Energy-momentum tensor, forces in nucleon, applications

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Outline

- **Introduction**
  GPDs, Ji sum rule, tomography, and more
- **Energy-momentum tensor**
  EMT form factors & D-term
  last unknown global property(!)
- **Physical interpretation**
  3D densities: limitations & uses
  stress tensor and stability
- **D-term**
  What do we know?
  theory & experiment
- **Applications**
  large-N_c baryons
  hadrocharmonia
- **Outlook**

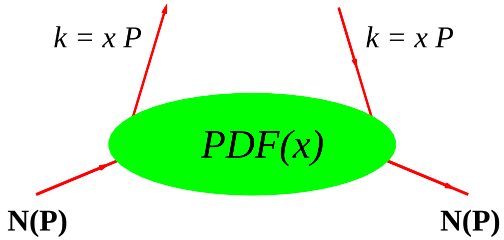
based on: PS, Boffi, Radici, PRD**66** (2002)
Goeke et al, PRD**75**, 094021; PRC**75**, 055207
Cebulla et al, Nucl. Phys. A**794**, 87 (2007)
Mai, PS, PRD**86**, 076001 & **86**, 096002 (2012)
Cantara, Mai, PS, Nucl. Phys. A**953**, 1 (2016)
Perevalova, Polyakov, PS, PRD**94**, 054024
Hudson, PS, PRD96 (2017) 114013
Hudson, PS, PRD97 (2018) 056003
Neubelt, Sampino, et al in progress
Polyakov, PS 1801.05858, 1805.06596
supported by: NSF

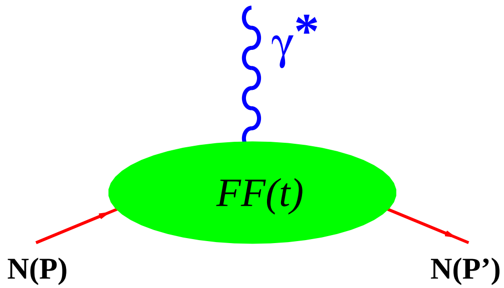
Introduction

- {form factors, PDFs} $\in$ GPDs

  $\int \mathrm{d}x \; H^q(x,\xi,t) = F_1^q(t)$

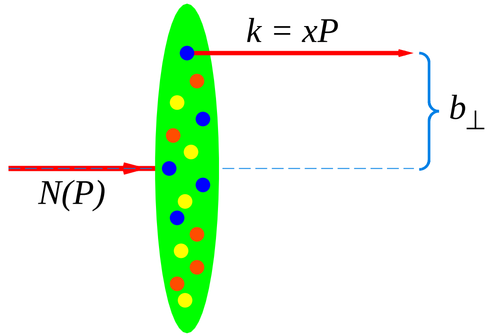
  $\lim\limits_{\Delta\to 0} H^q(x,\xi,t) = f_1^q(x)$





- do tomography (M. Burkardt)

$$H^q(x,b_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\left[\lim_{\xi\to 0} H^q(x,\xi,t)\right] e^{i\boldsymbol{\Delta}_T\boldsymbol{b}_T}$$

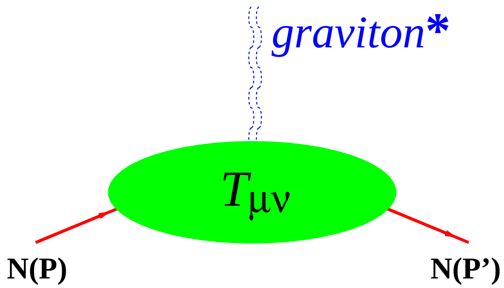


- **gravitational form factors** (polynomiality)

  $\int \mathrm{d}x \; x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$

  $\int \mathrm{d}x \; x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$

  Ji sum $A^q(t) + B^q(t) = 2J^q(t) \xrightarrow{t\to 0} 2J^q(0)$



- $\boldsymbol{T_{\mu\nu}}$ $\Rightarrow$ generators of Poincaré group

matrix elements of $\boldsymbol{T_{\mu\nu}}$: $\underbrace{\textbf{mass,}}_{T_{00}}$ $\underbrace{\textbf{spin},}_{\varepsilon^{ijk}x_j T_{0k}}$ $\underbrace{\boldsymbol{D}\textbf{-term}}_{T_{ij}}$

$\left.\begin{matrix}\boldsymbol{M}\\ \boldsymbol{J}\end{matrix}\right\}$ external properties

$\boldsymbol{D}$ "internal" property

nucleon EMT form factors

$$\langle p'|\hat{T}^{a}_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[\, A^a(t)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^a(t)\,\frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{4M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} \pm \bar{c}^a(t) g_{\mu\nu}\Bigg] u(p) \quad (a = q, g)$$

- $\hat{T}^q_{\mu\nu}$, $\hat{T}^g_{\mu\nu}$ both gauge-invariant (not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$ is conserved $\partial_\mu \hat{T}^{\mu\nu} = 0$ (and $\sum_a \bar{c}^a(t) = 0$, $a = q, g$)
- constraints: **mass** $\Leftrightarrow$ $A^q(0) + A^g(0) = 1$ (quarks + gluons carry 100 % of nucleon momentum)

  spin $\Leftrightarrow$ $B^q(0) + B^g(0) = 0$ (i.e. $J^q + J^g = \frac{1}{2}$ nucleon spin due to quarks + gluons)*
- property: **D-term** $\Leftrightarrow$ $D^q(0) + D^g(0) \equiv \boldsymbol{D}$ → unconstrained! **Unknown!** **Last global unknown!**

$$2P = (p' + p) \qquad \Delta = (p' - p) \qquad t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M_2^q(t)$

* also expressed as: vanishing of total gravitomagnetic moment

last global unknown: How do we learn about hadrons?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\mathbf{em}} = 0$	$\langle N'\|J^\mu_{\mathbf{em}}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\|J^\mu_{\mathbf{weak}}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\mathbf{grav}}\|N\rangle$	$\longrightarrow$	$M,\ J,\ D,\ \ldots$

global properties:

$$Q_{\text{prot}} = 1.602176487(40) \times 10^{-19}\text{C}$$
$$\mu_{\text{prot}} = 2.792847356(23)\mu_N$$
$$g_A = 1.2694(28)$$
$$g_p = 8.06(0.55)$$
$$M = 938.272013(23)\,\text{MeV}$$
$$J = \tfrac{1}{2}$$
$$\boldsymbol{D} = \ \textbf{??}$$

and more:
t-dependence ... ...
parton structure, etc ...

