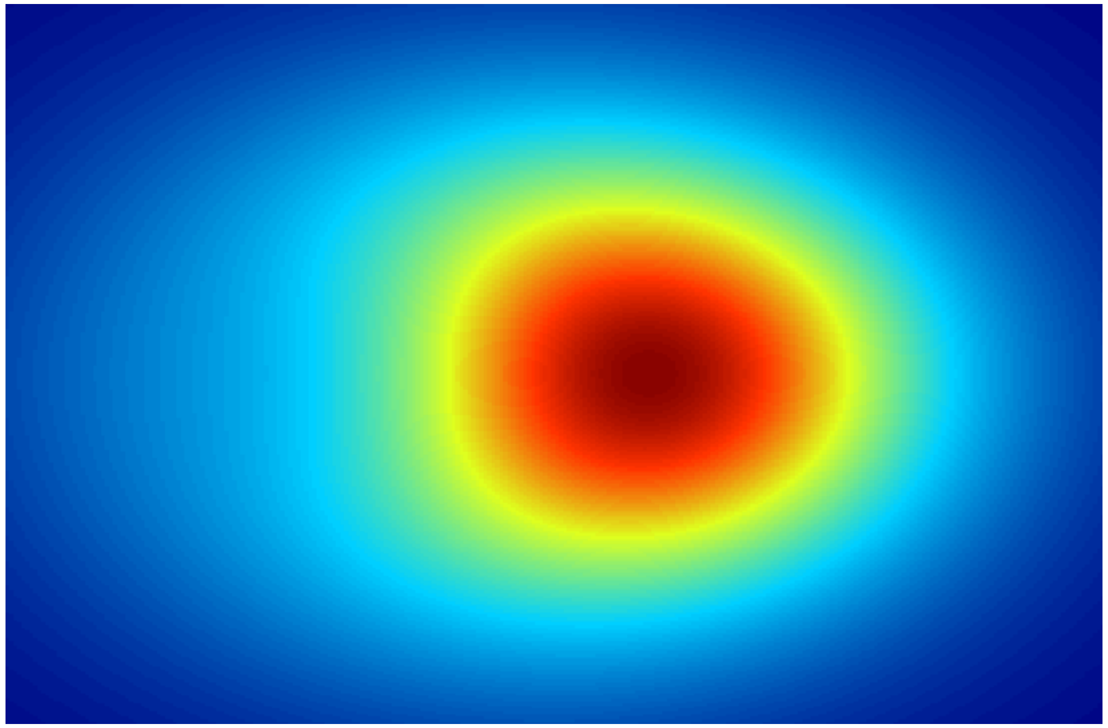
Phenomenology of TMDs

Alexei Prokudin

QCD Evolution 2018

PennState
Berks



The polarized proton in momentum space as "seen" by the virtual photon

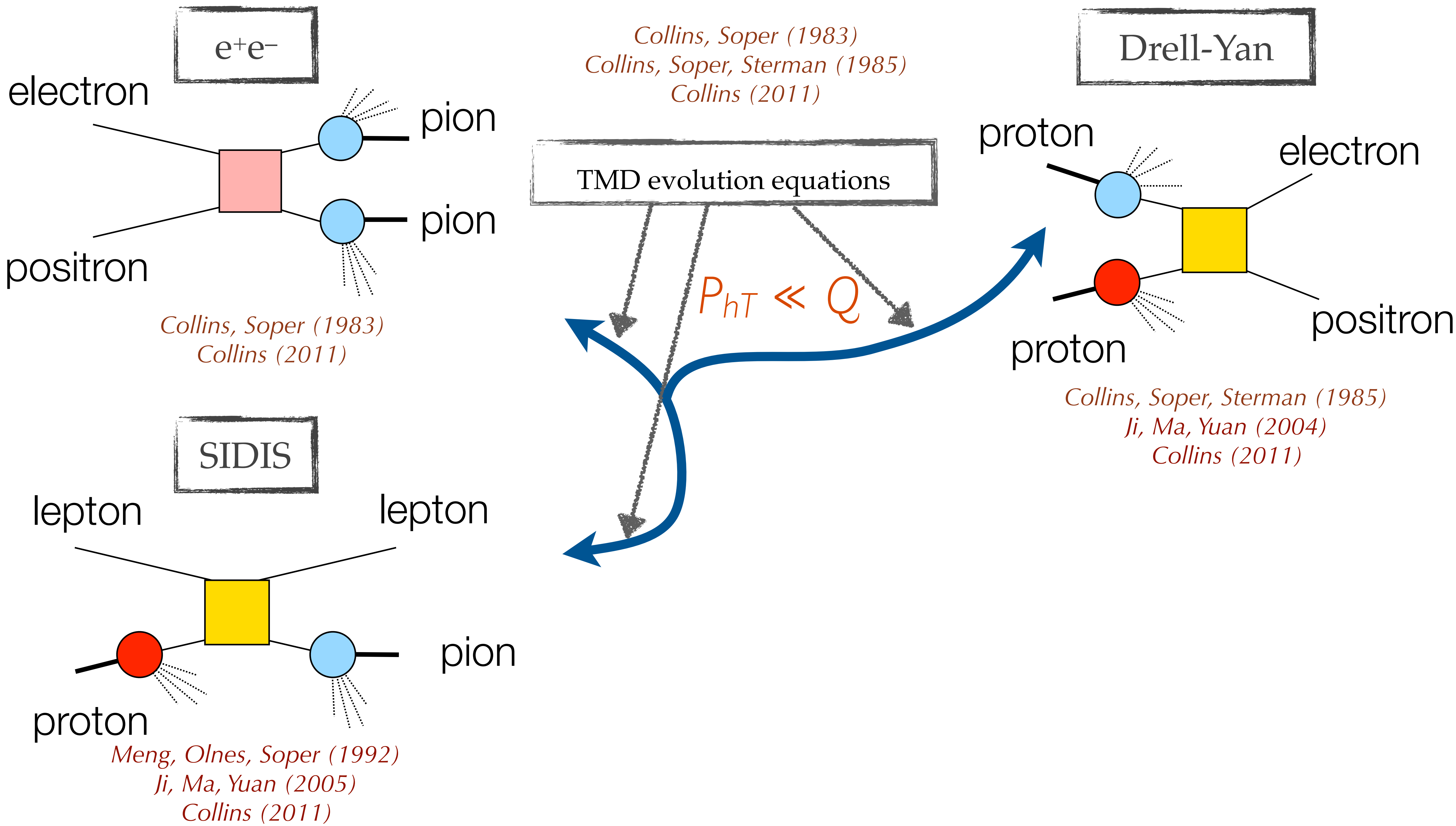
Factorization theorems help us to relate functions that describe the hadron structure and the experimental observables

Factorization is a *controllable approximation* and the goal of theorists and phenomenologists is to test and improve the region of applicability of factorization and/or construct new factorization theorems

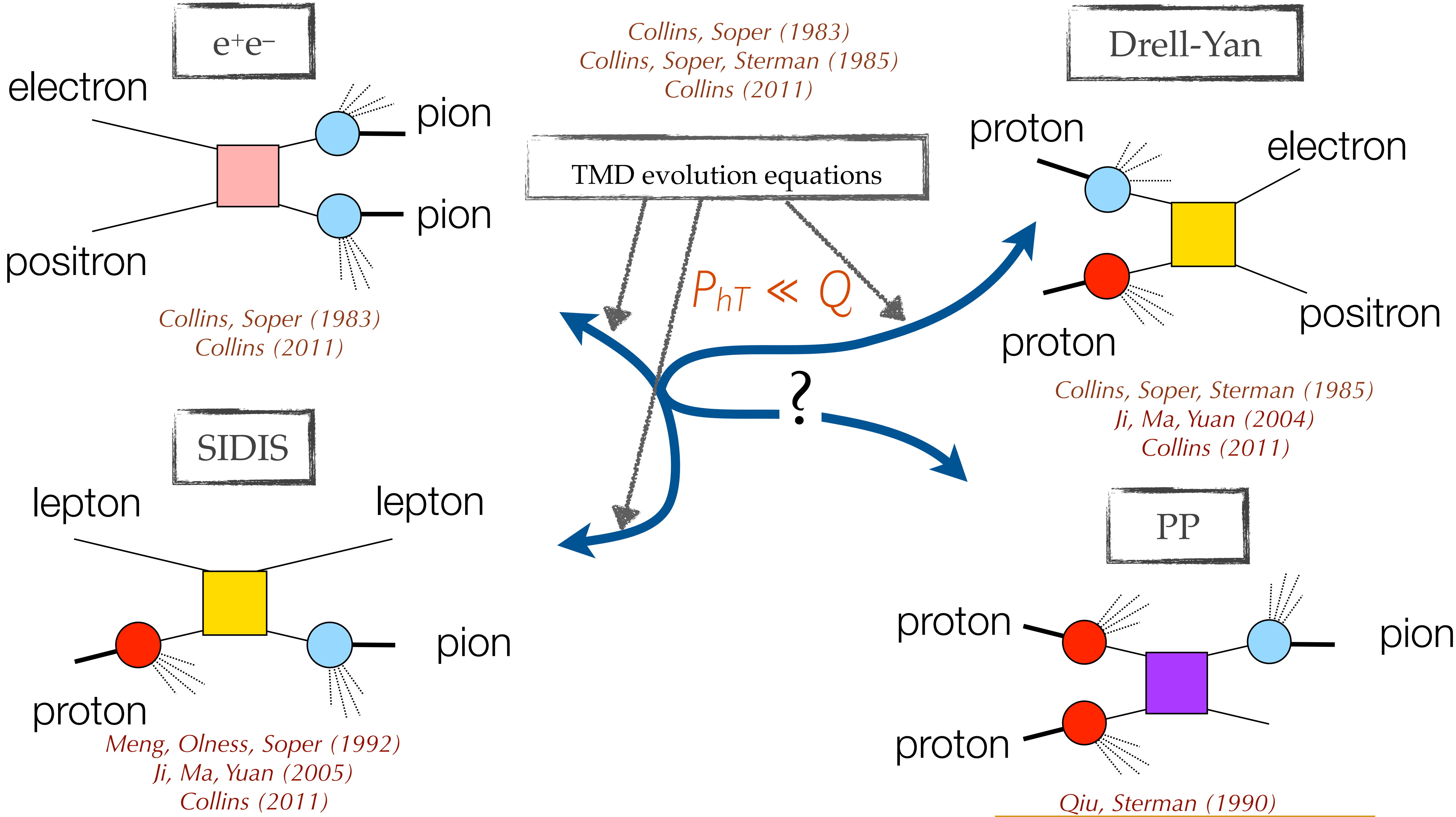
Hadron structure is the ultimate goal of measurements and phenomenology

