Examining Semi-Inclusive Electron-Positron Annihilation at Large Transverse Momentum

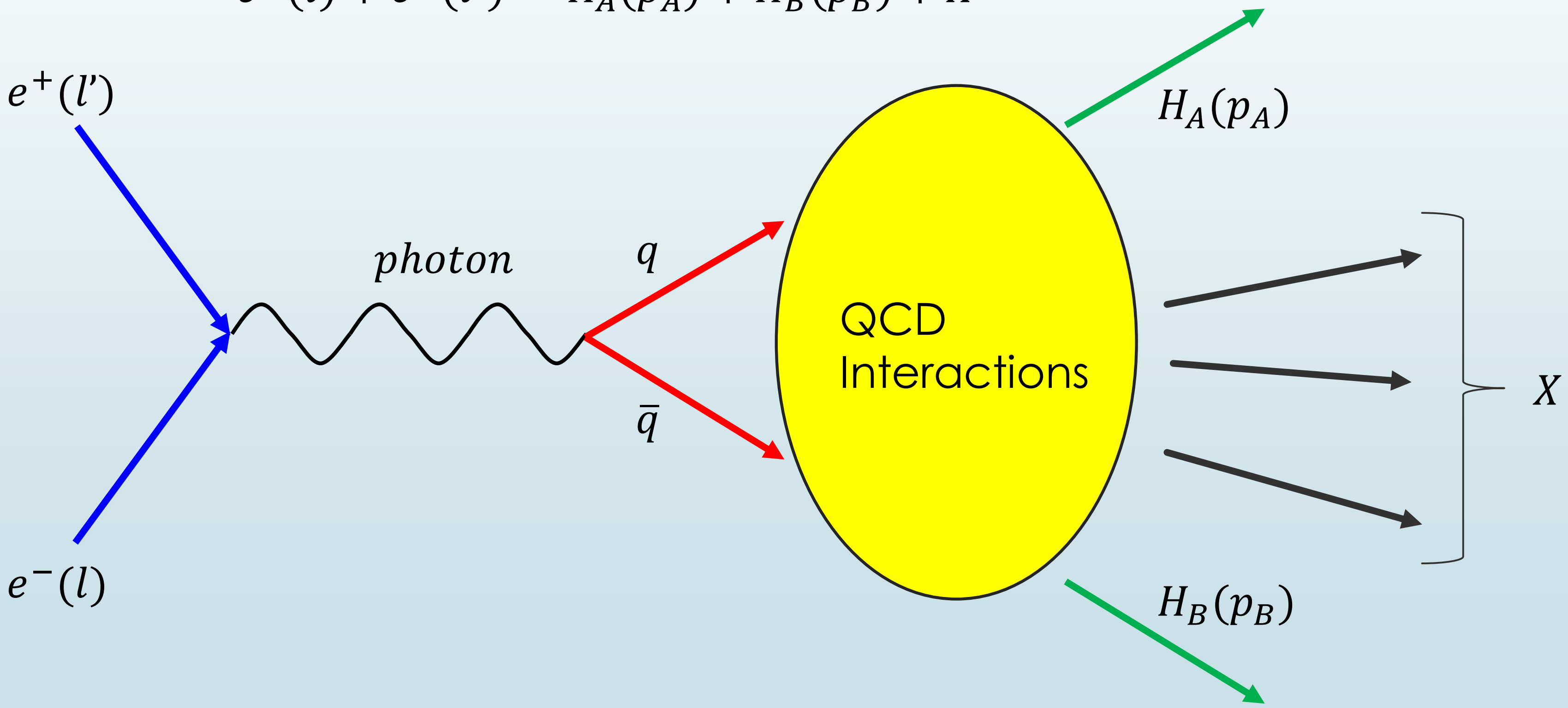
QCD Evolution 2018

Eric Moffat

Collaborators: Ted Rogers and Andrea Signori

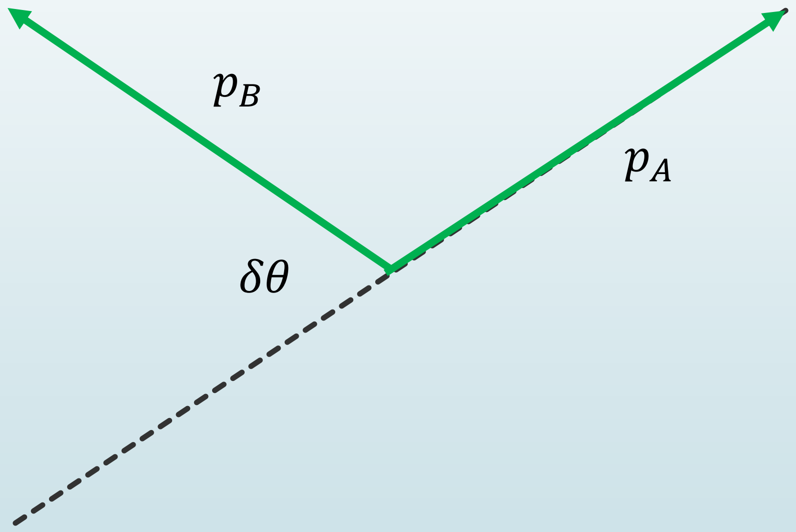
Semi-inclusive electron-positron annihilation with two hadrons

$$e^-(l) + e^+(l') \rightarrow H_A(p_A) + H_B(p_B) + X$$



Case of large transverse momentum

- In the center-of-mass frame:

