SIDIS in Wandzura-Wilczek-type approximation

Saman Bastami

- TMDs & FFs in SIDIS
- WW-approximation
- Asymmetries in WW-type-Approximation
- Remarks
- Conclusion

QCD evolution 2018

H. Avakian, A. V. Efremov, A. Kotzinian, B.U. Musch, B. Parsamyan, A. Prokudin, M. Schlegel, P. Schweitzer, K. Tezgin, W. VogelsangPeter

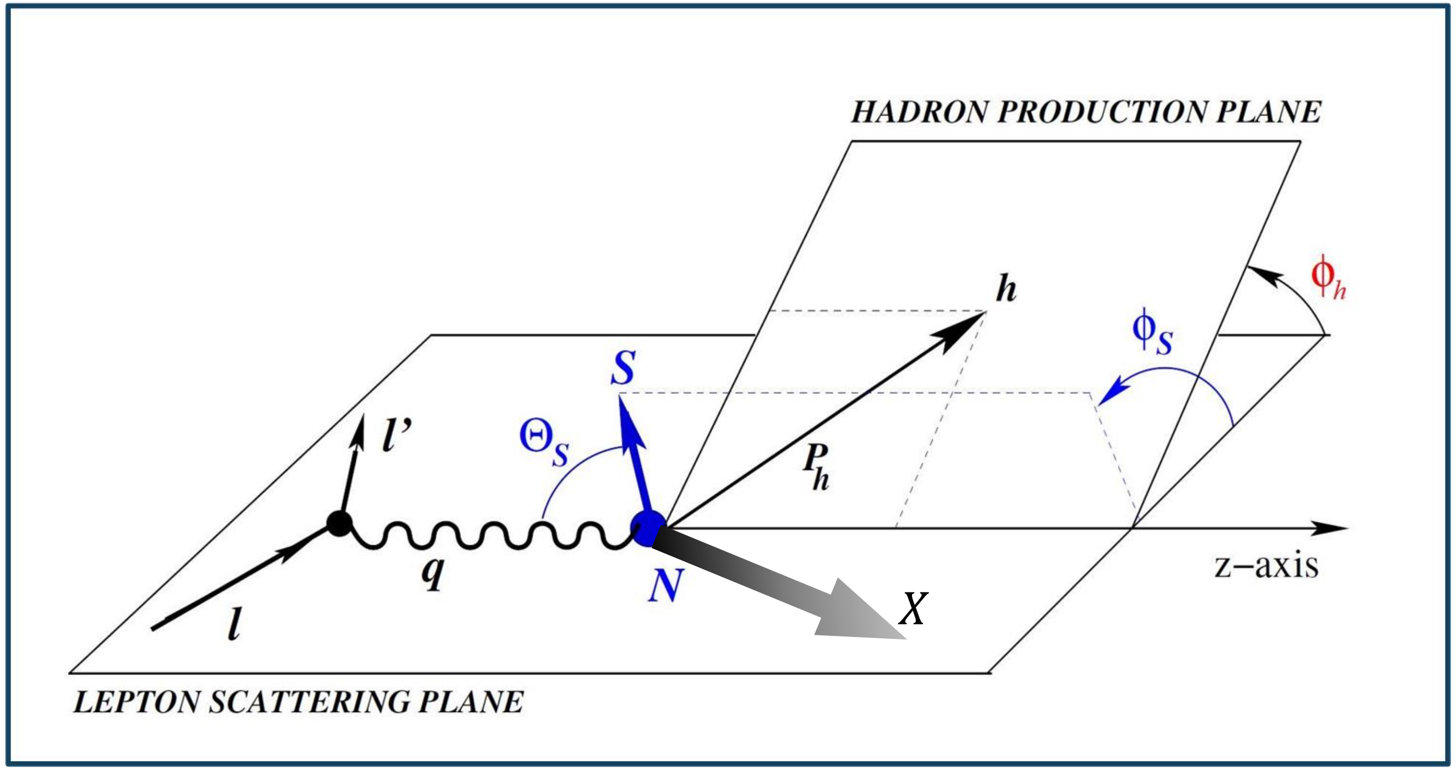
How TMDs show up?

Going beyond 1D picture of nucleon by generalizing the PDFs

$$l + N \rightarrow l + X$$

$$x = \frac{Q^2}{2P.q} \qquad y = \frac{P.l}{P.q}$$

$$s = (l + P)^2 \qquad W = (q + P)^2$$



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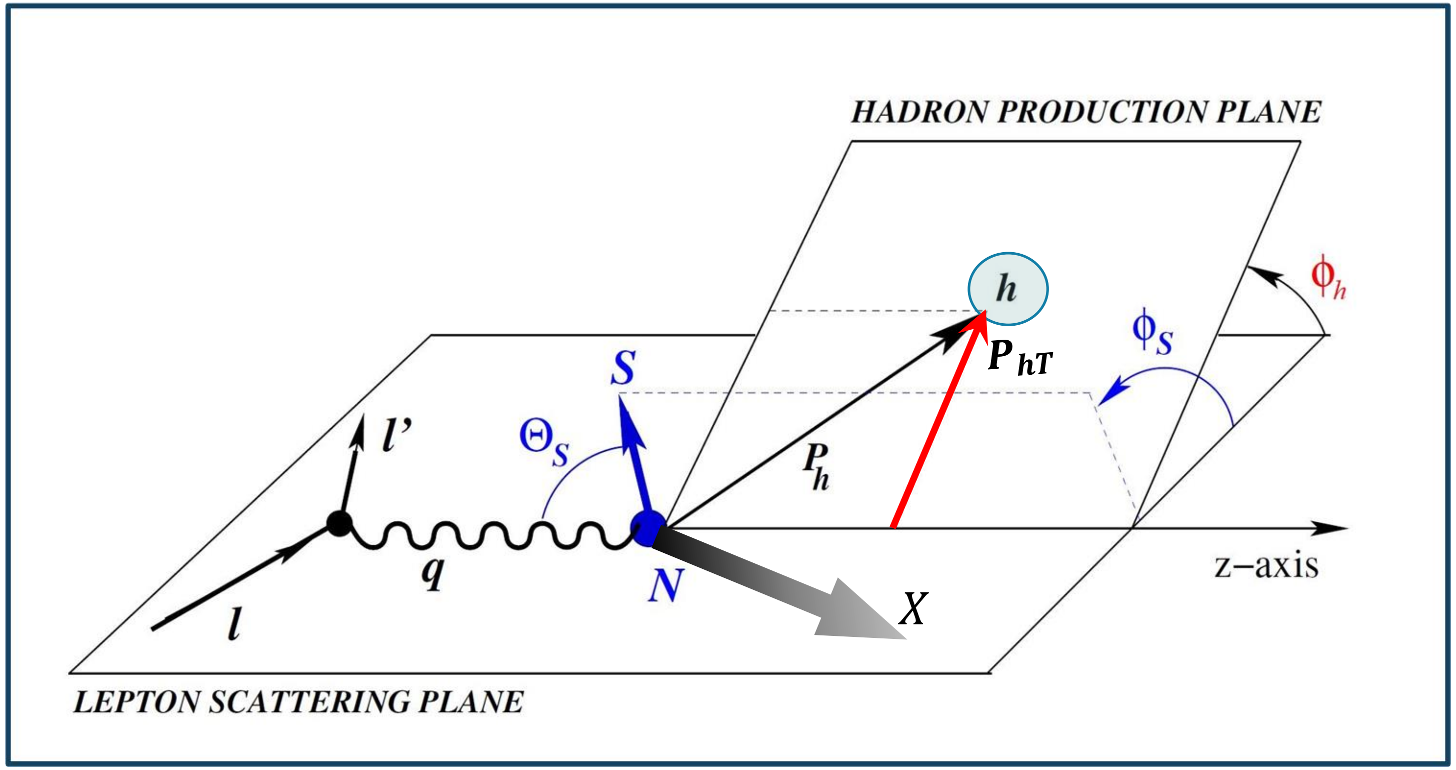

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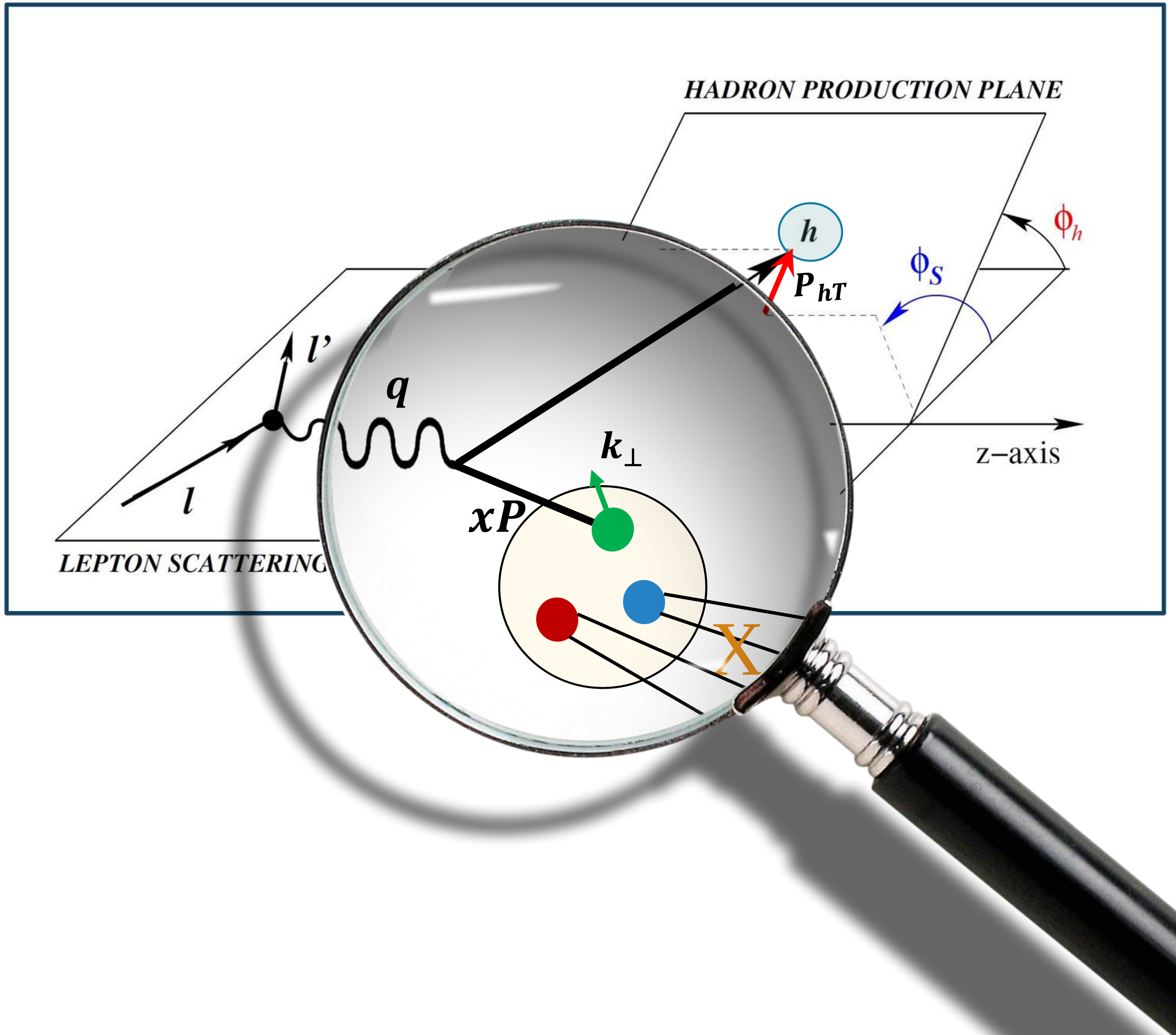

