Santa Fe, 20 May 2018

Collinear QCD as a starting point

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abstract

QCD evolution 2018 (Santa Fe, 20 May 2018):

Collinear QCD as a starting point

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Going beyond the collinearity of QCD may require more than just resummation. As an example, I will discuss Wilson loop GTMDs and the possible color entanglement and its unwinding in DY. I will also discuss an attempt to understand the transition from 1D to 3D in a different context, where not only hadrons are entangled states of partons, but also quarks as well as leptons are maximally entangled states. Even if it does not upset the present phenomenology, it might provide a new way to look at many aspects of electroweak and strong interactions as well as quark-hadron duality.

QCD – From Wilson loops to entangled states

- Parton-hadron duality in hard QCD scattering: PDFs x FFs
  - nucleon is pure state → ensemble of partons (good light-front states) [see also Kharzeev & Levin (1702.03489)]
  - Parton physics in essence collinear d.o.f. $\frac{1}{2}\gamma^-\gamma^+\psi$ and $g_T^{\alpha\mu} A_\mu^a$
  - hard (short distance) process: partons → partons
  - emerging partons are pure state(s) → ensemble of hadron states

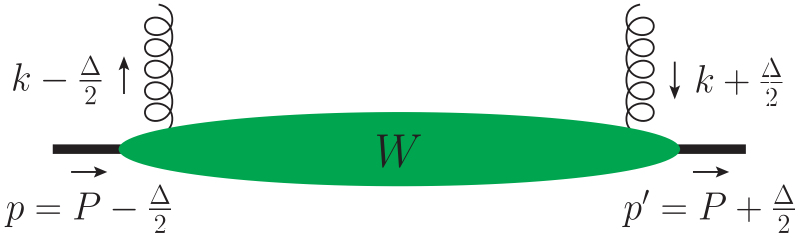
- For QCD Wilson loops and their expectation values (GTMDs, TMDs) may play a crucial role in the transition 1D → 3D (1805.05219)

- Color entanglement in DY (1709.04935)

- More speculative: maybe not only hadrons but also quarks and leptons live in a multipartite Hilbert space! (1801.03664)

WILSON LINES AND GLUON (G)TMDs

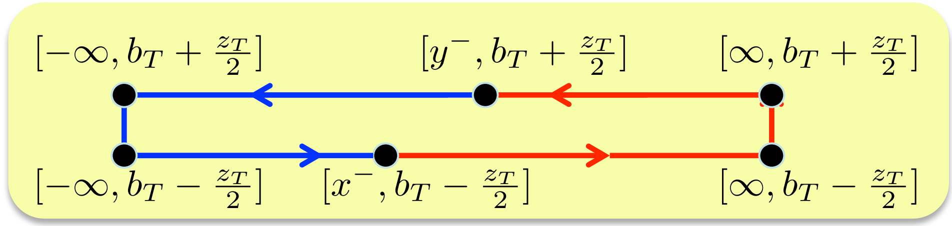
Simplification of gluon GTMDs (x → 0)



$$x = k^+/P^+$$
$$2\xi = \Delta^+/P^+$$

$$\langle p'|p\rangle = \int \underbrace{d(b{\cdot}P)d^2b_T}_{d^3b}\; e^{-i\Delta\cdot b} = (2\pi)^3\,\delta(\xi)\,\delta^2(\Delta_T)$$

$$x = b - z/2$$
$$y = b + z/2$$



$$G^{[+,-]\alpha\beta}(x,k_T,\xi,\Delta_T) = 4\int \frac{d^3z\,d^3b}{(2\pi)^3}\, e^{ik\cdot z - i\Delta\cdot b}\, \frac{\langle p'|F^{n\beta}(x)U^{[-]}_{[x,y]}F^{n\alpha}(y)U^{[+]}_{[y,x]}|p\rangle\Big|_{LF}}{\langle P|P\rangle}$$

x=ξ=0

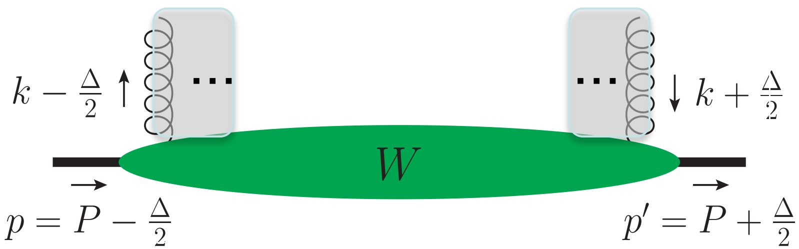
$$G^{[+,-]\alpha\beta}(k_T,\Delta_T) = 16\int \frac{d^2z\,d^2b}{(2\pi)^3}\, e^{ik_T\cdot z_T - i\Delta_T\cdot b_T}\, \frac{\langle p'|G^{\beta}_{T}(x)U^{[-]}_{[x,y]}G^{\alpha}_{T}(y)U^{[+]}_{[y,x]}|p\rangle\Big|_{LF}}{\langle P|P\rangle}$$

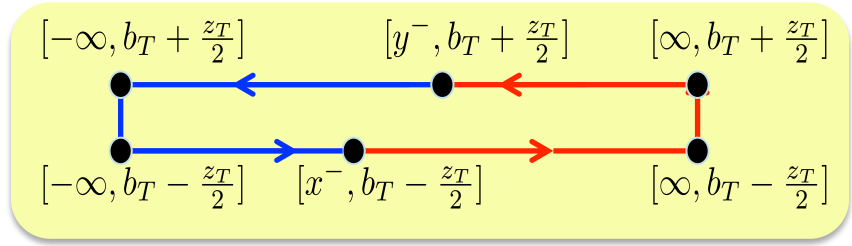
gluonic pole: $\left[i\partial^\alpha_x, U^{[\pm]}_{[a,x]}\right] = \pm g U^{[\pm]}_{[a,x]} G^\alpha_T(x)$

$$G^{[+,-]\alpha\beta}(k_T,\Delta_T) = \frac{2N_c}{\alpha_s}\left[k_T^\alpha k_T^\beta - \tfrac{1}{4}\Delta_T^\alpha\Delta_T^\beta - \tfrac{1}{2}k_T^{[\alpha}\Delta_T^{\beta]}\right] G^{[\Box]}(k_T,\Delta_T)$$

Boer, van Daal, PJM, Petreska 1805.05219

Relating GTMDs to Wilson loop GTMD (x → 0)





