A framework for TMDs in the $P + P \rightarrow \gamma + jet$ process

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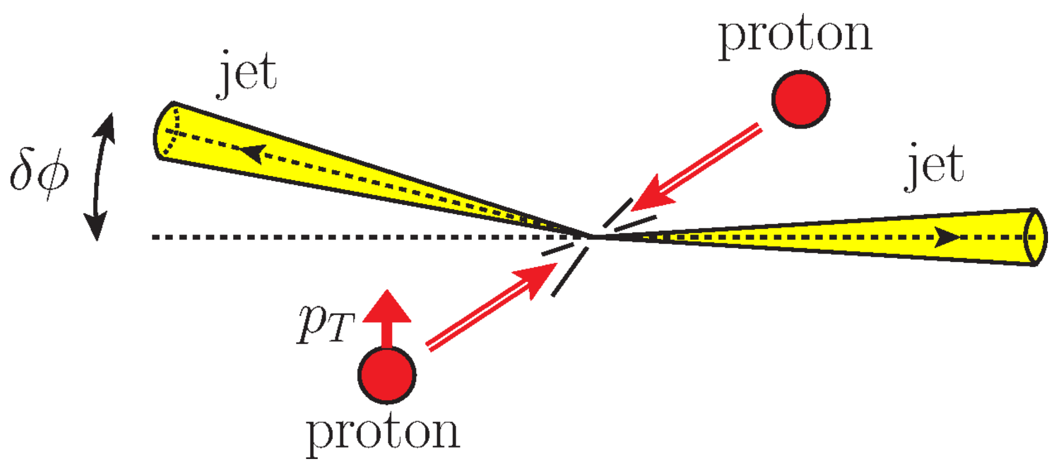
Content - outline

- Motivation – why study the $P + P \rightarrow \gamma + jet$ process?
- Ingredients for factorization form
  - Soft functions – global and jet
  - TMDs
  - Jet
  - …..
- Numerical studies
  - Unpolarized TMDs
  - Sivers
- Conclusions

Motivation

- Why the focus on DY, SIDIS and e^+e^- thus far?
  - Well established framework for these processes
  - Factorization theorems available (Collins, Soper, Sterman, 1980's; Collins, 2011 and others)
  - One of the simplest process in hadronic collisions; well understood
  - Simple color structure
  - Imbalances well defined

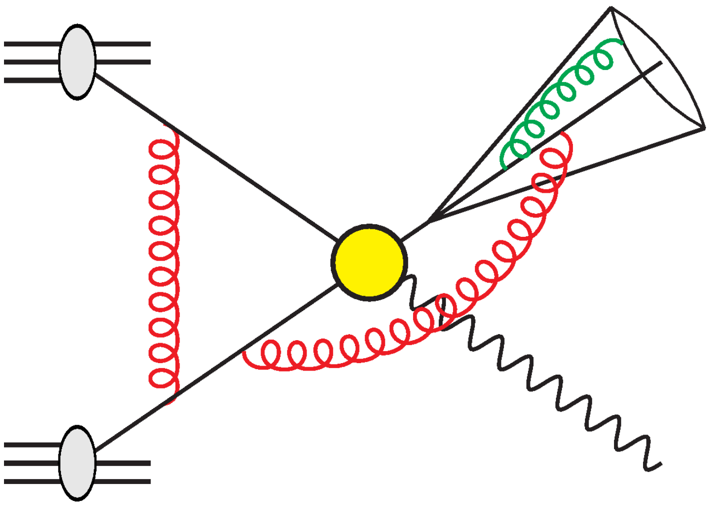


Rogers and Mulders, PRD **81** (2010) 094006; Rogers PRD **88** (2013), 014002

Motivation

- Currently, there is no TMD framework for spin-dependent observables in $\gamma + jet$ production
  - Of course seminal studies by

Qiu, Vogelsang and Yuan, PRD **76**, 074029 (2007), Bacchetta e.a., PRL **99** (2007) 212002



- No factorization theorem exists for TMDs in $P + P \rightarrow \gamma + jet$
  - Examples of factorization breaking effects, color entanglement in proton proton collisions with colored final state particles.

