

# QCD EVOLUTION

## Workshop

May 20 - 24, 2018  
Santa Fe, NM,  
Drury Plaza Hotel

### TOPICS:

Hadron structure:  
theory & experiment

Transverse momentum  
dependent distributions

Generalized parton distributions

Effective theories, SCET

Lattice QCD

Multi-parton interactions

Resummation techniques

Nuclear effects, small-x

### Organizing Committee

Ian Balitsky	Alexei Prokudin
Martha Constantinou	Anatoly Radyushkin
Leonard Gamberg	Matt Sievert
Chris Lee	Ivan Vitev (Chair)
Duff Neill	Shinsuke Yoshida

qcdevolution2018@lanl.gov  
[www.jlab.org/conferences/qcd-evolution2018](http://www.jlab.org/conferences/qcd-evolution2018)

**Sponsors:** DOE Office of Science  
DOE Early Career Program  
Jefferson Lab  
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Penn State University Berks

# First extraction of Transversity from a global analysis of lepton-hadron scattering and hadronic collisions



**Marco Radici**  
INFN - Pavia

in collaboration with  
A. Bacchetta (Univ. Pavia)



based on  
P.R.L. **120** (2018) 192001  
arXiv:1802.05212



**plus updates**

# a phase transition

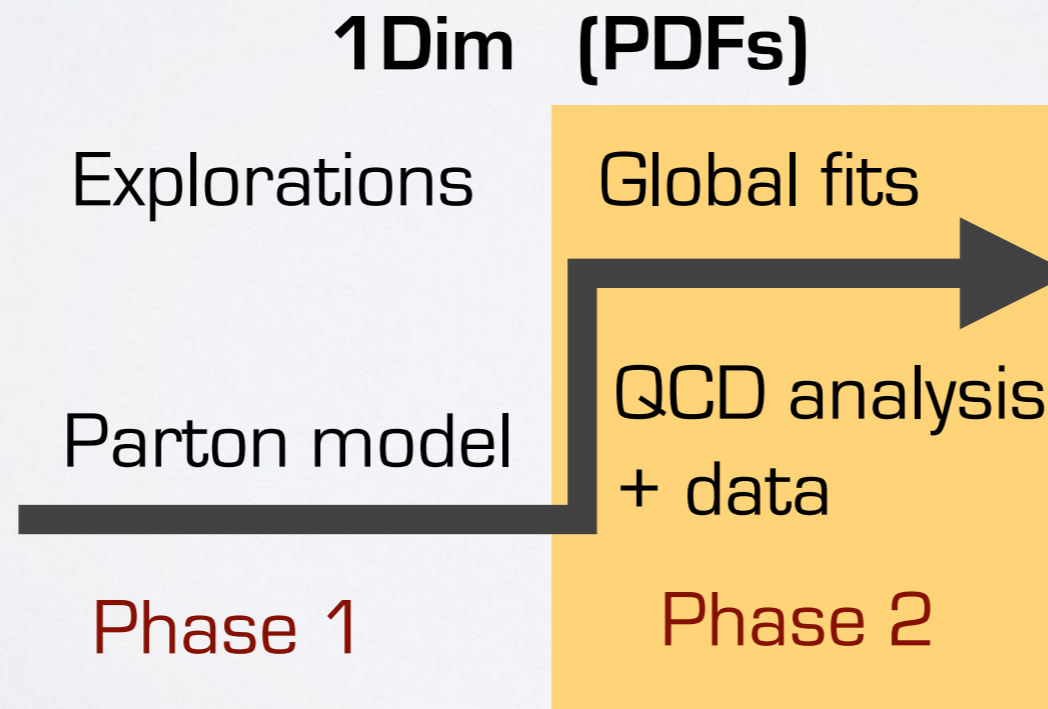
quark polarization

	U	L	<b>T</b>
U	<b>f<sub>1</sub></b>		$h_1^\perp$
L		<b>g<sub>1L</sub></b>	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	<b>h<sub>1</sub></b> $h_{1T}^\perp$

nucleon polarization

chiral-odd → **SIDIS**

first global fit  
 (= lepton-hadron scatt.  
 and hadron collisions)  
 of **PDF h<sub>1</sub>**



# 2-hadron-inclusive production

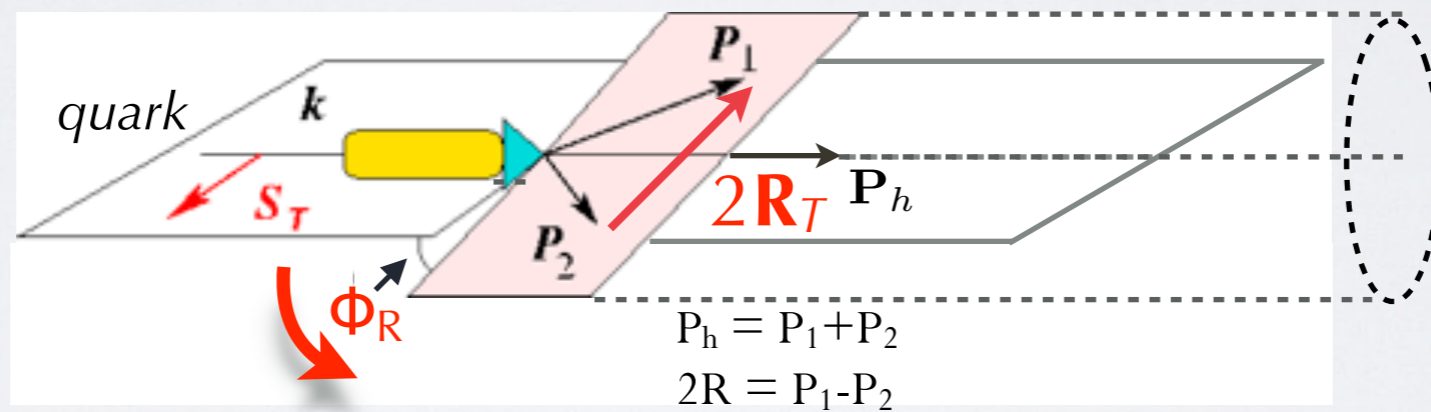
Collins, Heppelman, Ladinsky,  
N.P. **B420** (94)

$$R_T \ll Q$$

$$H_1^{\triangleleft}$$

$$M_h$$

invariant mass



correlation  $S_T$  and  $R_T \rightarrow$  **azimuthal asymmetry**

# 2-hadron-inclusive production

framework  
collinear  
factorization

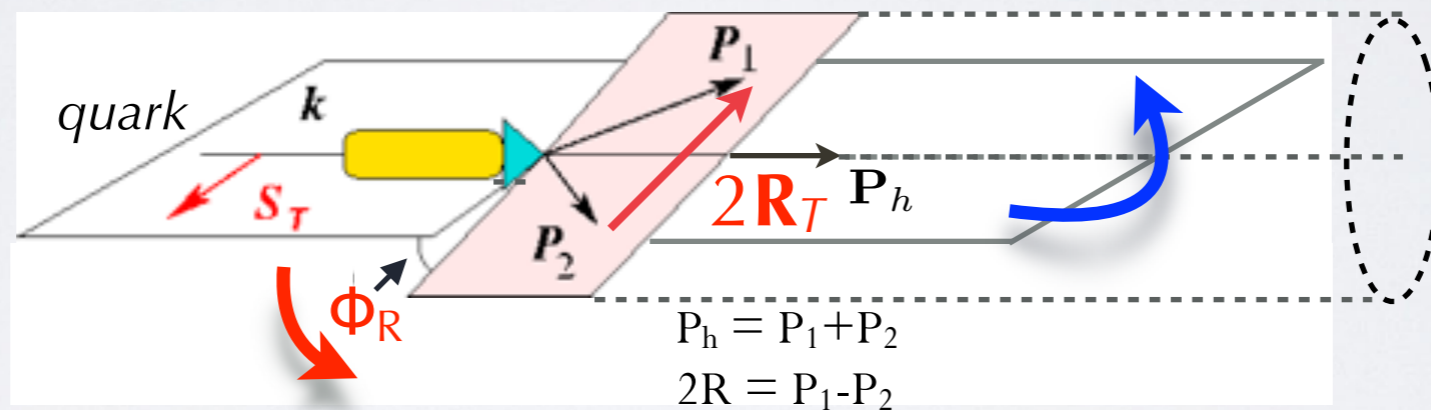
Collins, Heppelman, Ladinsky,  
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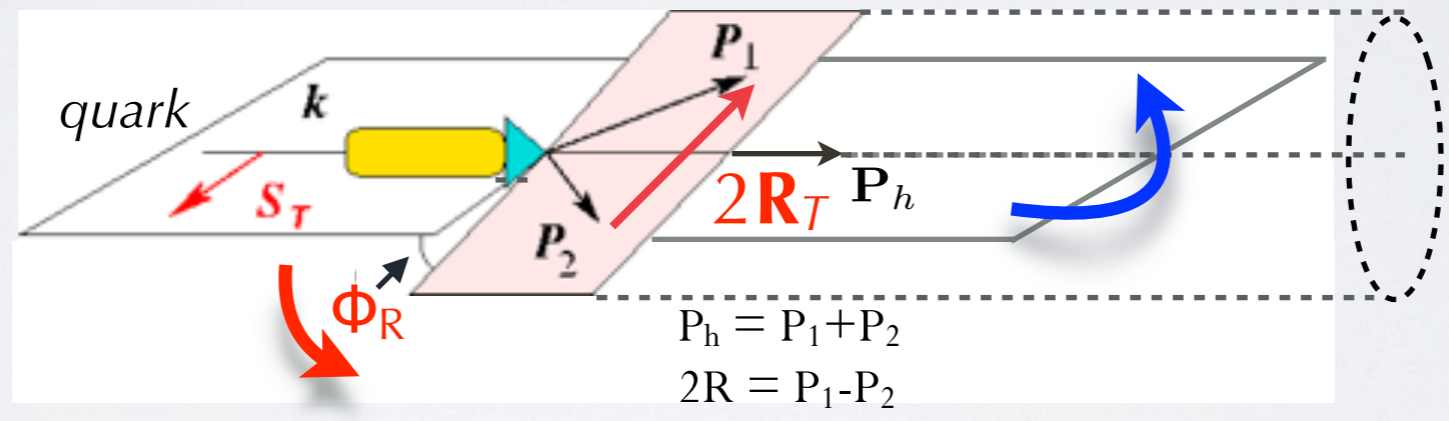
invariant mass



survives to  
**polar  
symmetry**  
(  $\int dP_{hT}$  )

correlation  $S_T$  and  $R_T \rightarrow$  **azimuthal asymmetry**

# 2-hadron-inclusive production



survives to polar symmetry  
 (  $\int dP_{hT}$  )

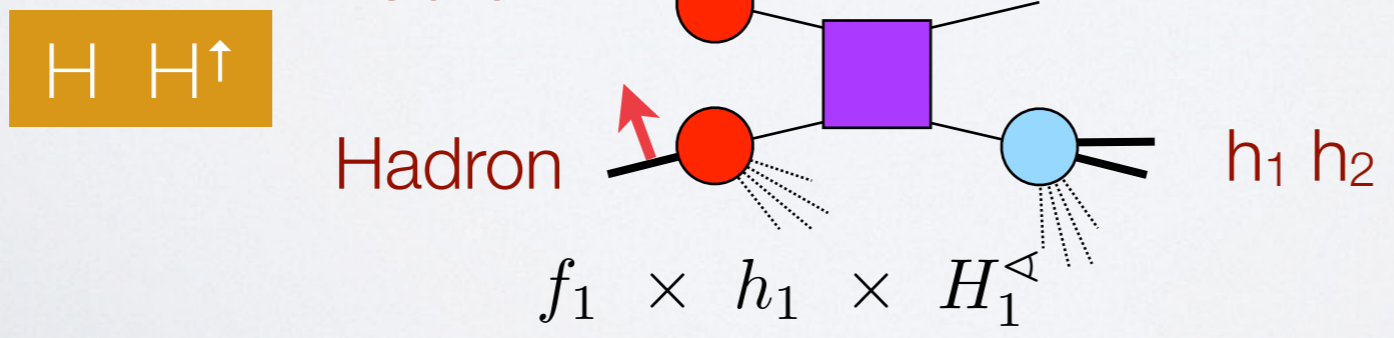
$R_T \ll Q$

$H_1^{\Delta}$

$M_h$

correlation  $S_T$  and  $R_T \rightarrow$  azimuthal asymmetry

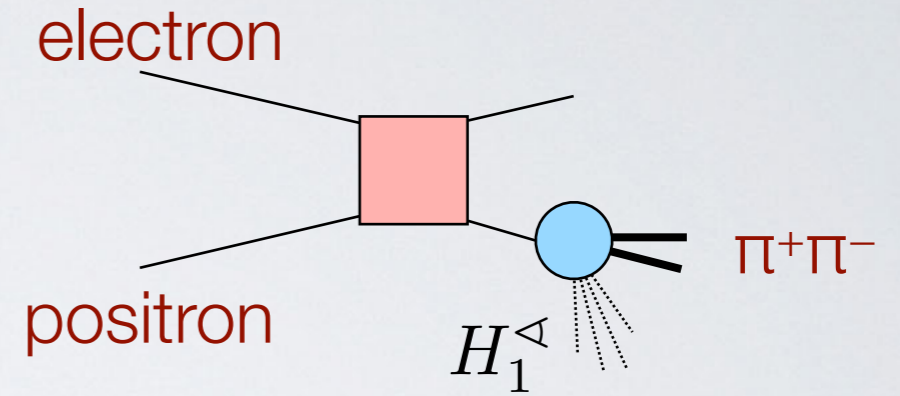
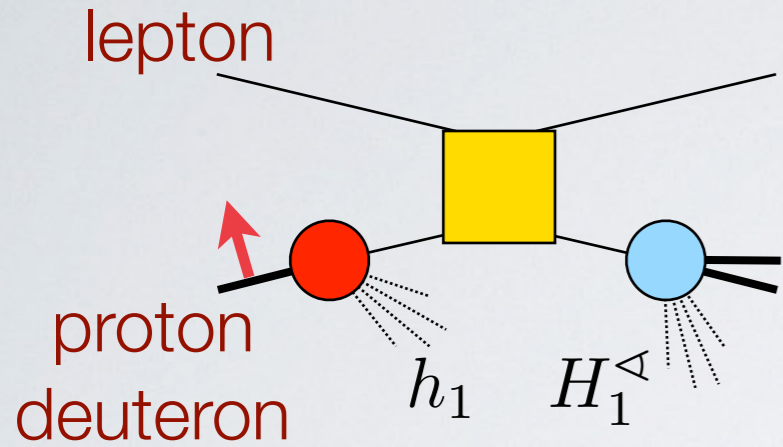
invariant mass



# take-away message

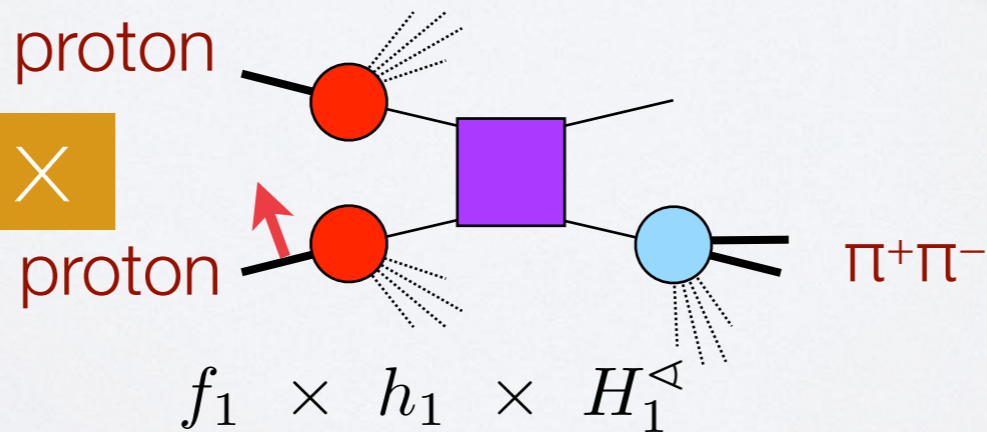
SIDIS  $l H^\uparrow \rightarrow l' (h_1 h_2) X$

$e^+e^- \rightarrow (h_1 h_2) X$



first extraction of transversity from a global fit of these data (at leading order - LO)

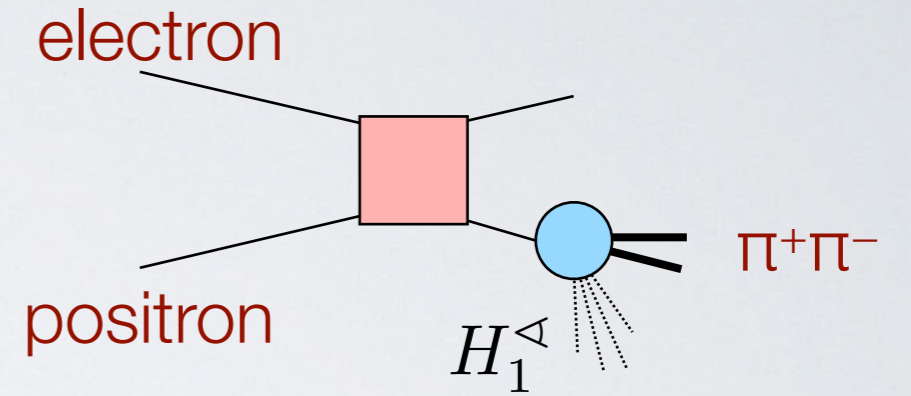
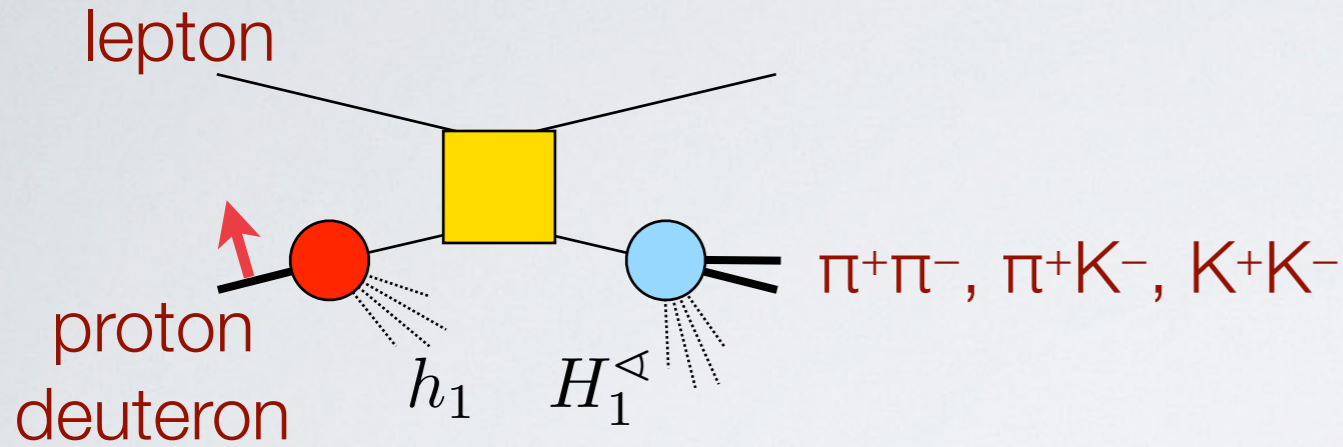
$H H^\uparrow \rightarrow (h_1 h_2) X$



# exp. data for 2-hadron-inclusive production

SIDIS  $l H^\uparrow \rightarrow l' (h_1 h_2) X$

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Airapetian et al.,  
JHEP **0806** (08) 017



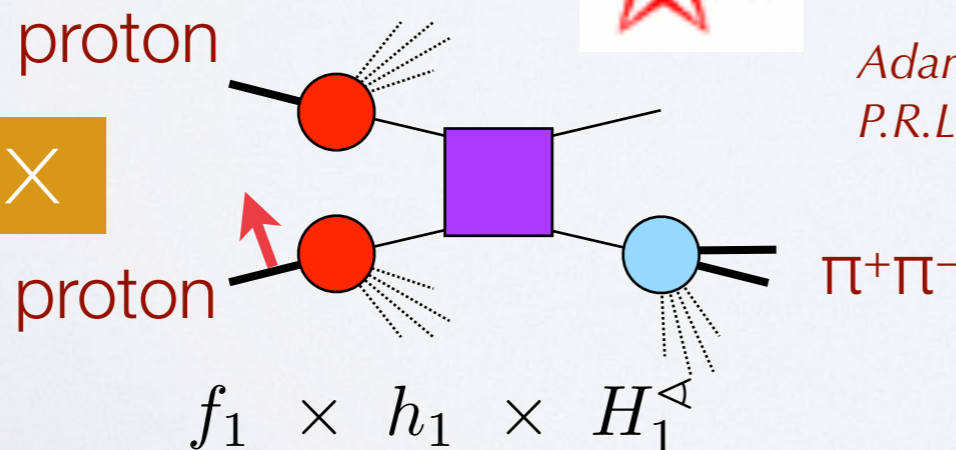
Adolph et al., P.L. **B713** (12)  
Braun et al., E.P.J. Web Conf. **85** (15) 02018