$\hookrightarrow$ $\boldsymbol{D}$ = *"last" global unknown*
which value does it have?
what does it mean?

Theoretical results

free spin 0 field

- free Klein-Gordon field $D = -1$
  (Pagels 1966; Hudson, PS 50 years later ...)

pions, kaons, η-meson

- Goldstone bosons of chiral symmetry breaking $D = -1$ in soft pion limit
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)

- chiral perturbation theory for Goldstone bosons
  Donoghue, Leutwyler (1991); Diehl, Manashov, Schäfer (2005); ...

$$
\begin{aligned}
D_\pi &= -1 + 16\,a\,\frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\,I_\pi - \frac{m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_K &= -1 + 16\,a\,\frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_\eta &= -1 + 16\,a\,\frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\,I_\pi + \frac{8m_K^2}{3F^2}\,I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4)
\end{aligned}
$$

where
$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}(\log\frac{\mu^2}{m_i^2} - 1)$
$i = \pi,\ K,\ \eta.$

$D_\pi = -0.97 \pm 0.01$
$D_K = -0.77 \pm 0.15$
$D_\eta = -0.69 \pm 0.19$ (estim. uncertainty, Hudson,PS 2017)

nuclei

- nuclei in liquid drop model $\boldsymbol{D = -0.2 \times A^{7/3}}$ $\rightarrow$ potential for DVCS with nuclei!
  Maxim Polyakov (2002) (see below)

- nuclei in Walecka model
  Guzey, Siddikov (2006)

$$\begin{aligned}
{}^{12}\text{C}: \quad & D = -6.2 \\
{}^{16}\text{O}: \quad & D = -115 \\
{}^{40}\text{Ca}: \quad & D = -1220 \\
{}^{90}\text{Zr}: \quad & D = -6600 \\
{}^{208}\text{Pb}: \quad & D = -39000
\end{aligned}$$

Q-balls (toy model laboratory)

- Q-balls, non-topological solitons in strongly interacting theory: $90 \leq -D =\leq \infty$
  Mai, PS PRD**86**, 076001 (2012)

- N^{th} excited Q-ball state (decay into ground states): $D = -\,\text{const}\, N^8$
  Mai, PS PRD**86**, 096002 (2012)

- Q-cloud limit, most extreme instability we could find: $D = -\,\text{const}/\varepsilon^2$ in the limit $\varepsilon \to 0$
  Cantara, Mai, PS NPA**953**, 1 (2016)

- Q-cloud excitations, even more extreme instability: $D < 0$ divergent and even more negative
  Bergabo, Cantara, PS, in preparation (2018)

free spin $\frac{1}{2}$ fermion

- $D = 0$ Dirac equation predicts $g = 2$ anomalous magnetic moment
  analoguously it predicts $D = 0$ for non-interacting fermion
  implicit: Donoghue, Holstein, Garbrecht, Konstandin, PLB**529**, 132 (2002)
  explicit in Hudson, PS Phys.Rev. D97 (2018) 056003

if $D_{\rm fermion} \neq 0 \;\leftarrow$ interactions!!

interacting fermion systems

- case study I: introduce boundary condition (bag model)

  "switch on interaction" $D = N_c^2 \underbrace{\left(\frac{-4\,\pi^2+15}{45}\right)}_{=-0.54\ldots\;<\;0}$ in limit $R \to \infty$

- case study II: chiral quark-soliton model

  $D = -F_\pi^2 M_N \int d^3r\; r^2\, P_2(\cos\theta)\, {\rm tr}_F[\nabla^3 U][\nabla^3 U^\dagger] + \mathcal{O}\big((\nabla U)^3\big)$ PS, Radici, Boffi

  "switch off chiral interaction" i.e. pion fields $U = \exp(i\tau^a\pi^a/F_\pi) \to 1 \;\Rightarrow\; D \to 0$

  Hudson, PS Phys.Rev. D97 (2018) 056003

D-term distinguishes free bosons and fermions

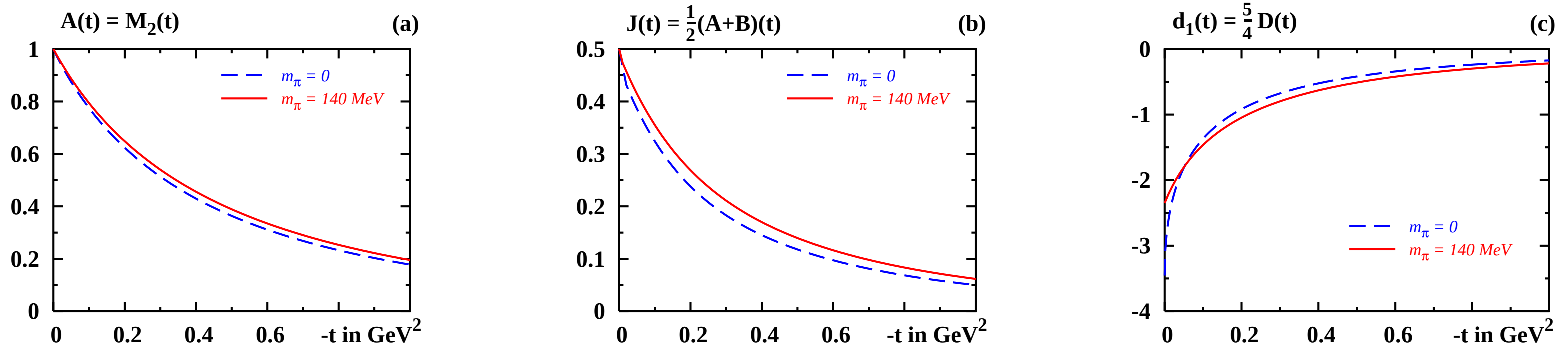
free spin-0 case $D = -1$ vs spin-$\frac{1}{2}$ case $D = 0$

visible universe made of fermions! Have to learn more about D-term!