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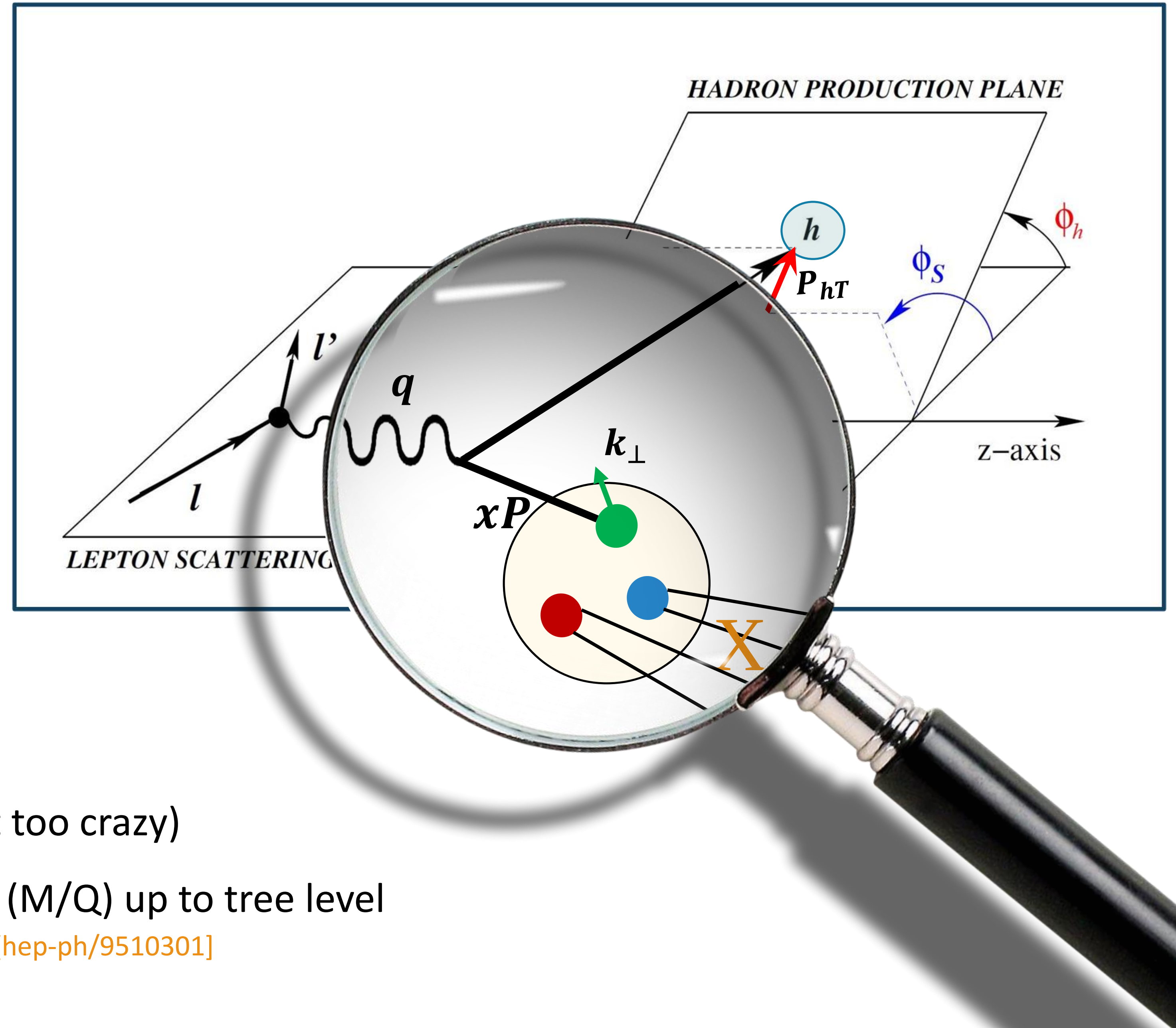

- ✓ One photon exchange approximation
- ✓ $Q \rightarrow \infty$ and small P_{hT}
- ✓ Factorization assumed to be working (not too crazy)
- ✓ Expansion of hadronic tensor in orders of (M/Q) up to tree level

P. J. Mulders and R. D. Tangerman, Nucl. Phys. B461 (1996) [hep-ph/9510301]

SIDIS cross section involves **18 structure functions** and as a result of **factorization**, they are such convolutions:

$$C\,[w\,f\,D] = \; x\sum_a e_a^2 \int d^2k_\perp d^2P_T\;\; \delta^2(zk_\perp - P_\perp - P_{hT})\;\, w(p_T\,,k_T)\;\, f^a(x,k_\perp)\;\, D^a\big(z,{P_\perp}^2\big)$$

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qq **correlator** parametrized up to **twist-3**:

8 Leading TMDs ★ $f_1\,,\; f_{1T}^{\perp}\,,\; g_1\,,\; g_{1T}^{\perp}\,,\; h_1\,,\; h_{1L}^{\perp}\,,\; h_{1T}^{\perp}\,,\; h_1^{\perp}$

16 Subleading TMDs $e\,,\; e_T^{\perp}\,, ..\,,\; f^{\perp}\,,\; f_T\,, ..\,,\; g^{\perp}\,,\; g_T\,, ..\,,\; h\,,\; h_T^{\perp}\,, ..$

2 Leading FFs ★ $D_1\,,\; H_1^{\perp}$

4 Subleading FFs $E\;\;,\; D^{\perp}, H\,,\, G^{\perp}$

A. Bacchetta et al., JHEP 02 (2007) 093, [hep-ph/0611265].

$\boldsymbol{\bar{q}gq}$ **correlator** parameterized up to **twist-3**: $\tilde{f}^{\perp},\; \tilde{g}_T\,,\;\; \tilde{h}_L\;\,,\; \tilde{f}_T'\,, \tilde{e}\,, \dots, \widetilde{D}^{\perp}\,,\;\; \widetilde{H}\;\,,\;\; \tilde{E}\;\;, \tilde{G}^{\perp}$

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***qgq* correlator** parameterized up to **<u>twist-3</u>**: $\tilde{f}^\perp,\ \tilde{g}_T\ ,\ \tilde{h}_L\ ,\ \tilde{f}_T'\ ,\ \tilde{e}\ ,\ ....\ ,\ \tilde{D}^\perp\ ,\ \tilde{H}\ ,\ \tilde{E}\ ,\ \tilde{G}^\perp$

Is there a way to make life easier?!

Wandzura-Wilczek Approximation

…Might be

1. Equations of motion

$$g_T = \overset{\text{Twist-3}}{\tilde{g}_T} + \frac{k_\perp^2}{2M^2}\overset{\text{Twist-2}}{g_{1T}^\perp} + \cancelto{\ll 1}{\frac{m_q}{M}} h_1 \quad , \quad \frac{H}{z} = \frac{\overset{\text{Twist-3}}{\tilde{H}}}{z} + \frac{P_\perp^2}{{m_h}^2} H_1^\perp \quad , \dots \dots$$

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2. Gaussian Ansatz

$$f_1^q\left(x, k_\perp^2\right) = f_1^a(x)\, \frac{1}{\pi\langle k_\perp^2\rangle}\, \exp\left(-\frac{k_\perp^2}{\langle k_\perp^2\rangle}\right)$$

$$D_1^h\left(z, P_\perp^2\right) = D_1^h(z)\, \frac{1}{\pi\langle P_\perp^2\rangle}\, \exp\left(-\frac{P_\perp^2}{\langle P_\perp^2\rangle}\right)$$

Widths of the Gaussians ?
- Regular extractions
- Lattice calculations
- Positivity bounds

Well-known and supported by both experimental data and effective models

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Twist-3 Twist-2 $\ll 1$ Twist-3

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$$g_T = \tilde{g}_T + \frac{k_\perp^2}{2M^2} g_{1T}^\perp + \cancel{\frac{m_q}{M} h_1} \quad , \quad \frac{H}{z} = \frac{\tilde{H}}{z} + \frac{P_\perp^2}{{m_h}^2} H_1^\perp \quad , \dots\dots$$

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Widths of the Gaussians ?
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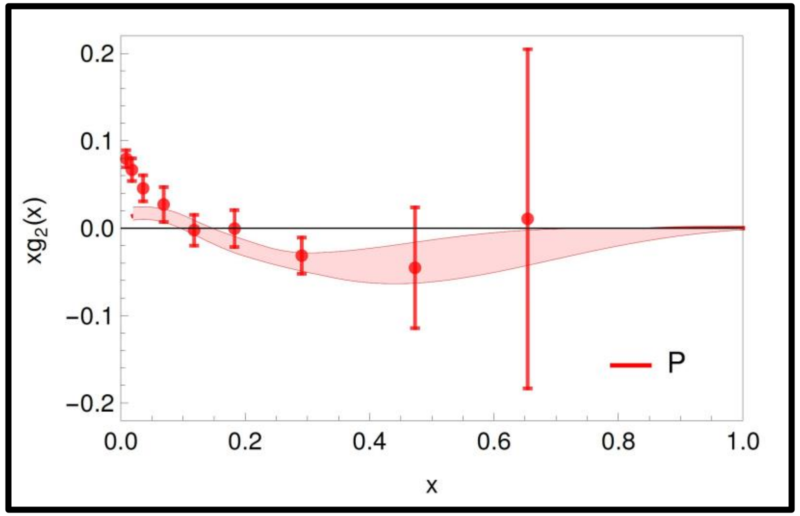
Well-known and supported by both experimental data and effective models)

3. WW-Approximation
$$g_T^a(x) = \int_x^1 \frac{dy}{y} g_1^a(y) + \cancel{\tilde{g}'^a_T(x)}$$
$$h_L^a(x) = 2x \int_x^1 \frac{dy}{y^2} h_1^a(y) + \cancel{\tilde{h}'^a_L(x)}$$

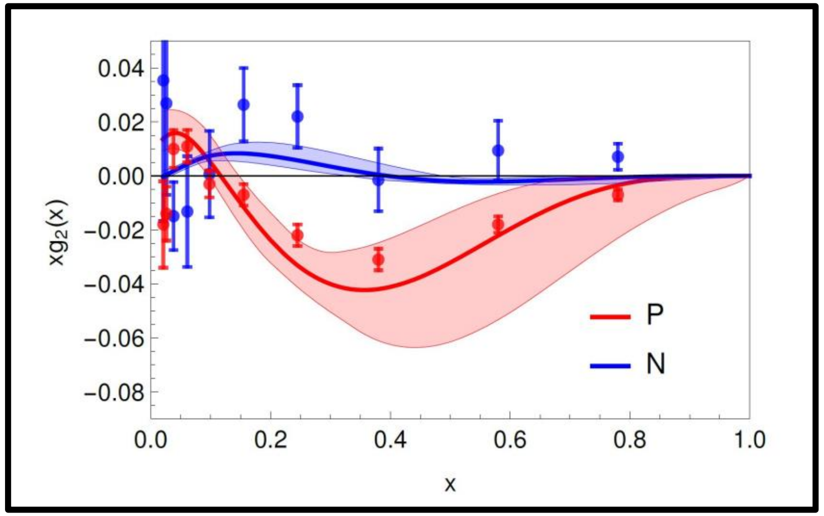
S. Wandzura and F. Wilczek, Phys. Lett. B72 (1977) 195.