Rogers and Mulders, PRD **81** (2010) 094006; Rogers PRD **88** (2013), 014002.

See also Peng Sun, Yuan and Yuan, PRD **92**, 094007 (2015) for a different situation.

Motivation

- Limiting TMD studies to DY and SIDIS meaning passing over a lot of potentially interesting things

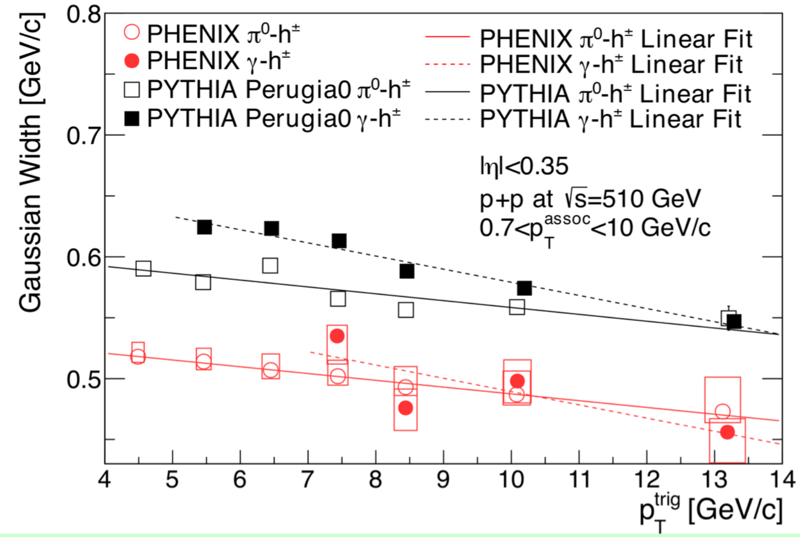
- As a direct follow-up: we also like to understand the effects of factorization breaking effects

Rogers and Mulders, PRD **81** (2010) 094006; Collins and Qiu, PRD 75 (2007) 114014

Motivation

- Processes like $P + P \to \gamma + jet$ can be studied experimentally though

- Example: Gaussian widths
  - Experimental results
  - $P + P \to \gamma + h$
  - Decreasing Gaussian width for increasing p_T^{trig}
  - TMD evolution says otherwise
  - We need a TMD framework for further studies



PNENIX collaboration, PRD **95**, 072002 (2017)

- Motivation: even though factorization breaking is present from a theoretical side, we would like to develop a framework for handling the $P + P \to \gamma + jet$ process.

- This talk: developing such a framework for $P + P \to \gamma + jet$.

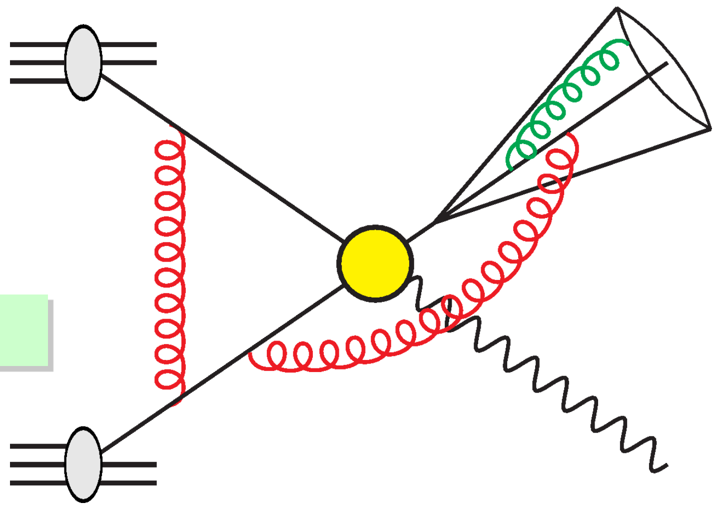
Factorized form

- Out starting points

$$\frac{d\sigma_{PP\to \text{jet}+\gamma}}{dp_\perp dq_\perp} = \int \prod_i^4 d^2k_{i\perp} H^{ab\to c\gamma}(p_\perp)\delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{3\perp} + \vec{k}_{4\perp} - q_\perp)$$