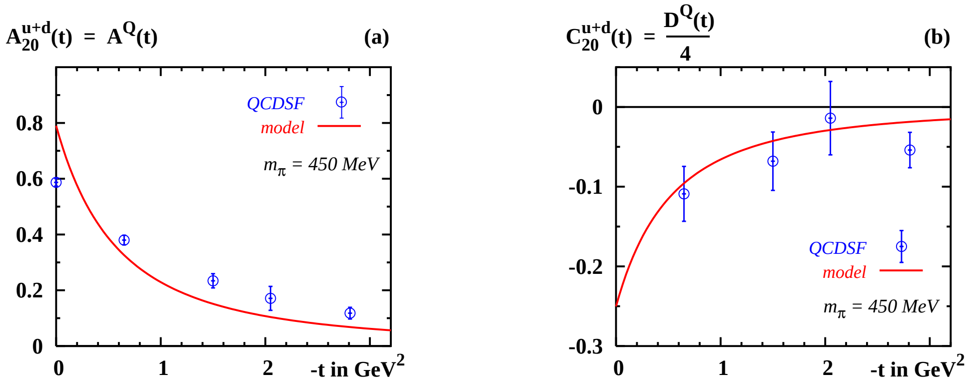
nucleon

- bag model (always good starting point!) $D = -1.145 < 0$ due to bag boundary!
  Ji, Melnitchouk, Song (1997); Neubelt, Sampino, et al (2018)

- chiral quark soliton model
  Petrov et al 1998, Goeke et al, PRD75 (2007) 094021



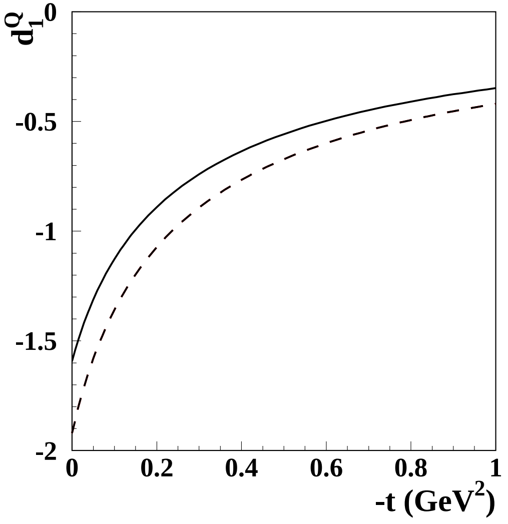
- lattice D^Q: QCDSF Collaboration, Göckeler et al, PRL92 (2004) 042002 & hep-ph/0312104



- χPT cannot predict D-term, but $d_1(m_\pi) = \mathring{d}_1 + \frac{5k\, g_A^2 M_N}{64\,\pi\, f_\pi^2}\, m_\pi + \ldots,\quad \mathring{d}_1'(0) = -\frac{k\, g_A^2\, M_N}{32\,\pi\, f_\pi^2\, m_\pi} + \ldots$
  $k = 1$ for finite N_c, and $k = 3$ for $N_c \to \infty$ Belitsky, Ji (2002), Diehl et al (2006), Goeke et al (2007)

nucleon dispersion relations

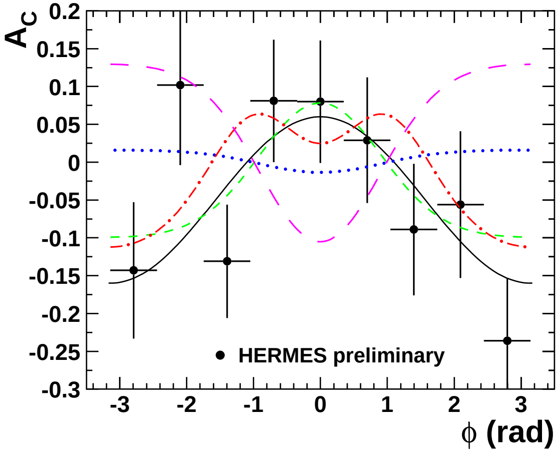
- unsubtracted t-channel dispersion relations (need pion PDFs) at $\mu^2 = 4\,\mathrm{GeV}^2$
  Barbara Pasquini, Maxim Polyakov, Marc Vanderhaeghen (2014)



... predictions are made. What does experiment say?

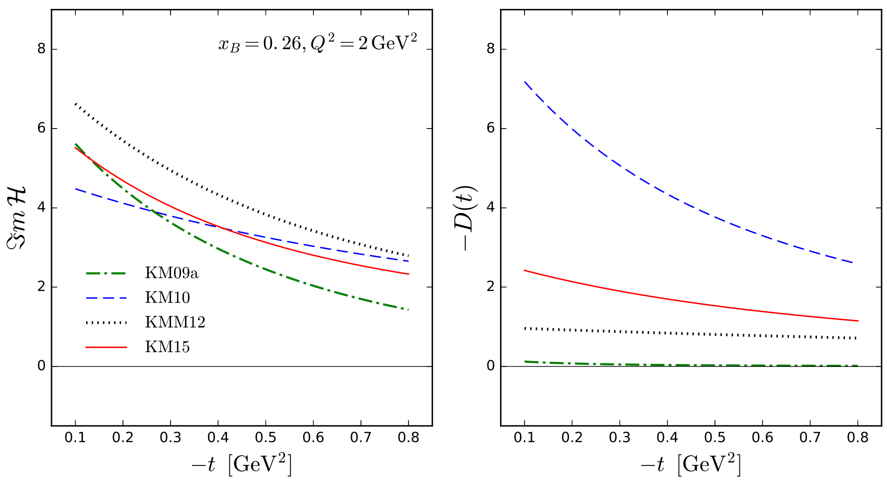
Experiment and phenomenology

- HERMES proceeding NPA711, 171 (2002); Airapetian et al PRD 75, 011103 (2007)



beam charge asymmetry
dotted line: VGG model without D-term (ruled out)
dashed line: VGG model + positive D-term (ruled out)
dashed-dotted: VGG model + **negative** D-term (yeah!)
Frank Ellinghaus, NPA711, 171 (2002)
model-dependent statement (!)
Belitsky, Müller, Kirchner, NPB629 (2002) 323