The main goal of phenomenology now is to have a well defined methodology that allows to study hadron structure

TMD factorization

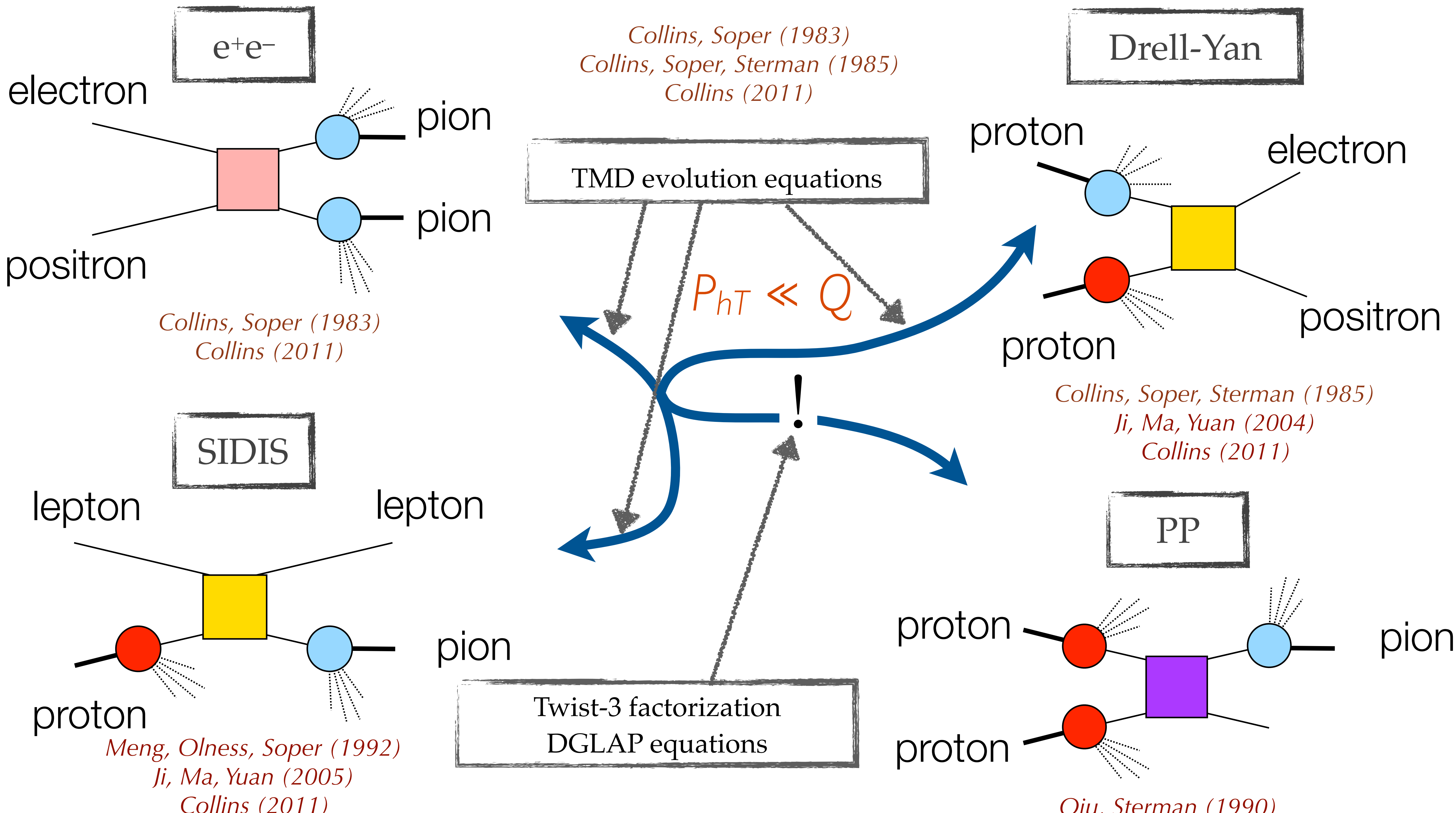


TMD factorization



Only one scale is measured in PP
TMD factorization is not applicable?

TMD factorization



- Twist-3 functions are related to TMD via OPE
- TMD and twist-3 factorizations are related in high QT region
- Global analysis of TMDs and twist-3 is possible:

All four processes can be used.

- Data are from HERMES, COMPASS, JLab,

BaBar, Belle, RHIC, LHC, Fermilab

Global fit is needed.
Work in progress

TMD distributions

Quark TMDs

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each depends on Bjorken-x, transverse momentum, the scale

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Quark TMDs

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This talk

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD Fragmentation Functions

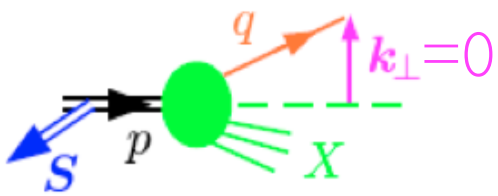
N \ q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$H_{1T}^\perp$	G_{1T}	H_1 $H_{1T}^\perp$

8 functions describing fragmentation of a quark into spin ½ hadron

This talk

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

Transversity:



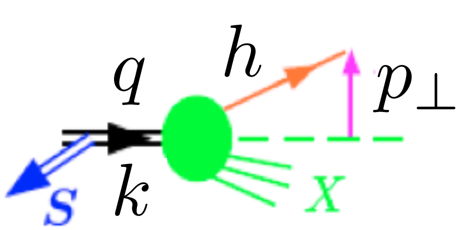
Ralston, Soper 1979

$$\Phi_{q/h}(x,P,S) = \frac{1}{2}\left(f_1(x)\,\not{P} + S_L g_1(x)\gamma_5\,\not{P} + \frac{1}{2} h_1(x)\gamma_5[\not{S}_T, \not{P}] \right)$$

Spin independent

Spin dependent

Collins function: unpolarized hadron from a transversely polarized quark



Collins 1992

$$D_{q/h}(z,\vec{p}_\perp,\vec{s}_q) = D_{q/h}(z,p_\perp^2) + \frac{1}{zM_h} H_1^{\perp q}(z,p_\perp^2)\,\vec{s}_q \cdot (\hat{k} \times \vec{p}_\perp)$$

Transversity: the source of information on tensor charge

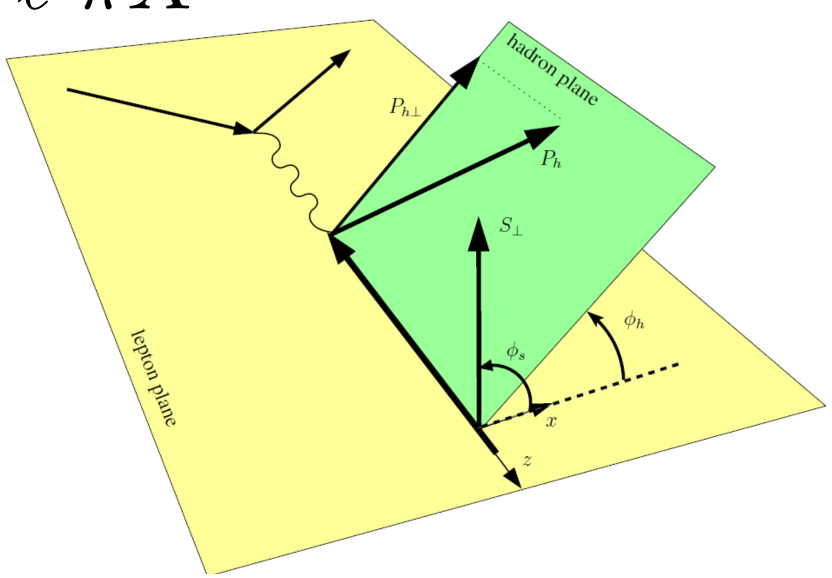
$$\delta q = \int_0^1 d\,x(h_1^q(x) - h_1^{\bar{q}}(x))$$