$$\times x_1 f_a(x_1, k_{1\perp})\ x_2 f_b(x_2, k_{2\perp})\ S^{ab\to c\gamma}_{\text{global}}(k_{3\perp}) S^{(1)}_{J_c}(k_{4\perp})\ J_c(p_\perp R)$$

- All of phase space is considered for global soft function
  - Like in e^+e^-
  - 3 instead of 2 colored particles



- Unpolarized case has been established in the past

Vogelsang, Sterman

- There are two soft functions

MGAB, Kang, Lee, Liu (in progress).

Factorized form

- Out starting points

$$\frac{d\sigma_{PP\to\text{jet}+\gamma}}{dp_\perp dq_\perp} = \int \prod_i^4 d^2k_{i\perp} H^{ab\to c\gamma}(p_\perp)\delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{3\perp} + \vec{k}_{4\perp} - q_\perp)$$

$$\times x_1 f_a(x_1, k_{1\perp})\ x_2 f_b(x_2, k_{2\perp})\ S^{ab\to c\gamma}_{\text{global}}(k_{3\perp}) S^{(1)}_{J_c}(k_{4\perp})\ J_c(p_\perp R)$$

- No TMD factorization theorems have been derived for $P + P \to \gamma + jet$

- Partonic channels are
  - $q\bar{q} \to g\,\gamma$
  - $qg \to q\,\gamma$
  - $gg \to g\,\gamma$ starts contributing at sub-leading contributions
  - The above channels, but with (anti)-quarks/gluons interchanged.

MGAB, Kang, Lee, Liu (in progress).

Factorized form

- Out starting points

$$\frac{d\sigma_{PP\to\text{jet}+\gamma}}{dp_\perp dq_\perp} = H^{ab\to c\gamma}(p_\perp)\int \frac{bdb}{2\pi} J_0(q_\perp b) x_1 f_a(x_1, b)$$
$$\times x_2 f_b(x_2, b)\ S^{ab\to c\gamma}_{\text{global}}(b)\ S^{(1)}_{J_c}(b)\ J_c(p_\perp R)$$

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Factorized form (Sivers TMD)

- The factorization form for Sivers TMD is given by

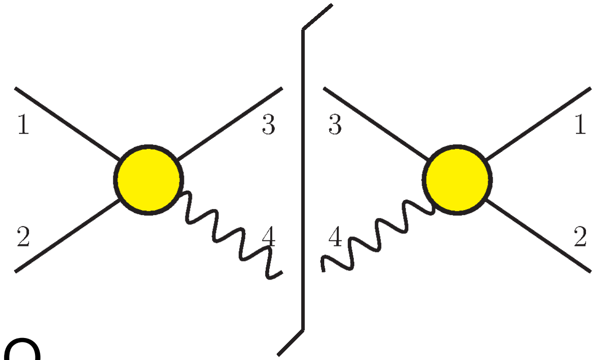
$$\frac{d\sigma_{PP\to \text{jet}+\gamma}}{dp_\perp dq_\perp} = \frac{\epsilon^{\alpha\beta} S_\perp^\alpha q_\perp^\beta}{\vec{q}_\perp^{\,2}} \int \prod_i^4 d^2 k_{i\perp} H_{\text{Sivers}}^{ab\to c\gamma}(p_\perp)\delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{3\perp} + \vec{k}_{4\perp} - q_\perp)$$
$$\times \frac{\vec{k}_{1\perp}\cdot\vec{q}_\perp}{M_P} x_1 f_{1T,a}^{\perp}(x_1, k_{1\perp})\; x_2 f_b(x_2, k_{2\perp}) S_{\text{global}}^{ab\to c\gamma}(k_{3\perp})\; S_{J_c}^{(1)}(k_{4\perp})\; J_c(p_\perp R)$$