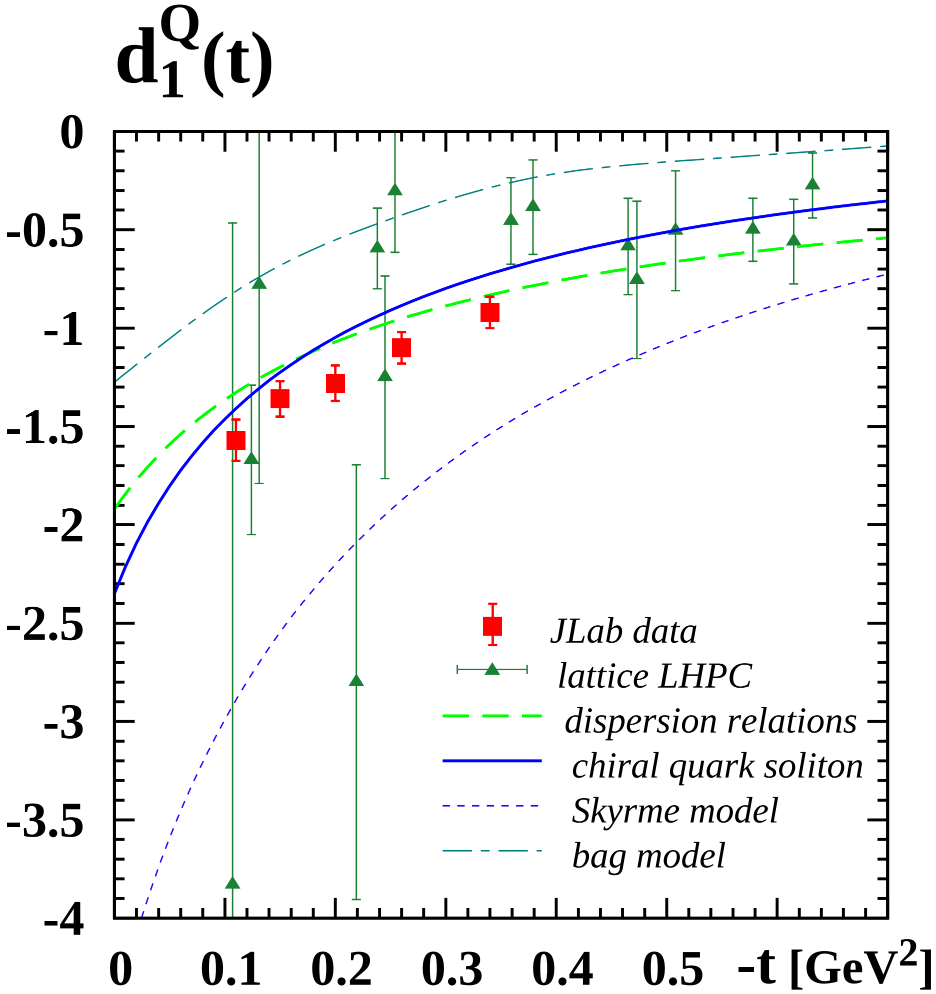
- fits by Kresimir Kumerički, Dieter Müller et al: $D < 0$ needed! model-independent evidence!



DVCS parametrizations from:
Kumerički, Müller, NPB 841 (2010) 1,
Kumerički, Müller, Murray, Phys. Part. Nucl. 45 (2004) 723
Kumerički, Müller, EPJ Web Conf. 112 (2016) 01012.
Fig. 9 from ECT* workshop proceeding 1712.04198
statistical uncertainty of D in KMM12: $\sim 50\,\%$,
statistical uncertainty of D in KM15: $\sim 20\,\%$.
unestimated systematic uncertainty
Kresimir Kumerički private communication

- **CLAS result**

based on: Girod et al PRL 100 (2008) 162002, Jo et al PRL 115 (2015) 212003
Burkert, Elouadrhiri, Girod, **Nature 557, 396 (2018)** ← Latifa (Monday)
see talk: V. Burkert, SPIN 2016 in Urbana-Champaign, Sep. 2016



D-term = subtraction term in fixed-t dispersion relations for $\mathcal{A}_{\rm DVCS}$
Teryaev hep-ph/0510031
Anikin, Teryaev, PRD76, 056007 (2007)
Diehl and Ivanov, EPJC52, 919 (2007)
Radyushkin, PRD83, 076006 (2011)

subtraction term $\sim d_1 + d_3 + d_5 + \ldots$
the $d_i \to 0$ for $i > 1$ with $Q^2 \to \infty$

assumed d_3, d_5, ... small compared to d_1
working assumption (do better → future data)

chiral quark-soliton $d_3^q/d_1^q = 0.3$, $d_5^q/d_1^q = 0.1$
Kivel, Polyakov, Vanderhaeghen, PRD63 (2001)

$D^q(t) = \frac{4}{5}\, d_1^q(t)$

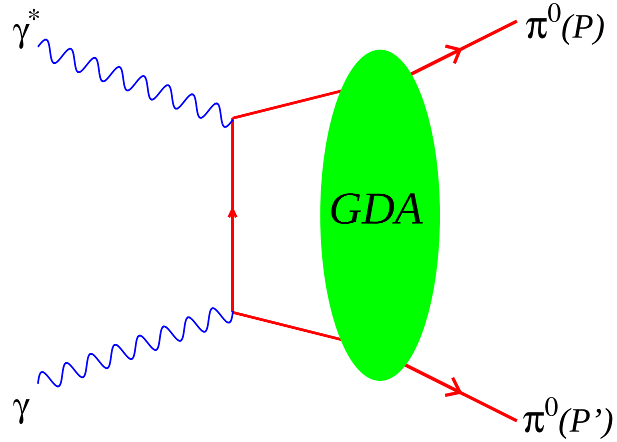
⇒ CLAS, KM-fits, dispersion relations, models, lattice: ***D*-term negative & sizeable!**

(double-checking if same normalization in analysis and calculations) Exciting! What do we learn?