Vossen et al., P.R.L. **107** (11) 072004

$D_1$  Seidl et al., P.R. **D96** (17) 032005

$H H^\uparrow \rightarrow (h_1 h_2) X$



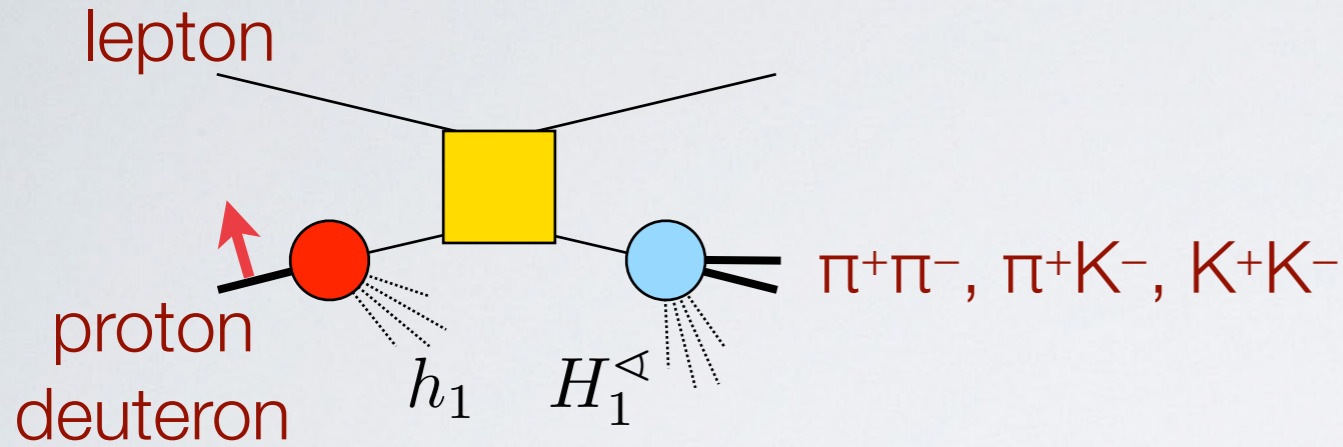
run 2006 (s=200)  
Adamczyk et al. (STAR),  
P.R.L. **115** (2015) 242501

run 2011 (s=500)  
Adamczyk et al. (STAR),  
P.L. **B780** (18) 332

$A_{UT}(\eta, M_h, P_T)$

# exp. data for 2-hadron-inclusive production

SIDIS  $e H^\uparrow \rightarrow e' (h_1 h_2) X$

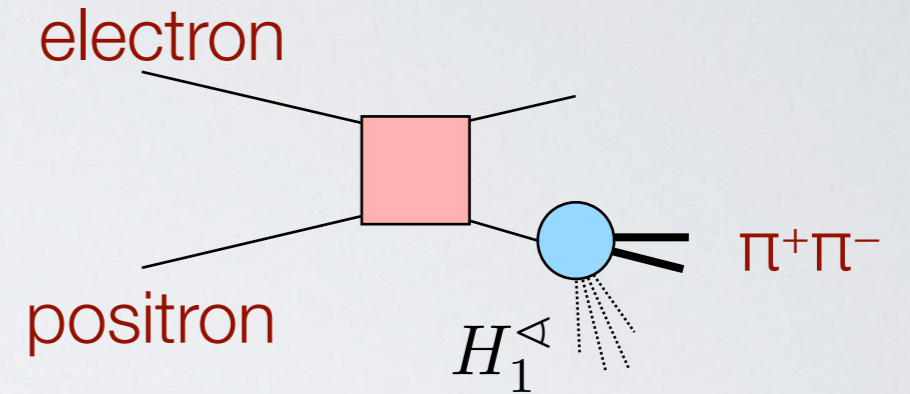


Airapetian et al.,  
*JHEP* **0806** (08) 017



Adolph et al., *P.L.* **B713** (12)  
Braun et al., *E.P.J. Web Conf.* **85** (15) 02018

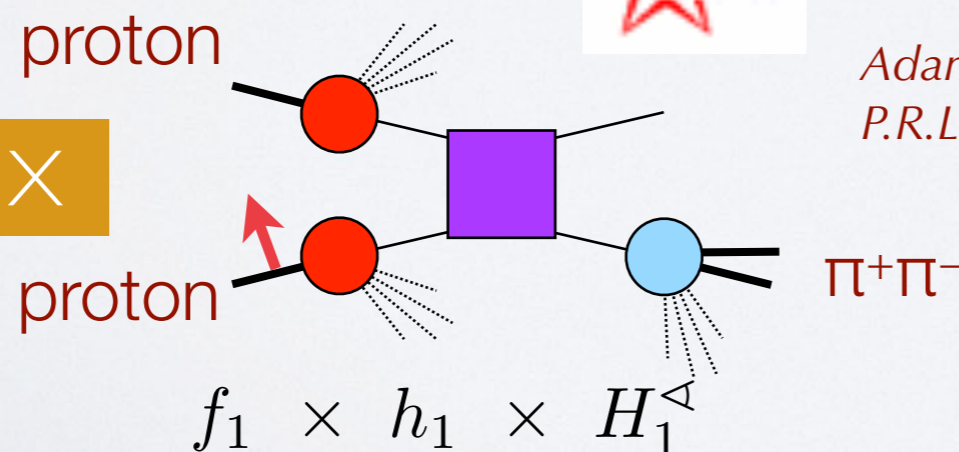
$e^+e^- \rightarrow (h_1 h_2) X$



Vossen et al., *P.R.L.* **107** (11) 072004

$D_1$  Seidl et al., *P.R. D96* (17) 032005  
from Montecarlo

$H H^\uparrow \rightarrow (h_1 h_2) X$



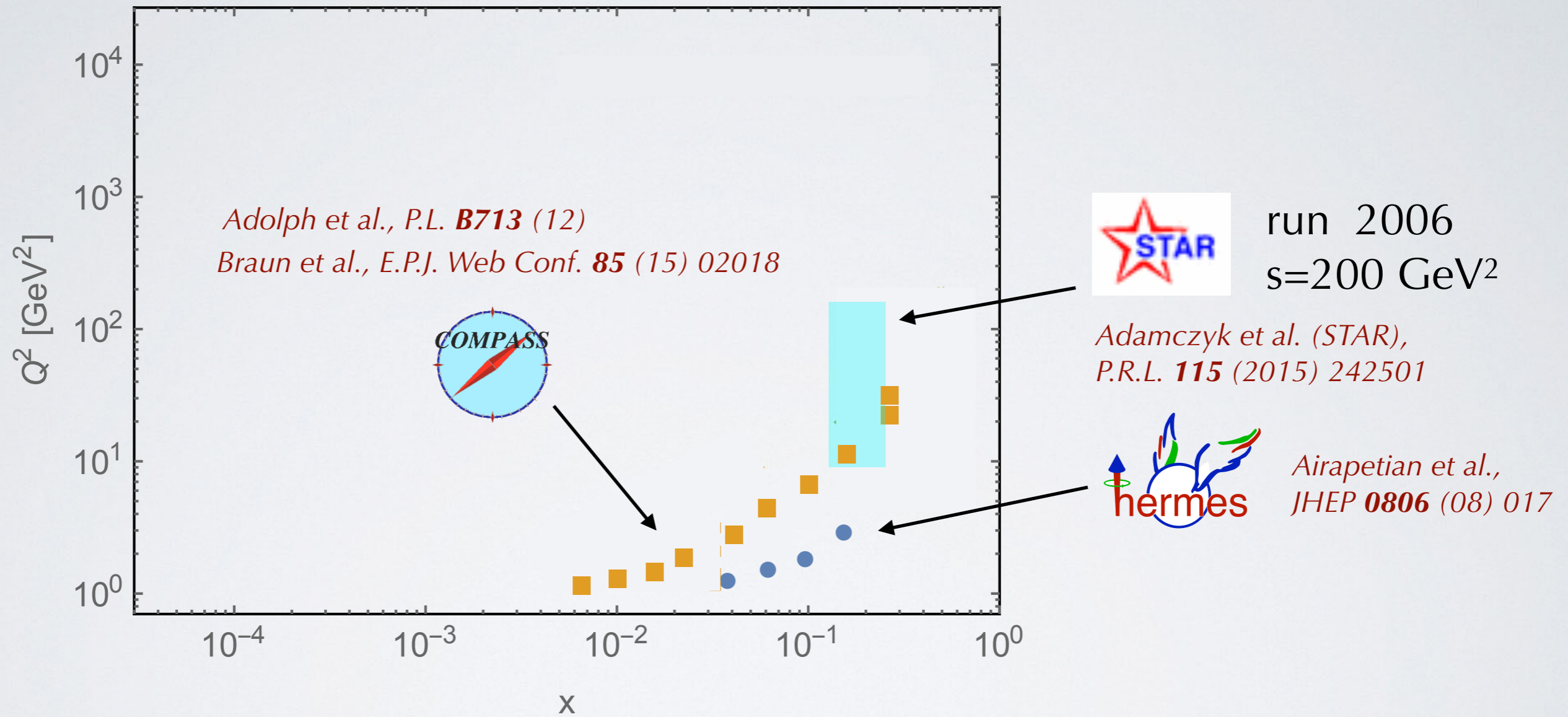
run 2006  
Adamczyk et al. (STAR),  
*P.R.L.* **115** (2015) 242501

run 2011  
Adamczyk et al. (STAR),  
*P.L.* **B780** (18) 332

$A_{UT}(\eta, M_h, P_T)$



# the kinematics



explore only valence quarks

# choice of functional form

different funct. form whose Mellin transform can be computed analytically  
but keep main feature: comply with Soffer Bound at any  $x$  and scale  $Q^2$

$$h_1^{qv}(x; Q_0^2) = F^{qv}(x) \left[ \text{SB}^q(x) + \overline{\text{SB}}^{\bar{q}}(x) \right]$$



Soffer Bound

$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}^q(x, Q^2) = |f_1^q(x, Q^2) + g_1^q(x, Q^2)|$$

MSTW08

DSSV

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MSTW08

DSSV

$$F^{qv}(x) = \frac{N_{qv}}{\max_x [|F^{qv}(x)|]} x^{A_{qv}} [1 + B_{qv} \text{Ceb}_1(x) + C_{qv} \text{Ceb}_2(x) + D_{qv} \text{Ceb}_3(x)]$$

$$|N_{qv}| \leq 1 \Rightarrow |F^{qv}(x)| \leq 1$$

Ceb<sub>n</sub>(x) Chebyshev polynomial

10 fitting parameters

**Soffer Bound satisfied at any  $Q^2$**

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MSTW08

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Ceb<sub>n</sub>(x) Chebyshev polynomial

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**Soffer Bound satisfied at any  $Q^2$**

$$\text{if } \lim_{x \rightarrow 0} x \text{SB}^q(x) \propto x^{a_q} \text{ then } h_1^q(x) \stackrel{x \rightarrow 0}{\approx} x^{A_q + a_q - 1}$$

$$A_q + a_q > \frac{1}{3} \text{ grants that tensor charge } \delta q(Q^2) = \int_{x_{\min}}^1 dx h_1^{q-\bar{q}}(x, Q^2) \text{ is finite and error } O(1\%)$$

MSTW08  $x_{\min}=10^{-6}$

**constrain parameter**

# theoretical uncertainties

quark  $D_1^q$  is well constrained by  $e^+e^-$  (Montecarlo) but

we don't know anything about the gluon  $D_1^g$  ( $e^+e^-$  doesn't help..)

Single-Spin Asymmetry  
in  $p$ - $p^\uparrow$  collisions

$$A_{UT}(\eta, M_h, P_T) = \frac{d\sigma_{UT}}{d\sigma_0}$$

chiral-odd  
not important  
important !

typical cross section for  $a+b \rightarrow c+d$  process

$$d\sigma_0 \propto \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} D_1^c(\bar{z}, M_h)$$

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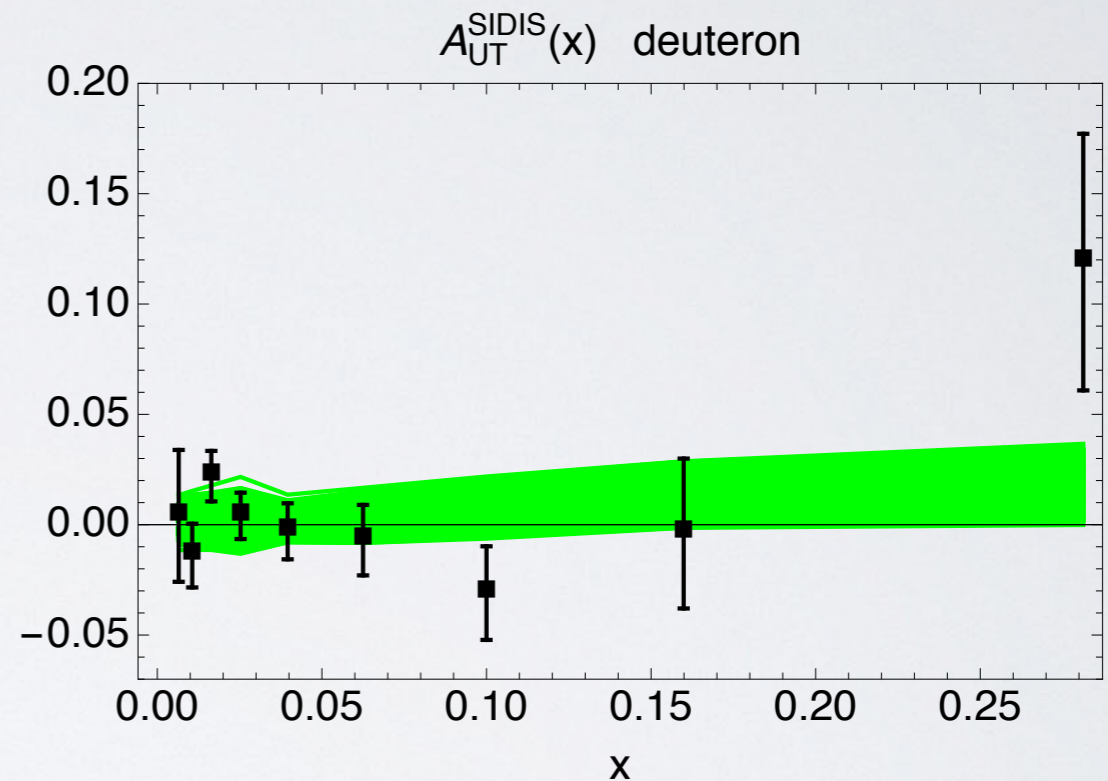
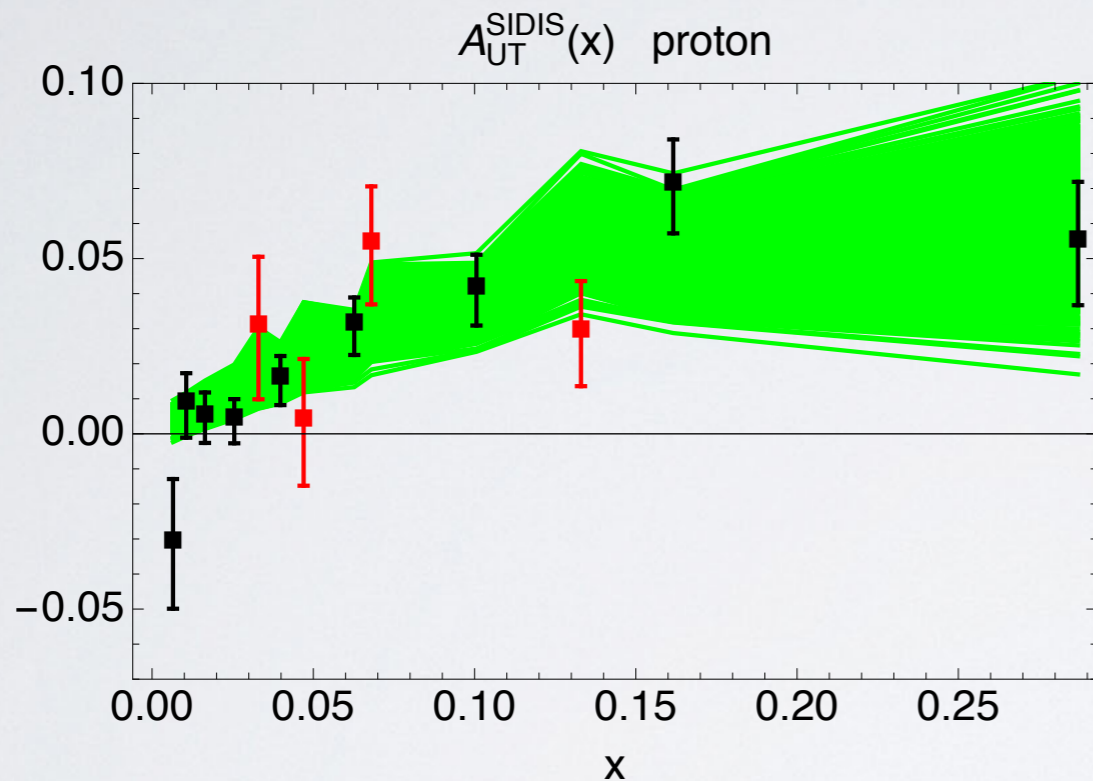
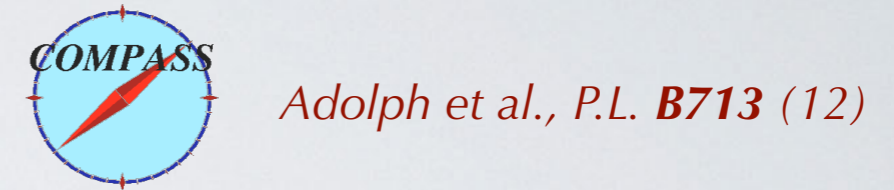
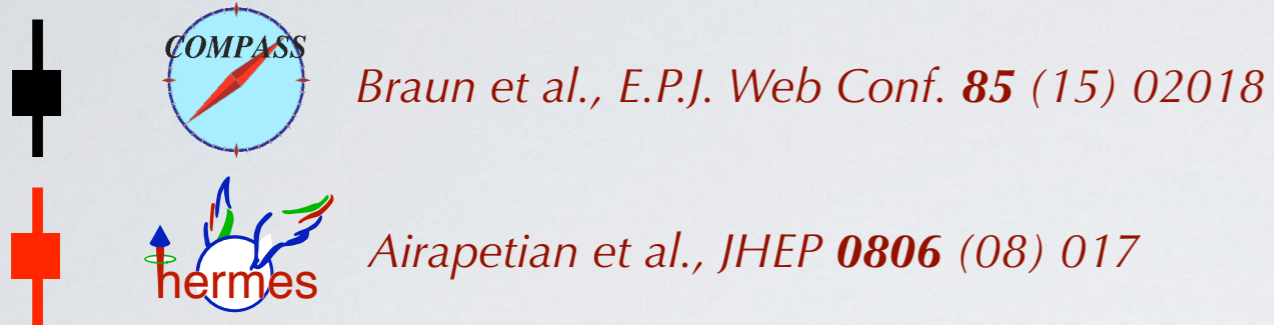
$$d\sigma_0 \propto \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} D_1^c(\bar{z}, M_h)$$

our choice: compute  $d\sigma_0$  with  $D_1^g(Q_0) = \begin{cases} 0 \\ D_1^u(Q_0) / 4 \\ D_1^u(Q_0) \end{cases}$

deteriorates our  $e^+e^-$  fit as  $\chi^2/\text{dof} = \begin{cases} 1.69 & 1.28 \\ 1.81 & 1.37 \\ 2.96 & 2.01 \end{cases}$

