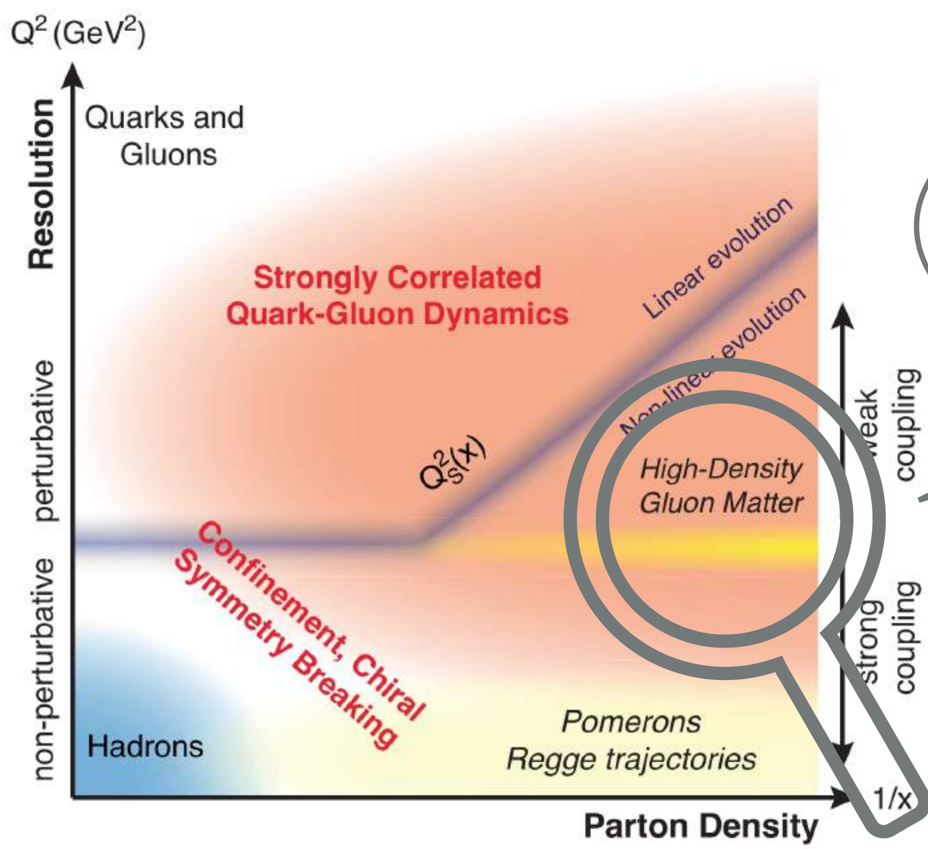
- Inclusive photon production in p+A at small x within the CGC framework has been computed to NLO accuracy (in dilute (**p**)-dense (**A**) regime).

Gelis, Jalilian-Marian, hep-ph/0205037; Dominguez, Marquet, Xiao, Yuan, arXiv:1101.0715

Benic, Fukushima, arXiv:1602.01989; Benic, Fukushima, Garcia-Montero, Venugopalan, arXiv:1609.09424

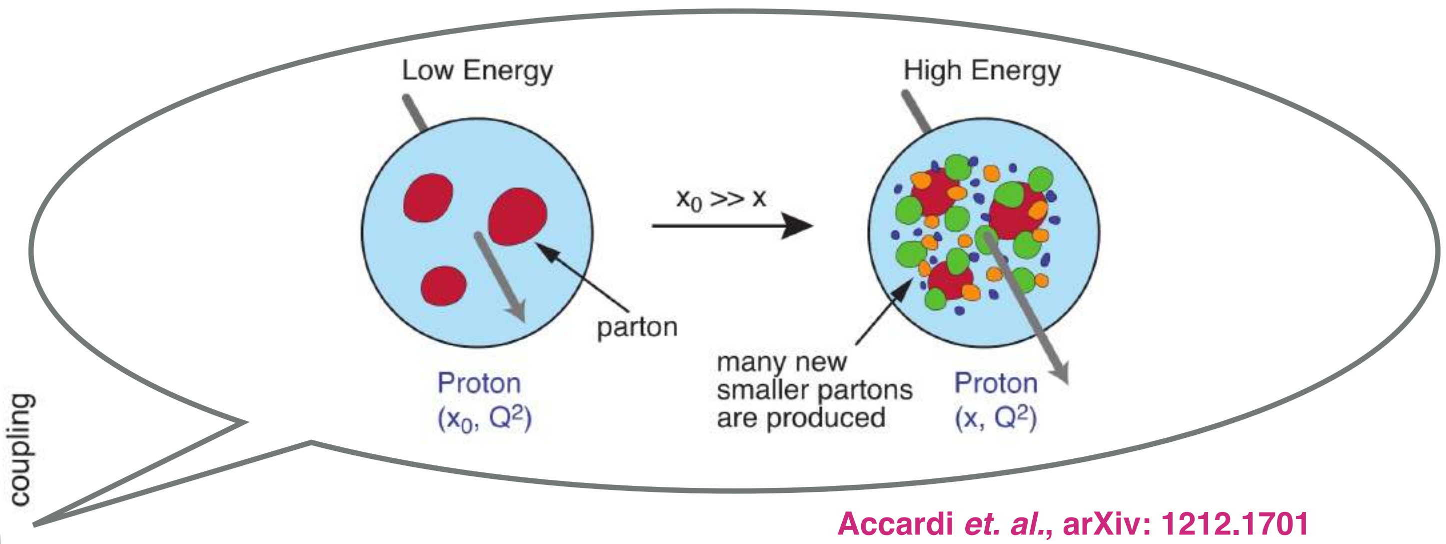
- Interesting to study its DIS counterpart; cleanest process after fully inclusive DIS; can be measured at the luminosities provided by a future **E**lectron **I**on **C**ollider (**EIC**).

Saturation physics overview

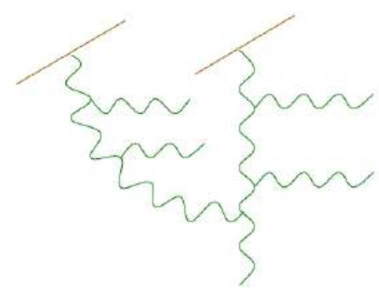


QCD Landscape

Aschenauer *et. al.* , arXiv:1708.01527



Accardi *et. al.*, arXiv: 1212.1701



Recombination vs. gluon bremsstrahlung generates a resolution scale growing with energy

$$Q_s^2 \sim A^{1/3} s^{\lambda}$$

- For high enough energy, $Q_s^2(x) \gg \Lambda_{QCD}^2$ and weak coupling techniques are applied.
- Physics is still non-perturbative; treat non-linear effects to all orders at each order in α_S.
- EFT description of such matter at extreme gluon densities is the **C**olor **G**lass **C**ondensate (**CGC**) framework.

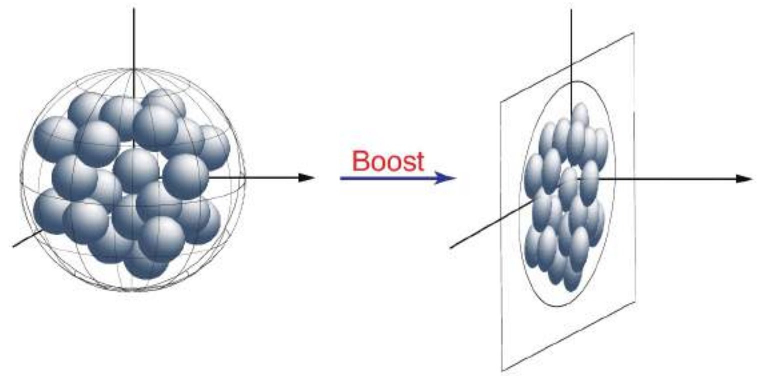
CGC essentials and power counting

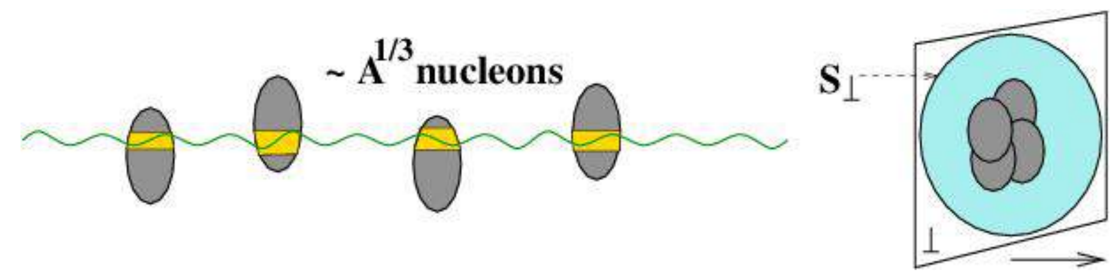
CGC = classical effective field theory in the non-linear regime of QCD describing **dynamical** gluon fields (**small x** partons) effected by **static** color sources (**large x** partons).

McLerran, Venugopalan, hep-ph/9309289
Iancu, Leonidov, McLerran, hep-ph/0011241
Iancu, Venugopalan, hep-ph/0303204

$$D_\nu F^{\nu\mu,a}(x) = \delta^{\mu+} \rho_A^a(x^-, \boldsymbol{x}_\perp)$$

Infinite Momentum Frame (IMF) of the nucleus



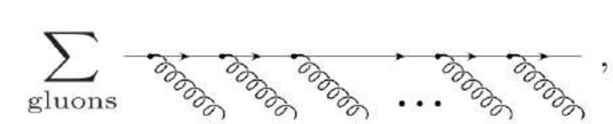


Sources and fields in the IMF

Kovchegov, Levin, *Quantum chromodynamics at high energy*

- In the saturation regime, gluon occupation number $\propto \langle \rho_A \rho_A \rangle \sim \mathrm{O}(1/\alpha_S)$ implying strong classical sources, $\rho_A \sim \mathrm{O}(1/g)$.
- Must resum eikonal interactions, $g\rho_A \sim O(1)$ to all orders **at each order** in α_S .
- So, in general for an observable, O, the following perturbative expansion holds

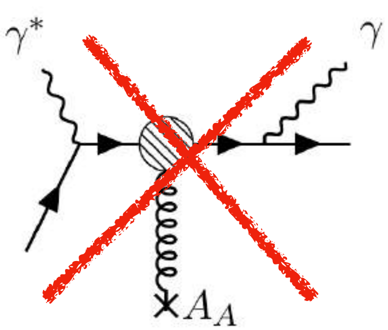
$$\mathrm{O} = \sum_{n=0} c_n \alpha_S^n \quad \text{where} \quad c_n = \sum_{j=1}^{\infty} d_{nj} (g\rho_A)^j$$



Inclusive photon production in small x DIS at LO

Kinematics:

$$e(\tilde{l}) + A(P) \rightarrow e(\tilde{l}') + Q(k) + \overline{Q}(p) + \gamma(k_\gamma) + X$$