Collins function: $H_1^{\perp q}$ describes strength of correlation $\vec{s}_q \cdot (\hat{k} \times \vec{p}_\perp)$

Collins 1992

Both functions extensively studied experimentally, phenomenologically, theoretically

$$\ell P \rightarrow \ell' \pi X$$



Transversity and Collins function can give rise to Single Spin Asymmetries in scattering processes. For instance in Semi Inclusive Deep Inelastic process

Kotzinian (1995),
Mulders, Tangerman (1995)

$$d\sigma(S) \sim \sin(\phi_h + \phi_S) h_1 \otimes H_1^\perp$$

Collins function

Schafer-Teryaev sum rule

Schafer Teryaev 1999
Meissner, Metz, Pitonyak 2010

→ Conservation of transverse momentum

$$\langle P_T^i(z)\rangle \sim H_1^{\perp(1)}(z) \qquad H_1^{\perp(1)}(z) = \int d^2p_\perp \frac{p_\perp^2}{2z^2M_h^2} H_1^\perp(z,p_\perp^2)$$

→ Sum rule

$$\sum_h \int_0^1 dz \langle P_T^i(z)\rangle = 0$$

→ If only pions are considered $H_1^{\perp fav}(z) \sim -H_1^{\perp unf}(z)$

Universality of TMD fragmentation functions

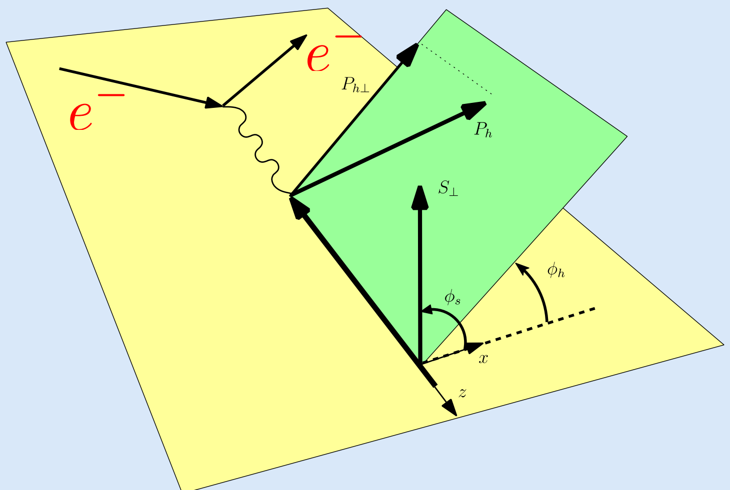
Metz 2002, Metz, Collins 2004, Yuan 2008
Gamberg, Mukherjee, Mulders 2011
Boer, Kang, Vogelsang, Yuan 2010

$$H_1^\perp(z)|_{SIDIS} = H_1^\perp(z)|_{e^+e^-} = H_1^\perp(z)|_{pp}$$

→ Very non trivial results

→ Agrees with phenomenology, allows global fits

- SIDIS and e+e-: combined global analysis

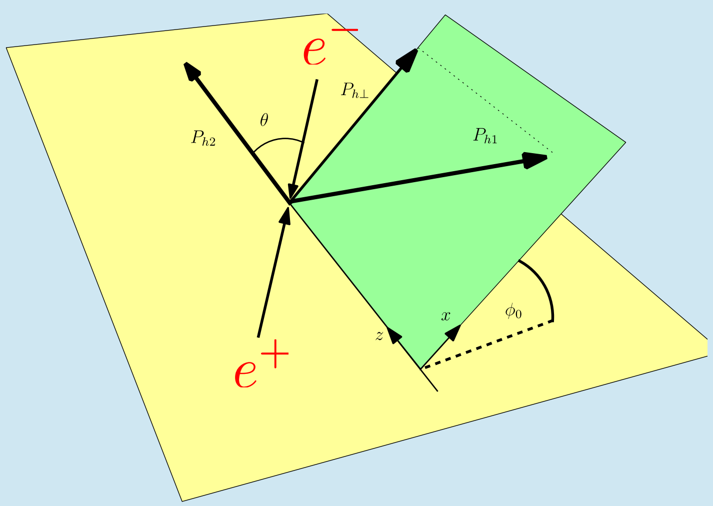


$$F_{UT}^{\sin(\phi_h+\phi_s)} \sim h_1(x_B,k_\perp)H_1^\perp(z_h,p_\perp)$$

transversity

Collins function

$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2P_{h\perp}} = \sigma_0(x_B,y,Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$



Boer, Jacob, Mulders (1997)

$$Z_{\text{collins}}^{h_1h_2} \sim H_1^\perp(z_1,p_{1\perp})H_1^\perp(z_2,p_{2\perp})$$

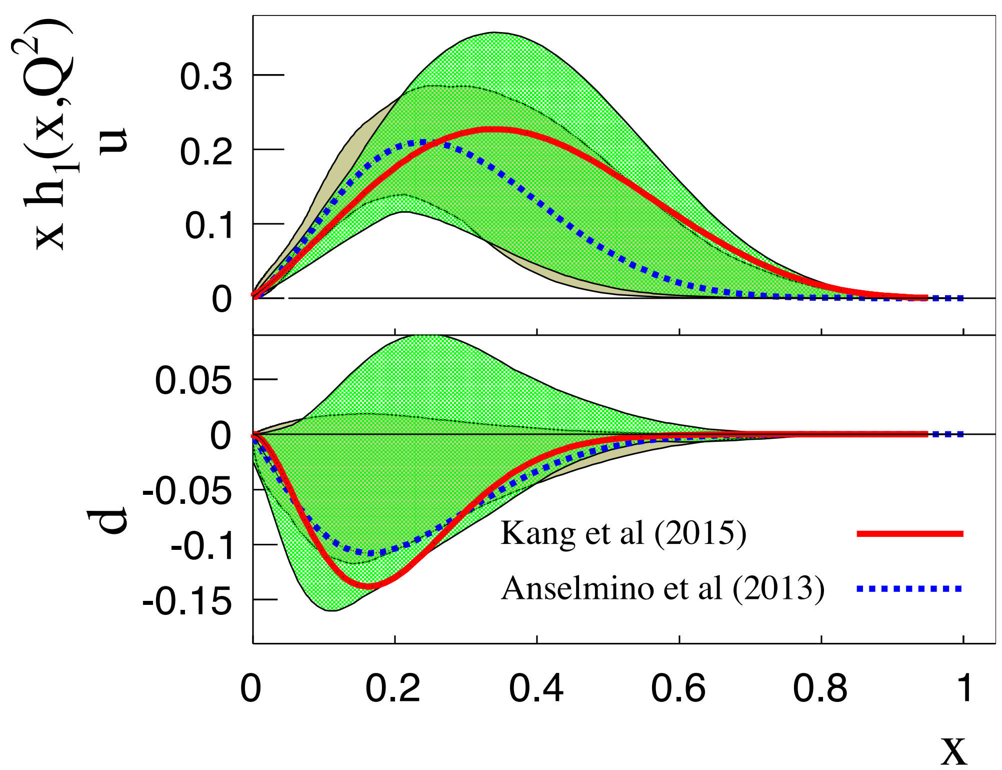
Collins function

Collins function

$$\frac{d\sigma^{e^+e^-\to h_1h_2+X}}{dz_{h1}dz_{h2}d^2P_{h\perp}\,d\cos\theta} = \frac{N_c\pi\alpha_{\text{em}}^2}{2Q^2}\left[\left(1+\cos^2\theta\right)Z_{uu}^{h_1h_2} + \sin^2\theta\cos(2\phi_0)Z_{\text{collins}}^{h_1h_2}\right]$$