Wandzura-Wilczek Approximation

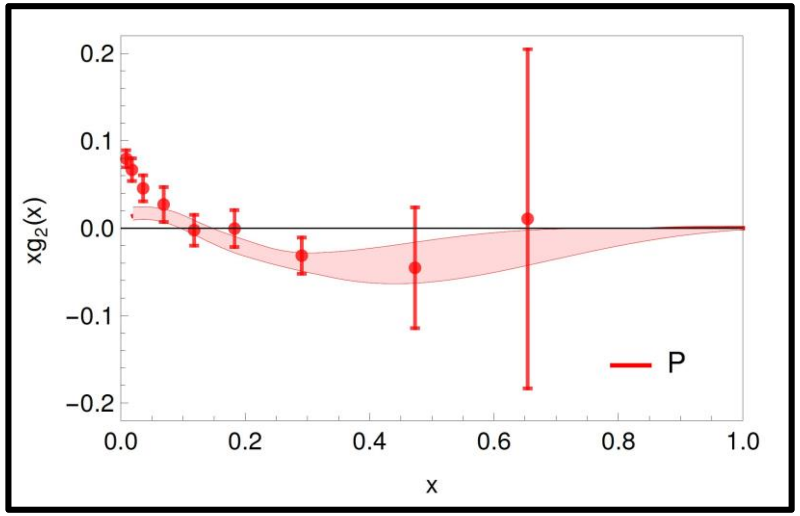


$$g_2(x) \approx -g_1(x) + \int_x^1 \frac{dy}{y} g_1^a(y)$$

E143 collaboration, [hep-ph/9802357]
E155 collaboration, [hep-ex/0204028]
HERMES collaboration, [hep-ex/1112.5584]



Wandzura-Wilczek Approximation



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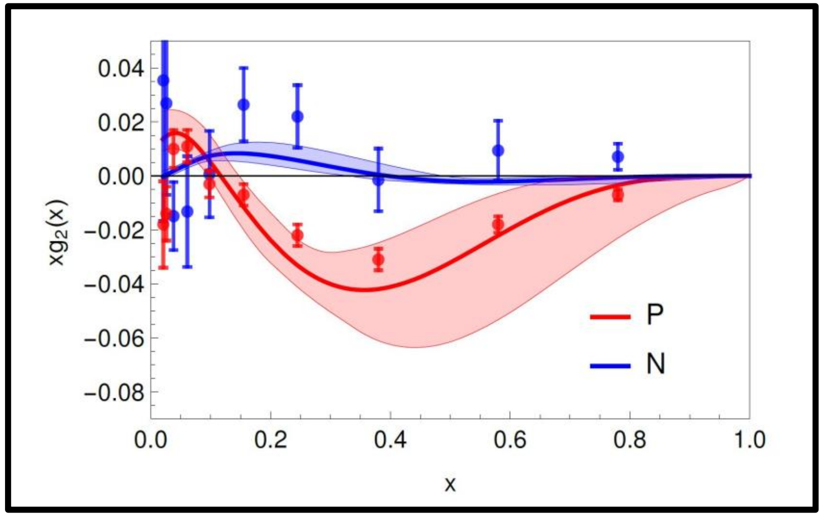
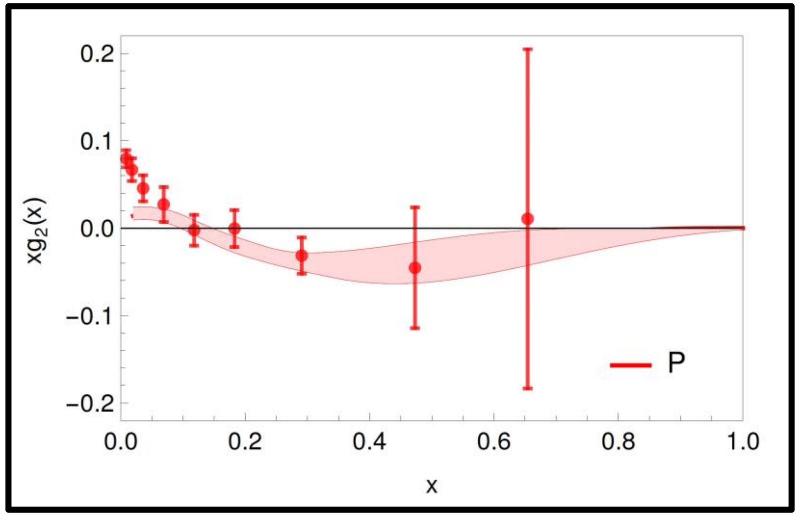


- ✓ Instanton vacuum model
- ✓ Lattice QCD
- ✓ Quark models (10-30%)

Support WW-aprroximation

Wandzura-Wilczek Approximation



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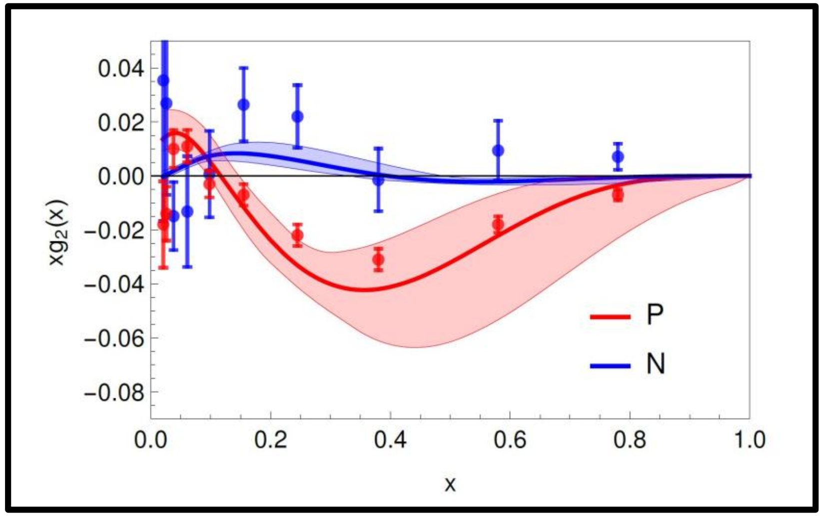

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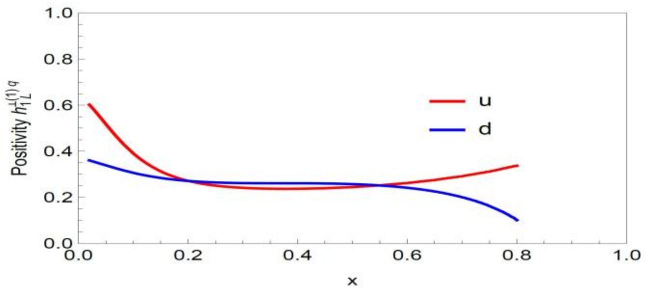
Support WW-aprroximation

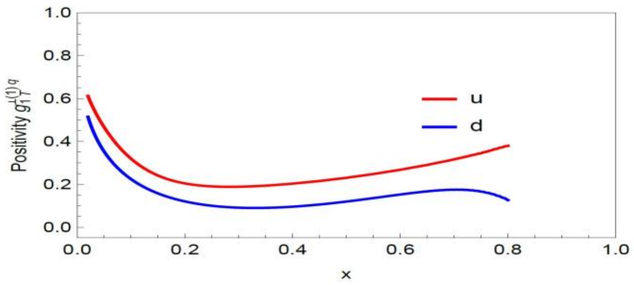
- ✓ Positivity bounds cross-check :

$$\frac{k_\perp^2}{4M^2}\left(f_1^a(x,k_\perp^2)\right)^2 - \left(h_{1L}^{\perp(1)a}(x,k_\perp^2)\right)^2 - \left(h_1^{\perp(1)a}(x,k_\perp^2)\right)^2 \geq 0$$

$$\frac{k_\perp^2}{4M^2}\left(f_1^a(x,k_\perp^2)\right)^2 - \left(f_{1T}^{\perp(1)a}(x,k_\perp^2)\right)^2 - \left(g_{1T}^{\perp(1)a}(x,k_\perp^2)\right)^2 \geq 0$$

A. Bacchetta et al., [hep-ph/9912490].





Imagine…

$$F_{UU,T} = C[f_1 , D_1]$$

$$F_{LL} = C[g_1 , D_1]$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = C[h_1 , H_1^{\perp}]$$

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DIS

LT SIDIS

Drell-Yann

e^+e^- *process*

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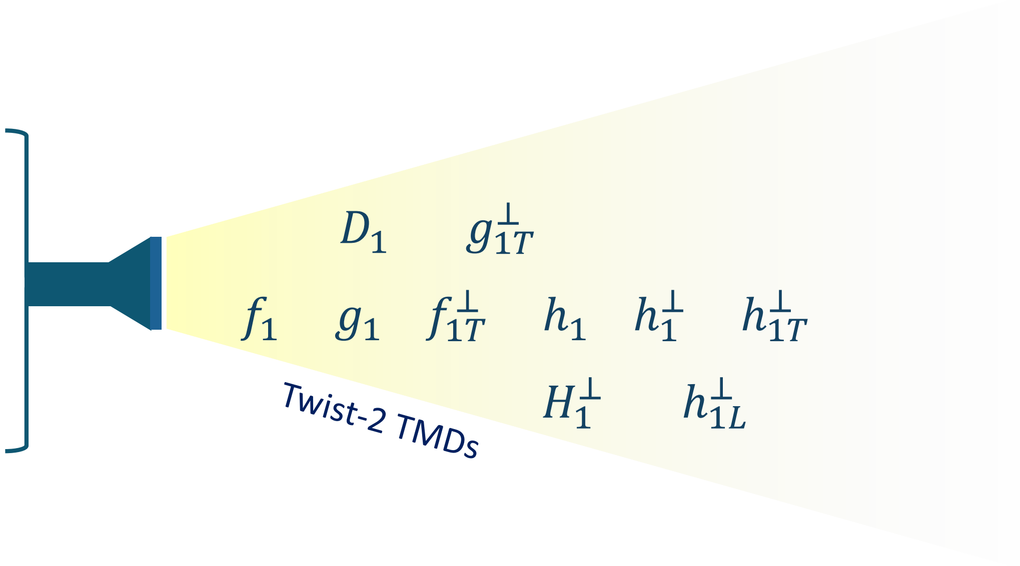

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D_1 $g_{1T}^\perp$

f_1 g_1 $f_{1T}^\perp$ h_1 $h_1^\perp$ $h_{1T}^\perp$

$H_1^\perp$ $h_{1L}^\perp$

Twist-2 TMDs

SLT SIDIS? Limited data are available; (M/Q) Suppression + too many terms which also make trouble in model calculations.

$$F_{UU}^{\cos(2\phi_h)} = \frac{2M}{Q} C\left[\frac{\hat{\boldsymbol{h}}\cdot P_\perp}{z m_h}\left(xh\, H_1^\perp + \frac{m_h}{zM}\, f_1\, \tilde{D}^\perp\right) - \frac{\hat{\boldsymbol{h}}\cdot k_\perp}{M}\left(x f^\perp\, D_1 + \frac{m_h}{zM}\, h_1^\perp \tilde{H}\right)\right]$$

and 6 more SFs with same or more number of unknowns TMDs

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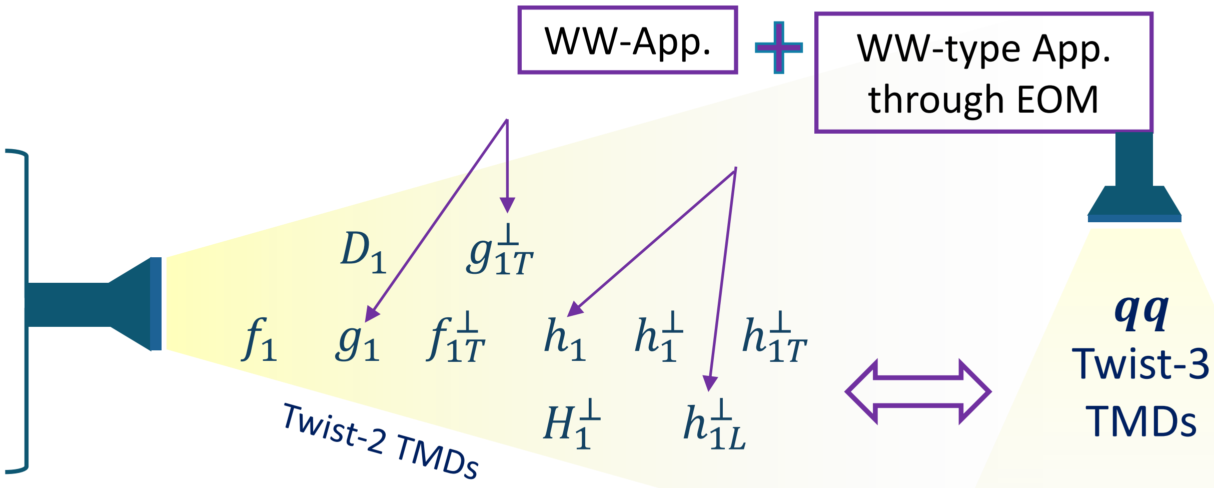