- Differences compared to unpolarized situation
  - Dirac structure reflecting other polarization structure
  - Hard function different

NLO global soft function

- The LO + NLO soft function can be written as

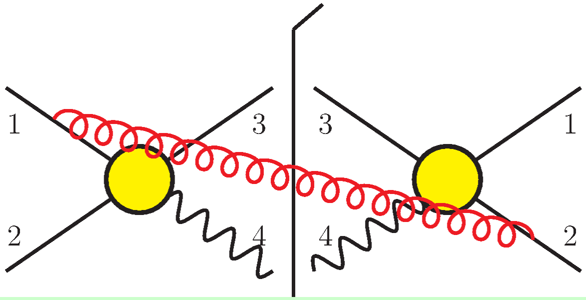
$$S_{\text{global}}^{ab\to c\gamma} = S_0\left(1 + \sum_{i<j}\left[\mathbf{T}_i \cdot \mathbf{T}_j S_{ij}^{(1)} + \text{h.c.}\right]\right)$$

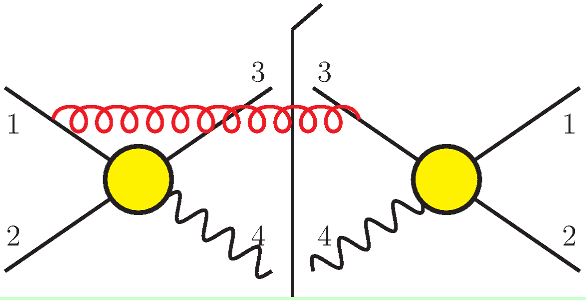


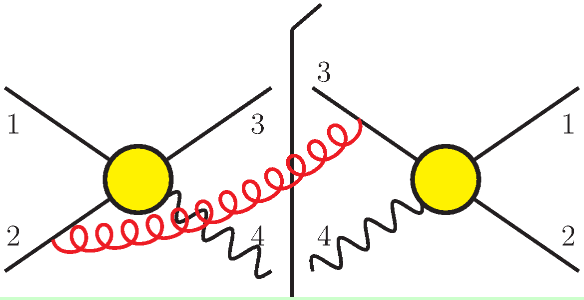
- S_0 is the LO contribution

- $\mathbf{T}_i \cdot \mathbf{T}_j$ are the NLO color parts relative to LO
  - *Generally a matrix in color configuration space*
  - *Different for each partonic channel*

- $S_{ij}^{(1)}$ are the NLO kinematic parts relative to LO







A similar situation has been considered in Hornig, JHEP **12** (2017) 043 for the transverse energy situation.

NLO global soft function - color

- Consider partonic channel with *i* partons in initial and *j* in final state
  - At NLO, color part captured by $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$

- Consider partonic field configuration and give all the *I* color config.

Examples

$\boldsymbol{q^{\alpha}\bar{q}^{\beta}g^{a_3}}$

$$\theta_1 = \bar{\xi}_2 T^{a_3}_{\beta\alpha} \xi A^{a_3}$$

$\boldsymbol{g^{a_1}g^{a_2}g^{a_3}}$

$$\theta_1 = i f^{a_1 a_2 a_3} A^{a_1} A^{a_2} A^{a_3}$$

$$\theta_2 = d^{a_1 a_2 a_3} A^{a_1} A^{a_2} A^{a_3}$$

- We don't require $g^{a_1} g^{a_2} g^{a_3}$: but more than 1 conf. is possible

- When more color configurations *I* and *J* exist, the soft (and hard) function become matrices in color configuration space

Similar and different uses can be found in e.g. Moult e.a., PRD **93** (2016), 094003.