- **D-term of π^0**

access EMT form factors of unstable particles
through generalized distribution amplitudes
(analytic continuation of GPDs)
via $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^-

Masuda et al (Belle), PRD 93, 032003 (2016)



best fit to Belle data $\rightarrow$ $D^Q_{\pi^0} \approx -0.7$
at $\langle Q^2 \rangle = 16.6$ GeV^2
compatible with soft pion theorem $D_{\pi^0} \approx -1$
(if gluons contribute the rest)

Kumano, Song, Teryaev, PRD97, 014020 (2018)

interpretation

- Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$
- analog to electric form factor $G_E(\vec{\Delta}^2) = \int d^3\vec{r}\, \rho_E(\vec{r})\, e^{i\vec{\Delta}\,\vec{r}} \to$ charge distribution

  Sachs, PR126 (1962) 2256

  $\hookrightarrow Q = \int d^3\vec{r}\, \rho_E(\vec{r})$

- static EMT $T_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{-i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle \to$ mechanical properties of nucleon

  M.V.Polyakov, PLB 555 (2003) 57

  $\hookrightarrow M_N = \int d^3\vec{r}\, T_{00}(\vec{r})$, etc

- limitations: 2D densities exact partonic probability densities.

  3D densities not exact, corrections for radius $\delta_{\rm rel} = \frac{1}{2m^2R^2}$, reservations for $r \ll \lambda_{\rm Compt} = \frac{\hbar}{mc}$

known since earliest days (Sachs, 1962) comprehensive studies, e.g. by

- Belitsky & Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2
- X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect)
- G. Miller, PRC80 (2009) 045210 (toy model, dramatic effect)

mathematically well-defined, correct and consistent

relative correction for $\langle r_E^2\rangle = \int d^3r\, r^2 T_{00}(r)/m$ is $\delta_{\rm rel} = 1/(2m^2R^2)$ Hudson, PS PRD (2007)

numerically $\underbrace{\text{pion}}_{220\,\%}$, $\underbrace{\text{kaon}}_{25\,\%}$, $\underbrace{\text{nucleon}}_{3\,\%}$, $\underbrace{\text{deuterium}}_{1\times10^{-3}}$, $\underbrace{{}^4\text{He}}_{5\times10^{-4}}$, $\underbrace{{}^{12}\text{C}}_{3\times10^{-5}}$, $\underbrace{{}^{20}\text{Ne}}_{6\times10^{-6}}$, $\underbrace{{}^{56}\text{Fe}}_{5\times10^{-7}}$, $\underbrace{{}^{132}\text{Xe}}_{6\times10^{-8}}$, $\underbrace{{}^{208}\text{Pb}}_{2\times10^{-8}}$

- important distinction:

  2D densities = partonic probability densities (unitarity)

  must be exact! → M. Burkardt (2000) is exact $\surd$

  apply to any particle including pion

  vs

  3D densities = mechanical response functions

  correlation functions subject to corrections (which is ok) → M. Polyakov (2002)

  can be studied for nucleon or heavier where corrections acceptably small $\surd$

- besides:

  no 2D interpretations exist for stress tensor and pressure

  inherently 3D concepts, have to pay a prize (and pay attention to corrections)

- **interpretation as 3D-densities** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $\boldsymbol{T_{\mu\nu}(\vec{r})} = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$

all formulae correct, interpretation in terms of 3D-densities has limitations (see above)

$$\int d^3r\, T_{00}(\vec{r}\,) = M_N \qquad \text{known}$$

$$\int d^3r\, \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r}, \vec{s}\,) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5}\, M_N \int d^3r \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T_{ij}(\vec{r}\,) \equiv \boldsymbol{D} \qquad \textbf{new!}$$

with: $$\boldsymbol{T_{ij}(\vec{r}\,) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}} \qquad \textbf{stress tensor}$$

$\boldsymbol{s(r)}$ related to distribution of *shear forces*
$\boldsymbol{p(r)}$ distribution of *pressure* inside hadron
$\Big\} \longrightarrow$ **“mechanical properties”**

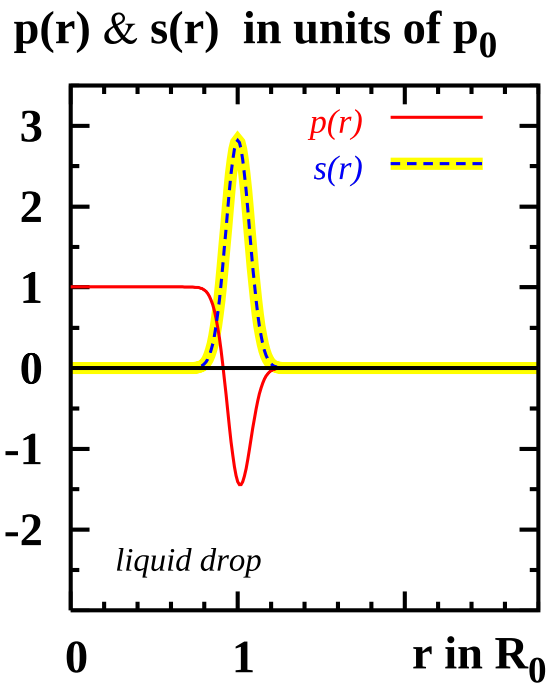
relation to stability: EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

$\hookrightarrow$ necessary condition for stability $\int_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

$$D = -\frac{16\pi}{15}\, m \int_0^\infty \mathrm{d}r\, r^4 s(r) = 4\pi m \int_0^\infty \mathrm{d}r\, \boldsymbol{r^4\, p(r)}$$ $\rightarrow$ shows how internal forces balance

let's gain intuition from models:

- **liquid drop model of nucleus**



radius $R_A = R_0\, A^{1/3}$, $m_A = m_0\, A$

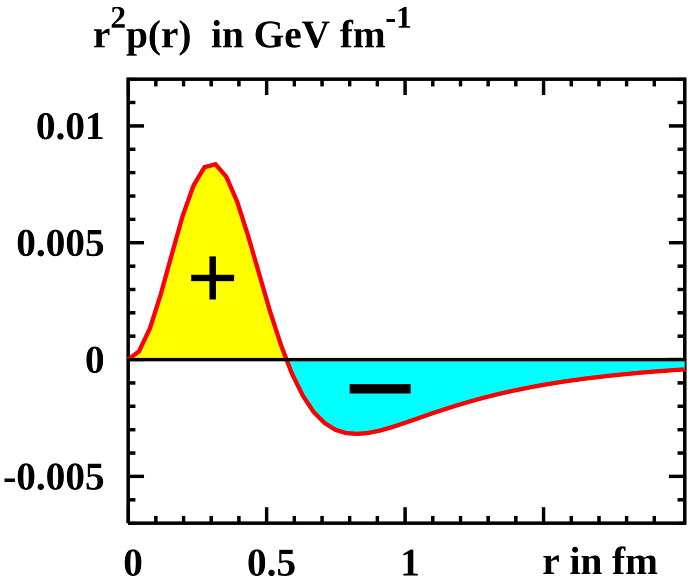
surface tension $\gamma = \frac{1}{2} p_0 R_A$, $s(r) = \gamma\, \delta(r - R_A)$

pressure $p(r) = p_0\, \Theta(R_A - r) - \frac{1}{3} p_0 R_A\, \delta(r - R_A)$

D-term $\boldsymbol{D = -\frac{4\pi}{3}\, m_A\, \gamma\, R_A^4 \approx -0.2\, A^{7/3}}$

M.V.Polyakov PLB555 (2003);
tested in Walecka model Guzey, Siddikov (2006)

- **chiral quark soliton model of nucleon**



- $p(0) = 0.23\,\text{GeV/fm}^3 \approx 4 \times 10^{34}\,\text{N/m}^2$

  $\gtrsim$ 10-100×(pressure in center of neutron star)

- $p(r) = 0$ at $r = 0.57\,\text{fm}$ change of sign in pressure
- $p(r) = \left(\frac{3g_A^2}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large r in chiral limit $m_\pi \to 0$

  Goeke et al, PRD75 (2007) 094021

- **How does it look like in nature?** Look in **Nature article** ☺

  see Burkert, Elouadrhiri, Girod Nature 557, 396 (2018)

  beware: additional assumptions!
  (early state of art, will be improved)

- **technical remark** on additional assumption in Nature-article

JLab sensitive only to quarks! (also other experiments so far, see KM fits)

now one can define D-term $D^q = -\frac{16\pi}{15}\, m \int_0^\infty \mathrm{d}r\, r^4 s^q(r)$ and D^g for quarks and gluons

"partial" (quark, gluon) contributions to shear forces can be defined

but pressure **only** defined for total (quark + gluon) system!

"partial" (quark, gluon) contributions to pressure cannot be defined

reason: $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij}$ such that $\begin{cases} \text{shear forces} \propto \text{traceless part} \\ \text{pressure} \propto \text{trace of stress tensor} \end{cases}$

but remember: $\langle p'|\hat{T}^q_{\mu\nu}|p\rangle = \bar{u}(p')\left[\cdots + \boldsymbol{D^q(t)}\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} \pm \bar{c}^q(t)g_{\mu\nu}\right]u(p)$

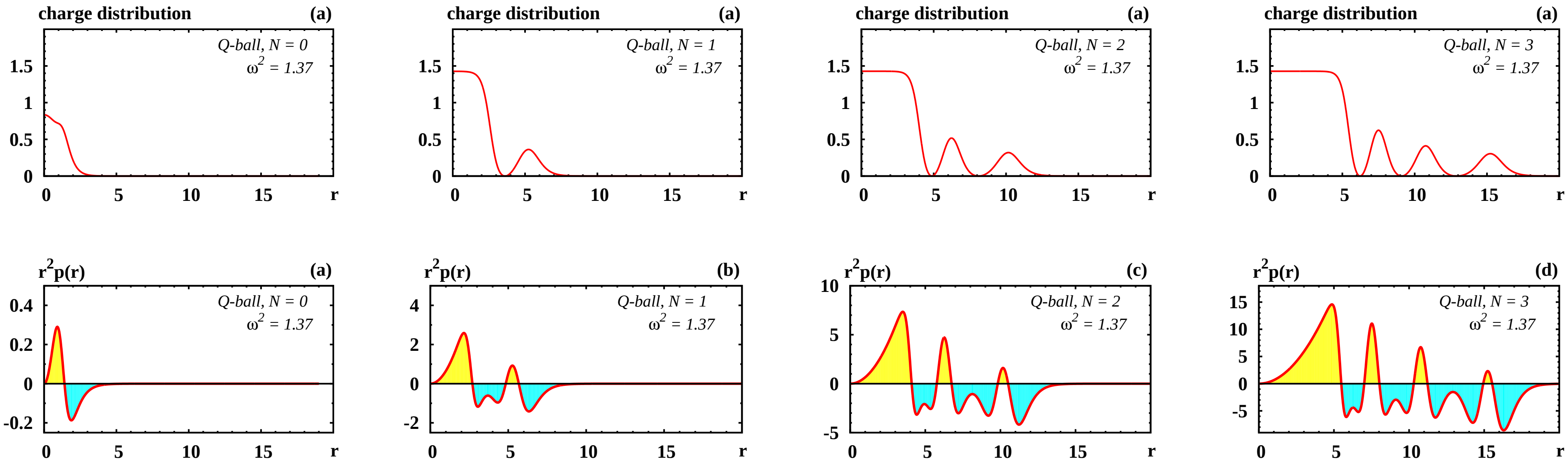
form $D^q(t)$ alone one cannot obtain pressure! Some implicit assumption on $D^g(t)$ in Nature.
Keep in mind: very first, very model-dependent look; will improve!