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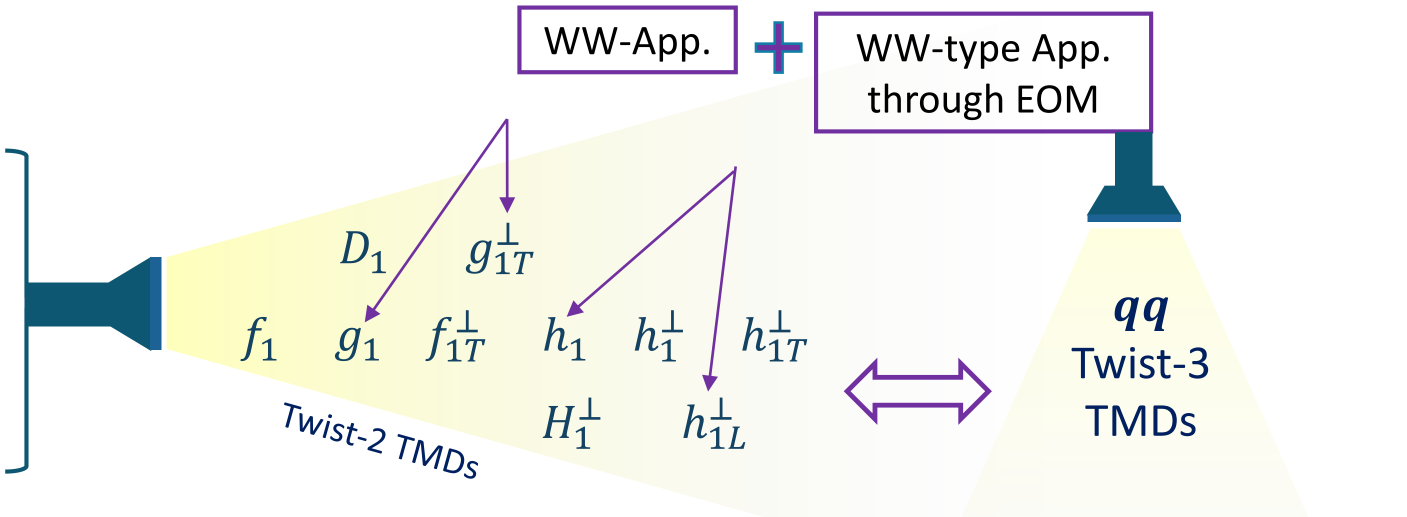

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and 6 more SFs with same or more number of unknowns TMDs

All SIDIS SFs up to subleading twist in terms of 6 twist-2 TMD PDFs and 2 twist-2 FFs!

f_1 , g_1 , $f_{1T}^\perp$, h_1 , $h_1^\perp$, $h_{1T}^\perp$, D_1 , $H_1^\perp$ ***Isn't it amazing!?***

OK!

SIDIS differential cross section in terms of all single/double spin asymmetries :

Leading Twist

$$\frac{d^6\sigma_{\text{leading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{d^6\sigma_0}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2}\Big\{ 1 + \cos(2\phi_h)\,p_1\,A_{UU}^{\cos(2\phi_h)} + S_L\sin(2\phi_h)\,p_1\,\boxed{A_{UL}^{\sin(2\phi_h)}} + \lambda\,S_L\,p_2\,A_{LL} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,\boxed{A_{LT}^{\cos(\phi_h-\phi_S)}} + S_T\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)} + S_T\sin(\phi_h+\phi_S)\,p_1\,A_{UT}^{\sin(\phi_h+\phi_S)} + S_T\sin(3\phi_h-\phi_S)\,p_1\,A_{UT}^{\sin(3\phi_h-\phi_S)}\Big\}$$

Subleading Twist

$$\boxed{\frac{2M}{Q}}$$

$$\frac{d^6\sigma_{\text{subleading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{d^6\sigma_0}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2}\Big\{ \cos(\phi_h)\,p_3\,\boxed{A_{UU}^{\cos(\phi_h)}} + \lambda\sin(\phi_h)\,p_4\,\boxed{A_{LU}^{\sin(\phi_h)}} + S_L\sin(\phi_h)\,p_3\,\boxed{A_{UL}^{\sin(\phi_h)}} + S_T\sin(\phi_S)\,p_3\,\boxed{A_{UT}^{\sin(\phi_S)}} + S_T\sin(2\phi_h-\phi_S)\,p_3\,\boxed{A_{UT}^{\sin(2\phi_h-\phi_S)}} + \lambda\,S_L\cos(\phi_h)\,p_4\,\boxed{A_{LL}^{\cos(\phi_h)}} + \lambda\,S_T\cos(\phi_S)\,p_4\,\boxed{A_{LT}^{\cos(\phi_S)}} + \lambda\,S_T\cos(2\phi_h-\phi_S)\,p_4\,\boxed{A_{LT}^{\cos(2\phi_h-\phi_S)}}\Big\}$$

With
$$\frac{d^6\sigma_0}{dx\,dy\,dz\,d\phi_h\,d\psi_l\,dP_{hT}^2} = \frac{2\alpha^2}{xyQ^2}\left(1-y+\frac{y^2}{2}\right)F_{UU,T}(x,z,P_{hT})$$

Azimuthal angle dependent weight

$$A_{XY}^{W}(x,z,P_{hT}) = \frac{F_{XY}^{W}(x,z,P_{hT})}{F_{UU,T}(x,z,P_{hT})}$$

Target polarization

Beam polarization

$p_i(y)$'s are known kinematic prefactors

SIDIS differential cross section in terms of all single/double spin asymmetries :

Leading Twist

$$\frac{d^6\sigma_{\text{leading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{d^6\sigma_0}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2}\Big\{1 + \cos(2\phi_h)\,p_1\,A_{UU}^{\cos(2\phi_h)} + S_L\sin(2\phi_h)\,p_1\,\boxed{A_{UL}^{\sin(2\phi_h)}} + \lambda\,S_L\,p_2\,A_{LL} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,\boxed{A_{LT}^{\cos(\phi_h-\phi_S)}} + S_T\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)} + S_T\sin(\phi_h+\phi_S)\,p_1\,A_{UT}^{\sin(\phi_h+\phi_S)} + S_T\sin(3\phi_h-\phi_S)\,p_1\,A_{UT}^{\sin(3\phi_h-\phi_S)}\Big\}$$

Subleading Twist

$$\boxed{\frac{2M}{Q}}$$

$$\frac{d^6\sigma_{\text{subleading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{d^6\sigma_0}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2}\Big\{\cos(\phi_h)\,p_3\,\boxed{A_{UU}^{\cos(\phi_h)}} + \lambda\sin(\phi_h)\,p_4\,\boxed{A_{LU}^{\sin(\phi_h)}} + S_L\sin(\phi_h)\,p_3\,\boxed{A_{UL}^{\sin(\phi_h)}} + S_T\sin(\phi_S)\,p_3\,\boxed{A_{UT}^{\sin(\phi_S)}} + S_T\sin(2\phi_h-\phi_S)\,p_3\,\boxed{A_{UT}^{\sin(2\phi_h-\phi_S)}} + \lambda\,S_L\cos(\phi_h)\,p_4\,\boxed{A_{LL}^{\cos(\phi_h)}} + \lambda\,S_T\cos(\phi_S)\,p_4\,\boxed{A_{LT}^{\cos(\phi_S)}} + \lambda\,S_T\cos(2\phi_h-\phi_S)\,p_4\,\boxed{A_{LT}^{\cos(2\phi_h-\phi_S)}}\Big\}$$

With $$\frac{d^6\sigma_0}{dx\,dy\,dz\,d\phi_h\,d\psi_l\,dP_{hT}^2} = \frac{2\alpha^2}{xyQ^2}\left(1-y+\frac{y^2}{2}\right)F_{UU,T}(x,z,P_{hT})$$

Azimuthal angle dependent weight

$$A_{XY}^{W}(x,z,P_{hT}) = \frac{F_{XY}^{W}(x,z,P_{hT})}{F_{UU,T}(x,z,P_{hT})}$$

Target polarization

Beam polarization

$p_i(y)$'s are known kinematic prefactors

$F_{UU,L}(x,z,P_{hT})$ and $F_{UT,L}^{\sin(\phi_h-\phi_S)}$ are dropped systematically from our calculations since we agreed to stop at $\mathcal{O}(\frac{1}{Q^2})$.

$$F_{LT}^{\cos(\phi_h-\phi_S)} \overset{\text{WW}}{=} \mathcal{C}\left[\frac{\mathbf{h}_\perp \mathbf{p}_T}{M} g_{1T} D_1\right]\Bigg|_{g_{1T}^a \to g_1^a \text{ using Eq. (23)}}$$

$$F_{UL}^{\sin 2\phi_h} \overset{\text{WW}}{=} \mathcal{C}\left[-\frac{2\,(\mathbf{h}_\perp \mathbf{k}_T)\,(\mathbf{h}_\perp \mathbf{p}_T) - \mathbf{k}_T \mathbf{p}_T}{M m_h} h_{1L}^\perp H_1^\perp\right]\Bigg|_{h_{1L}^{\perp a} \to h_1^a \text{ using Eq. (24)}}$$

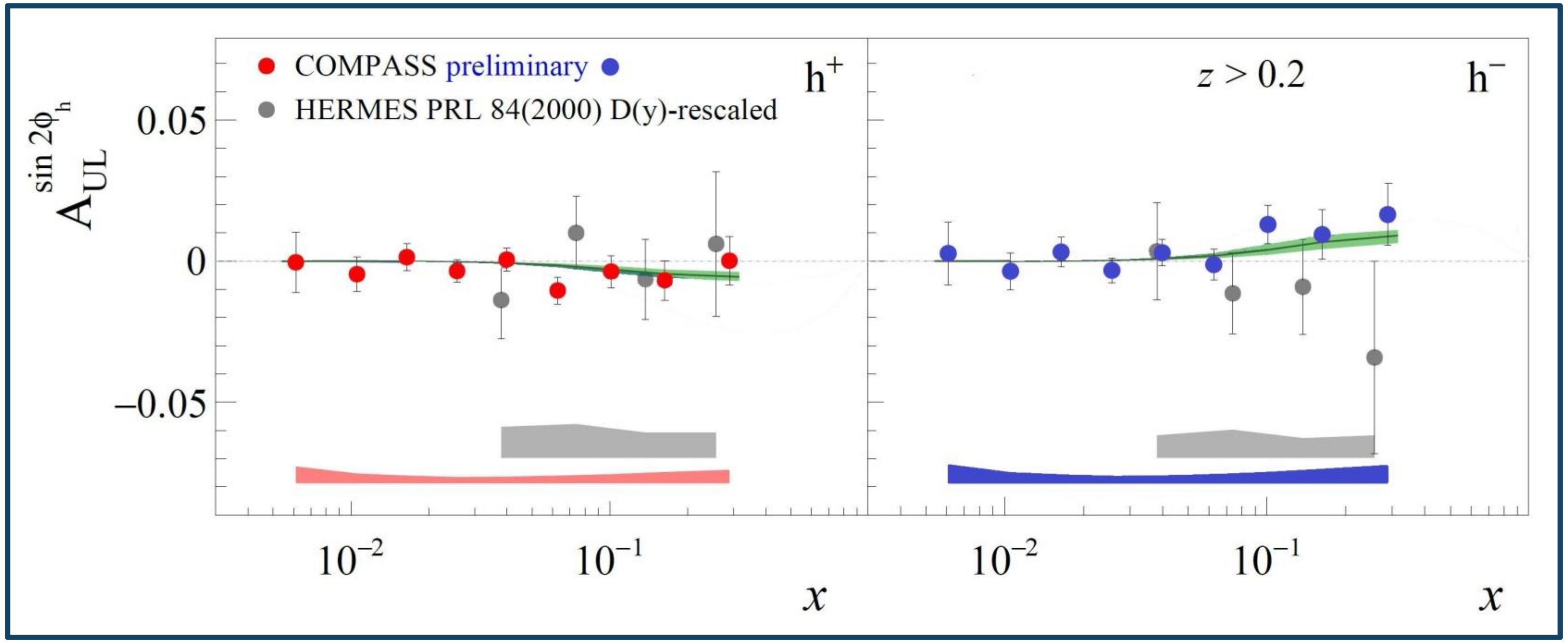
Every survived TMD or FF will go with a Gaussian Ansatz of its own width along with a longitudinal part.