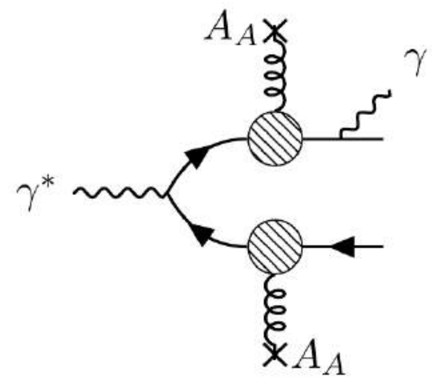
Class I: LO

Suppressed at small x

Cross-section accompanied by

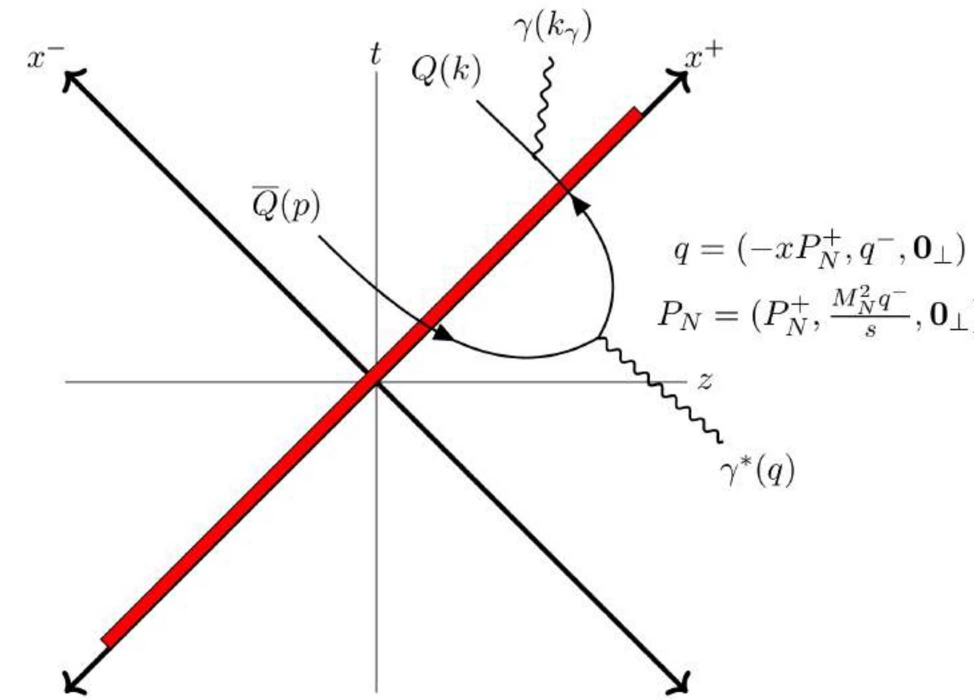
valence quark distribution

$xf(x,Q^2) << xG(x,Q^2)$ at small x



Class II: LO

Consider processes of this class



The right moving nucleus with large P_N^+ has its x^- extent Lorentz contracted

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi y^2}{64\pi^3 Q^2}\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2E_k}\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3 2E_p}\frac{\mathrm{d}^3\boldsymbol{k}_\gamma}{(2\pi)^3 2E_{k_\gamma}}\frac{1}{2q^-}\left(\frac{1}{2}\sum_{\text{spins},\lambda}\left\langle|\widetilde{\mathcal{M}}|^2\right\rangle_{Y_A}\right)(2\pi)\delta(P^- - q^-)$$

$$\frac{1}{2}\sum_{\text{spins},\lambda}|\mathcal{M}|^2 = L^{\mu\nu}X_{\mu\nu}$$

$L_{\mu\nu}$ **: Well-known lepton tensor in DIS**

$X_{\mu\nu}$ **: Hadron tensor for our process which we compute. How??**

KR, Venugopalan, arXiv: 1802.09550

CGC inputs:

Working gauge: $\partial_\mu A^\mu = 0$ (Lorenz gauge)

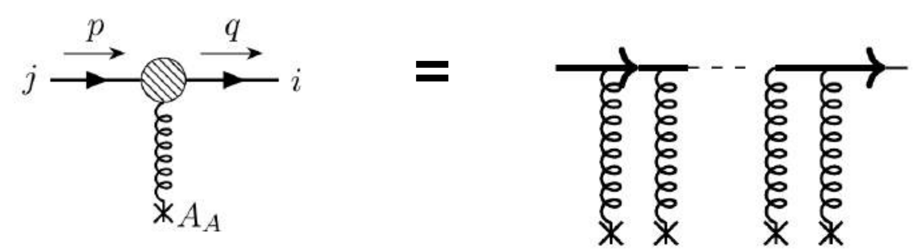
Background classical field: $A^{-,a} = A^{i,a} = 0, \quad A^{+,a} = \int \mathrm{d}^2 \boldsymbol{y}_\perp \langle \boldsymbol{x}_\perp | \frac{1}{-\nabla_\perp^2} | \boldsymbol{y}_\perp \rangle \, \rho_A^a(x^-, \boldsymbol{x}_\perp)$

Momentum space fermion propagator in this background field has a simple form

$$S_{Lor.}(q,p) = (2\pi)^4 \delta^{(4)}(q-p) S_0(p) + S_0(q) \mathcal{T}(q,p) S_0(p)$$

McLerran, Venugopalan, hep-ph/9402335, hep-ph/9809427

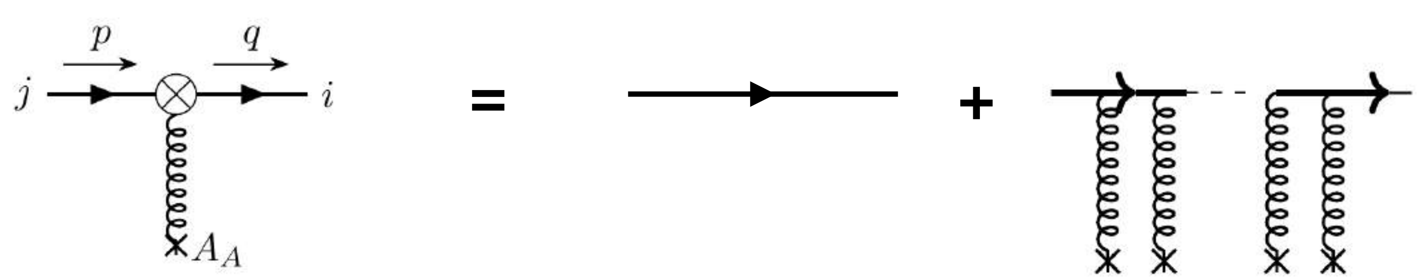
Diagrammatically represented by an effective vertex



Vertex factor $\mathcal{T}_{ij}(q,p) = 2\pi\delta(q^- - p^-)\,\mathrm{sign}(p^-)\gamma^- \int \mathrm{d}^2 \boldsymbol{z}_\perp e^{-i(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp).\boldsymbol{z}_\perp} [\tilde{U}^{\mathrm{sign}(p^-)}(\boldsymbol{z}_\perp) - \mathbb{1}]_{ij}$

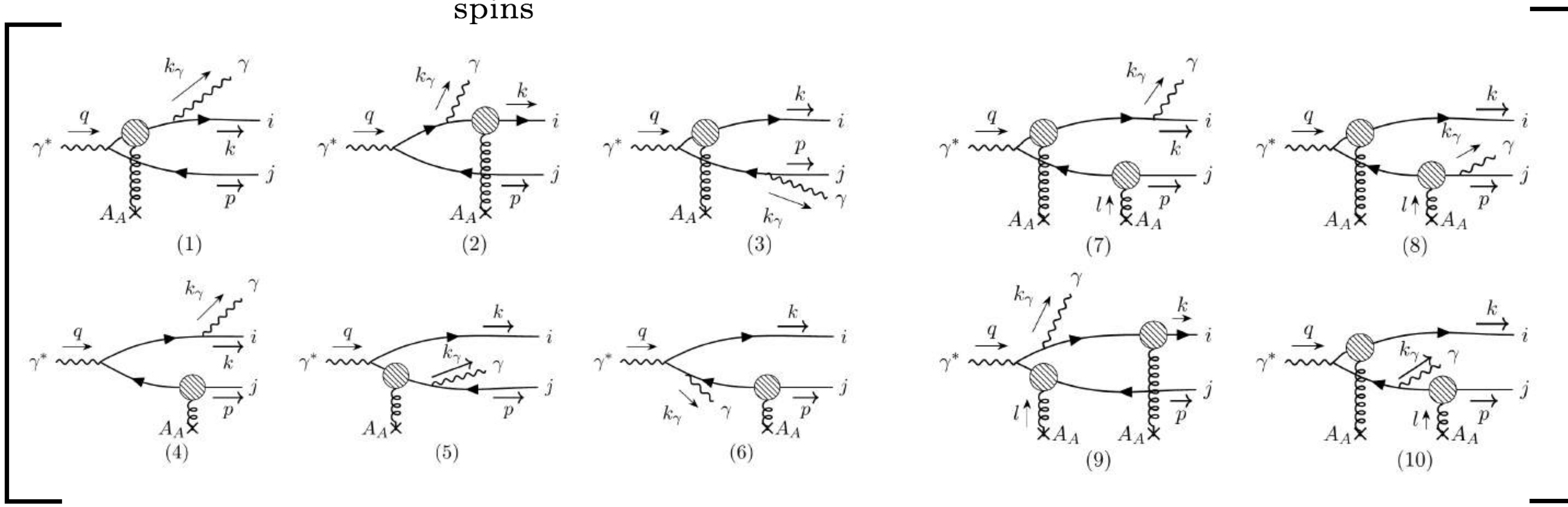
Fundamental Wilson line $\tilde{U}(\mathbf{x}_\perp) = \mathcal{P}_- \exp\left[-ig \int_{-\infty}^{+\infty} \mathrm{d}z^- \frac{1}{\nabla_\perp^2} \rho_A^a(z^-, \mathbf{x}_\perp) t^a\right]$

What we show and utilize: **Use modified vertices (same as above modulo identity)**
Subtract "no-scattering" contribution at the end. Simple and equivalent



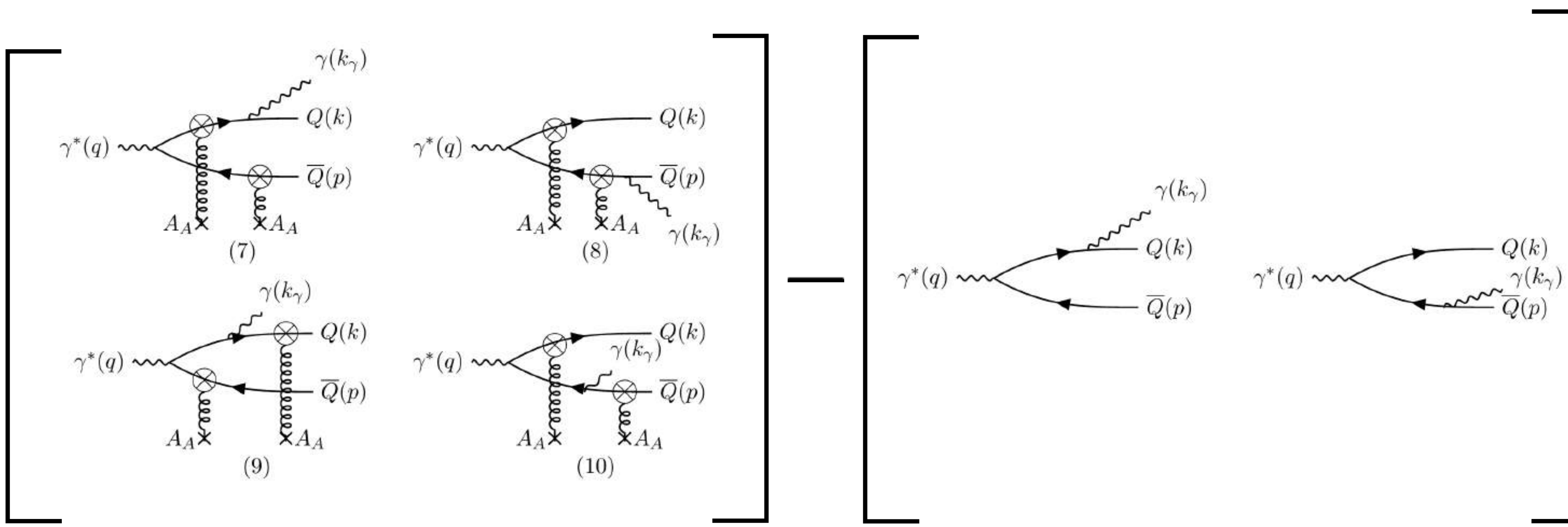
Hadron tensor: $X_{\mu\nu} = -\sum_{\text{spins}} \mathcal{M}^{*}_{\mu\alpha}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma)\mathcal{M}_{\nu}{}^{\alpha}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma)$