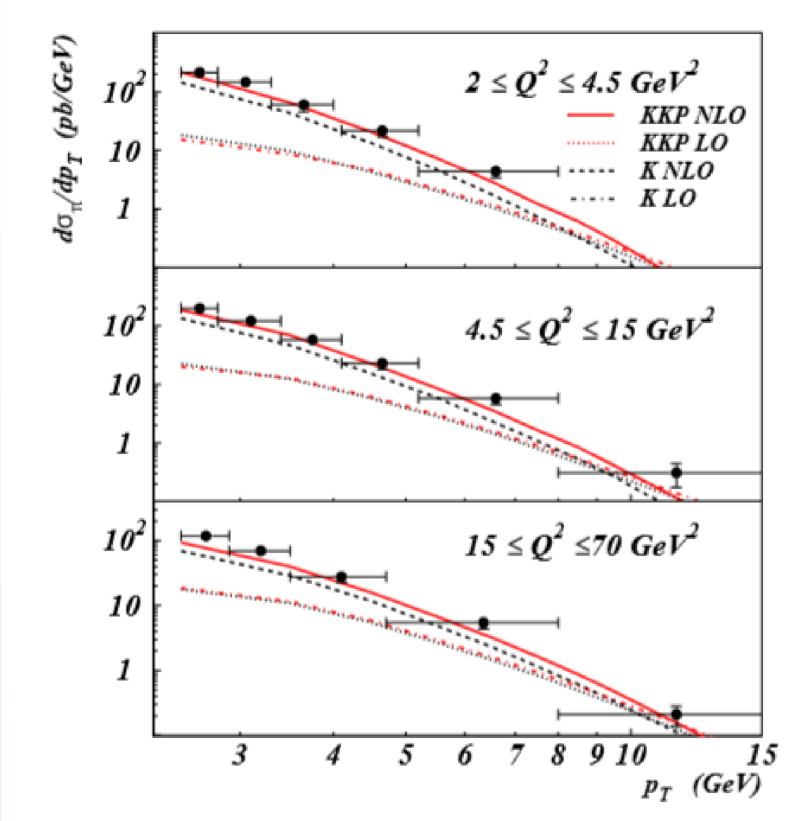


- Requires emission of a gluon in the hard part

Motivation

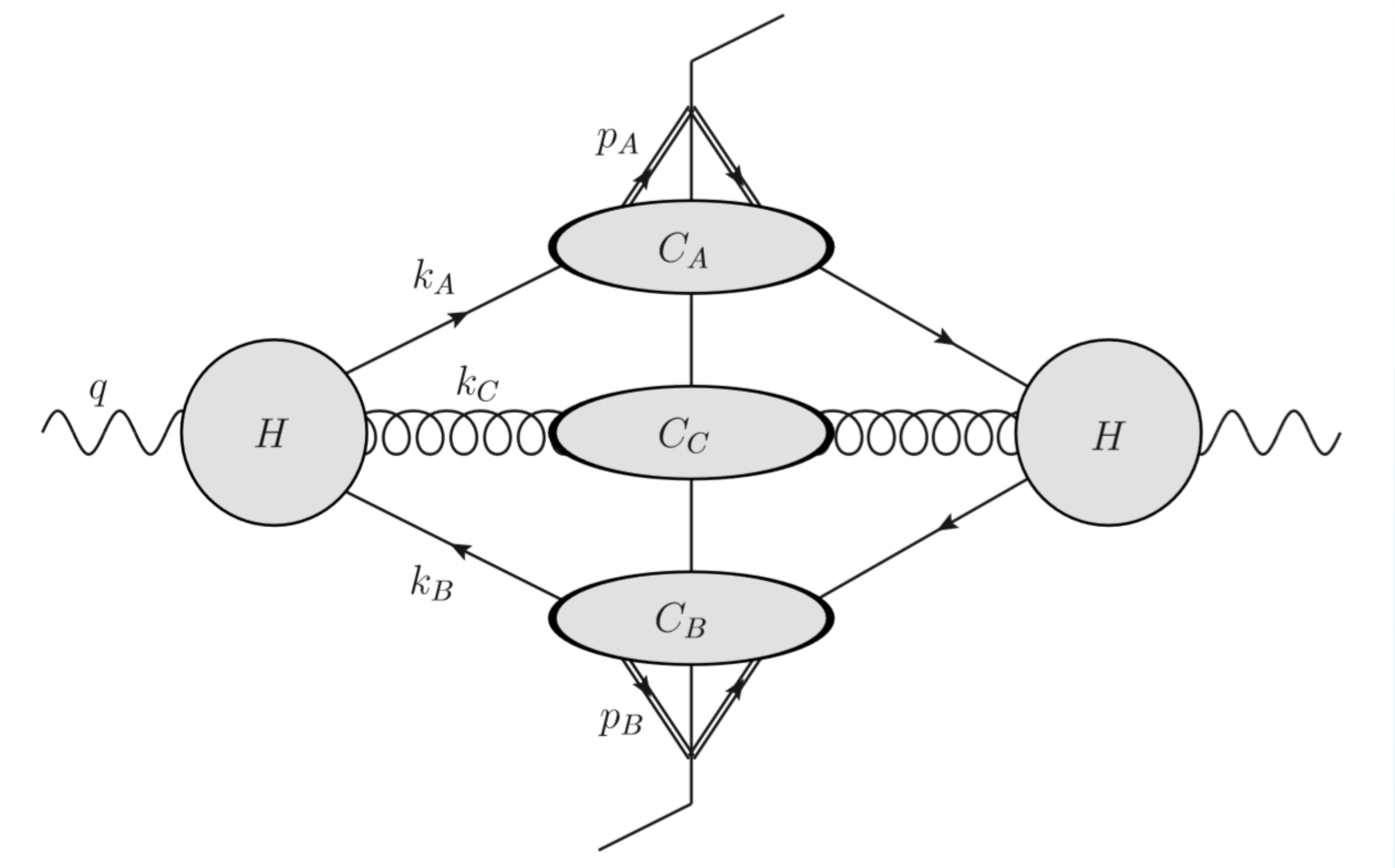
- Large transverse momentum calculations for SIDIS gave unexpected results
  - Large discrepancy between LO contribution and data