Transversity from global fits

Single hadron, TMD

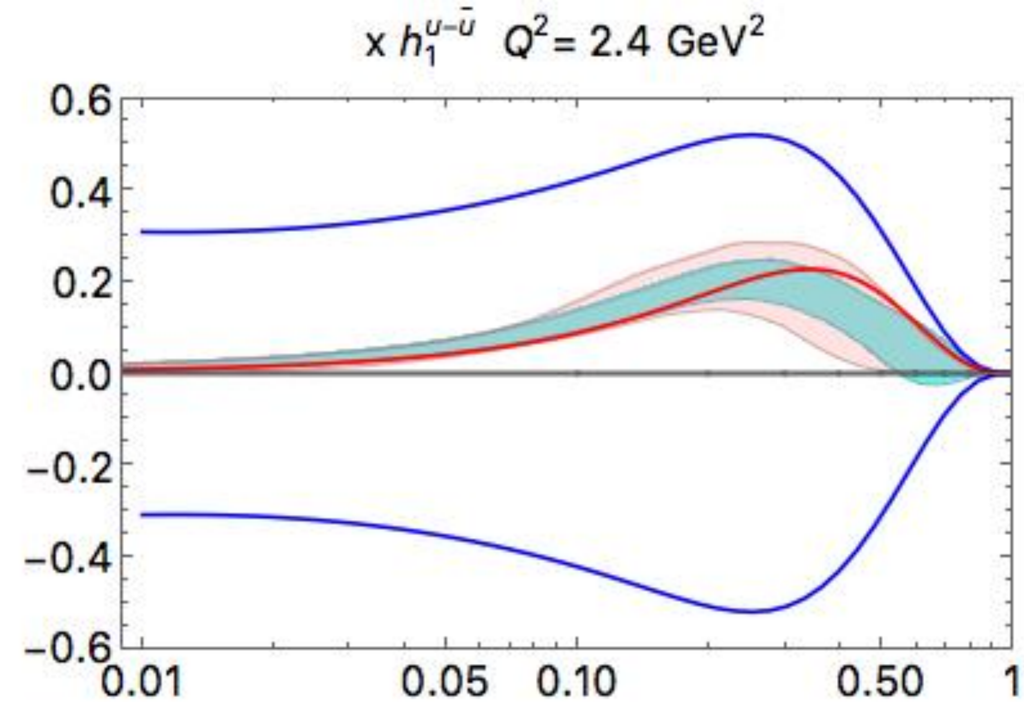


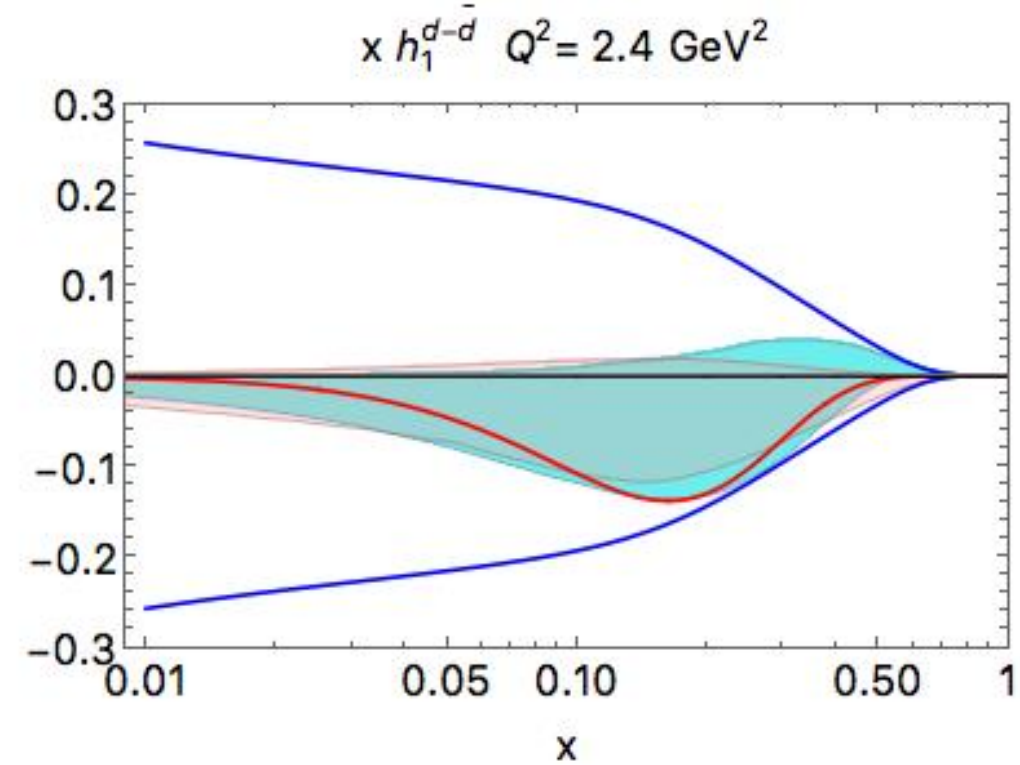
*Kang et al. (“**TMDfit**”), P.R. D**93** (16) 014009*

*Anselmino et al. (**Torino**), P.R. D**87** (13) 094019*

*Radici et al. (“**global fit**”) PRL 120 (18) n.19*

Di-hadron, collinear





up

global fit

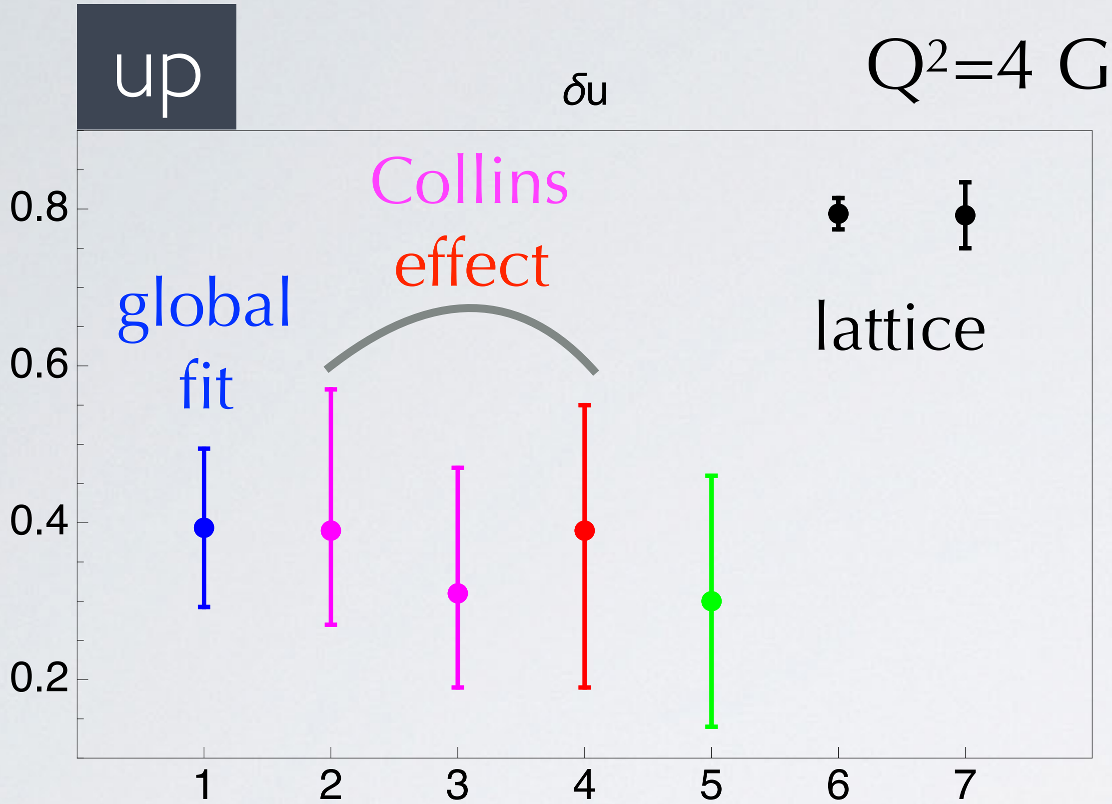
Torino

“TMDfit”

down

tensor charge $\delta q(Q^2) = \int dx\, h_1^{q-\bar{q}}(x, Q^2)$

Q^2=4 GeV^2 *



incompatibility for up

compatible for down
but with large errors
(except JAM)