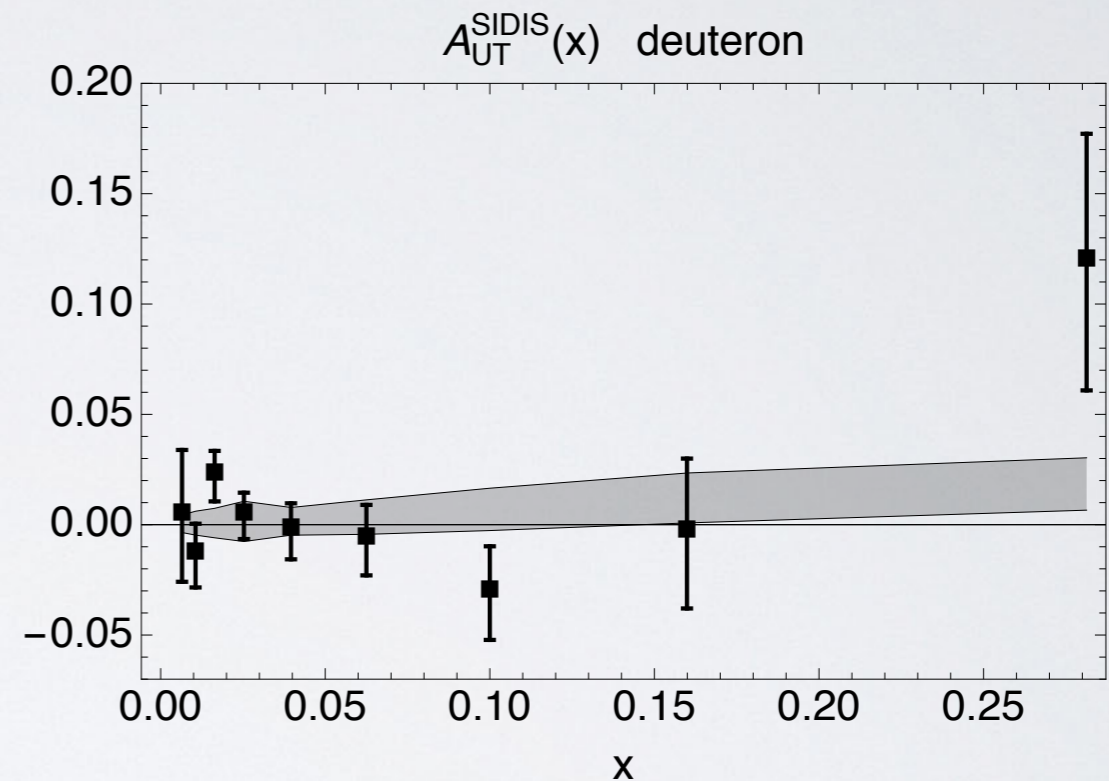
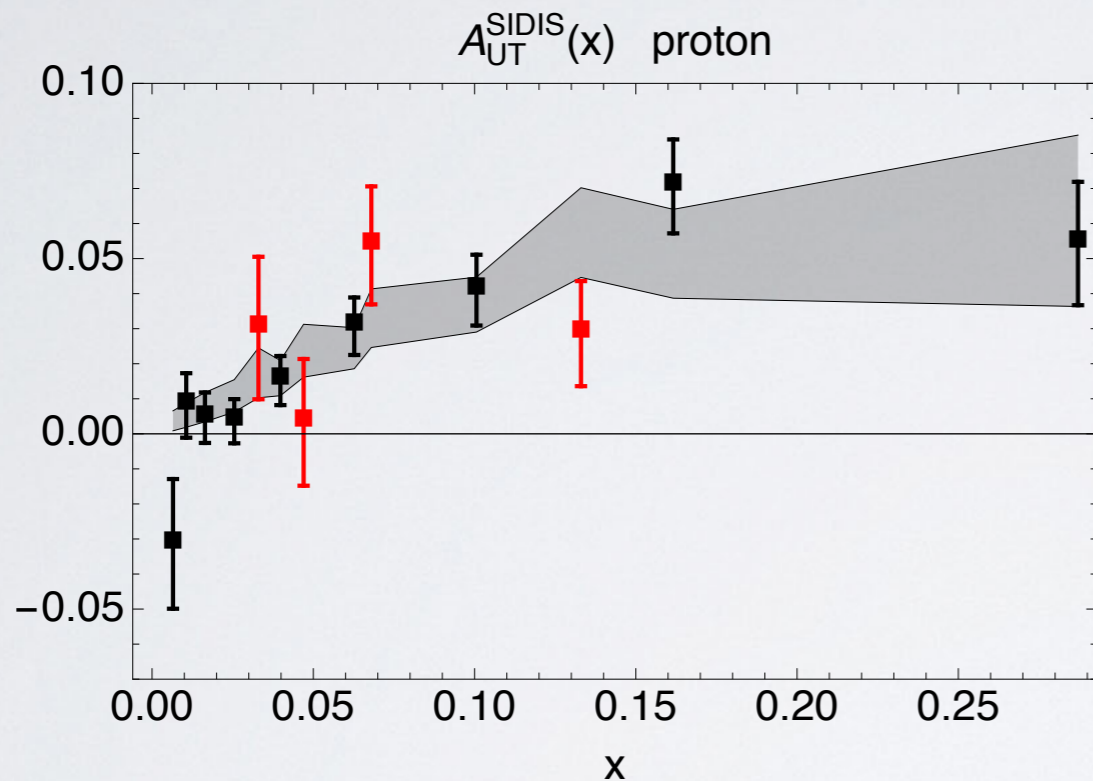
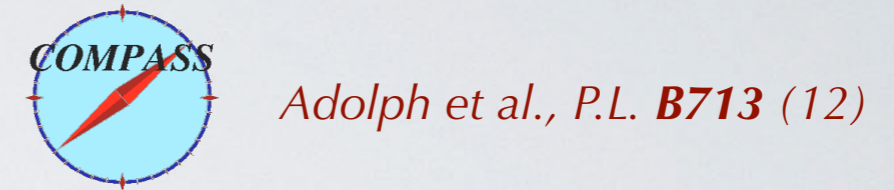
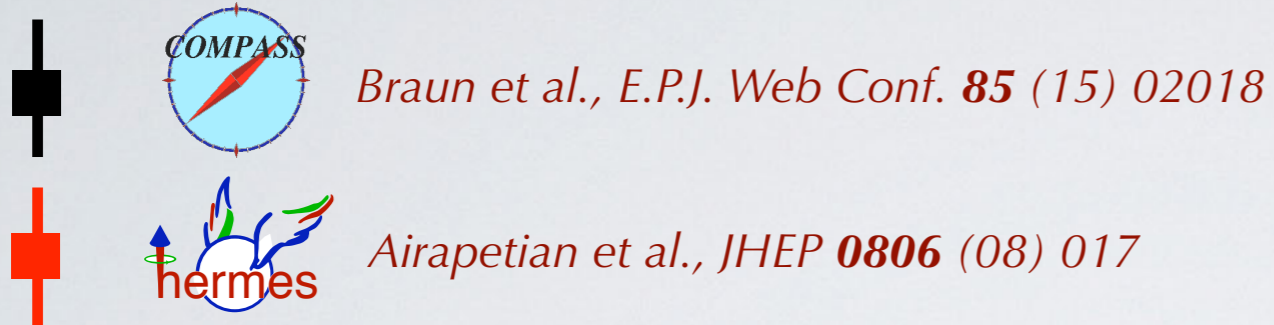
background       $\rho$       channels

# statistical uncertainty: the bootstrap method



all 600 replicas

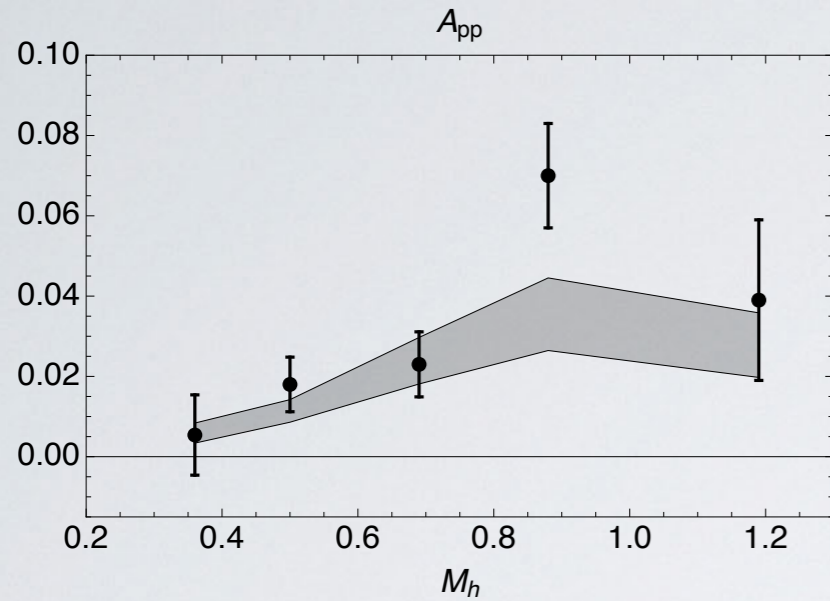
# statistical uncertainty: the bootstrap method



90% replicas



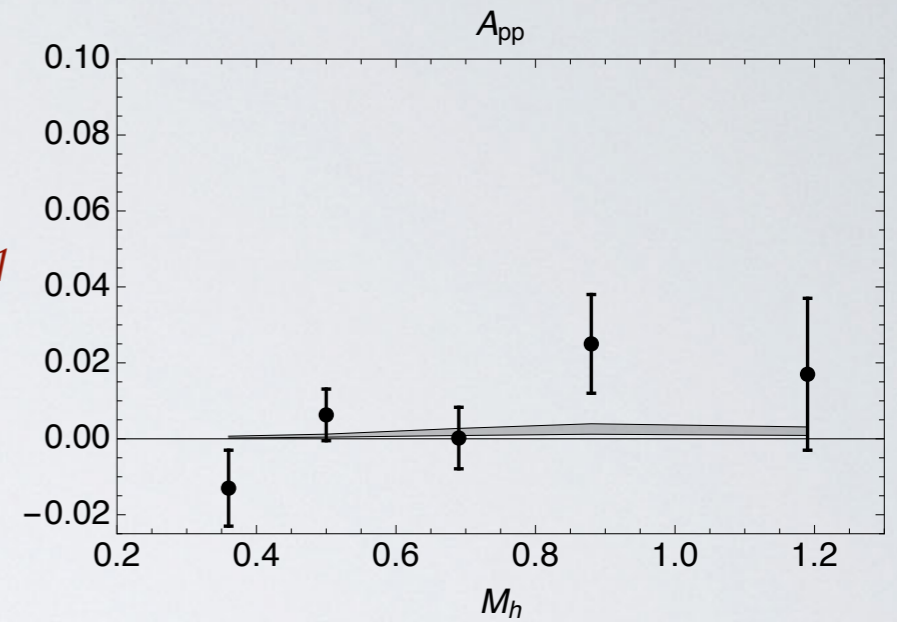
# fit STAR asymmetry



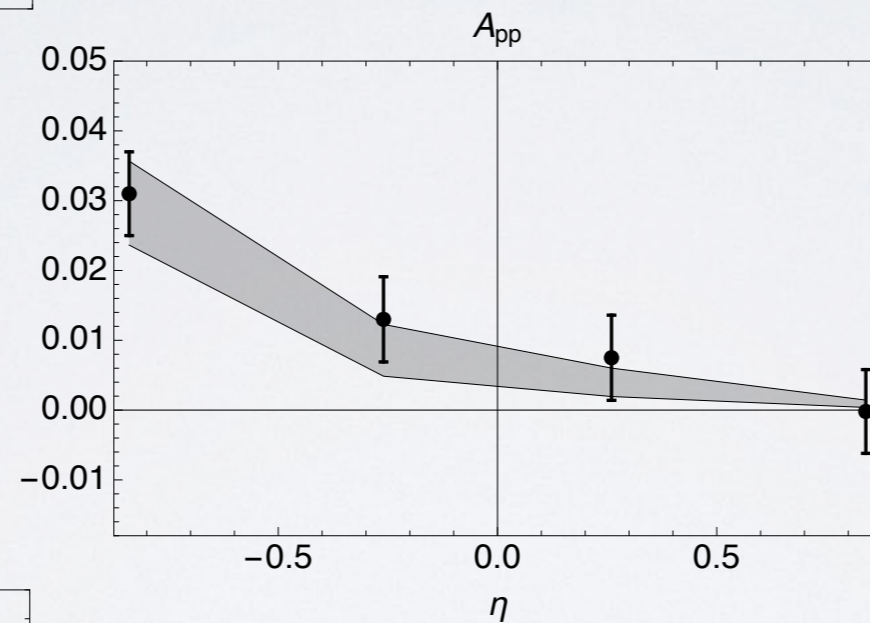
$\eta < 0$



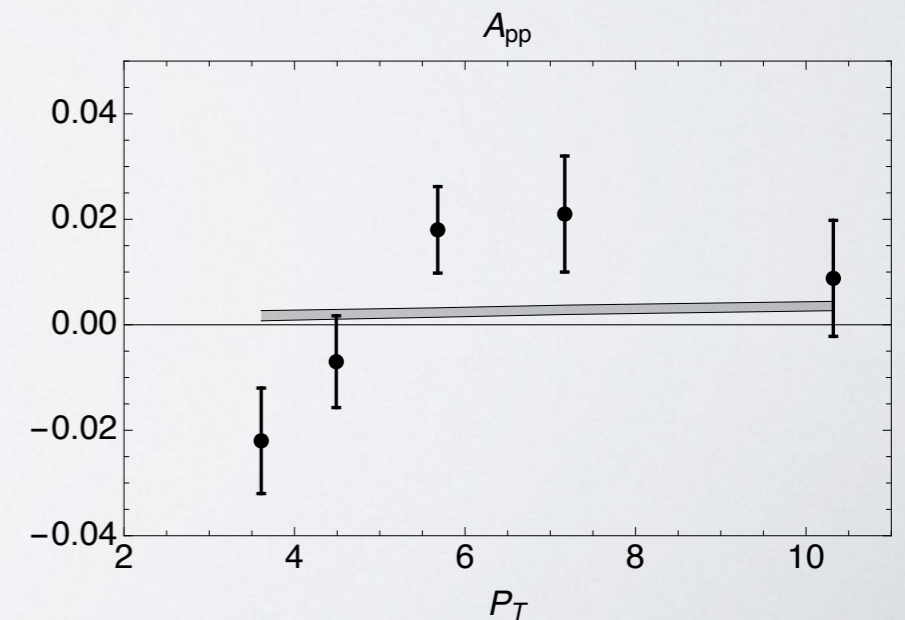
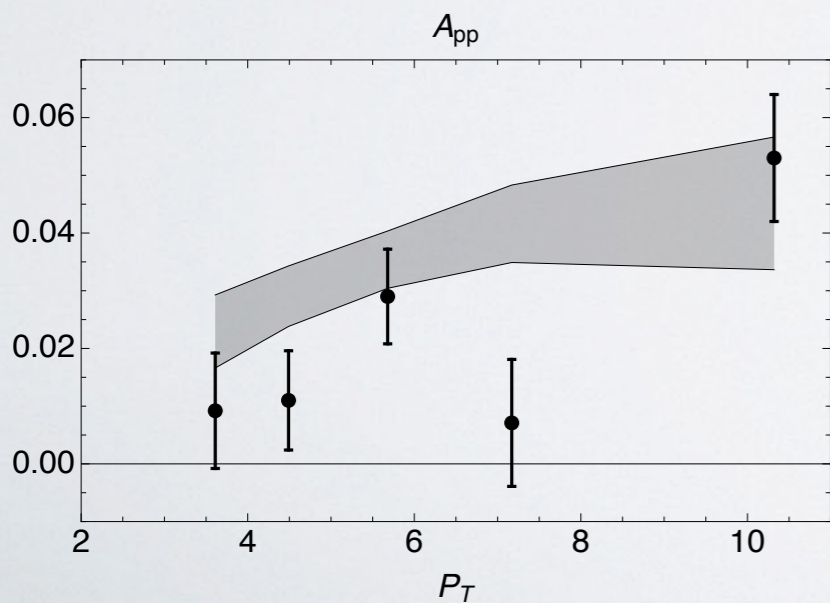
*Adamczyk et al. (STAR),  
P.R.L. 115 (2015) 242501*



$\eta > 0$



90% uncertainty band



# $\chi^2$ of the fit

**46** data points, **10** parameters  
 global  $\chi^2/\text{dof} = 2.08 \pm 0.09$

$\approx 38\%$

$\approx 62\%$

**SIDIS**

**STAR**

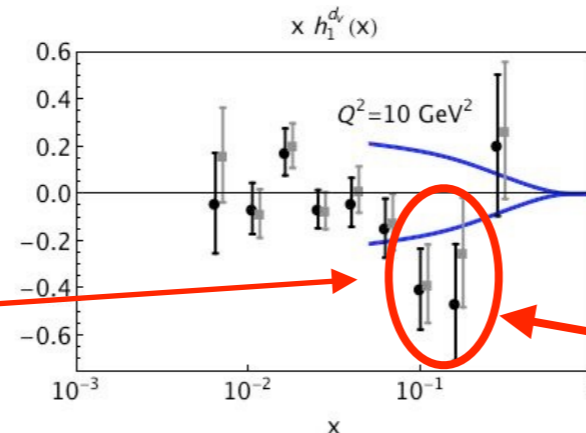
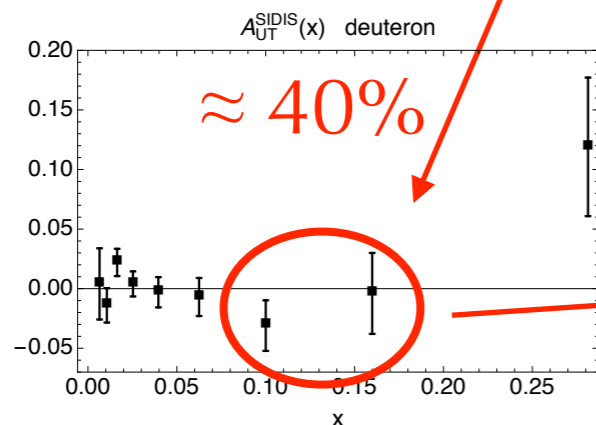
$\approx 24\%$

$\approx 76\%$

- $P_T$  bins  $\approx 70\%$
- $M_h$  bins  $\approx 28\%$
- $\eta$  bins  $\approx 2\%$



$\approx 60\%$   
rest

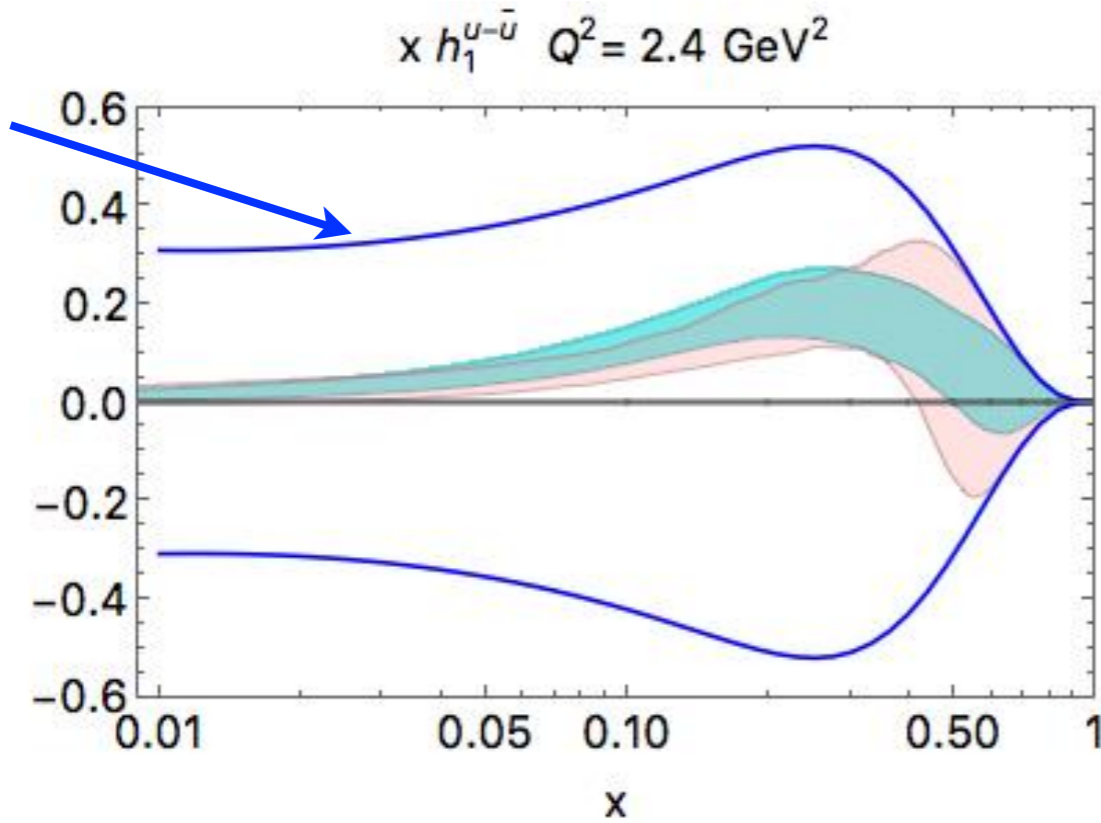


Soffer bound  
@10 GeV<sup>2</sup>

( $x=0.1, Q^2 \sim 9 \text{ GeV}^2$ )  
 ( $x=0.16, Q^2 \sim 15 \text{ GeV}^2$ )