- Dipole Gluon GTMD correlator → Wilson loop GTMD correlator:

$$G^{[\Box]}(k_T, \Delta_T) = \int \frac{d^2z\, d^2b}{(2\pi)^4}\, e^{ik_T\cdot z_T - i\Delta_T\cdot b_T}\, \frac{\langle P - \frac{\Delta_T}{2}|\frac{1}{N_c}\mathrm{Tr}\left(U^{[\Box]}(x,y)\right)|P - \frac{\Delta_T}{2}\rangle\Big|_{LF}}{\langle P|P\rangle}$$

$$= \frac{\alpha_s}{2N_c\, M^2}\mathcal{E}(k_T^2, \Delta_T^2, k_T\cdot\Delta_T)$$

- ... and gluon GTMD is

$$G^{[+,-]\alpha\beta}(k_T, \Delta_T) = \left[\frac{k_T^\alpha k_T^\beta}{M^2} - \frac{\Delta_T^\alpha \Delta_T^\beta}{4M^2} - \frac{k_T^{[\alpha}\Delta_T^{\beta]}}{2M^2}\right]\mathcal{E}(k_T^2, \Delta_T^2, k_T\cdot\Delta_T)$$

$$\overset{\Delta\to 0}{\Longrightarrow} \quad \frac{k_T^\alpha k_T^\beta}{M^2}\, e(k_T^2)$$

- $G^{[\Box]}(k_T, \Delta_T)$ has a real part (pomeron structure) and an imaginary part (odderon structure, odd powers of $k_T\cdot\Delta_T$), with for TMDs no odderon structure in unpolarized nucleon

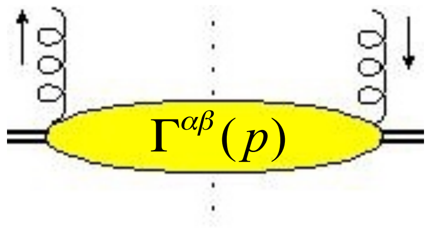
Hatta, Xiao and Yuan 2016

- gluon TMD correlator: $\epsilon^{\alpha}(k)\epsilon^{\beta *}(k) \Longrightarrow$

$$\Gamma^{[U,U']\,\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\; e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,U_{[0,\xi]}\,F^{n\nu}(\xi)\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$

parametrized (for unpolarized hadrons)

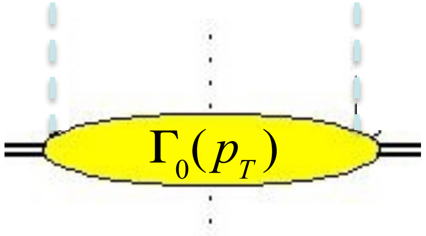
$$\Gamma^{\alpha\beta[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{\alpha\beta}\, f_1^{[U]}(x,k_T^2) + \frac{k_T^{\alpha\beta}}{M^2}\, h_1^{\perp\,[U]}(x,k_T^2)\right\}$$



- Comparison with Wilson loop correlator

$$\delta(x)\,\Gamma_0^{[U,U']}(k_T;n) = \int \frac{d\,\xi{\cdot}P\, d^2\xi_T}{(2\pi)^3}\; e^{ik\cdot\xi}\,\langle P,S|U_{[0,\xi]}\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$

$$\Gamma_0^{[+,-]}(k_T) = \frac{1}{2M^2}\, e^{[+,-]}(k_T^2)$$



- Small x behavior of (dipole) gluon TMDs

$$x\, f_1^{[+,-]}(x,k_T^2) \overset{x\to 0}{\Longrightarrow} \frac{k_T^2}{2M^2}\, e^{[+,-]}(k_T^2)$$

$$x\, h_1^{\perp[+,-]}(x,k_T^2) \overset{x\to 0}{\Longrightarrow} e^{[+,-]}(k_T^2)$$

D Boer, S Cotogno, T van Daal, PJM, A. Signori, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Structure of gluon **TMD PDFs** in spin 1 target

PARTON SPIN / TARGET SPIN

GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$
LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
LT	f_{1LT}^g	g_{1LT}^g	h_{1LT}^g $h_{1LT}^{\perp g}$
TT	f_{1TT}^g	g_{1TT}^g	h_{1TT}^g $h_{1TT}^{\perp g}$ $h_{1TT}^{\perp\perp g}$

Jaffe & Manohar, Nuclear gluonometry, PL B223 (1989) 218

PJM & Rodrigues, PR D63 (2001) 094021

Meissner, Metz and Goeke, PR D76 (2007) 034002

D Boer, S Cotogno, T van Daal, PJM, A. Signori, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Gluon TMDs in polarized nucleon

- Polarized target (vector polarization)

$$\Gamma_L^{ij[U]}(x,k_T) = \frac{x}{2}\left\{ i\epsilon_T^{ij}\, S_L g_1^{[U]}(x,k_T^2) + \frac{\epsilon_{T\ \alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, S_L h_{1L}^{\perp\,[U]}(x,k_T^2) \right\}$$

$$\Gamma_T^{ij[U]}(x,k_T) = \frac{x}{2}\left\{ \frac{g_T^{ij}\epsilon_T^{kS_T}}{M}\, f_{1T}^{\perp\,[U]}(x,k_T^2) - \frac{i\epsilon_T^{ij}k_T\cdot S_T}{M}\, g_{1T}^{[U]}(x,k_T^2) \right.$$
$$\left. - \frac{\epsilon_T^{k\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,k_T^2) - \frac{\epsilon_{T\ \alpha}^{\{i}k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp}(x,k_T^2) \right\}$$

- Cf. Wilson loop TMDs in polarized nucleon (no TMD for L polarization)

$$\Gamma_0(k_T) = \frac{1}{2M^2}\left\{ e(k_T^2) - \frac{\epsilon^{kS_T}}{M}\, e_T(k_T^2) \right\}$$

‘pomeron’ ‘odderon’

Dominguez, Xiao, Yuan 2011; Hatta, Xiao, Yuan 2016
D Boer, MG Echevarria, PJM, J Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485
D Boer, S Cotogno, T van Daal, PJM, A. Signori, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