$\mathcal{M}_{\mu\alpha}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma) = \sum$



Equivalent results with vertex definitions modified slightly

$\mathcal{M}_{\mu\alpha}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma) = \sum$



Offers significant advantage for NLO computation

Key analytical results:

Single differential cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2\mathrm{d}^2\boldsymbol{k}_{\gamma\perp}\mathrm{d}\eta_{k_\gamma}} = \frac{\alpha^2 q_f^4 y^2 N_c}{512\pi^5 Q^2}\frac{1}{2q^-}\int_0^{+\infty}\frac{\mathrm{d}k^+}{k^+}\int_0^{+\infty}\frac{\mathrm{d}p^+}{p^+}\int_{\boldsymbol{k}_\perp,\boldsymbol{p}_\perp} L^{\mu\nu}\widetilde{X}_{\mu\nu}(2\pi)\delta(P^- - q^-)$$

Lepton tensor

$$L^{\mu\nu} = \frac{2e^2}{Q^4}\left[(\tilde{l}^\mu\tilde{l}'^\nu + \tilde{l}^\nu\tilde{l}'^\mu) - \frac{Q^2}{2}g^{\mu\nu}\right]$$

Hadron tensor

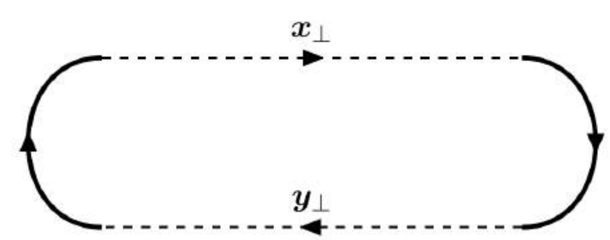
Complicated trace over gamma matrices

$$\widetilde{X}_{\mu\nu} = \int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp}\int_{\boldsymbol{l}_\perp,\boldsymbol{l}'_\perp} e^{-i(\boldsymbol{P}_\perp-\boldsymbol{l}_\perp).\boldsymbol{x}_\perp - i\boldsymbol{l}_\perp.\boldsymbol{y}_\perp + i(\boldsymbol{P}_\perp-\boldsymbol{l}'_\perp).\boldsymbol{x}'_\perp + i\boldsymbol{l}'_\perp.\boldsymbol{y}'_\perp}\ \tau_{\mu\nu}{}^{q\bar{q},q\bar{q}}(\boldsymbol{l}_\perp,\boldsymbol{l}'_\perp|\boldsymbol{P}_\perp)\times\Xi(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$

Non-perturbative input about strongly correlated gluons is contained in

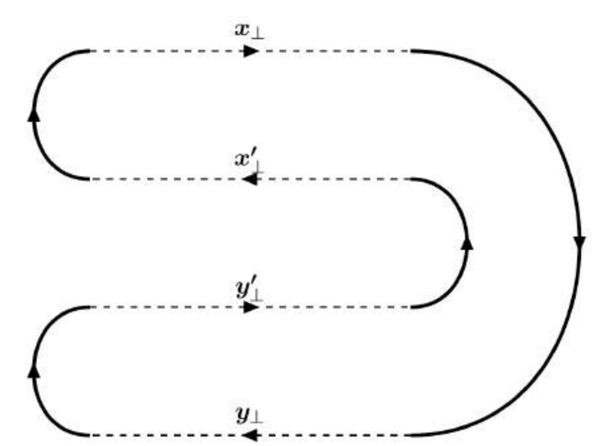
$$\Xi(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) = 1 - D(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) - D(\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp) + Q(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp)$$

Dipole Wilson line correlator $D(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = \frac{1}{N_c}\langle\mathrm{Tr}\left(\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp)\right)\rangle_{Y_A}$



Quadrupole Wilson line correlator

$$Q(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{y}'_\perp,\boldsymbol{x}'_\perp) = \frac{1}{N_c}\langle\mathrm{Tr}\left(\tilde{U}(\boldsymbol{y}'_\perp)\tilde{U}^\dagger(\boldsymbol{x}'_\perp)\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp)\right)\rangle_{Y_A}$$



Ubiquitous building blocks of high energy QCD

KR, Venugopalan, arXiv: 1802.09550

Interesting limits:

KR, Venugopalan, arXiv: 1802.09550

In the limit of $k_\gamma \to 0$ the amplitude satisfies the Low-Burnett-Kroll theorem

$$\mathcal{M}_\mu(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma) \to -(eq_f)\epsilon^*_\alpha(\boldsymbol{k}_\gamma,\lambda)\Big(\frac{p^\alpha}{p.k_\gamma} - \frac{k^\alpha}{k.k_\gamma}\Big)\mathcal{M}^{NR}_\mu(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p})$$

Polarization vector **x** Vectorial structure depending only on momenta of emitted particles **x** Non-radiative DIS amplitude

Recover existing results on inclusive dijet production in DIS

$$\frac{\mathrm{d}\sigma^{L,T}}{\mathrm{d}^3k\mathrm{d}^3p} = \alpha\, q_f^2 N_c \delta(q^- - p^- - k^-)\int \frac{\mathrm{d}^2\boldsymbol{x}_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{x}'_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{y}_\perp}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{y}'_\perp}{(2\pi)^2}\; e^{-i\boldsymbol{k}_\perp.(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)}e^{-i\boldsymbol{p}_\perp.(\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}$$
$$\times \sum_{\alpha,\beta}\psi^{L,T}_{\alpha\beta}(q^-,z,|\boldsymbol{x}_\perp-\boldsymbol{y}_\perp|)\;\psi^{L,T*}_{\alpha\beta}(q^-,z,|\boldsymbol{x}'_\perp-\boldsymbol{y}'_\perp|)\times \Xi(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$

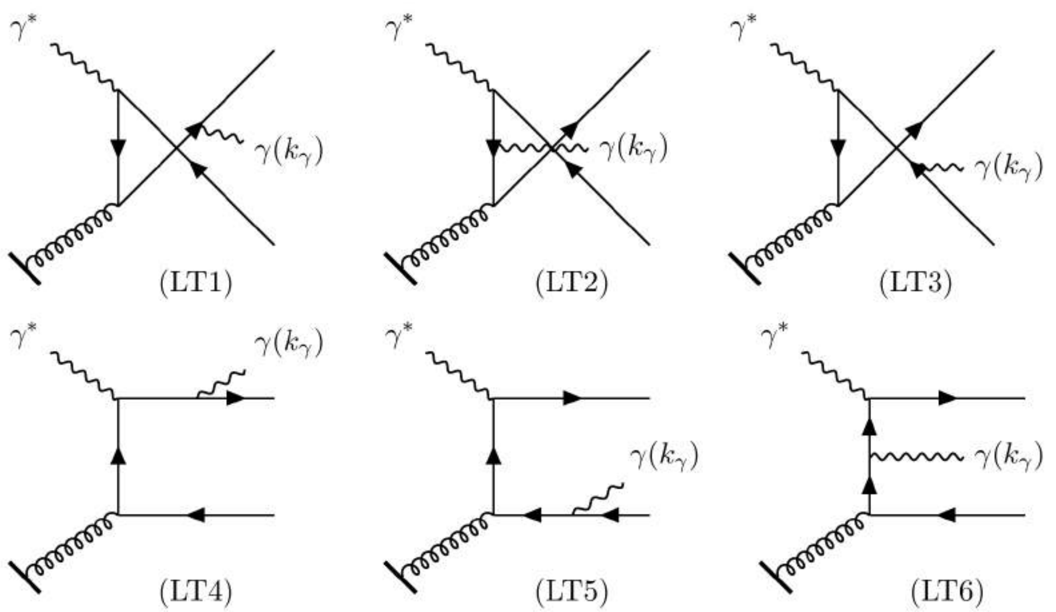
Dominguez, Marquet, Xiao, Yuan, arXiv:1101.0715

Important check because as they show, it is sensitive to the Weizsäcker-Williams UGD in the back-to-back correlation limit $|\boldsymbol{k}_\perp + \boldsymbol{p}_\perp| \ll |\boldsymbol{k}_\perp - \boldsymbol{p}_\perp|/2$.

Golden channel for probing and understanding the WW distribution at a future EIC or LHeC

In the limit of large $P_\perp$, we recover leading twist $k_\perp$ and collinear factorization expressions

$k_\perp$



Unintegrated gluon distribution (UGD)

$$\tilde{X}^{\mathrm{LT}}_{\mu\nu} = \frac{2\alpha_S}{N_c}\frac{\phi_{Y_A}(\boldsymbol{P}_\perp)}{\boldsymbol{P}_\perp^2}\ \Theta_{\mu\nu}(\boldsymbol{P}_\perp)$$

→ Modified trace over Dirac matrices in this limit

$$\tilde{X}^{\mathrm{coll.}}_{\mu\nu} = \frac{2\alpha_S\pi^2}{N_c}\ (2\pi)^2\delta^{(2)}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp + \boldsymbol{k}_{\gamma\perp})\ x_A f_{g,A}(x_A, Q^2) \lim_{\boldsymbol{P}_\perp \to 0} \frac{\Theta_{\mu\nu}(\boldsymbol{P}_\perp)}{\boldsymbol{P}_\perp^2}$$

Collinear factorized result directly sensitive to nuclear gluon distribution at small x

Agreement with small x limit of results by Aurenche *et. al.*

Aurenche, Douiri, Baier, Fontannaz Schiff, Z. Phys. C 24, 309 (1984)

KR, Venugopalan, arXiv: 1802.09550

Progress towards NLO calculation

CGC inputs: Shockwave gluon propagator

- Convenient to work in the "wrong" light-cone gauge $A^- = 0$ for the kinematics of this problem. (Gauge links appearing in PDF definitions are unity in the conventional LC gauge $A^+ = 0$.)
- Resulting momentum space expression is simple and similar to the shockwave fermion propagator.