# comparison with previous fit

Soffer bound



*Radici & Bacchetta,*  
*P.R.L. **120** (18) 192001*

global fit

up

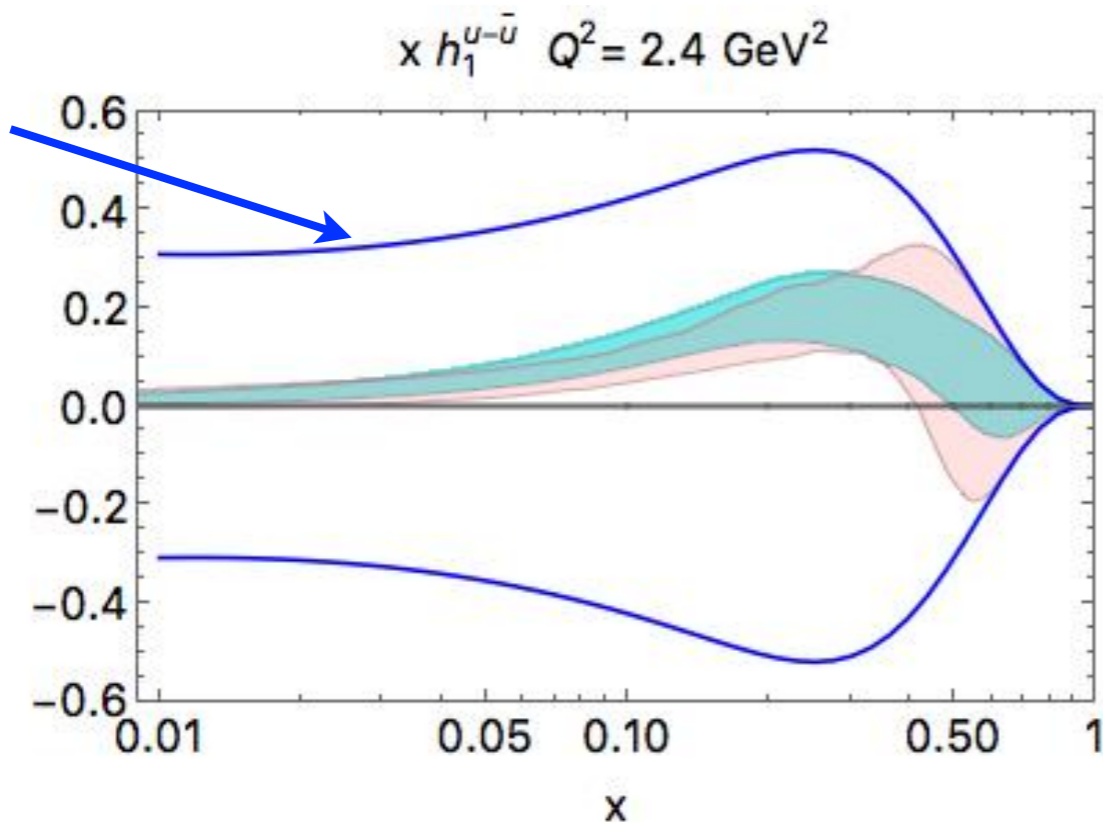
higher  
precision

old fit (only SIDIS data)

*Radici et al.,*  
*JHEP **1505** (15) 123*

# comparison with previous fit

Soffer bound



*Radici & Bacchetta, P.R.L. 120 (18) 192001*

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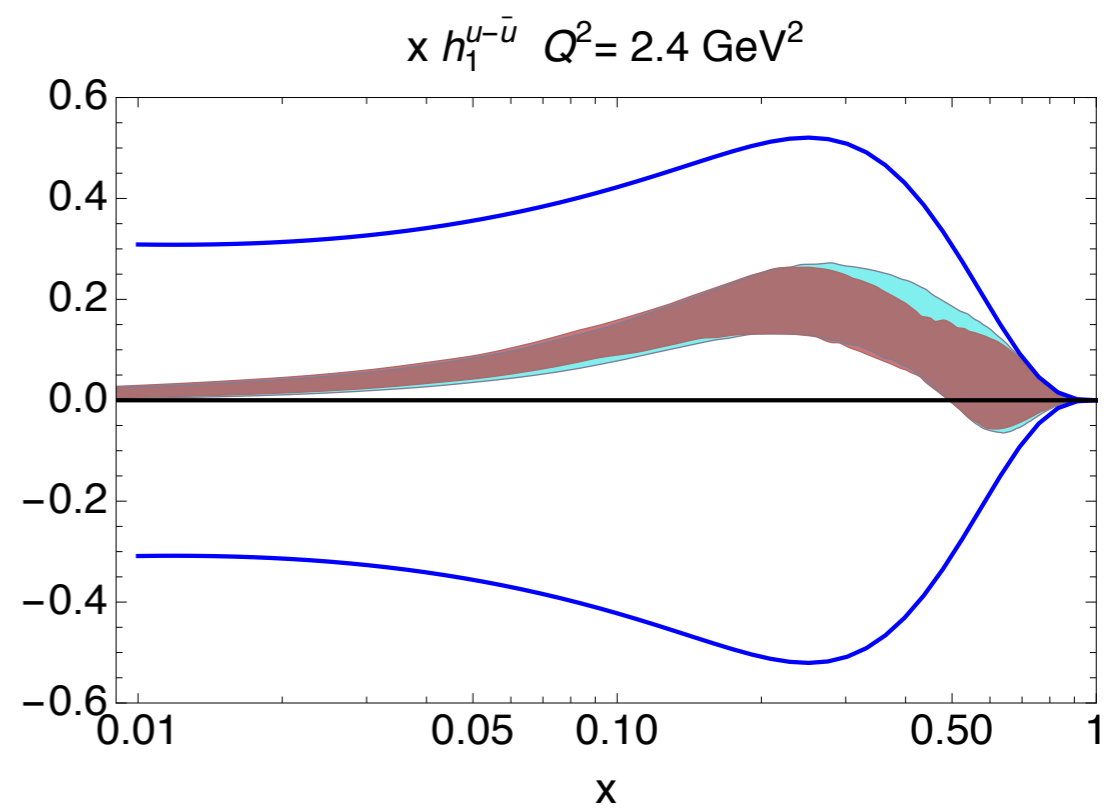
old fit (only SIDIS data)

*Radici et al., JHEP 1505 (15) 123*

up

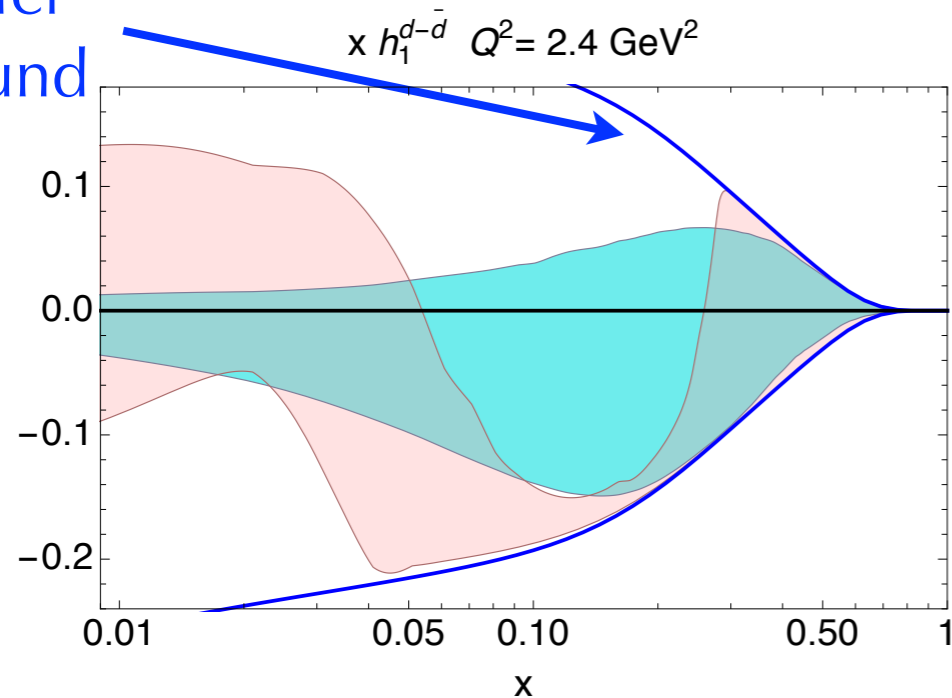
insensitive to uncertainty on gluon  $D_1$

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_1^u/4 \\ D_1^u \end{cases}$$



# comparison with previous fit

Soffer bound



*Radici & Bacchetta,*  
*P.R.L. **120** (18) 192001*

global fit

old fit

*Radici et al.,*  
*JHEP **1505** (15) 123*

down

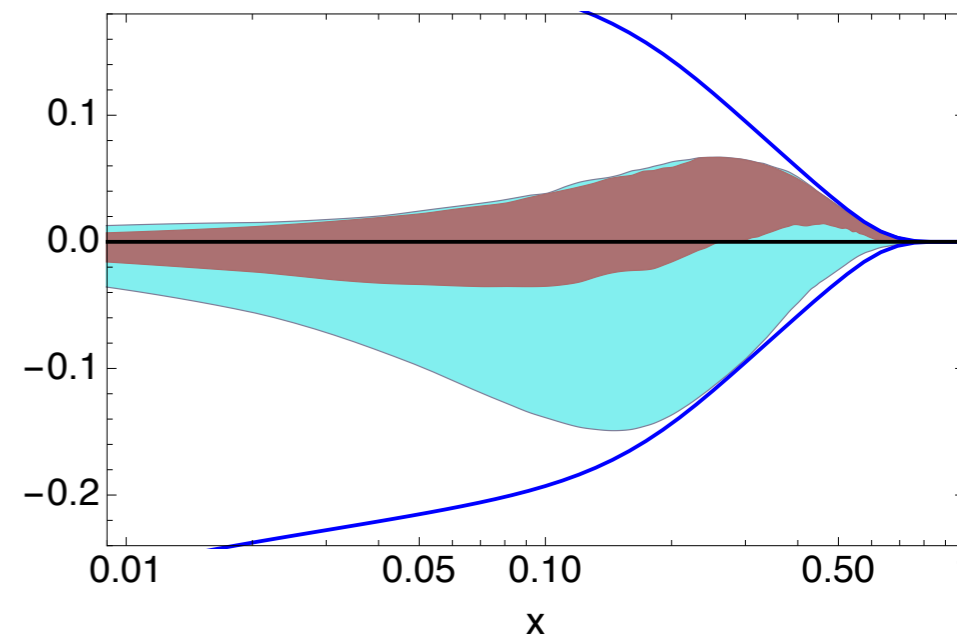
down

sensitive to  
uncertainty on  
gluon  $D_1$

$$D_{1g}(Q_0) = 0$$

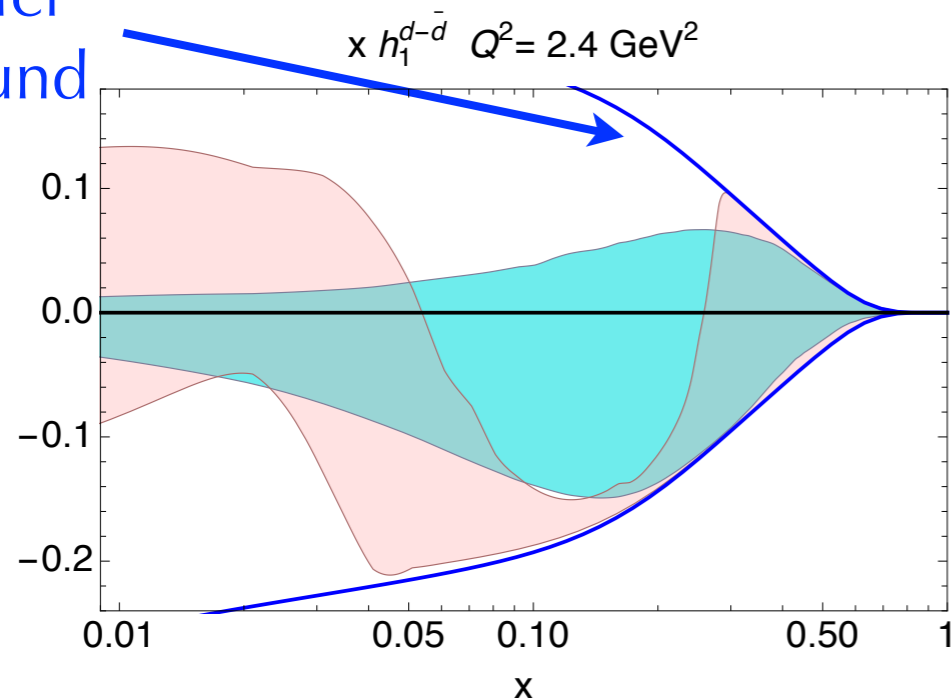
$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_1^u / 4 \\ D_1^u \end{cases}$$

$x h_1^{d-\bar{d}} \quad Q^2 = 2.4 \text{ GeV}^2$



# comparison with previous fit

Soffer bound



Radici & Bacchetta,  
*P.R.L.* **120** (18) 192001

global fit

down

old fit

Radici et al.,  
*JHEP* **1505** (15) 123

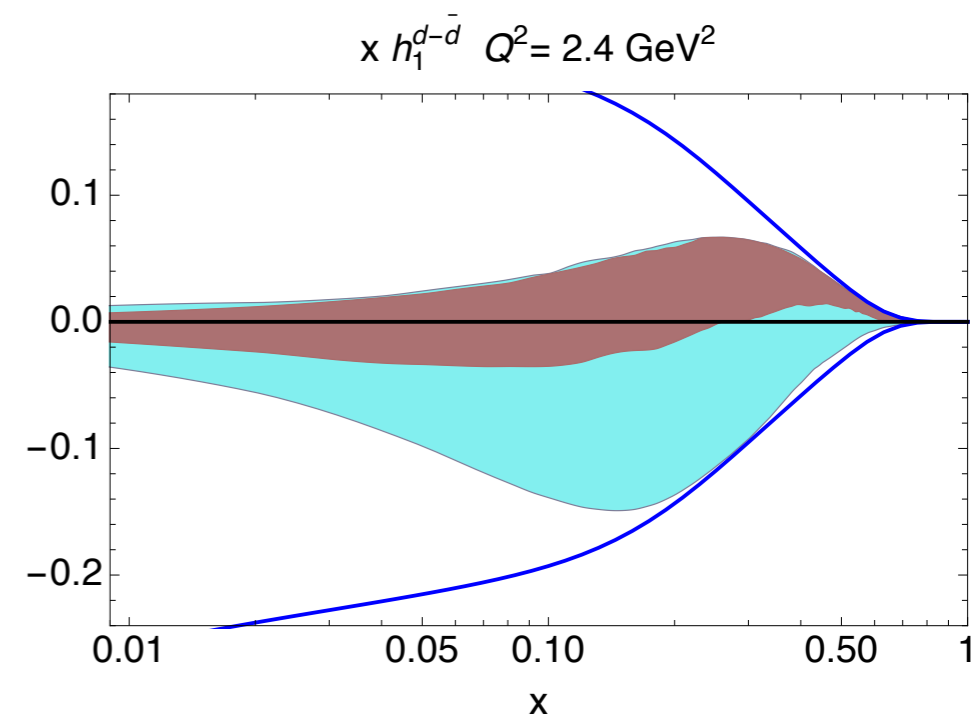
need dihadron multiplicities  
from RHIC

down

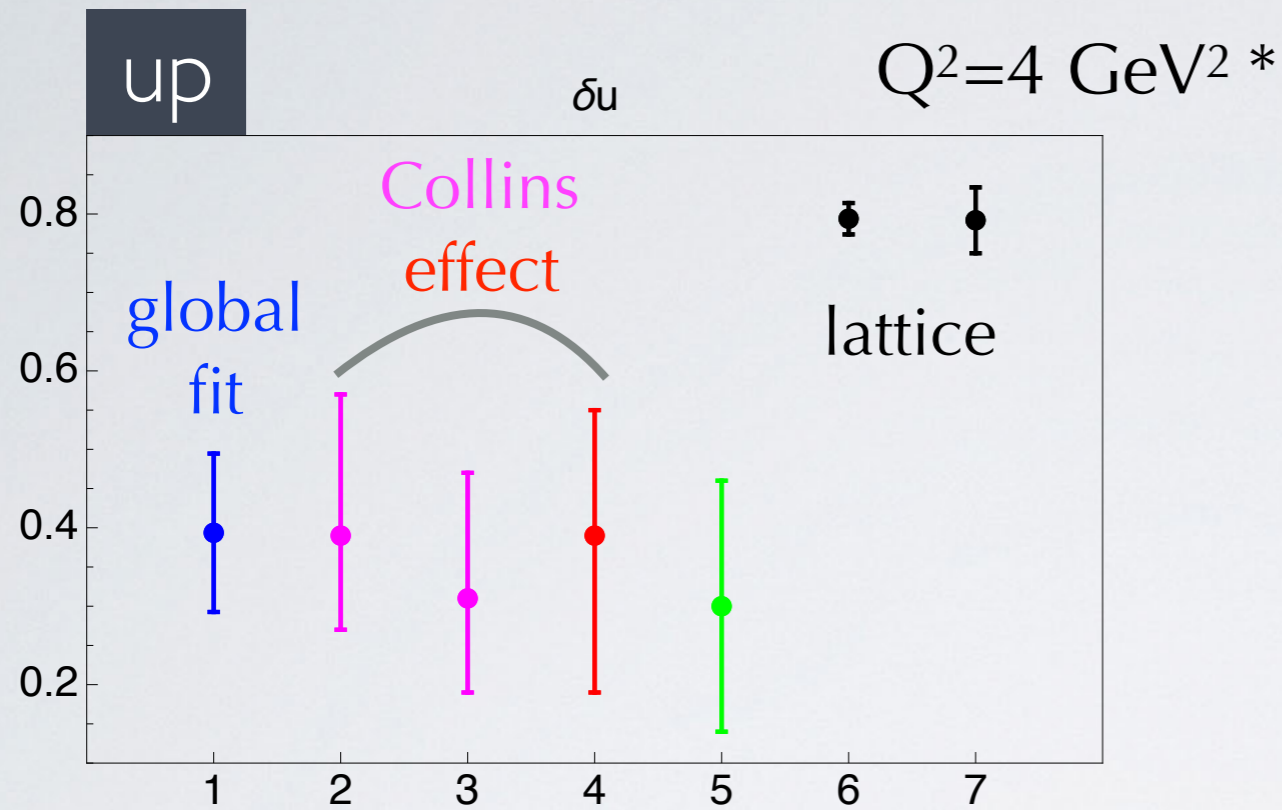
sensitive to  
uncertainty on  
gluon  $D_1$

$$D_{1g}(Q_0) = 0$$

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_1^u / 4 \\ D_1^u \end{cases}$$