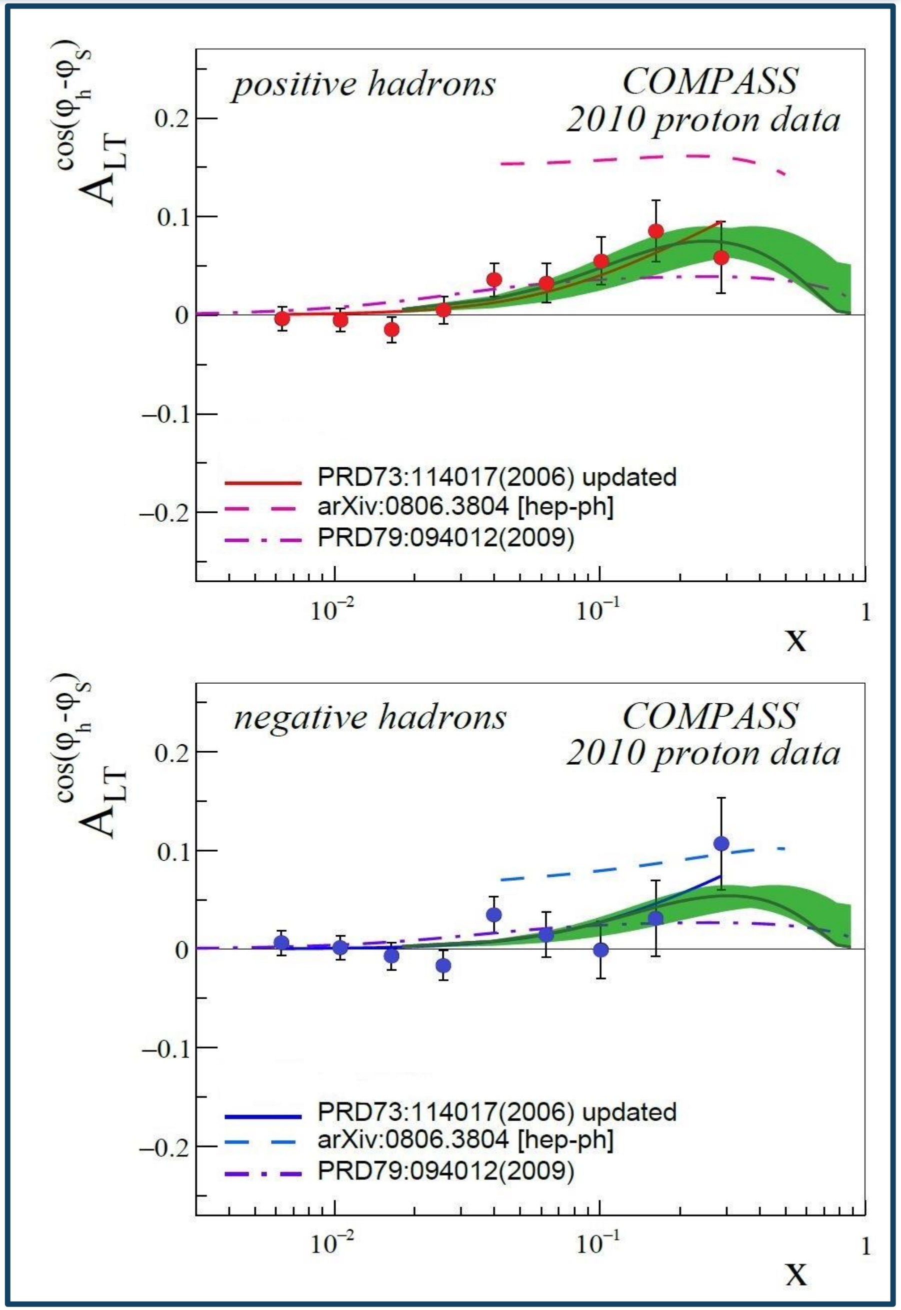
$$\begin{aligned}
F_{LU}^{\sin\phi_h} &= 0 \\
F_{LT}^{\cos\phi_S} &= \frac{2M}{Q}\,\mathcal{C}\left[-x\,g_T D_1\right]\Bigg|_{g_T^a \to g_1^a \text{ using Eq. (1)}} \\
F_{LL}^{\cos\phi_h} &= \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\mathbf{h}_\perp \mathbf{p}_T}{M} x g_L^\perp D_1\right]\Bigg|_{g_L^{\perp a} \to g_1^a \text{ using Eq. (8)}} \\
F_{LT}^{\cos(2\phi_h-\phi_S)} &= \frac{2M}{Q}\,\mathcal{C}\left[-\frac{2\,(\mathbf{h}_\perp \mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2} x g_T^\perp D_1\right]\Bigg|_{g_T^a \to g_1^a \text{ using Eqs. (1, 9)}} \\
F_{UL}^{\sin\phi_h} &= \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\mathbf{h}_\perp \mathbf{k}_T}{m_h} x\,h_L H_1^\perp\right]\Bigg|_{h_L^a \to h_1^a \text{ using Eq. (2)}} \\
F_{UU}^{\cos\phi_h} &= \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\mathbf{h}_\perp \mathbf{k}_T}{m_h} x\,h\,H_1^\perp - \frac{\mathbf{h}_\perp \mathbf{p}_T}{M} x\,f^\perp D_1\right]\Bigg|_{f^{\perp a} \to f_1^a,\ h^a \to h_1^{\perp a} \text{ using Eqs. (7, 21)}} \\
F_{UT}^{\sin\phi_S} &= \frac{2M}{Q}\,\mathcal{C}\left[\ x\,f_T D_1 - \frac{\mathbf{k}_T \mathbf{p}_T}{2M m_h}(x h_T - x h_T^\perp) H_1^\perp\right]\Bigg|_{f_T^a \to f_{1T}^{\perp a},\ (h_T^a - h_T^{\perp a}) \to h_1^a \text{ using (20, 12, 13)}} \\
F_{UT}^{\sin(2\phi_h-\phi_S)} &= \frac{2M}{Q}\,\mathcal{C}\left[\frac{2\,(\mathbf{h}_\perp \mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2} x\,f_T^\perp D_1 - \frac{2\,(\mathbf{h}_\perp \mathbf{k}_T)\,(\mathbf{h}_\perp \mathbf{p}_T) - \mathbf{k}_T \mathbf{p}_T}{2M m_h}\right. \\
&\qquad \left.\times x(h_T + h_T^\perp) H_1^\perp\right]\Bigg|_{f_T^{\perp a} \to f_{1T}^{\perp a},\ (h_T^a + h_T^{\perp a}) \to h_{1T}^{\perp a} \text{ using Eqs. (19, 12, 13)}}.
\end{aligned}$$

WW-approximation results on leading twist:

$$F_{LT}^{\cos(\phi_h-\phi_S)} \overset{\text{WW}}{=} \mathcal{C}\left[\omega_{\text{B}}^{\{1\}}\, g_{1T}^{\perp} D_1\right]\Bigg|_{g_{1T}^{\perp a}\to g_1^a} \quad \text{Eq. (3.6a)}$$

$$F_{UL}^{\sin 2\phi_h} \overset{\text{WW}}{=} \mathcal{C}\left[\omega_{\text{AB}}^{\{2\}}\, h_{1L}^{\perp} H_1^{\perp}\right]\Bigg|_{h_{1L}^{\perp a}\to h_1^a} \quad \text{Eq. (3.6b)}$$