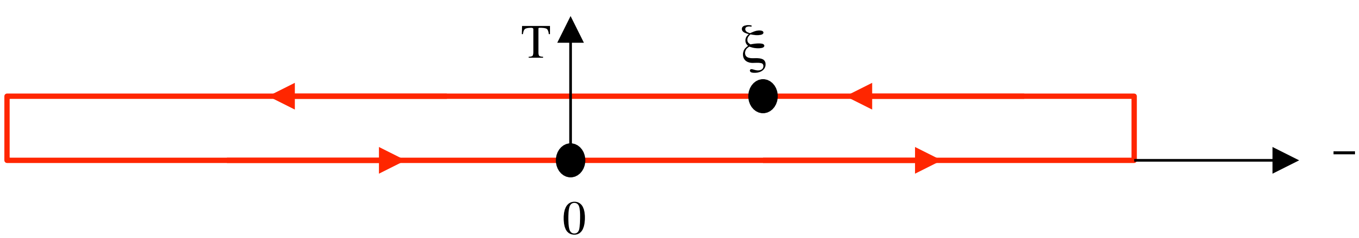
Small x physics in terms of TMDs

- Dipole gluon TMDs: at small x only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

$$x\,f_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\,e^{[+,-]}(k_T^2)$$
$$x\,h_1^{\perp[+,-]}(x,k_T^2) \longrightarrow e^{[+,-]}(k_T^2)$$

$$x\,f_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\,e_T^{[+,-]}(k_T^2)$$
$$x\,h_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\,e_T^{[+,-]}(k_T^2)$$
$$x\,h_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow e_T^{[+,-]}(k_T^2)$$

- Reflects role of Wilson loop in 1D → 3D transition where gluons emergence



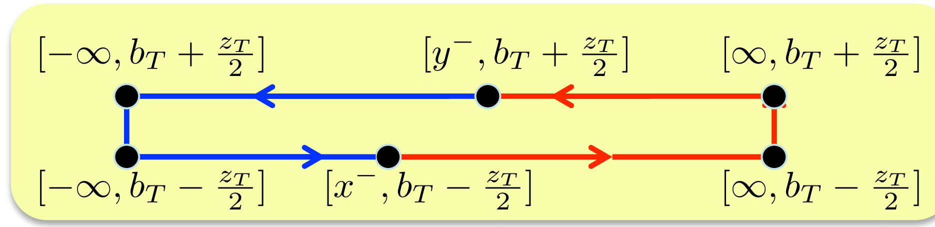
$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta \sigma_{\alpha\beta}}$$

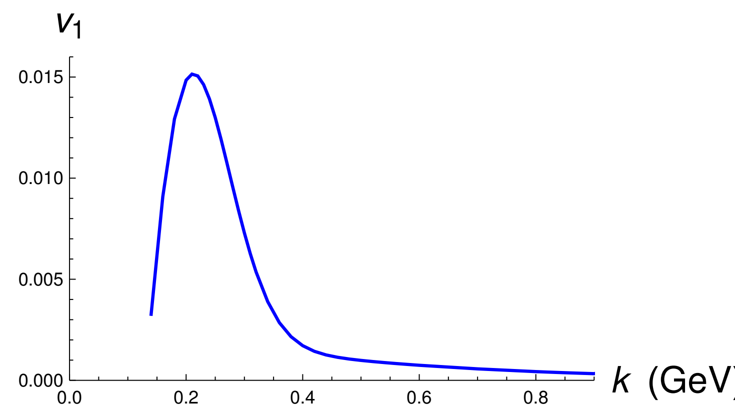
D Boer, S Cotogno, T van Daal, PJM, A. Signori, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

C-odd gluon correlations in peripheral pA collisions

- Odderon type of correlations are C-odd!
- In a nucleus the z-dependence is restricted to nucleon size, while the b-dependence follows the nucleon density profile
- The odd $k_T.\Delta_T$ dependence can show up as a non-vanishing v_1 elliptic flow parameter in pA peripheral collisions

  (Boer, van Daal, PJM, Petreska, ArXiv 1805.05219)



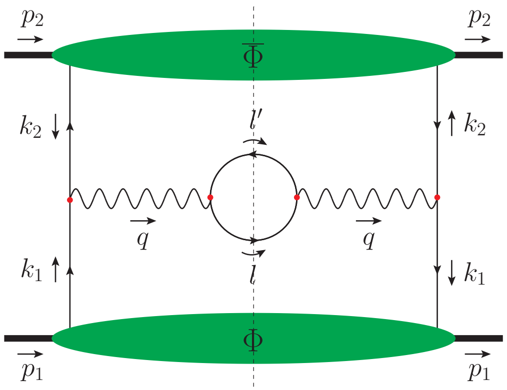


Directed flow $v_1(k)$ for p Pb scattering using CGC

Kovner, Rezaeian; Zhou, Voloshin, Zhang; Gelis, Pelier; Kovchegov, Szymanowski, Wallon; Hatta, Iancu, Itakura, McLerran, Jeon, Venugopalan; Kovchegov, Sievert; ...

COLOR ENTANGLEMENT (IN DY)

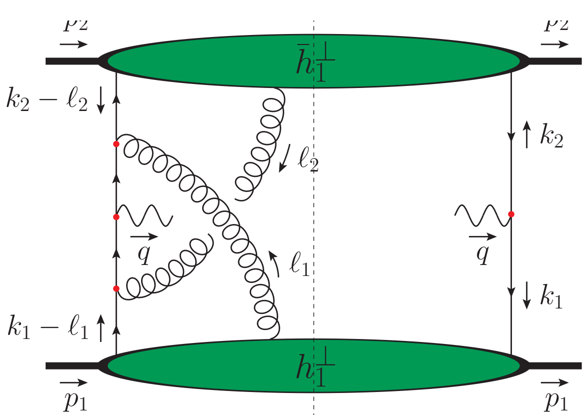
Color factors for (entangled) multiple T-odd TMDs



$$d\sigma_{\rm DY} \sim {\rm Tr}_c\left[\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma\right]$$
$$= \frac{1}{N_c}\Phi(x_1,p_{1T})\Gamma^*\overline{\Phi}(x_2,p_{2T})\Gamma,$$

- Complications if the transverse momentum of two initial state hadrons is involved, resulting for DY at measured Q_T in

$$d\sigma_{\rm DY} = {\rm Tr}_c\Big[U_-^\dagger[p_2]\Phi(x_1,p_{1T})U_-[p_2]\Gamma^*$$
$$\times U_-^\dagger[p_1]\overline{\Phi}(x_2,p_{2T})U_-[p_1]\Gamma\Big]$$
$$\neq \frac{1}{N_c}\Phi^{[-]}(x_1,p_{1T})\Gamma^*\overline{\Phi}^{[-^\dagger]}(x_2,p_{2T})\Gamma,$$