    (See Nobuo's talk)
- Does this discrepancy occur in e+/e- annihilation too?
- Necessary calculation to be able to extract TMD FFs under the complete W+Y framework



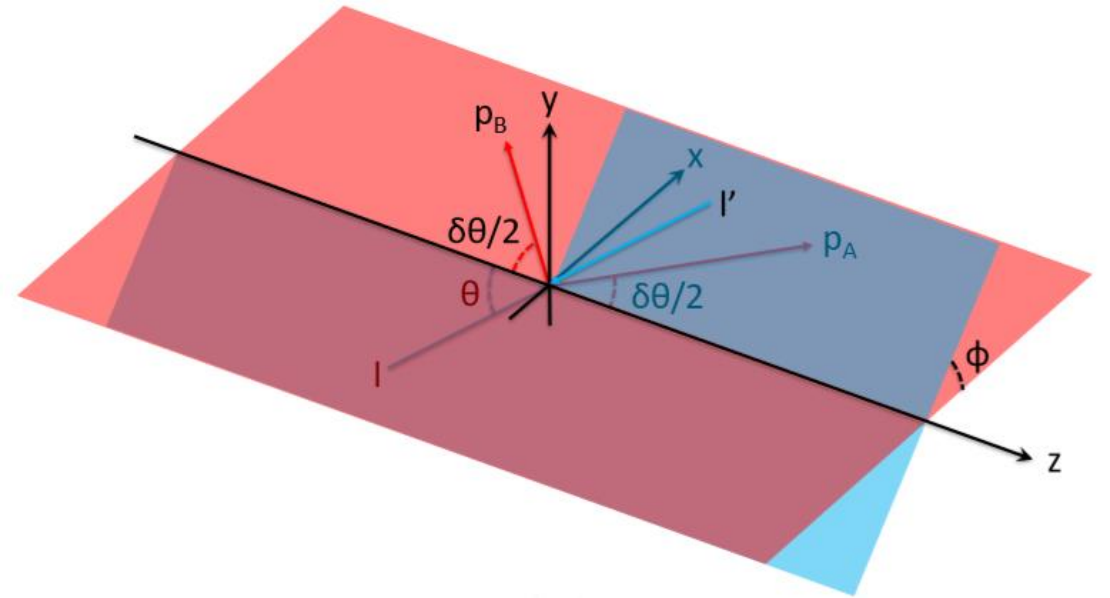
Daleo, de Florian, Sassot, arXiv:hep-ph/0411212