NLO global soft function - color

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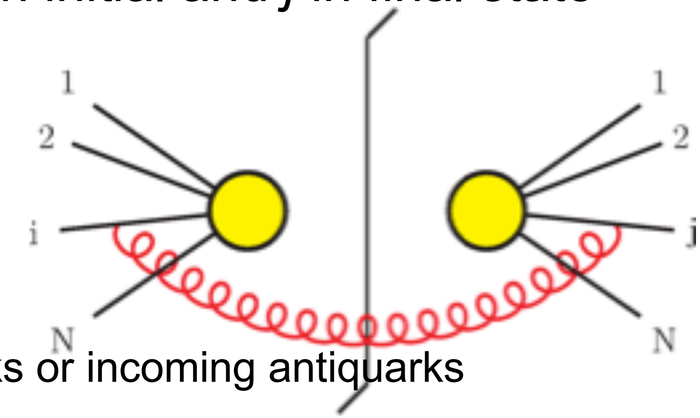


- For the calculation of $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$, use

$$(T_i^c \theta_I)_{\dots\alpha_i\dots} = T^c_{\alpha_i\beta_i}\theta_{I,\dots\beta_i\dots}$$ outgoing quarks or incoming antiquarks

$$(T_i^c \theta_I)_{\dots\alpha_i\dots} = -T^c_{\beta_i\alpha_i}\theta_{I,\dots\beta_i\dots}$$ incoming quarks or outcoming antiquarks

$$(T_i^c \theta_I)_{\dots\alpha_i\dots} = i f^{\alpha_i c \beta_i}\theta_{I,\dots\beta_i\dots}$$ gluons

- Then, $\left(\boldsymbol{T}_i \cdot \boldsymbol{T}_j\right)_{I\,J} \propto \theta_I^{T_i}\bar{\theta}_J^{T_j}$ (proportionality)

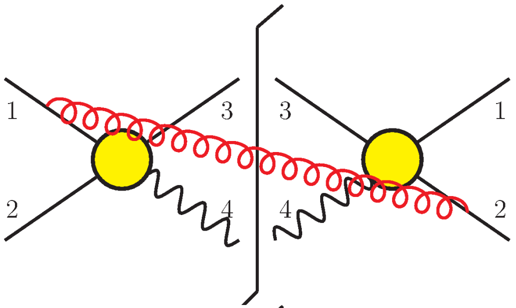
- When more color configurations *I* and *J* exist, the soft (and hard) function become matrices in color configuration space

Similar and different uses can be found in e.g. Moult e.a., PRD **93** (2016), 094003.

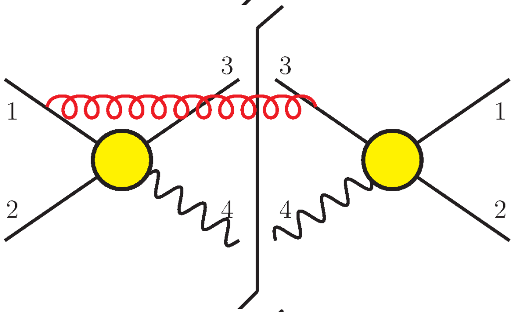
NLO global soft function - color

- At NLO the color part is captured by $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$

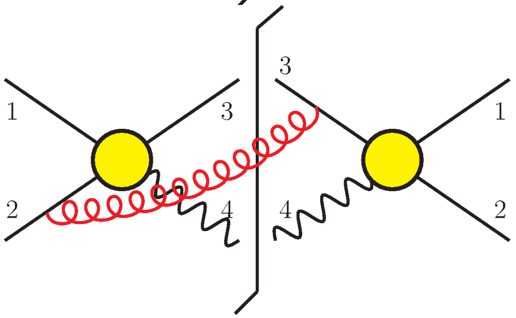
- $$\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 = \begin{cases} -\left(C_F - \frac{1}{2}C_A\right) & \text{for } q\bar{q} \to g\,\gamma \\ -\frac{1}{2}C_A & \text{for } qg \to q\,\gamma \end{cases}$$