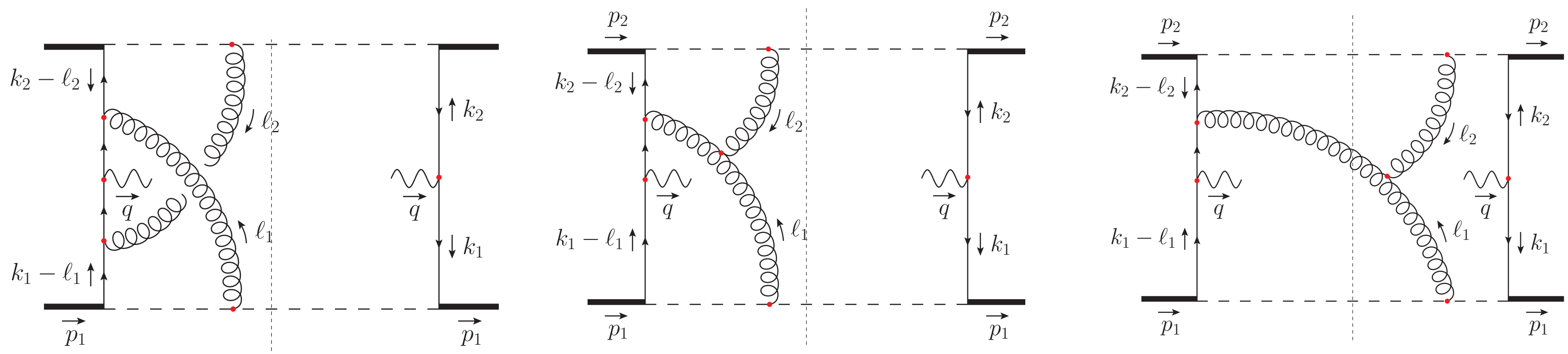
- This leads to color factors just as for twist-3 squared in collinear DY

$$\sigma_{DY}(x_1,x_2,q_T) = \frac{1}{N_c} f_1(x_1,p_{1T})\otimes \overline{f}_1(x_2,p_{2T})$$
$$- \frac{1}{N_c}\frac{1}{N_c^2-1} h_1^\perp(x_1,p_{1T})\otimes \overline{h}_1^\perp(x_2,p_{2T})\cos(2\varphi)$$

Buffing, M, PRL 112 (2014), 092002

Unwinding color entanglement in DY

- A model calculation: gluons needed for T-odd BM TMD functions
- Inclusion of Glauber gluons



- First two diagrams give entanglement with additonal color factor beyond 1/N

$$(a) \Longrightarrow -\frac{1}{N_c^2 - 1}\,\frac{1}{N_c}$$

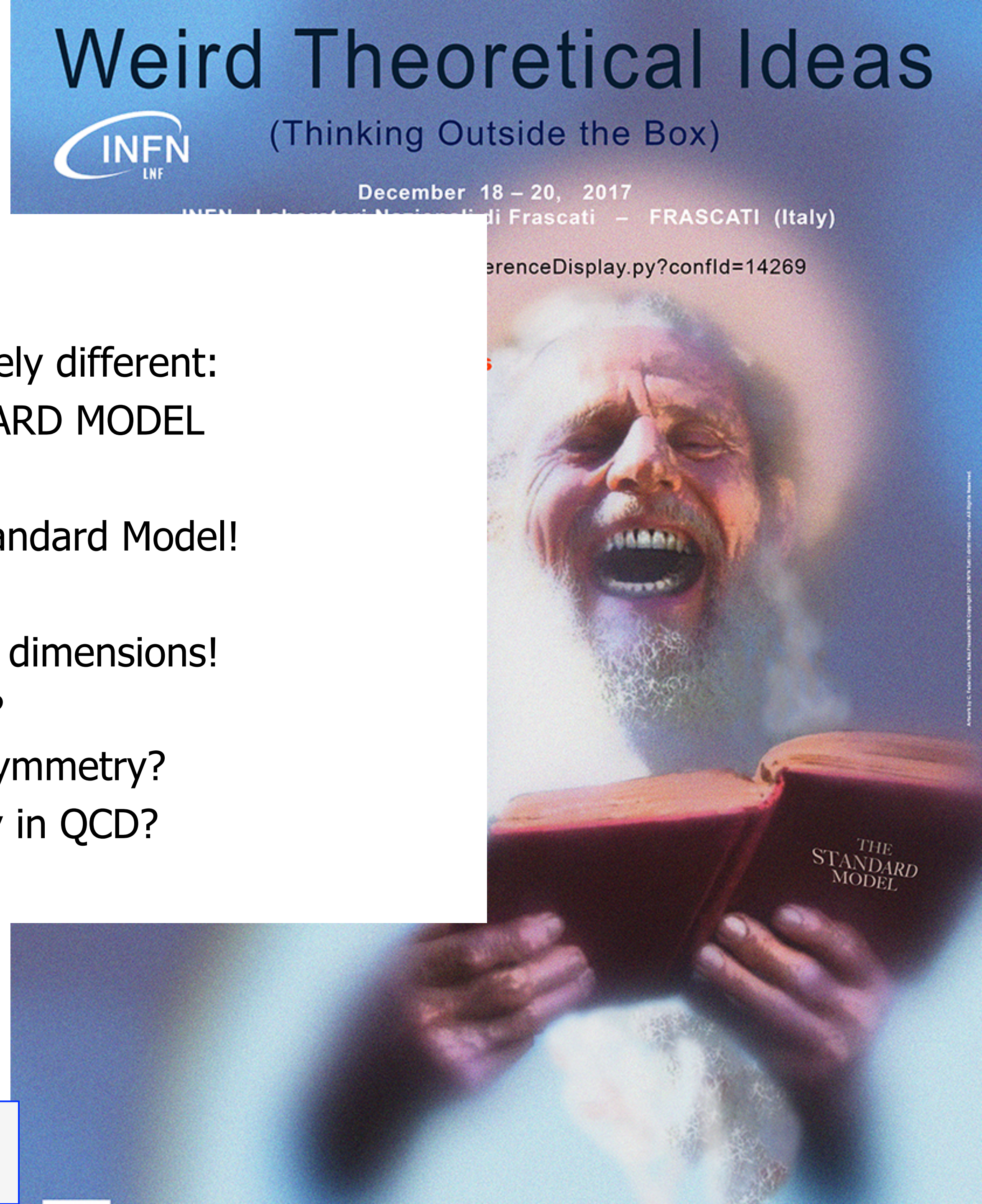
- Effect is cancelled by third (right) diagram!

$$(a) + (b) + (c) \Longrightarrow \frac{1}{N_c}$$

- Essential (in model unavoidable) is perturbative treatment and color in final state!

Boer, van Daal, Gaunt, Kasemets, PJM, SciPost 3 (2017) 040

QUARKS AND LEPTONS AS ENTANGLED MULTIPARTITE STATES



- Then for something completely different:
  (NOT) HAPPY WITH STANDARD MODEL

- In spite of the success of Standard Model!