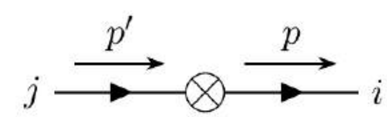
McLerran, Venugopalan, hep-ph/9402335
Ayala, Jalilian-Marian, McLerran, Venugopalan, hep-ph/9501324
Balitsky, Belitsky, hep-ph/0110158

$$G^{\mu\nu;ab}(p,p') = (2\pi)^4\delta^{(4)}(p-p')G_0^{\mu\nu;ab}(p) + G_0^{\mu\rho;ac}(p)\mathcal{T}_{\rho\sigma;cd}(p,p')G_0^{\sigma\nu;db}(p')$$

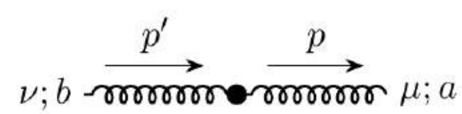
$G_0^{\mu\nu;ab}$ **: Free gluon propagator in** $A^- = 0$ **gauge**

Vertex structures identical to quark-quark-reggeon and gluon-gluon-reggeon in Lipatov's Reggeon EFT

Bondarenko, Lipatov, Pozdnyakov, Prygarin , arXiv: 1708.05183
Hentschinski, arXiv: 1802.06755



$$\mathcal{T}_{ij}(p,p') = (2\pi)\delta(p^- - p'^-)\gamma^- \mathrm{sign}(p^-)\int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp).\boldsymbol{z}_\perp}\ \tilde{U}_{ij}^{\mathrm{sign}(p^-)}(\boldsymbol{z}_\perp)$$



$$\mathcal{T}_{\mu\nu;ab}(p,p') = -2\pi\delta(p^- - p'^-) \times (2p^-)g_{\mu\nu}\ \mathrm{sign}(p^-)\int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp).\boldsymbol{z}_\perp}\ U_{ab}^{\mathrm{sign}(p^-)}(\boldsymbol{z}_\perp)$$

The Wilsonian RG ideology: LO and NLO JIMWLK evolution

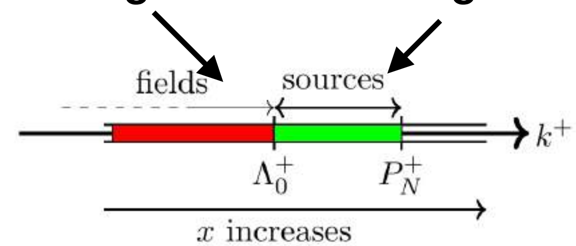
The expectation value of an observable in CGC EFT at a momentum(rapidity) scale is given by

$$\langle \mathcal{O} \rangle_{\Lambda^+(Y_A)} = \int [\mathcal{D}\rho_A]\, W_{\Lambda^+(Y_A)}[\rho_A]\, \mathcal{O}[\rho_A]$$

ρ_A **and** $W_\Lambda[\rho_A]$ **reproduce the effects of the fast gluons**

At LO, the separation scale Λ_0^+ **is arbitrary**

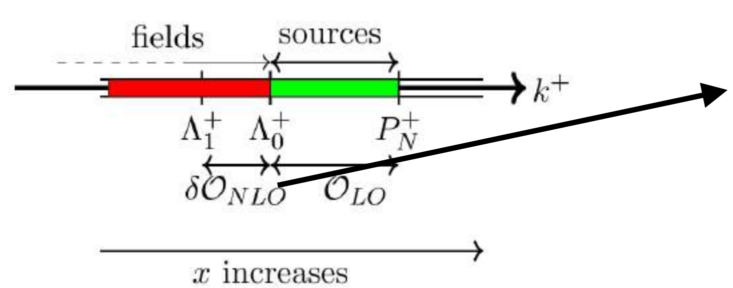
Slow gluons **Fast gluons**



Non-perturbative gluon distribution described by a model eg. MV model

McLerran, Venugopalan, hep-ph/9309289, hep-ph/9311205, hep-ph/9402335
Iancu, Venugopalan, hep-ph/0303204

Evolve to smaller x



Quantum effects (large logarithms) induced by Semi-fast gluons = Nearly on-shell fluctuations with

Momenta: $\Lambda_1^+ (= b\Lambda_0^+) \ll |l^+| \ll \Lambda_0^+$

Energies: $\Lambda_0^- \ll |l^-| \ll \Lambda_0^-/b$

All these effects can be absorbed in a redefinition of $W_\Lambda[\rho_A] \equiv$ **Change with scale governed by JIMWLK equation.**

Jalilian-Marian, Kovner, Leonidov, Weigert, hep-ph/9701284
Jalilian-Marian, Kovner, Weigert, hep-ph/9709432
Iancu, Leonidov, McLerran, hep-ph/0102009, hep-ph/0011241
Ferreiro, Iancu, Leonidov, McLerran, hep-ph/0109115

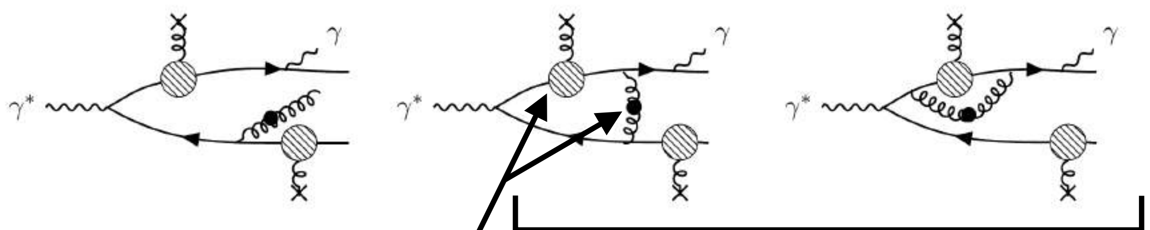
KR, Venugopalan, in preparation

High energy $\ln(1/x)$ resummation:

Observable of interest at LO: $\langle \mathrm{d}\sigma_{LO} \rangle = \int [\mathcal{D}\rho_A]\, W_{\Lambda_0^-}[\rho_A]\; \mathrm{d}\hat{\sigma}_{LO}[\rho_A]$

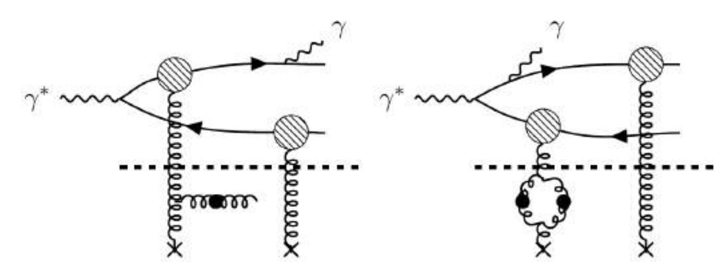
Representative NLO (=O(α_S)) processes in CGC power counting

NLO:1



Real emissions **Virtual emissions**

NLO:2



Instantaneous eikonal interactions with the shockwave

Gluon fluctuations building up the shockwave

Match leading logarithmic pieces $\propto \alpha_S \ln(\Lambda_1^-/\Lambda_0^-)$

(Arises from $\int \mathrm{d}k_g^-/k_g^-$ **with gluon field modes in the strip** $\Lambda_0^- \ll |k_g^-| \ll \Lambda_1^- (\lesssim \min(k^-, p^-))$ **)**

Observable at LO+LLx accuracy: $\langle \mathrm{d}\sigma_{LO} + \delta\sigma_{NLO} \rangle = \int [\mathcal{D}\rho_A] \Big\{ \underbrace{\Big(1 + \ln(\Lambda_1^-/\Lambda_0^-)\mathcal{H}_{LO}\Big) W_{\Lambda_0^-}[\rho_A]} \Big\} \mathrm{d}\hat{\sigma}_{LO}[\rho_A]$

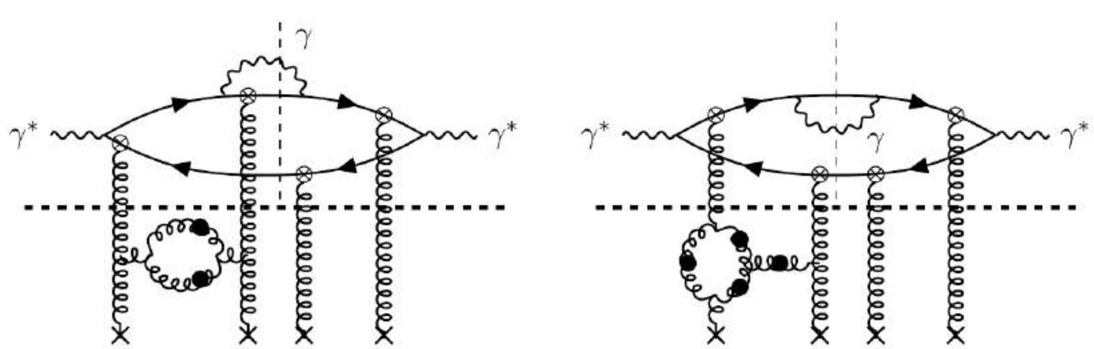
$W_{\Lambda_1^-}[\rho_A]$ **obeys** $\dfrac{\partial}{\partial(\ln \Lambda^-)} W_{\Lambda^-}[\rho_A] = \mathcal{H} W_{\Lambda^-}[\rho_A]$

(JIMWLK)

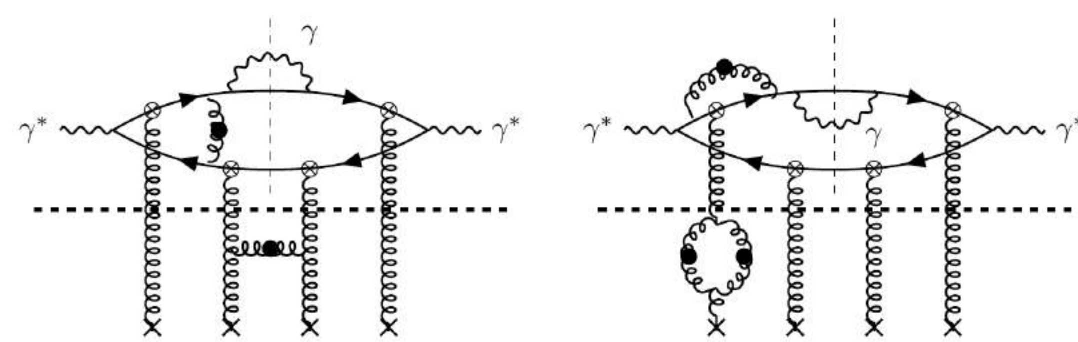
What more can we do?

We are also computing the non-trivial part $\propto \alpha_S$: the "impact factor"
So, go one step further; consider relevant NNLO diagrams
with contributions $\propto \alpha_S^2 \ln(\Lambda_1^- / \Lambda_0^-)$

NNLO:1



NNLO:2



Resum large logarithms with LO+LLx result above

$$\langle \mathrm{d}\sigma_{LO} + \delta\sigma_{NLO} + \mathrm{d}\sigma_{NNLO:1} \rangle = \int [\mathcal{D}\rho_A]\, W^{NLLx}_{\Lambda_0^-}[\rho_A]\, \mathrm{d}\hat{\sigma}_{LO}[\rho_A]$$

LLx evolution of pure α_S suppressed NLO result

$$\langle \mathrm{d}\sigma_{NLO} + \delta\sigma_{NNLO} \rangle = \int [\mathcal{D}\rho_A]\, W^{LLx}_{\Lambda_0^-}[\rho_A]\, \mathrm{d}\hat{\sigma}_{NLO}[\rho_A]$$

$$W^{NLLx}_{\Lambda_0^-}[\rho_A] = \left\{ 1 + \ln(\Lambda_1^- / \Lambda_0^-)(\mathcal{H}_{LO} + \mathcal{H}_{NLO}) \right\}$$ **(NLLx resummed weight functional)**

$\mathcal{H}_{NLO}$ **= NLO JIMWLK Hamiltonian =** $O(\alpha_S^2)$

NLO BK [Balitsky, Chirilli, arXiv:0710.4330
Kovchegov, Weigert, hep-ph/0609090

NLO JIMWLK [Kovner, Lublinsky, Mulian, arXiv:1310.0378
Grabovsky, arXiv:1307.5414
Caron-Huot, arXiv:1309.6521

Final expression for observable at NLO+NLLx accuracy:

Combining these resummed results we obtain

$$\begin{aligned}\langle \mathrm{d}\sigma_{NLO+NLLx}\rangle &= \int [\mathcal{D}\rho_A] \left\{ W^{NLLx}_{\Lambda_0^-}[\rho_A]\, \mathrm{d}\hat{\sigma}_{LO}[\rho_A] + W^{LLx}_{\Lambda_0^-}[\rho_A]\, \mathrm{d}\hat{\sigma}_{NLO}[\rho_A] \right\} \\ &= \int [\mathcal{D}\rho_A] \left(W^{NLLx}_{\Lambda_0^-}[\rho_A] \left\{ \mathrm{d}\hat{\sigma}_{LO}[\rho_A] + \mathrm{d}\hat{\sigma}_{NLO}[\rho_A] \right\} + O(\alpha_S^3 \ln(\Lambda_1^- / \Lambda_0^-)) \right)\end{aligned}$$

Correction terms are higher order compared to the relevant accuracy of the problem

KR, Venugopalan, in preparation

NLO impact factor for inclusive photon production:

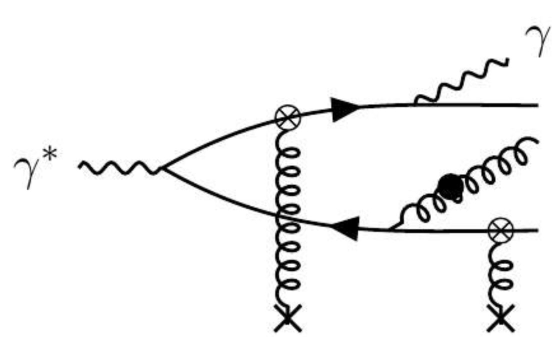
$\int \mathrm{d}\Pi_R$ → Transverse phase space integrals

Real contributions at NLO: **The amplitude can be written as**

$$\mathcal{M}^{\mathrm{real}}_{\mu\alpha;b} = 2\pi(eq_f)^2 g\,\delta(q^- - P^-_{tot}) \int \mathrm{d}\Pi_R\, \bar{u}(\boldsymbol{k})\Big(T^{(1)}_{R\ \mu\alpha;b}\Big((\tilde{U}(\boldsymbol{x}_\perp)t^a\tilde{U}^\dagger(\boldsymbol{y}_\perp))_{ij}\,U_{ba}(\boldsymbol{z}_\perp) - (t_b)_{ij}\Big)$$

$$+ T^{(2)}_{R\ \mu\alpha;b}\Big((t_b\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp))_{ij} - (t_b)_{ij}\Big) + T^{(3)}_{R\ \mu\alpha;b}\Big((\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp)t_b)_{ij} - (t_b)_{ij}\Big)\Big)v(\boldsymbol{p})$$

The diagrams are grouped based on the Wilson line structures (emission of gluon before or after the quark-antiquark scattering)



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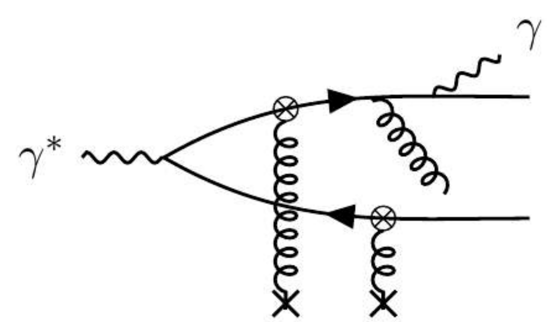

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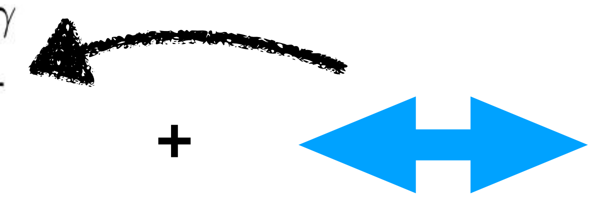


$T^{(1)}_R$ $T^{(2)}_R$ $T^{(3)}_R$

Set of 10 diagrams.
Gluon crosses the shockwave

Set of 10 diagrams
Gluon does not cross the shockwave

Move location of photon and gluon emission to get all possible graphs from representative diagrams

Virtual contributions at NLO:

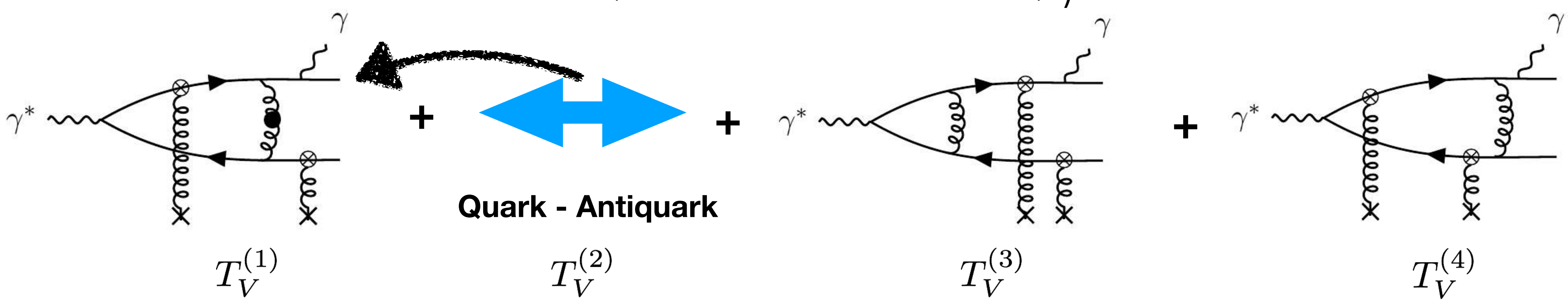
Vertex corrections

$$\mathcal{M}^{\text{vertex}}_{\mu\alpha} = 2\pi(eq_f g)^2\,\delta(q^- - P^-)\;\mathrm{d}\Pi_V\;\bar{u}(\boldsymbol{k})\bigg(T_V^{(1)}{}_{\mu\alpha}\Big((t^a\tilde{U}(\boldsymbol{x}_\perp)t^b\tilde{U}^\dagger(\boldsymbol{y}_\perp))_{ij}\,U_{ab}(\boldsymbol{z}_\perp) - C_F\delta_{ij}\Big)$$

$$+\,T_V^{(2)}{}_{\mu\alpha}\Big((\tilde{U}(\boldsymbol{x}_\perp)t^a\tilde{U}^\dagger(\boldsymbol{y}_\perp)t^b)_{ij}\,U_{ba}(\boldsymbol{z}_\perp) - C_F\delta_{ij}\Big) + T_V^{(3)}{}_{\mu\alpha}\Big(C_F(\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp) - \mathbb{1})_{ij}\Big)$$

6 diagrams in each group

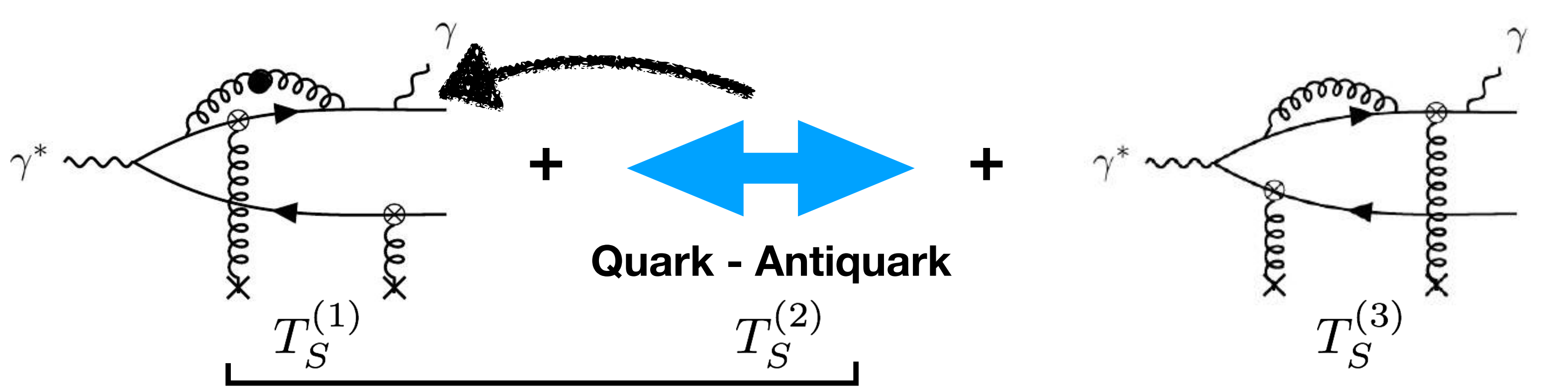
$$+\,T_V^{(4)}{}_{\mu\alpha}\Big((t^a\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp)t_a)_{ij} - C_F\delta_{ij}\Big)\bigg)v(\boldsymbol{p})$$



Self- energy corrections

$$\mathcal{M}^{\text{self-energy}}_{\mu\alpha} = 2\pi(eq_f g)^2\,\delta(q^- - P^-)\;\mathrm{d}\Pi_S\;\bar{u}(\boldsymbol{k})\bigg(T_S^{(1)}{}_{\mu\alpha}\Big((t^a\tilde{U}(\boldsymbol{x}_\perp)t^b\tilde{U}^\dagger(\boldsymbol{y}_\perp))_{ij}\,U_{ab}(\boldsymbol{z}_\perp) - C_F\delta_{ij}\Big)$$

$$+\,T_S^{(2)}{}_{\mu\alpha}\Big((\tilde{U}(\boldsymbol{x}_\perp)t^a\tilde{U}^\dagger(\boldsymbol{y}_\perp)t^b)_{ij}\,U_{ba}(\boldsymbol{z}_\perp) - C_F\delta_{ij}\Big) + T_S^{(3)}{}_{\mu\alpha}\Big(C_F(\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp) - \mathbb{1})_{ij}\Big)\bigg)$$



6 diagrams in each

24 diagrams in total

Assembling different contributions in the amplitude squared:

KR, Venugopalan, in preparation

- **Novel and rich structure in terms of 2-point and 4-point Wilson line correlators obtained.**

$|\mathcal{M}^R|^2$ $\quad\quad$ $\longleftarrow$ $\mathcal{M}_{NLO}\mathcal{M}^*_{LO} + c.c$ $\longrightarrow$

Wilson line factor	Real emission	Virtual: Vertex	Virtual: Self-energy
$\frac{N_c^2}{2}\left(1 - D_{xz}D_{zy} - D_{y'z}D_{zx'} + D_{y'y}D_{xx'}\right) - \frac{1}{2}\Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$	$T_R^{(1)*}T_R^{(1)}$		
$C_F N_c \Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$	$T_R^{(2)*}T_R^{(2)} + T_R^{(3)*}T_R^{(3)}$	$T_{LO}^*T_V^{(3)} + c.c$	$T_{LO}^*T_S^{(3)} + c.c$
$\frac{N_c^2}{2}[(1 - D_{xy})(1 - D_{y'x'})] - \frac{1}{2}\Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$	$T_R^{(2)*}T_R^{(3)} + c.c$	$T_{LO}^*T_V^{(4)} + c.c$	
$\frac{N_c^2}{2}\left(1 + (Q_{zy;y'x'} - D_{zy})D_{xz} - D_{y'x'}\right) - \frac{1}{2}\Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$	$T_R^{(2)*}T_R^{(1)}$	$T_{LO}^*T_V^{(1)}$	$T_{LO}^*T_S^{(1)}$
$\frac{N_c^2}{2}\left(1 + (Q_{y'x';xz} - D_{xz})D_{zy} - D_{y'x'}\right) - \frac{1}{2}\Xi(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$	$T_R^{(3)*}T_R^{(1)}$	$T_{LO}^*T_V^{(2)}$	$T_{LO}^*T_S^{(2)}$

+conjugates of last two rows
Wilson line factors obtained by permutations of coordinates

Collinear divergences cancel between real and interference contributions

No collinear singularities due to transverse momentum kicks from nucleus

Rapidity and UV divergent pieces: absorb into the NLLx JIMWLK expressions using a suitable subtraction scheme.

Similarities with computation of exclusive diffractive cross-section for one or two jets production in DIS

Boussarie, Grabovsky, Szymanowski, Wallon, arXiv:1405.7676, arXiv:1606.00419

Summary

- We present a first computation of inclusive photon production in deeply inelastic electron-nuclear scattering at small x in the CGC framework. Clean way of studying the emergent regime of saturation physics that has aspects of both weak and strong interactions.
- The LO result is proportional to universal 2-point and 4-point Wilson line correlators in the nucleus. Extant results on fully inclusive DIS dijet are obtained in the soft photon limit. In the leading twist limit, the cross-section is directly proportional to the nuclear gluon distribution.
- The simple structure of the dressed quark and gluon propagators in the “wrong” light cone gauge enables higher order computations in momentum space using otherwise standard covariant perturbation theory (pQCD) techniques.
- The techniques employed in NLO calculation are also discussed, with emphasis on the high energy resummation of high energy logarithms. Our goal now is to mould existing NLO JIMWLK results into the methodology adopted for our calculation.