• more intuition from toy system: Q-ball

$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$ with U(1) global symm., $V = A\,(\Phi^*\Phi) - B\,(\Phi^*\Phi)^2 + C\,(\Phi^*\Phi)^3$, $\Phi(t,\vec{r}) = e^{i\omega t}\,\phi(r)$

$N = 0$ ground state, $N = 1$ first excited state, etc Volkov & Wohnert (2002), Mai, PS PRD86 (2012)

charge density exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros



excited states unstable, but $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0$ always valid, and D-term always negative!

so far **all D-terms** negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q-balls, Q-clouds

could perhaps the Roper resonance look like this? (possible to measure??)
However e.g. Δ-resonance, similar to nucleon! (lowest state for $J = T = \frac{3}{2}$, see below)

stress tensor and mechanical radius

- $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij} =$ symmetric 3×3 matrix

  $\rightarrow$ can be diagonalized with eigenvalues:

  $\frac{2}{3}\,s(r) + p(r) =$ normal force (eigenvector $\vec{e}_r$)

  $-\frac{1}{3}\,s(r) + p(r) =$ tangential force ($\vec{e}_\theta$, $\vec{e}_\phi$, degenerate for spin 0 and $\frac{1}{2}$)

- mechanical stability $\Leftrightarrow$ normal force directed towards outside

  $\Leftrightarrow T^{ij} e_r^j\,dA = \underbrace{[\frac{2}{3}\,s(r) + p(r)]}_{>0} e_r^i\,dA \quad \Rightarrow \quad \boldsymbol{D < 0}$

- define: $\langle \boldsymbol{r^2} \rangle_{\mathbf{mech}} = \dfrac{\int d^3r\; r^2 [\frac{2}{3}\,s(r) + p(r)]}{\int d^3r\; [\frac{2}{3}\,s(r) + p(r)]} = \dfrac{6D(0)}{\int_{-\infty}^{0} \mathrm{d}t\, D(t)}$ vs $\langle r^2_{\text{ch}} \rangle = \dfrac{6\,G'_E(0)}{G_E(0)}$

  intuitive result for large nucleus $\frac{2}{3}\,s(r) + p(r) = p_0\,\Theta(R_A - r) \;\rightarrow\; \langle r^2 \rangle_{\text{mech}} = \frac{3}{5}\,R_A^2$

  M.Polyakov, PS arXiv:1801.05858 (Kumano, Song, Teryaev PRD (2018) used $D'(0)$ but inadequate)

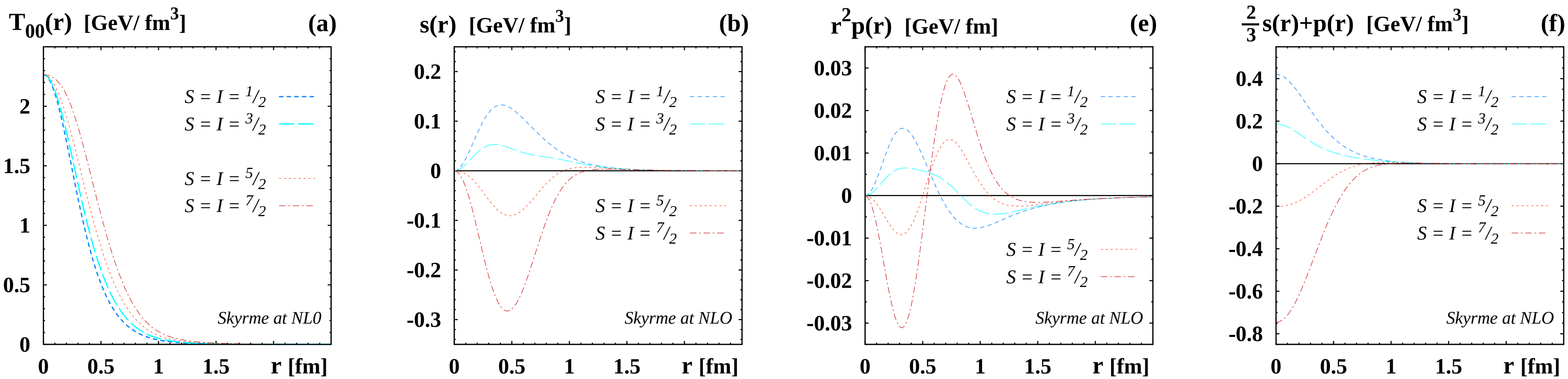
- proton: $\langle r^2 \rangle_{\text{mech}} \approx 0.75\,\langle r^2_{\text{ch}} \rangle$ for $m_\pi = 140\,\text{MeV}$ (chiral quark soliton model)

  Notice: in chiral limit $\langle r^2 \rangle_{\text{mech}}$ finite vs $\langle r^2_{\text{ch}} \rangle$ which diverges

more on normal/tangential forces in future from Arek Trawinski (see talk Lightcone 2018) Lorcé, Moutarde

Application I: nucleon, Δ, large-N_c artifacts Witten 1979

in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \ldots}_{\text{artifacts}}$



$M_B = M_{\text{sol}} + \dfrac{S(S+1)}{2\Theta}$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$
stability needs more:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

mechanical stability
$T^{ij}\, da^j \geq 0$
$\Leftrightarrow \quad \frac{2}{3} s(r) + p(r) \geq 0$
artifacts do not satisfy!
⇒ **have positive D-term!!**
That's why they do not exist!
EMT: dynamical understanding
Perevalova et al (2016)

⇒ particles with positive D unphysical!!!