1- global fit *Radici et al. PRL 120 (18) n.19*

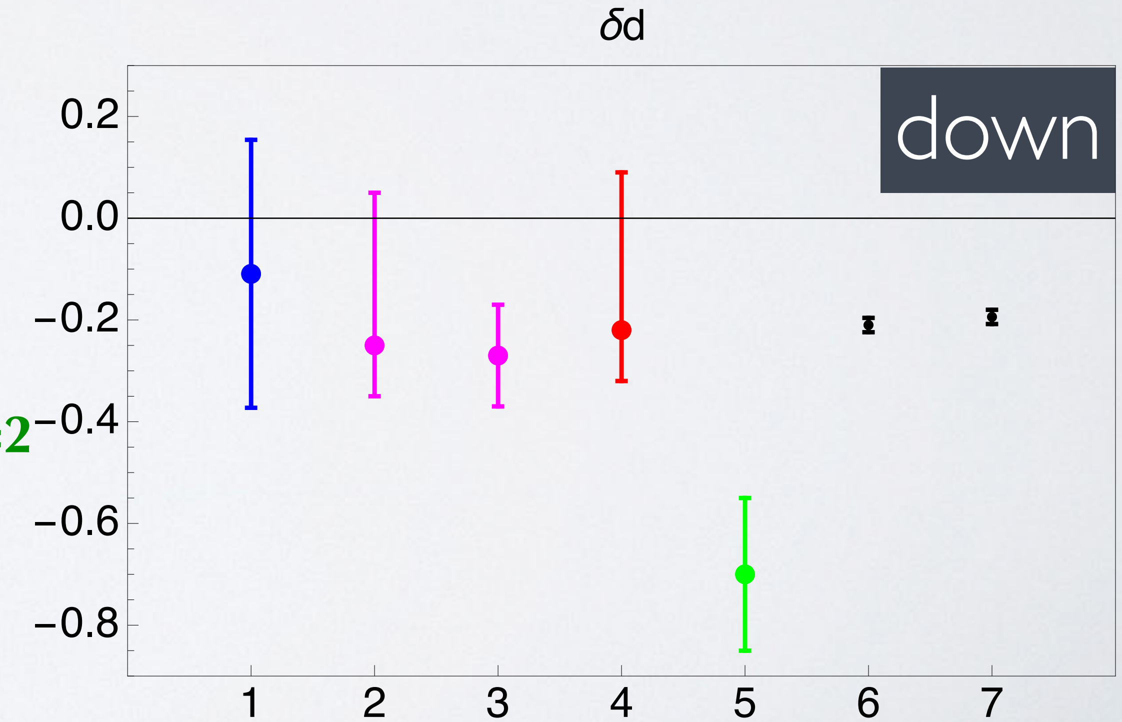
2,3- Torino *Anselmino et al., P.R. D**87** (13) 094019* * Q^2=1

4- TMD fit *Kang et al., P.R. D**93** (16) 014009* * Q^2=10

5- JAM fit *Lin et al., PRL 120 (18) n.15* { **Collins effect + lattice g_T=δu-δd** } * Q_0^2=2

6- ETMC17 *Alexandrou et al., P.R. D**95** (17) 114514; E P.R. D**96** (17) 099906*

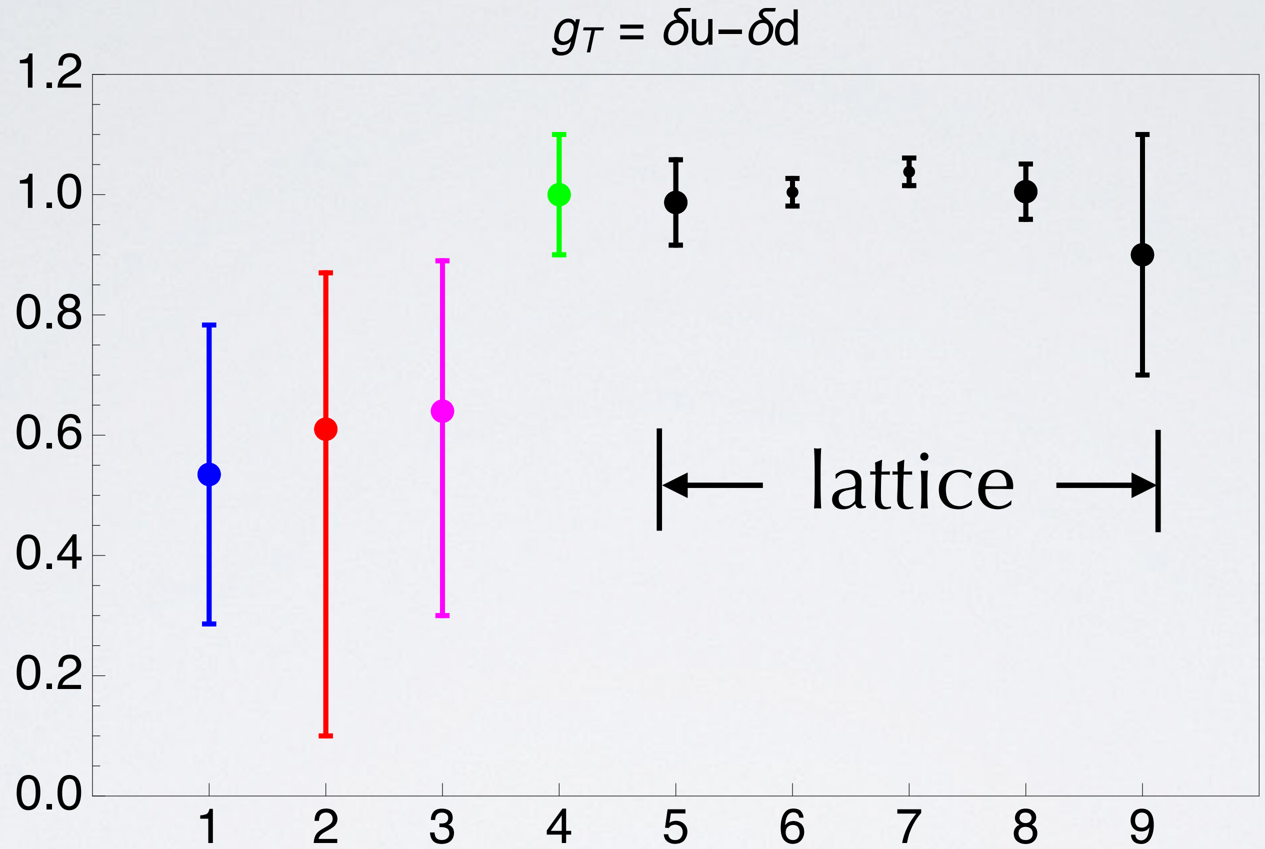
7- PNDME16 *Bhattacharya et al., P.R. D**94** (16) 054508*



Slide courtesy of M. Radici

isovector tensor charge $g_T = \delta u - \delta d$

Q^2=4 GeV^2 *



incompatibility (except JAM)

Radici et al. PRL 120 (18) n.19	**1) global fit '17**	5) PNDME '16	*Bhattacharya et al., P.R. D**94** (16) 054508*
*Kang et al., P.R. D**93** (16) 014009*	**2) "TMD fit" * Q^2=10**	6) ETMC '17	*Alexandrou et al., P.R. D**95** (17) 114514; E P.R. D**96** (17) 099906*
*Anselmino et al., P.R. D**87** (13) 094019*	**3) Torino fit * Q^2=1**	7) LHPC '12	*Green et al., P.R. D**86** (12)*
Lin et al., PRL 120 (18) n.15	**4) JAM fit '17 *Q_0^2=2**	8) RQCD '14	*Bali et al., P.R. D**91** (15)*
		9) RBC-UKQCD	*Aoki et al., P.R. D**82** (10)*

Slide courtesy of M. Radici

“transverse-spin puzzle” ?

there is no simultaneous compatibility
about δu, δd, $g_T = \delta u - \delta d$
between lattice and
phenomenological extractions
of transversity

Slide courtesy of M. Radici

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?

Slide courtesy of M. Radici

JAM fitting methodology

Jefferson Lab Angular Momentum Collaboration has developed a robust fitting methodology based on Bayesian statistical methods and machine learning algorithms

Such methodology may prove crucial and essential for our future endeavors in studies of the structure of the nucleon and beyond.

→ Expectation value and variance estimates:

$$E[\mathcal{O}] = \int d^n a \mathcal{P}(\vec{a}|data)\mathcal{O}(\vec{a}) \qquad V[\mathcal{O}] = \int d^n a \, \mathcal{P}(\vec{a}|data)[\mathcal{O}(\vec{a}) - E[\mathcal{O}]]^2$$

→ Bayes' theorem defines probability density $\mathcal{P}$ as

$$\mathcal{P}(\vec{a}|data) = \frac{1}{Z}\mathcal{L}(\vec{a}|data)\, \pi(\vec{a})$$