Example diagram



Reference Frames

- Photon Frame:



- Momenta:

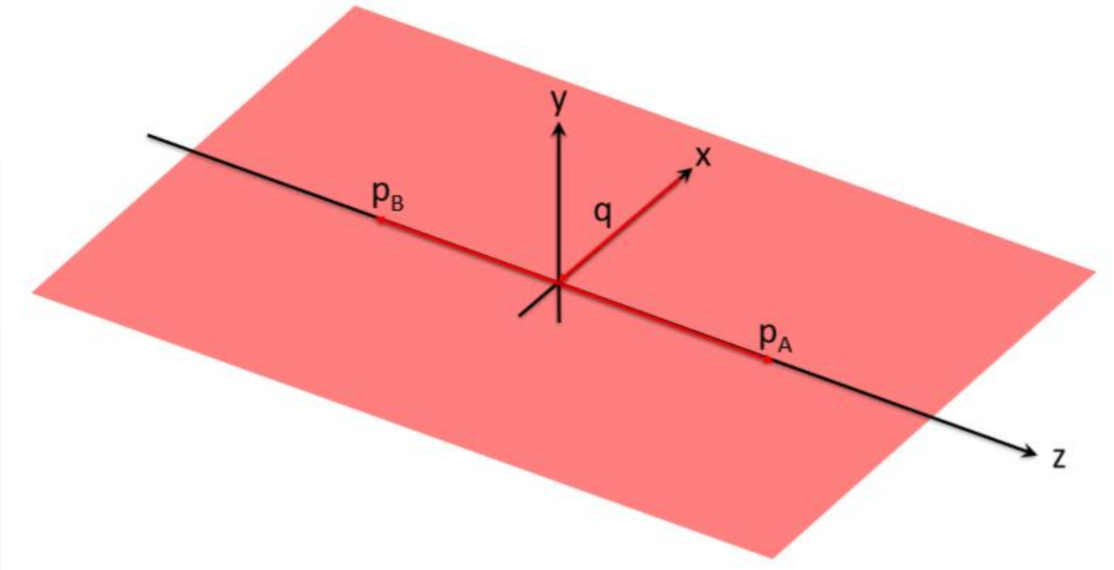
$$q_\gamma = (Q, \mathbf{0})) \quad p_{A,\gamma} = |\boldsymbol{p}_{A,\gamma}|(1, \boldsymbol{n}_{A,\gamma}) \quad p_{B,\gamma} = |\boldsymbol{p}_{B,\gamma}|(1, \boldsymbol{n}_{B,\gamma})$$

- Coordinate Axes:

$$X_\gamma^\mu = \frac{(0,\ \boldsymbol{n}_{A,\gamma}+\boldsymbol{n}_{B,\gamma})}{|\boldsymbol{n}_{A,\gamma}+\boldsymbol{n}_{B,\gamma}|} \qquad Z_\gamma^\mu = \frac{(0,\ \boldsymbol{n}_{A,\gamma}-\boldsymbol{n}_{B,\gamma})}{|\boldsymbol{n}_{A,\gamma}-\boldsymbol{n}_{B,\gamma}|}$$

Reference Frames

- Hadron Frame:



- Momenta:

$$q_h = \left(\sqrt{\frac{Q^2+q_{T,h}^2}{2}}, \sqrt{\frac{Q^2+q_{T,h}^2}{2}}, \boldsymbol{q}_{T,h}\right) \qquad p_{A,h} = \left(p_{A,h}^+, 0, \boldsymbol{0}\right) \qquad p_{B,h} = \left(0, p_{B,h}^-, \boldsymbol{0}\right)$$

- Useful variables:

$$z_A = \frac{p_{A,h}^+}{q_h^+} = \frac{p_A \cdot p_B}{q \cdot p_B} \qquad z_B = \frac{p_{B,h}^-}{q_h^-} = \frac{p_A \cdot p_B}{q \cdot p_A} \qquad q_{T,h}^2 = \frac{2\, p_A \cdot q\, p_B \cdot q}{p_A \cdot p_B} - Q^2$$