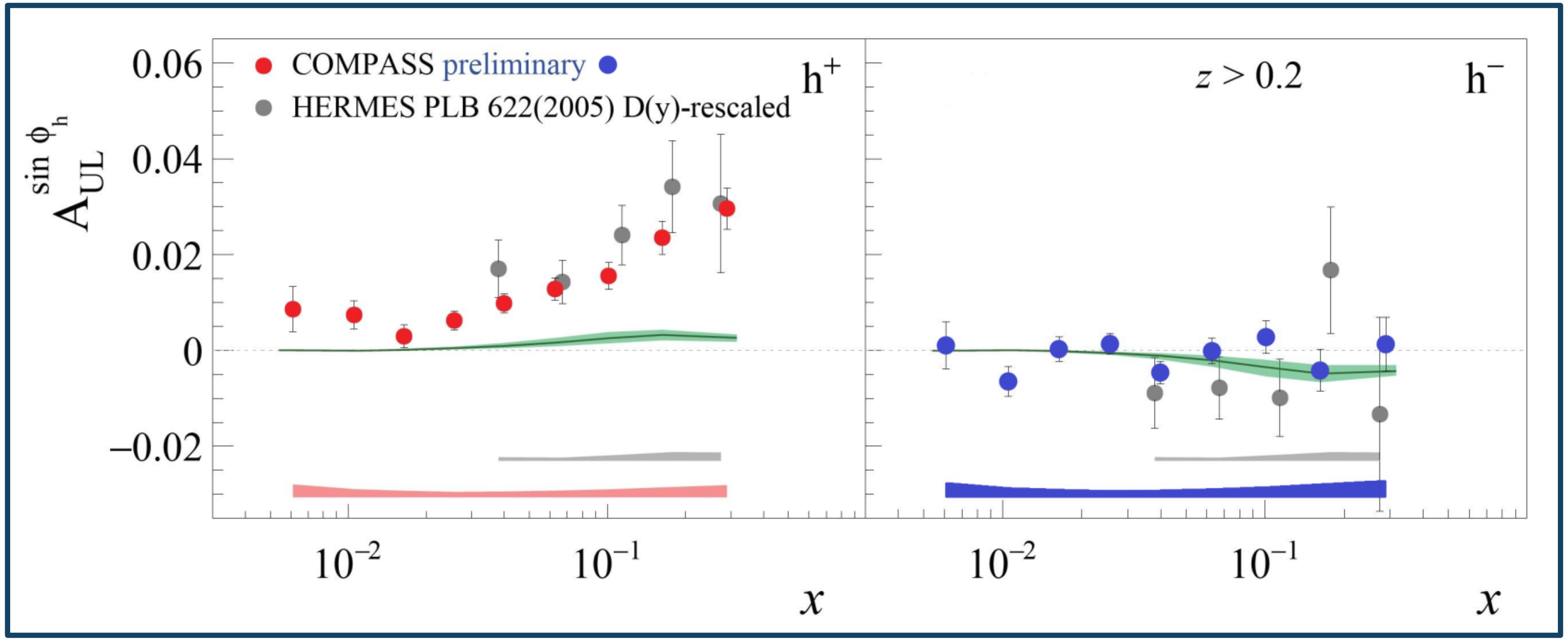


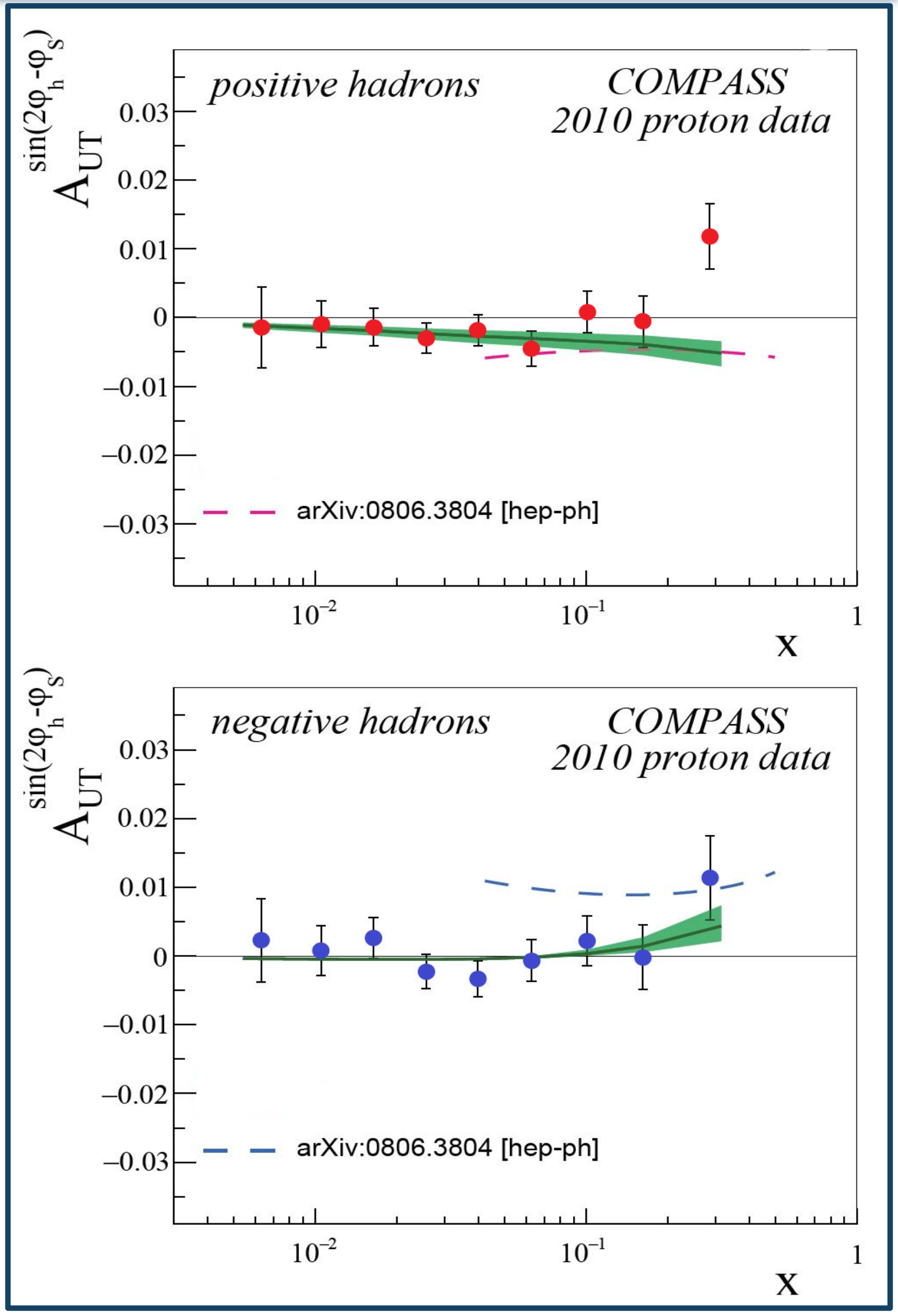


WW-approximation results on subleading twist:

$$F_{UL}^{\sin\phi_h} \overset{\mathrm{WW}}{=} \frac{2M}{Q}\,\mathcal{C}\Big[\;\omega_{\mathrm{A}}^{\{1\}}\,x\,h_L H_1^{\perp}\Big]\Bigg|_{\substack{h_L^a \to h_{1L}^{\perp a} \\ \text{with Eq. (3.3f),}}}$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} \overset{\mathrm{WW}}{=} \frac{2M}{Q}\,\mathcal{C}\Big[\omega_{\mathrm{C}}^{\{2\}}\,x\,f_T^{\perp} D_1 + \frac{\omega_{\mathrm{AB}}^{\{2\}}}{2} x(h_T + h_T^{\perp})H_1^{\perp}\Big]\Bigg|_{\substack{f_T^{\perp a} \to f_{1T}^{\perp a}, \\ (h_T^a + h_T^{\perp a}) \to h_{1T}^{\perp a} \\ \text{with (3.4f, 3.3g, 3.3h).}}}$$

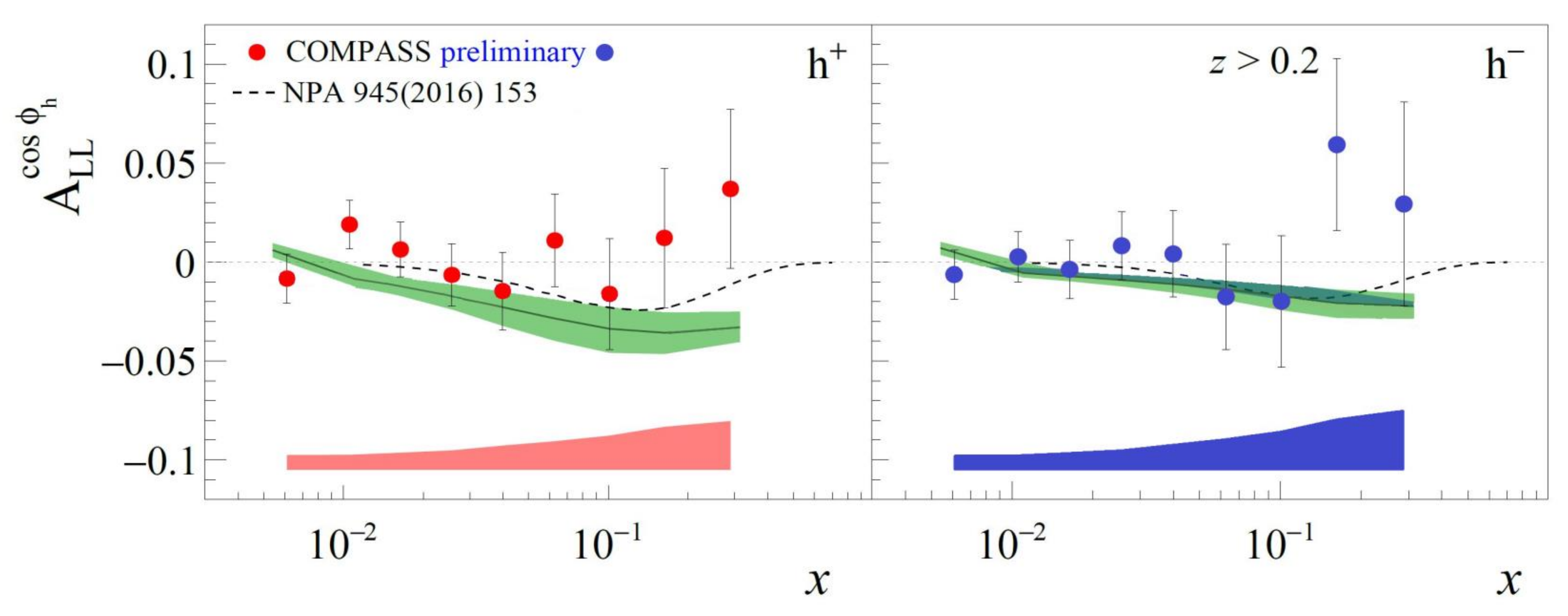


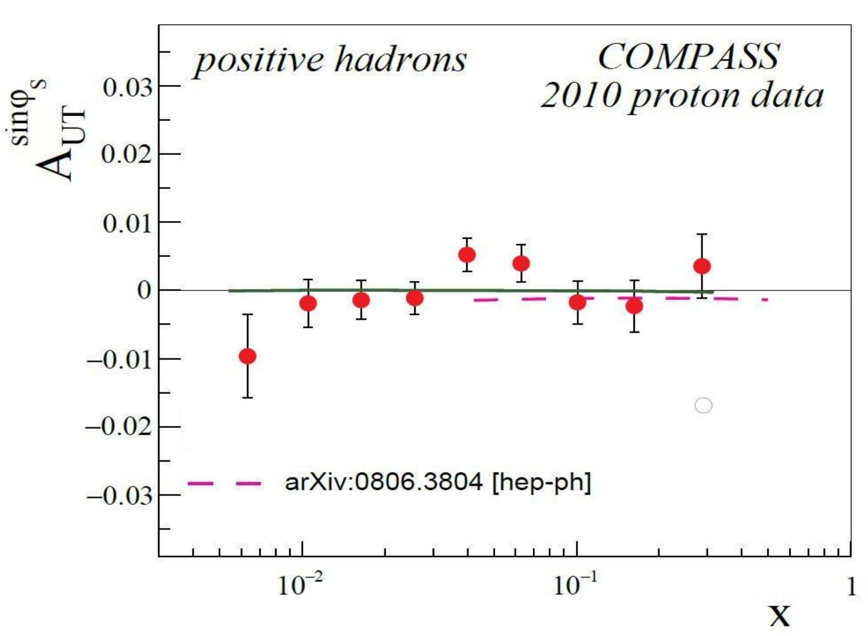


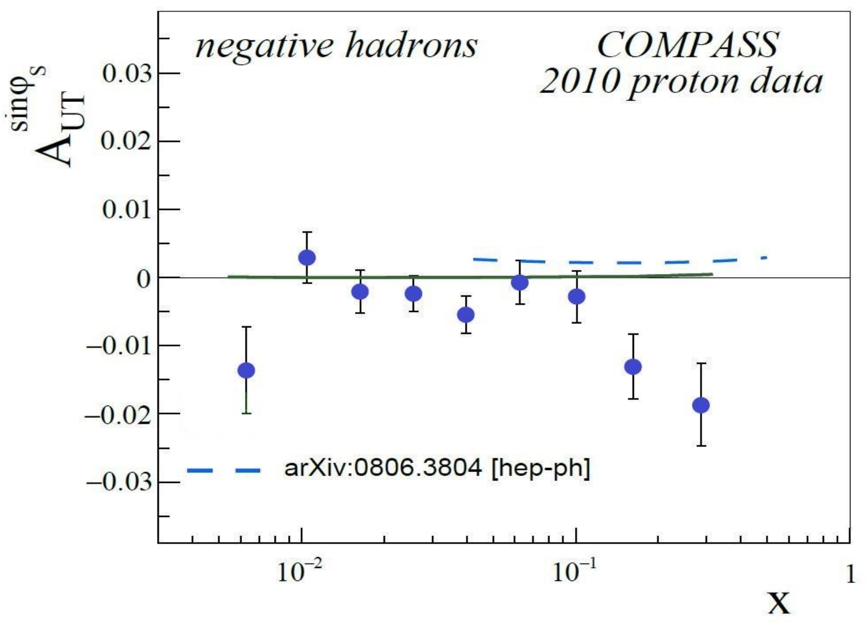
WW-approximation results on subleading twist:

$$F_{UT}^{\sin\phi_S} \overset{\text{WW}}{=} \frac{2M}{Q}\,\mathcal{C}\left[\ \omega^{\{0\}}\,x\,f_T D_1 - \frac{\omega_{\text{B}}^{\{2\}}}{2}\,(xh_T - xh_T^{\perp})\,H_1^{\perp}\right]\Bigg|_{\substack{f_T^a \to f_{1T}^{\perp a},\\ h_T^a - h_T^{\perp a} \to h_1^a\\ (3.4\text{g},\ 3.3\text{g},\ 3.3\text{h})}}$$

$$F_{LL}^{\cos\phi_h} \overset{\text{WW}}{=} \frac{2M}{Q}\,\mathcal{C}\left[-\omega_{\text{B}}^{\{1\}}\,xg_L^{\perp}D_1\right]\Bigg|_{\substack{g_L^{\perp a} \to g_1^a\\ \text{Eq. (3.3c)}}}$$







1. There are two classes of WW-type relations:

twist-3 $\longrightarrow$ twist-2 $\qquad x\, f^{\perp a}(x, k_\perp^2) \approx f_1^a(x, k_\perp^2)$

twist-3 $\longrightarrow$ transverse moment of a twist-2 $\qquad x\, g_T^a(x, k_\perp^2) \approx \dfrac{k_\perp^2}{M^2}\, g_{1T}^{\perp a}(x, k_\perp^2)$

Both Gaussians?
Not beautiful!

Do the convolutions without WW then use the integrated WW-type relations.

Remarks

1. There are two classes of WW-type relations:

twist-3 $\longrightarrow$ twist-2 $\qquad x\, f^{\perp a}(x, k_\perp^2) \approx f_1^a(x, k_\perp^2)$

twist-3 $\longrightarrow$ transverse moment of a twist-2 $\qquad x\, g_T^a(x, k_\perp^2) \approx \frac{k_\perp^2}{M^2} g_{1T}^{\perp a}(x, k_\perp^2)$

Both Gaussians?
Not beautiful!

Do the convolutions without WW then use the integrated WW-type relations.