Evidence

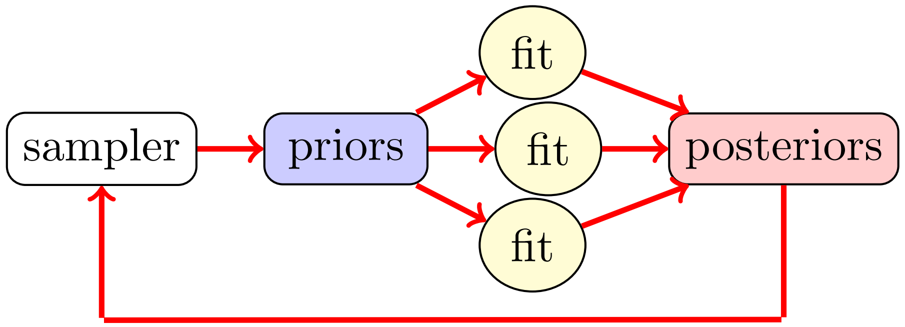
Likelihood function

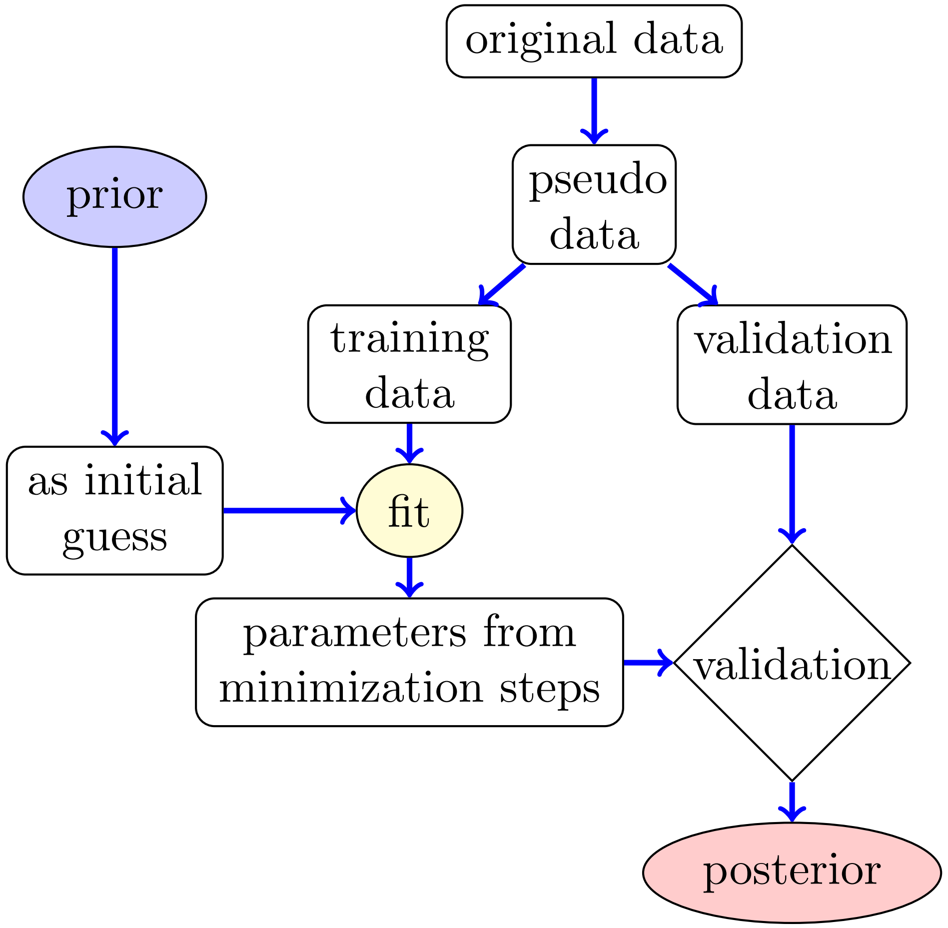
Prior

$$Z = \int d^n a \, \mathcal{L}(\vec{a}|data)\, \pi(\vec{a}) \qquad \mathcal{L}(\vec{a}|data) = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$$

See talk of Jake Ethier at Light Cone 18

JAM fitting methodology





Iterative Monte Carlo is then used to perform the fit

Large parameter space is sampled

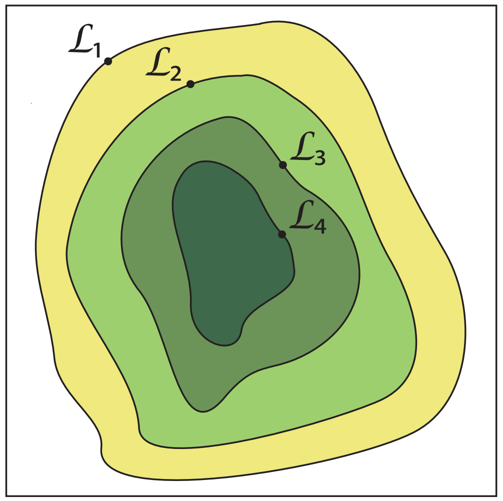
Data is partitioned in validation and training sets

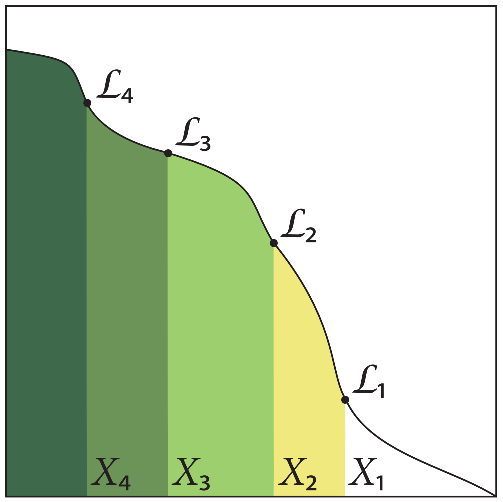
Training set is fitted via chi-square minimization