Calculating the cross section

- Want to calculate: $\frac{d\sigma_{AB}}{dz_A dz_B dq_T}$
- In terms of leptonic and hadronic tensors:

$$\frac{d\sigma_{AB}}{dz_A dz_B dq_T d\cos\theta} = \frac{\alpha_{EM}^2 z_A z_B q_T}{4Q^4} L^{\mu\nu} W_{\mu\nu}$$

  - where

$$L^{\mu\nu} = l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu} l \cdot l'$$

Calculating the cross section

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2} - Z^\mu Z^\nu\right) W_T + Z^\mu Z^\nu W_L$$

- Structure Function Extraction Tensors:

$$W_T = P_T^{\mu\nu} W_{\mu\nu} \quad W_L = P_L^{\mu\nu} W_{\mu\nu}$$

  - where

$$P_T^{\mu\nu} = \frac{1}{2}(X^\mu X^\nu - Z^\mu Z^\nu - g^{\mu\nu}) \qquad P_L^{\mu\nu} = Z^\mu Z^\nu$$

Calculating the cross section

- Contract leptonic and hadronic tensors:

$$\frac{d\sigma_{AB}}{dz_A dz_B dq_T d\cos\theta} = \frac{\alpha_{EM}^2 z_A z_B q_T}{8Q^2}\left[(1+\cos^2\theta)W_T + \sin^2\theta W_L\right]$$

- Integrate over θ

$$\frac{d\sigma_{AB}}{dz_A dz_B dq_T} = \frac{\alpha_{EM}^2 z_A z_B q_T}{6Q^2}\left[2W_T + W_L\right]$$

Calculating the cross section

- Large q_T factorized cross section:

$$\frac{d\sigma_{AB}}{dz_A dz_B dq_T} = \sum_f \int_{z_A}^{1} \frac{d\zeta_A}{\zeta_A^2} \int_{z_B}^{1} \frac{d\zeta_B}{\zeta_B^2} \frac{d\hat{\sigma}_{AB,f}}{d\hat{z}_A d\hat{z}_B dq_T}(Q, q_T, z_A/\zeta_A, z_B/\zeta_B)\delta(k_C^2)$$
$$\times [d_{A/f}(\zeta_A) d_{B/\bar{f}}(\zeta_B) + d_{A/\bar{f}}(\zeta_A) d_{B/f}(\zeta_B)$$
$$+ d_{A/g}(\zeta_A) d_{B/f}(\zeta_B) + d_{A/g}(\zeta_A) d_{B/\bar{f}}(\zeta_B)$$
$$+ d_{A/f}(\zeta_A) d_{B/g}(\zeta_B) + d_{A/\bar{f}}(\zeta_A) d_{B/g}(\zeta_B)]$$

$$\zeta_A = \frac{p_A}{k_A} \qquad \zeta_B = \frac{p_B}{k_A} \qquad \hat{z}_A = \frac{z_A}{\zeta_A} = \frac{k_A \cdot k_B}{q \cdot k_B} \qquad \hat{z}_B = \frac{z_B}{\zeta_B} = \frac{k_A \cdot k_B}{q \cdot k_A}$$

Calculating the cross section

- Conservation of momentum and $\delta(k_C^2)$ gives:

$$(q - k_A - k_B)^2 = 0$$

- Integrate the cross section over ζ_B:

$$\zeta_B = \frac{z_B(Q^2 + q_T^2)(\zeta_A - z_A)}{Q^2(\zeta_A - z_A) - z_A q_T^2}$$

- Cross section becomes

$$\frac{d\sigma_{AB}}{dz_A dz_B dq_T} = \sum_f \int_{\zeta_{Amin}}^{1} \frac{d\zeta_A}{\zeta_A z_B(Q^2 + q_T^2)(\zeta_A - z_A)} \frac{d\hat{\sigma}_{AB,f}}{d\hat{z}_A d\hat{z}_B dq_T}(Q, q_T, z_A/\zeta_A, z_B/\zeta_B)$$
$$\times [d_{A/f}(\zeta_A) d_{B/\bar{f}}(\zeta_B) + d_{A/\bar{f}}(\zeta_A) d_{B/f}(\zeta_B)$$
$$+ d_{A/g}(\zeta_A) d_{B/f}(\zeta_B) + d_{A/g}(\zeta_A) d_{B/\bar{f}}(\zeta_B)$$
$$+ d_{A/f}(\zeta_A) d_{B/g}(\zeta_B) + d_{A/\bar{f}}(\zeta_A) d_{B/g}(\zeta_B)]$$

$$\zeta_{Amin} = \frac{z_A(Q^2 + q_T^2)(1 - z_A)}{Q^2(1 - z_A) - z_A q_T^2}$$

Calculating the cross section

- Partonic cross section

$$\frac{d\hat{\sigma}_{AB,f}}{d\hat{z}_A d\hat{z}_B dq_T} = \frac{\alpha_{EM}^2 \hat{z}_A \hat{z}_B q_T}{6Q^2}\left[2\widehat{W}_{T,f} + \widehat{W}_{L,f}\right]$$