# tensor charge $\delta q(Q^2) = \int dx h_1^{q-\bar{q}}(x, Q^2)$



## 1- global fit

*Radici & Bacchetta, P.R.L. 120 (18) 192001*

## 2,3- Torino

*Anselmino et al., P.R. D87 (13) 094019*

\*  $Q^2=1$

## 4- TMD fit

*Kang et al., P.R. D93 (16) 014009*

\*  $Q^2=10$

## 5- JAM fit

*Lin et al., P.R.L. 120 (18) 152502*

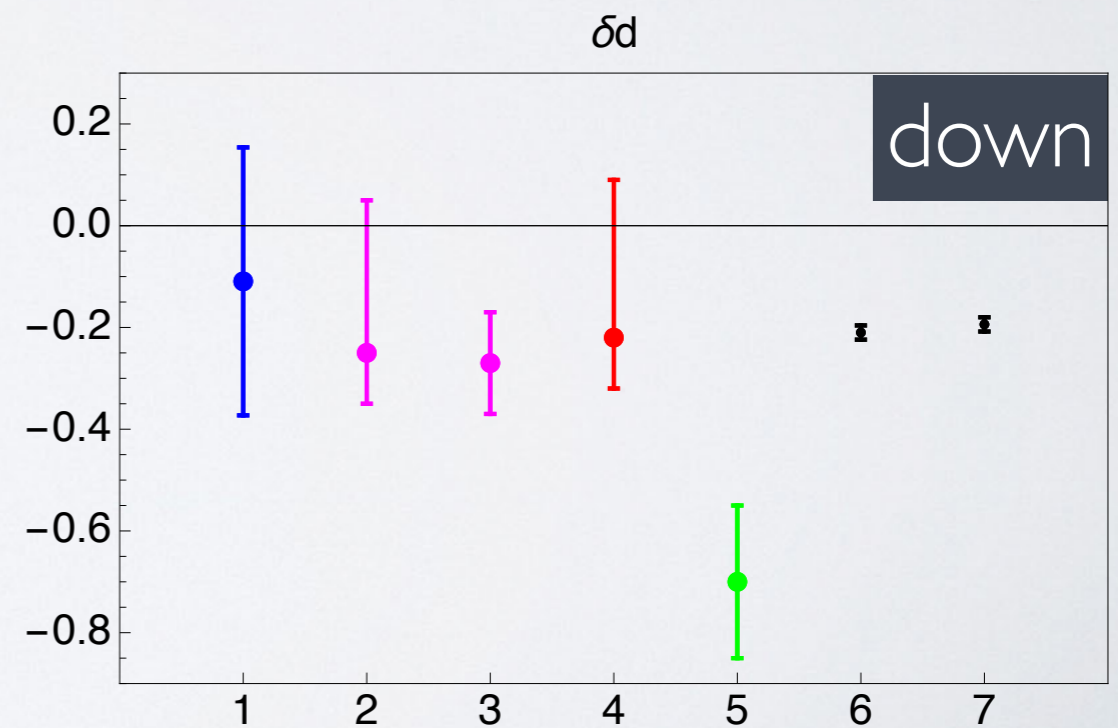
{ Collins effect +  
lattice  $g_T = \delta u - \delta d$  \*  $Q_0^2=2$

## 6- ETMC17

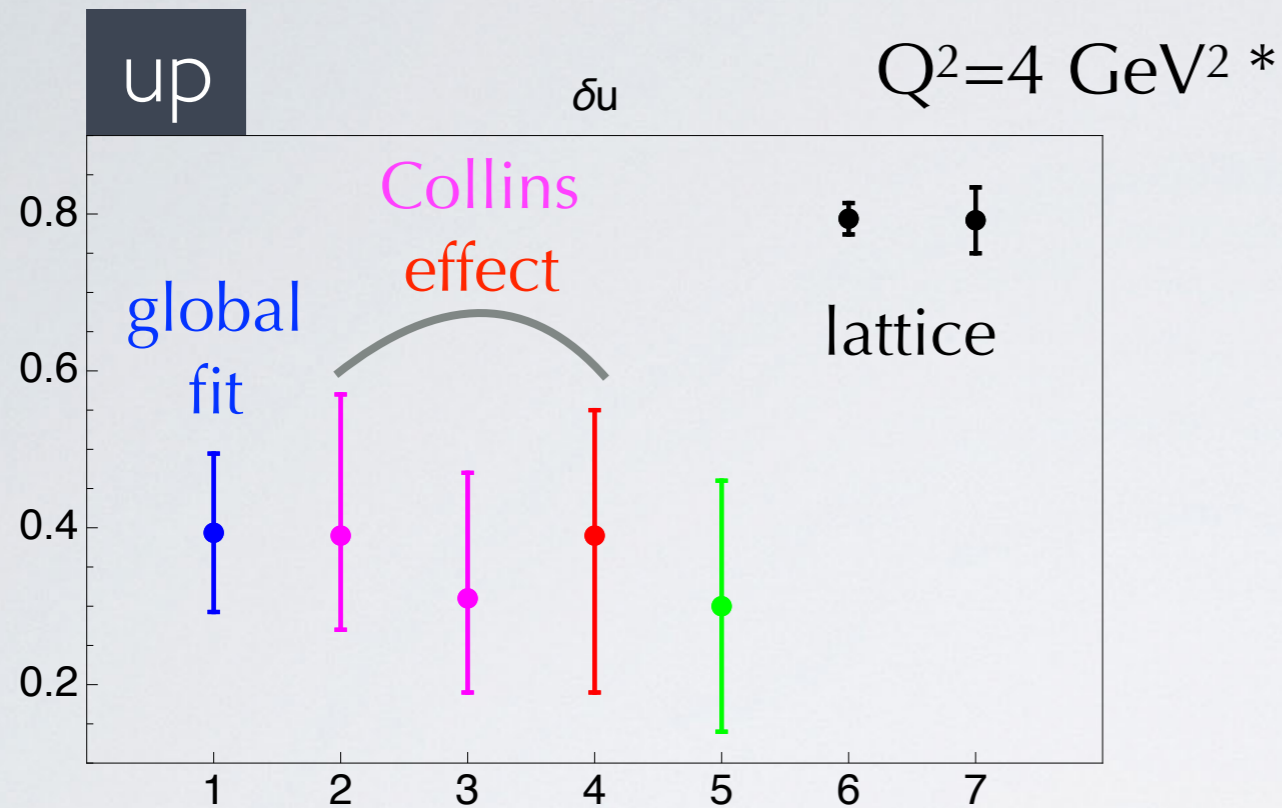
*Alexandrou et al., P.R. D95 (17) 114514;  
E P.R. D96 (17) 099906*

## 7- PNDME16

*Bhattacharya et al., P.R. D94 (16) 054508*

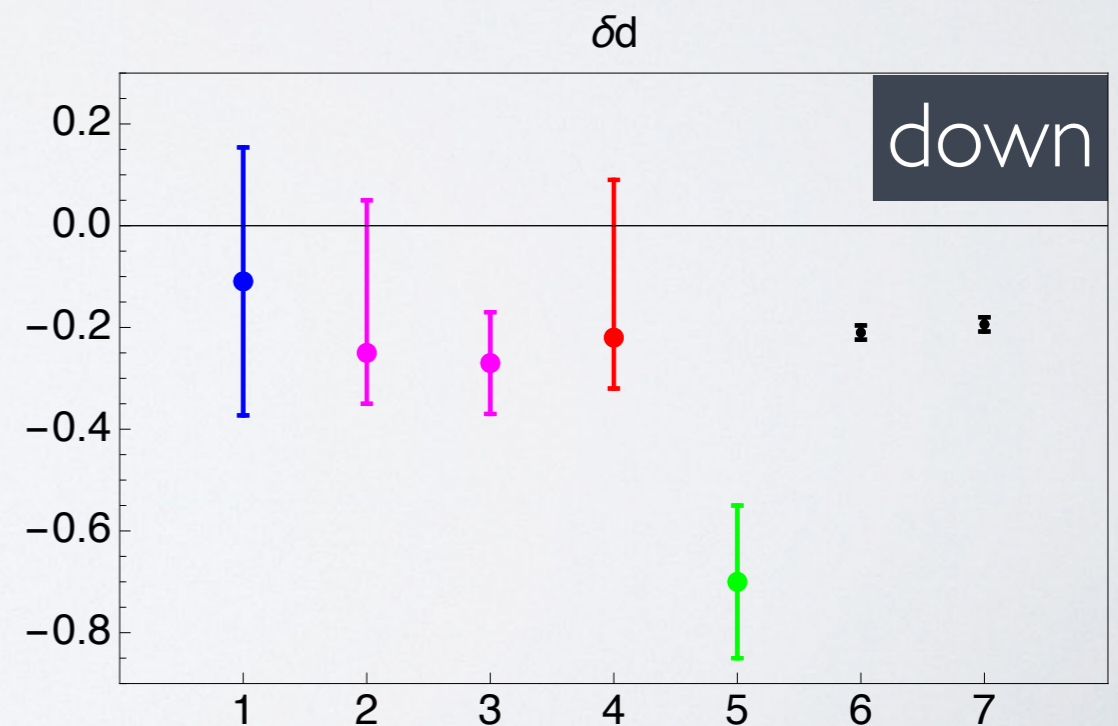


# tensor charge $\delta q(Q^2) = \int dx h_1^{q-\bar{q}}(x, Q^2)$



incompatibility for up  
compatible for down  
but with large errors  
(except JAM)

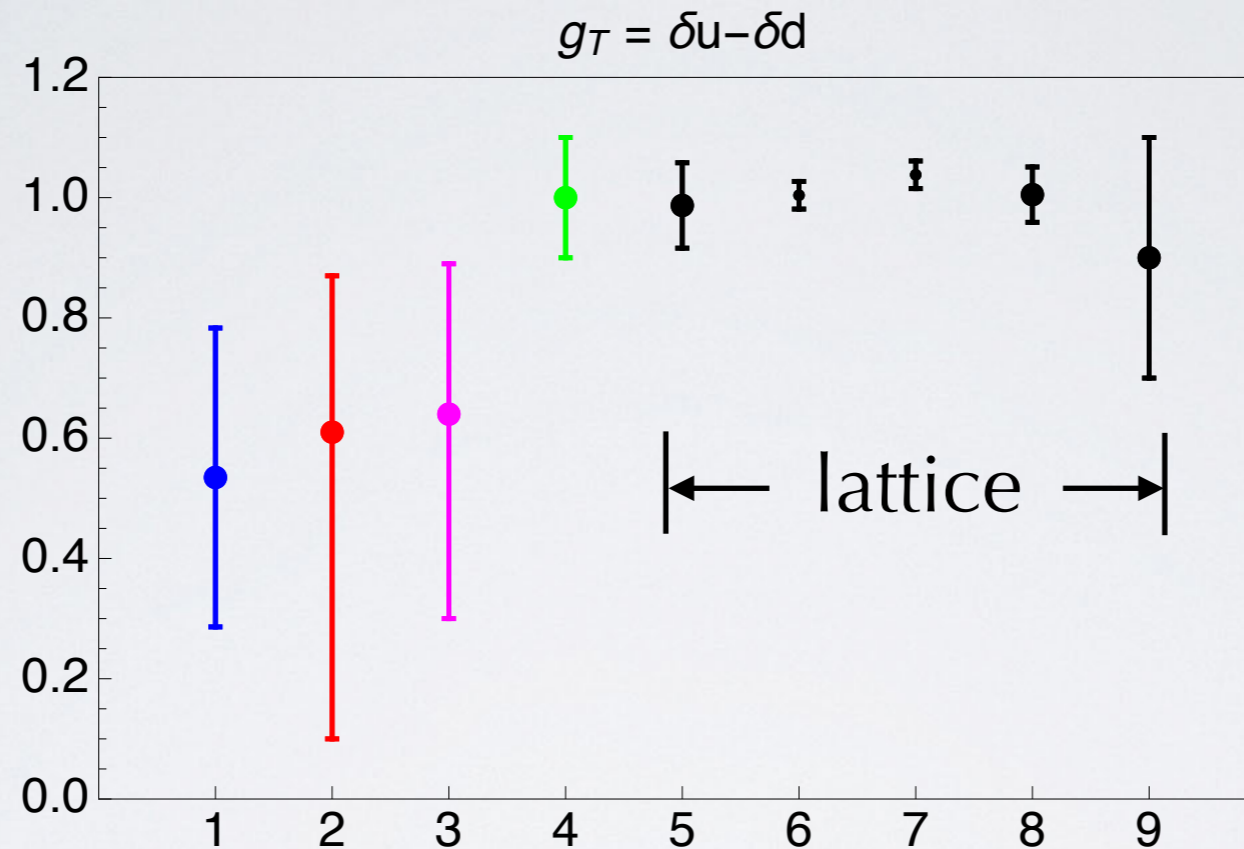
- 1- global fit *Radici & Bacchetta, P.R.L. 120 (18) 192001*
- 2,3- Torino *Anselmino et al., P.R. D87 (13) 094019* \*  $Q^2=1$
- 4- TMD fit *Kang et al., P.R. D93 (16) 014009* \*  $Q^2=10$
- 5- JAM fit *Lin et al., P.R.L. 120 (18) 152502* { Collins effect + lattice  $g_T = \delta u - \delta d$  \*  $Q_0^2=2$
- 6- ETMC17 *Alexandrou et al., P.R. D95 (17) 114514; E P.R. D96 (17) 099906*
- 7- PNDME16 *Bhattacharya et al., P.R. D94 (16) 054508*





# isovector tensor charge $g_T = \delta u - \delta d$

$Q^2=4 \text{ GeV}^2 *$



incompatibility  
(except JAM)

Radici & Bacchetta,  
*P.R.L.* 120 (18) 192001

1) **global fit '17**

Kang et al., *P.R.* D93 (16) 014009

2) **"TMD fit" \*  $Q^2=10$**

Anselmino et al., *P.R.* D87 (13) 094019

3) **Torino fit \*  $Q^2=1$**

Lin et al., *P.R.L.* 120 (18) 152502

4) **JAM fit '17 \*  $Q_0^2=2$**

5) PNDME '16

*Bhattacharya et al., P.R. D94 (16) 054508*

6) ETMC '17

*Alexandrou et al., P.R. D95 (17) 114514;  
E P.R. D96 (17) 099906*

7) LHPC '12

*Green et al., P.R. D86 (12)*

8) RQCD '14

*Bali et al., P.R. D91 (15)*

9) RBC-UKQCD

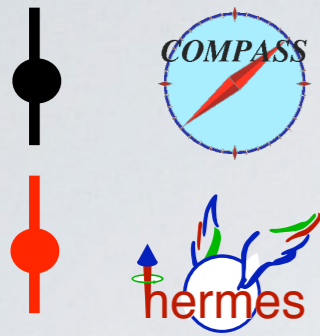
*Aoki et al., P.R. D82 (10)*

# “transverse-spin puzzle” ?

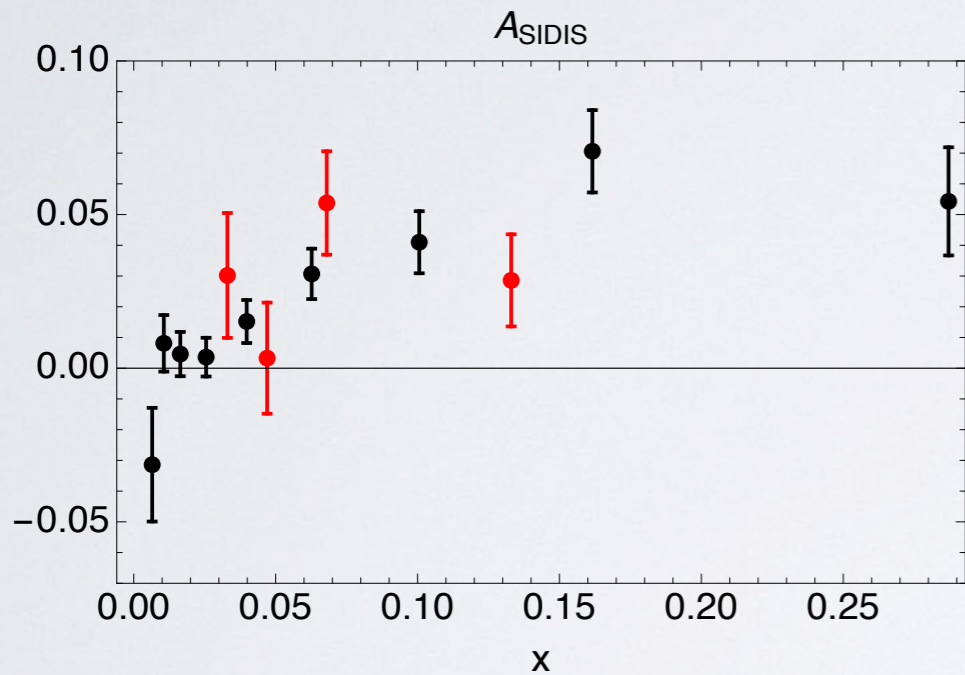
there seems to be no simultaneous compatibility  
about  $\delta u$ ,  $\delta d$ ,  $g_T = \delta u - \delta d$   
between lattice and  
phenomenological extractions  
of transversity

**BUT**

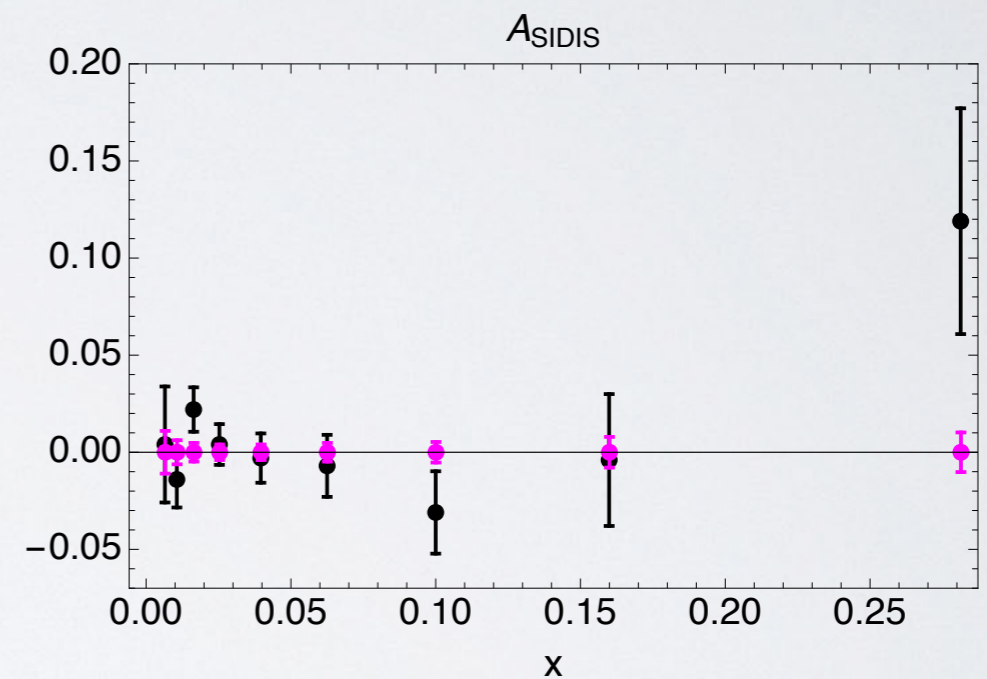
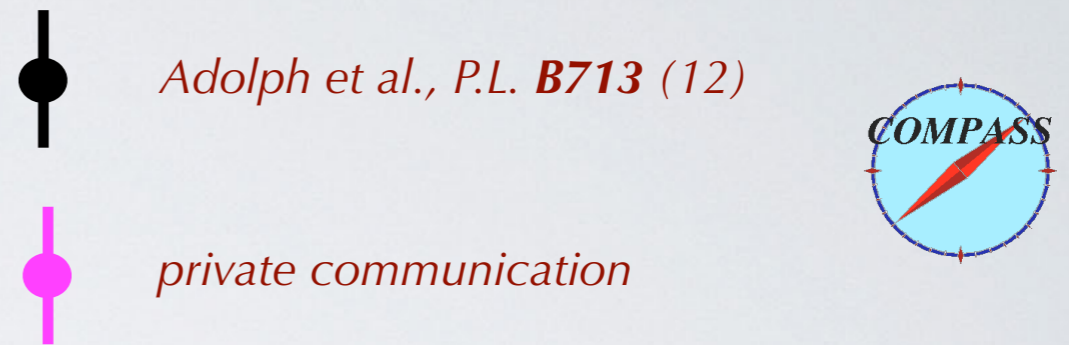
# add Compass pseudodata for future deuteron run



proton



$$A_{\text{SIDIS}} \sim 4h_1^{u_v} - h_1^{d_v}$$

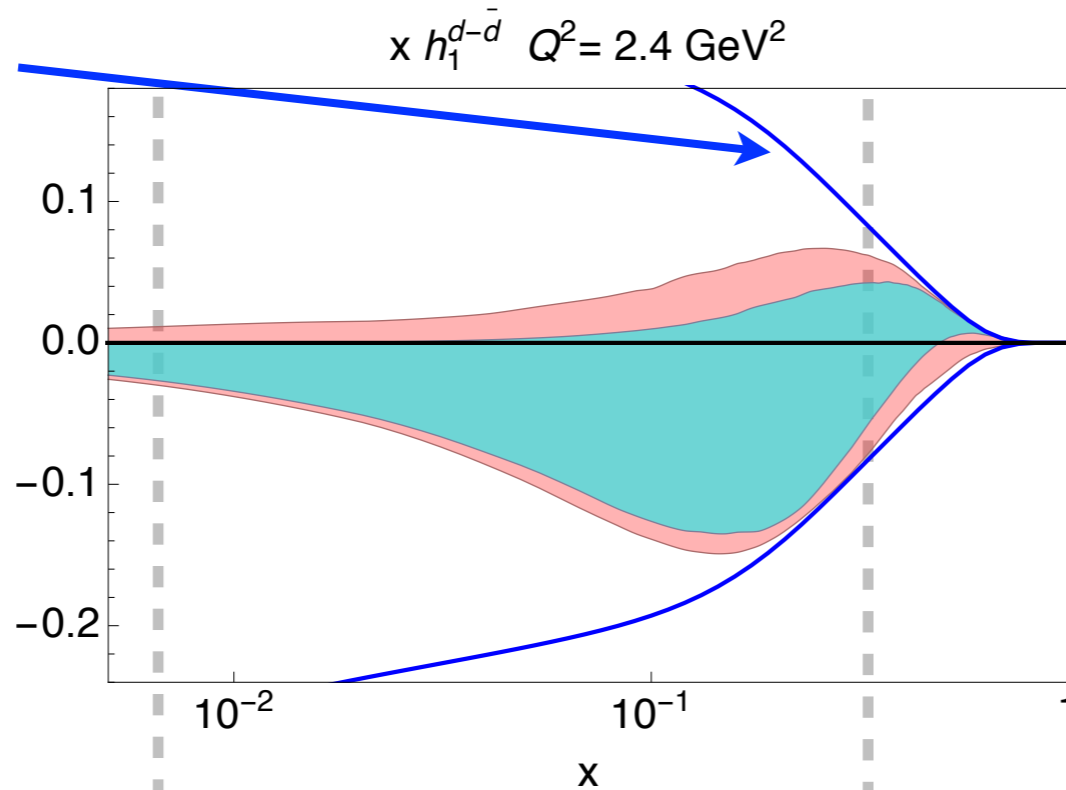


$$A_{\text{SIDIS}} \sim h_1^{u_v} + h_1^{d_v}$$

# pseudodata impact on down

Soffer bound

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1^u}/4 \\ D_{1^u} \end{cases}$$