- Three families, colors, space dimensions!
- Left-right (a)symmetry? B-L?
- Naturalness? Missing supersymmetry?
- Confinement and Collinearity in QCD?

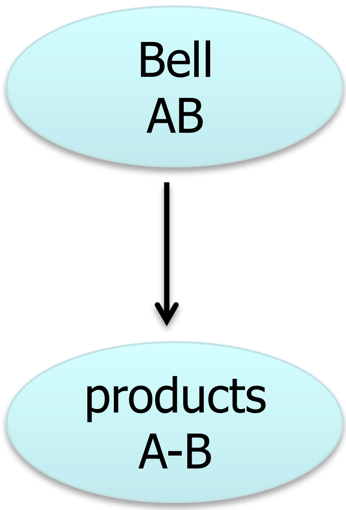
PJM 1601.00300
PJM 1801.03664

QCD – entangled states and QIT

- Parton-hadron duality in hard QCD scattering: PDFs x FFs
  - nucleon is pure state → ensemble of partons (good light-front states) [see also Kharzeev & Levin (1702.03489)]
  - hard (short distance) process: partons → partons
  - emerging partons are pure state(s) → ensemble of hadron states
- Entangled (pure) states $|\Phi>$ in $\mathcal{H}^A \otimes \mathcal{H}^B$ with a density matrix $\rho = |\Phi><\Phi|$ lead to ensembles (non-pure state) in the reduced spaces.
  - EPR bipartite pure state leads to a 50% - 50% ensemble in both subspaces.
  - Relevant dimensionality: PDF ~ N_c while FF ~ $1/N_c$
- Maybe both hadrons and partons live in a multipartite Hilbert space !
- Possibly combined with a principle of maximal entanglement (MaxEnt), such as hinted at in Cervera-Lierta, Latorre, Rojo & Rottoli (1703.02989): maximally entangled chiral left/right two-particle states are consistent with QED (g_A=0) & electroweak (g_V=0), at least if sin Θ_W = ½

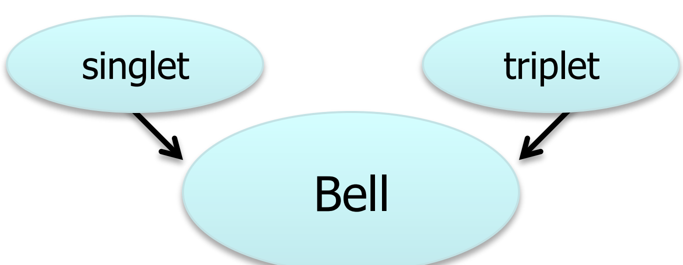
Bipartite entangled states

- Bell states are maximally entangled states: $|RR\rangle \pm |LL\rangle$ or $|RL\rangle \pm |LR\rangle$
- They belong to the same class (SLOCC, for us local unitary, local = subspace)



$$\rho = |\mathrm{Bell}\rangle\langle\mathrm{Bell}| \implies \rho_A = \frac{1}{2}\left(|R\rangle\langle R| + |L\rangle\langle L|\right)$$

- Symmetry eigenstates are in general entangled



Tripartite entangled states

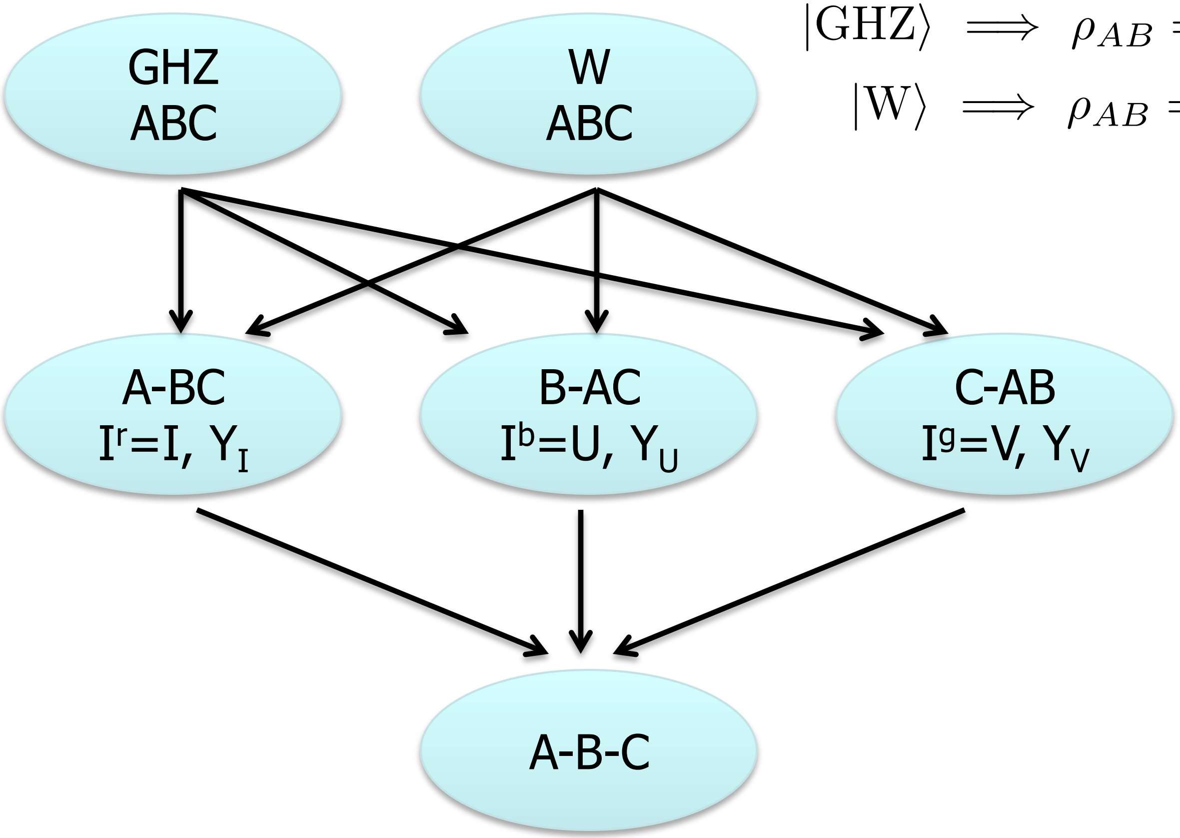
- Two classes of maximally entangled states: (Dur, Vidal, Cirac 2000)