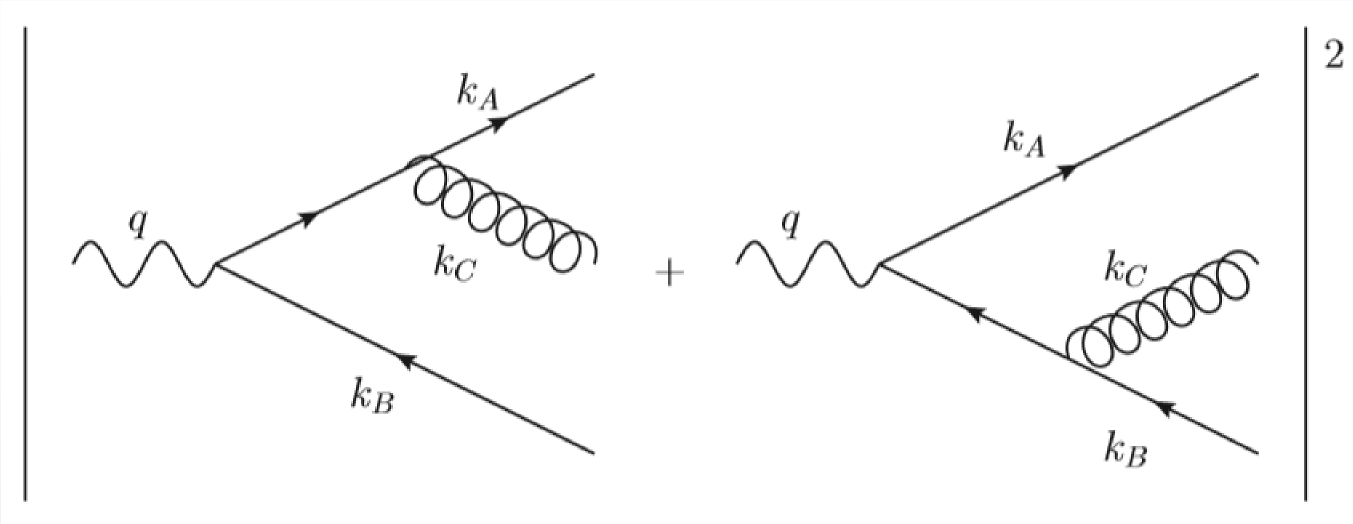
- where

$$\widehat{W}_{T,f} = P_T^{\mu\nu}\widehat{W}_{\mu\nu,f} \qquad \widehat{W}_{L,f} = P_L^{\mu\nu}\widehat{W}_{\mu\nu,f}$$

- Partonic tensor

$$\widehat{W}_f^{\mu\nu} = \frac{8\pi^3 \hat{z}_A \hat{z}_B}{Q^2}\left|\widehat{M}_f^{\mu\nu}\right|^2$$

Calculating the cross section



- $$\frac{d\hat{\sigma}_{AB,f}}{d\hat{z}_A d\hat{z}_B dq_T} = \frac{256\pi^4 \alpha_{EM}^2 \alpha_s e_f^2 \hat{z}_A^2 \hat{z}_B^2 q_T (Q^2+q_T^2)(3Q^2+q_T^2)(z_A^2+z_B^2)}{3Q^4 (Q^2(1-z_A)-z_A q_T^2)(Q^2(1-z_B)-z_B q_T^2)}$$