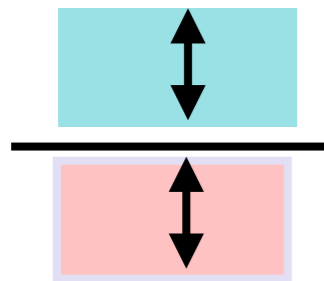


global fit + pseudodata

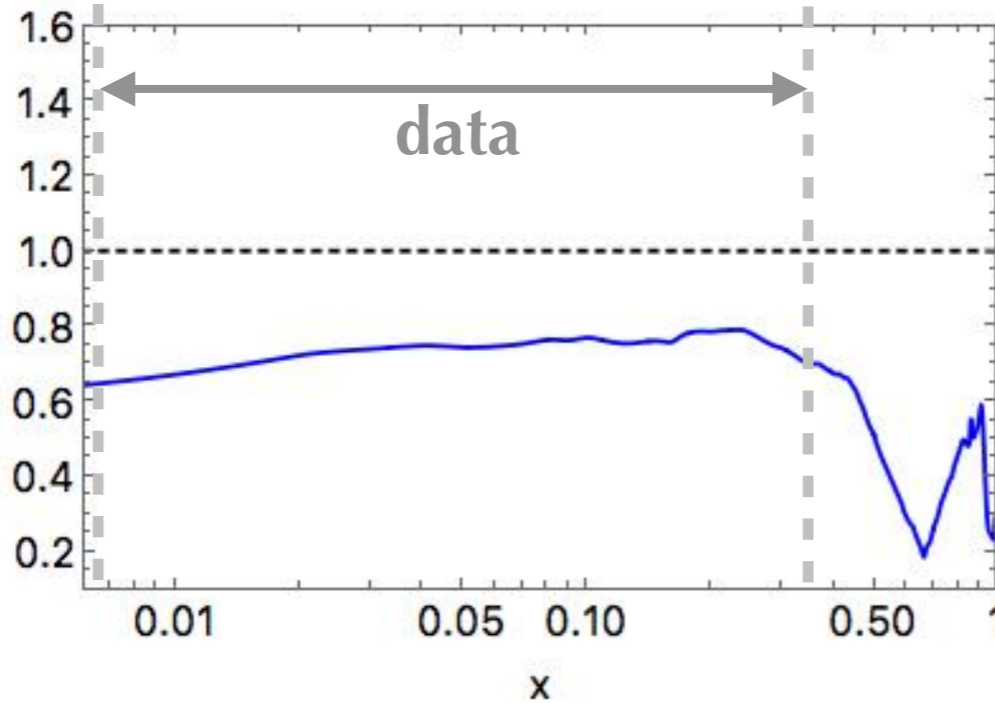
global fit

*Radici & Bacchetta,*  
*PRL 120 (18) 192001*

down



ratio of widths

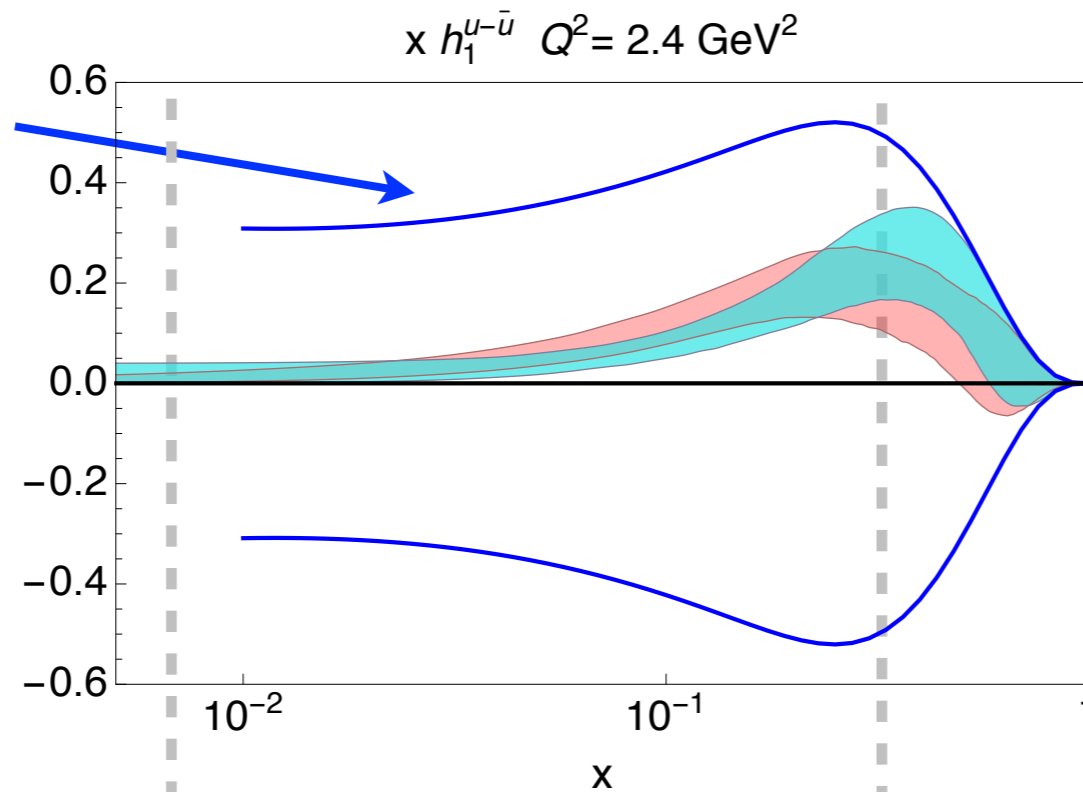


< 20% >  
increase in  
precision

# pseudodata impact on up

Soffer bound

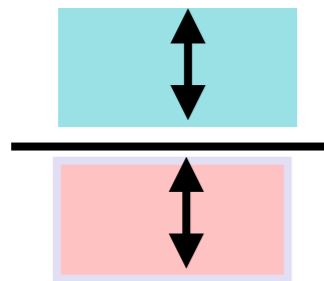
$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1^u}/4 \\ D_{1^u} \end{cases}$$



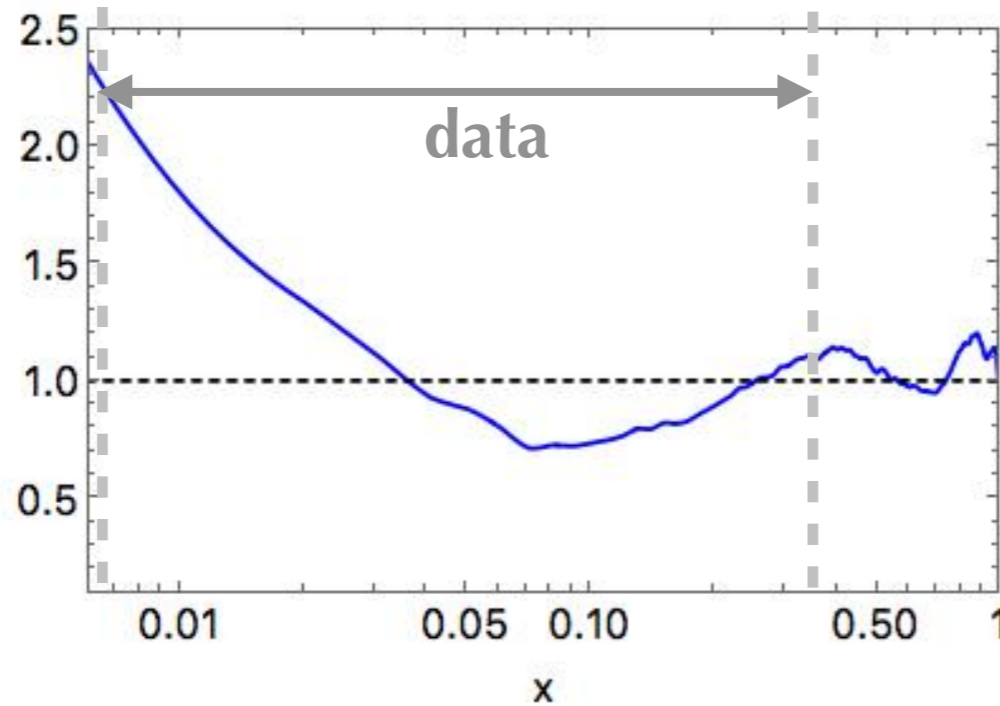
global fit + pseudodata

global fit

*Radici & Bacchetta,  
PRL **120** (18) 192001*



ratio of widths

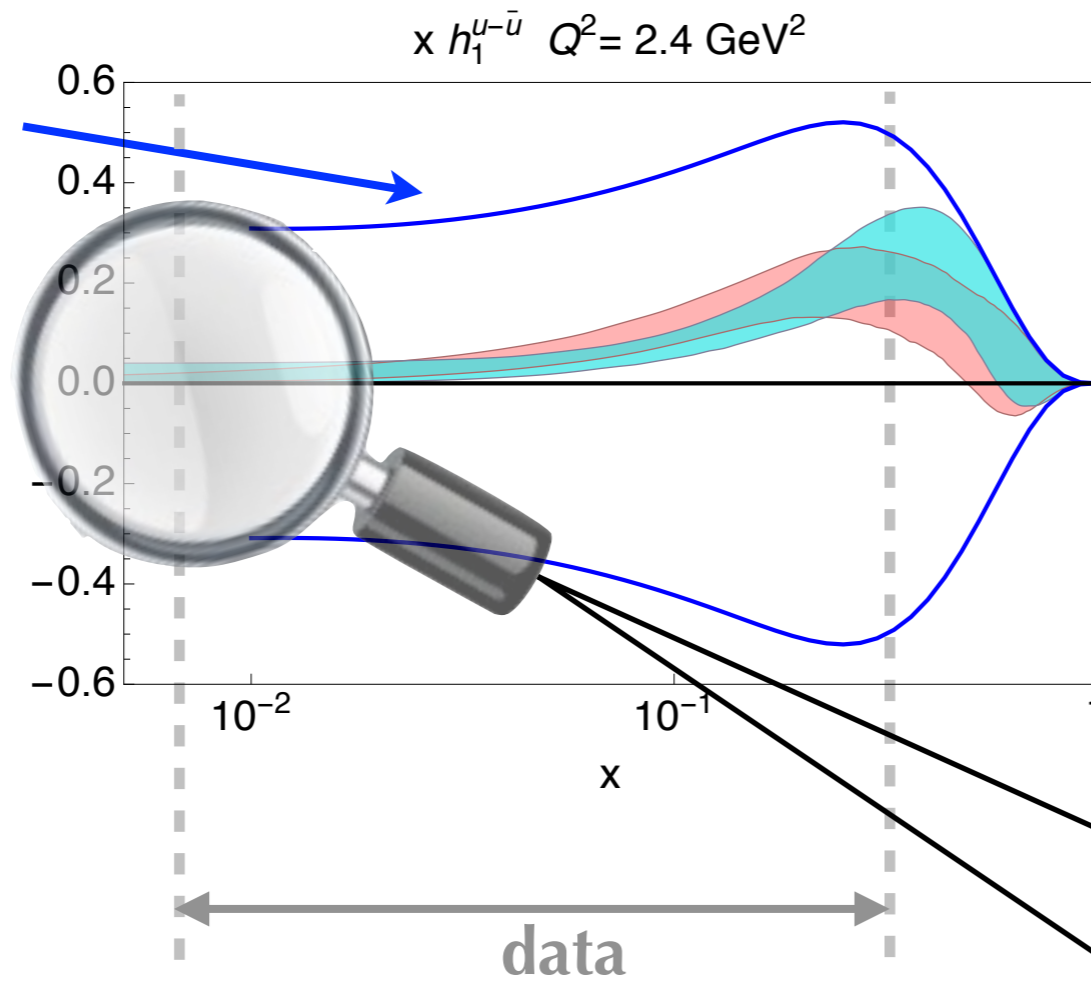


up

# pseudodata impact on up

Soffer bound

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1^u}/4 \\ D_{1^u} \end{cases}$$

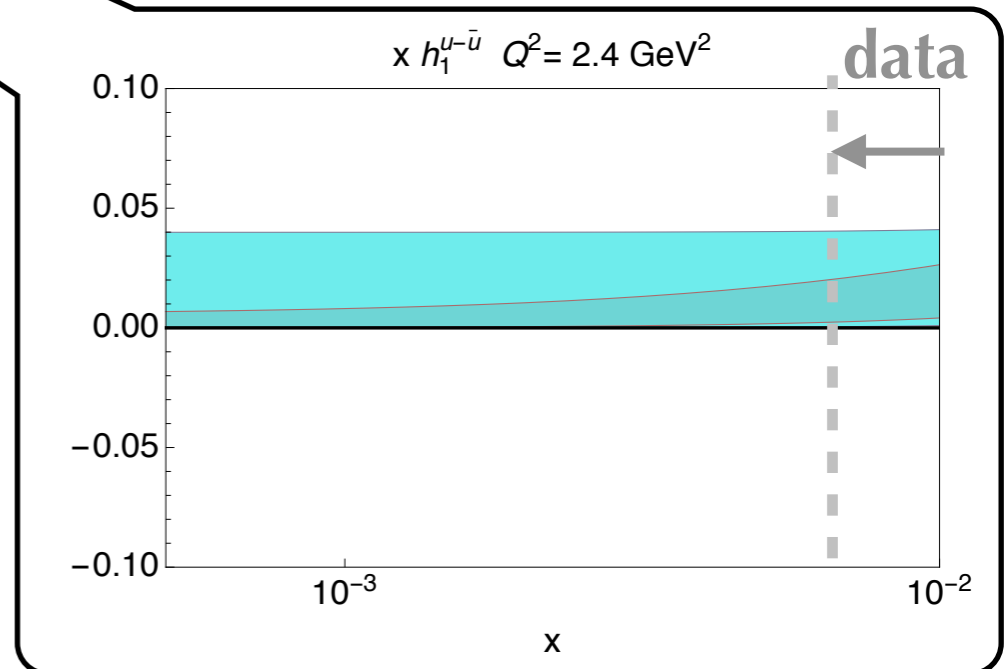


global fit + pseudodata

global fit

*Radici & Bacchetta,  
PRL **120** (18) 192001*

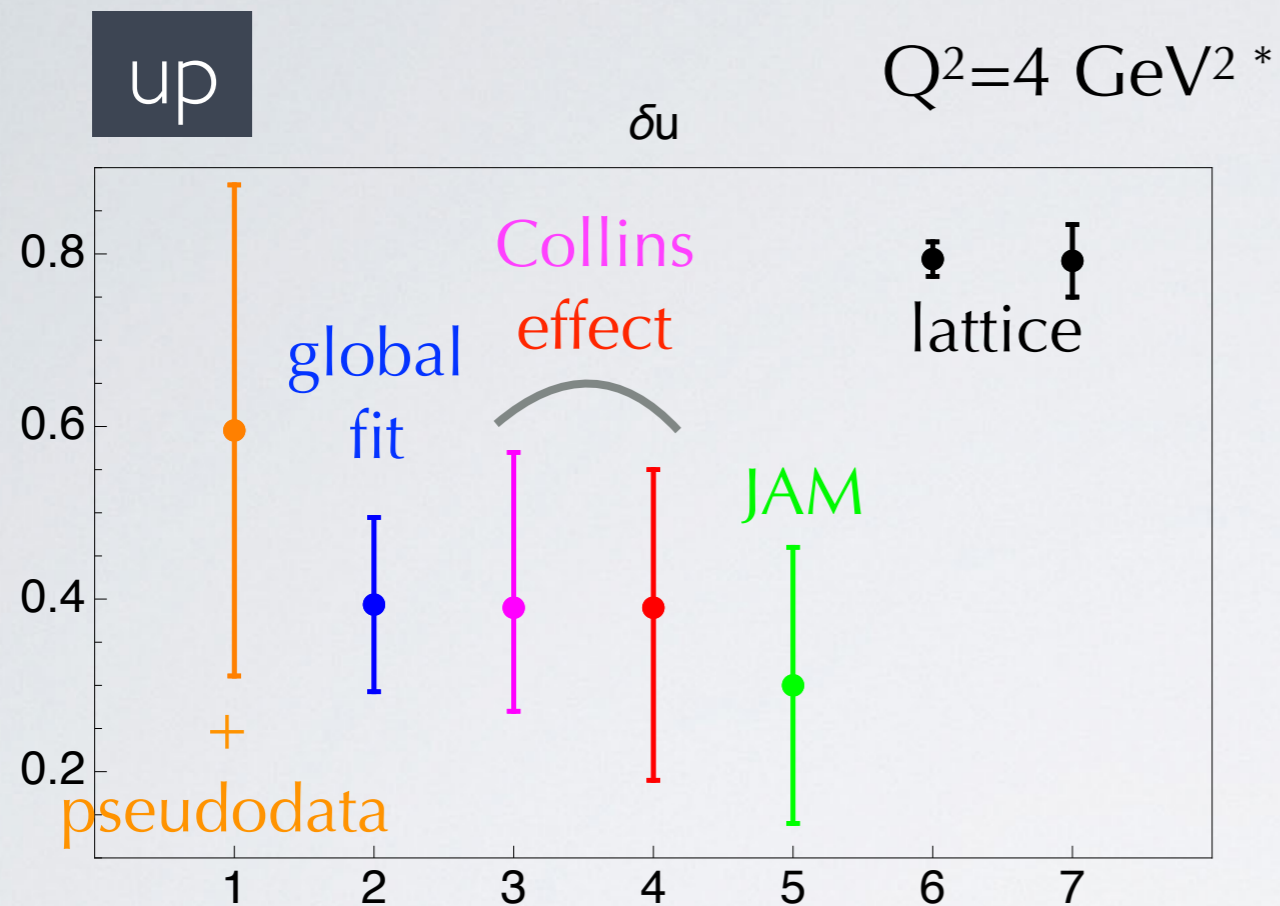
up



$$h_1^q(x) \stackrel{x \rightarrow 0}{\approx} x^{A_q + a_q - 1} \quad A_q + a_q > \frac{1}{3}$$