Hänninen, Lappi, Paatelainen , arXiv: 1711.08207
Lublinsky, Mulian, arXiv:1610.03453
Balitsky, Chirilli, arXiv:0710.4330

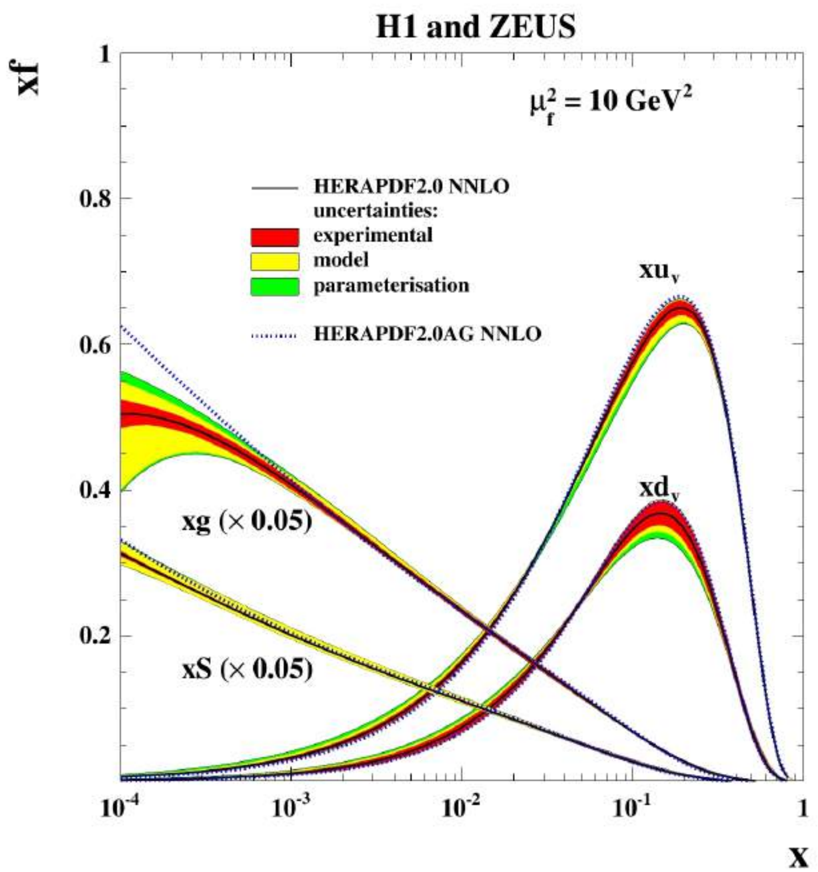
- The computation of the NLO impact factor is the missing non-trivial piece and is nearing completion.

We thank Ian Balitsky, Renaud Boussarie, Al Mueller and Yair Mulian for useful discussions

Thank you…

Backup

The small x problem in QCD:



H. Abramowicz *et. al.* , arXiv:1506.06042

In IMF and $A^+ = 0$ gauge, at high Q^2, PDFs are related to the number of partons per unit rapidity in the hadron wavefunction.

$$xG(x, Q^2) = \left(\frac{\mathrm{d}N}{\mathrm{d}y}\right)_{\mathrm{gluons}}$$

$$y = y_{\mathrm{hadron}} - \ln(\frac{1}{x})$$

$$\tau = \text{rapidity interval}$$

Observation:

- Rapidity distribution of sea quarks and gluons grow rapidly as x decreases.
- Larger than τ or τ^2.

Problem:

- Growth cannot be explained by BFKL ($\exp(\tau)$) or double-log DGLAP ($\exp(\sqrt{\tau})$).
- Exceeds Froissart bound (violates unitarity)

McLerran-Venugopalan (MV) Model:

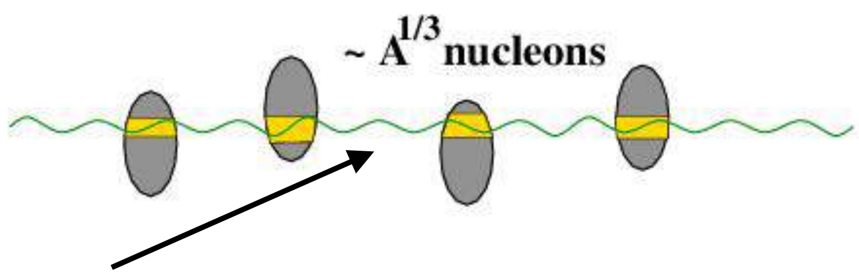
McLerran, Venugopalan, hep-ph/9309289, hep-ph/9311205, hep-ph/9402335

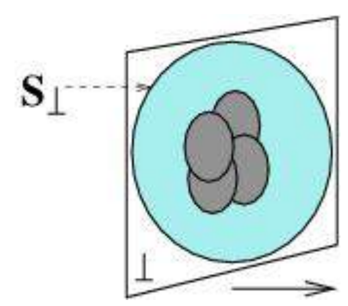
Originally formulated for a large nucleus in the IMF

$$|\Psi\rangle = c_1|qqq\rangle + c_2|qqqq\rangle + \ldots + c_n|qqqggg\ldots q\bar{q}gg\rangle$$

Higher Fock-components dominate the hadronic wave function at small x. Predominantly gluons.

Based on natural separation of energy (time) scales into 'wee' (small x) and valence (large x) partons





$$\lambda wee \approx \frac{1}{k^+} = \frac{1}{xP^+} >> \lambda val. = \frac{Rm_p}{P^+} \Rightarrow x << A^{-1/3}$$

'Wee' partons 'see' a large density of color 'sources' at small transverse resolutions

- Slow/`wee' partons (small-x) have short lifetimes, $\Delta x^+ \equiv \frac{xP_N^+}{m_\perp^2}$ and must be treated as standard gauge fields.
- Fast/valence partons (large-x) appear to live forever on the time-scale of wee partons.
- Consider them as static sources of color charge, $J^\mu(x) = \delta^{\mu+}\delta(x^-)\rho(\boldsymbol{x}_\perp)$.
- When `wee' partons couple to a large number of these `random' color sources simultaneously, we can consider the charge distribution to be classical.
- Gauge fields of small-x gluons 'eikonally' couple to J^+.

Iancu, Venugopalan, hep-ph/0303204

Total charge in transverse area

$$Q^a = \int_{\Delta S_\perp} \mathrm{d}^2\boldsymbol{x}_\perp \rho^a(\boldsymbol{x}_\perp) = \int_{\Delta S_\perp} \mathrm{d}^2\boldsymbol{x}_\perp \int \mathrm{d}x^- \rho^a(x^-, \boldsymbol{x}_\perp)$$

Equal LC time charge correlators

$$\langle \rho_a(x^-, \boldsymbol{x}_\perp)\rho_b(y^-, \boldsymbol{y}_\perp)\rangle_A = \delta_{ab}\delta^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)\delta(x^- - y^-)\lambda_A(x^-)$$

$$\int \mathrm{d}x^- \lambda_A(x^-) = \mu_A^2 \equiv \frac{g^2 A}{2\pi R_A^2}$$

Average color charge squared of valence quarks per unit transverse area per color

$$\mu_A^2 \propto A^{1/3}$$

For large nucleus, $\alpha_S(\mu^2) \ll 1$

Nuclear wavefunction at small x is perturbative (not totally); must treat non-linear effects due to large gluon density to all orders

Criterion for gluon recombination = $\rho\sigma_{gg\to g} \geq 1$

$$\rho \sim \frac{xG(x,Q^2)}{\pi R^2} \qquad \sigma_{gg\to g} \sim \frac{\alpha_S}{Q^2}$$

$$Q^2 \leq Q_S^2 \equiv \frac{\alpha_S\,(xG(x,Q^2))}{\pi R^2}$$

Saturation momentum scale = avg. color charge squared of gluons/rapidity/transverse area

In MV model, the non-trivial correlators shown in the previous slide are generated by the weight functional

$$W_{\Lambda_0^+}[\rho_A] = \mathcal{N}\, exp\Big\{ -\frac{1}{2}\int \mathrm{d}x^- \mathrm{d}^2\boldsymbol{x}_\perp \frac{\rho_A(x^-,\boldsymbol{x}_\perp)\rho_A(x^-,\boldsymbol{x}_\perp)}{\lambda_A(x^-)}\Big\}$$

Gauge invariant (since local) and Gaussian in ρ_A

Valid by construction for a large nucleus and for kinematical range

$$\Lambda^2_{QCD} \ll Q^2 (= 1/\Delta S_\perp) \ll \Lambda^2_{QCD} A^{1/3}$$

Details of the LO result:

KR, Venugopalan, arXiv: 1802.09550

LO amplitude:

$$\mathcal{M}_{\mu\alpha}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma) = \sum_{\beta=1}^{10} \mathcal{M}^{\beta}_{\mu\alpha}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}_\gamma) = 2\pi(eq_f)^2\delta(P^- - q^-)\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp}\int_{\boldsymbol{l}_\perp} e^{-i\boldsymbol{P}_\perp.\boldsymbol{x}_\perp + i\boldsymbol{l}_\perp.\boldsymbol{x}_\perp - i\boldsymbol{l}_\perp.\boldsymbol{y}_\perp}$$
$$\times\, \overline{u}(\boldsymbol{k})\Big[T^{(q\bar{q})}_{\mu\alpha}(\boldsymbol{l}_\perp,\boldsymbol{P}_\perp)\big[\tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(\boldsymbol{y}_\perp) - 1\big]\Big]v(\boldsymbol{p})$$

$$T^{(q\bar{q})}_{\mu\alpha}(\boldsymbol{l}_\perp,\boldsymbol{P}_\perp) = \sum_{\beta=7}^{10} R^{\beta}_{\mu\alpha}(\boldsymbol{l}_\perp,\boldsymbol{P}_\perp)$$

R-factors:

$$R^{(7)}_{\mu\alpha}(\boldsymbol{l}_\perp,\boldsymbol{P}_\perp) = \frac{\gamma_\alpha \not{k}_\gamma + 2k_\alpha}{(k+k_\gamma)^2 - m^2 + i\varepsilon}\gamma^- \frac{\not{q} - \not{p} + \not{\boldsymbol{l}}_\perp + m}{N(\boldsymbol{l}_\perp)}\gamma_\mu(\gamma^- \not{\boldsymbol{l}}_\perp + 2p^-)$$

$$R^{(8)}_{\mu\alpha}(\boldsymbol{l}_\perp,\boldsymbol{P}_\perp) = -\frac{\gamma^-(\not{q} - \not{P} + \not{\boldsymbol{l}}_\perp) + 2k^-}{(p+k_\gamma)^2 - m^2 + i\varepsilon}\gamma_\mu(\not{p} + \not{k}_\gamma - \not{\boldsymbol{l}}_\perp - m)\gamma^- \frac{(\not{k}_\gamma\gamma_\alpha + 2p_\alpha)}{S(\boldsymbol{l}_\perp)}$$

$$R^{(9)}_{\mu\alpha}(\boldsymbol{l}_\perp,\boldsymbol{P}_\perp) = \Big(\gamma^-(\not{q} - \not{P} + \not{\boldsymbol{l}}_\perp) + 2k^-\Big)\gamma_\alpha(\not{q} - \not{p} + \not{l}_a + m)\gamma_\mu \frac{(\gamma^- \not{\boldsymbol{l}}_\perp + 2p^-)}{V(\boldsymbol{l}_\perp)}$$

$$R^{(10)}_{\mu\alpha}(\boldsymbol{l}_\perp,\boldsymbol{P}_\perp) = \Big(\gamma^-(\not{q} - \not{P} + \not{\boldsymbol{l}}_\perp) + 2k^-\Big)\gamma_\mu(\not{l}_b - \not{p} - \not{k}_\gamma + m)\gamma_\alpha \frac{(\gamma^- \not{\boldsymbol{l}}_\perp + 2p^-)}{W(\boldsymbol{l}_\perp)}$$