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$

$$|\mathrm{GHZ}\rangle \implies \rho_{AB} = \frac{1}{2}\left(|RR\rangle\langle RR| + |LL\rangle\langle LL|\right)$$

$$|\mathrm{W}\rangle \implies \rho_{AB} = \frac{2}{3}|\mathrm{Bell}\rangle\langle\mathrm{Bell}| + \frac{1}{3}|RR\rangle\langle RR|$$



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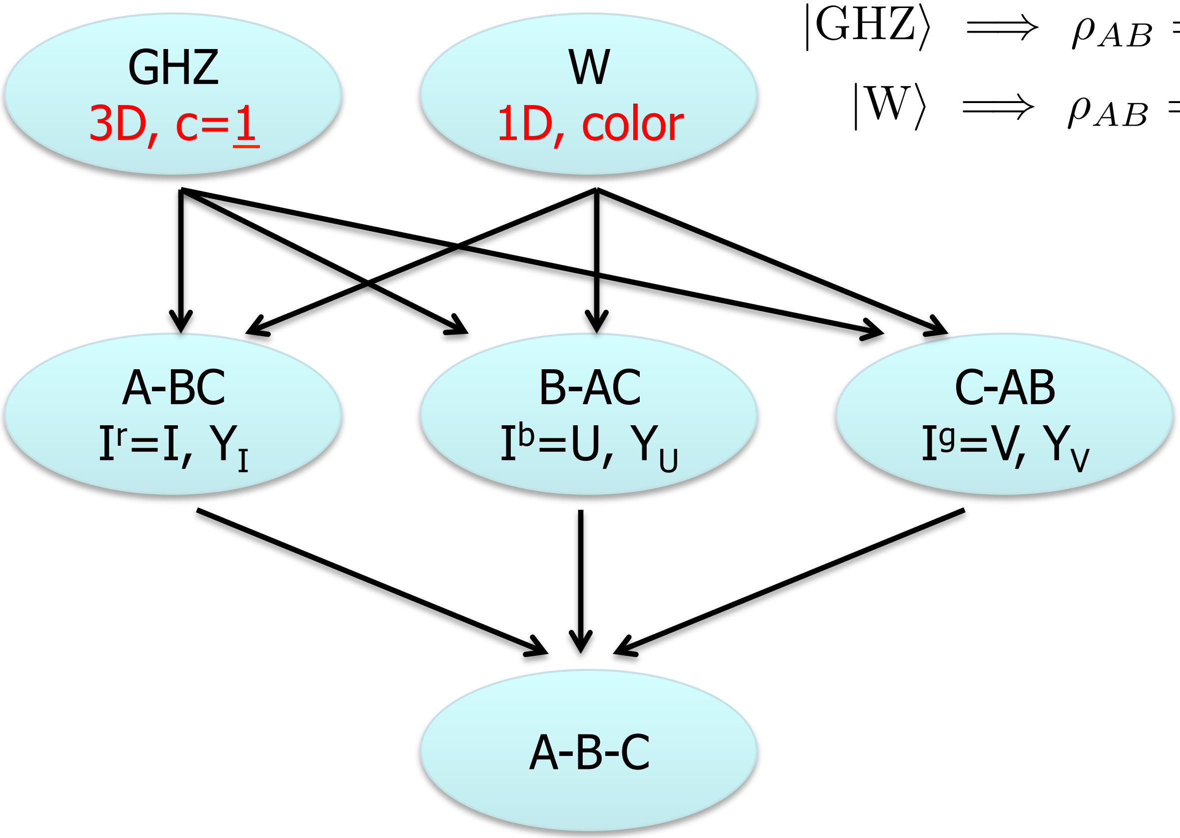
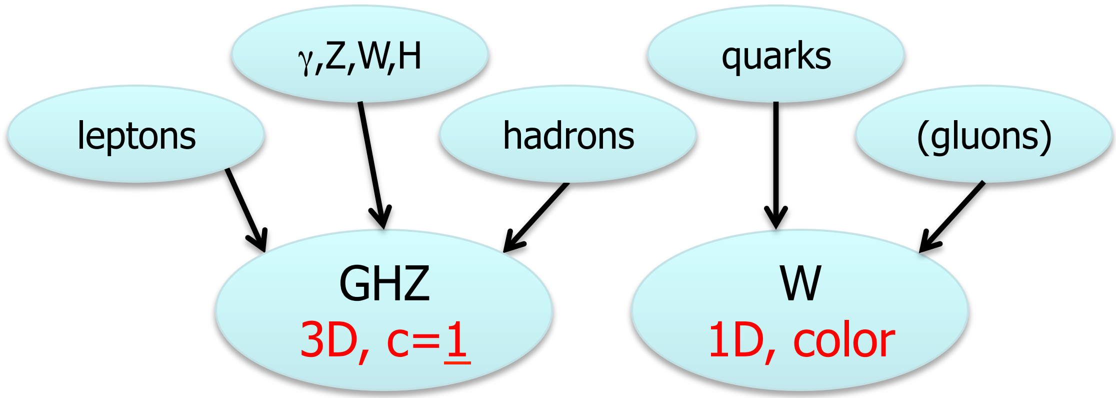
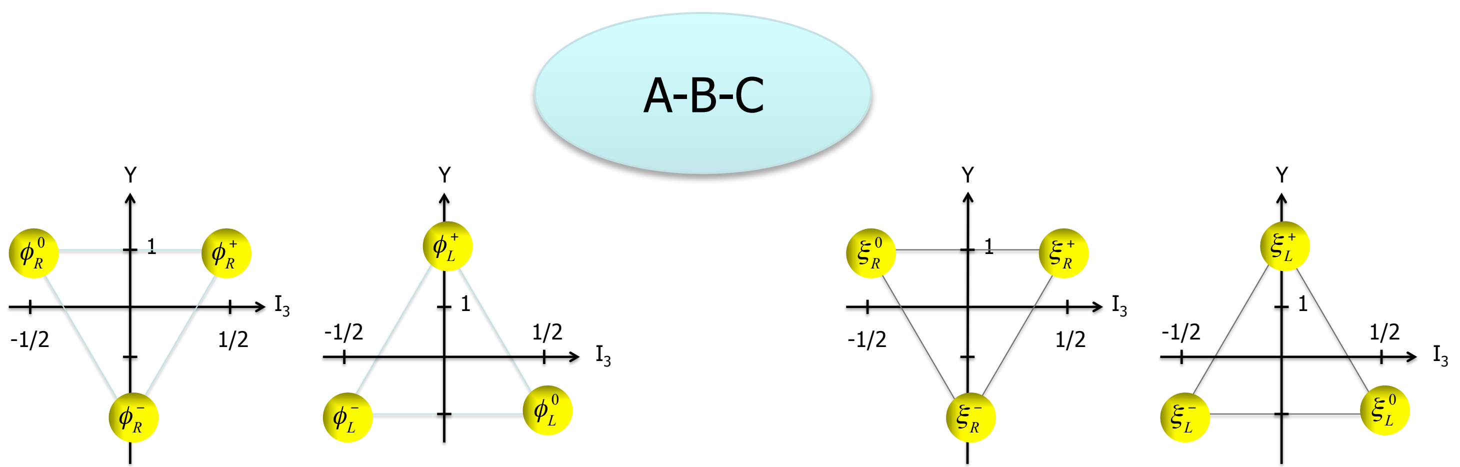