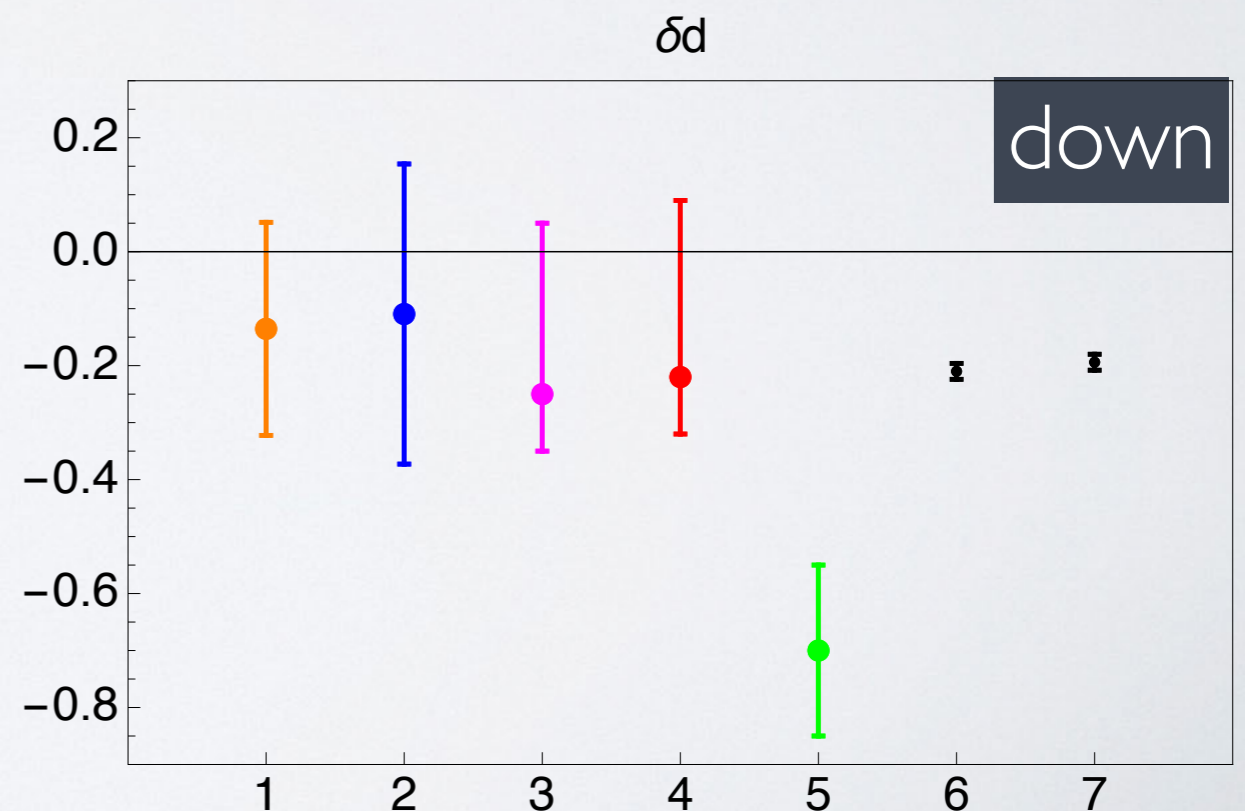
constraint not enough to avoid re-arrangement of replicas outside the range of new pseudodata

# pseudodata impact on tensor charge

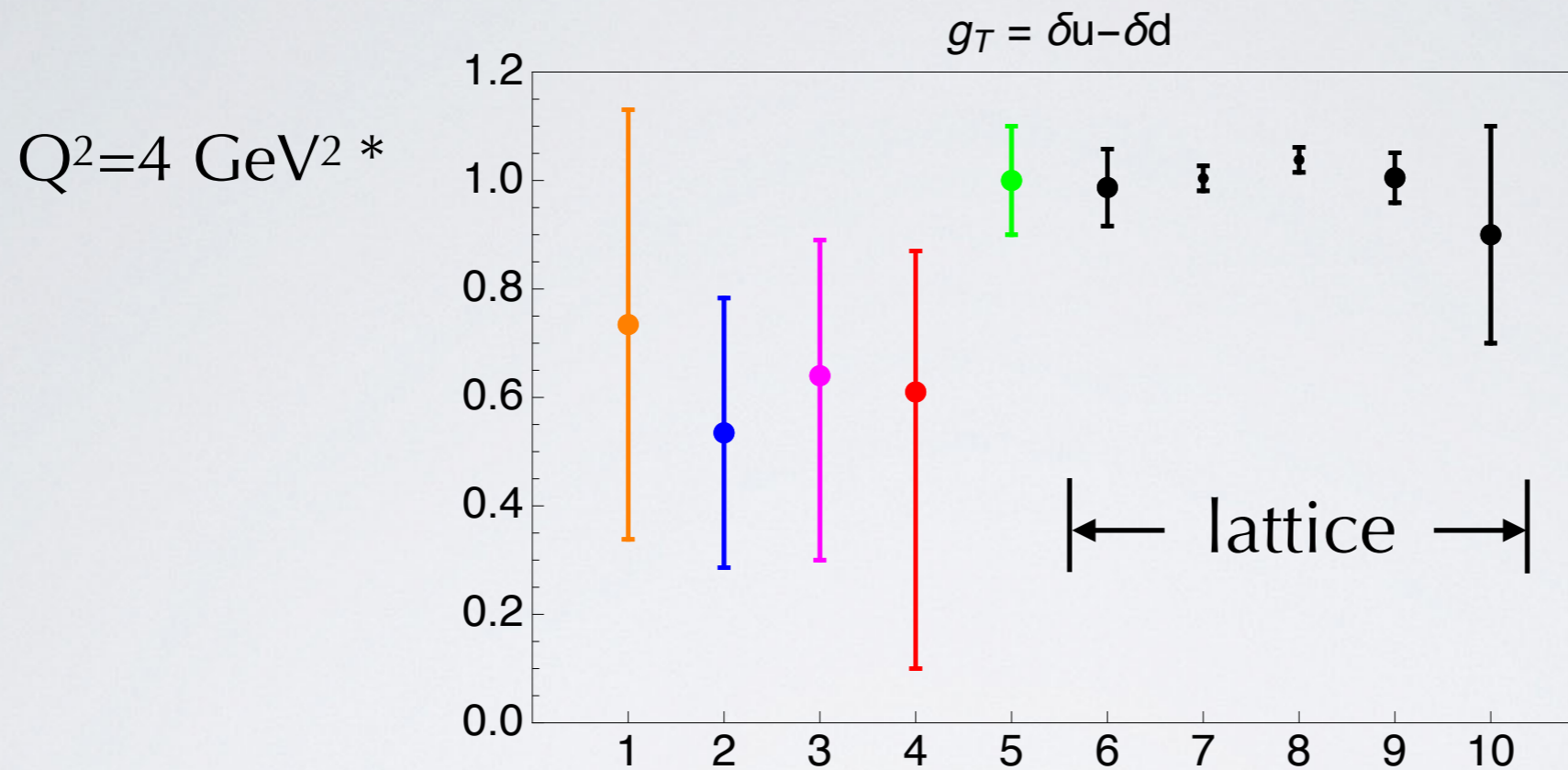


better precision on down  
larger uncertainty on up  
("reversed role" of flavors..)

but global fit re-establishes  
full compatibility  
with lattice



# pseudodata impact on isovector tensor charge

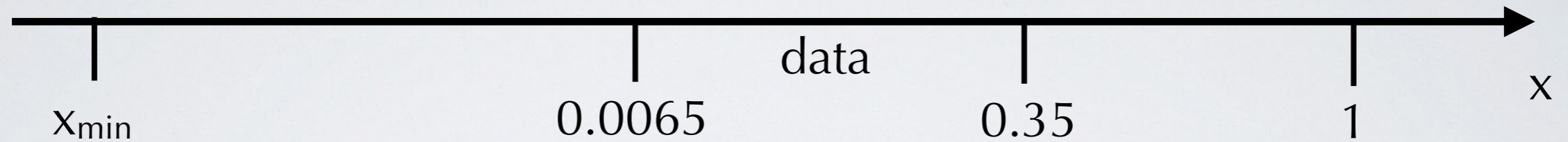


**we can have** simultaneous compatibility about  $\delta u$ ,  $\delta d$ ,  $g_T$  between lattice and our global fit, but because of large uncertainties coming from extrapolation outside the  $x$ -range of data (mainly at low  $x$ )

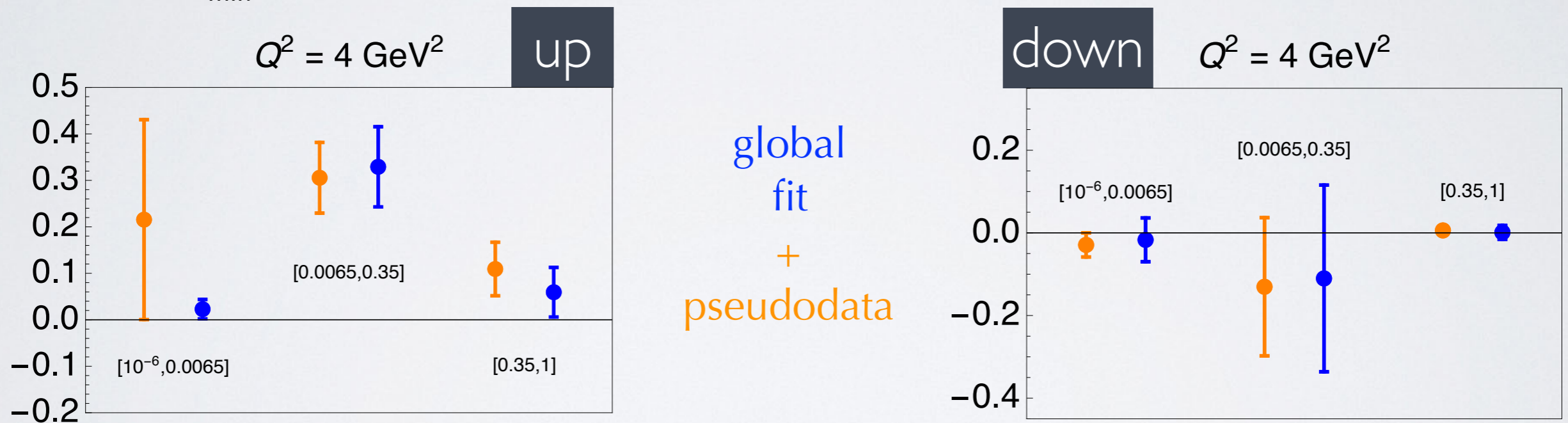


# impact of extrapolation outside data

$$\delta q(Q^2) = \int_{x_{\min}}^1 dx h_1^{q-\bar{q}}(x, Q^2)$$

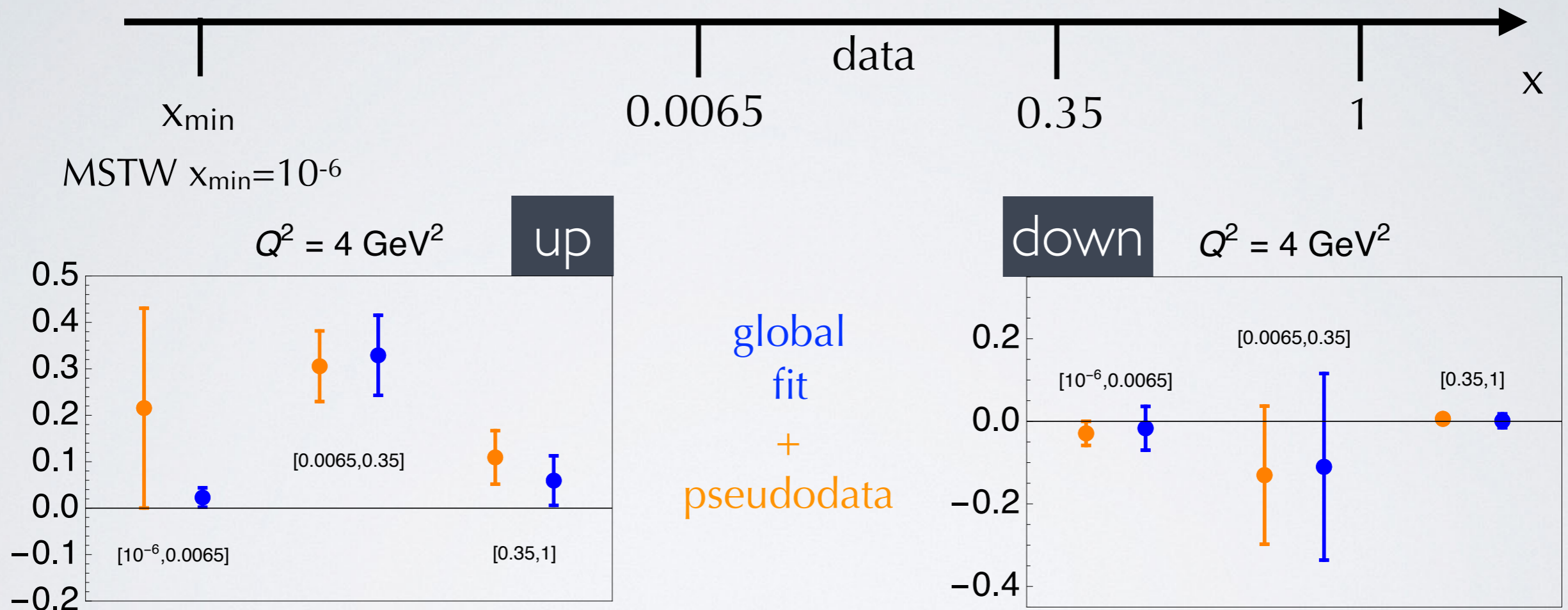


MSTW  $x_{\min}=10^{-6}$



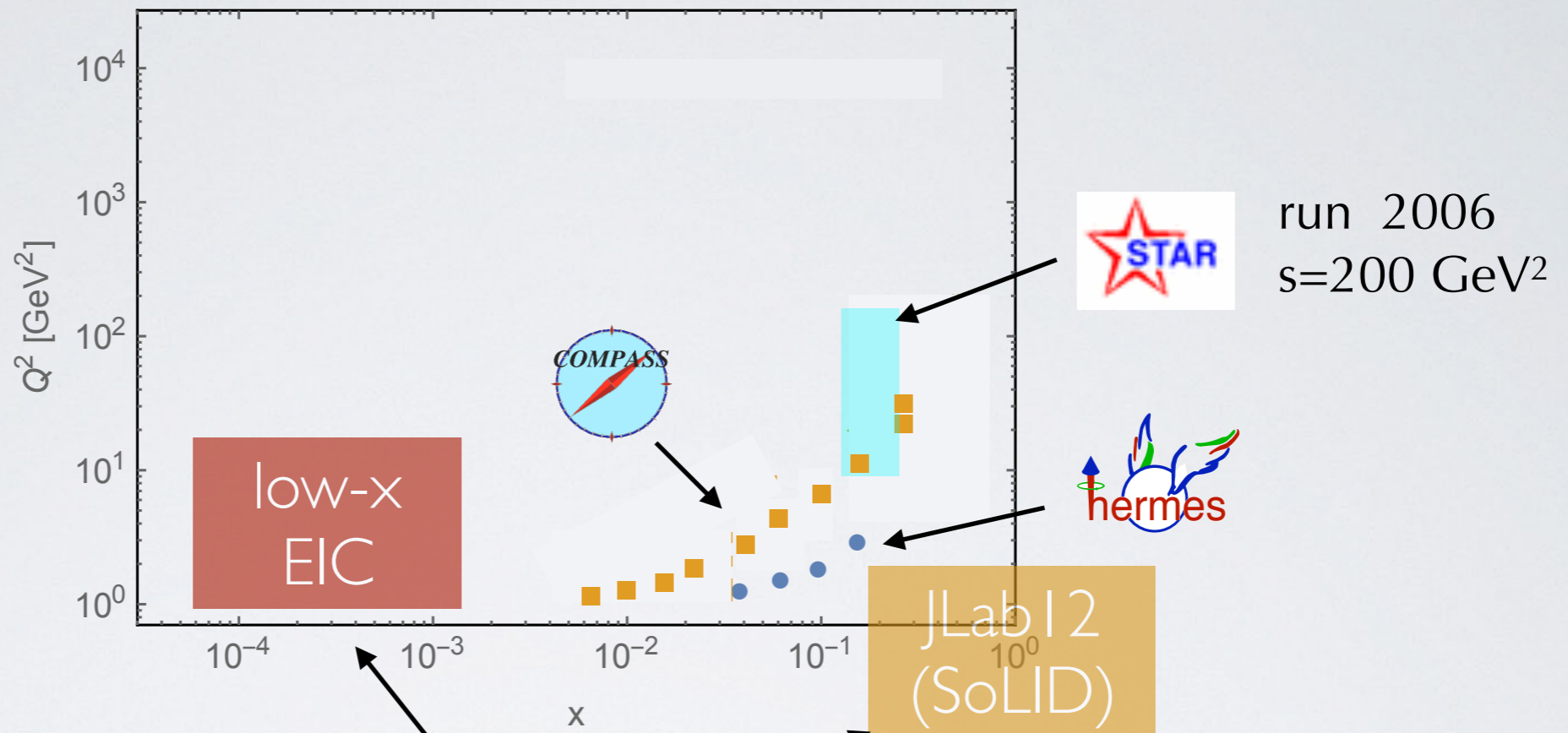
# impact of extrapolation outside data

$$\delta q(Q^2) = \int_{x_{\min}}^1 dx h_1^{q-\bar{q}}(x, Q^2)$$



impact of pseudodata  
 for down: better precision everywhere  
 for up: large uncertainties in extrapolation at low  $x$

# more constraints on extrapolation



- of course, need more data

- theoretical constraints from low-x behavior in dipole picture  
(generalize work on helicity  $\Delta q^S(x, Q^2) \approx \left(\frac{1}{x}\right)^{\alpha_h}$   $\alpha_h = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}$  by

*Kovchegov et al., P.L. **B772** (17) 136*

- theoretical constraints from Burkardt-Cottingham sum rule  $h_1^q(x) \approx x^{A_q+a_q-1}$   $\begin{cases} A_q + a_q > \frac{1}{3} \\ A_q + a_q > 1 \end{cases}$

*Accardi & Bacchetta, P.L. **B773** (17) 632*

# Conclusions

- first global fit of di-hadron inclusive data leading to extraction of transversity as a PDF in collinear framework
- inclusion of STAR  $p$ - $p^\uparrow$  data increases precision of up channel; large uncertainty on down due to unconstrained gluon unpolarized di-hadron fragmentation function
- no apparent simultaneous compatibility with lattice for tensor charge of up, down, and isovector
- adding Compass pseudodata for deuteron reverses the scenario: better down but larger uncertainties on up from extrapolation (mainly at low  $x$ ); reach overall compatibility with lattice
- need data spanning larger  $x$  range; meantime, look for other theoretical constraints on extrapolation (mostly, at low  $x$ )