Factors in R:

$$N(\boldsymbol{l}_\perp) = 4p^-(q^- - p^-)\Big[q^+ - \frac{M^2(\boldsymbol{l}_\perp - \boldsymbol{p}_\perp)}{2p^-} - \frac{M^2(\boldsymbol{l}_\perp - \boldsymbol{p}_\perp)}{2(q^- - p^-)}\Big]$$

$$S(\boldsymbol{l}_\perp) = 4(p^- + k_\gamma^-)(q^- - p^- - k_\gamma^-)\Big[q^+ - \frac{M^2(\boldsymbol{p}_\perp + \boldsymbol{k}_{\gamma\perp} - \boldsymbol{l}_\perp)q^-}{2(p^- + k_\gamma^-)(q^- - k_\gamma^- - p^-)}\Big] \ ,$$

$$V(\boldsymbol{l}_\perp) = 8p^-(q^- - p^-)(q^- - p^- - k_\gamma^-)\Big[q^+ - \frac{M^2(\boldsymbol{l}_\perp - \boldsymbol{p}_\perp)q^-}{2p^-(q^- - p^-)}\Big]$$
$$\times \Big[q^+ - k_\gamma^+ - \frac{M^2(\boldsymbol{l}_\perp - \boldsymbol{p}_\perp)}{2p^-} - \frac{M^2(\boldsymbol{p}_\perp + \boldsymbol{k}_{\gamma\perp} - \boldsymbol{l}_\perp)}{2(q^- - p^- - k_\gamma^-)}\Big] \ ,$$

$$W(\boldsymbol{l}_\perp) = 8p^-(p^- + k_\gamma^-)(q^- - p^- - k_\gamma^-)\Big[q^+ - \frac{M^2(\boldsymbol{p}_\perp + \boldsymbol{k}_{\gamma\perp} - \boldsymbol{l}_\perp)q^-}{2(p^- + k_\gamma^-)(q^- - p^- - k_\gamma^-)}\Big]$$
$$\times \Big[q^+ - k_\gamma^+ - \frac{M^2(\boldsymbol{l}_\perp - \boldsymbol{p}_\perp)}{2p^-} - \frac{M^2(\boldsymbol{p}_\perp + \boldsymbol{k}_{\gamma\perp} - \boldsymbol{l}_\perp)}{2(q^- - p^- - k_\gamma^-)}\Big]$$

Complicated Dirac trace:

$$\tau_{\mu\nu}{}^{q\bar{q},q\bar{q}}(\boldsymbol{l}_\perp, \boldsymbol{l}'_\perp|\boldsymbol{P}_\perp) = \mathrm{Tr}\Big[(\not{k} + m)T_\nu^{(q\bar{q})\,\alpha}(\boldsymbol{l}_\perp, \boldsymbol{P}_\perp)(m - \not{p})\hat{\gamma}^0 T_{\mu\alpha}^{(q\bar{q})\,\dagger}(\boldsymbol{l}'_\perp, \boldsymbol{P}_\perp)\hat{\gamma}^0\Big]$$

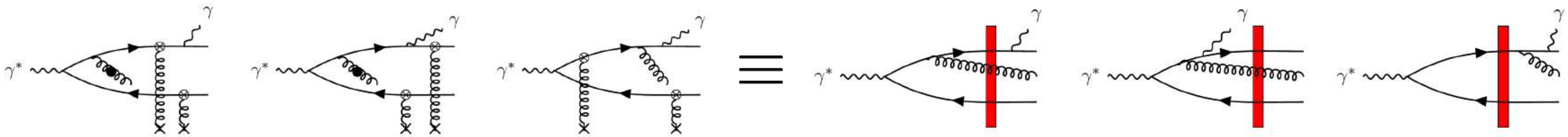
UGD satisfying BK equation:

$$\phi_{Y_A}(\boldsymbol{l}_{1\perp}) = \frac{2\pi N_c C_F g^2}{\boldsymbol{l}_{1\perp}^2}\int_{\boldsymbol{x}_\perp} \mu_A^2(Y_A, \boldsymbol{x}_\perp)$$

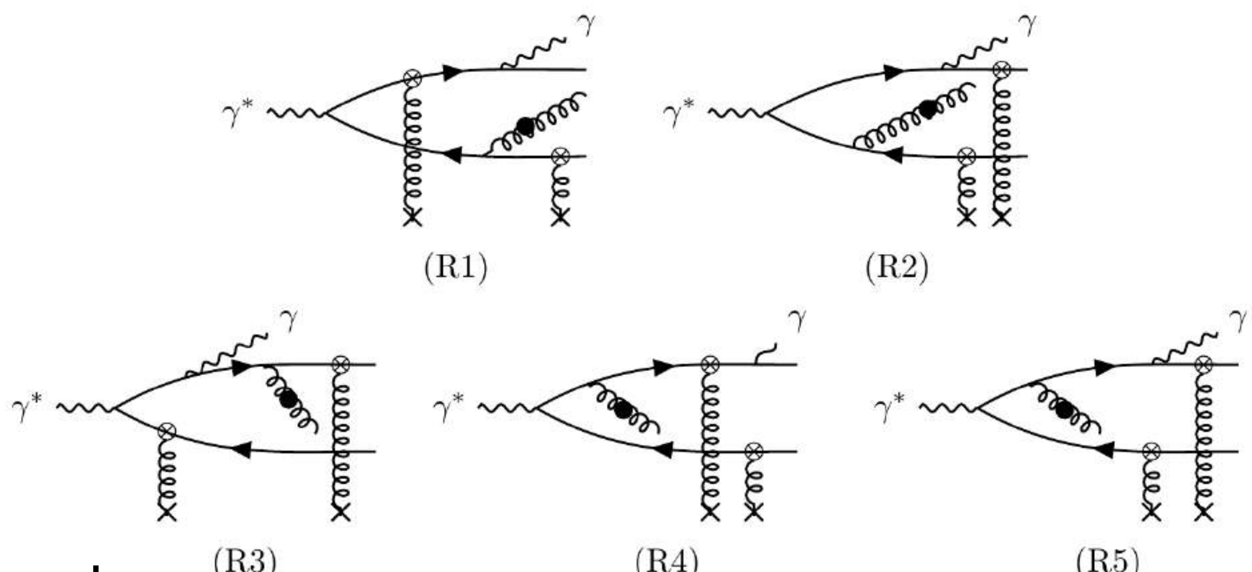
Modified Dirac trace:

$$\Theta_{\mu\nu}(\boldsymbol{P}_\perp) = \mathrm{Tr}\Big[(\not{k} + m)\big(T^{(\bar{q})}(\boldsymbol{P}_\perp) - T^{(q)}(\boldsymbol{P}_\perp)\big)_\nu{}^\alpha(m - \not{p})\hat{\gamma}^0\big(T^{(\bar{q})\,\dagger}(\boldsymbol{P}_\perp) - T^{(q)\,\dagger}(\boldsymbol{P}_\perp)\big)_{\mu\alpha}\hat{\gamma}^0\Big]$$

Equivalent diagrams in shockwave representation:



NLO Feynman graphs in entirety: KR, Venugopalan, in preparation



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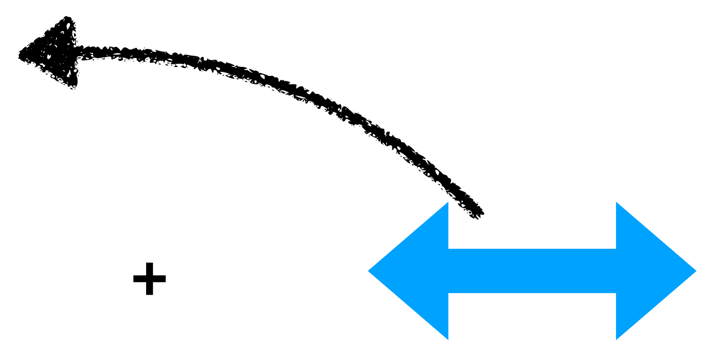
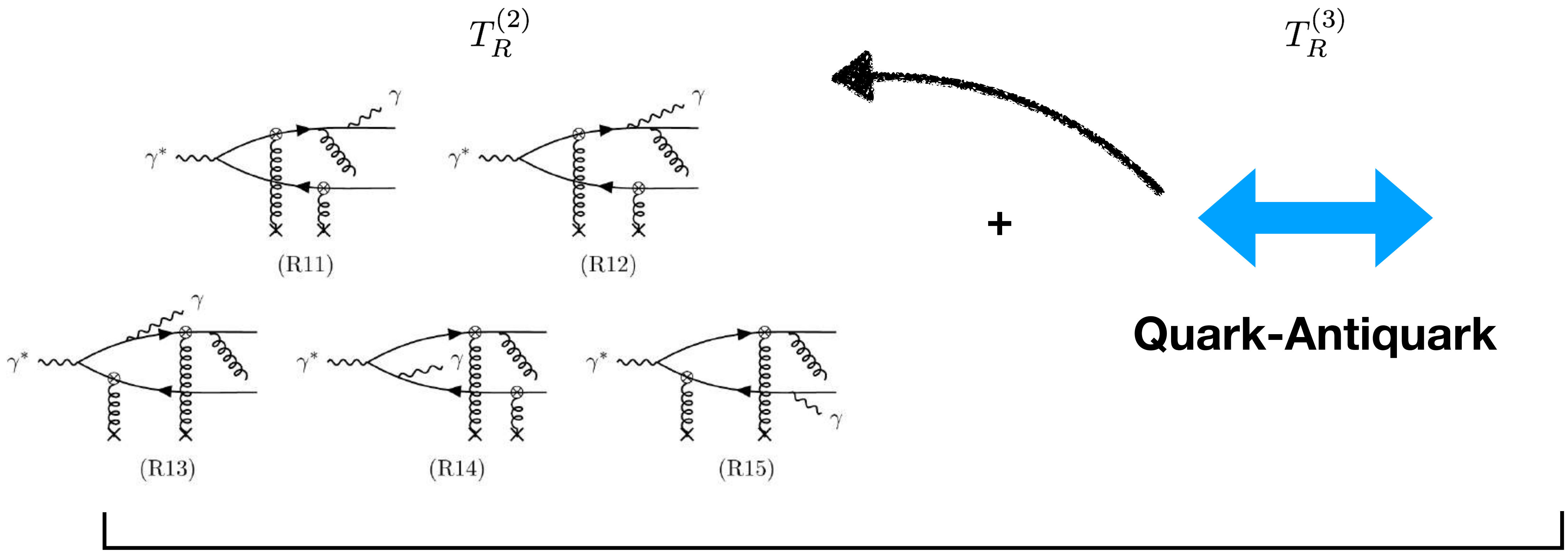