Quarks and leptons as entangled states

- Symmetry eigenstates are in general entangled



- Hope is that basic product states are simple and even may exhibit SUSY

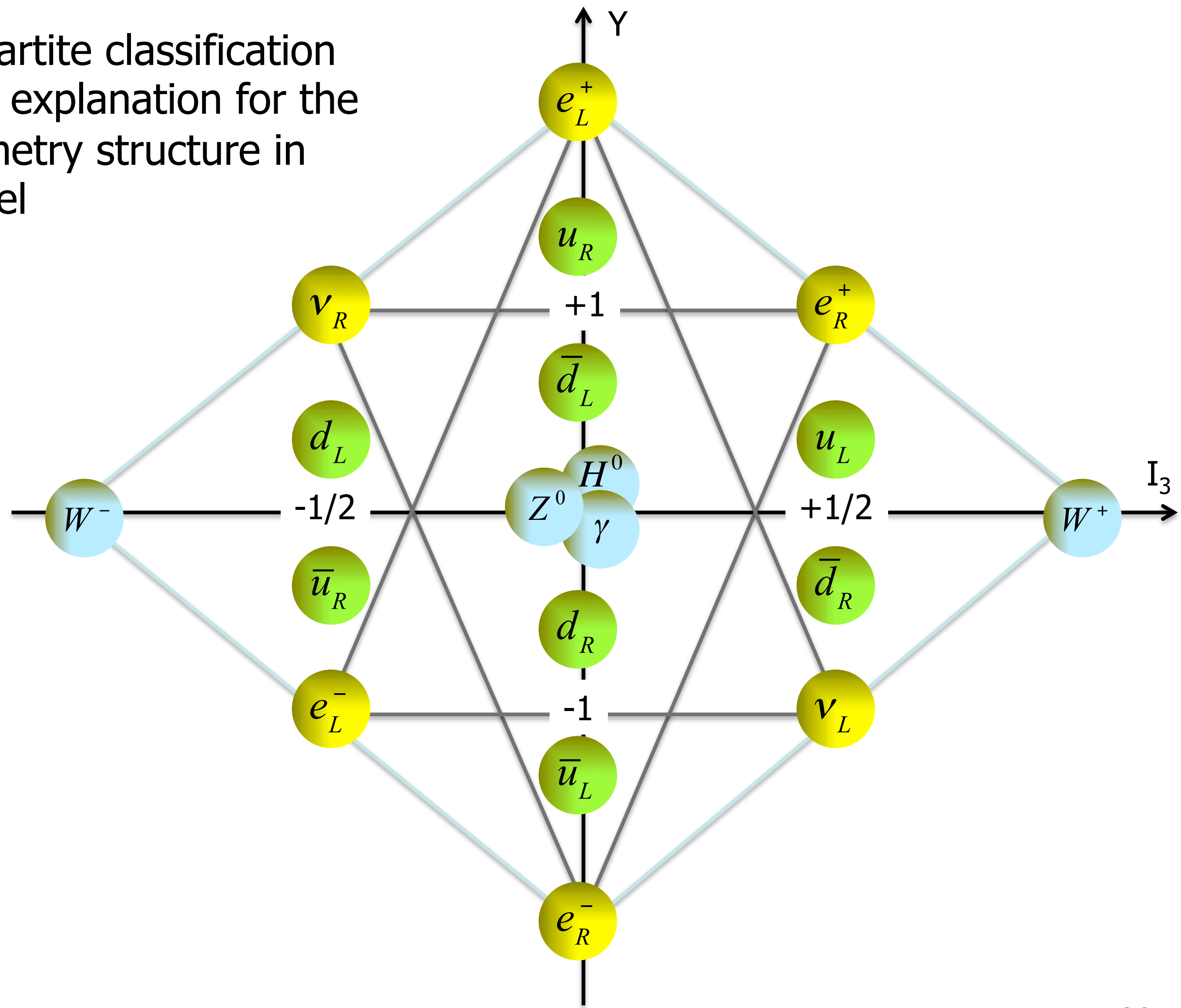


PJM 1601.00300, PJM 1801.03664

MULTIPARTITE FERMIONS IN THE STANDARD MODEL

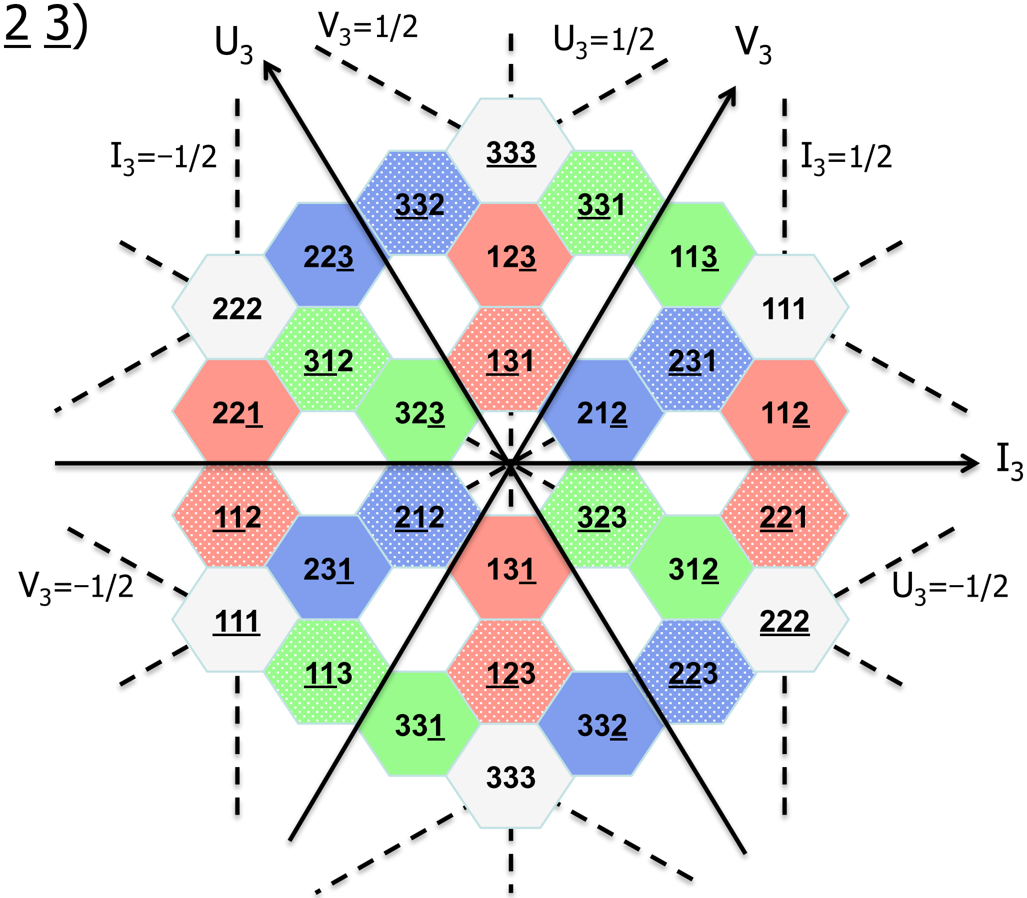
Standard model particle content

Important: multipartite classification can give a natural explanation for the electroweak symmetry structure in the standard model



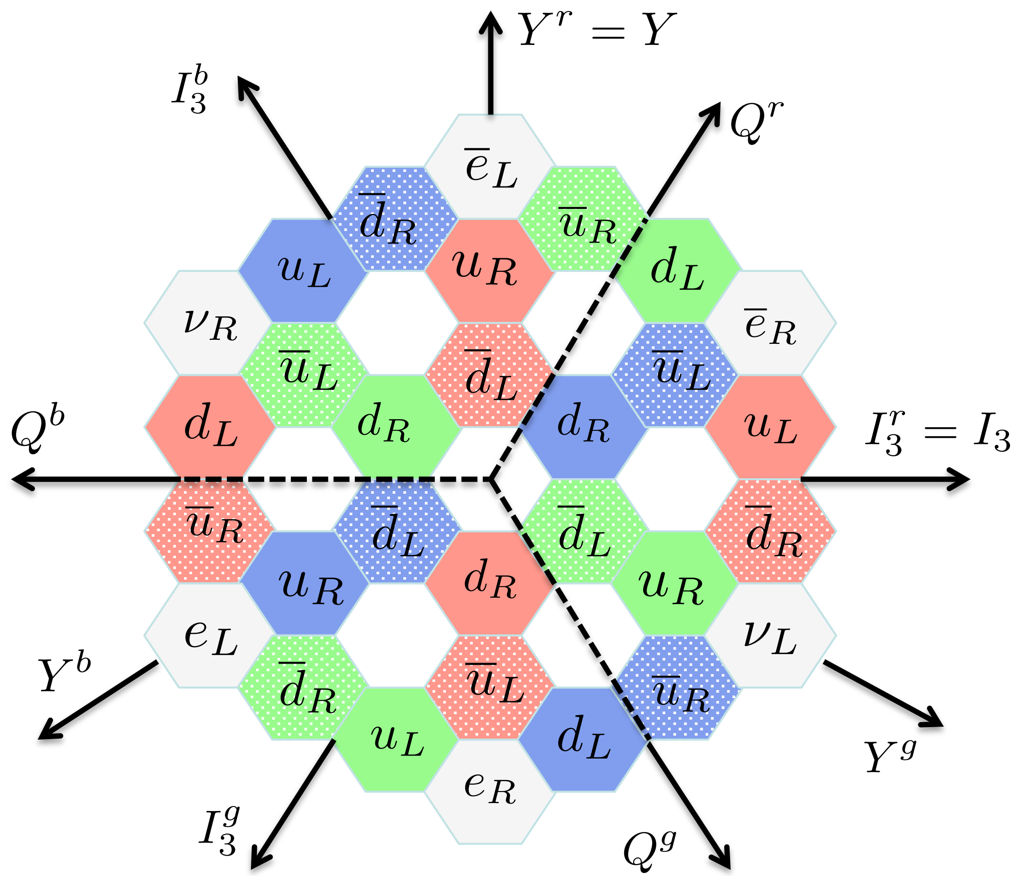
Fermionic excitations: leptons and quarks

- Tripartite states (R: 1 2 3 & L: 1 2 3)
- Aligned (RRR, LLL) GHZ states
  - SO(3) → asymptotic/space
    I, U, *and* V allowed
  - Three A(4) singlets
- Mingled (RRL, RLL) W-states
  - non-asymptotic
    I, U, *or* V allowed
  - Three A(4) triplets



Fermionic excitations: electroweak identification

- LEPTONS
  Aligned (RRR, LLL)
  - SO(3) → asymptotic/space
    I, U, *and* V allowed
  - Three A(4) singlets (families)
  - Family mixing is
    tri-bimaximal
- QUARKS
  Mingled (RRL, RLL)
  - non-asymptotic
    I, U, *or* V allowed
  - Three A(4) triplets (families)
  - Just one heavy quark!



- Resembles the rishon model
  (Harari & Seiberg 1982)

A paradigm shift?