Maximum q_T

- Calculations showed that above a particular value of q_T the cross section becomes negative

$$\zeta_{Amin} = \frac{z_A(Q^2+q_T^2)(1-z_A)}{Q^2(1-z_A)-z_A q_T^2} < 1$$

- Solve for q_T^2

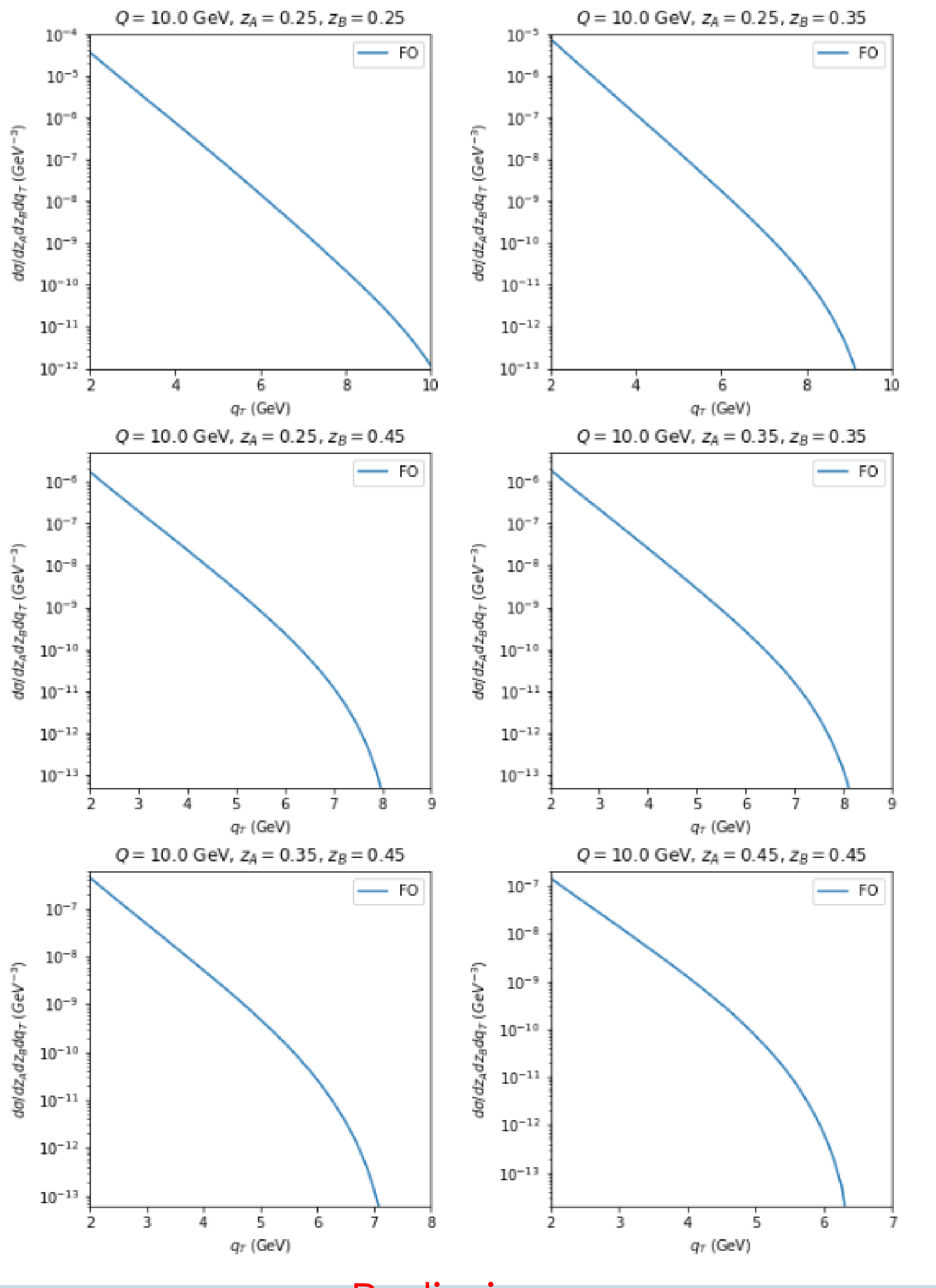
$$q_T^2 < \frac{(1-z_A)(1-z_B)}{z_A + z_B - z_A z_B} Q^2$$

- This maximum could give us a means of delineating the boundaries between the large, small, and intermediate q_T

Preliminary Results

- Plotting results for π^+/π^-
  - Using DSS14 fragmentation functions
  - $Q = 10$ GeV (Belle II energy)
  - Various values of z_A, z_B