2. The alternative treatment is bulkier but OK most of the times except when it comes to respect **sum rules of T-odd TMDs**. This happens e.g. in case of $F_{UT}^{\sin(\phi_S)}$ where the T-odd Sievers function imposes sum rule to vanish.

- Presented a full treatment of SIDIS asymmetries based on 6 TMDs and 2 FFs basis ready for phenomenology and event generators.
- Tested the applicability of WW-approximation type with available data.
- Made prediction for upcoming data from JLab, COMPASS, HERMES & the future EIC.
- Opened some how a window to investigate the $\boldsymbol{\bar{q}gq}$ TMDs.

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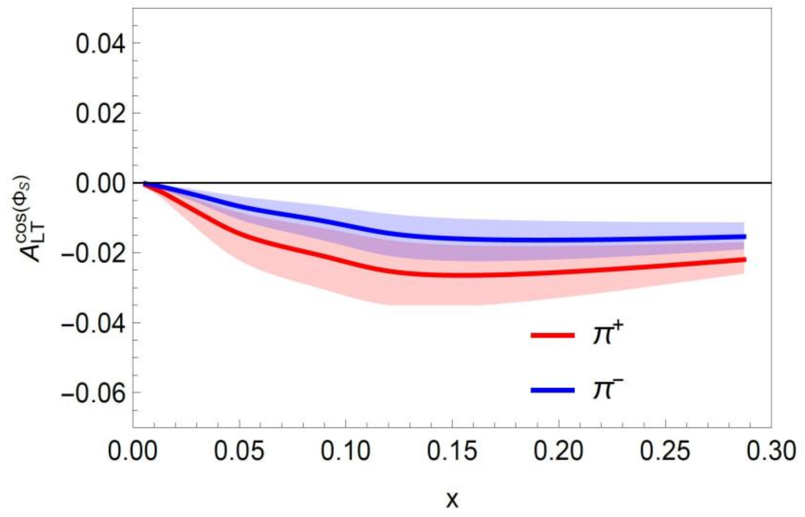
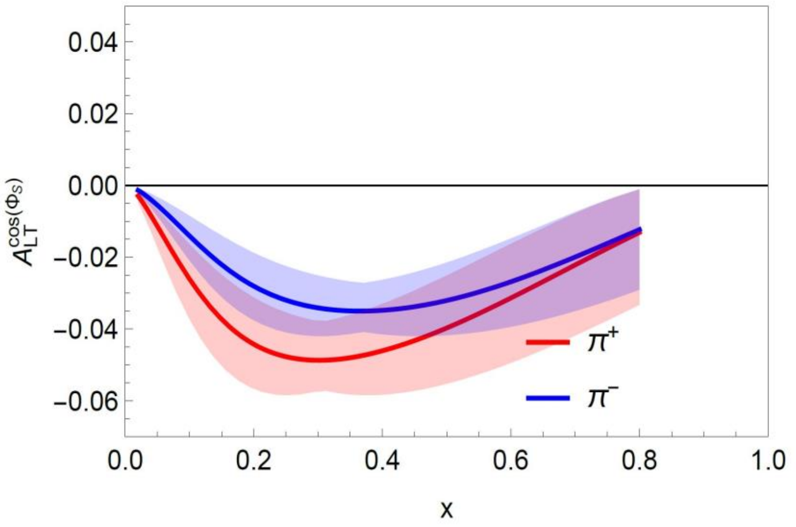
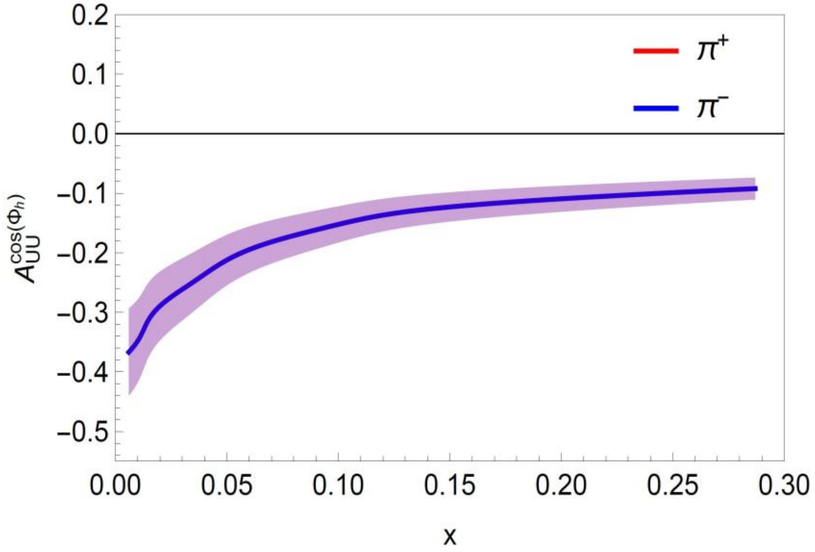
Thank You

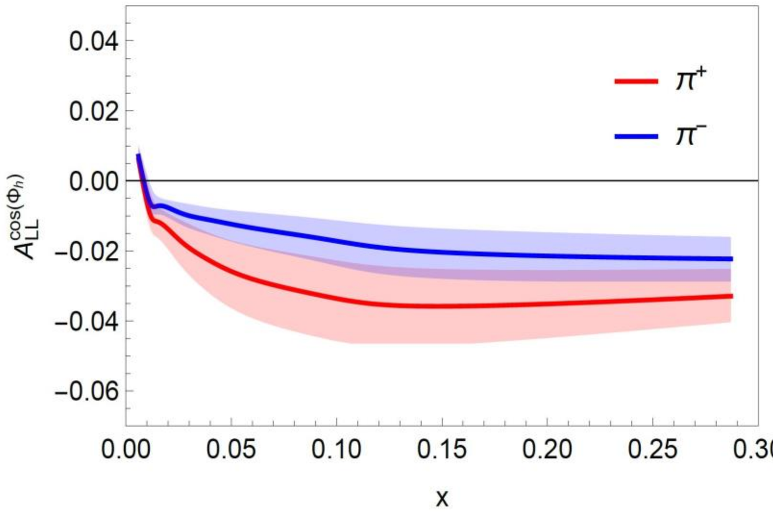
Backups

Spin Asymmetries in WW-type-Approximation

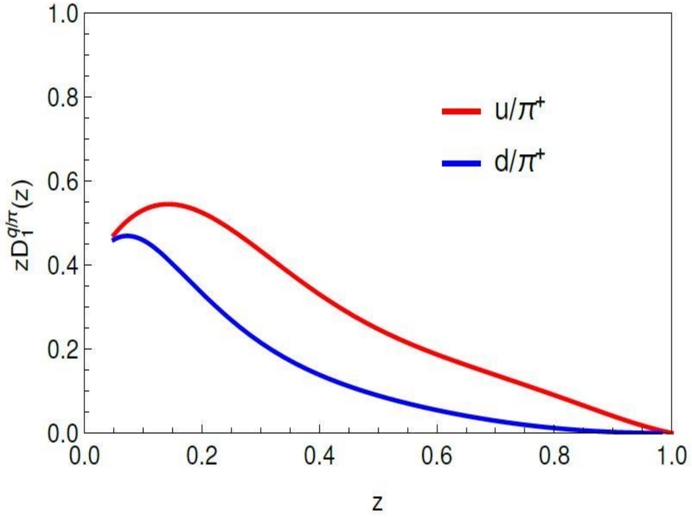


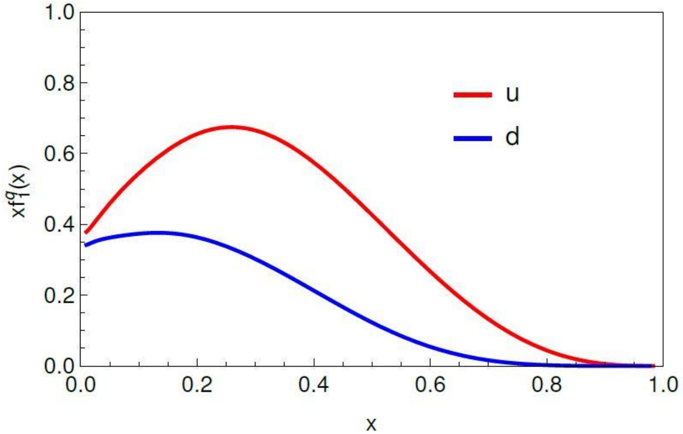


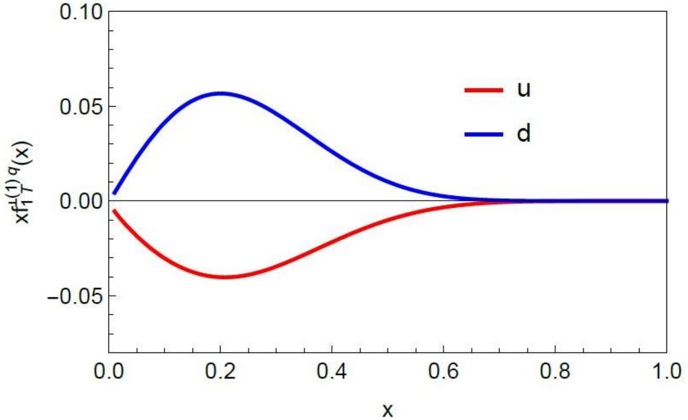


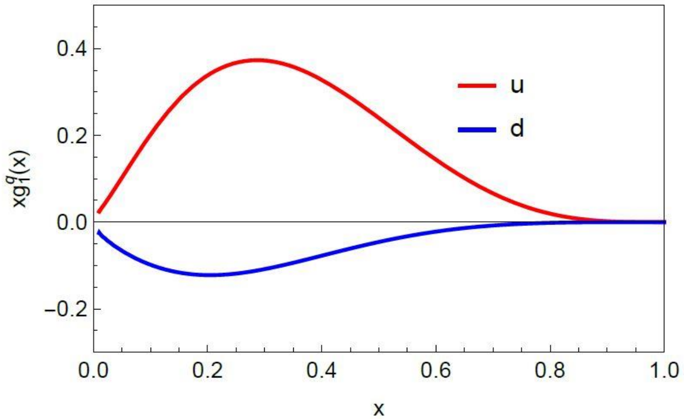


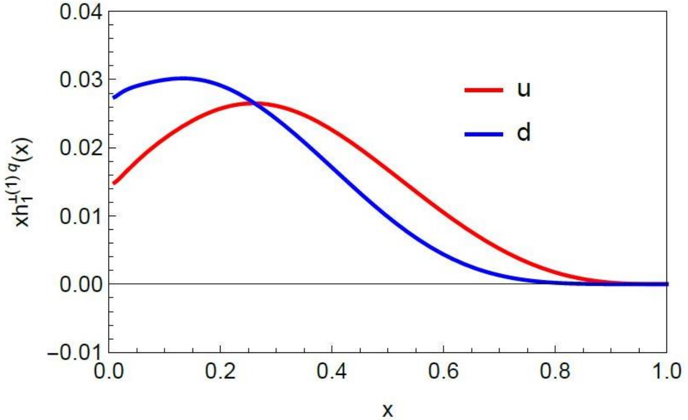
Introduction : SIDIS SF’s & spin asymmetries