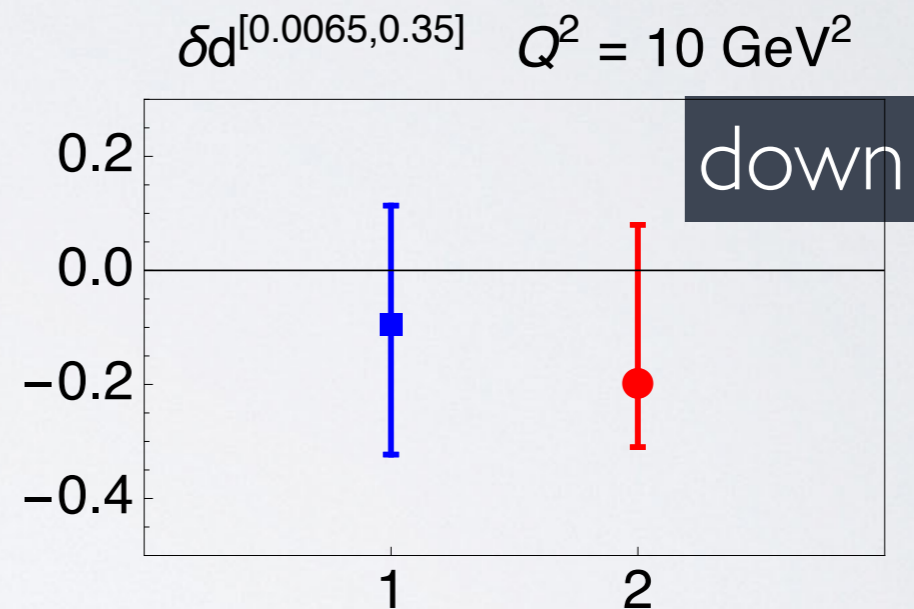
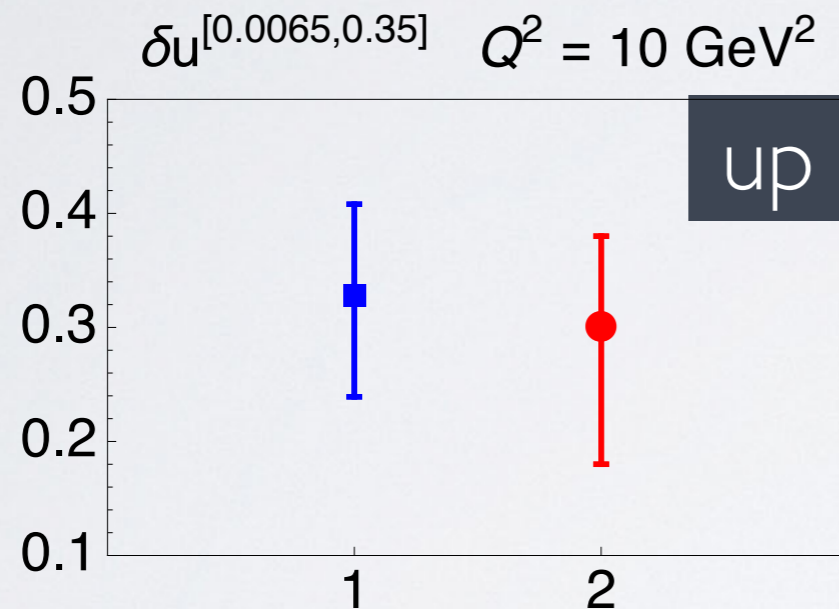
**THANK YOU**

# Back-up

tensor charge  $\delta q(Q^2) = \int dx h_1^{q-\bar{q}}(x, Q^2)$

truncated

$$\delta q^{[0.0065, 0.35]} \quad Q^2 = 10$$



global fit

*Radici & Bacchetta,*  
*P.R.L. 120 (18) 192001*

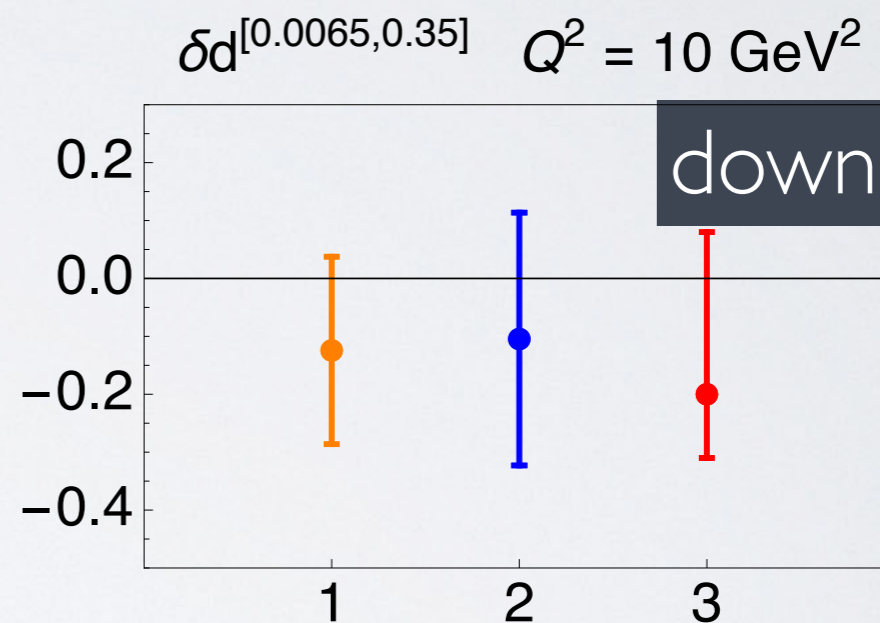
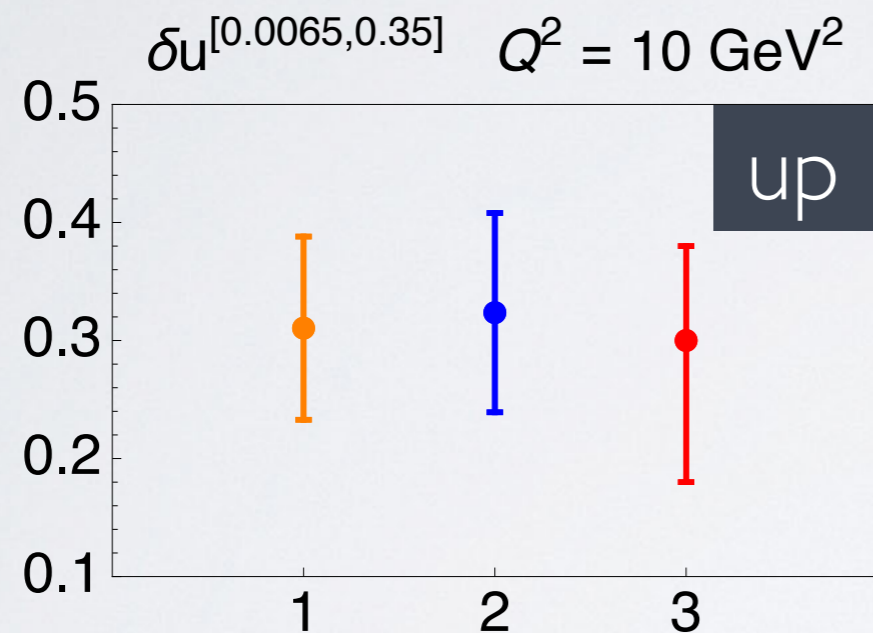
TMD fit

*Kang et al.,*  
*P.R. D93 (16) 014009*

tensor charge  $\delta q(Q^2) = \int dx h_1^{q-\bar{q}}(x, Q^2)$

truncated

$\delta q^{[0.0065, 0.35]} \quad Q^2 = 10$



+  
pseudodata

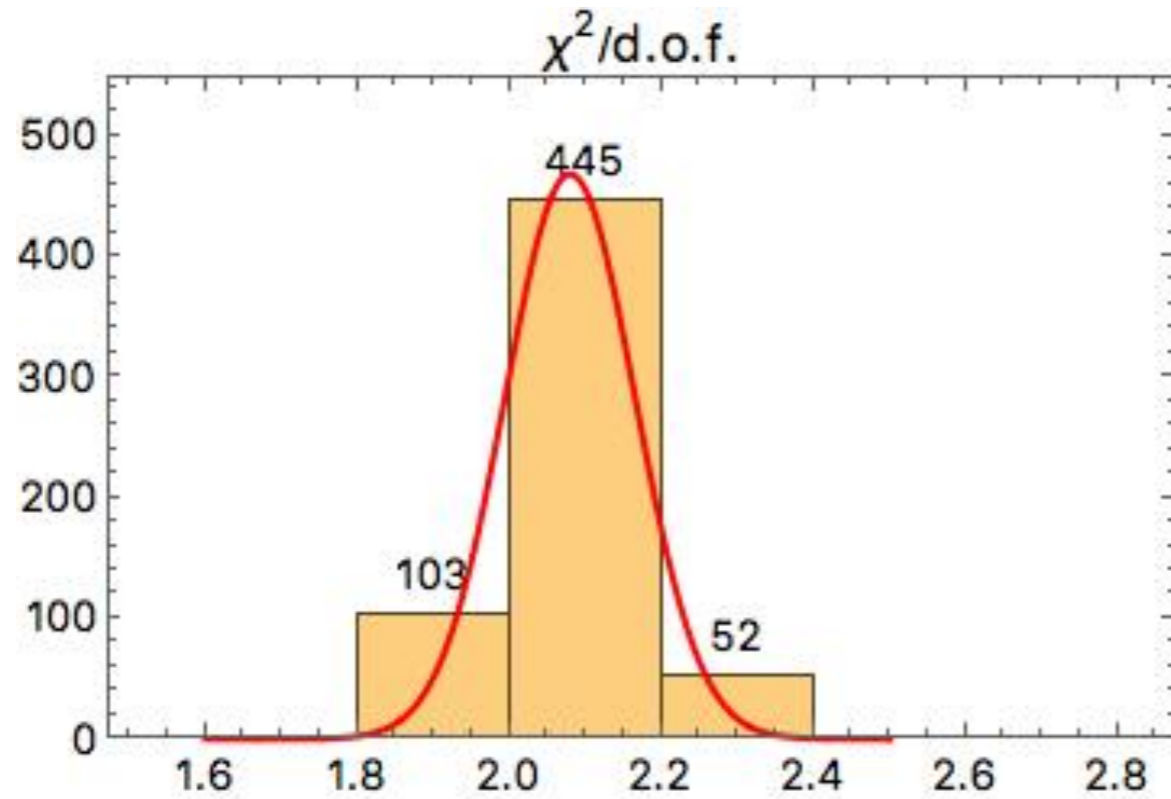
global fit

*Radici & Bacchetta,*  
*P.R.L. 120 (18) 192001*

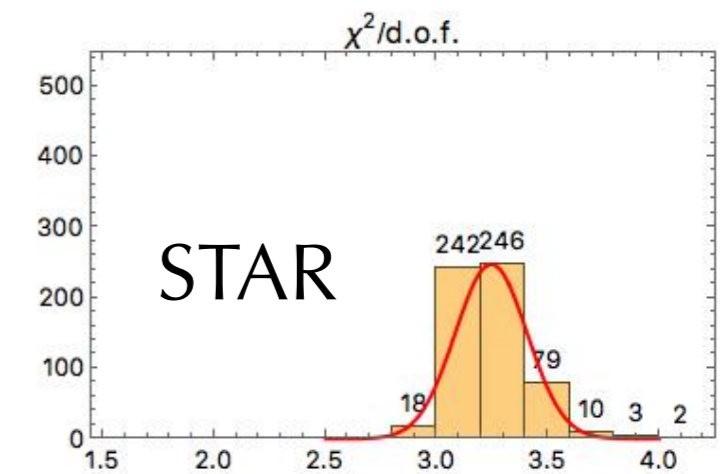
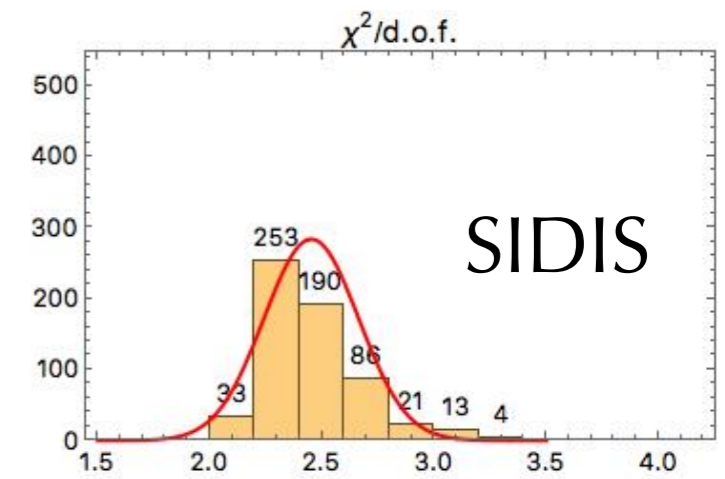
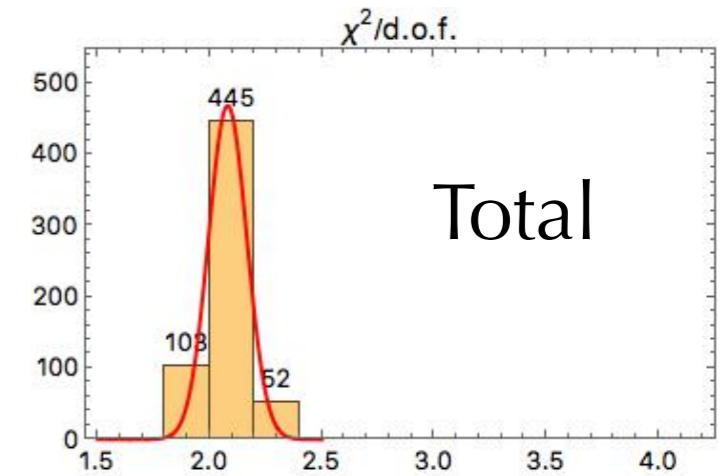
TMD fit

*Kang et al.,*  
*P.R. D93 (16) 014009*

# $\chi^2$ of the fit



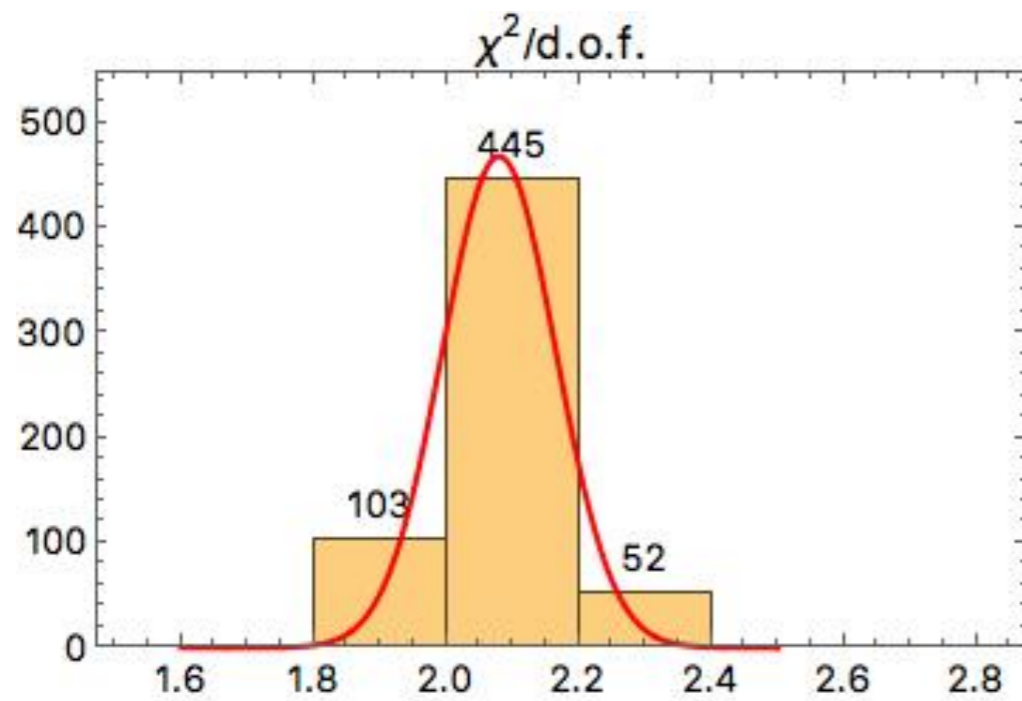
$$\chi^2/\text{dof} = 2.08 \pm 0.09$$





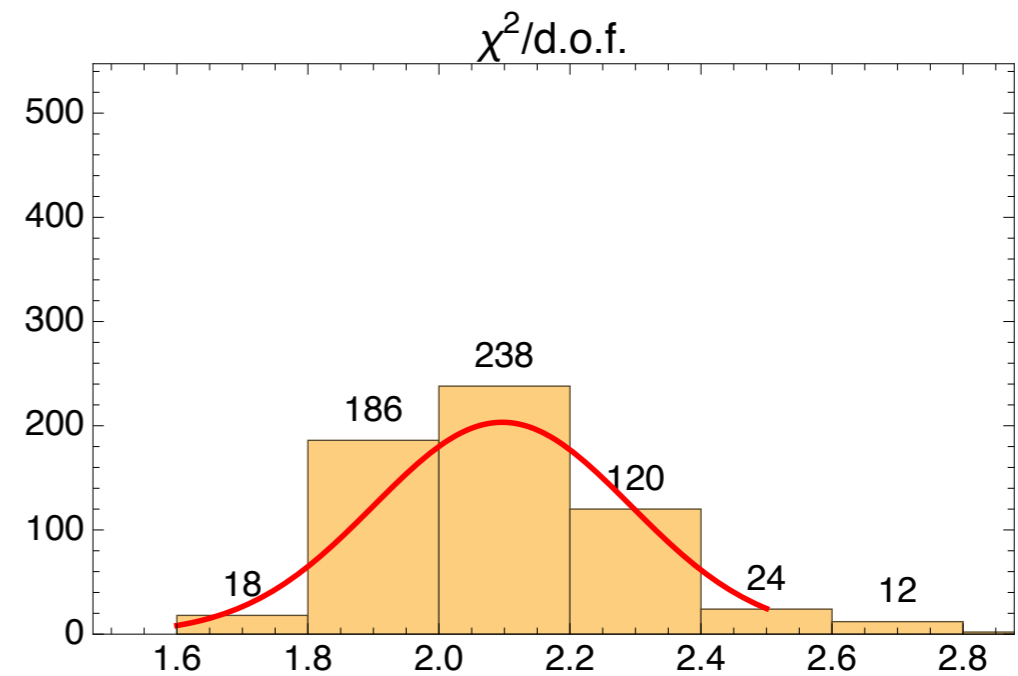
# $\chi^2$ of the fit

global fit



$$\chi^2/\text{dof} = 2.08 \pm 0.09$$

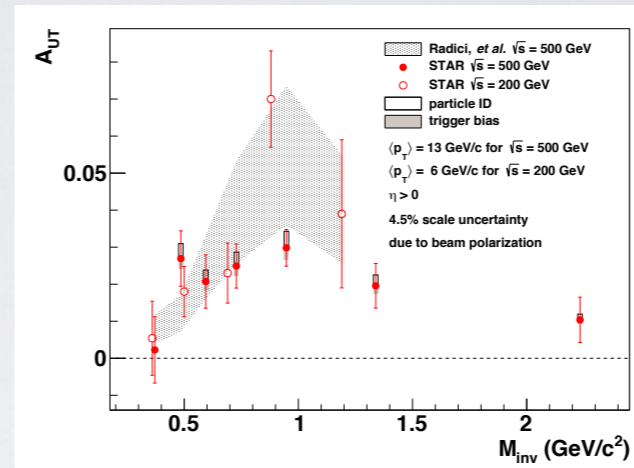
+ pseudodata



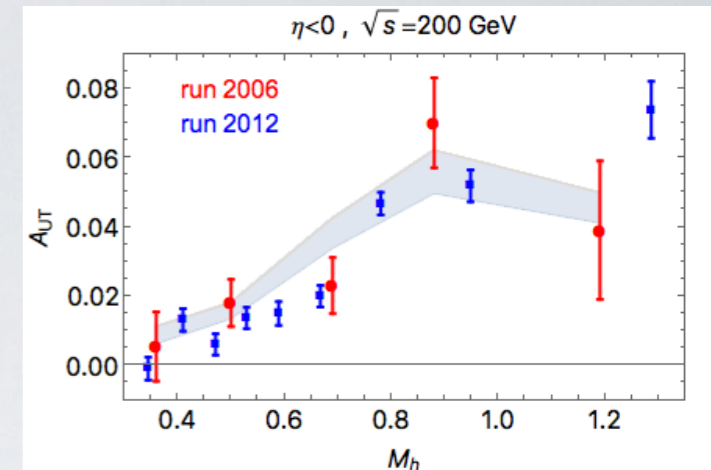
$$\chi^2/\text{dof} = 2.10 \pm 0.20$$

# To do list

- use also other (multi-dimensional) data from STAR run 2011 ( $\sqrt{s}=500$ ) and (later) run 2012 ( $\sqrt{s}=200$ )



Adamczyk et al. (STAR), P.L. **B780** (18) 332



Radici et al., P.R. **D94** (16) 034012

- need data on  $p+p \rightarrow (\pi\pi) X$  constrains gluon  $D_{1g}$
  - refit di-hadron fragmentation functions using new data:  
 $e^+e^- \rightarrow (\pi\pi) X$  constrains  $D_{1q}$   
 (currently only by Montecarlo)
- 
- Seidl et al.,  
P.R. **D96** (17) 032005
- use COMPASS data on  $\pi K$  and  $KK$  channels, and from  $\Lambda^\uparrow$  fragmentation: constrain strange contribution ?
  - explore other channels, like inclusive DIS via Jet fragm. funct.'s