Preliminary

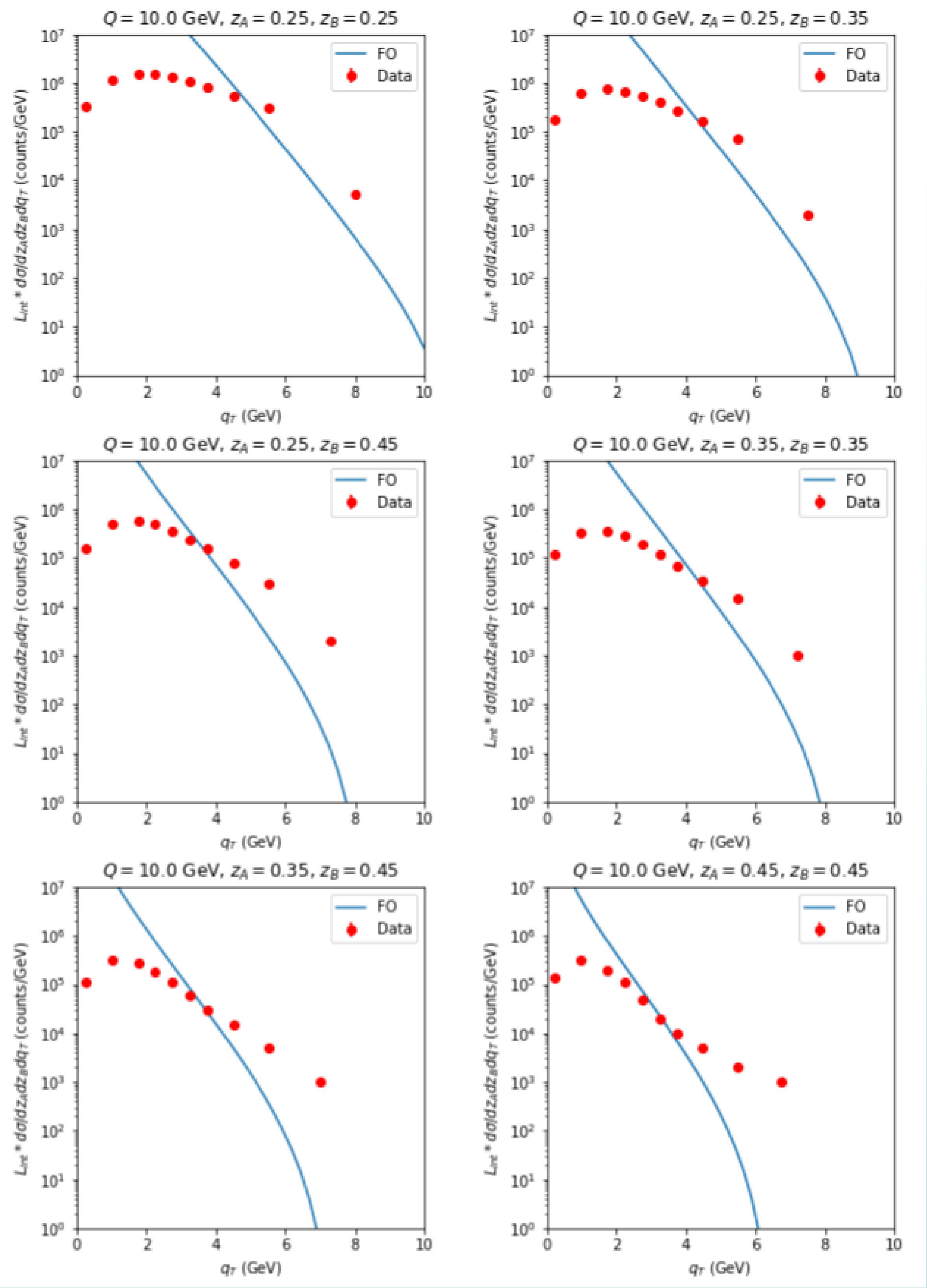


Preliminary

Preliminary comparison with simulated data

- PYTHIA simulated Belle data
- Significant discrepancy between the prediction and the simulation
- Possible reasons:
  - Limitations of PYTHIA accuracy at low Q
  - Differences between the fragmentation function q_T behavior (from ζ_{Amin} and ζ_B) which may differ from that of the PYTHIA hadronization
  - Need smaller bins at large q_T
  - Prediction does not yet include uncertainties

Preliminary



Preliminary

Conclusion and next steps

- Conclusion
  - Obtained an order α_s theoretical prediction for the fixed order contribution to electron-positron annihilation to two hadrons
  - Comparison with simulated data shows disagreement and we've identified several possible sources of the discrepancy that we need to investigate
- Next steps
  - Calculate the uncertainties in our prediction
  - Compare results and PYTHIA data at larger Q
  - Compare PYTHIA results at $Q = 10$ GeV to observables for which data already exists
  - Fit fragmentation functions to the PYTHIA data and use those in the theory calculation
  - Compare prediction with Belle II data once available