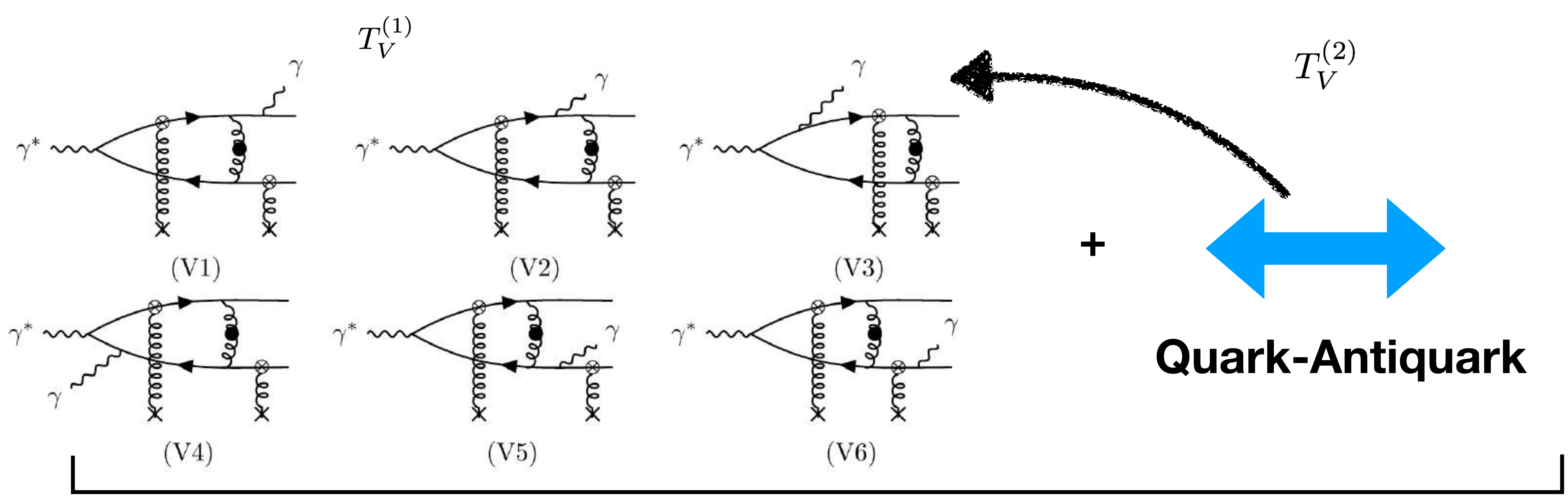
Quark-Antiquark

Set of 10 diagrams

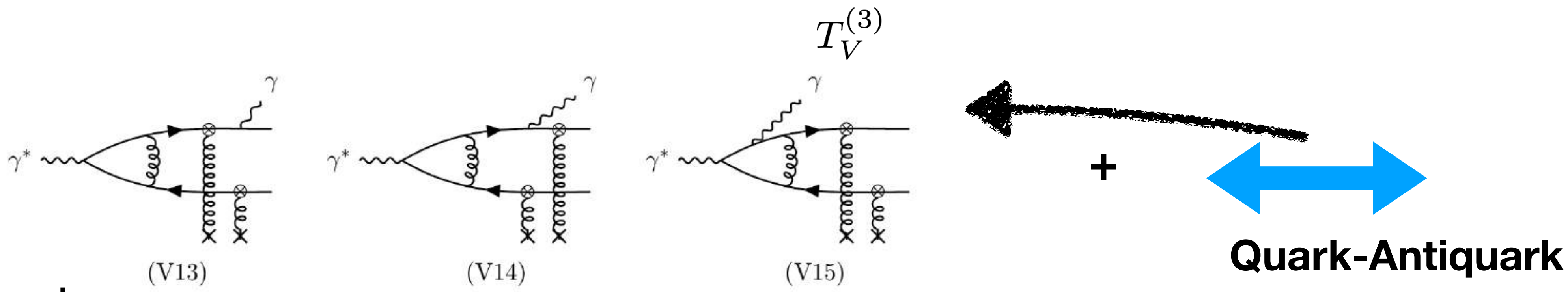
$$T_R^{(1)}$$



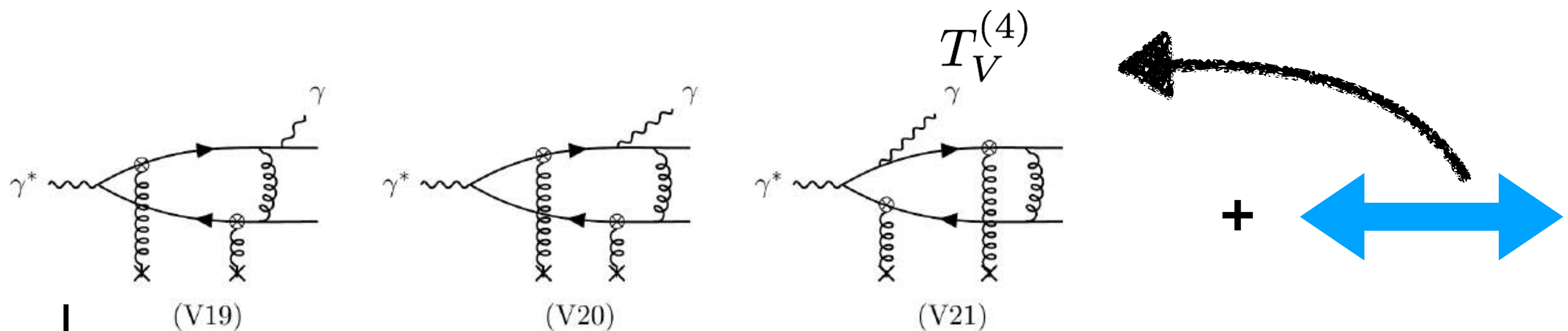
Set of 10 diagrams with 5 in each sub-category



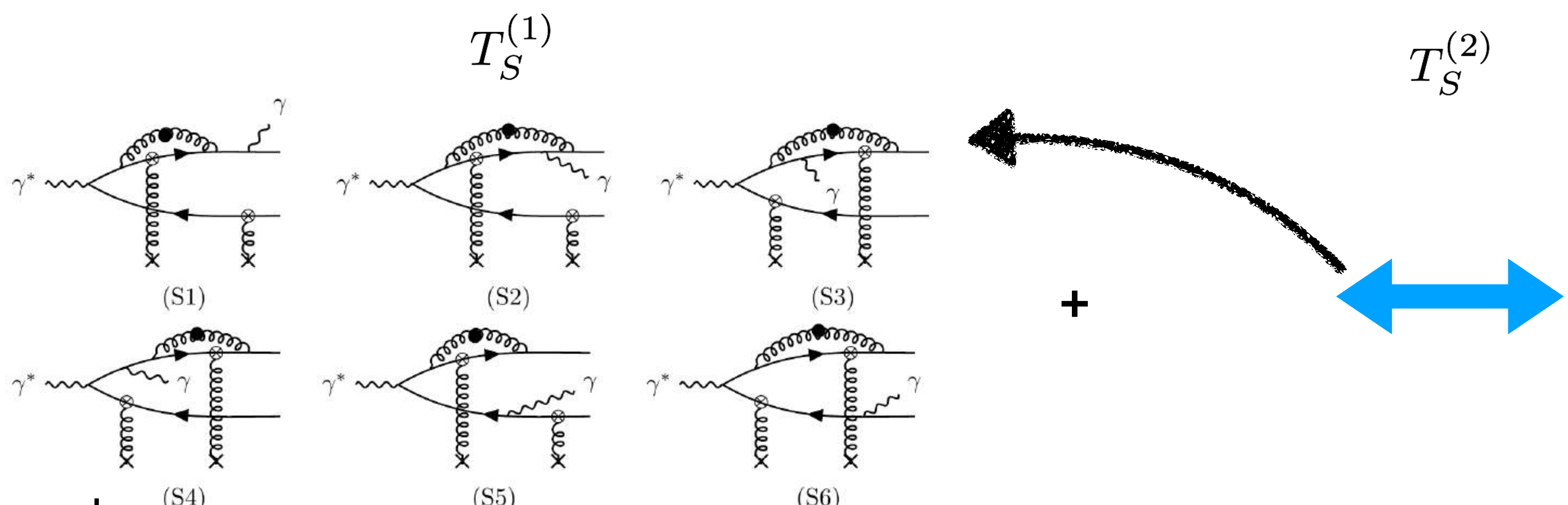
Set of 12 diagrams with 6 in each sub-category



Set of 6 diagrams

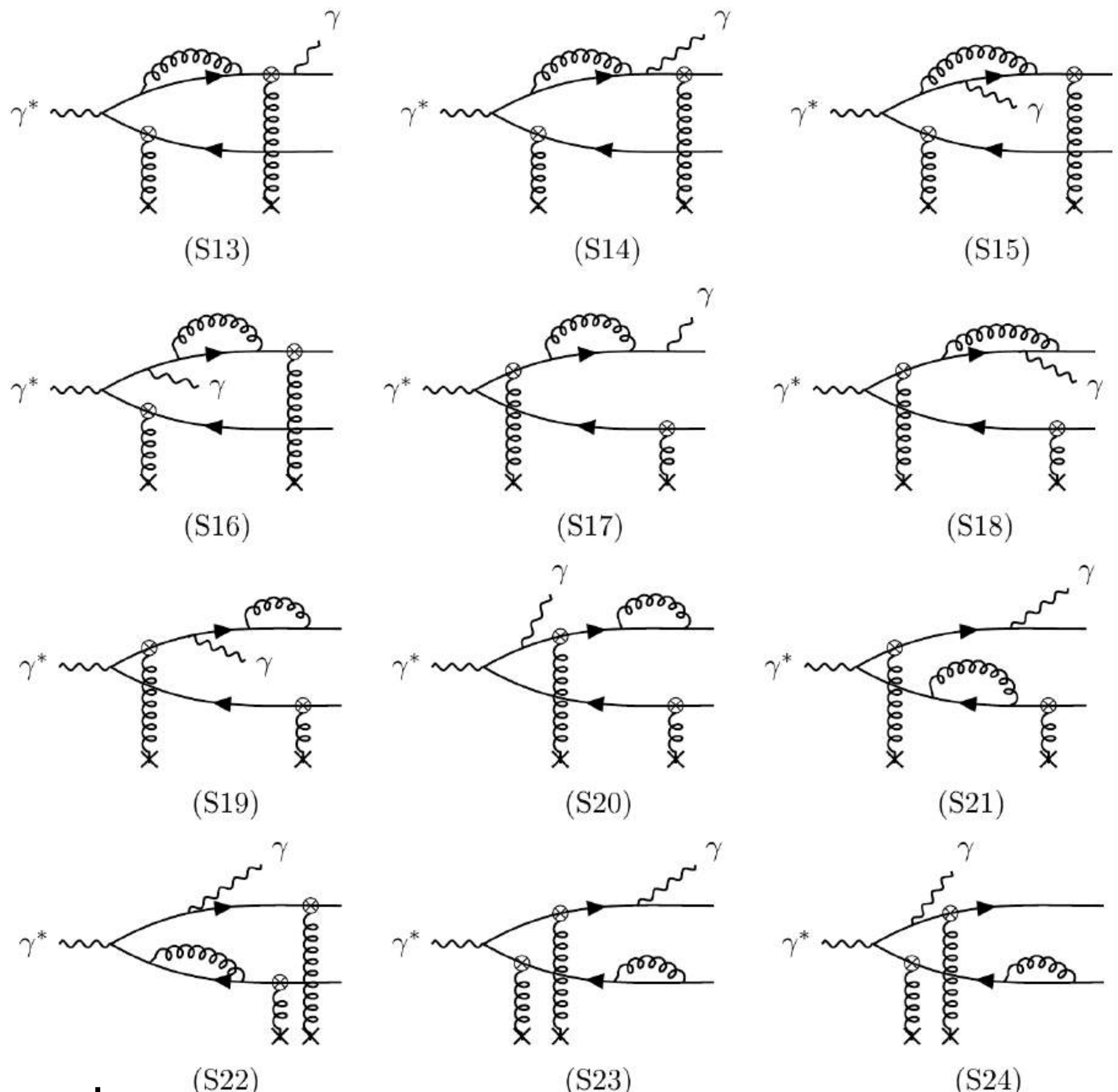


Set of 6 diagrams



Set of 12 diagrams with 6 in each sub-category

$$T_S^{(3)}$$



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Set of 24 diagrams