Posteriors are used to feed the next iterations

See talk of Jake Ethier at Light Cone 18

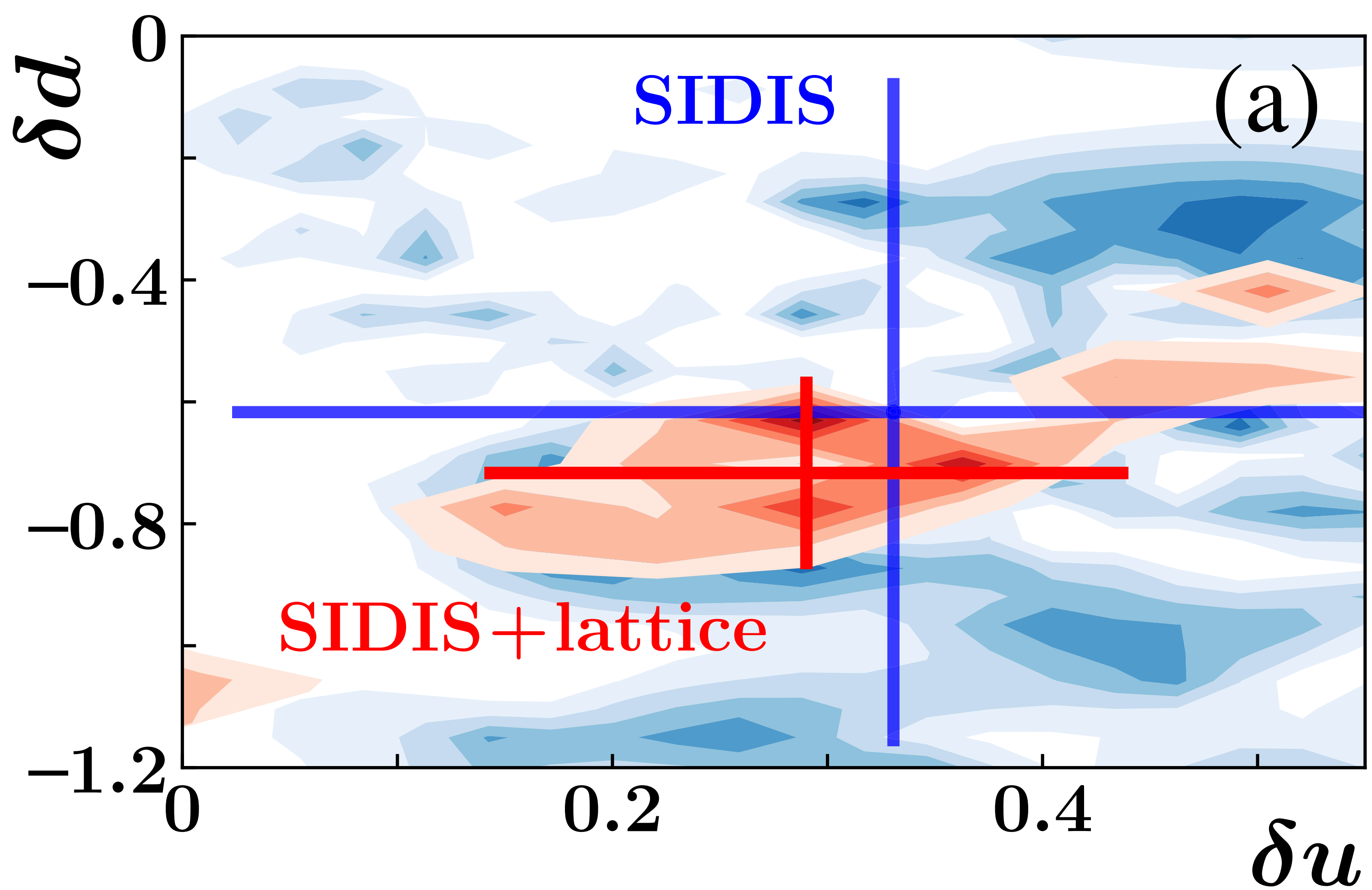
JAM fitting methodology





Nested sampling is essential to map multidimensional integral to 1-D integral

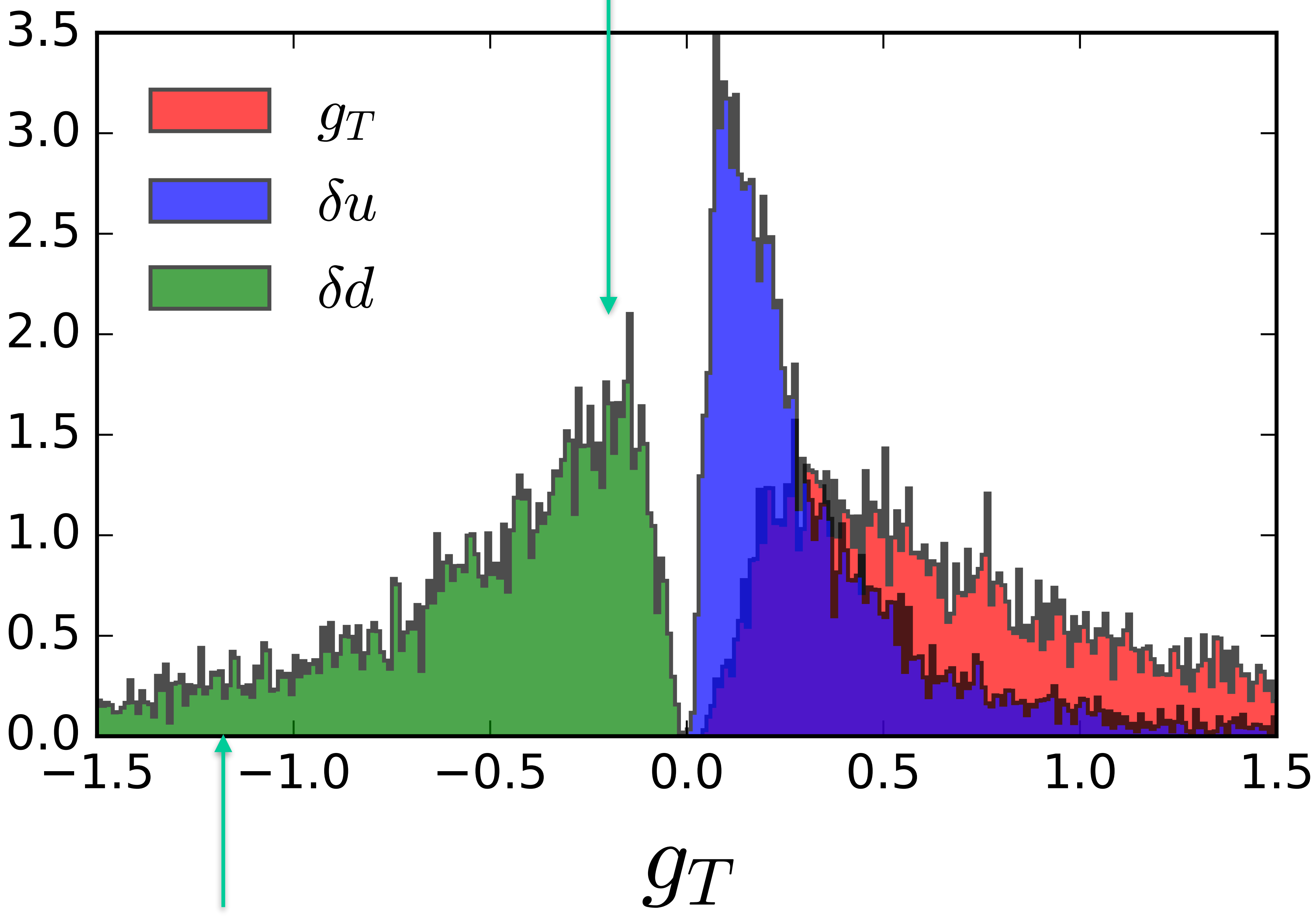
$$Z = \int d^n a \, \mathcal{L}(\vec{a}|data) \, \pi(\vec{a}) = \int_0^1 dX \mathcal{L}(X)$$



First combined new methodology fit of SIDIS data using lattice constraints: Lattice and SIDIS data are compatible and including lattice data improves extraction of g_T

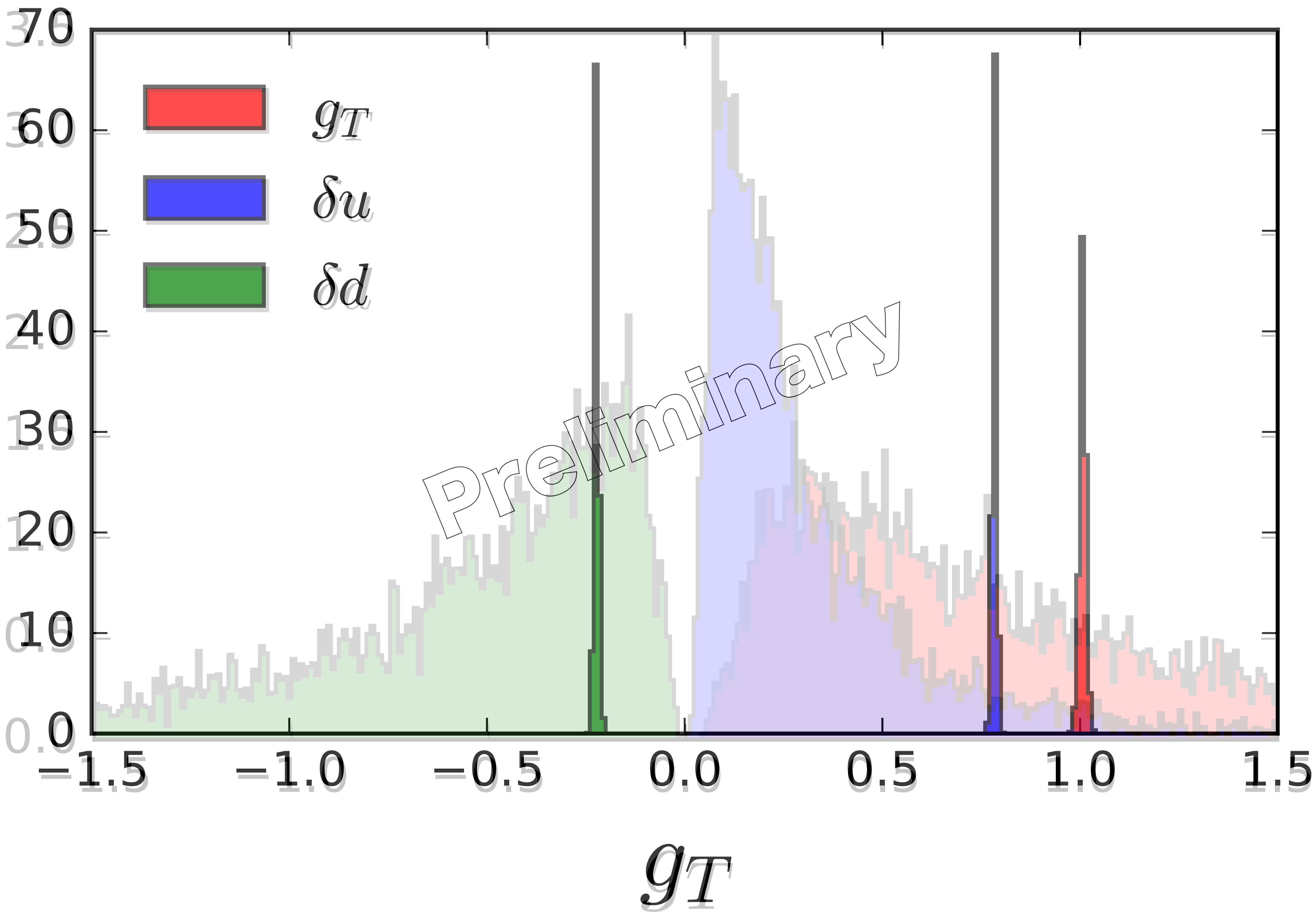
Lin, Melnitchouk, AP, Sato, PRL 120 (18) n.15

Peaks correspond to "single fit results"



The tails of distributions are very wide, usual methods would not give reliable errors

Simultaneous fit of SIDIS, e^+e^- and lattice δu, δd, $g_T=\delta u-\delta d$



First combined fit of
SIDIS data: HERMES, COMPASS, JLab
e+e- data: Belle and BaBar
and lattice results for δu, δd, g_T
Alexandrou et al., P.R. D**95** (17) 114514

Lattice:
$\delta u = 0.782 \pm 0.021$
$\delta d = -0.219 \pm 0.017$
$g_T = 1.004 \pm 0.021$