- $$\boldsymbol{T}_1 \cdot \boldsymbol{T}_3 = \begin{cases} -\frac{1}{2}C_A & \text{for } q\bar{q} \to g\,\gamma \\ -\frac{1}{2}C_A & \text{for } qg \to q\,\gamma \end{cases}$$



- $$\boldsymbol{T}_2 \cdot \boldsymbol{T}_3 = \begin{cases} -\frac{1}{2}C_A & \text{for } q\bar{q} \to g\,\gamma \\ -\left(C_F - \frac{1}{2}C_A\right) & \text{for } qg \to q\,\gamma \end{cases}$$



NLO global soft function

- The kinematic NLO contributions $S_{ij}^{(1)}$ are calculated in the η regulator scheme

$$S_{12}^{(1)}(\vec{b}) = -\frac{\alpha_s}{4\pi}\left[\frac{4}{\eta}\left(-\frac{1}{\epsilon} - \ln\frac{\mu^2}{\mu_b^2}\right) + \mathcal{O}(\epsilon^{-2}) + \mathcal{O}(\epsilon^{-1}) + \mathcal{O}(\epsilon^{0})\right]$$

$$S_{13}^{(1)}(\vec{b}) = -\frac{\alpha_s}{4\pi}\left[\frac{2}{\eta}\left(-\frac{1}{\epsilon} - \ln\frac{\mu^2}{\mu_b^2}\right) + \mathcal{O}(\epsilon^{-2}) + \mathcal{O}(\epsilon^{-1}) + \mathcal{O}(\epsilon^{0})\right]$$

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- We use the η regulator scheme

$$\int \frac{dk^-}{k^-} \to \int \frac{dk^-}{k^-}\left(\frac{\nu}{2k_z}\right)^{\eta}$$

Duff Neill e.a., JHEP **1205** (2012) 084

MGAB, Kang, Lee, Liu (in progress). A similar situation has been considered in Hornig, JHEP **12** (2017) 043 for the transverse energy situation.

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- Global soft function is then given by

$$S_{\text{global}}^{(0)+(1)} = S_0\left(1 + C_{12}S_{12}^{(1)}(\vec{b}) + C_{13}S_{13}^{(1)}(\vec{b}) + C_{23}S_{23}^{(1)}(\vec{b})\right)$$

where the C_{ij} are color factors, different for each partonic channel

MGAB, Kang, Lee, Liu (in progress). A similar situation has been considered in Hornig, JHEP **12** (2017) 043 for the transverse energy situation.

Consistency check: rapidity divergences

- From the global soft function

$$S_{\text{global}}^{(0)+(1)} = S_0 \left(1 + C_{12} S_{12}^{(1)}(\vec{b}) + C_{13} S_{13}^{(1)}(\vec{b}) + C_{23} S_{23}^{(1)}(\vec{b})\right)$$

we then get for the rapidity divergent part for $q\bar{q} \to g\,\gamma$

$$\frac{\alpha_s}{4\pi}(2C_{12} + C_{13} + C_{23})\frac{2}{\eta} + \text{TMD PDF}\ \eta \text{ dependent part} = 0$$

and similarly for $qg \to q\,\gamma$.

Divergences cancel!

Consistency checks: anomalous dimensions

- Also, the anomalous dimensions should cancel in a particular way. This is required for having a consistent factorization form.

- For $q\bar{q} \to g\,\gamma$

$$2\gamma_\nu^{f_q} + \gamma_\nu^{\text{global}} = 0$$

$$\gamma_\mu^{f_q} + \gamma_\mu^{f_{\bar{q}}} + \gamma_\mu^{\text{global}} + \gamma_\mu^{S_{J_g}} + \gamma_\mu^{J_g} + \gamma_\mu^{H_{q\bar{q}}} = 0$$

- For $qg \to q\,\gamma$

$$\gamma_\nu^{f_q} + \gamma_\nu^{f_g} + \gamma_\nu^{\text{global}} = 0$$

$$\gamma_\mu^{f_q} + \gamma_\mu^{f_g} + \gamma_\mu^{\text{global}} + \gamma_\mu^{S_{J_q}} + \gamma_\mu^{J_q} + \gamma_\mu^{H_{qg}} = 0$$

- This works out fine too!