- Start with less dimensions (1+1 → 1+3) advantageous
  - Convergence: d[ϕ] = (d-2)/2 → 0, d[ξ] = (d-1)/2 →½, naturalness, …
    [see Stojkovic – 1406.2696]
- Take the appropriate quantum states in multipartite space: ontological basis [see 't Hooft – 1405.1548]
- Tripartite space for quarks naturally has color dual to space/electroweak.
  - Can explain why color naturally is decoupled from electroweak interactions
  - Full color invisible in 3D: local gauge invariance! No free quarks or gluons!
  - Global color visible in 3D via valence quarks, N vs 1/N, f x D (distribution x fragmentation), color flow (future and past pointing gauge links), …
  - Natural role for Wilson loops generating (gluon) TMDs (1805.05219)
- Natural arena for light-front approach with a 'preferred' space direction: quantization of good fields, dominating the OPE at high energies, these are asymptotic (free) fields (Kogut & Soper): $\frac{1}{2}\gamma^-\gamma^+\psi \quad \text{and} \quad g_T^{\alpha\mu}A_\mu^a$
- New ways to look at color-kinematic duality, soft collinear effective theory (SCET), ...

Concluding remarks

- 3D tomography: role of Wilson loop GTMDs
- Test case: color entanglement in DY
- Conjecture: quarks, leptons and hadrons in multipartite spaces