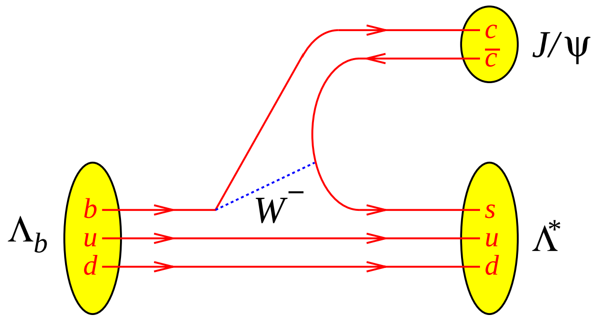
Application II: hidden-charm pentaquarks as hadrocharmonia

$\Lambda_b^0 \longrightarrow J/\Psi\, p\, K^-$ seen

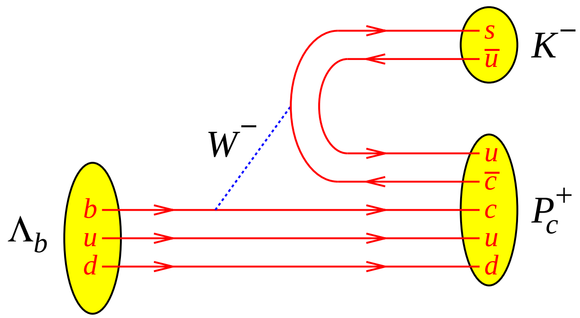
Aaij *et al.* PRL 115 (2015)

Λ_b^0 $m = 5.6$ GeV, $\tau = 1.5$ ps
J/Ψ $m = 3.1$ GeV, $\Gamma = 93$ keV, $\Gamma_{\mu^+\mu^-} = 6\,\%$
Λ^* $m = 1.4$ GeV or more, $\Lambda^* \to K^- p$ in 10^{-23}s

$\longrightarrow J/\Psi\, \Lambda^*$



$\longrightarrow K^-\, P_c^+$



state	m [MeV]	Γ [MeV]	Γ_{rel}	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1)\,\%$	$J/\psi\, p$	$\frac{3}{2}^{\mp}$ or $\frac{5}{2}^{+}$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2)\,\%$	$J/\psi\, p$	$\frac{5}{2}^{\pm}$ or $\frac{3}{2}^{-}$

appealing approach to new pentaquarks

M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

- **theoretical approach**

$R_{c\bar{c}} \ll R_N \;\; \Rightarrow$ non-relativistic multipole expansion Gottfried, PRL 40 (1978) 598
baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

$V_{\text{eff}} = -\frac{1}{2}\,\alpha\,\vec{E}^2$ Voloshin, Yad. Fiz. **36**, 247 (1982)

- **chromoelectric polarizability**

$$\alpha(1S) \approx \quad 0.2\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S) \approx \quad 12\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S \to 1S) \approx \begin{cases} -0.6\,\text{GeV}^{-3}\ \text{(pert)}, \\ \pm 2\,\text{GeV}^{-3}\ \text{(pheno)}, \end{cases}$$

in heavy quark mass limit & large-N_c limit
$\rightsquigarrow$ "perturbative result" Peskin, NPB 156 (1979) 365

value for $2S \to 1S$ transition from
phenomenological analysis of $\psi' \to J/\psi\,\pi\,\pi$ data
Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455

- **chromoelectric field strength:**

$$\vec{E}^2 = g^2\left(\frac{8\pi^2}{bg^2}\,T^{\mu}{}_{\mu} + T_{00}^{G}\right)$$

$b = \frac{11}{3}\,N_c - \frac{2}{3}\,N_F$ leading coeff. of β-function
$g =$ strong coupling at low (nucleon) scale $\lesssim 1$ GeV
$g_s =$ strong coupling at scale of heavy quark ($g_s \neq g$)
$T_{00}^{G} = \xi T_{00}$ with $\xi =$ fractional contributions of gluon to M_N
$T^{\mu}{}_{\mu} = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i}{r}\frac{r^j}{r} - \frac{1}{3}\,\delta^{ij}\right) s(r) + \delta^{ij}\,p(r)$

- **universal effective potential**

$$\boldsymbol{V_{\text{eff}} = -\tfrac{1}{2}\,\alpha\,\frac{8\pi^2}{b}\,\frac{g^2}{g_s^2}\Big[\nu\,T_{00}(r) + 3p(r)\Big]\,,\quad \nu = 1 + \xi_s\,\frac{b\,g_s^2}{8\pi^2}}$$

$\boldsymbol{\nu \approx 1.5}$ estimate by Eides et al, op. cit.
Novikov & Shifman, Z.Phys.C8, 43 (1981);
X. D. Ji, Phys. Rev. Lett. **74**, 1071 (1995)

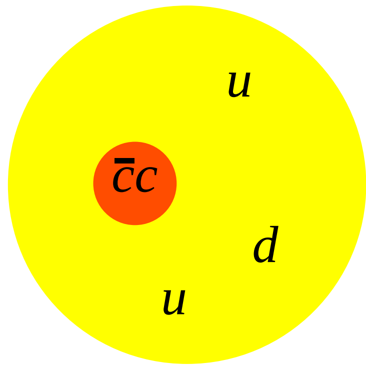
- **in future GPDs can help:** GPDs $\Rightarrow$ EMT form factors $\Rightarrow$ EMT densities
  $\Rightarrow$ universal potential V_{eff} for quarkonium-baryon interaction!

- **currently:** use chiral quark soliton model (Eides et al, 2015); Skyrme (Perevalova et al 2016)

- **compute quarkonium-nucleon bound state**

  solve $\left(-\frac{\vec{\nabla}^2}{2\mu} + V_{\text{eff}}(r)\right)\psi = E_{\text{bind}}\,\psi$

  $\mu =$ reduced quarkonium-baryon mass



- **results:**

  nucleon and J/ψ form no bound state

  nucleon and $\psi(2S)$ form 2 bound states with nearly degenerate masses around 4450 MeV
  in $L = 0$ channel, $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ if $\alpha(2S) \approx 17$ GeV^{-3} (consistent with guideline from pert. calc.)
  with partial width $\Gamma \sim$ few tens of MeV

- predictions for bound states of $\psi(2S)$ with Δ and hyperons $\leftarrow$ test approch

Summary & Outlook (see review arXiv:1805.06596)

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  mass decomposition, spin decomposition, and D-term!

- **D-term**: last unknown global property, related to forces
  attractive and physically appealling $\rightarrow$ "motivation"

- **first results**(!) from experiment/phenomenology for proton, π^0
  compatible with results from theory and models

- define **pressure & mechanical radius** $\rightarrow$ complementary information!

- development: apply to **hadrocharmonia** pentaquarks & tetraquarks
  rich potential, new predictions, ongoing work

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Thank you!