Soft functions vs imbalance

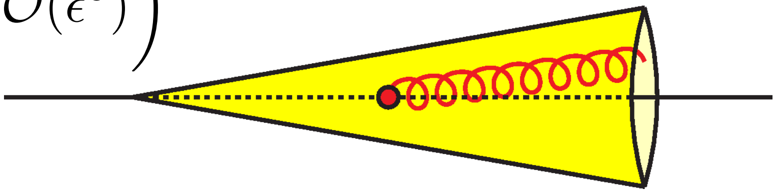
- The imbalances of the final state products (jet + γ) is caused only by the radiation outside of the jet.

- However, the global soft functions have no phase space restrictions.

- We have to subtract all the soft radiation inside the jet.

- Soft-collinear function describes the radiation inside the jet necessary to subtract from the global soft.

$$S_{J_i}^{(1)}(\vec{b}) = \frac{\alpha_s C_i}{2\pi}\left(-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{\mu^2}{\mu_b^2 R^2} + \mathcal{O}(\epsilon^0)\right)$$



- *R* is the jet-radius and there is no rapidity divergence

See also Y-T. Chien. et al, PRD 93, 014033 (2015)

Jet

- Depending on channel, we have the NLO quark or gluon jet

$$J^q(p_T, R, \mu) = 1 + \frac{\alpha_s}{\pi}\Big[L^2 - \frac{3}{2}L + \frac{13}{4} - \frac{3\pi^2}{8}\Big]$$
$$J^g(p_T, R, \mu) = 1 + \frac{\alpha_s}{\pi}\Big[L^2 - \frac{\beta_0}{2C_A}L + \frac{67}{18} - \frac{3\pi^2}{8} - \frac{T_F n_f}{C_A}\frac{23}{18}\Big]$$

with $L = \ln \frac{p_T R}{\mu}$.

- *R* is jet radius

- As mentioned before: radiation inside the jet has to be subtracted
  - This radiation does not cause the imbalance

Ellis, Vermilion, Walsh, Hornig and Lee, JHEP **11** (2010) 101

TMDs - unpolarized

- The LO expressions are
  $f_{q/q}^{(0)}(x,\vec{k}_\perp^2) = \delta(1-x)\delta(\vec{k}_\perp^2)$

- Fourier transformation

  $$f_{q/q}^{(0)}(x,b) = \int d^2\vec{k}_\perp \, e^{-i\,\vec{k}_\perp\cdot\vec{b}}\delta(1-x)\delta(\vec{k}_\perp^2) = \delta(1-x)$$

- The NLO expression is given by

$$f_{q/q}^{(1)}(x,b,\mu,\nu) = \frac{\alpha_s}{2\pi}C_F\Big\{\Big(-\frac{1}{\epsilon}-L_T\Big)P_{qq}(x)\Big\}$$
$$+\frac{\alpha_s}{2\pi}C_F\Big\{(1-x)-\delta(1-x)\Big[-\frac{1}{2}L_T^2-\frac{3}{2}L_T+L_T\ln\frac{(p^-)^2}{\mu_b^2}+\frac{\pi^2}{12}\Big]\Big\}$$

with $L_T = \ln\frac{\mu^2}{\mu_b^2}$ and $\mu_b = 2e^{-\gamma_E}/b$.