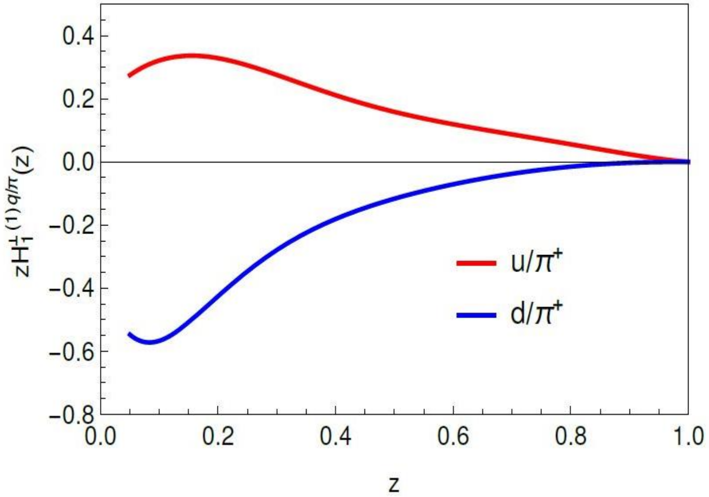


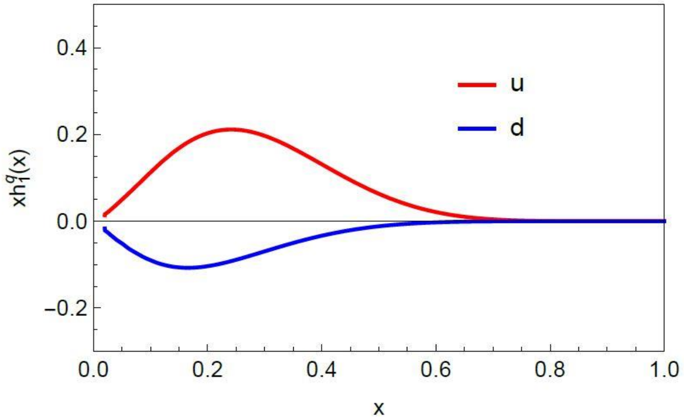


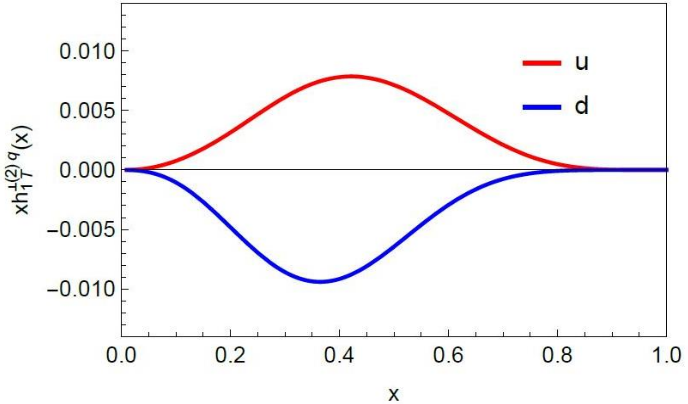




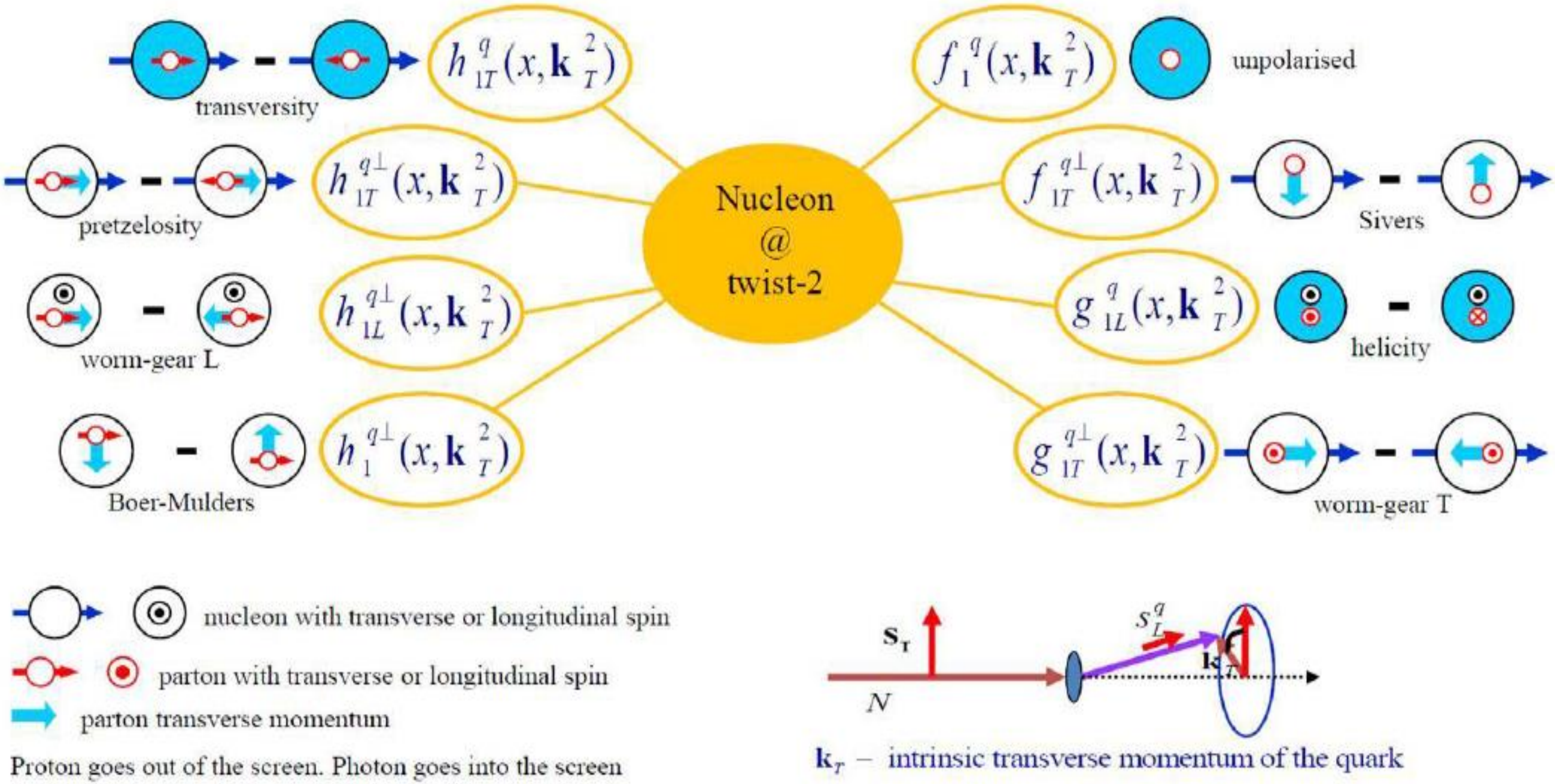








TMDs physical interpretations



$$xe = x\tilde{e} + \frac{m}{M} f_1,$$

$$xf^{\perp} = x\tilde{f}^{\perp} + f_1,$$

$$xg'_T = x\tilde{g}'_T + \frac{m}{M} h_{1T},$$

$$xg_T^{\perp} = x\tilde{g}_T^{\perp} + g_{1T} + \frac{m}{M} h_{1T}^{\perp},$$

$$xg_T = x\tilde{g}_T - \frac{p_T^2}{2M^2} g_{1T} + \frac{m}{M} h_1,$$

$$xg_L^{\perp} = x\tilde{g}_L^{\perp} + g_{1L} + \frac{m}{M} h_{1L}^{\perp},$$

$$xh_L = x\tilde{h}_L + \frac{p_T^2}{M^2} h_{1L}^{\perp} + \frac{m}{M} g_{1L},$$

$$xh_T = x\tilde{h}_T - h_1 + \frac{p_T^2}{2M^2} h_{1T}^{\perp} + \frac{m}{M} g_{1T}$$

$$xh_T^{\perp} = x\tilde{h}_T^{\perp} + h_1 + \frac{p_T^2}{2M^2} h_{1T}^{\perp}.$$

$$xe_L = x\tilde{e}_L,$$

$$xe_T = x\tilde{e}_T,$$

$$xe_T^{\perp} = x\tilde{e}_T^{\perp} + \frac{m}{M} f_{1T}^{\perp},$$

$$xf'_T = x\tilde{f}'_T + \frac{p_T^2}{M^2} f_{1T}^{\perp},$$

$$xf_T^{\perp} = x\tilde{f}_T^{\perp} + f_{1T}^{\perp},$$

$$xf_T = x\tilde{f}_T + \frac{p_T^2}{2M^2} f_{1T}^{\perp},$$

$$xf_L^{\perp} = x\tilde{f}_L^{\perp},$$

$$xg^{\perp} = x\tilde{g}^{\perp} + \frac{m}{M} h_1^{\perp},$$

$$xh = x\tilde{h} + \frac{p_T^2}{M^2} h_1^{\perp}.$$

$$\frac{E}{z} = \frac{\tilde{E}}{z} + \frac{m}{M_h} D_1,$$

$$\frac{D^{\perp}}{z} = \frac{\tilde{D}^{\perp}}{z} + D_1,$$

$$\frac{G^{\perp}}{z} = \frac{\tilde{G}^{\perp}}{z} + \frac{m}{M_h} H_1^{\perp},$$

$$\frac{H}{z} = \frac{\tilde{H}}{z} + \frac{k_T^2}{M_h^2} H_1^{\perp}.$$

EOMs

$$F_{LT}^{\cos(\phi_h-\phi_S)} \stackrel{\rm WW}{=} \mathcal{C}\left[\frac{\mathbf{h}_\perp \mathbf{p}_T}{M} g_{1T} D_1\right]\Bigg|_{g_{1T}^a \to g_1^a \;\text{ using Eq. (23)}}$$

$$F_{UL}^{\sin 2\phi_h} \stackrel{\rm WW}{=} \mathcal{C}\left[-\frac{2\,(\mathbf{h}_\perp \mathbf{k}_T)\,(\mathbf{h}_\perp \mathbf{p}_T) - \mathbf{k}_T \mathbf{p}_T}{M m_h} h_{1L}^\perp H_1^\perp\right]\Bigg|_{h_{1L}^{\perp a} \to h_1^a \;\text{ using Eq. (24)}}$$

$$F_{LU}^{\sin\phi_h} = 0$$

$$F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\mathcal{C}\left[-x\, g_T D_1\right]\Bigg|_{g_T^a \to g_1^a \;\text{ using Eq. (1)}}$$

$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\mathbf{h}_\perp \mathbf{p}_T}{M} x g_L^\perp D_1\right]\Bigg|_{g_L^{\perp a} \to g_1^a \;\text{ using Eq. (8)}}$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\mathcal{C}\left[-\frac{2\,(\mathbf{h}_\perp \mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2} x g_T^\perp D_1\right]\Bigg|_{g_T^a \to g_1^a \;\text{ using Eqs. (1, 9)}}$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\mathbf{h}_\perp \mathbf{k}_T}{m_h} x\, h_L H_1^\perp\right]\Bigg|_{h_L^a \to h_1^a \;\text{ using Eq. (2)}}$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\mathbf{h}_\perp \mathbf{k}_T}{m_h} x\, h\, H_1^\perp - \frac{\mathbf{h}_\perp \mathbf{p}_T}{M} x\, f^\perp D_1\right]\Bigg|_{f^{\perp a} \to f_1^a,\; h^a \to h_1^{\perp a} \;\text{ using Eqs. (7, 21)}}$$

$$F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\mathcal{C}\left[\;x\, f_T D_1 - \frac{\mathbf{k}_T \mathbf{p}_T}{2M m_h}\,(x h_T - x h_T^\perp)\, H_1^\perp\right]\Bigg|_{f_T^a \to f_{1T}^{\perp a},\; (h_T^a - h_T^{\perp a}) \to h_1^a \;\text{ using (20, 12, 13)}}$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\mathcal{C}\left[\frac{2\,(\mathbf{h}_\perp \mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2}\, x\, f_T^\perp D_1 - \frac{2\,(\mathbf{h}_\perp \mathbf{k}_T)\,(\mathbf{h}_\perp \mathbf{p}_T) - \mathbf{k}_T \mathbf{p}_T}{2M m_h}\right.$$
$$\left.\times x(h_T + h_T^\perp) H_1^\perp\right]\Bigg|_{f_T^{\perp a} \to f_{1T}^{\perp a},\; (h_T^a + h_T^{\perp a}) \to h_{1T}^{\perp a} \;\text{ using Eqs. (19, 12, 13)}}.$$

Further considerations

$$2MW^{\mu\nu} = \frac{1}{(2\pi)^3} \sum_X \int \frac{d^3 P_X}{2P_X^0} \, \delta^4(q + P - P_X - P_h) \, \langle P | J^\mu(0) | h, X \rangle \langle h, X | J^\nu(0) | P \rangle$$