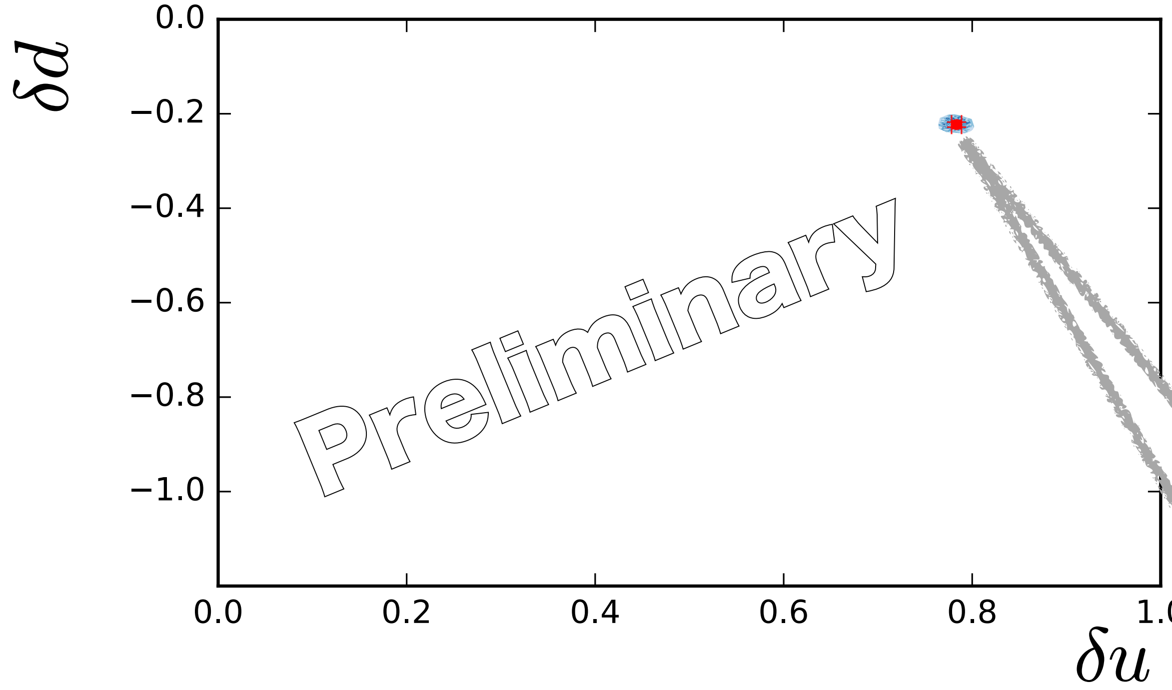
Courtesy of M. Constantinou

After the fit:
$\delta u = 0.79 \pm 0.01$
$\delta d = -0.22 \pm 0.01$
$g_T = 1.01 \pm 0.01$

Melnitchouk, AP, Sato, 2018

PennState Berks

Simultaneous fit of SIDIS, e^+e^- and lattice **δ**u, **δ**d, g_T=**δ**u-**δ**d



Melnitchouk, AP, Sato, 2018

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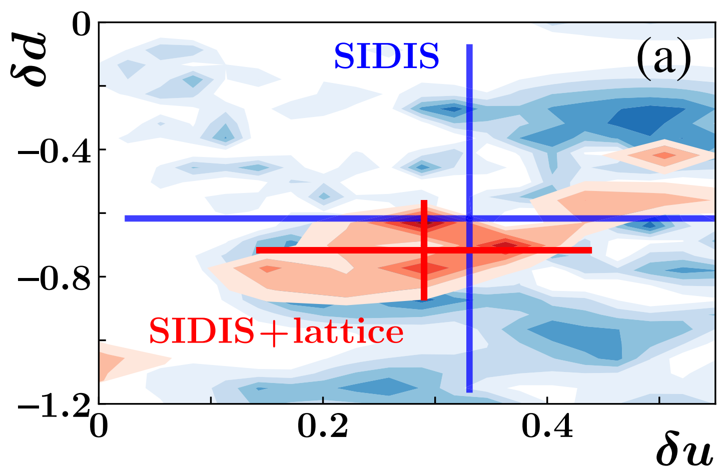
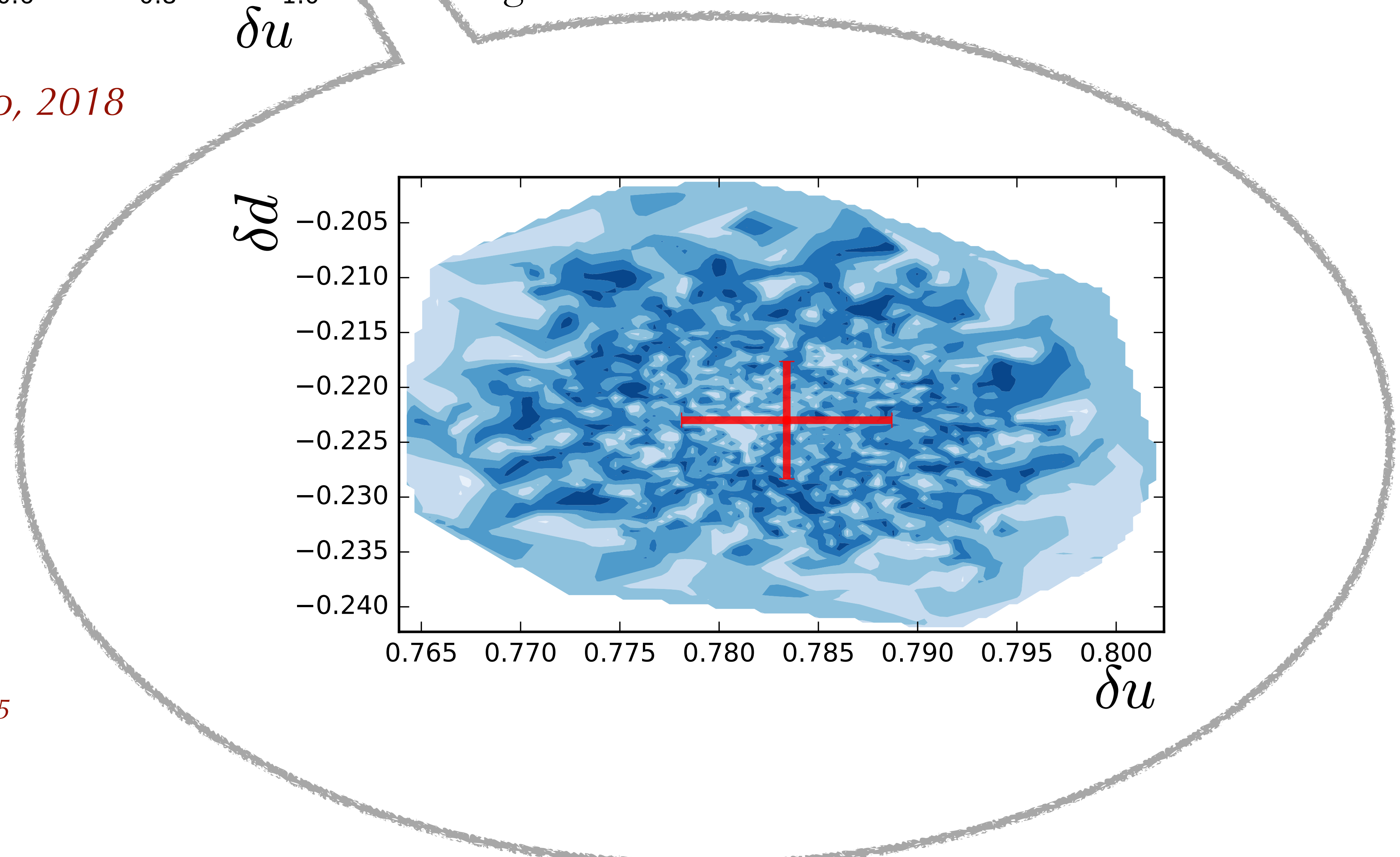


Lin, Melnitchouk, AP, Sato, PRL 120 (18) n.15



Complementarity of SIDIS, e+e- and Drell-Yan, and hadron-hadron

Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them

$$f(x) \otimes D(z)$$

Semi Inclusive DIS – convolution of distribution functions and fragmentation functions

$$\ell + P \to \ell' + h + X$$

$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

$$P_1 + P_2 \to \bar{\ell}\ell + X$$

$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \to h_1 + h_2 + X$$

$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and fragmentation functions

$$h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

Last but not least: Lattice QCD can also provide valuable input for our analysis!

- Robust methodology may prove crucial and essential for future endeavors in studies of the structure of the nucleon and beyond.

- A dedicated effort for sharpening our tools is needed and is provided in part by TMD Collaboration, Jefferson Lab and other labs, and by NSF and DOE grants.

- We plan to develop a comprehensive framework that will be available for the nuclear physics community and could be used by other groups in the USA and abroad.

- The framework will include machine learning techniques, sharing via open source platforms such as GitHub, and flexible Python implementation via Jupyter notebooks.

- Many people involved:

N. Sato, W. Melnitchouk, J. Ethier, A. Signori, T. Liu, J. Terry, Z. Kang, A. Metz, L. Gamberg, AP, D. Pitonyak, M. Albright, J. Qiu, A. Vladimirov, I. Scimemi, K. Tezgin, D. Riser, …

- Join us if you are interested!

- New methods allow to resolve "transverse spin puzzle" and show consistency of the experimental data on Collins asymmetries in SIDIS and e^+e^- with lattice computations of tensor charge.