TMDs - Sivers

- Quark Sivers function is known

$$f_{1T}(x, b_\perp) = \frac{\alpha_s}{2\pi}\left(\frac{-ib_\perp^\alpha}{2}\right)\left\{\dots\dots\dots\right\}$$

- For Sivers, hard function needs rescaling with different color factors compared to unpolarized TMDs
  - $\frac{N_c^2+1}{N_c^2-1}$ for $q\bar{q} \to \gamma g$ and $\bar{q}q \to \gamma g$
  - $-\frac{N_c^2+1}{N_c^2-1}$ for $qg \to \gamma q$

- Two separate methods to calculate these color factors
  - Through gauge links calculations
  - The method described in

MGAB, PhD thesis (2015) and references therein; Bomhof and Mulders JHEP **02** (2007) 029

Qiu, Vogelsang and Yuan, PRD **76**, 074029 (2007)

Evolution

- Evolution has to be included
- There is evolution for the rapidity scale parameter ν
- There is evolution for the parameter μ
- TMD evolution considered before for other processes: standard in SCET
- We only show perturbative here, obviously also non-perturbative parts should be included

Evolution

- For TMDs, we use

$$f_{q/P}(\vec{b},\mu,\nu) = \exp\Big[\int_{\mu_f}^{\mu} d\ln\mu' \Big[\frac{\alpha_s C_F}{\pi}\Big(\ln\frac{\nu^2}{Q^2}+\frac{3}{2}\Big)\Big]\Big(\frac{\nu}{\nu_f}\Big)^{\gamma_\nu^f(\mu_f)}\Big]$$

- Evolution global soft function $S^{\text{global}}(\vec{b},\mu,\nu)$ different for $q\bar{q}\to\gamma g$ and $qg\to\gamma q$

$$\exp\Big(\int_{\mu_{\text{global}}}^{\mu} d\ln\mu' \frac{\alpha_s}{\pi}\Big(-2C_F\ln\frac{\nu^2}{\mu^2}+C_A\ln\frac{\mu^2}{\mu_b^2}\Big)\Big)\Big(\frac{\nu}{\nu_{\text{global}}}\Big)^{\gamma_\nu^{\text{global}}(\mu_{\text{global}})}$$

$$\exp\Big(\int_{\mu_{\text{global}}}^{\mu} d\ln\mu' \frac{\alpha_s}{\pi}\Big(-(C_F+C_A)\ln\frac{\nu^2}{\mu^2}+C_F\ln\frac{\mu^2}{\mu_b^2}-2(C_A-C_F)y_J\Big)\Big)\Big(\frac{\nu}{\nu_{\text{global}}}\Big)^{\gamma_\nu^{\text{global}}(\mu_{\text{global}})}$$

Evolution

- The evolved cross section contribution would be

$$\int \frac{b\,db}{2\pi} J_0(q_\perp b) f_{q/P}(\vec{b},\mu,\nu_f) f_{\bar{q}/P}(\vec{b},\mu,\nu_f) S^{\rm global}(\vec{b},\mu_b,\nu_{\rm glo.}) S^{J_g}(\vec{b},\mu_b R,\nu_{S_J})$$

$$\times \exp\Big[\int_{\mu_b}^{\mu} d\ln\mu' \Big(\frac{2\alpha_s C_F}{\pi}\Big(\ln\frac{\mu^2}{Q^2}+\frac{3}{2}\Big)+\frac{\alpha_s C_A}{2\pi}\Big(\ln\frac{\mu^2}{\mu_b^2}-\ln\frac{\mu^2}{\mu_b^2 R^2}\Big)\Big)\Big]$$

$$\times \exp\Big[\int_{\mu_b R}^{\mu_b} d\ln\mu' \Big(-\frac{\alpha_s C_A}{\pi}\ln\frac{\mu^2}{\mu_b^2 R^2}\Big)\Big]$$

- and a similar expression for the other partonic channel

Numerics

- There are two competing effects
  - The relative contributions of quarks and gluons changes
  - As is well known, TMD evolution will make the Gaussian width larger

- Because we now have both quarks and gluon TMDs simultaneously, this introduces the interplay between various effects.

- Stay tuned

Conclusions

- Motivation: even though factorization breaking is present from a theoretical side, we would like to develop a framework for handling the $P + P \to \gamma + jet$ process.

- This talk: developing such a framework for $P + P \to \gamma + jet$.

- Stay tuned for numerical NLO results for Sivers TMD in this process.

- Opens up new possibilities for the future